Pure and Applied Geometry in Kant

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1 Introduction

The standard objection to Kant's epistemology of geometry as expressed in the CPR is that he neglected to acknowledge the distinction between pure geometry and applied geometry. The result of this insensitivity, critics say, is that Kant ends up ascribing a priority to empirical applications of geometry that ought to be reserved for truths of pure geometry. The criticism, that is, is that he failed to appreciate that although theorems of pure geometry can be known a priori to be true within the Euclidean system, the assertion that space is itself Euclidean is an empirical claim[1]—one that, in light of Einstein's theory of general relativity, is widely regarded as false. What is more, the development of non-Euclidean geometries showed how the Euclidean system as a whole was on equal footing with incompatible geometrical models. Non-Euclidean geometrical truths could be known a priori within their respective formal systems. In light of these developments, the idea that Euclidean geometry was epistemologically privileged and rooted in our intuition of space seemed rather implausible.

In a classic paper, Michael Friedman has worked to vindicate Kant's position by explaining how his incorporation of spatial intuition into geometry (and mathematics in general) was an unavoidable outcome of the limited logical apparatus that 18th century geometry was couched in [2]. Our modern conception

[1] See [2, pp.455-56], [6, p.29]

of pure geometry as entirely formal and independent of any physical or diagrammatic interpretation is one that requires more powerful logical machinery than had been developed in Kant's time. Thus, since there did not yet exist the logical framework adequate for what we now think of as pure geometry, Kant can hardly be to blame for regarding a spatial interpretation of geometry as part of geometry proper.

On the other hand, that Kant did not have *our* distinction between pure and applied geometry does not entail that he did not possess *any* such distinction. Kant was clearly well aware of the generality of geometrical concepts (or schemata) with respect to their particular concrete instantiations and of the difference between the pure science of geometry and its use in physics. Moreover, he took seriously the task of establishing the applicability of pure geometry to the world of appearances through transcendental philosophy.

Indeed, Friedman notes that Kant conceives of pure geometry wherein we reason about figures in an "empty" space constructed via the motion of mathematical points as distinct from applied geometry, which treats the properties that sensible objects have in virtue of their being given in space [2, p.482, footnote 36]. The key textual evidence that Friedman cites in support of the existence of a Kantian distinction between pure and applied geometry is the footnote appearing on B155:

> Motion of an **object** in space does not belong in a pure science, thus also not in geometry; for that something is movable cannot be cognized *a priori* but only through experience. But motion, as **description** of a space, is a pure act of the successive synthesis of the manifold in outer intuition in general through productive imagination, and belongs not only to geometry but even to transcendental philosophy.

Kant here explicitly distinguishes geometry as a pure science from the study of motions of sensible objects through space. A serious question naturally arises regarding this distinction, however, concerning how to make sense of the difference between the two in the context of Kant's epistemology of geometry together with his metaphysical views regarding the ideality of space. How is such a distinction possible, given that the very space through which we intuit empirical objects is the same space that can be known a priori and which thereby grounds all geometrical knowledge? This will be the guiding question for this paper.

In what follows, I will explicate the Kantian distinction between pure geometry and applied geometry in concert with Kant's distinction between a *representation* of space as a formal intuition and *space* as a form of intuition. My interpretation is that the representation of space "as object" (that is, as a formal intuition) refers neither to bounded spaces enclosed by particular geometrical figures nor to space as a pure form of intuition[2] Rather, I interpret "space as a formal intuition" as a representation of space as an empty and unbounded arena in which geometrical construction/reasoning can take place. Space as formal intuition can then be thought of as a methodologically indispensable representation for the purpose of doing geometry, while space as form of intuition is a fundamental feature of human sensibility. Pure geometry can then be understood as reasoning about constructions in a representation of space as a formal intuition. Applied geometry, on the other hand, would deal with space as a pure form of intuition. This is because appearances are given in and through this form, and so a priori (geometry based) knowledge of objects within it would be knowledge *of* empirical appearances, though it would not itself be *empirical knowledge*, since it is a priori. Characterizing the pure/applied distinction in this way is consistent with the basic idea that a theory's being "applied" relays

