

FREGE ON THOUGHTS AND THEIR STRUCTURE*

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Thoughts are essentially structured. That much is agreed by almost everybody who finds it useful to talk about thoughts. Consequently, any account of the nature of thoughts must incorporate an account of their structure. Frege's is the most comprehensive and worked-out account of the nature of thoughts, but recent work has cast doubt on whether he can be credited with a coherent conception of how they are structured. In an extremely interesting series of exchanges David Bell and Michael Dummett have investigated Frege's views on the relation between thoughts and the concepts of which they are composed.¹ Both authors have identified tensions in Frege's views in this important area and proposed emendations to smooth out the apparent inconsistencies. The difficulties stem from Frege's simultaneously holding both that the structure of a thought is isomorphic to the structure of a sentence and that two structurally different sentences can express the same thought. In the case of Bell, the proposed cure looks as if it might well kill the patient. Bell ends his latest contribution by proposing a distinction between thoughts and the senses of sentences such that the analysis of a sentence no longer reveals the intrinsic nature of a thought (Bell 1996). The consequence of this, as Bell willingly admits, is that thoughts no longer have a determinate, intrinsic structure. Dummett proposes, on Frege's behalf, to abandon the idea that two structurally different sentences can express the same thought. This is also a drastic revision of Frege's expressed views.

In this paper I show, against both Bell and Dummett, that Frege does have a coherent conception of the structure of thoughts, and that neither of the proposed revisions is either

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¹ The key papers are Bell 1987a, 1987b, Dummett 1991a and Bell 1996. There is also highly relevant material in Chapter 14 of Dummett 1991d. The background for the debate is Dummett's discussion in Chapter 2 of his 1973 of Frege's doctrine of analysis, pursued further in Chapters 15, 16 and 17 of his 1981. Interesting discussion of related topics will also be found in Geach 1975, Hodes 1982, Currie 1985.

necessary or justified. Along the way I discuss the following topics: the relation between statements about equinumerosity and statements about numbers; the formation of complex concepts via the formation of complex predicates from complete sentences; Frege's views on the relation between definiens and definiendum; the possibility of multiple analyses of sentences; Frege's views on truth; and what Frege himself took to be the upshot of Grundlagen 64-67.

I

Bell finds Frege committed to two inconsistent theses concerning the structure of thoughts. On the one hand Frege accepts

THESIS A We distinguish parts in a thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a model of the structure of the thought.

On the other hand, however, he also accepts

THESIS B Two structurally different sentences can express one and the same thought.

Thesis A maintains a structural isomorphism between a sentence and the thought it expresses that seems to deny precisely the possibility maintained by Thesis B, namely, that a given thought might be expressed by structurally different sentences.

There can be no question but that Frege endorsed Thesis A. It was a corner-stone of his approach to philosophical logic. Thesis B is more tricky, however. We need to distinguish two different ways in which sentences can differ from each other structurally. Frege's own discussion in 'On Concept and Object' gives a clear illustration of one such way in which structurally different sentences can express the same thought. He notes (1892, p.188) that there is one thought that can be expressed by any of the following three sentences:

1a) There is at least one square root of 4

1b) The concept square root of 4 is realized

1c) There is something which has the property of giving the result 4 when multiplied by itself.

Although he is not explicit about the matter there, these three sentences can all express the same thought because they would all be translated the same way into primitive notation (cf Van

Heijenoort 1977a). Sentences (1a) – (1c) are differently structured at the level of natural language, but they share a common structure in primitive notation. That Frege held Thesis (B) with respect to differently structured sentences of natural language is beyond dispute. In fact, it is hard to see how anybody could deny that differently structured natural language sentences could express the same thought. The interesting question, however, is whether Frege was of the view that differently structured sentences in primitive notation could express one and the same thought. In the following I shall discuss this stronger and more interesting version of Thesis B.

That Frege espoused Thesis B seems to be implied by the well-known views on concept formation by contextual definition that Frege expressed in Grundlagen. Bell draws attention to the following two familiar pairs of sentences:

- 2a) Line A is parallel to line B
- 2b) The direction of line A = the direction of line B

- 3a) There are just as many Fs as Gs
- 3b) The number of Fs is the same as the number of Gs

Frege explicitly maintained in Grundlagen §64 that (2b) represents a different way of “carving up the content” of (2a). His discussion of (2a) and (2b) in Grundlagen §§64-67 is intended to motivate the parallel with the introduction of numbers by abstraction from the equivalence relation of equinumerosity. The idea is that (3b) is just a different way of carving up the content of (3a), one that introduces the abstract object of a number in exactly the same way as (2b) introduces the abstract object of a direction by abstraction from the equivalence relation of parallelism. The direction of explanation, and the direction of concept formation, goes from the first member of each pair to the second. Bell suggests that if two sentences are to offer different ways of carving up the same content then they must be synonymous – that is to say, they must express the same thought.² And, moreover, the transformational equivalence between the first

² The suggestion that, for Frege, two sentences are synonymous iff they express the same thought needs to be qualified. Frege certainly held that some pairs of sentences which we would not intuitively think of as synonyms could nonetheless express the same thought. But these sentences involve demonstrative expressions (like the 'today'/'yesterday' and 'here'/'now' pairs discussed in 'Thoughts') and for the purposes of this paper I will prescind from the tricky question of how, if at all, demonstratives can be accommodated within Frege's theory of sense. For further discussion see Perry 1977, Evans 1981 and Dummett 1991c. Conversely, Frege also recognised that there are elements in the meaning of a sentence that are not part of the sense or thought it expresses – what he called 'tone' or 'colouring'. These elements are not part of the thought expressed because they do not have any implications for the truth-value of the sentence. As I will be using the term, two sentences are synonymous when they have the same sense/express the same thought. So, 'A and B' and 'A but B' count as synonyms. In 'Thoughts' (1918-1919a) Frege

and the second members of each pair will only motivate the formation of a new concept if the apparent difference in surface syntax reflects a difference at the level of deep structure, or logical form – that is to say, if they have different translations into the primitive notation. But then, it seems, the validity of transformational equivalence as a means of concept formation directly entails Thesis B in the sense we have identified.

