

Hilary Putnam on Meaning and Necessity

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Abstract

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In this dissertation on Hilary Putnam's philosophy, I investigate his development regarding meaning and necessity, in particular mathematical necessity. Putnam has been a leading American philosopher since the end of the 1950s, becoming famous in the 1960s within the school of analytic philosophy, associated in particular with the philosophy of science and the philosophy of language. Under the influence of W.V. Quine, Putnam challenged the logical positivism/empiricism that had become strong in America after World War II, with influential exponents such as Rudolf Carnap and Hans Reichenbach. Putnam agreed with Quine that there are no absolute a priori truths. In particular, he was critical of the notion of truth by convention. Instead he developed a notion of relative a priori truth, that is, a notion of necessary truth with respect to a body of knowledge, or a conceptual scheme. Putnam's position on necessity has developed over the years and has always been connected to his important contributions to the philosophy of meaning. I study Hilary Putnam's development through an early phase of scientific realism, a middle phase of internal realism, and his later position of a natural or commonsense realism. I challenge some of Putnam's ideas on mathematical necessity, although I have largely defended his views against some other contemporary major philosophers; for instance, I defend his conceptual relativism, his conceptual pluralism, as well as his analysis of the realism/anti-realism debate.

Keywords: philosophy of language, philosophy of science, philosophy of mathematics, Hilary Putnam, W.V. Quine, Rudolf Carnap, realism, anti-realism.

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Introduction with a Summary

1. General background

Hilary Putnam has been a leading American philosopher since the end of the 1950s, becoming famous in the 1960s within the school of analytic philosophy, associated in particular with the philosophy of science and the philosophy of language. In later years, he has become increasingly devoted to modifying his ideas in a way that takes broader perspectives into consideration. For instance, he has developed an interest for ethical issues, not in the sense that he is interested in “solving” ethical problems by means of narrow analytical techniques, but in a way that brings ethical considerations in a wider sense to the philosophical forefront. He has also broadened his philosophical frame of reference to include such names as Derrida, Foucault, Gadamer and Heidegger, although usually with a critical purpose. His main influences today are Kant, Wittgenstein and pragmatists such as William James and John Dewey, but he keeps returning to the problems posed by Rudolf Carnap and W.V. Quine, his early and lifelong influences.

Hilary Putnam was born in Chicago in 1926¹, but the family lived in Paris until 1934, when they moved back to the United States. Putnam started his career in philosophy in the 1940s, influenced as an undergraduate at the University of Pennsylvania. Putnam finished his PhD on the concept of probability in 1951 at UCLA under the guidance of Hans Reichenbach, but he was strongly influenced by Quine at Harvard. The young Putnam was in close contact with logical positivists/empiricists from continental Europe; for instance, he worked on inductive logic with Carnap at Princeton in the early 1950s. He contributed with some outstanding philosophical papers, such as *The Analytic and the Synthetic* (published in 1962, but available already in 1957) and *It Ain't Necessarily So* (1962). He subsequently became a Professor of the Philosophy of Science at MIT in 1961, but moved permanently to Harvard in 1965, where he is now emeritus.

¹In this introduction, which contains a bibliographical sketch of Putnam's work, I have relied heavily on Y. Ben-Menahem's Introduction to her (ed.) *Hilary Putnam* (2005) as well as from the short biographical note in Putnam's *Pragmatism* (1995), together with a very general background knowledge I have gleaned from Putnam's other writings.

Putnam's father, Samuel Putnam, was an intellectual and worked as a translator, writer and a columnist for the *Daily Worker*. Hilary Putnam shared his father's political activism for a long time, although he became disillusioned with the far left in the beginning of the 1970s. He has also been critical of leftist "ir-responsibility" in the wake of Derrida's and Foucault's works, in that he views "postmodern" skepticism about reason-talk as dangerous, as well as mistaken, since it presupposes that the "defenders of reason" believe that our considerations in ethics and politics are free from historical contingency. To counter the criticism of an absolute reason, Putnam defends an enlightenment ideal that was formulated by Dewey. An important pragmatist insight for Putnam is that both problems and their solutions are contingent. This is one reason why the idea of *corrigibility*, an idea Putnam identifies with the classical American pragmatists, is important for him. Another important idea for the later Putnam is that of the interpenetration between fact and value, i.e., that we cannot separate the analysis of facts from that of values. This is an idea that makes it difficult to separate economic theory from political views, a theme of *The Collapse of the Fact/Value Dichotomy and other Essays* (2002).

The fact/value dichotomy may be viewed as parallel to another dichotomy that has been a major concern for Putnam over the years, that between analytic and synthetic truths, originally proposed by Kant in order to give a philosophical account of the certain truth of Newton's physics. For Kant, some truths were assumed to be *a priori*, that is, true independent of experience. One would think that there are merely *analytic* truths of this kind, that is, statements that are true in a substitutional sense; but for Kant, there were also the important *synthetic a priori* truths that frame our experiences. Examples of such truths are those of Euclidean geometry that describe Newtonian absolute space, and certain other principles, such as the principle of causality (every effect has a cause). After the Einsteinian revolution in physics, there was a big stir among philosophers, since Einstein's physics implied, among other things, that Euclidean geometry did not describe our physical world, which seemed to threaten our "intuitive" picture of the world, and certainly the *a priori* status of Euclidean geometry. It became urgent to come to terms with the new world view that Einstein's theories seemed to suggest.

Reichenbach (1920) proposed that Kant had conflated two separate ideas in his notion of synthetic *a priori* truths. These may be constitutive for a theory, but that should not necessarily mean that they are absolute necessary truths. The idea of a relative notion of *a priori* truth was first launched by Reichenbach (later to be forgotten); modern proponents of such a view are Hilary Putnam and Michael Friedman, although in very different ways. Putnam sided early on with W.V. Quine against the ideas of the logical positivists, usually represented by Rudolf Carnap, who had fled continental Europe to the United States before

the Second World War because of his socialist sympathies. In the United States, Carnap was well taken care of, notably due to efforts of Quine, but his logical positivism was heavily challenged by Quine and other American philosophers. Carnap was originally trained in theoretical physics and logic (by Frege) and was one of the members of the Vienna Circle who were inspired by the new physics. The Vienna Circle consisted of a group of young philosophers in the mid 1920s who wanted to adapt philosophy to the emerging new science; its members included Moritz Schlick and Hans Hahn, among others, and on a few occasions they met with the young Ludwig Wittgenstein. The ensuing interpretations of *Tractatus Logico-Philosophicus* were and still are a source of philosophical inspiration and contention. One historically important interpretation was that it seemed to throw out everything but science from intelligible discourse; in particular, it seemed to throw out philosophy. However, it soon was clear that Wittgenstein did not embrace the idea that philosophy should become a “theory of science”, something that was more in line with Carnap’s point of view. Carnap tried to pursue the idea of philosophy as a *Wissenschaftslogik*, suggesting a dichotomy between analytic and synthetic truths within (reconstructed, formalized) scientific theories. The analytic truths were now thought of as framing what was expressed through factual statements, but in Carnap’s more mature work there was a difference between an external level, on which we are free to propose a logic, and an internal level, on which we are bound within a specific theory by the choice of our logic. In particular, Carnap thought that we may choose between intuitionistic logic and classic logic, depending on our pragmatic needs.

Quine launched an attack on Carnap’s scheme in *Two Dogmas of Empiricism* (1951), which was seen for a long time as the final blow to Carnap’s project. Quine attacked the analytic-synthetic distinction, which he regarded as untenable, since the notion of “analytic” is left totally unexplained. He also proposes that the scientific revolutions of Kepler, Darwin and Einstein should lead us to the conclusion that “no statement is immune to revision”. In fact, not even the laws of logic should be viewed in this definite way, as he thought that quantum mechanics may suggest. Furthermore, Quine challenged the second dogma, that statements have meaning by being reducible to statements about sense experience, i.e., the verification theory of meaning.

The young Putnam supported Quine’s picture. In *It Ain’t Necessarily So*, he argues that there are principles within a theory that we treat as necessary with respect to our current body of knowledge, but which, as this body of knowledge changes in confrontation with a rival theory that perhaps explains phenomena better, we may come to revise or reject earlier “necessary” truths. Not even definitions have prevailed in science. In *The Analytic and the Synthetic*, Putnam gives the example of Newtonian kinetic energy, thought of as stipulated in the

tradition of logical positivism. Whether this is true or not, kinetic energy is not definitional in Einstein's physics; the new expression for kinetic energy is now an empirical law on par with other empirical laws. This change, from a purported definition to an empirical law, suggests that there is no such thing as a conventional or stipulated truth. Euclidean geometry is not viewed by Putnam as only a formal mathematical system that is disconnected from statements such as "I cannot come back to the same place by traveling in a straight line". The overthrow of Euclidean geometry was a long process according to Putnam, occurring roughly between 1815 and 1915, and ending with the falsity of the just cited statement. A statement can only be necessarily true with respect to a body of knowledge, and it was this body of knowledge that changed. It is not because we have changed the meaning of the word "straight" or "straight line" after Einstein that we came to think that "I can come back to the same place" is a true statement. For the same reason the statement, "there is a triangle with angle sum larger than 180° " was seen to be false in the year 1700, say, but it is now viewed to be a true statement—there are triangles with an angle sum greater than 180° . It is not that we have just decided to include some objects on a sphere to be called triangles. Triangles are made up of straight lines, but straight lines were not what we earlier thought them to be. Mathematical necessity is therefore connected to other true scientific statements, and there is no statement that can be said to be unrevisable.

Later Putnam changes this view. He finds it somewhat strange to call statements that eventually become false *necessary*. Instead he suggests, in *Rethinking Mathematical Necessity* (1990), that such a statement should perhaps be called quasi-necessary with respect to a conceptual scheme, the latter notion replacing "body of knowledge". It is still the case that we only change the truth values of quasi-necessary statements by replacing a whole theory, and for this to occur, we have to have access to a new theory. It is not possible to imagine what such a change would amount to in advance. It is not *intelligible* to propose that a given statement that we hold true can change its truth value. Here Putnam has become influenced by the later Wittgenstein's view on logic, a view that has roots in Kantian thought, that a change of the truths of logical laws is not an intelligible thought. Putnam uses this Kantian picture to say that a belief B cannot intelligibly be said to be a statement that can be revised, if not- B has not been confirmed, or if we cannot describe the circumstances under which such a confirmation could occur. In this way, Putnam views "statements" of arithmetic, such as $5+7=12$, as examples of truths to which alternatives at present we are unable to make any sense of; it is senseless to question the truth of $5+7=12$. On the other hand, he continues to speculate as to under what circumstances, if any, there could be a sense to $5+7\neq 12$, for instance, when we count electrons.

I find this Wittgenstein-inspired position, which Putnam takes from Cora Diamond and James Conant to be essentially mistaken; it creates a picture that makes it impossible to come to terms with mathematical necessity, in the sense that we take $5 + 7 = 12$ to express a necessary result of addition. But Putnam's use of Wittgenstein regarding the general function of our language is more successful, and ultimately leads him to give up the language/reality dichotomy that he claims has haunted philosophy since the 17th century.

In this thesis, the main topics are the notions of *meaning* and *necessity* in Putnam's works from the 1950s to the *Ethics without Ontology* (2004)² The main focus throughout is on certain important problems regarding his view of meaning, from his rehabilitation of Quine's denouncement of meaning and necessity up to his Wittgensteinian view of meaning as use.

2. Carnap and Quine

During the 1950s and 1960s, Putnam developed a realist metaphysics with respect to the claims of science, physics in particular, inspired to a large extent by Quine. Putnam has tried to modify some of the more extreme consequences of Quine's arguments, such as Quine's claim in *Two Dogmas of Empiricism* that there are no analytic truths whatsoever. In *Two Dogmas*, not even "all bachelors are unmarried" is regarded as an analytic statement, that is, a statement that holds true by definition, or by "virtue of the meanings of the words". An early Putnam (1957) took great pains to disprove Quine on this point, and won Quine's acceptance of this particular modification. Quine's views were considered quite extreme in the eyes of many philosophers in the 1950s, notably some Oxford philosophers. Putnam concurred that an example such as "all bachelors are unmarried" should be regarded as an analytic statement, but at the same time he felt that the critical Oxford philosophers supplied the wrong arguments. What was right in Quine's insights, according to Putnam, was that the notions of analytic truths or truths by stipulation, as used by the logical positivists, were of no substantial use in a logical reconstruction of science. In particular, our scientific descriptions of the world cannot be intelligibly formalized in frameworks with a factual or empirical part, and another part which constitutes the particular logic that is chosen by convention.

For Putnam, Quine's thesis that "no statement is immune to revision" is a principle which, correctly analyzed, should be a rejection of *a priori* truths (in the absolute sense) and not of analytic truths, which, in a trivial way, may be seen as substitution rules in the few cases they occur. In fact, Putnam views

²Including comments to this work together with replies by Putnam published in an issue of *Contemporary Pragmatism* in 2006.

Quine as subscribing to a picture that is sometimes attached to the logical positivists, that it is only the rejected *a priori* statements that would have been the analytic ones, had they existed. An *a priori* statement is verified no matter what experience tells us, and the view that Putnam thinks that Quine shared with the logical positivists is the mistaken view that an analytical truth has such a universal confirmation range. It should rather be the other way round: an analytic statement may be viewed as *a priori*, but not absolutely; it may be revised in the future. He agrees with Quine that there are no absolute *a priori* truths, in particular not within our scientific theories. This does not mean that there is nothing to a distinction between analytic and synthetic truths; “all bachelors are unmarried” may certainly be regarded an analytic truth. However, such statements play no significant role in the sciences, and here Quine was right. In fact, most statements in physical science are neither analytic nor synthetic, according to Putnam.

Warren Goldfarb has pointed out that it is wrong to attach the view of “universal confirmation range” of analytic truth to the logical positivists, at least in general. It certainly misrepresents the views of Carnap on analytic truths; these are regarded as constitutive for other claims in a scientific theory. In Carnap’s later works in particular, the analytical truths are arbitrary rules which constitute a linguistic framework. There are always two questions as to whether a statement such as “if one has as many fingers as toes, then the number of one’s fingers is the same as the number of one’s toes” is necessary (the example is borrowed from Burgess (2004)³), since we may on an external level always ask if it is appropriate to include the number concept. However, if we do add the number concept to our linguistic framework, then the statement is analytic. Quine and Putnam have difficulties in seeing two questions here. When Putnam considers mathematical necessity, he is stuck with a picture that all our well-formed expressions are fundamentally of the same type, in the sense that, for example, “ $2 + 2 = 4$ ”, “all bachelors are unmarried” and that “there is a deer on the meadow” are all propositions that may be true or false. But these three expressions are all of a different nature. The last one is an empirical statement, the model for a true or false statement. The analyticity of “all bachelors are unmarried”, I will argue, is due to the way we look at this expression as a calculus, comparable to the way we read $2 + 2 = 4$. This is Putnam’s insight here, I claim, but when he says that we may in the future revise the analyticity (the general truth) of “all bachelors are unmarried”, this does not mean that we could in this sense also revise $2 + 2 = 4$. There may be circumstances, a linguistic drift as it were, that could account for a change in the use of the words “bachelor”, “unmarried” and “man”, which may

³I would like to thank Sten Lindström for drawing my attention to this important paper.

make possible exceptions to “all bachelors are unmarried”. But we would no longer be adding 2 and 2, if we would get another answer than 4.