[2]Cf. Longuenesse who identifies space as a form of intuition with space as a formal intuition.This view will be addressed below.

that it is being used to learn about certain features of empirical reality; namely those kinds of features that the theory aims to characterize. To apply geometry to a system of physical bodies in motion, for example, is to study the geometrical features of elements in the system (e.g. the feature *being parabolic* of a body's trajectory). Similarly, to apply biology to an empirical system consisting of a human being is to study the biological features of that person. Thus, in Kant's system, if applied geometry is to be understood as studying the non-empirical (geometrical) features of empirical objects, it makes sense to think of applied geometry as studying the spatial form of intuition which the appearances given within it have their geometrical features in virtue of. To engage in study within a theory in its "pure" form, on the other hand, is usually thought of as involving idealized *representations* of some kind or another. Thus, my interpretation respects basic common sense notions about the terms "pure" and "applied" in a theoretical context, in addition to, I'll argue, being consistent with Kant's philosophy of space and geometry.

2 The Ideality of Space and the A Priority of Geometrical Knowledge

The ideality of space on Kant's view and his resulting epistemology for geometry is what makes the question of what distinction he could have had in mind between pure and applied geometry so interesting. A Plato-influenced view of geometry as being the product of wholly non-empirical perception (or recollection) of a realm of mathematical forms, tied to a more traditional conception of space as a subject-independent substrate for physical things is, I think, behind the more commonplace construals of the pure/applied distinction of geometry (and of mathematics in general). Broadly, geometrical knowledge is considered to have its source elsewhere than experience (e.g. the divine, pure rationality, etc.), and the study of bare geometrical concepts is thus considered to be "pure" as in untainted by the vulgarity of the physical world. And yet, physical things appear to exhibit geometrical features, at least approximately, which allows us to make correct inferences about them based on our knowledge of pure geometry. Insofar as a concept from pure geometry (e.g. triangularity) seems applicable to a physical thing, we can draw on theorems regarding the concept (e.g about relationships between angles and side lengths) and apply them to that thing as well. And although, on this more traditional conception, the possibility of this applicability remains mysterious, and invites many distinct explanations and dissolutions[3], the *distinction* between pure and applied geometry is marked and clear. Pure geometry is about mathematical points, lines, and figures, and is done a priori, while applied geometry is about appearances, and depends (at least in part) on experience, and has a concrete subject matter.

For Kant, however, all geometrical knowledge arises *with* experience, though not *from* the content of experience[4]. Rather, it arises as a result of the form that any possible appearance must take in order to be sensible for us. Kant argues in the Transcendental Aesthetic that geometry, as a science about space, is not simply a matter of pure reason acting on concepts alone, because geometrical theorems assert propositions that contain more than what is contained in the original subject concept—they are ampliative, in a sense. Thus, geometrical theorems are not analytic, but synthetic.

[3]In fact, Kant's theory about the ideality of space and his epistemology of geometry can be seen as motivated by the puzzle of the applicability of geometry that results from the traditional picture of mathematics and the natural world: "Empirical intuition is possible only through the pure intuition (of space and time); what geometry says about the latter is therefore undeniably valid of the former, and evasions, as if objects of the senses did not have to be in agreement with the rules of construction in space (e.g., the rules of infinite divisibility of lines or angles), must cease."[1, B206]

[4]Kant foreshadows this kind of a priori knowledge in the introduction to the 2nd edition of the CPR: "But although all our cognition commences **with** experience, yet it does not on that account all arise **from** experience." [1, B1]

> For I am not to see what I actually think in my concept of a triangle (this is nothing further than its mere definition), rather I am to go beyond it to properties that do not lie in this concept but still belong to it. [1, B746]

Kant is here explaining what is involved in geometrical reasoning, and distinguishing it from the conceptual analysis that characterizes the metaphysics of his time. The possibility of these a priori but synthetic propositions in geometry is owed to the ideality of its subject matter.