In his response to an earlier paper of Bell's (Bell, 1987a) Dummett proposed rejecting Thesis B on Frege's behalf (Dummett 1991). More precisely, he proposed denying that there is a single thought expressed by (2a) and (2b) or by (3a) and (3b), but his grounds for so doing offer an argument against Thesis (B) in general.³ The basic proposal is that Thesis B is incompatible with Frege's conception of how the sense of a complex expression (like a sentence) is determined by the senses of its constituent expressions. The compositionality of thoughts seems to entail that one cannot grasp the thought expressed by a sentence without grasping its constituent senses, namely, the senses of the words that go to make up the sentence.⁴ From which one can easily derive

PRINCIPLE K If one sentence involves a concept that another sentence does not involve, the two sentences cannot express the same thought or have the same content.⁵

If transformational equivalence is a genuine means of concept formation then the second member of each pair will contain a concept not to be found in the first member. Principle K therefore entails that (2a) cannot be synonymous with (2b) nor (3a) with (3b).

distinguishes the thought a sentence expresses from its content, where the content of a sentence includes these extra components.

³ The actual claim Dummett discusses is that the two sentences in each of five pairs discussed by Bell express the same sense. In his 1996 Bell dropped two of the five pairs. I am dropping one of the three pairs remaining. There seems very little plausibility in the thesis that Frege thought that the two halves of Basic Law V express the same content, given his requirement (discussed further below as Principle R) that anyone who grasps the thought expressed by each of a given pair of synonymous sentences must immediately recognise one as true if he recognises the other as true. Compare Dummett 1991a, p.293 and, for an opposing view, §7 of Sluga 1986.

⁴ This interpretation of Frege's conception of the compositionality of thoughts has been queried in Hale 1997. Hale defends a weak conception of sense according to which it is possible to understand the thought expressed by a sentence even though one does not understand the senses of the words of which that sentence is composed. This is a puzzling thesis. It seems undeniable that one understands a sentence in virtue of understanding the expressions of which it is composed. But to understand an expression is to grasp the sense of that expression. There doesn't seem to be room for grasping the sense of the whole sentence without grasping the senses of its sub-sentential parts.

⁵ Concepts are here, and in the remainder of this paper, being taken to be thought constituents or senses, rather than in Frege's technical sense on which concepts are functions from objects to truth-values. I take it that a sentence involves a concept when its sense (the thought it expresses) has that concept as one of its constituents.

So why did Frege fail to see this? Dummett makes two conjectures. The first is that the synonymy of the first and second member of each pair seems to be guaranteed by Frege's favoured criterion for synonymy, namely;

CRITERION R Anyone who grasps the thought expressed by each of a given pair of synonymous sentences must immediately recognise one as true if he recognises the other as true.⁶

But, maintains Dummett, Frege made the understandable mistake of failing to see that Criterion R is a necessary, but not sufficient, condition of synonymy. No two sentences can be synonymous unless they satisfy Criterion R, but two sentences can satisfy Criterion R and yet fail to be synonymous. Criterion R fails to be a sufficient condition precisely because it is trumped by Frege's conception of the compositionality of thoughts, as expressed in Principle K.⁷

Dummett's second suggestion is that Frege drew a false analogy between the transformational equivalence of (2a) and (2b) and (3a) and (3b), on the one hand, and the mode of concept formation that he (Dummett) calls decomposition, on the other. In essence, decomposition is the formation of a complex predicate (and corresponding complex concept) from a complete sentence by taking one or more occurrence of a singular term in that sentence and conceiving it as replaceable by any other singular term.⁸ Thus, to take an example that both Bell and Dummett discuss and to which we shall return, from the sentence

(5) 13 is greater than 1 and, for any number n , if n divides 13, then either $n = 13$ or $n = 1$

we can derive the complex predicate

(5*) \square is greater than 1 and, for any number n , if n divides \square then either $n = \square$ or $n = 1$.

⁶ See Frege 1906 and, for discussion, Van Heijenoort 1977b and Dummett 1981 Ch. 17.

⁷ It should be recognised, despite what Dummett says, that in the text where he discusses at greatest length the synonymy of two sentences (or what he terms their equipollence) he offers a version of criterion R as one part of a two-pronged definition. The second component is, effectively, that neither sentence should be a logical truth. Should one draw from this the conclusion that criterion R was not, in Frege's view, a sufficient condition for synonymy? Wolfgang Künne has drawn that conclusion (Künne 1997 p.232) and Dummett has followed him in this (Dummett 1997 p. 247). But one could, of course, equally say that Frege offers criterion R as a sufficient condition for the synonymy of sentences that are not logical truths – and it is this category of sentences that we are currently interested in.