In the first two chapters, I present Putnam’s discussion of analyticity as well as some of his contributions to the topic of mathematical necessity, which are related to his views on *a priori* truth. In recent years, Putnam has been seen as a forerunner of a notion of a relative *a priori*, in particular after Michael Friedman’s work on this topic. I will compare Putnam’s notion of a relative or contextual *a priori* with that of Friedman’s version of a relative *a priori*, which is closer to Carnap’s and Reichenbach’s, since it is a notion of a constitutive *a priori*, which emphasizes conventional principles that frame the empirical part of a theory.

3. Internal realism

In the mid-1970s, Putnam gradually changed his mind about the nature of philosophy and its relation to the sciences. The position he took immediately following his scientific realist period is sometimes called *internal realism*, but later also pragmatic realism, or realism with a small “r”. These positions acknowledged the presence of cognitive equivalence between different scientific descriptions. For instance, within a physical theory we may use a description that employs individual space-time points, but we may also disregard the existence of these points and instead view such points as limits of sequences of spheres converging to such “points”. Behind this defense of a notion of *conceptual relativity* was a criticism of the view that there are fixed sets of objects and properties, as well as a fixed relation of correspondence upon which truth depends. Putnam became increasingly influenced by Wittgenstein, but was during this period to a large extent also influenced by Michael Dummett, culminating with his highly influential book *Reason, Truth and History* (1981). An essential part of the internal realism presented there is a view that truth is “warranted assertability in ideal epistemic circumstances”. Putnam defended such a position because he found it difficult to reconcile two pictures: the picture that language fixes the interpretation of words, and his model theoretic considerations that seemed to imply that there is a multitude of different realities with which our descriptions are consistent.

The notion of conceptual relativism presupposes that there are different uses of words such as “exist” and “object”, so that we may describe the same reality by saying that there exist three objects on the table, or that there are seven objects, if we count the (mereological) sum of these objects as well. There has been an extensive debate on the tenability of Putnam’s conceptual relativism. Davidson and Blackburn, for example, have criticized this notion. Blackburn views the difference between a description of whether we have three or seven objects as matter of difference of meaning of the words, i.e., that we use a more inclusive or more exclusive notion of “exist” in these cases. For Putnam, the everyday

use of the word “meaning” is not at home in scientific contexts, of which the mereological counting is intended as a toy example. The reason for not calling it a matter of change of word meaning is that Putnam wants to defend the view that we have two cognitively equivalent descriptions of the same reality, and that these descriptions are in conflict only with respect to their “surface grammar”.

The point is that we may describe the same reality by different uses of “exist” and “object” in such a way that we may have three objects on the table but also seven objects on the table, if we in the latter case choose to count the mereological sums as objects. That we obtain “three” and “seven” objects is just an apparent contradiction, and not of the same type as stating that there are three or four chairs in a room. Davidson and his followers do not like this explanation, because they want “there are three objects” in English to come out as “there are three objects” in any other natural language (say Swedish or French). We cannot have the answer “there are seven objects” by using standard translation practice, since, in Putnam’s interpretation of Davidson, we attach a particular meaning in our language to the word “object”.

Jennifer Case has helped Putnam to clarify that when we translate three objects into seven as in the case of translating our usual way of counting objects to the mereological one, we are using a (relative) translation of optional languages, rather than a translation in the usual way between natural languages. Case defends Davidson’s view that it is not because we have adopted a particular conceptual scheme that we may say such things as “there is a computer on the desk in front of me”, an example given by Putnam when he acknowledges this particular criticism. Putnam says that his notion of conceptual schemes was not explained well enough to avoid such misunderstandings. But Case thinks that Putnam is right to employ a wider notion of language, adding the *optional languages* to our natural languages, the latter including tables and chairs. For these optional languages, Putnam’s examples of conceptual relativity, formulated in terms that involves the notion of conceptual schemes, are correct. Here Case finds Davidson to be mistaken, in that he defends a too narrow view of language.

The conceptual relativity Putnam defends has ontological implications for the philosophy of mathematics. In Quine’s *On What There Is*, talk of numbers (*a façon de parler*) may be reduced to talk of sets, and these must then be assumed to exist, if we are not to evade the problem of existence in mathematics. However, sets may be replaced by functions, which again are connected to the notion of number, so there seems to be some redundancy here. Putnam’s view is that we may talk of numbers, and we may talk of sets, or of functions, but there is no concept here on which the other concepts rest. In fact, as Putnam argues in *Ethics without Ontology*, we may replace all existence talk in an equivalent way with modalities (i.e., that we only assert the mathematical *possibility* of certain structures), which is not to cover up ontological commitments as Quine thought.

The modal logic interpretation of mathematics is very much an example of conceptual relativity for Putnam. The equivalence between our ordinary way of talking about existence within mathematics and the modal logic interpretation shows that existence assumptions are not required at all in mathematics.

But Putnam does not stop here. In the *Ethics without Ontology*, he goes further than Case, criticizing Davidson for thinking that we may succeed in translating other languages into our own. The intelligibility of another language may be something we project onto this language. In fact, the speakers of the language may even adopt our way of understanding their language, thereby distorting the original sense. I argue that Putnam accepts Case's terminology of optional languages, but when he adopts Wittgenstein's notion of meaning as use, he also finds natural languages to be continuous with optional languages in the sense that words such as *Geist* in German and *mind* in English are hopelessly untranslatable into each other.

4. Meaning and sense

The verificationist (or anti-realist) position was later abandoned at the end of the 1980s, very much as a result of Putnam's study of Wittgenstein. I find this transition period particularly difficult to come to terms with in Putnam's thought, because he retains many ideas from earlier periods, and these are mixed with the new ideas in a problematic way. A particularly difficult work to evaluate is *Rethinking Mathematical Necessity* from 1990, where Putnam embraces a certain view of Wittgenstein's thoughts on logic in order to modify the Quinean picture that no statement is immune to revision. Putnam does not claim that there are certain statements that are immune to revision, but rather that there are certain statements that it is not *intelligible* to question, at least now, in the absence of an alternative theory. This picture of Putnam's is quite similar to his earlier one in *It Ain't necessarily So*, but he now wishes to emphasize the methodological (rather than psychological) impossibility of imagining any alternative. This is a view he continues to emphasize in the *Ethics without Ontology*.

It is not clear, however, that this new variant of a relative *a priori* truth is substantially different from his earlier view, since there is a sense in which everything still may be revised, even if we now find it unintelligible. One complication in Putnam's position is his reliance on a distinction between the lexical (or linguistic) meaning of a word, and its sense, the use of a word and how we understand it in a particular context, such as, for example, a technical scientific term. Putnam gives the example of the word "momentum", which he argues, did not change its meaning from Newton's mechanics to that of Einstein's; rather, the sense has changed, which illustrates Putnam's point that we were earlier unable to understand a statement such as "momentum is not the product of mass and velocity". Einstein's theory made sense of this statement.

I find Putnam's position here to be difficult to understand and evaluate. He retains a notion of "meaning", the lexical notion of meaning, that he finds to be distinct from the use- and understanding-oriented "sense", essentially "meaning as use" in Wittgenstein's later philosophy. I think that his main reason for defending a distinction between meaning and sense was that Putnam wanted to defend a realist position; it is important that the same phenomena are explained within the new theories. "Meaning" is external, in a way that was explained in *The Meaning of 'Meaning'* (1975). When Putnam later adopts Wittgenstein's notion of "meaning as use" to include both "meaning" and "sense", this is connected, I think, to giving up a language/reality dichotomy, which is a main theme of the 1994 *Dewey Lectures*.

In keeping the notion of "sense", albeit under the notion of "meaning as use" in the *Ethics without Ontology*, Putnam also retains the view that there are certain statements that we are not able to question intelligibly at a certain point in time, but which (perhaps) in future may have intelligible alternatives, in a new and different context. One recurrent example that Putnam gives, even in the *Ethics without Ontology*, is the example that we could earlier (say in the year 1700) not understand what it would mean to say that "there is a triangle whose angle sum is greater than 180° ", but that we now, given non-Euclidean geometry, can understand such a statement.

I will argue that it is misleading to associate such a change with the claim that it may now be unintelligible to question $2 + 2 = 4$, but, by analogy with the geometrical statement of the angle sum of a triangle, we may perhaps in the future come to see that there are exceptions to $2 + 2 = 4$. This view, expressly argued in *Rethinking Mathematical Necessity*, seems to be retained in *Ethics without Ontology*, now as an example of a conceptual truth that may be corrigible. My argument relies on a criticism of Putnam's treatment of mathematical calculations, such as $2 + 2 = 4$, as statements which have the property of being true or false, comparable to a statement such as "there is a triangle whose angle sum is greater than 180° ". Putnam does not distinguish between statements that may be true and false (in *our language*) and calculations according to calculation rules, but this is an important difference in Wittgenstein's philosophy of mathematics. It is not merely unintelligible to question $2 + 2 = 4$; the very idea of questioning a mathematical calculation is not applicable. This does not mean that mathematics as we practice it, whether in social contexts or in research, consists only of calculations. On the contrary, the very example "there is a triangle whose angle sum is greater than 180° " shows well that we may be talking of mathematics in an informal but meaningful and important way. In fact, the later Wittgenstein thought that mathematical "prose" is essential for the applications (and hence the meaning) of mathematics. Putnam has certainly understood the importance of

mathematical *propositions* as related to applications (and the important informal talk, as when we make conjectures). He argues, for example that

*Mathematical propositions wouldn't be propositions—that is, meaningful statements—if mathematics were not applied outside of mathematics.*⁴

This does not mean, however, that the *calculations* on which these prosaic assertions are based should be viewed as *propositions*. Such a view inevitably leads to untenable ideas about the revisability of mathematical calculations, so that one is led to suggest, as Putnam does, that to view alternatives to 4 as a result of the calculation $2 + 2$ is (merely) unintelligible.

A somewhat similar confusion appears in Imre Lakatos' *Proofs and Refutations* (1976), but in a different and stronger form than in Putnam's philosophy of mathematics. Lakatos attacks what he broadly identifies as "formalism" (which he thinks distorts the quasi-empirical *content* of mathematics) and Euclidean deductive methodology in mathematics. Lakatos suggests that mathematics is fallible in very much the same sense as the empirical sciences, and he is clearly influenced by Karl Popper in this respect. Lakatos suggests that the mathematicians should use the specific method of looking for counterexamples as their primary method of investigation. But Lakatos is unable to give a satisfactory argument for how proved theorems can have counterexamples. In particular he has no philosophy of language to account for such a suggestion.

In Putnam's philosophy of language, the sense (but not the meaning) of words may change; I argue that this way of reasoning has no application to calculations such as $2 + 2 = 4$, since these are not propositions. The similarity between Lakatos and Putnam consists in their refutation of a Carnapian view that calculations can be treated as rules that we may choose to adopt or not. In Putnam's case, I think this refutation has to do with his essentially Quinean picture of Carnap's project. For Putnam, Carnap's view on conventions became a position to keep at a safe distance, once he had become convinced that the notion of truth by convention was untenable. I think Putnam has always been too hostile to conventions, even if they do not presuppose the notion of analyticity that Carnap defended. This hostility to conventions is always beneath the surface as he argues for the interpenetration between fact and convention, or when he suggests that there are no incorrigible truths, not even the conceptual ones, such as $2 + 2 = 4$. For Putnam, early and late, essentially all "statements" are on a par, very much as in Quine's works, although Putnam has supplied important modifications to this picture.

⁴Putnam (2001), p. 188.

5. The Dewey Lectures

In the *Dewey Lectures*,⁵ Putnam proposes yet a new position, that of *natural realism* or *commonsense realism*. He is now influenced by his extensive study of Wittgenstein especially, but also by the American Pragmatists, such as William James, C.S. Peirce and John Dewey, as well as by the English philosopher J.L. Austin, in particular the latter's *Sense and Sensibilia* (1962). Here Putnam has developed a realistic view which is very different from what one normally finds in the spectrum between anti-realists and metaphysical realists. He still imports arguments from science (especially physics) and from mathematics, but his philosophical project is now very different from that of his early days, when he saw philosophy as continuous with science, contributing to a global scientific picture of our world. Philosophy is a distinctively humanistic discipline for Putnam in his later years, and he views science and its projects from a distance, often in a "critical" fashion. This passage from the *Dewey Lectures* is typical for the late period.

Today the humanities are polarized as never before, with the majority of the 'new wave' thinkers in literature departments celebrating deconstruction cum Marxism cum feminism [...] and the majority of the analytic philosophers celebrating materialism cum cognitive science cum the metaphysical mysteries just mentioned. And no issue polarizes the humanities—and increasingly, the arts as well—as much as realism described as 'logocentrism' by one side and as the 'defense of the idea of objective knowledge' by the other. If, as I believe, there is a way to do justice to our sense that knowledge claims are responsible to reality without recoiling to metaphysical phantasy, then it is important that we find that way. For there is, God knows, irresponsibility enough in this world, including irresponsibility masquerading as responsibility, and it belongs to the vocation of the thinker, now as always, to try to teach the difference between the two.⁶

Under attack in the *Dewey Lectures* is the prevailing adherence within analytic philosophy to what Putnam calls "Cartesianism cum materialism", a position that in a quite unreflected way combines two pictures:

(1) That perception involves an interface (impressions, sensations, experiences, sense data, qualia) between the mind and the external objects perceived.

⁵Re-published as Part I of *The Three-fold Cord* (1999).

⁶Putnam (1999), p. 4.

(2) The interface consists of brain processes.⁷

This view, to which Putnam says he adhered to up to and including his period of internal realism, makes our contact with the world a mystery. This picture sustains, for example, a view held earlier by Putnam, that brain inputs are *caused* by an external reality. These inputs are treated by us in language, detaching notions such as “meaning as use” from reality. Such a picture is in line with a use-oriented notion of meaning found in “cognitive science”, rather than within a serious philosophy of language. As I argued above, I think that Putnam’s defense of a distinction of “meaning” and “sense” is part of such a picture, which I refer to as a language/reality dichotomy. According to Putnam’s view in the *Dewey Lectures*, we have to acknowledge that thought and language connect us with reality. Of course, one has to do the philosophical work of defending such a view and not just claim a direct realism (as he says that Searle has done) by making a “linguistic reform” to the effect that instead of saying that we *perceive* visual experiences, we *have them*.⁸ The philosophical work in this direction has been accomplished by Wittgenstein and Austin according to Putnam.

Here language is no longer viewed as merely providing representations of the world (or “copies” in the crudest versions of such a picture). Language is part of our interactions with the real world, and is not distinct from other ways of coping with our world, ways that we share with other animals, although our human language alters the range of experiences we can have.⁹ In particular, language is not merely “marks and noises” into which interpretations have to be read. Putnam writes:

When we hear a sentence in a language we understand, we do not associate a sense with a sign design; we perceive the sense *in* the sign design. Sentences that I think, and even sentences that I hear or read, simply do refer to whatever they are about—not because the ‘marks and noises’ that I hear (or hear ‘in my head’, in the case of my own thoughts) intrinsically have the meanings they have but because the sentence in use is not just a bunch of ‘marks and noises’ (*Philosophical Investigations* 508).¹⁰

The view expressed here does not fit well with the usual debate between realism and anti-realism that has been structured by Michael Dummett, who places Wittgenstein in the anti-realist camp. In fact, Wittgenstein is usually misinterpreted (on this view) as an extreme anti-realist (a “full-blooded” conventionalist),

⁷Ibid., p. 43.