On the assumption of transcendental idealism, space itself is nothing other than a feature of human sensibility; it is the form which all outer intuitions have, and must have due to their being intuited by us. It is this very claim, the ideality of space, that Kant uses to explain the possibility of a priori knowledge about *our intuition of space* (geometrical knowledge, in particular) in the Transcendental Exposition. On my reading of the Transcendental Exposition, Kant shifts between talking about space itself and our representation (or intuition) of it[5]. Kant begins his argument there (the "argument from geometry") with the following claim.

> Geometry is a science that determines the properties of space synthetically, and yet *a priori*. [1, B40]

Here, Kant is talking about space itself, as a feature of human sensibility[6]. He is stating a peculiar state of affairs that seems to cry out for an explanation: that the space (that seems to be) around us can be characterized a priori. Kant then suggests in a rhetorical question that the explanation lies in the nature of our *representation of* space:

[5] In this respect, my interpretation closely resembles that of Waldemar Rohloff [6].

[6] Of course, since he is here still working to establish that space is transcendentally ideal, he wants "space" to refer to what his contemporaries regarded as a subject-independent substrate for physical bodies. He wants to argue, against both substantivalists and relationalists, that what they call "space" is actually a feature of human perception and not of the external world.

> What then must the representation of space be for such a cognition of it to be possible? [1, B40]

It makes sense that Kant shifts at this point from space itself to our representation of it. As I'll argue below, Kant sees our representation of space, rather than space itself, as the arena for the pure science of geometry. Geometry proceeds a priori, using a representation of space, to generate propositions that turn out to be necessary and yet synthetic truths about space itself—How? What connection must there be between our representation of space and space itself that enables geometry, a discipline that consists of constructing mathematical concepts in the representation (or intuition) of space, to yield synthetic truths about space itself through a priori methods? Kant begins to narrow down the answer:

> It [the representation of space] must originally be intuition; for from a mere concept no propositions can be drawn that go beyond the concept, which, however, happens in geometry (Introduction, V). But this intuition must be encountered in us *a priori*, i.e. prior to all perception of an object, thus it must be pure, not empirical intuition. [1, B40-41]

His answer is that our representation of space has to be an intuition that is not provided by the content of experience, but which we somehow find already within ourselves. That this must be the case, Kant argues, is indicated by the self-evidence of geometric propositions.

> For geometrical propositions are all apodictic, i.e. combined with consciousness of their necessity, e.g., space has only three dimensions; but such propositions cannot be empirical or judgments of experience, nor inferred from them (Introduction II). [1, B41]

Kant gets to the heart of the matter in the second paragraph of his argument from geometry. How is it that this intuition of space lies ready within the subject, antecedent to all empirical content?

> Now how can an outer intuition inhabit the mind that precedes the objects themselves, and in which the concept of the latter can be derived *a priori?* Obviously not otherwise than insofar as it has its seat merely in the subject, as its formal constitution for being affected by objects and thereby acquiring **immediate representation,** i.e., **intuition,** of them, thus only as the form of outer **sense** in general. [1, B41]

Kant's answer to this question, the second sentence of this crucial paragraph, seems open to two obvious interpretations, depending on what is made of the expression "...has its seat in ..." The first interpretation is to understand Kant as saying that the outer intuition (our intuition/representation of space) is located only in the subject *as* both its susceptibility to noumenal affection and its particular mode of representing noumena as appearances. The second interpretation, which is the one that I favour, is that Kant is saying that this pure intuition (or representation) is somehow *derived from* or *abstracted from* the feature of sensibility that enables noumenal affection and spatial representation. On the former interpretation, "has its seat in" is equivalent to "is in", while on the latter it means something closer to "comes from" or "is metaphysically dependent on." On this latter interpretation, then, it is "*the seat*" of our intuition of space (whatever is in the subject and that our intuition of space is derived from) that Kant is saying is the subject's "formal constitution for being affected by objects and thereby acquiring **immediate representation**, i.e., **intuition**, of them, thus only as the form of outer **sense** in general."

Which of these interpretations we ascribe to Kant has consequences for how we ought to understand his epistemology of geometry, and consequently for how we should understand Kant's distinction between pure and applied geometry. This is because Kant regards geometrical propositions as being knowledge of our *intuition of space*. According to the first interpretation, on which our intuition of space *just is* our ideal form of outer intuition, geometrical knowledge is knowledge of space itself. But on the second interpretation, even though geometrical knowledge is *applicable* to space itself, it is knowledge *of* something distinct from, though dependent on, space. This distinction between our intuition of space and space itself will be explored further in the next section.