⁸ The key texts in Frege for the doctrine of decomposition are Begriffsschrift (Frege 1879) §9 and the posthumously published article 'Boole's Logical Calculus and the Concept-script' (Frege 1880-1). Dummett's 1991b is a sustained discussion of decomposition.

The transformational equivalence between the first and second members of the two pairs is not an instance of concept formation by decomposition. There is no way of transforming a statement of parallelism like (2a) into an identity statement between directions by conceiving the proper names of the two lines as replaceable by other singular terms. How could there be, since the sign for identity does not feature in (2a)? Similarly for (3a) and (3b).

Dummett's proposal certainly solves the problem. Bell objects to it, however. He is unconvinced by Principle K. He objects, first, that the appeal to Principle K is strategically questionable, and second, that Principle K ought to be rejected. The strategic problem arises, according to Bell, "because Principle K is only compelling if one already assumes that the structure of a sentence is a good guide to the intrinsic structure of the thought it expresses" (Bell 1996, p. 593). We cannot conclude that a difference in surface syntactical structure betokens a difference in conceptual structure, and hence bring Principle K to bear, unless we have already decided that the structure of the sentence is isomorphic to the structure of the thought.

There seems to be an immediate reply to the strategic objection. The objection holds if surface syntax is the only reason for thinking that (2a) and (2b), for example, involve different concepts. And that doesn't seem to be the case. The first and second members of each pair have correlated with them dissociable abilities/capacities, which seems to provide at least prima facie evidence for difference in conceptual structure. It seems conceivable that a creature might be able to perform one-one mappings and parallelism-detections without being able to abstract from these operations to the determination of identities between abstract objects. Such a creature would correspondingly understand the first member of each pair but not the second. In fact, this seems a plausible description of what goes on in the early stages of children's learning to count. But then does this not give us a criterion for determining when two sentences have or fail to have the same conceptual structure, namely, that they cannot have the same conceptual structure if it is possible to understand one without understanding the other?

As a general criterion for difference of conceptual structure this fails. One can fail to understand a sentence that has the same conceptual structure as a sentence that one does understand simply because one is unfamiliar with with one or more of the expressions it

contains. This can happen when the sentences contain two expressions that are definitionally equivalent with each other. In order to get a firm grip on the issues here it will be helpful, I think, to put the strategic objection to one side for the moment to concentrate on Bell's substantive objection.

Bell finds Principle K “intrinsically objectionable”. His objection rests upon the example we have already used to illustrate the strategy of decomposition – namely the formation of the concept of a prime number. He asks us to consider the following two predicate expressions, one of which we have already encountered:

(5*) x is greater than 1 and, for any number n , if n divides x then either $n = x$ or $n = 1$.

(5**) x is prime.

The defender of Principle K is committed, Bell claims, to holding that (5*) and (5**) express different concepts. But then Principle K tells us that the corresponding sentences

(6*) 13 is greater than 1 and, for any number n , if n divides 13 then either $n = 13$ or $n = 1$.

(6**) 13 is prime

must express different thoughts. But this seems absurd, given that the concept of a prime number just is the concept of a number that is divisible only by itself and one.

The key claim that (6*) and (6**) cannot be synonymous is also accepted by Dummett as a consequence of Principle K, although he does not take it as a reductio of that principle. Dummett states:

The sentence ‘13 is prime’ and its definitional expansion are, of course, intimately connected: the sole difference between them is that what, in the terminology of Begriffsschrift, was not integral to the content of the expanded sentence, but was merely one out of different possible ways of regarding it, has become integral to the content of its definitional abbreviation, ‘13 is prime’. The predicate ‘. . . is prime’ has indeed the same sense as the complex predicate serving as its definiens; but this sense is not a constituent of the thought expressed by the expanded sentence. (Dummett 1991 pp.298-299)

As is clear in this passage, Dummett differs from Bell in maintaining that the predicate expressions (5*) and (5**) are synonymous⁹, even though he agrees with Bell that the sentences

⁹ Dummett is not entirely consistent on this. The sentence just before the quoted passage reads: "In fact, Principle K disallows identity of content even for definitionally equivalent predicates, and sentences containing them, at least

(6*) and (6**) obtained by completing those predicate expressions with the name of a number will not be synonymous. The lack of synonymy at the level of the sentence comes about because (6**) picks up only one of the ways in which (6*) should be analysed, and it does not identify what one might term the canonical analysis of (6*). The canonical analysis of (6*) represents the structure that must be grasped by anyone who is to understand the sentence, and the whole point of the doctrine of decomposition is that (6*) can be understood by somebody who does not understand that it contains the concept expressed in full by (5*) and in abbreviated form by (5**).

It seems to me, however, that both Dummett and Bell are on the wrong track here. Dummett is right against Bell that Frege is committed to holding that (5*) and (5**) are synonymous. It seems absurd to say that one predicate expression can be a definitional abbreviation of a predicate expression with which it is not synonymous. Frege himself clearly states that definition requires sameness of sense. In the posthumously published 'Logic in Mathematics' Frege states:

The introduction of a simple sign adds nothing to the content; it only makes for ease and simplicity of expression. So definition is really only concerned with signs. We shall call the simple sign the definiendum, and the complex group of signs which it replaces the definiens. The definiendum acquires its sense only from the definiens. This sense is built up out of the sense of the parts of the definiens. (Frege 1914, p.208)

Clearly, if Frege thinks that the definiendum acquires its sense only from the senses of the parts of the definiens then he cannot deny that they must be synonymous.¹⁰

According to Bell the non-synonymy of (6*) and (6**) follows from the non-synonymy of their respective predicate components (5*) and (5**). This cannot be right. But what about Dummett's suggested derivation of the non-synonymy of (6*) and (6**) from the doctrine of decomposition? The position Dummett attributes to Frege is unattractive. For one thing it is straightforwardly incompatible with the following passage from 'Logic in Mathematics':

We have to distinguish between a sentence and the thought it expresses. If the definiens occurs in a sentence and we replace it by the definiendum, this does not affect the thought at all. It is true we get a different sentence if we do this but we do not get a different thought. (PW, p.208)

given Frege's doctrine of analysis" (Dummett 1991, pp.298). But this denial that definitionally equivalent predicates are synonymous seems to be a slip of the pen.