⁸Ibid., p. 10.

⁹Ibid., p. 48.

¹⁰Ibid., p. 46.

because of his arguments against the view that there must be something that constitutes correctness in our following of rules, such as the rule of addition. There are no mental or Platonic entities that guarantee that rule-following is always done correctly, for instance, for very large numbers. This does not mean that Wittgenstein is suggesting that we have to make a decision for every particular application of addition. Neither are we in need of a communitarian notion of practice that takes the place of these other Platonist or mentalist justifications.

To ask whether the rule itself is correct is meaningless. Our possibilities of following the addition rule (so that we for numbers too large to check by counting may rely on it) is something that occurs in our (ordinary) language, where there is no need for a philosophical justification of what we say. But to clarify that is no trivial matter, and it is part of Wittgenstein's famous rule-following paragraphs in the *Philosophical Investigations* to show that our way of expressing ourselves in *our* language is in no need of justification, as Putnam also acknowledges in the extensive treatment of rule-following in *Was Wittgenstein Really an Anti-realist about Mathematics?*

But what does it mean that "our language is in place" and that we are in no need of philosophical justification for what we say? One example that Putnam discusses himself in the paper just mentioned is the twin prime conjecture, the hypothesis that there are infinitely many pairs of prime numbers p , so that $p + 2$ is also a prime number. We have as yet no proof or disproof of this conjecture. Could we then say that it is either true or false? Putnam's answer is yes, and here I agree with Putnam. What other alternatives are there? The conjecture is stated in *our* language, in its "prose", and it is not part of any calculus. Dummett, on the other hand, rejects the principle of bivalence, that is, that a statement, in mathematics for example, is either true or false. This is why his position is included among the verificationistic ones: a statement is meaningful if we know what it means to have a proof of it. But his view of our language is also one of a calculus, one could say.

A proof or disproof of the twin prime conjecture may be surprising to us, but we should remember that a result on this conjecture in either direction would have us saying in our ordinary (prosaic) language that the conjecture is true or false. *Truth* is no abstract mathematical notion here; a statement is either true or false *in our language*. These are correct conclusions, arrived at by Putnam, but he does not connect these arguments to a calculus/prose distinction, but refers instead to Stanley Cavell's talk of "the ordinary".¹¹ That is, Putnam bases his insight that statements in our language, such as mathematical conjectures, are phrased in our ordinary language (which should not be viewed as the language

¹¹Putnam (2001), p. 176.

of “the ordinary man”). Cavell’s point is that we use ordinary talk as mathematicians within mathematics, talking about the Riemann zeta functions and what not. But simply to refer to “the ordinary” in this way, as Putnam does, makes it difficult to explain why there is no harm in speaking as we (in particular the mathematicians) do, in that no metaphysical assumptions of bivalence are made when we use the ordinary talk of stating conjectures or looking for counterexamples. It is important to realize that ordinary talk within mathematics is phrased in the “prose” of mathematics, but that does not mean that when the mathematicians do this, they are (entirely) confined to this ordinary language or “prose” when they do mathematics; they *work* with a “calculus”.

I do not think that Carnap, as he is portrayed by Putnam, would object to what has been said here regarding a statement being either true or false, not because he would appreciate that the formulation is made in *our* (ordinary) *language*, but rather because it is a matter of linguistic convention to say that either we have p or not- p . That is, Carnap would turn the tables and emphasize the calculus part only, and save practices by saying that the twin prime conjecture is either true or false by recourse to the linguistic conventions we make. Carnap’s picture certainly has its merits, but the problem with his global philosophical view is that one would like to know from which position these conventional logical choices are made. These are the external questions in Carnap’s scheme, which are formed by pragmatic considerations and employ the principle of tolerance, a principle Putnam thinks redundantly presupposes the verification principle and, more generally, a dogmatic empiricism. I will discuss this problem, but in general I will refrain from taking a definite stand on the interpretation of Carnap’s philosophy, since my investigation is confined to the philosophy of Hilary Putnam and to connections he makes to other philosophers, e.g., Carnap, but not of Carnap’s philosophy as such.

The merit of Carnap’s philosophy is that he understands mathematical necessity well, that $2 + 2 = 4$ may be viewed as an application of the stipulated rule of addition. The problem may be that he does not see that mathematics also contains the informal talk and pictures of our ordinary language and thus that it is impossible to reduce or formalize this important part of mathematical activity into a logical apparatus.

One could very briefly summarize the views of Putnam, Wittgenstein and Carnap as follows (a triangulation as it were): Carnap understands and appreciates the level of rules and calculations, and Putnam the level of our prosaic language, while Wittgenstein’s position takes care of both levels of our mathematical activities. It is true, as I have said, that Carnap does distinguish between external and internal questions, referring to the pragmatic choices of linguistic schemes we make and these schemes themselves, respectively, but I think that Putnam is correct in his criticism that we do not understand what the external

is if it is indeed not cognitively empty, a mere reformulation of the principle of verification.

It is important to understand that for Putnam, following Wittgenstein, notions such as “proposition” and “true” are not technical notions in a mathematical or logical calculus. Putnam explains well how Wittgenstein in the *Philosophical Investigations* is no deflationist when he says that saying that ‘*p*’ is true is the same thing as saying *p*, for a particular proposition (*Satz*) in *our language*. The point is not to define “truth” or “proposition” in terms of the other notion, assuming one of these to be a primitive notion. Putnam writes that “we do not recognize that something is a proposition by seeing that it ‘fits’ with the concept of ‘truth’, where truth is conceived of as a freestanding property”.¹² Neither can we explain truth by saying that for any proposition *p*, ‘*p*’ is true = *p*. The deflationist interpretation of Wittgenstein is an interpretation that bases (and usually accepts) formulations such as ‘*p*’ is true, as the same as *p*, on a formalized view of language, very much in the spirit of Carnap.

In view of the *Dewey Lectures*, Putnam would now be equipped with a different picture of language, one which allows for “conventional” talk *in our language*, something entirely different from the (formalized) notion of convention for which he has criticized Carnap throughout his career. And indeed Putnam employs such a different notion of convention in *Ethics without Ontology*. His view of a convention is that it is merely a solution to a coordination problem,¹³ such as when we decide to drive on the left *or* right hand side of the road. He connects this view of convention to what he views as a convention of deciding whether or not “mereological sums” exist. This puts something of a new twist on his notion of *conceptual relativity*, but Putnam stresses that no metaphysical notion of analytic or *unrevisable* truth is involved here.¹⁴ A little later in the same work, he concludes that his notion of *conceptual truths* (of which “ $2 + 2 = 4$ ” is an example) presupposes neither analyticity nor unrevisability, an insight he finds in Hegel and the American pragmatists.¹⁵ I think a remaining problem for Putnam is that he views $2 + 2 = 4$ as a (true) *proposition* rather than an application of a *rule* of arithmetic. Although he succeeds in doing justice, to *our language*, instead of reducing it to a calculus, he seems to assume that any notion of a calculus must necessarily rely on some metaphysical conception of unrevisable truth, a notion which, according to Putnam, Quine destroyed. The problem is that the notion of truth does not belong here since we are no longer talking about “propositions”, not even in our language. I think Putnam could very well keep his “commonsense realism” if he just took the reality of $2 + 2 = 4$ to be an

¹²Putnam (1999), p. 67.

¹³Putnam (2004), p. 44.

¹⁴Ibid., p. 44.

¹⁵Ibid., pp. 60–63.

acknowledgement that the left hand side may be transformed into the right hand side, in accordance with the rule of addition.

6. Ethics and mathematics without Ontologies

I have already mentioned that Putnam proposes that mathematics is in no need of existence assumptions, since we may choose to express every existence assumption in mathematics with an equivalent modal way of talking, which should not be seen as a foundation of mathematics, but simply an equivalent way of speaking. But this denouncement of “existence talk” in mathematics runs deeper in Putnam’s philosophy.

Putnam’s has recently become an advocate of a “third enlightenment”, arguing against currents in contemporary continental and analytic philosophy that are hostile to Enlightenment ideas. This is a central theme of the *Ethics without Ontology*. A central question in this important albeit somewhat sketchy book is the clearly emphasized view that Ontology with a “capital O” cannot be sustained in ethics or in the philosophy of mathematics. Putnam goes through ontological positions such as inflationary ones, where it is assumed that there is something that guarantees moral truth or mathematical truth, e.g., Platonism, G.E. Moore on the *good*, Hegel on *history* and Quine in *On What There Is*. Putnam writes that Quine’s paper “had disastrous consequences for just about every part of analytic philosophy.”¹⁶ Putnam is equally hostile to deflationary positions, such as reductionism (“A is nothing but B”) and eliminationism (that we are talking about mythical entities). Instead of choosing between such alternatives, Putnam defends a *pragmatic pluralism*,

the recognition that it is no accident that in everyday language we employ many different kinds of discourses, discourses subject to different standards and possessing different sorts of applications, with different logical and grammatical features—different “language games” in Wittgenstein’s sense—no accident because it is an illusion that there could be just one sort of language game which could be sufficient for the description of all reality! [...] [P]ragmatic pluralism does not require us to find mysterious and supersensible objects *behind* our language games; the truth can be told in language games we actually play when language is working, and the inflations that philosophers have added to those language games are examples, as Wittgenstein said—using a rather pragmatist turn of phrase—of ‘the engine idling’.¹⁷

¹⁶Ibid., p. 2.

¹⁷Ibid., p. 22.

Although Putnam has embraced a Wittgensteinian view on many philosophical issues, I find his attachment to the New Wittgenstein school somewhat problematic. I am thinking in particular of his appreciation of the interpretations of Cora Diamond and James Conant. I think one problem is Diamond's influence on Putnam's idea that certain "statements", the truth of which we do not question, rely for their truth on the unintelligibility to questioning them. But even Conant's view is wrong, i.e., that mathematical "truths" such as $2 + 2 = 4$ are true because they fit our descriptions and "we cannot do without them" (on his interpretation of Wittgenstein), a *pragmatic* motivation for the truth of $2 + 2 = 4$. I will now describe some of the deeper problems that motivated me to engage in Putnam's thought in the first place, which will lead me to criticize part of Putnam's use of Diamond's metaphors "the face of meaning" and "the face of necessity", in particular in the way these notions are applied to do justice to the necessity of arithmetic calculations.

7. A philosophical problem

There are many good reasons to read Putnam. I have myself become increasingly inspired by Putnam's thought, although it was a mere coincidence that I began to read his works systematically in the Fall of 2007 at Amherst College. He is an excellent writer who establishes close contact with his reader in a way that gives one a sense of joining him on a philosophical journey. His openness to new ideas I find important and encouraging. These are reasons alone to read Putnam, who has been one of the central figures of analytic philosophy for more than 50 years. But I had very specific reasons for reading Putnam, having to do with my own philosophical problems.

One such problem was: is it possible to apply Kuhn's theory of paradigms and/or revolutions in science to mathematics? Is there an "incommensurability" between mathematics as we practice it now and in antiquity? In *Reason, Truth and History* (1981), Putnam criticizes both Kuhn and Feyerabend and the notions of incommensurability they defend. Of course, one way of solving my problem of incommensurability between mathematical theories over time is to reject the view that there is any intelligible notion of incommensurability at all, a position that Putnam held for a long time. More recently, in the *Ethics without Ontology*, he has defended a more complicated Wittgensteinian view of the difficulties involved in understanding another language in terms of one's own in a way that does not distort what was originally said in the other language, but I have not emphasized this line in my investigation.

A similar philosophical problem has arisen in discussions within the historiography of mathematics since the 1970s. This version of the problem is sometimes referred to as the "Unguru debate": did the Greeks have an algebra? During the second half of the 1970s, there was a debate in the *Archive for History of Exact*

Sciences, a journal which is mainly devoted to the history of mathematics, concerning whether one could rightly say that the Babylonians and the Greeks had an algebra, which in the case of Greek mathematics was hidden behind a geometrical veil. In his paper *On the Need to Rewrite the History of Greek Mathematics* (1975), Sabetai Unguru claimed that the received opinion, which was based on research by Tannery, Zeuthen, Neugebauer and van der Waerden, was incorrect and based on an anachronistic reading of ancient texts, in which they were translated into a modern algebraic notation; hence algebra is imposed on the Greek texts and not discovered in them. Unguru writes the following about the use of modern symbolism in an example from Euclid's *Elements*:

History? Perhaps, but certainly not sound, acceptable history. It is rather 'logical history', i.e., in more cases than not, *non-history*. *It is history as it should be* rather than an honest attempt to establish it as it was; it is, in other words, a logical rather than a historical reconstruction.¹⁸

It is today sometimes regarded as a scandal in the field of history of mathematics that the leading mathematician André Weil was allowed to dismiss Unguru's critical paper in his own "Who Betrayed Euclid (Extract from a letter to the Editor)" (1978), by accusing Unguru of not knowing enough mathematics and appealing to his own authority and claiming, without much justification, that Euclid just used a somewhat cumbersome notation in his algebra.

In later years, the debate has gone beyond either saying that it is obvious that the Greeks had an algebra, or the equally crude way of claiming that they did not. Of course, they did not have an algebra in the sense that we have simple symbolic treatments of the "corresponding" cumbersome geometrical versions in Euclid's *Elements*. But one problem here is what we mean by the "corresponding" theorem. Should we say that they proved something different, or should we say that they proved the same theorem with different means?

My treatment of Putnam's criticism of verificationism in mathematics, essentially the view that mathematical truth cannot be accounted for by mathematical proof, is connected with this problem. Putnam defends Cora Diamond's reading of Wittgenstein that goes against the view that we prove *different* theorems when we prove a theorem employing modern algebra or by classical geometrical means, that is, by employing different methods of proof, or simply different proofs. Diamond suggests a face-metaphor, giving rise to the two notions "the face of necessity" and "the face of meaning". In Putnam's view, we may understand that two different proofs prove the same theorem, just as we may recognize

¹⁸Unguru (1975), p. 92

a face,¹⁹ that is not through identifying the parts piece by piece, such as a particular shape of the mouth and the eyes, etc., i.e., not by identifying the statement of a theorem with a particular proof of it. I find this claim quite reasonable, but it is important that this recognition of “same theorem” is based on a “norm”, in the sense that it refers to something outside the calculus part of mathematics. What I mean by this is that I find it to be very similar to the case of saying “there is a triangle with an angle sum greater than 180° ” outside the calculus part, which in turn may make this a true statement. Putnam, however, wants to explain even the necessities of arithmetic, such as $2 + 2 = 4$, through the face metaphor in the third part of his *Dewey Lectures*,²⁰ an explanation which conflates calculus with prose.²¹ I will argue that Putnam certainly has the resources to mend these problematic parts of his philosophy of mathematical necessity, which are based on a too excessive application of Diamond’s notions of “the face of meaning” and “the face of necessity”.

In general, I try to give a positive account of Putnam’s later philosophy, and I defend him on several points. In particular, I defend his notion of conceptual relativity. But I focus a lot of attention to criticism also, since I think that one could supply Putnam’s solutions with improved answers, within the reach of Putnam’s own philosophy of language.