In the paragraph following the argument from geometry, Kant extols the explanatory virtue of his theory of space:

> Thus our explanation alone makes the **possibility** of geometry as a synthetic a priori cognition comprehensible. [1, B41]

The argument from geometry takes it as given that we can know synthetic propositions about our intuition of space a priori via the pure science of geometry, and regresses from this knowledge to the conclusion that this intuition is rooted in an ideal form of intuition, since this alone accounts for the possibility of geometrical knowledge. Kant agrees with the prevailing supposition that geometrical truths (and other mathematical theorems) are a priori since they are necessarily true.[7] But since they are also synthetic, they cannot be had by mere reason acting on its own concepts. The only remaining option is that truths of geometry are true of our representation (as object) of the pre-existing conditions that any appearance must meet in order that it conforms to human sensibility. Kant thus argues that space itself is the structure imposed on appearances which makes possible our representation of objects as a result of our

[7] "It must first be remarked that properly mathematical propositions are always *a priori* judgments and are never empirical, because they carry necessity with them, which cannot be derived from experience." [1, B15]

being affected by the things in themselves. That is, space is contributed by the perceiving subject as a pure form of intuition. Space (and time) are "relations that only attach to the form of intuition alone, and thus to the subjective constitution of our mind, without which these predicates could not be ascribed to anything at all" [1, A23/B37-38]. Since space is a subject-contributed form of intuition, it is ideal; and since any possible appearance necessarily conforms to the inherent structure of spatial intuition, space is objective.

Now, while the ideality of space, and thus the purity of our intuition of it, explains why geometry, as a science of space, seems to be objectively true of everything in experience, it does seem to complicate our basic conception of the distinction between pure and applied geometry. Traditionally, pure geometry is independent of experience of the natural world and has a more secure (rationalist) epistemology than that of empirical science. However, on Kant's view, it seems that geometrical truths come indirectly from experience, insofar as it signals what formal properties all of it has in common necessarily and thus must be attributed to the form of all outer intuition. These properties are then reified in our intuition of space, and these reified properties are what are revealed through the practice of pure geometry. This makes for an interesting picture on which pure and applied geometry are intimately related, but nevertheless remain distinct (as will be clarified in the final section of this paper).

In a footnote, Friedman characterizes Kant's distinction between pure and applied geometry in the following way.

> In pure geometry we consider figures generated in an "empty" space by the motion of mere mathematical points; in applied geometry we consider the actual sensible objects contained "in" this space. That what holds for mere mathematical points in "empty" space holds also for actual sensible objects found "in" this space (that pure math-

> ematics can be applied) can only be established in transcendental philosophy. [2, p.482, n36]

For me at least, this brief note—while tantalizing—leaves much unresolved. The words "empty" and "in" are in scare-quotes for reasons that are not entirely clear to me, and I have difficulty with his identification of the space in which figures are considered (in the context of pure geometry) with the space that contains "actual sensible objects". I agree with Friedman that there is good reason to think that Kant had a distinction between pure and applied geometry (though clearly not our contemporary distinction), and that Kant's note at B155 (quoted above) is especially telling. But I think that the key to really drawing out the difference between the two kinds of geometry lies in another distinction in Kant's CPR—one that comes with its own interpretive issues. The distinction between *space as a formal intuition* and *space as a form of intuition* is, I think, at the heart of Kant's distinction between pure and applied geometry.

3 Space as a pure form of intuition and space as a formal intuition.

Before I can provide my full interpretation of Kant's distinction between pure and applied geometry, however, I must defend a particular interpretation of the distinction between space as a formal intuition and space as a form of intuition. In §26 of the Transcendental Deduction, Kant introduces this distinction in his discussion of the synthesis of the manifold of experience.

> But space and time are represented a priori not merely as **forms** of sensible intuition, but also as **intuitions** themselves (which contain a manifold), and thus with the determination of the unity of this manifold in them (see the Transcendental Aesthetic).