¹⁰ Compare Grundgesetze I, 27: "By means of a definition we introduce a new name by stipulating that it is to have the same sense and the same Bedeutung as a name composed of already known signs".

But secondly, and perhaps this is what underlies the point Frege is making here, one might suggest that the sentence (6*) can be the vehicle of two different thoughts. One of those thoughts is the non-compound thought expressed unambiguously by (6**). This is the non-compound thought that attributes to the number 13 a property that it shares with a range of other numbers (namely, all those numbers that are divisible only by themselves or by 1). But, alternatively, (6*) can express a compound thought. This is the conjunctive thought that, first, 13 is a number greater than 1 and, second, that any number that divides 13 without remainder must be either 13 or 1. Frege (at least as interpreted by Dummett) is committed to holding that the compound thought is more fundamental than the simple thought. The compound thought reflects the basic composition of the sentence as constructed in a series of stages from atomic sentences.¹¹

There's nothing puzzling about the proposal that (6*), canonically construed as a compound thought, should express a different thought from (6**) – how, after all, could a compound thought be identical to a non-compound thought? Nor about the proposal that if we take (6*) on its non-canonical construal then it must express the same thought as (6**). Of course, if we assume that a sentence (strictly, a sentence-type) that is neither lexically nor syntactically ambiguous and contains no demonstrative elements can only express one thought, then it is natural to identify that one thought with the thought that is isomorphic with its canonical analysis. But the assumption seems profoundly incompatible with the doctrine of decomposition. The idea that a sentence can express a range of thoughts is a natural interpretation of the thesis that a given sentence can be analysed in different ways.

Consider the sentence that Frege used to illustrate the doctrine of multiple analyses in *Begriffsschrift*, namely, 'Cato killed Cato' (Frege 1879, §9). As he notes this sentence is susceptible to four different function-argument analyses, which we might convey as follows:

- (1) Cato, \square killed \square , Cato (i.e. 'Cato' fills both argument places in the function-name ' \square killed \square ')
- (2) Cato, \square killed Cato (i.e. 'Cato' is the argument of the function-name ' \square killed Cato')

¹¹ Dummett's position, and in particular its dependence on the distinction between simple and complex predicates has been contested in Geach 1975 and Currie 1985. Dummett defends the distinction in Ch. 16 of his 1981.

(3) Cato, Cato killed \square (i.e. 'Cato' is the argument of the function-name ' \square is killed by Cato')

(4) Cato, \square killed \square (i.e. 'Cato' is the argument of the function-name ' \square killed himself').¹²

To each of these function-argument analyses there corresponds a distinct thought. Since each analysis is an analysis of the sentence 'Cato killed Cato' it follows that 'Cato killed Cato' can express four distinct thoughts.

It might be wondered just how faithful this proposal is to Frege's conception of thoughts. Frege repeatedly remarks that a thought is the sense of a sentence, but at no point does he say that a thought is the sense of a sentence under a particular analysis or under a particular decomposition. So why should he be credited with the view that a single sentence can express multiple thoughts?¹³ In order to see why this *has* to be Frege's position we need to consider the alternative way of understanding decomposition. What is it for a sentence to be analysable in different ways, if not for it to be capable of expressing a range of different thoughts? The only possibility is that the sentence expresses a single thought which can be analysed in different ways. But what could it mean to say that a thought can be analysed in different ways? It must mean that new thoughts are generated from the original thought. But then, by Frege's thesis of the isomorphism between sentences and thoughts (Thesis A), there must be a sentence corresponding to each one of these thoughts. Unless the view I am advocating is correct, these sentences must be different from the original sentence. But there is absolutely no reason to think that there will always be such a sentence, as the examples we have already considered clearly indicate. Consider (6*). This can be decomposed in two different ways, one corresponding to a compound thought (its canonical analysis) and the other to a non-compound thought (corresponding essentially to the thought that 13 is a prime number. The possibility of this decomposition does not depend upon the existence of an expression ' \square is prime' to mark the non-compound reading. Similarly of the four possible decompositions we have noted of the sentence 'Cato killed Cato' only the final one can properly be said to have a distinct sentence ('Cato killed

¹² Frege actually only identified three function-argument analyses of 'Cato killed Cato', not distinguishing between (1) and (4). This was an oversight.

¹³ I am grateful to Michael Dummett for urging me to address this point.

himself') corresponding to it.¹⁴ Each of the other three decompositions has the same sentence corresponding to it, namely, 'Cato killed Cato'. The different decompositions are marked by the different inferences they make possible. So, for example, on the third decomposition one can infer that someone was killed by Cato, while on the second one can infer that someone killed Cato. In each case the sentence that, under Frege's Thesis A, corresponds to each decomposition is none other than the original sentence 'Cato killed Cato'.