8. Summary

In **Chapter 1**, I introduce the early Putnam’s views of the relative *a priori* principles, that is, principles that are *a priori* relative a body of knowledge, and I contrast this picture with Friedman’s more recent emphasis on a constitutive *a priori*, roughly in the spirit of Carnap. I also comment on Tsou’s recent work on Putnam’s views (also in comparison with Friedman’s). I continue with a longer background to the views of Carnap and some fellow logical positivists of the Vienna Circle, as laid out by Goldfarb. Then I plunge into the depths of Quine’s *Two Dogmas of Empiricism*, paying special attention to Putnam’s accounts of Quine’s views, in order to better explain Putnam’s own position. In addition, I comment on Friedman’s reaction to Quine’s epistemological stance.

In **Chapter 2**, I explain Putnam’s position under the influence of the Conant/Diamond reading of Wittgenstein, where the revision of a true statement is unintelligible if we have no methodological tools available to explain such a revision. To this end, I contrast Putnam’s new views with those of Quine’s. I explain Putnam’s use of two notions of meaning: a linguistic or lexical notion

¹⁹Putnam (2001), p. 156.

²⁰Putnam (1999), p. 63.

²¹In *Was Wittgenstein Really an Anti-realist about Mathematics?*, these applications of the face metaphor are reiterated. (Putnam (2001), p. 184.)

of meaning, and *sense*, connected with the specific use of a word. I criticize Putnam's analysis, an analysis that winds up in the claim that not even $5 + 7 = 12$ may escape revision if physical theory demands this, in the sense that we may have to accept that, in an imagined example from quantum mechanics, $5 + 7 \neq 12$, in spite of his view that such a claim is unintelligible. The problem with Putnam's New Wittgenstein approach is that he wants to reject Quine's epistemological point of view, but has very little or nothing to offer in its place.

In **Chapter 3**, the main theme is Putnam's development with respect his view of *meaning*, in particular in the context of a central tenet of his middle and later philosophy, i.e., his *conceptual relativism* and later additions of *conceptual pluralism*. I discuss and essentially defend Putnam's notion of conceptual relativism against the critiques of Davidson, Blackburn and others, who think that conceptual relativism can only be understood such that we change the meaning of our words, such as "object" and "exist", and that the different employment of these words then gives us non-equivalent descriptions of our world. Goodman has a version of this view in which he prefers to say that we obtain different worlds instead. I argue that Putnam's notion of conceptual relativism, of cognitively equivalent descriptions of the same world, is possible by pointing to different *uses* of the words, which should not be viewed as employing different "meanings" in the sense that "table" and "chair" are words with different meanings. This is connected to Putnam's eventual assimilation of Wittgenstein's notion of "meaning as use".

In **Chapter 4**, I discuss the important *Dewey Lectures* with particular emphasis on Putnam's acceptance of Wittgenstein's meaning as use, but also on his analysis of "proposition" (*Satz*) and truth and how these words are dependent on each other *in our language*. I present Putnam's arguments against viewing Wittgenstein as an anti-realist, for instance in the case of rule following, an analysis correctly laid out by Putnam. I also discuss Putnam's deployment of Diamond's metaphors "the face of meaning" and "the face of necessity" and argue here that these metaphors have a point in connection with understanding the same mathematical theorem through different proofs, but that Putnam wants to "explain" too much through these metaphors, in particular he uses these metaphors to give a general description of mathematical necessity. This move starts from his view of expressions such as $2 + 2 = 4$ as propositions, conflating in general mathematical calculus with prose. This is connected to the topics of Chapter 1 and 2, and I conclude with a discussion of the difference between revising a *proposition* in our language such as "the angle sum of a triangle is 180° " and an expression of a calculation, such as $2 + 2 = 4$. However, I find Putnam's later philosophy of language to be essentially correct, and my criticism here should be viewed as an attempt of improving Putnam's philosophy regarding mathematical necessity.

In the **Appendix**, I offer another example of the problem of mathematical prose in connection to certain views of mathematical revisability, namely Lakatos' *Proofs and Refutation*.

CHAPTER 1

Putnam, Quine and Carnap

In this chapter, I discuss the early Putnam's ideas on analytic truth, *a priori* truth, and problems involved in the application of those terms to describe scientific theories. Putnam developed a notion of relative *a priori* truth. In a scientific theory there may be statements that are necessary relative a body of knowledge. These necessities may be replaced, according to Putnam, if we find a theory that explains phenomena better. This means that the total body of knowledge is replaced.

Today there are other competing ideas on relative *a priori* truths, notably Michael Friedman's. These ideas, however, are inspired by the much earlier work of Rudolf Carnap and Hans Reichenbach. In particular, there has been a revival of interest in Carnap during the last two decades. Thus it can be illuminating to compare Putnam's notion of relative *a priori* truth with Friedman's, as well as Carnap's, whose work has been an important source of inspiration to Friedman. Carnap was so severely criticized by W.V. Quine, and later also by Putnam, that his philosophical contributions were largely disregarded, or considered important only as a historical artifact, of little or no value for current philosophical concerns. Quine's attack on Carnap's notion of analytic truth was particularly devastating. Putnam supported Quine's criticism of Carnap, with slight modifications, and both Quine and Putnam became hostile to any notion of truth by convention.

These considerations alone make it worthwhile to reconsider the arguments leveled by Quine and Putnam against Carnap. I begin with the early Putnam's notion of necessity relative a body of knowledge. I proceed to describe Friedman's notion of a relative *a priori*, which differs from Putnam's in that the *a priori* truths are viewed as presuppositions for a physical theory. Friedman attributes a similar view to Carnap, in particular in *Logical Syntax of Language* and in the later *Empiricism, Semantics, and Ontology*, where Carnap distinguishes between external and internal questions. This distinction makes it possible for Carnap to separate the external question of whether or not a rule should be adopted from the internal question of following such a rule. Carnap made a distinction between analytic and synthetic statements. The analytic statements are the linguistic or logical rules of the particular framework that is set up in order to consider factual, or synthetic, statements. Quine found the analytic-synthetic

distinction untenable, since he rejected the very notion of analyticity. Quine had two types of arguments for this. One was that the attempt to base analyticity on synonymity was simply begging the question; synonymity is not better explained than analyticity. The other argument was motivated by the history of scientific revolutions. In Quine's view we should have learnt by now that "no statement is immune to revision". Putnam examines Quine's arguments, and arrives at a position in which he saves a small but rather unimportant class of analytic statements, such as the classical example "all bachelors are unmarried". He also argues that Quine conflates *a priori* truth with analytic truth, since the slogan "no statement is immune to revision" has nothing to do with the notion of analyticity, but is rather a statement that is equivalent to saying that there are no statements that are *a priori* true in an absolute sense. In fact, some of the logical positivists held the view that a tautology is a statement that has a "universal confirmation range", a view which can be seen to be adopted by Quine in his argument that there are no analytical truths, since "no statement is immune to revision". According to Putnam, Quine, like the logical positivists, turns the question of analyticity into an epistemological matter. Carnap, however, did not hold such a view, in particular not in his later philosophy, when he distinguishes between external and internal questions, as Goldfarb has shown. I argue that both Putnam and Quine have an epistemological view regarding what I would like to call "well-formed formulas". Putnam defends the possibility of analytical statements, but he does not think that they play any decisive role; further, for the young Putnam especially, "no statement is immune to revision". As we will see in the following chapters, this is a position that he will continually re-examine and modify, but which, I argue, he never abandons.

I will also consider a debate on the nature of Carnap's external questions. On this level, we choose the linguistic rules or the logical system we want to employ within our linguistic framework. It is natural then to ask what status our considerations have on this external level. Carnap's answer is that we use the pragmatic principle of tolerance in selecting our logical system. The point is that we are free to choose any logical system (even an inconsistent one) when we construct our framework. Putnam has objected that Carnap's principle of tolerance presupposes the principle of verification. The principle of verification is therefore not optional. Thus the problem of how we should justify the verification principle arises. Putnam argues that it certainly cannot be supported by its own standards of justification. Thomas Ricketts has rebutted Putnam, who has in turn responded to Ricketts' critique. I have borrowed extensively from Goldfarb's invaluable paper, *The Philosophy of Mathematics in Early Positivism* (1996), which elucidates the philosophy of Carnap and the Vienna Circle in my discussion here. Carnap's system proves to be far more sophisticated than usually acknowledged. Nonetheless, Putnam's criticism concerning the intelligibility of

the external level seems justified. Carnap seems to be positing a position in which we are free of any ontological or epistemological commitments. One could of course interpret Carnap's project limited to the reconstruction of scientific theories, rather than as a grand philosophical vision. In any case, there are a number of philosophical questions raised by Putnam's notion of relative *a priori* for future considerations, since there seems to be problems with Putnam's position, in so far that he has difficulties to separate the questions of choosing and applying a rule. The problem with Carnap's model, on the other hand, is that it seems to introduce a metaphysical position in positing an "external" level.

In this chapter, I have used two of Putnam's fundamental early papers, *The Analytic and the Synthetic* (1957) and *It Ain't Necessarily So* (1962). I have also used Quine's *Two Dogmas of Empiricism* (1951) and Putnam's *Two dogmas revisited* (1976). I have made extensive use of Friedman's book, *Dynamics of Reason* (2001) as well as Goldfarb's paper mentioned above.

1. The relative *a priori*

In this section I will focus mainly on the early Putnam to set the background. I refer to views published in the two collections of papers *Mathematics, Matter and Method* (Putnam, (1975)[1979a]) and *Mind, Language and Reality* (1975)[1979b], including the papers *It Ain't Necessarily So* (1962) and *The Analytic and the Synthetic* (1957)[1962].

Let us begin with Putnam's overview of his main ideas in the Introduction to the collection *Mathematics, Matter and Method*. Although Putnam's views underwent changes during the 15 year period in which the papers were written, he claims that there is a unity. In particular, there are four major themes:

- (1) Realism, not just with respect to material objects, but also with respect to 'universals' as physical magnitudes and fields, and with respect to mathematical necessity and mathematical possibility (or equivalently between mathematical objects).
- (2) The rejection of the idea that any truth is absolutely *a priori*.
- (3) The complementary rejection that 'factual' statements are all and at all times 'empirical', i.e., subject to experimental or observational test.
- (4) The idea that mathematics is not an *a priori* science, and an attempt to spell out what its empirical and quasi-empirical aspects really are, historically and methodologically.¹

"Realism" is for Putnam connected to mathematical necessity in that we have to be "realistic about the objectivity of mathematical necessity and mathematical possibility (or equivalently about the existence of mathematical objects),"² and

¹Putnam (1975)[1979a], p. vii.

²Ibid., vii–viii.

it is clear that Putnam's view on the relationship between mathematics and the physical sciences is deeply influenced by Quine. In no area, not even in classical logic, do we find any absolute a priori truths, in the sense of, for example, Descartes' 'clear and distinct ideas'.³

There are two issues which, though not independent, have been major philosophical problems for Putnam since the early 1960s:

- (1) What does it mean (if anything) to say that a statement is unrevisable?
- (2) Does the overthrow of a theory like Euclidean geometry consist only of a change in the meaning of the words? (In the sense of logical positivism.)

Putnam's views have changed over the years as regard the first of these intertwined questions, but regarding the second, his answer has always been an unrevised *no*. In fact, in *It Ain't Necessarily So*, Putnam argues at length that it is not because the meaning of *straight* or *straight line* has changed that we now regard it as possible to travel in a straight line (in the universe) and return to the starting position. It is rather an overthrow of a conceptual system, a total body of knowledge, that took place during a longer period of time (1815-1915), completed by Einstein's general theory of relativity. Einstein's new physics enabled us to make sense of "returning to the starting position as we travel in a straight line", because close to the sun the influence of gravitation would make "the shortest path between two points" a geodesic in a Riemannian sense. Any attempt at saying that this is not what I mean by a straight line will be confusing. Putnam says he would embarrass anyone trying to say that he means something different by a straight line in the vicinity of the sun.⁴ There is no external Euclidean reference system in which Einsteinian space is imbedded; our space affected by gravitation is the only space there is.

Hence, it is not a change of the meaning of the words that is involved when we now, in the wake of the of non-Euclidean geometry, say for example that there is a triangle whose angle-sum is greater than 180° . It is not a matter of changing the meaning of triangle, because it would be trivial to find another geometrical object, let's say a square, and say that "I also include this figure among my triangles and it has an angle sum greater than 180° ". In *It Ain't Necessarily So*, Putnam takes great pain to try to make the reader see what is involved in the overthrow of Euclidean geometry. It is not that we have defined a triangle on the sphere and have a conception of angle which allows for its computation a value larger than 180° . Rather, it is a matter of having to give in to the pressure from

³Ibid., p. x.

⁴Ibid., p. 242.

physical theory (Einstein's general theory of relativity) that we now have to regard this object on the sphere as a genuine triangle. We have not changed, e.g. by stipulation, the meaning of a triangle; we have rather been forced to extend the concept to a context which was previously alien to us. Euclidean geometry is no longer a preferred geometry through which we view toy examples, such as something that looks like a triangle on a sphere (imbedded in Euclidean space).

For the 1962 Putnam, the *a priori* status of Euclidean geometry was due to its unique position within a body of knowledge. It was a theory of the world, but one that could not be questioned within a certain body of knowledge. The overthrow of Euclidean geometry does not consist in the early 19th century investigations of the independence of the parallel postulate from other working geometries, but rather in that Euclidean geometry is no longer part of a true description of the world. Putnam finds the received view on this to be a "philosophical scandal".⁵ Putnam writes:

The received view is that the temptation to think that statements of Euclidean geometry are necessary truths about actual space just arises from confusion. One confuses, so the story goes, the statement that one can't come back to the same place by traveling in a straight line (call this statement 'S'), with the statement that S is a theorem of Euclid's geometry. That the axioms of Euclid's geometry imply S is indeed a necessary truth; but there is all the difference in the world between saying that S would be true if space were Euclidean, and saying that S is necessarily true. To put it bluntly, I find this account of what was going on simply absurd.⁶

Putnam argues that even after the work of Lobachewsky, we still would have been entitled to think that it is impossible to travel in a straight line in space and return to the origin.⁷ In fact it was, say in 1800, a necessary statement, relative a certain body of knowledge, that one cannot return to the origin by traveling in space in a straight line. Putnam remarks that by saying that a statement is necessary relative a body of knowledge,

we imply that it is included in that body of knowledge and that it enjoys a special role in that body of knowledge. For example, one is not expected to give much of a reason for that kind of statement. But we do not imply that the statement is necessarily *true*, although, of course, it is thought to be true by someone whose knowledge that body of knowledge is.⁸

⁵Ibid, p. x.

⁶Ibid., p. ix.

⁷Ibid., p. ix.

⁸Ibid., p. 240.

Putnam brings up a second example, probably in order to convince those unconvinced that we have not changed the meaning of “straight” or “straight line”. As we travel along any *path* in Riemannian space (a physical space with the geometric properties of Riemannian geometry, as in the general theory of relativity), we can visit at most finitely many *places*, where these are designed to be of a certain volume, and this is also an unexpected turn in the overturn of Euclidean geometry, in which we would hold that we can visit infinitely many places in this sense. Since we cannot say that we have changed the meaning of “path” or “place”, it follows that we have a statement which was previously (given another body of knowledge) a necessary statement, i.e., that we can visit infinitely many “places”, and this is now seen to be a false statement (given a new body of knowledge, a new theory). Those who are unconvinced that we have retained the meaning are asked by Putnam to show us where these other places are.