Here Kant is broaching the idea that space, in addition to being the pure form of all outer intuition, is sometimes represented as an intuition itself. Kant goes on, in a footnote, to link space as (formal) intuition to the science of geometry.

> Space, represented as **object** (as is really required in geometry), contains more than the mere form of intuition, namely the **comprehension** of the manifold given in accordance with the form of sensibility in an **intuitive** representation, so that the **form of intuition** merely gives the manifold, but the **formal intuition** gives unity of the representation. [1, B160-61]

So our representation of space as object is a methodological prerequisite for the practice of geometry, and goes beyond the feature of human sensibility described in the Transcendental Aesthetic in that it is a unified representation. What more can be said to make sense of space as a formal intuition as opposed to the space as a form of intuition that Kant argued so strongly for in the Transcendental Aesthetic?

One view, attributed to Michel Fichant, is that space as a formal intuition (as object, that is) refers to bounded space enclosed by geometrical figures. That is, a formal intuition of space is a determination or restriction of infinite space (space as form of intuition, presumably). That Kant thinks objects have to be determinate supports this interpretation of space as formal intuition, since he refers to it also as "space as object". Also, that Kant says that representing space as object is required in geometry is consistent with this reading; drawing geometrical figures thereby delimiting an enclosure of space is indeed required in geometry.

Another view, advocated by Longuenesse, is that space as formal intuition is identical with space as a form of intuition; that is, the two share a single referent. Longuenesse regards the passage on B160 quoted above, and the footnote appearing there, which is partially quoted above, as essential evidence for her interpretation. Here is the remainder of that footnote.

> In the Aesthetic I ascribed this unity merely to sensibility, only in order to note that it precedes all concepts, though to be sure it presupposes a synthesis, which does not belong to the senses but through which all concepts of space and time first become possible. For since through it (as the understanding determines the sensibility) space or time are first **given** as intuitions, the unity of this *a priori* intuition belongs to space and time, and not to the concept of the understanding. [1, B160-61]

Now, considering this latter half of the footnote together with the first half certainly opens up more avenues of interpretation, because it's difficult to see how the note considered in its entirety does not contain an outright contradiction. Kant seems to first say that the unity of the formal intuition (of space) has a unity not to be found in the "mere" form of intuition, and then to say that space as he described in the Transcendental Aesthetic—as form of intuition—"presupposes a synthesis" that is prior to any processing by the understanding. It is easy to see how attempts to make sense of this can go in either of at least two different ways.

Longuenesse's approach is to understand Kant as saying that space both as form of intuition and as formal intuition has unity, and to credit this unity to the act of figurative synthesis of the imagination. This allows her to maintain that "space as form of intuition" and "space as formal intuition" have the same referent. Below she defends her view against Michel Fichant, who holds the position that "space as formal intuition" refers to space enclosed by geometrical figures, which was mentioned above.

> As we can see, Kant expressly states here that the same unity of

> intuition that he had attributed, in the Transcendental Aesthetic, to sensibility alone because it is anterior to all concepts must now be held to suppose a synthesis by which 'space and time are first given as intuitions.' Michel Fichant probably thinks that the unity in question here is the unity of particular figures in space, resulting form the construction of geometrical concepts. But the text, it seems to me, does not allow this interpretation, since it expressly states that the unity in question precedes all concepts. What is in question here is space as one whole, and time as one whole, within which all particular figures and durations are delineated. [4, p.68]

On Longuenesse's account, "space as form of intuition" and "space as formal intuition" are used interchangeably depending on context (that is, on which of space's features Kant wants to emphasize) and refer to the same thing: a unified representation of space as a totum. I concur with much of what Longuenesse says here, especially in her argument against Fichant. However, I also think that her interpretation is problematic because it doesn't distinguish between space itself and our *representation* of it. It seems quite clear to me that in various places Kant wishes to speak of our representation of space separately from space itself. In the Metaphysical Exposition, for example, his overall argument for the ideality of space seems to depend on various sub-arguments, some of which deal with space itself, and some of which deal with our representation of it. In fact, the overarching argument, though obviously very difficult to pin down, seems to rely on inferring properties about space itself *from* facts about our representation of it. For instance, in the Transcendental Aesthetic he uses the claim from the Metaphysical Exposition that the *representation of space* is an a priori intuition [1, A23/B38-A25/B40] to infer in the second part of the Transcendental Exposition that what this representation represents—space itself—is "nothing other than merely the form of all appearances of outer sense, i.e., the subjective condition of sensibility, under which alone outer intuition is possible to us." [1, A26/B42] And in general, it seems that the distinction between space and our representation of it is required for Kant's signature style of transcendental argumentation as it occurs in the Transcendental Aesthetic.