The suggestion does not seem to be in conflict with Frege's fundamental idea (Thesis A) that the structure of a sentence can serve as a model of the structure of a thought. Thesis (A) does not say that sentences ought to have a single semantic structure. All it says is that to each such structure there corresponds a thought and, conversely, that every thought corresponds to some construal of the structure of a sentence. Nor is it in conflict with Frege's insistence that every token sentence, if it expresses a thought that is true or false at all, must express one that is true or false tout court. The different thoughts for which a single sentence may be the vehicle are all logically equivalent, in the sense that they necessarily always have the same truth-value. The way in which a sentence is decomposed cannot affect its truth-value. The decomposition of a sentence merely affects the inferences that can be drawn from that sentence. Nonetheless, although the different thoughts for which a single sentence may be the vehicle are logically equivalent, they are not semantically equivalent. For any sentence that can be decomposed (that is to say, for any non-atomic sentence) only one of the thoughts that it expresses must be grasped if that sentence is to be understood. This fundamental thought is the one whose structure reflects the basic composition of the sentence – either as an atomic sentence or as constructed in a series of stages from atomic sentences.

I cannot see how to interpret all this other than by attributing to Frege the view that a single sentence can express distinct thoughts. Nonetheless, the fact remains that Frege did not explicitly adopt the view that I am ascribing to him. It is hard to strike a balance between textual fidelity

¹⁴ And this, in fact, is a contingent feature of our language. We would still be able to grasp the concept of suicide even if our language did not contain the machinery of reflexive pronouns or the word 'suicide'. We would be able to grasp it in terms of the two argument-places in the relational predicate '- killed -' necessarily being occupied by names of the same individual when we wish to use it to express that concept rather than the standard concept of killing. That is, we would grasp the concept as \square killed \square rather than as \square killed \square .

and the principle of charity, and it may well be that my proposal counts as an emendation of Frege's doctrine rather than an interpretation of it. But, even if it is an emendation, it is not one that does violence to Frege's other views in this area - and arguably it is a necessary emendation if we are to preserve consistency in the face of his account of decomposition.

In any case, to return to the thread of the argument, the idea that a single sentence can express different thoughts gives us the tools to rebut Bell's substantive objection to Principle K. Even if we agree with Bell that any principle entailing that (6*) and (6**) must express different thoughts should indeed be rejected, it is not clear that Principle K has that consequence. It is true that (6*) and (6**) can express different thoughts. But they can equally express the same thought. And this ambiguity is exactly what we would expect on Frege's view, given his commitment to Thesis (A) and to the doctrine of decomposition.

Nor does the suggestion that the sentence (6*) can be the vehicle of two distinct thoughts bring us into conflict with Criterion R (if two sentences express the same thought then anyone who grasps the thought expressed by both must immediately recognize one as true if he recognizes the other as true). On the contrary, it is inconceivable that someone should understand both (6*) and (6**) and accord them different truth-values. Nor could anyone understand (6**) without ipso facto understanding (6*).¹⁵ But one can understand (6*) without understanding (6**), because (6**) picks out just one of the thoughts that might be expressed by (6*).

We might extract a general principle from this by noting that, when expression A is an expression being defined (say '□ is bachelor' or '□ is prime') and B the expression in terms of

¹⁵ It might be suggested that this is clearly false, since there are ways by which one could come to understand (6**) that would not put one in a position to understand (6*). For example, a monolingual speaker of German who has been authoritatively told that (6**) means the same as "13 ist prim" would understand (6**) without knowing what on earth to make of (6*). It seems to me, however, that this objection lacks force. Most accounts of linguistic understanding expressly distinguish what is often termed 'phrasebook mastery' of a segment of a language from genuine mastery of a language (or segment thereof). The difference is that genuine understanding is compositional and structured. When we genuinely understand a sentence we do so in virtue of our understanding of its constituent elements. When, in contrast, we have phrasebook mastery of a sentence we "understand" the sentence only as an unstructured whole. Our "understanding" is not genuine precisely because it is not a function of distinct abilities that can be put to work to make sense of further sentences of the same language. Although, of course, in the current example the phrasebook understanding is grounded in a compositional understanding of a German sentence, the English sentence is not itself understood compositionally. I would say, therefore, that there is a very real sense in which the English sentence is not actually understood by the monolingual speaker of German, although the German speaker is in a position to assign it a truth-value – and indeed, from an epistemological point of view, he may be perfectly justified in so doing.

which it is being defined (say ' x is an unmarried adult male' or ' x is greater than 1 and, for any number n , if n divides x then either $n = x$ or $n = 1$ '), anyone who understands A will of necessity understand B . The converse does not hold, which is why (as we noted earlier) from the fact that one can understand one but not the other of a pair of sentences it is fallacious to conclude that they are not synonymous.. One can learn something by being told of a definitional equivalence — but only if one is in the situation of understanding the definiens but not the definiendum.

Let us return now to Bell's strategic argument against the deployment of Principle K. The objection, it will be remembered, was that Principle K can be applied only if we have some criterion independent of syntactic structure for determining when two or more sentences have different conceptual structure. The fact that the first member of each pair can be understood without the second being understood does not immediately give us such a criterion — because of the possibility of unknown definitional equivalence. All that the proposed criterion shows is that either the two sentences are non-synonymous or that the second sentence contains some unfamiliar expression definitionally equivalent to some expression in the first. From the general principle that emerged in the previous paragraph, however, we can conclude that in the case where the two expressions are definitionally synonymous the relation of definitional equivalence must hold between a definiens in the first sentence and a definiendum in the second. Applying this to our two pairs of sentences yields the result that, when we apply the criterion proposed earlier, we should conclude that either the first and second members of each pair have different conceptual structures or some expression in the second sentence is a definitional abbreviation of an expression in the first.