These are the formulations of the 1962 publication *It Ain't Necessarily So*, but Putnam changes his formulations concerning mathematics in *Mathematics, Matter and Method*. In *It Ain't Necessarily So*, he wrote that he was considering “synthetic necessary truths” and that “the general area of necessary truths might be broken up [...] into three main subareas: the area of analytic truths, the area of logical and mathematical truths, and the area of ‘synthetic *a priori*’ truths”.⁹ According to Putnam, Euclidean geometry as a theory of physical space was always a synthetic theory. In the Introduction to *Mathematics, Matter and Method*, Putnam now says that the principles of *mathematics* are relatively *a priori*:

[I]n mathematics too there is an interplay of postulation, quasi-empirical testing, and conceptual revolution leading to the formation of contextually *a priori* paradigms. Mathematics is not an experimental science; that is the first thing that every philosopher learn. Yet the adoption of the axiom of choice *was* an experiment, even if the experiment was not performed by men in white coats in a laboratory. And similar experiments go all the way back in the history of mathematics. Of course, mathematics is much more *a priori* than physics. The present account does not deny the obvious difference between the sciences. [...] In particular, we can recognize that principles of mathematics are relatively *a priori*, without having to conclude that they are conventions or rules of language and therefore say nothing. In particular we do not have to choose between Platonism—the view that mathematics is about objects of which we have *a priori* knowledge—and Nominalism—the view that mathematics is not about real objects, that most of it is just make believe.¹⁰

⁹Ibid., p. 237.

¹⁰Ibid., p. xi.

Here we clearly see that Putnam insists on refuting Carnap's conventionalism regarding mathematics. We also see that Putnam's *realism* does not presuppose mathematical objects.

Tsou (2010) has recently analyzed Putnam's view of the *a priori* as a reaction to Quine's refutation of analytic sentences (and later of *a priori* sentences). Tsou has concluded that Putnam's attempt is still within a kind of Quinian setting, since, among other things, he in 1962 (the paper was written in 1957) in *The Analytic and the Synthetic* stresses the "monolithic character of our conceptual system, the idea of our conceptual system as a massive alliance of beliefs which face the tribunal of experience collectively and not independently".¹¹ Tsou gives examples of what Putnam argues have worked as *a priori* principles. The examples are the formula for kinetic energy, $e = \frac{1}{2}mv^2$, and Newton's law of gravitation. Tsou cites part of a passage in *The Analytic and the Synthetic*, which I cite in full:

The principle ' $e = \frac{1}{2}mv^2$ ' may have been introduced, at least in our fable, by stipulation; the Newtonian law of gravity may have been introduced on the basis of induction from the known satellite system and the solar system (as Newton claimed); but in subsequent developments these two famous formulas were to figure on a par. Both were used in innumerable physical experiments until they were challenged by Einstein, without ever being regarded as themselves subject to test in the particular experiment. If a physicist makes a calculation and gets an empirically wrong answer, he does not suspect that the mathematical principles used in that calculation may have been wrong (assuming that those principles are themselves theorems of mathematics) nor does he suspect that the law ' $f = ma$ ' may be wrong. Similarly he did not frequently suspect that the law ' $e = \frac{1}{2}mv^2$ ' might be wrong or that the Newtonian gravitational law might be wrong. (Newton himself did, however suspect the latter.) These statements, then, have a kind of preferred status. They can be overthrown, but not by a single experiment. They can be overthrown only if someone incorporates principles incompatible with those statements in a successful conceptual system.¹²

Putnam argues in the passages before the one cited above that the principle $e = \frac{1}{2}mv^2$ may have changed from being a stipulation of "kinetic energy" to become, in Einstein's relativistic physics, an empirical natural law. This is to show that not even stipulated definitions in the logical positivists' sense can withstand revision, if these "conventions" (if they indeed exist outside this "fable") are parts of a scientific theory. For Putnam, these physical principles, as well as the principles of Euclidean geometry, are called *framework principles* and they

¹¹Tsou (2010), 433; Putnam (1975)[1979b], p. 40

¹²Putnam (1975)[1979b], p. 45.

[...] have the characteristic of being so central that they are employed as auxiliaries to make predictions in an overwhelming number of experiments, without themselves being jeopardized by any possible experimental results.¹³

Tsou then goes on to discuss the “contemporary relevance of Putnam’s analysis” (one of the section headings of Tsou’s paper), and compares Putnam to what he thinks is the more detailed and comprehensive account of the history of physics¹⁴ worked out by Michael Friedman. Friedman argues, contra Quine, that some of the mathematical and basic physical principles in Newtonian and Einsteinian physics, respectively, are *a priori* in the sense that they are *constitutive* for the empirical part. But as Friedman argues against Quine (and Tsou in the same fashion against Putnam): these constitutive principles are not necessarily *entrenched* (protected in the center of our beliefs) in the Quinian sense.

One of the main arguments against the Quinian picture delivered by Friedman, and supported by Tsou, against Putnam, is that entrenchment is not characteristic of a notion of a relative *a priori*. It is not because the mathematical parts of a physical theory are entrenched, or resist revision, that we should regard these as *a priori*. It is because of their constitutive role in setting up an experiment, of posing questions and framing the possible interpretations of the outcome of an experiment.

Friedman’s arguments in support of his picture is a historical investigation of Newton’s physics, in particular his law of gravitation, and the corresponding theory by Einstein that later refuted Newton’s. Friedman’s investigation suggests that the new differential *calculus* that is implicitly used in the *Principia*, in order to define the laws of motion (force is the time derivative of momentum¹⁵), *was not better entrenched* than any other part of Newton’s theory. In fact, the calculus was quite controversial in Newton’s time. Similarly, Friedman claims that Riemannian geometry was not well entrenched either, but was rather fully developed after Einstein’s general theory of relativity.¹⁶

¹³Tsou (2010), p. 434; Putnam (1975)[1979a], p. 48.

¹⁴Tsou (2010), p. 437.

¹⁵Newton does not use such an explicit use of the calculus, which I have here also formulated in a slightly different way. Instead he uses synthetic Euclidean geometry and limit considerations. It is clear, however, that what is behind these calculations are the very ideas in the theory of fluxions that Newton had already developed. Friedman has been criticized for neglecting that one cannot really talk of a calculus here at all, since it was not fully developed until the latter half of the 19th century. This, however, only supports Friedman’s claim that Newton’s fundamental ideas about instantaneous change, functioning as a constitutive part in Newton’s theory, was not well entrenched. Not even at the time Kant.

¹⁶Friedman (2001), pp. 39–40, p. 47n.

It is not only the calculus that has this constitutive function in Newton's theory, according to Friedman. There are also coordinating principles that set up a correspondence between the mathematical part and the empirical part of a physical theory such as Newton's, in this case, the laws of motion. Hence Friedman (2001) aims to show that in Newton's theory of gravity, we have the calculus and the laws of motion as constitutive *a priori* parts of the law of gravitation, which states that two bodies in the universe are attracted towards each other with a force that is proportional to the square of the reciprocal distance between them. Thus that any two bodies accelerate towards each other relative an absolute space, is encoded by the coordinating principles, given by the laws of motion, which are dependent for their formulation on the calculus.

So what is the difference between this picture and Putnam's? Didn't Putnam claim that the role played by the relative *a priori* principles is that of auxiliary hypotheses that function as framework principles? Tsou writes that "on a general level Putnam and Friedman's accounts are remarkably similar",¹⁷ but his main conclusion of Putnam's views is that he "formulates his analysis through Quinian insights, whereas Friedman formulates his account in explicit opposition to the sort of Quinian web-of-belief holism endorsed by Putnam".¹⁸ Tsou claims "that a feature of Putnam's *a priori* principles is their resistance to revision (or entrenchment)".¹⁹

I agree with Tsou that Putnam endorses a strong kind of holism in the 1960s quotes cited by Tsou to support his claim that Putnam remains in a Quinian setting of formulating the problems of the analytic/synthetic distinction, or the *a priori* versus the *a posteriori*. And I will argue that Putnam has an epistemological view similar to that of Quine and of some of the logical positivists (although not Carnap). This will create major problems for Putnam when he explains the necessity of mathematics, since, as we shall see, this "epistemological view" makes mathematical "statements" essentially of the same kind as any other statement, in the sense that their function is only explained through their resistance to revision.

Note, for example, that Putnam in *The Analytic and the Synthetic*, as cited above, uses Newton's law of gravitation as an example of a principle that received an *a priori* status (recasting his view in his 1970s terms) within our body of knowledge before Einstein, since nothing could overthrow it; one could not think of an experiment or observation within the Newtonian body of knowledge that would disconfirm it. This is an example (not stressed by Tsou) that supports Tsou's claim that what for Putnam is characteristic of *a priori* principles is their ability to resist revision; in Quinian language, their entrenchment. Friedman,

¹⁷Tsou (2001), p. 437.

¹⁸Ibid., p. 438.

¹⁹Ibid., p. 441.

on the other hand, stresses the functional status of the law of gravity, in that it is meant to be an empirical statement in contrast to other principles that enable us to formulate the law of gravity in terms of concepts that rely for their meaningfulness on the calculus (to give us a mathematical notion of instantaneity) and the laws of motion (the coordination principles that makes it possible to connect force and acceleration in an absolute space which acts as a frame of reference for all motions).

Tsou's main arguments for his claim regarding this difference between Putnam and Friedman's view rely on citations from Putnam's *Revisiting Mathematical Necessity* (1990)[1994], which is very different from the works from the early 1960s, but which Tsou either does not note or chooses to disregard. In particular, Tsou cites the following passage from the 1990 paper:

[C]all a statement *empirical relative to a body of knowledge B* if possible observations [...] would be *known* to disconfirm the statement [...] Statements which belong to a body of knowledge but which are not empirical relative to that body of knowledge I called 'necessary relative to the body of knowledge.' [...] The point of this new distinction was [...] to emphasize that there are at any given time some accepted statements which cannot be overthrown merely by *observations*, but can only be overthrown by thinking of a whole body of alternative theory as well [...] I would [...] emphasize the nonpsychological character of the distinction by pointing out that the question is not a *mere* question of what some people can imagine or not imagine; *it is a question of what, given a conceptual scheme, one knows how to falsify or at least disconfirm.* Prior to Lobachevski[sic], Riemann, and others, no one knew if anything *could* disconfirm it.²⁰

Tsou remarks that

Putnam qualifies this distinction by stating that framework principles should properly be characterized as '*quasi-necessary*' relative a conceptual scheme' given the abnormalities of calling potentially *false* statements 'necessary' or 'a priori' (whether these statements are contextualized or not).²¹

Tsou's way of citing Putnam is somewhat misleading, since what Putnam is trying to explain is the difference between *It Ain't Necessarily So* and the new paper regarding *necessity*. Here is what Putnam writes:

I suggested [in *It Ain't Necessarily So*] that to identify 'empirical' and 'synthetic' is to lose a useful distinction. The way I proposed to draw the distinction is as follows: call a statement *empirical relative to a body*

²⁰Tsou (2010), p. 436; Putnam (1990)[1994], p. 251.

²¹Tsou (2010), p. 436.

of knowledge *B* if possible observations (including observations of the results of experiments people with that body of knowledge could perform) would be *known* to disconfirm the statement (without drawing on anything outside that body of knowledge). It seemed to me that this pretty well captures what the traditional notion of an empirical statement. Statements which belong to a body of knowledge but which are not empirical relative to that body of knowledge I called ‘necessary relative to the body of knowledge.’ The putative truths of Euclidean geometry were, prior to their overthrow, simultaneously synthetic and necessary (in this relativized sense). The point of this new distinction was [...] to emphasize that there are at any given time some accepted statements which cannot be overthrown merely by *observations*, but can only be overthrown by thinking of a whole body of alternative theory as well. And I insist (and still insist) that this is a distinction of methodological significance. If I were writing ‘It Ain’t Necessarily So’ today, I would alter the terminology somewhat. Since it seems odd to call statements which are false ‘necessary’ (even if one adds ‘relative to the body of knowledge *B*’), I would say ‘*quasi*-necessary relative to the body of knowledge *B*.’ Since a ‘body of knowledge,’ in the sense in which I used the term, can contain (what turned out later to be) false statements, I would replace ‘body of knowledge’ with ‘conceptual scheme.’ And I would further emphasize the nonpsychological character of the distinction by pointing out that the question is not a *mere* question of what some people can imagine or not imagine; it is a question of what, given a conceptual scheme, one *knows* how to falsify or at least disconfirm. Prior to Lobachevski, Riemann, and others, no one knew if anything *could* disconfirm it.²²

The point now in the 1990 paper is the new emphasis on the “nonpsychological character of what one knows how to falsify or at least disconfirm”. Putnam changes his old view from *It Ain’t Necessarily So* and *The Analytic and the Synthetic* to one influenced a by a certain reading of Wittgenstein, in order to distance himself from Quine’s views regarding the possibility of revising statements. Putnam wants to show that what Quine is up to in his talk of “revisions of any statement” is wrong in that the question of whether theorems of classical logic are revisable “is one which we have not succeeded in giving a sense.” In fact, it is unintelligible.

Tsou omits any consideration of differences between the 1962 papers and the 1990 paper. In fact, he never mentions any influence from Wittgenstein, which Putnam himself explicitly names as a main new influence in his attempt (which I will argue is at least partly unsuccessful) to challenge his old views that mathematics is best viewed on a par with its applications in physics and other

²²Putnam (1990)[1994], p. 251.

sciences.²³ Tsou seems to regard Putnam's contribution of the relative *a priori* as something from the past with influences on a more recent development. In what follows, however, I will discuss important insights offered by Putnam which are contemporary with some of Friedman's main contributions, e.g. Friedman (2001), or even post-dates them.

2. Friedman's Kantianism and Carnap

In the *Critique of Pure Reason*, one of Kant's main tasks is to give a philosophical account of why Newton's physics can be regarded as certainly true. What Kant aims for is to explain the new role of philosophy in the wake of Newton's global theory of a terrestrial and celestial mechanics of the world that now with the same equations explains physical phenomena, such as how stones fall to the ground and at the same time explains earth's revolution around the sun, as well as the movements of the other planets. Kant sees the role of (his) philosophy to explain the possibility of Newton's physics and the mathematics involved in it. There is no longer any place for the speculative philosophy that preceded the Newtonian way of doing physics; natural philosophy belongs to an embarrassing past. Kant's answer to the question of how the new mathematics and physics can accurately describe, predict and explain celestial and terrestrial motion, is essentially the notion of *the synthetic a priori* truths, that is, a conception of *a priori* that allows for truths about the world that are constitutive for the way we describe it. The familiar examples includes the law of causality and the principles of Euclidean geometry. We can think of a world where these principles are violated (in contrast to a world where logic is not valid), but we cannot formulate any theory of this world in which they do not hold. Newton's law of gravitation, for instance, means that any two bodies in the universe accelerate towards each other (for instance me and Jupiter), and acceleration depends on new mathematical principles (acceleration is instantaneous change in velocity, which is in turn the instantaneous change of position; implicitly, the notion of a second and a first derivative of a function, respectively, is involved) and the notion of an absolute space, since the acceleration has to be relative something that is fixed. Euclidean geometry describes this absolute space, and its principles are furthermore *synthetic a priori*; and it consists of truths that are constitutive of Newtonian physics.