It is, however, difficult to track which of space and our representation of space Kant means to talk about in every case, since he often speaks of our spatial form of intuition (space itself) *as* a representation. On my reading, there could perhaps be a weak sense in which space itself is a representation, since "it has its seat merely in the subject, as its formal constitution for being affected by objects..." [1, B41] But if it is a representation, it represents only an organizational framework for all outer representations, and I'm not so sure that this is really *content* of the kind that a bona fide representation must contain. The outer intuitions that space makes possible have material content, but space itself has no such content. I would thus opt to regard space itself as a *mode of representation* understood as a feature of human sensibility itself, rather than as a genuine representation[8].

Acknowledging that Kant sometimes wishes to speak of space itself and sometimes wishes to speak of our representation of it is key for understanding his distinction between space as form of intuition and space as formal intuition—and subsequently, as we will see, his distinction between applied geometry and pure geometry. Specifically, I think that Kant uses "space as a formal intuition" (and "space as object") to refer to our representation of space, and "space as a pure form of intuition" to refer to space itself.

[8] I should note, though, that whether or not space itself can be properly regarded as a genuine representation is not crucial for my major premise that Kant maintains a distinction between space and our representation of it; there is nothing contradictory about the notion that we have a representation of one of our representations. That is, even if space is itself a representation, that would in no way preclude that we have a representation of our representation that is space. Moreover, it would not preclude that the two representations are distinct; a representation of a representation is not the original representation.

One piece of evidence for this interpretation is a remark Kant makes toward the end of the Transcendental Exposition.

> The constant form of this receptivity, which we call sensibility, is a necessary condition of all the relations within which objects can be intuited as outside us, and, if one abstracts from these objects, it is a pure intuition, which bears the name of space. [1, A27/B43]

Here Kant is explaining that from space as a feature of sensibility—as form of intuition—we can represent, via a process of abstraction, an intuition *of* space. I believe that he is here foreshadowing the distinction between space as form of intuition and space as formal intuition that he will make explicit in §26 of the Transcendental Deduction.

Let's now attempt to apply this interpretation to that section, the passage at B160 and its footnote cited above in our discussion of Longuenesse. In the main body of text, Kant is discussing the "forms of our outer and inner sensible intuition" and goes on to say that these forms are *represented as intuitions.* Also, Kant here refers back to the Transcendental Aesthetic, where in arguing that space cannot be (or come from) a concept he asserts that space as a whole is metaphysically prior to any of its parts (that is, is a totum as opposed to a compositum)[9], to justify his claim here (in the B160 footnote) of the unity of our representation (intuition) of space.

In the footnote, Kant contrasts the representation of space "as **object**" with "the mere form of intuition," saying that the former contains "the **comprehension** of the manifold which corresponds to the "form of sensibility" inherent in the latter. Understanding the former as being our representation of the latter makes it clear how our *comprehending* the form of our own sensibility amounts

[9] "For, first, one can only represent a single space, and if one speaks of many spaces, one understands by that only parts of one and the same unique space. And these parts cannot as it were precede the single all-encompassing space as its components (from which its composition would be possible), but rather are only thought **in it**." [1, A35/B39]

to representing this form as a unified manifold which constitutes space as a formal intuition. He also remarks in parentheses that the former is required in geometry. This too makes sense on my reading, because what is required for geometry is a unified representation of space itself which captures only the *formal* properties of which space itself is exhaustively constituted.