This gives us enough to block Bell's argument that Principle K can only be applied by question-beggingly presupposing that the structure of a sentence is a good guide to the structure of the thought that it expresses. Clearly, we can understand the first member of each pair without understanding the second. This tells us that one branch or other of a disjunction holds. Let us consider, then, the first branch of the disjunction. If the thoughts expressed by the first and second members of each pair are composed of different concepts then by Principle K the two sentences cannot express the same thought. There is no threat here to Thesis A. On the second

branch, however, some expression in the second sentence is a definitional equivalent of some expression in the first sentence. So, for example, talk about identity of directions might just be seen as a fancy way of saying what could equally well be said by talking about lines being parallel. But this is no threat to Thesis A either, because if this is the case then the first and second members of each pair do indeed express the same thought, but there are no interesting differences of structure between them. In fact, on this branch of the disjunction, the criterion itself settles the issue without any need at all to bring Principle K into play.

We can, therefore, draw three substantive conclusions from the discussion so far:

- (1) Dummett is correct, as against Bell, that the pairs of sentences Frege discusses in Grundlagen §64-7 are not examples of differently structured sentences expressing one and the same thought.
- (2) Dummett's reasons for holding that two sentences differing from each other only in that a definiens is substituted for a definiendum cannot be synonymous are unconvincing. His argument can be accommodated by recognising (as indeed one needs to do on independent grounds) that a single sentence can express a range of different thoughts.
- (3) We must reject Bell's argument that it is question-begging to appeal to the principle that two sentences involving different concepts must express different thoughts. It is possible to get an independent grip on whether two thoughts involve different concepts without presupposing that the structure of a sentence is a good guide to the structure of the thought it expresses.

II

What about the more general question of the tenability or otherwise of attributing to Frege Thesis (B), that two structurally different sentences can express one and the same thought? Obviously everything depends upon what 'structurally different' means. There is a sense in which Frege's notion of constructive definition allows structurally different sentences to express one and the same thought. As we have seen a constructively defined simple sign is synonymous with a complex sign so that they are intersubstitutable salva significatione. Since a sentence containing a simple sign is obviously different in structure from a sentence containing a complex sign, it looks very much as if sentences with different structures can express one and the same thought. But this does not address, I think, the real issue posed by Thesis (B). As Frege stresses in 'Logic in Mathematics', the difference in sentential structure between definiens and definiendum is

essentially a matter of convenience. But, as we have seen, the real question concerns differences in sentential structure that entail differences in conceptual structure. Definiens and definiendum can be written differently in primitive notation – but they do not have to be.

When we consider only differences in sentential structure that entail differences in conceptual structure it looks as if Thesis (B) and Principle (K) are directly incompatible. So, if Dummett is right in affirming Principle (K) then he must be right in rejecting Thesis (B). And Principle (K) does seem extremely plausible. But there do seem to be cases in which, by Frege's own lights, Principle (K) does not seem to hold. In a footnote to the essay 'Compound Thoughts', for example, Frege suggests that 'A & A' expresses the same thought as 'A' (1918-1919c, p.393 n.21). Clearly, though, since one sentence involves the concept of conjunction and the other does not, Principle (K) entails that they express different thoughts. A similar problem arises with double negation. It has seemed obvious to many philosophers that there can be no difference in content between a thought and the negation of the negation of that thought. Currie, for example, has used this to argue (1985, p.297) that Frege's account of the composition of thoughts is incoherent. Yet the obvious differences in conceptual structure between a thought and its double negation will, by Principle (K), entail that they are two different thoughts.

The footnote about conjunction should not be taken too seriously, however. Frege cites the supposed synonymy of 'A' and 'A & A' as analogous to the indubitable synonymy of 'A & B' and 'B & A'. But it is highly plausible that, if he had thought about it a bit more, he would have realised that there is no analogy. The difference between the two sentences 'A & B' and 'B & A' is purely notational - but the same can hardly be said of 'A' and 'A & A', given that 'A' is a simple thought and 'A & A' a compound thought.

As far as double negation is concerned, the situation is less clearcut. On the one hand, in the essay 'Negation' he says pretty clearly that, of the two thoughts 'A' and ' $\sim\sim A$ ', either both are true or neither is (1918-1919b, p.389). Clearly if there are two thoughts then they can't be the same. The form of words seems incompatible with a belief in the synonymy of 'A' and ' $\sim\sim A$ ', particularly since he summarises the discussion by saying: "Wrapping up a thought in double negation does not alter its truth-value" (ibid.). Perhaps it is unfair to put too much weight on the

metaphor, but it is hard to see one can wrap something up and still be left with what one started with. On the other hand, however, in the more or less contemporaneous 'Compound Thoughts' (1918-1919c) he commits himself unequivocally to the synonymy of 'A' and ' $\sim\sim A$ ' (ibid. p. 399). What are we to make of this?