As Friedman²⁴ (2001) has shown, Kant's notion of synthetic *a priori* essentially consists of two parts which Kant never really separates, and this, Friedman writes, was noted by Reichenbach already in 1920 (Reichenbach (1920)[1965]) when he was engaged in the urgent task of trying to understand what Einstein's

²³Ibid., p. 245.

²⁴I have made extensive use of Friedman's *Dynamics of Reason* (2001) in this section.

relativity theory now meant for our view of the world, the *a priori*, physics and philosophy. What Kant did not discern, but which perhaps seems important to separate now that Einstein's theory has overthrown Newtonian physics and our earlier view of the physical world, is that the constitutive function of certain principles does not entail that these are absolute necessary truths about the world. The principles of Euclidean geometry may be constitutive of Newtonian physics and Riemannian geometry may be constitutive of Einsteinian physics, but neither geometries need to express truths about the world. This view was of course already present in Einstein's own motivation for his relativity theories, and Friedman notes that he got these insights, e.g., from studying Poincaré. Since alternative geometries had been studied axiomatically in the 19th century, in particular in the late 19th century and the beginning of the 20th century, Poincaré suggested that Euclidean geometry should be viewed as one alternative geometry among others: it is a free choice, depending only on the pragmatic needs of the scientist. Which geometry should be chosen is a matter of *convention*. But it was first after the success (and confirmation) of Einstein's theory that it was clear that philosophy (of science) had to accommodate to the new situation and try to explain the relation between the mathematical principles underpinning physical theory and the empirical part. Reichenbach's answer in 1920, which Friedman in principle endorses, was a distinction between "axioms of coordination" (constitutive principles) and "axioms of connection" (empirical laws).²⁵

According to Friedman, Moritz Schlick in 1921 made it clear that his special concern was to do for Einstein's physics what Kant had done in relation to Newton. That is, Schlick wanted to understand what made Einstein's physics a paradigm for rational knowledge. It was clear that Kant was mistaken in thinking that the principles he had pointed out as synthetic *a priori* were absolute truths, and that Poincaré was right that the mathematical first principles that are needed in physics are conventions, in the sense that they are free choices that we make.²⁶

In 1922, Schlick moved to Vienna and became the leader of what was to become known as the Vienna Circle. The members of the group tried to combine the insights of the new results in physics with the new methods in logic and Wittgenstein's *Tractatus*. Friedman now tells the following story. The Vienna Circle's reading of Wittgenstein's *Tractatus* (in 1926-27) made it difficult for the Circle to develop a philosophy of science that answered the problems that Schlick had originally set out to investigate, since Wittgenstein, according

²⁵Friedman (2001), p. 32.

²⁶Ibid., p. 14.

to perhaps both the Circle and Friedman, did not have a particularly constructive attitude as to what philosophy can contribute to science. Friedman cites the *Tractatus*:

The correct method of philosophy would properly be the following:
To say nothing but what can be said, that is, the propositions of natural science – and thus something that has nothing to do with philosophy – and then, whenever another wanted to say something metaphysical, to demonstrate to him that he had given no meaning to certain signs in his propositions.²⁷

This is the very paragraph that exponents of the New Wittgenstein interpretation rely on when they emphasize that the aim of *Tractatus* is to say nothing—that the final line of the *Tractatus*, quite literally, says nothing, not a pregnant nothing that alludes, for instance, to something mystical that can only be shown. One does not have to choose between these opposing schools of reading Wittgenstein, that is between the resolute reading of the *Tractatus* versus P.M.S. Hacker and other interpreters' views. One has the choice of viewing Wittgenstein as here coming to a certain insight which is in conflict with earlier thoughts in the *Tractatus*. This rhymes well with Wittgenstein's attempts around 1930 first to try to improve upon the *Tractatus*, and then later to abandon certain features to overcome this tension between an "explanatory" part of the *Tractatus* and the insight that philosophy cannot explain things in the way that science gives explanations.

This "non-scientific sentiment" of the *Tractatus* was clearly felt by the members of the Vienna Circle, whose intended project was to contribute to the development of science by working out a scientific philosophy.

Friedman now writes that the rescue came in 1926, when the young Rudolf Carnap joined the Circle, after an education in Jena involving theoretical physics and mathematical logic (under Frege). According to Friedman, Carnap's intention was to extend the role metamathematics had in Hilbert's program of treating mathematics as syntactical systems of sentences and proofs. In *Logical Syntax of Language*, Carnap explicitly takes issue with Wittgenstein's ideas in the *Tractatus*, and refutes these as mystical and unscientific. Instead he offers his own *Wissenschaftslogik*, the extension of Hilbert's metamathematics in the form of a "meta-logical investigation of the logical syntax of scientific language".²⁸

Philosophy is now to be viewed as part of science, indeed as a part of mathematical logic. For example, traditional debates between realists and idealists should now be viewed as different proposals that lead to different axiomatic formulations within a certain language which entails its own "standards of logical correctness and truth". *Internal questions* are those which are relative such a

²⁷Ibid., p. 15; *Tractatus* §6.53.

²⁸Ibid., p. 16.

language and its standards of correctness and truth, whereas *external questions* concern which language or *linguistic framework* one should choose—but these questions are not given within a framework where truth and correctness are determined; instead pragmatic or conventional decisions have to be made at this level. As Friedman points out, this particular form of the ideas in *Logical Syntax of Language* were elaborated after Carnap had arrived in America, and in particular after his discussions with Quine and Tarski at Harvard in 1939–41 and finally published in 1950 as *Empiricism, Semantics, and Ontology*, the most mature version of the logical positivists' new view, according to Friedman. Carnap makes a distinction between analytic statements and synthetic statements relative to a given framework. The internal questions follow rational considerations based on the rules laid down in the analytic statements, which are separated from the synthetic statements of empirical reality. This is the analytic/synthetic dichotomy which is criticized in *Two Dogmas of Empiricism*, although the analytic statements are not absolutely analytic, they are rules chosen for the linguistic framework. In Friedman's view, the rules of mathematics and logic then function as *constitutive* in Carnap's scheme. In particular, whether one uses a classical or intuitionistic logic to build up one's mathematics is a matter of convention; for Carnap it is an external question, whereas once such matters have been decided upon, one may start to ask internal questions within a given framework.²⁹

3. Carnap and the Vienna Circle on Mathematics

In this section, I will rely almost entirely on Warren Goldfarb's *The Philosophy of Mathematics in Early Positivism* (1996) with the purpose of showing in particular the sophistication of Carnap's view on the philosophy of mathematics, especially in the *Logical Syntax of Language*. This will be important in several ways. One is that this presentation will clarify the structure of Quine's critique in *Two Dogmas of Empiricism*, which together with Carnap's publications, forms the very background for Putnam's investigations. Another aim is to provide the background to Carnap's ideas in *Logical Syntax* in order to clarify some of Friedman's writings on the relative *a priori*, which has been compared with Putnam's view on a conception of a relative or contextual *a priori*. Friedman expresses sympathy with many of Carnap's ideas, and thinks that although Quine's critique is appropriate, Carnap's work nonetheless deserves attention, since he may be on the right track, if we disregard the particular logical setting which was later criticized by Quine. But the most important part of this section is to clarify how the picture that both Quine and Putnam rely on in their critique of the logical positivists, namely, their view that the logical positivists thought that the analytical truths are those with a universal range of confirmation, i.e., confirmed “no

²⁹Ibid., pp. 17–18; pp. 31–33.

matter what”, is mistaken with respect to Carnap. Goldfarb shows that Carnap does not hold such a view. Instead, the rules of mathematics are parts of other rules, such as logical rules, laid down in a linguistic framework—and their role is to function as constitutive for the empirical claims. However, there is no fixed scheme, and even empiricism itself should be viewed as merely a hypothesis, formulated externally to any linguistic framework on pragmatic grounds.

Wittgenstein’s well-known influence on the Vienna Circle began in the academic year 1926-27. Goldfarb examines the influence of Wittgenstein in the works of Carnap, Hahn and Schlick, and he makes a particularly interesting comparison (for our present purposes) with the different ways these three philosophers are influenced in their own treatments of the problems concerning the status of mathematics.

Briefly, around 1930, all three, Carnap, Hahn and Schlick, embrace the view that mathematics, like logic, consists of tautologies. Hahn and Schlick say that logical truth is not really a species of truth; it is a mere artifact of the representational system. They both view Wittgenstein’s “discovery” (as they see it) of the tautologous nature of logic to be essential in order to justify empiricism and to reconcile it with the apodictic certainty of logic.³⁰ There is however no mentioning of the truth-table analysis of tautologies in the works of Schlick and Hahn; instead, and this is particularly interesting for us, they rely on an epistemic view. Hahn writes (1933):

There is no material a priori, i.e., no a priori knowledge of facts; for we cannot know of any observation how it must come out before we have actually made it.³¹

Schlick (1932) also stresses that tautologies are “compatible with *any* observation”³² and he writes

Although there is at present still considerable disagreement about the ultimate foundations of mathematics, nobody can nowadays hold the opinion anymore that “arithmetical propositions” communicate any knowledge about the real world. [...] Their validity is that of mere tautologies; they are true because they assert nothing of any fact. [...] I repeat: arithmetical rules have tautological character [...] (no matter whether arithmetic is just part of logic—as Bertrand Russell will have it—or not).³³

³⁰Goldfarb (1996), p. 220.

³¹Ibid., p. 220.

³²Ibid., p. 222.

³³Ibid., p. 222.

That is, tautologies are true because they assert *nothing of any fact*. Goldfarb writes that Schlick's view

that mathematics *must* be tautologous—without so much as a glance at the logicist reduction—is an upshot of the epistemological twist that both Hahn and Schlick give the notion of tautology. ‘Tautologous’ winds up meaning true no matter what the *experiential* facts are, or true but not the subject to empirical verification. The underlying picture seems to be this: the empirical world, the world of experience, is given; it is talk of that world that has content.³⁴

Carnap (1929-30), on the contrary, sticks to a more *Tractarian* view of the tautologies characterizing logical truth. He stresses truth-table analysis and holds that tautologies lack content, since any notion of content requires a contrast between what would make propositions true or false, and such a contrast is lacking for the tautologies.³⁵ Hence, on Goldfarb's reading, Carnap does not epistemologize the notion of tautology; when Carnap speaks of tautologies as “true under all possibilities”, no particular epistemological cast is presupposed, in particular not empiricism.³⁶

Goldfarb concludes that for Hahn and Schlick, the tautologous nature of mathematics is a by-product of their epistemological view of tautologies, whereas for Carnap it is his commitment to logicism that is the basis for viewing mathematical propositions as tautologous. In the 1930 work *Die Mathematik als Zweig der Logik*, Carnap mentions Wittgenstein's position in the *Tractatus*, that mathematics is *not* tautologous and that Wittgenstein's view is rather that mathematics is a method of transforming identities.³⁷ However, Carnap sees Wittgenstein's

³⁴Ibid., p. 222.

³⁵Ibid., pp. 220–221.

³⁶Ibid., p. 223.

³⁷Already in the *Tractatus*, Wittgenstein remarked that mathematical propositions are not tautologies, indeed that they are not propositions at all, but pseudo-propositions, *Scheinsätze*. (Shanker (1987), 274.) Wittgenstein continues in this direction in the *Philosophical Remarks*. Shanker writes the following in *Wittgenstein and the Turning-Point in the Philosophy of Mathematics*:

While the Vienna Circle was preoccupied with the complications involved in treating mathematics as ‘tautological’, Wittgenstein embarked on a radical new conception. He remained committed to the general principle that mathematical propositions are not descriptive, but rejected the *Tractatus* argument that mathematical propositions are nonsensical (ill-formed) expressions. As opposed to this conception he introduced the bold new suggestion that mathematical propositions are ‘rules of syntax’ [...], not tautologies, for the latter are senseless,

view as preliminary and he thinks that when the logicist reduction of mathematics is accepted, mathematics will definitely be regarded as tautologous.³⁸

Not only is Carnap closer to *Tractatus* than Hahn and Schlick, Goldfarb also finds that Carnap is relatively immune to the type of criticism delivered by Gödel in the 1950s, where Gödel criticizes the view that mathematics could be viewed as “merely” syntax of language. Gödel’s critique is that the positivist’s view that there are no mathematical facts is based on a circle-argument, since one assumes that “fact” must mean *empirical* fact from the very beginning. Gödel thinks that the empiricists have a prejudiced view of content, and therefore cannot acknowledge “conceptual” content for mathematical propositions. But Carnap does not “epistemologize the notion of tautology” according to Goldfarb. Of course, Carnap presupposes some realm of facts, but the notion of “truth-possibilities” does not require a *particular* “epistemological cast”.³⁹

However, the pressing implications of Gödel’s two incompleteness theorems is perhaps the main reason for Carnap to finally alters his view on the status of mathematics in a radical way. In the *Logical Syntax of Language* (1934)[1937], there are no tautologies. The truths of logic and mathematics are now called analytic, but this notion is now relativized. As mentioned earlier, Carnap introduces the notion of a “linguistic framework” in 1950, which we have already described. In *Logical Syntax*, the frameworks are implicit in his construction, and analytic statements are statements true in some artificial language *L*. The introduction of a meta-level is linked to the critique of Wittgenstein in the *Logical Syntax*, since such a meta-level may be considered as alien (and Carnap stresses this) to Wittgenstein’s view of language, but the motivation of the meta-level is quite possibly an attempt of avoiding the incompleteness theorems.⁴⁰ Gödel stressed that

a rule about the truth of sentences can be called *syntactical* only if it is clear from its formulation, or if it somehow can be known beforehand, that it does not imply the truth or falsehood of any ‘factual’ sentence.⁴¹

Only if a rule is consistent will it fulfill this requirement, Goldfarb writes, since otherwise it will imply all sentences. Mathematics not captured by a rule in question must be used in order to prove that the rule is consistent and hence

whereas mathematical propositions are norms of representation [...].
(Shanker (1981), pp. 274–275.)

³⁸Goldfarb (1996), p. 221.

³⁹Ibid., p. 222–223.

⁴⁰Ibid., p. 225.

⁴¹Ibid., p. 226.

“additional mathematics must be invoked in order to legitimize the rule, and the claim that mathematics is solely a result of rules of syntax is refuted”.⁴²

Goldfarb stresses that Gödel’s argument, that further mathematics has to be invoked than what is supported by any syntactical system, presupposes that we have a factual or empirical realm available in advance,

independently of and prior to the envisaged rules of syntax. As Gödel characterizes the positivist view, first there are empirical sentences, which are true or false by virtue of the facts of the world; mathematics is then added, by means of conventional syntactical rules. Gödel’s argument is that the addition has to be known not to affect the empirical sentences given at the start, and, by this theorem, to ascertain that requires more mathematics.⁴³

Goldfarb now reminds us that the picture attacked by Gödel is one defended by Hahn and Schlick, and he suggests that Gödel’s argument is “very effective, perhaps conclusive, against the claim that their notion of tautology could provide a foundation of mathematics”.⁴⁴ Even Carnap’s earlier position is not unproblematic in this sense, and Goldfarb believes that Carnap’s transition to dropping the label “tautology”, together with the introduction of the *principle of tolerance* (any system of rules may constitute a linguistic framework, even the perhaps not very useful inconsistent ones) is intended, at least in part, to meet the demands of the incompleteness theorems. This is because sense can only be made of “fact” and “empirical world” within and by virtue a linguistic framework. Gödel’s argument presupposes a transcendence across different linguistic frameworks, but this is rejected in *Logical Syntax* as being a central ingredient of the *principle of tolerance*. Goldfarb writes that

the notion of empirical fact is given *by way* of the distinction between what follows from the rules of a particular language and what does not, so that different languages establish different domains of fact.⁴⁵

Goldfarb’s description of Carnap’s undercutting in the *Logical Syntax* of Gödel’s argument is important in view of Thomas Ricketts’ argument in *Carnap’s Principle of Tolerance, Empiricism, and Conventionalism* (1994). Here Ricketts attacks some of Putnam’s standard critiques of Carnap, for instance that the *principle of tolerance* presupposes the *verification principle*, and in particular the claim that Carnap’s view of the verification principle as a mere *proposal* (and indeed not a *principle*), granted by the principle of tolerance, is untenable. The argument

⁴²Ibid., p. 226.