Kant continues, still in the footnote, explaining that although in the Transcendental Aesthetic he attributed the unity of space to sensibility alone, he did so for didactic reasons. He reveals here, however, that the unity of space as a formal intuition requires a synthesis—which belongs neither to the senses nor to the understanding or its concepts. Ostensibly, as I mentioned above, this footnote seems to contradict what Kant said in the Transcendental Aesthetic. Kant said there that space (and time) are wholly the province of sensibility.

> In the transcendental aesthetic we will therefore first **isolate** sensibility by separating off everything that the understanding thinks through its concepts, so that nothing but empirical intuition remains. Second, we will then detach from the latter everything that belongs to sensation, so that nothing remains except pure intuition and the mere form of appearances, which is the only thing that sensibility can make available *a priori*. In this investigation it will be found that there are two pure forms of sensible intuition as principles of *a priori* cognition, namely space and time, with the assessment of which we will now be concerned. [1, A22/B36]

So, in the Transcendental Aesthetic Kant says that we can identify space and time by only *subtracting* from what is given through sensibility, whereas in the footnote at B160 he says that we must *add* to our form of sensibility a presupposed synthesis in order to arrive at a unified intuition of space. This contradiction dissolves, however, when we interpret Kant as saying that space *itself* as form of intuition is contributed entirely by our faculty of sensibility, and that what requires a synthesis is our formal intuition *of* space.[10] Since space itself is nothing other than the mere form of sensibility, it clearly owes its character to nothing other than sensibility itself. As an intuition *of* this form of sensibility, however, space is not just an organizational framework for intuition but rather a unified representation of that framework. The proposition that the former depends on nothing outside of our faculty of sensibility, as Kant did in the Transcendental Aesthetic, does not contradict the proposition that the latter requires a synthesis that makes possible the *concept* of space.

I have argued in this section, against Longuenesse who interprets Kant as identifying space as form of intuition with space as formal intuition, that "space as form of intuition" refers to space itself while "space as formal intuition" refers to our unified representation of space. This reading is what motivates and enables my interpretation of Kant's distinction between applied and pure geometry, which will be discussed in the next section.

4 Pure geometry and applied geometry.

Now that I have explained in detail what I take the difference between space as formal intuition and space as form of intuition to be, describing my interpretation of Kant's distinction between pure geometry and applied geometry becomes more straightforward. In short: pure geometry studies our representation of space as formal intuition via constructions in intuition of mathematical concepts, and applied geometry concerns the necessary features that appearances have in virtue of their being sensible for us.

[10] Kant also says in this footnote that this synthesis is also independent of the understanding, which presents its own puzzles. Making sense of this particular aspect of the unity of our representations of space and time is beyond the scope of this paper, but see Longuenesse who ascribes this unity to the productive imagination [3, p.208-9,213], and McClear who appeals to a non-discursive unity of space and time [5, pp.12-23].

How does this interpretation accord with Friedman's suggestive but somewhat cryptic brief note (quoted above) on Kant's distinction between the two kinds of geometries? Friedman says that "[i]n pure geometry we consider figures generated in an 'empty' space by the motion of mere mathematical points." My interpretation of pure geometry as taking place within our representation of space fits well with this description; space itself is not empty as it is full of the material content of appearances. We can however *represent* space as being empty in order to have a blank environment in which to construct intuitions of mathematical concepts. His use of scare quotes here is to prevent any misunderstanding caused by assuming that he is referring to space itself, as pure form of intuition, that is somehow voided of all material content. Such a misunderstanding, though unlikely, would lead one to think that in order to do their work geometers (on Kant's view) must enter a state of sensory deprivation. Friedman goes on to say that "in applied geometry we consider the actual sensible objects contained 'in' this space." Again, the use of scare quotes is needed because to take the sentence literally would be to take Friedman as saying that actual sensible objects are contained in the very same space that geometers construct their concepts in. But this too would be ridiculous—there are no perfect mathematical triangles hovering alongside trees and coffee cups in nature.