I think, on balance, that we should be cautious of putting too much weight on Frege's views on double negation. This is partly because he seems to say different things. More importantly, he could adopt either of the two views he offers in Logical Investigations without any serious implications for the rest of his thought. For example, although in 'Compound Thoughts' he does rely on the supposed synonymy of 'B' and ' $\sim\sim B$ ' in inferring from

$$(\text{not}-(\text{not}-B)) \ \& \ (\text{not}-A)$$

to

$$B \ \& \ (\text{not}-A),$$

it is clear that he does not need to hold that 'B' and its double negation are synonymous. The inference goes through perfectly well on the logical equivalence of 'B' and ' $\sim\sim B$ '. I suspect that, as with 'A & A', this is just not something to which he gave an enormous amount of thought. If we want compelling grounds for ascribing to Frege the view that differently structured sentences could express a single thought we had better look elsewhere.

A better place to look for evidence that Frege was indeed committed to Thesis B is his views on truth. Frege famously argued that 'true' is not an adjective in the normal sense of the word and truth not a property. Part of his reason for saying this is that there is no difference in sense between 'A' and 'A is true'. Yet he was also of the view that the predicate expression '- is true' has a sense. In 'My Basic Logical Insights', for example, he writes:

If I assert 'it is true that sea-water is salt', I assert the same thing as if I assert 'sea-water is salt'. This enables us to recognize that the assertion is not to be found in the word 'true', but in the assertoric force with which the sentence is uttered. This may lead us to think that the word 'true' has no sense at all. But in that case a sentence in which 'true' occurred as a predicate would have no sense either. All one can say is: the word 'true' has a sense that contributes nothing to the sense of the whole sentence in which it occurs as a predicate. (Frege 1915, p.252)

The conflict is clear enough. If the predicate '- is true' has a sense then the conceptual structure of 'A is true' differs from that of 'A'. Hence by Principle (K) the two sentences cannot express the same thought. Yet Frege says they do express the same thought.

In the case of 'A' and 'A & A' it seemed most plausible to deny any thesis of identity of content. This strategy is clearly unavailable for 'A' and 'A is true'. That these two sentences express the same thought is integral to Frege's account of assertoric force, as well as to much of what he says about truth. It is hard to see how Frege could abandon it. If we are to reconcile the tension between synonymy and difference in conceptual structure then it must be the second we reject on Frege's behalf. But how could 'A' and 'A is true' have the same conceptual content? Let us look again at Frege's argument. He argues that, were they to have identical contents, this would entail that any sentence in which the truth predicate featured also lacked a sense, and hence that it would be possible to transform any meaningful sentence 'A' into a sense-less sentence by prefixing it with 'It is true that -'. Clearly this is absurd. One might feel, however, that Frege is being too hasty here. He seems to be tacitly assuming that all grammatically distinct parts of a sentence must have a sense for the sentence as a whole to have a sense. And it might be objected that the whole point of the redundancy theory of truth (of which we are considering a special instance) is that the truth predicate cannot be a semantically relevant part of a sentence. This doesn't mean that the thought expressed by a sentence predicating truth of another sentence has a gap in it where the sense of '- is true' ought to go. Rather, it means that the difference between 'A is true' and 'A' is in some sense akin to the difference between a sentence that contains the adverb 'regrettably' and the sentence that results from removing the adverb. The word 'regrettably' does not have a sense (Frege 1918-1919), but that hardly prevents sentences in which it features from expressing a thought.

Frege's reason for holding that the word 'regrettably' has no reflection at the level of the thought expressed is that there is no difference in truth condition between 'A' and 'regrettably A'. He makes the same claim for 'A and B' and 'A but B'. So why can we not simply resolve the problem on his behalf by maintaining that the same holds of 'A' and 'A is true'?¹⁶ The reason is

¹⁶ With the difference, of course, that the distinction between 'A' and 'A is true' (unlike that between 'A and B' and 'A but B' is unlikely to be analysed as a difference in tone.

straightforward. Adopting any version of the redundancy theory of truth would be incompatible with holding, as Frege of course did, that we understand the sense of a sentence through grasping its truth-condition. Or rather, it would be incompatible with holding that any such account could be informative. The point has been elegantly put by Dummett:

If all it means to say that 'Frege died in 1925' is true is that Frege died in 1925, then the knowledge that 'Frege died in 1925' is true just in case Frege died in 1925 is the 'knowledge' that Frege died in 1925 just if Frege died in 1925. In the same way, if the whole explanation of the sense of the word 'win' consisted in a stipulation, for each game, of the conditions under which one player or another was said to have won, then a knowledge of what a particular game is could not involve knowing what it is to win that game: for the knowledge that, e.g., one wins a game of chess when one either check-mates one's opponent or he resigns would amount to no more than the 'knowledge' that either one checkmates one's opponent or he resigns when one checkmates one's opponent or he resigns, which is no knowledge at all. (Dummett 1973, p. 459)

The point is devastating. It is no use trying to respond by holding that there is a difference between understanding the sentence ' "Frege died in 1925" is true' and understanding that 'Frege died in 1925' is true. What could this difference possibly consist in?

It follows, then, that Frege's commitment to the informativeness of explaining the sense of a sentence in terms of grasp of a truth condition rules out the possibility of his consistently holding a redundancy theory of truth. This in turn entails that he must hold that there is a difference in conceptual structure between 'A' and 'A is true'. But then, given his insistence that 'A' and 'A is true' express the same thought, it follows that Frege is committed to Bell's Thesis (B). To return to the main thread of the argument, although Dummett is quite right to argue, against Bell, that Frege's discussion of numbers and directions in Grundlagen does not provide examples of differently structured sentences expressing one and the same thought, he is wrong to maintain that Frege's theory is incompatible with Thesis (B). Quite the contrary. For every sentence, Frege is committed to the existence of a further sentence with a different structure but expressing the same thought. Indeed, he is committed to indefinitely many such sentences, if one takes into account iterations like 'It is true that it is true that A is true'.