⁴³Ibid., p. 225.

⁴⁴Ibid., p. 227.

⁴⁵Ibid., p. 227.

Putnam puts forward in *Philosophers and Human Understanding* (1981b)[1983] is that the verification principle *is* precisely that the external questions are without cognitive sense.⁴⁶ Recall that the external questions are the questions *outside* of the “linguistic framework”, that is, outside the system where a logic has been decided upon and where truth and correctness may be determined. This external level is then confined to the pragmatic choice of linguistic systems, by virtue of the principle of tolerance, a principle that does not discriminate between the linguistic systems, in the sense that they are true or false. Thus, Putnam claims that the verification principle is nothing but the claim that there are no true or false statements at the external level in Carnap’s system. Thus, the very possibility of a principle of tolerance seems to be dependent on the verification principle, and so the verification principle cannot be optional in Carnap’s system, according to Putnam. (I think that one of Putnam’s reasons for rejecting Carnap’s dichotomy of external and internal questions, is that it does not make the point of view of the external level intelligible, and this is also the type of criticism that a Wittgensteinian would have.)

In short, Putnam turns Carnap into what Ricketts calls an “empiricist epistemologist” (in very much the same sense of Hahn and Schlick in Goldfarb’s description of early logical positivism). Ricketts claims, however, that this is “an example of the sort of *philosophical* position—in Carnap’s pejorative sense of ‘philosophical’—that Carnap disparages in *Logical Syntax*”.⁴⁷ One should remember that the philosopher’s task is now not to defend this or that position, but rather to contribute to science with interesting linguistic frameworks. It is clear that Carnap’s project is not a “philosophical project” in the sense of trying to understand our language and our life in any general sense, as is typical of the ambitions of Wittgenstein and Putnam. Ethical questions do not seem to have a place in Carnap’s system, for instance. The ambition is rather that we should now be able to reconstruct science within linguistic frameworks and that we should not necessarily take a stand for or against intuitionistic or classic logic; they may both be useful in different contexts. These are important insights, but both critics (Putnam) and defenders (Ricketts) of Carnap should note that Carnap’s ambitions are perhaps not those of the later Putnam, nor those of Wittgenstein and Quine. These philosophers are philosophers of language in a more general sense than Carnap, and for Putnam this leads to an increasingly wider perspective also on other problems in philosophy than those of the philosophy of science.

Ricketts and Goldfarb both argue that, for Carnap, “empiricism” is a proposal compatible with some linguistic frameworks, but not with others, and that

⁴⁶Putnam (1981)[1983], p. 191n.

⁴⁷Ricketts (1995), p. 179.

one cannot charge Carnap's *Logical Syntax* with the same objections as are commonly leveled against logical positivism.⁴⁸ In *Logical Syntax*, there is no longer any need to explain mathematics via logic in this new picture. The logicism is retained, but is now only used to support the idea that pure mathematics also devises rules for linguistic frameworks in the same fashion as any logic. Whether mathematics is also reducible to logic, that is, whether it is logic in a more narrow sense (rather than taken as primitive) is, as Carnap writes, "not a question of philosophical significance, but only one of technical expedience".⁴⁹ Furthermore, there are no longer any foundational questions in the sense that such questions are addressed by Kant, Frege, Russell, Hilbert or the earlier Carnap.

Putnam has argued against Ricketts' suggestion that we should not treat Carnap as an empiricist epistemologist. In his book *The Collapse of the Fact/Value Dichotomy* (2002), Putnam expresses "strong disagreement" with Ricketts' view, that Carnap's position is free of such metaphysical commitments. More precisely, Putnam says that Carnap in 1928 (in *Der logische Aufbau der Welt*) held that

factual statements are transformable into statements about the subjects' own sense experiences or *Elementarerlebnisse*. Indeed, some of the members of the Vienna Circle even insisted that a meaningful statement must be *conclusively verified* by confrontation by direct experience! [...] Carnap, however, held out against the requirement of *conclusive* verifiability; and in 1936 [in *Testability and Meaning*], he slightly liberalized the requirement that all factual predicates must be definable by means of observation terms. But still it remained the case that (1) a necessary condition that a statement had to meet to be counted as 'cognitively meaningful' was that it be expressible in the 'language of science' (as formalized by the logical positivists) and (2) the predicates admitted into the 'factual' part of the language of science had to be 'observation terms' or reducible (by specified and limited means) to observation terms.⁵⁰

According to Putnam, the problem with this conclusion was how one should account for statements about entities difficult to observe, such as bacteria, electrons or gravitational fields. Should such statements be counted as nonsense, or should they be part of "cognitively meaningful" discourse? Going for the latter alternative, the doctrine had to be revised. This happened in Carnap's *Foundations of Logic and Mathematics* (1938), where *electrons*, etc., are taken as primitive,

⁴⁸Ibid., p. 177.

⁴⁹Goldfarb (1996), p. 226.

⁵⁰Putnam (2002), pp. 22-23.

and it is now the system of scientific statements as a whole that has factual content. Putnam then asks: but what about individual statements? Are they factual or not? It is here that Carnap remained influenced by classical empiricism:

In his subsequent writings, Carnap continued to distinguish sharply between the ‘observation terms’, in other words the vocabulary that refers to ‘observable properties’, which he now said are ‘completely interpreted’, in other words, which have freestanding meaning, and ‘theoretical terms’, such as ‘bacterium’, ‘electron’, and ‘gravitational field’, which he said are ‘only partially interpreted’. [...] [These were] regarded as mere devices for the sentences that *really* state the empirical facts, namely the observation sentences.⁵¹

Thus, we have observation sentences that state empirical facts, observation terms with a freestanding meaning (meaning as *intension*), which according to Putnam, depends for its intelligibility on the verification principle, as we will see in Chapter 3. Thus empiricism, together with a not further motivated verification principle (the principle cannot be justified by itself), is fundamental to Carnap’s position as a whole. In Chapter 3, I will briefly survey how Putnam’s externalism with respect to meaning in *The Meaning of Meaning* (1975) is used to criticize Carnap’s view of meaning as intension. One could argue, however, that the semantical doctrines in Carnap’s work came long after the *Logical Syntax of Language*, and this work is currently under a lot of attention; see, for example, Wagner (ed.) (2010).

4. Quine’s *Two Dogmas of Empiricism*

In *Two Dogmas of Empiricism* (1951)[1953], Quine delivers his famous critique against the distinction, or even a dichotomy, between analytic and synthetic statements, at least relative a linguistic framework. It is not always clear what version of logical positivism (or Carnap) Quine has in mind, but his aim is quite explicitly to refute in general, as the title suggests, the two dogmas of empiricism he identifies: that a statement can be “true by virtue of the meanings and independent of fact”⁵², i.e., the notion of analyticity, and reductionism, in particular the version associated with the Vienna Circle, namely, that statements have meaning by being reducible to statements about sense experience, the verification theory of meaning: “the meaning of a statement is the method of empirically confirming or infirming it”.⁵³ (The reader should recall from the previous section that there are current interpretations of Carnap’s *Logical Syntax* that claims that Carnap does not endorse the verification principle; it should rather be seen as a *proposal*

⁵¹Ibid., p. 24.

⁵²Quine (1951)[1953], p. 21.

⁵³Ibid., p. 37.

in virtue of the principle of tolerance; in fact empiricism itself should be regarded as a pragmatic proposal.)

Burgess has claimed in *Quine, Analyticity and Philosophy of Mathematics* (2004) that “[t]he latter dogma implies the former, or at least a theory of meaning of the kind indicated gives one way of making sense of the analytic/synthetic distinction: *analyticity* is the limiting or degenerate case in which *every* potential observation counts in favor of the statement, and none against it. So for Quine, rejection of the second doctrine is a corollary to rejection of the first”.⁵⁴

This is essentially a correct picture, and it is in fact a good characterization of Quine’s epistemological point of view, similar to that of Hahn and Schlick, although Putnam makes a more detailed analysis. In Section 7, I will discuss Putnam’s elucidation of Quine’s different arguments against analyticity in *Two Dogmas*.

According to Putnam, the main point is now that it is the logical positivist view of meaning that is primarily under attack in the *Two Dogmas of Empiricism*. Quine writes:

Things had essences, for Aristotle, but only linguistic forms have meanings. Meaning is what essence becomes when it is divorced from the object or reference and wedded to the word. From the theory of meaning a conspicuous question is the nature of its objects: what sort of things are meanings? A felt need for meant entities may derive from an earlier failure to appreciate that meaning and reference are distinct. Once the theory of meaning is sharply separated from the theory of reference, it is a short step to recognizing as the primary business of the theory of meaning simply the synonymy of linguistic forms and the analyticity of statements; meanings themselves, as obscure intermediary entities, may well be abandoned.⁵⁵

Quine argues that we cannot rely on metaphysical entities of meaning, which are remnants of earlier metaphysical positions in a new disguise, after the “linguistic turn”. Since what becomes of the notion of meaning once one abandons a metaphysical view of “meaning-entities” is just the *linguistic* notions of analyticity and synonymy of statements, Quine continues to deal with the notion of analyticity of a statement, a linguistic version of Kant’s notion of analyticity.

The analytic statements fall into two classes, Quine writes, the first class containing logically true statements such as

(1) No unmarried man is married.

“The relevant feature of this example is that it not merely is true as it stands, but remains true under any and all reinterpretations of ‘man’ and ‘married’. If we

⁵⁴Burgess (2004), pp. 48–49.

⁵⁵Quine (1951)[1953], p. 22.

suppose a prior inventory of *logical* particles, comprising ‘no’, ‘un-’, ‘not’, ‘if’, ‘then’, ‘and’, etc., then in general a logical truth is a statement which is true and remains true under all reinterpretations of its components other than the logical particles.”⁵⁶

The second class of analytic statements are of the type

(2) No bachelor is married.

“The characteristic of such a statement is that it can be turned into a logical truth by putting synonyms for synonyms; thus (2) can be turned into (1) by putting ‘unmarried man’ for its synonym ‘bachelor’.”⁵⁷

Quine now remarks that we have a problem of characterizing the analyticity of (2), since its analyticity (and analyticity in general) relies on the notion of “synonymy”, “which is no less in need of clarification than analyticity itself.”⁵⁸ Quine’s key argument against a class of analytic statements (excluding possibly the logically true statements) is that “synonymy” relies in turn on “analyticity”.

Furthermore, it cannot be that the analyticity of (2) holds in virtue of a *definition*, that is, that (2) is analytic because we have defined bachelor to be an unmarried man. The lexicographer is an empirical scientist, Quine says, and if he “glosses ‘bachelor’ as ‘unmarried man’ it is because of his belief that there is a relation of synonymy between those forms, [. . .]. The notion of synonymy presupposed here has still to be clarified, presumably in terms relating to linguistic behavior.”⁵⁹ Definition cannot be taken as a ground for the synonymy needed in order to explain analyticity.

What we need in order to give a non-redundant explanation of analyticity is a notion of *cognitive synonymy* not presupposing analyticity, Quine claims. Assuming analyticity, we could define such a notion; the problem is whether we can come up with a conception of cognitive synonymy otherwise. Quine considers interchangeability *salva veritate* (after Leibniz) as a conception of cognitive synonymy; “synonymy” then consists of the interchangeability in all contexts without change of truth value. Quine notes that “synonymy so conceived need not even be free from vagueness, as long as the vaguenesses match”.⁶⁰ Thus the argument that by assuming analyticity we may explain cognitive synonymy is as follows: “bachelor” and “unmarried man” are cognitively synonymous if and only if

(3) All and only bachelors are unmarried men

⁵⁶Ibid., pp. 22–23.

⁵⁷Ibid., p. 23.

⁵⁸Ibid., p. 23.

⁵⁹Ibid., p. 24.

⁶⁰Ibid., p. 27.

is analytic. If we do not assume analyticity, the statement

(4) Necessarily all and only bachelors are bachelors

is still evidently true, Quine says, and if “bachelor” and “unmarried man” are interchangeable *salva veritate* we also have

(5) Necessarily all and only bachelors are unmarried men.

“But to say that (5) is true is to say that (3) is analytic.”⁶¹ Hence, we have that by virtue of interchangeability *salva veritate* that “bachelor” and “unmarried man” is cognitively synonymous. However, this presupposes that the adverb “necessarily” is unproblematic. “Does the adverb really make sense? To suppose that it does is to suppose that we have already made satisfactory sense of ‘analytic’.”⁶²

Quine’s argument so far is that analyticity cannot be based on a metaphysically overblown notion of *meaning*, nor on synonymy relying on definitions, since synonymy is presupposed in all linguistic definitions. If we do not want to rely on a “empirical” synonymy, we may perhaps use the notion of interchangeability which preserves the truth of statements (in all contexts), but this presupposes the notion of necessity, which in turn cannot but rely on the notion of analyticity sought.

5. Putnam’s argument in favor of analyticity

In *‘Two Dogmas’ revisited* (1976)[1983], Putnam argues that the type of arguments that Grice and Strawson (1956) put forward against Quine’s arguments may very well be correct (Putnam was less impressed by these arguments in 1957, when he wrote the *Analytic and the Synthetic*, to which I will return below). In particular, couldn’t we think of “analytic” as defining a family of notions (of which “synonymy” is one) and which are not reducible to other non-linguistic notions?⁶³ The gist of this argument is that maybe “analytic truth” is in no need of an explanation, for instance in terms of the synonymy of words; maybe such notions are primitive in some way. In addition, Putnam thinks that the argumentation provided by Quine against analyticity (so far as treated here) are not very good, since they seem to fall prey on Quine’s *inability* to define analyticity through synonymy. However, Putnam partly defends (1976) Quine’s project by recourse to the fact that Quine’s real target is the notion of *meaning*, so overworked by Carnap and other logical positivists that the notion of analyticity through stipulated “meaning postulates” became unclear, together with many other notions (such as “conceptually necessary”), etc., as *used by philosophers*. Quine’s demonstration of the circularities of the definitions of analyticity and synonymy was “Quine’s

⁶¹Ibid., p. 29.

⁶²Ibid., p. 30.

⁶³Putnam (1976), p. 88.

way of calling attention to the alarming looseness of the *use* these philosophers were making of the notions, and to their exaggerated confidence in the clarity of the notion of *meaning*".⁶⁴

Already in 1957, however, Putnam strongly criticized Quine's arguments that there are no analytical truths. The arguments of this critique have had a longstanding effect on Putnam's thought on the matter, and it also influenced Quine to slightly alter his view on analyticity in *Word and Object*.