In concluding his note, Friedman says "that what holds for mere mathematical points in 'empty' space holds also for actual sensible objects found 'in' this space (that pure mathematics can be applied) can only be established in transcendental philosophy." I take Friedman to be saying here that Kant sees his transcendental philosophy as functioning in part to demonstrate a sort of *correspondence* between the behaviour of (and constraints on) mathematical points (and the figures generated by their motions) in a representation of space and the behaviour of (and constraints on) physical objects in space itself. Now, Kant has shown that this correspondence exists because our formal intuition of space (within which we study mathematical points and figures) is rooted in space as a form of intuition (within which physical objects have their form and move around). The principles behind the form of our receptivity to external things translate to axioms about our representation of space, and thus space as formal intuition is a faithful representation of space as form of intuition. Therefore, the behaviour of the concepts we construct as intuitions in our representation of space will mirror that of actual objects in space itself.

We now have an answer to the guiding question with which we began, which was this: How is a Kantian distinction between pure and applied geometry possible, given that the very space through which we intuit empirical objects is the same space that can be known a priori and which thereby grounds all geometrical knowledge? The key, we have seen, is to be cognizant of Kant's distinction between our representation of space and space itself. Space itself, which is the form of the sensibility by which we intuit objects, and which together with the objects contained in it is the subject matter of applied geometry, provides the basis of our representation of space—space as a formal intuition, that is—which in turn is required for geometry as it is the arena within which geometrical concepts are constructed and studied.

On Kant's picture, then, pure geometry and applied geometry have an interestingly reciprocal relationship. Applied geometry is about appearances[11]. Specifically, it is about the properties that all appearances share in virtue of the possibility of their being represented spatially by us; namely, the geometrical

[11]As I'm using the term, "applied geometry" is unlike "pure geometry" in that it does not denote a well-defined academic discipline or practice. Rather, it is a more general term that refers to any investigation or discussion of the geometrical properties of objects. This can include everyday observations, e.g. of the fact that the shortest distance between any two objects follows the unique straight line connecting them, as well as the (not strictly Euclidean) geometry-based Newtonian method of kinematics.

properties *of appearances*. Appearances thus owe their geometrical properties (the province of applied geometry) to *space as a pure form of intuition*. And our *formal intuition of space*, which is used in practicing geometry, is grounded in that which gives rise to to those features that fall under the umbrella of applied geometry. On the other hand, in the course of constructing mathematical concepts in *space as a formal intuition*, we arrive at propositions of pure geometry, and these propositions are necessarily true of *space as a pure form of intuition*, and thus of any possible appearance, leading to necessary truths of applied geometry. Broadly, pure geometry precedes applied geometry in epistemology; but the subject matter of applied geometry is metaphysically prior to that of pure geometry. This interplay hinges entirely on our extricating Kant's notion of space from his notion of our intuition of space, especially in our reading of the Transcendental Exposition. Without attention to this subtlety, the Kantian distinction between pure and applied geometry collapses.

Before I conclude, I offer one further piece of evidence for my interpretation. Kant acknowledges that geometers carry out their work without understanding that their subject matter is a representation of the pure form of outer intuition. That is, geometers don't know—and don't need to know—about the ideality of space.

> We have above traced the concepts of space and time to their sources by means of a transcendental deduction, and explained and determined their *a priori* objective validity. Geometry nevertheless follows its secure course through strictly *a priori* cognitions without having to beg philosophy for any certification of the pure and lawful pedigree of its fundamental concept of space. [1, A87/B119-A87/B120]

So Kant believes that geometers work with a concept or representation of space and do perfectly well without any foundational theory about the basis of (or justification for) their fundamental concept. However, Kant goes on to note that, in the minds of geometers, truths of geometry are self-evident, and he *explains* this phenomenon of self-evidence by referring back to space as a pure form of intuition as the *grounds* for geometric knowledge.

> Yet the use of the concept in this science concerns only the external world of the senses, of which space is the pure form of its intuition, and in which therefore all geometrical cognition is immediately evident because it is grounded on intuition *a priori*, and the objects are given through the cognition itself *a priori* in intuition (as far as their form is concerned). [1, A87/B120-A88/B120]

So geometry as a science that has a representation of space as its subject matter can proceed perfectly well without its practitioners knowing anything about space itself—about their own sensibility and the form it imposes on any possible experience. They need only know about the axioms that govern space as formal intuition, which they know via their self-evident character, and ignorance of the explanation *for* this self-evidence is no hindrance.

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