It might be thought that Frege's position here is inherently unstable. How can there be an indefinite hierarchy of differently structured sentences expressing the same thought? As we have seen, saying that 'A is true' is differently structured from 'A' implies that the predicate '– is true'

has a genuine sense. But how can it have a genuine sense if it makes no contribution to the thought expressed by the sentence within which it features? One would be forgiven for finding it hard to see what exactly it is that distinguishes Frege's theory from the redundancy theory that we have seen he cannot accept.

The problem is partly Frege's own fault. Like many philosophers, when he discusses truth he tends to consider only predications of truth to assertoric sentences of the most straightforward type, like the example we have already considered of 'sea water is salt'. These are predications of truth in which what is said to be true is explicitly supplied. It is certainly true that, in a language in which all predications of truth were of this form, and in which 'A' and 'it is true that A' were deemed synonymous, there would be no way of attributing a sense to the predicate '- is true'. But our language is not, of course, such a language. We predicate truth of things that are only indirectly supplied, as in sentences like 'the last thing Frege said about truth was true'. What distinguishes these indirect truth predications is that when we remove from them the truth predicate we are not left with the sentence (or sentences) expressing the thought whose truth we wished to assert. We are left merely with a name of that sentence. In such cases it seems plausible that, unless the predicate '- is true' had a sense, no thought would be expressed.¹⁷ But these occurrences of '-is true' are the ones that give it its sense. When we take such sentences into account we see why Frege holds both that the predicate '- is true' has a sense and that the sentences 'A' and 'A is true' are synonymous.

Of course, if the predicate '- is true' has a sense then that sense features in any thought expressed by a sentence in which the predicate features. But the sense of the predicate '-is true' is such that, when it is predicated of a standard assertoric sentence, the truth condition of the ensuing sentence is identical to the truth-condition of the original sentence. In the standard case (i.e. when the predication is direct) this is not particularly useful. But when the predicate is indirect the predicate allows us to assert that a truth-condition is satisfied (a state of affairs holds) without knowing what that truth-condition/state of affairs is.

¹⁷ These case have for this very reason always been the Achilles heel of redundancy theories of truth.

Hence we see both why Frege was committed by his views on truth to Thesis B (that differently structured sentences can express the same thought) and why this does not bring him into any conflict with Theses A (that the structure of a sentence serves as a model for the structure of a thought). The conclusion of this section, therefore, is:

(4) Pace Dummett, Frege is committed to holding that differently structured sentences can express one and the same thought. This is a consequence of his views about truth. But it is not in tension with Thesis A.

III

There is a further respect in which Dummett's interpretation of Frege seems unstable. Dummett himself thinks, and puts it forward as an interpretation of Frege, that the process of abstraction from the first to the second member of each of the two pairs with which we began generates a new concept:

We indeed have, in the transition from the first member of such a pair to the second, a mode of concept formation, and one that follows a pattern common to many distinct instances; but it is a different mode from that which Frege expounded in Begriffsschrift and so strongly emphasised in 'Booles rechnende Logik'. (Dummett 1991 p.301)

But what concepts can be introduced by such a move? Surely not the concepts direction and number. For those concepts we need the explicit definitions offered in Grundlagen §68, whereby the direction of line a is explained as the extension of the concept parallel with line a and the number of Fs is explained as the extension of the concept equinumerous with the concept F. But if those concepts are not introduced then which ones are? Do we have the partial introduction of the concepts number and direction? Or do we have the introduction of some partial concepts number* and direction*, standing in some as yet unspecified relation to the concepts number and direction? Surely it is better to abandon all talk of concept formation here, as indeed Frege himself does at the beginning of §68 of Grundlagen. Frege there sums up the discussion in the preceding three sections as follows:

Seeing therefore that we cannot by these methods obtain any concept of direction with sharp limits to its application, nor therefore, for the same reasons, any satisfactory concept of Number either, let us try another way.

The most plausible interpretation seems to be that Frege ought not to be credited with the view that the move from (2a) to (2b) and from (3a) to (3b) introduces a new concept. That view of concept formation was proposed, considered and rejected in Grundlagen §64-7.

IV

To sum up, then, we can make the following comments on the debate between Bell and Dummett about the structure of Fregean thoughts.

(1) Dummett is correct, as against Bell, that the pairs of sentences Frege discusses in Grundlagen §64-7 are not examples of differently structured sentences expressing one and the same thought.

(2) Dummett's reasons for holding that two sentences differing from each other only in that a definiens is substituted for a definiendum cannot be synonymous are unconvincing. His argument can be accommodated by recognising (as indeed one needs to do on independent grounds) that a single sentence can express a range of different thoughts.

(3) We must reject Bell's argument that it is question-begging to appeal to the principle that two sentences involving different concepts must express different thoughts. It is possible to get an independent grip on whether two thoughts involve different concepts without presupposing that the structure of a sentence is a good guide to the structure of the thought it expresses.

(4) Pace Dummett, Frege is committed to holding that differently structured sentences can express one and the same thought. This is a consequence of his views about truth. But it is not in tension with Thesis A.

(5) Both Bell and Dummett are mistaken in holding that Frege believed the process of abstracting from an equivalence relation to be a genuine form of concept formation.

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