Putnam argues that there should be some trivial examples of analyticity, but that this class (larger than the logically true statements, which Quine himself seems not to have excluded) is a small class, and not very interesting. It is simply not enough of a foundation upon which we can build a positivist theory of truth by convention.

Putnam says that Quine is wrong in refuting the *distinction* between the analytic and the synthetic. Surely there is a distinction to be made, but it is a thin one, one which does not have much to offer, except for examples of analytic statements such as "all bachelors are unmarried".

There is very little (if anything) to be made use of in philosophy by such a *linguistic* distinction, which certainly is no dichotomy between statements into analytic and synthetic ones; we cannot say that any sentence is either analytic or synthetic. No philosophical bread is baked and no philosophical windows are cleaned, says Putnam, by the distinction between the analytic and the synthetic. However, what seems important to Putnam is rather that there are many statements that do *not* fall into the categories of analytic statements or synthetic statements. In fact, this class of statements is much bigger than philosophers generally have supposed it to be. Furthermore, there may be no general class to which these statements belong, nor some common feature that all such statements can be said to share. In particular, the laws of science may not be categorized as either "analytic" or "synthetic", and we should *not* call principles which Carnap referred to as analytic, or "*L*-true", as a particular class of statements, e.g. as *framework principles*, "as if one were to take seriously the label I have been using".⁶⁵

The way in which Putnam (1957)[1962] tries to save this "uninteresting class" of analytic statements from Quine's arguments is by his famous introduction of *law-cluster concepts*, a modification of *cluster concepts*. (Putnam also refers to a metaphor by Wittgenstein of a rope with a great many strands, no one which runs the length of the rope.) Putnam asks first whether it is an analytic statement that all men are rational? And is it an analytic statement (as Aristotle thought) that all men are featherless? Surely we can think of an irrational man,

⁶⁴Ibid., p. 89n.

⁶⁵Putnam (1975)[1979b], p. 39.

etc. One could continue in this way making a list of attributes P_1, P_2, \dots , asking whether there is a man without P_1 ? Could there be a man without P_2 ? Putnam writes that we cannot abandon a large number of these extensions without holding that we have not changed the meaning of “man” in a significant way. For cluster concepts, we say that the meaning is given by a cluster of properties. If most properties in the cluster are present, then we would say that we were dealing with a “man”, for example.⁶⁶ Putnam now introduces a *law-cluster* concept, in analogy with cluster concepts, but with the difference that the law-cluster concepts are not constituted by a bundle of properties, but by a cluster of laws which determine the identity of the concept. Putnam uses as an example the concept of energy: “It enters a great many laws. It plays a great many roles, and these laws and inference roles constitute its meaning collectively, not individually.”⁶⁷ Most of the terms in highly developed science are law-cluster concepts, says Putnam, and we should always be suspicious of claims that a principle “whose subject term is a law-cluster concept is analytic”. In general, any law can be abandoned without destroying the identity of the law-cluster concept, in very much the same way as a man can have feathers grown all over his body.⁶⁸

The statement that kinetic energy is *defined* through $e = \frac{1}{2}mv^2$ was perfectly fine before Einstein; the formula and “kinetic energy” may always be substituted for each other in the pre-relativistic physics. But after Einstein, all physical laws must be Lorentz-invariant. For Einstein kinetic energy becomes

$$e = mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}mv^4 + \dots$$

One could of course argue here, Putnam says, that mc^2 stands for rest energy and that we only refer to kinetic energy the other part, but this part still contains other and higher powers of v , and these terms are small (if we set $c = 1$ and $v \ll c$). But the most important observation is that once upon a time kinetic energy was given by definition, stipulated as it were, but after Einstein it became a physical natural law: “there was a whole set of pre-existing physical and mechanical laws which had to be tested for compatibility with the new body of theory. Some stood the test unchanged—others only with some revision. Among the equations that had to be revised [...] was the equation $e = \frac{1}{2}mv^2$.”⁶⁹ It is not that Einstein has *changed the definition* of kinetic energy, it is rather that the concept has been placed on a par with other natural laws in Einstein’s new theory.

Putnam now considers the difference between a statement such as “all bachelors are unmarried” and a stipulation of the kind $e = \frac{1}{2}mv^2$ in physics. The

⁶⁶Ibid., p. 52.

⁶⁷Ibid., p. 52.

⁶⁸Ibid., p. 52.

⁶⁹Ibid., p. 45.

point is that we cannot reject “all bachelors are unmarried” unless we make some change in the meaning of “bachelor”, in fact the extension has to be changed. In the example of kinetic energy, however, we have given up the formula $e = \frac{1}{2}mv^2$, but the extension of kinetic energy is still the same; the kinetic energy of a particle is *literally the energy due to its motion*, since “kinetic energy” is not an idiom, but consists of the words “kinetic” and “energy”, two words which have not changed their meaning.⁷⁰

The difference between the two examples, “all bachelors are unmarried” and “ $e = \frac{1}{2}mv^2$ ”, is that “bachelor” is not a law-cluster concept; instead there is an *exceptionless* “law” that someone is a bachelor if and only if he has never been married. Such an exceptionless law has two characteristics:

- (1) there are no other exceptionless “if and only if” statements associated with the noun (bachelor);
- (2) the exceptionless “if and only if” statement is a *criterion* in the sense that speakers can tell whether someone is a bachelor by verifying if he has been unmarried or not.⁷¹

In the trivial cases of analytic statements, such as “all bachelors are unmarried” and “all vixens are foxes”, a revision of the truth of such a statement would come down to changing the meaning of a word, let’s say the meaning of “bachelor”, to include some married men in a trivial way. But for the vast majority of non-trivial statements, revision of principles in conceptual systems does not follow from the change in the meaning of words. Putnam writes that he would like, with Quine,

[...] to stress the extent to which the meaning of an individual word is a function of its place in a network, and the impossibility of separating, in the actual use of a word, that part of the use which reflects the ‘meaning’ of the word and that part of the use which reflects deeply imbedded collateral information.⁷²

As mentioned earlier, in *Word and Object* Quine adopted Putnam’s view:

Putnam in “The Analytic and the Synthetic” has offered an illuminating account of the synonymy intuition in terms of a contrast between terms that connote clusters of traits and terms that do not. My account fits with his and perhaps adds to the explanation.⁷³

⁷⁰Ibid., p. 52–53.

⁷¹Putnam (1976), p. 89.

⁷²Putnam (1975)[1979b], pp. 40–41.

⁷³Quine (1960b), p. 57n.

6. Quine and Putnam on *Logical Syntax*

Quine now more directly focuses on Carnap's *Logical Syntax* and considers whether it would indeed be possible to make a distinction between analytic statements and synthetic ones in an *artificial language* with explicit "semantical rules", since ordinary language is imprecise. Quine considers the Carnapian context that a statement S is analytic in such an artificial language L . Quine's critique is that, just as in the cases we have already treated, the word "analytic" is unclear, even if it is applied in a very formal context, such as assuming that we have an artificial language L_0 whose semantical rules have the explicit form of a specification, by recursion for instance, of all the analytic statements of L_0 . The point is that we do not *understand* the word "analytic"; even if we understand what expressions the rules attribute analyticity to, we do not know what the rules attribute to those expressions.⁷⁴ Even if we give a conventional definition to "analytic for L_0 ", we have not explained the word "analytic", or "analytic for". If, on the other hand, we say that we have a certain class of statements whose truths are *stipulated* by semantical rules, a rule of truth, then perhaps we can have a clear case of analyticity, Quine rhetorically suggests, then "a statement is analytic if it is (not merely true but) true according to the semantical rule". But this again means that we appeal to an unexplained phrase, that of "semantical rule".⁷⁵

Furthermore, Quine thinks that an additional problem is that one has been

tempted to suppose that the truth of a statement is analyzable into a linguistic component and a factual component. Given this supposition, it next seems reasonable that in some statements the factual component should be null; and these are the analytic statements. But for all its a priori reasonableness, a boundary between analytic and synthetic statements simply has not been drawn. That there is such a distinction to be drawn at all is an unempirical dogma of empiricists, a metaphysical article of faith.⁷⁶

Here Quine is critical of Carnap's idea that in an artificial language L we can clearly separate the analytic statements, which are true by virtue of meaning alone, from the purely synthetic statements, which purports to describe reality. Even if Carnap can say that in an artificial language L he has postulated that two statements have the same meaning, and consequently he can say that every A is a B by virtue of the meaning of the words in an artificial language L , this does not mean that such a statement would have an usefulness in science. In the *Analytic and the Synthetic* (1957)[1962], Putnam argues that Quine's main argument is that any trivial stipulation ($A = B$) could of course be seen as an analytic statement,

⁷⁴Quine (1951)[1953], p. 33.

⁷⁵Ibid., p. 34.

⁷⁶Ibid., pp. 36–37

but not as part of any interesting statement, as a part of *science*.⁷⁷ In science, there are for Quine no stipulated truths that can withstand revision, as we will see. And, of course, *theories in science* are what Carnap has in mind when he builds up his notion of “true in *L*” in *Logical Syntax*.

In *Convention: a theme in philosophy* (1981)[1983], Putnam discusses Carnap’s project of using artificially constructed languages as rational reconstructions of science. In Carnap’s *Wissenschaftslogik*, “Conventionalism was a form of ‘as-ifisms’,” Putnam writes, and explains Quine’s position as a reaction to this as an incoherent make-believe, which cannot recapture the way logic and mathematics have come into existence. Of course, Carnap does not suggest that it does, but he clearly thinks that we may in principle treat science as if it had come into existence in this way. The point for Quine is that we would not have our present mathematics and logic as they actually are, and function within science, as a result of the adoption of conventions.⁷⁸

One of Putnam’s main themes in his earlier philosophy was to argue that Quine’s position is correct in this criticism of conventional truths, but with fleshed out arguments, examples, but also to correct of Quine’s view that there are no analytic truths. For Putnam, there are; but they play no role in the revisability of scientific statements.

7. Quine’s “historical argument” and Putnam

Quine suggests next that the verification principle of the logical positivists (essentially by reference to Carnap’s *Aufbau* (1928)) is at bottom the very same dogma as the distinction between the analytic and the synthetic. He now gives the following “historical argument”, which, as Putnam writes, has a very different flavor. Here are Quine’s own words, as he formulates his view of a web-of-belief and the possibility of overturning any (scientific) statement.

The totality of our so-called knowledge of beliefs, from the most casual matters of geography and history to the profoundest laws of physics or even of pure mathematics and logic, is a man-made fabric which impinges experience only the edges. [...] A conflict with experience at the periphery occasions readjustments in the interior [...]. Truth values have to be redistributed over some of our statements. Reevaluation of some statements entails reevaluation of others, because of their logical interconnections—the logical laws being in turn simply certain further statements of the system, [...]. [T]here is much latitude of choice of as to what statements to reevaluate in the light of any single contrary experience. [...] Any statement can be held true come what may if we make drastic enough adjustments in the system. Even

⁷⁷Putnam (1975)[1979b], p. 55.

⁷⁸Putnam (1981)[1983], p. 173.

a statement very close to the periphery can be held true in the face of recalcitrant experience by pleading hallucination or by amending certain statements of the kind called logical laws. Conversely, by the same token, no statement is immune to revision. Revision even of the logical law of excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?⁷⁹

It is the last part of the passage cited above that is sometimes called Quine's "historical argument" against analytic truths. The reader should note, however, that it is not a corollary to the earlier arguments purporting to show that analyticity cannot be explained in terms of something we understand better, because that argument presupposed that we indeed had something as a logical truth to build on. Here Quine suggests that even logic (as suggested by theories of quantum mechanics) should be possible to revise in the same sense as has been the case in the overthrow of other major beliefs, such as for instance that of Newton's physics. In fact, the cited argument above does not at all have the same status as the previous arguments against analyticity.

In *'Two Dogmas' revisited* (1976)[1983], Putnam suggests that Quine should have said that his "historical" argument is in fact a refutation of *a priori* truths. Why did Quine view this argument as another argument against analytic truths? Putnam argues that he did so, because the logical positivists held that

to fix a statement's range of confirming experiences is to fix its *meaning*, and that this meaning-fixing is done by *stipulation*. As a part of their view, the positivists held that *a priori* statements (statements with the universal range of confirming experiences) are *true* by *meaning alone*. And since truth by virtue of meaning is analyticity, it followed (for the positivists) that the apriority is analyticity.⁸⁰

The view that apriority is a form analyticity is something we have met earlier, for instance in connection with Hahn and Schlick, according to Goldfarb's description, but also in Burgess' interpretation of Quine (recall the quote in Section 4). In Putnam's eyes, Quine was confusing analyticity and apriority because he took on the alleged assumptions of the logical positivists he was attacking.⁸¹ The confusion arises from thinking of analytical truths as those having a universal range of confirming experiences,⁸² that is, that they are confirmed no matter what our experiences are, but this is to epistemologize the notion of analyticity

⁷⁹Quine (1951)[1953], p. 43.

⁸⁰Putnam (1976)[1983], p. 92.

⁸¹Ibid., p. 92.

⁸²Ibid., p. 90.

in the way Hahn and Schlick epistemologized tautologies. In Goldfarb's description of Carnap, we do not meet this conflation of tautologies with an epistemic notion such as apriority, and neither is Carnap's later notion of analyticity an epistemic notion. Putnam, however, describes all the positivists as subscribing to an epistemological view as regarding analytic truth, and that Quine did not distance himself from it. However, Putnam thinks it was fortunate, in spite of this confusion, that it did not invalidate Quine's argument against apriority.⁸³

The story now becomes more complicated, because Quine, as we have seen, thinks that no statements are confirmed once and for all no matter what (that is, there are no statements with a "universal range of confirmation"), with specific reference to the sciences, but at the same time he uses logical truths (which he thinks may undergo revision) as a basis for his analysis of analytic truths, in that he gives a "linguistic" definition of analyticity. Putnam writes that the definition of analyticity proposed by Quine is that a statement is analytic if it can be obtained from a truth of logic by substituting synonyms for synonyms. But then the logical truths become, in Putnam's words, "analytically analytic", since their analyticity relies only on the identity substitution; hence the notion of analyticity cannot be a substantive thesis about logic,⁸⁴ since not even logic is immune to revision. Hence, in Putnam's view (1976), Quine's investigation of analyticity by means of linguistic terms, such as synonymy, presupposes logic. The "historical argument" shows that these logical truths may themselves undergo revision, but this is an argument against their apriority, Putnam says.

Although Putnam is not impressed by Quine's detailed argument against analytic truths based on the redundancy arguments, he believes (1976) that Quine's argument against (absolute) *a priori* truths is valid, but at the same time we may have to accept the existence of linguistically analytic truths. This has importance for Putnam's 1970s project of developing Quine's suggestion that quantum logic is indeed a correct violation of classical logic.

It is important to note here that, although Putnam is sensitive to the fact that Quine in his critique of the notion of analyticity, subscribes to a picture of the logical positivism, essentially similar to that of Hahn and Schlick, this awareness does not in itself mean that Putnam has freed himself entirely from its influence, even though he adopts a more traditional terminology of the *a priori*. Putnam emphasizes the epistemological picture that all statements may be revised due to new information (or a new theory supplying this information); hence there is essentially no functional difference between statements of a theory, as in Freedman's Carnap-influenced view of a relative notion of the *a priori*.

⁸³Ibid., p. 92.

⁸⁴Ibid., p. 94.

8. Friedman's critique. Carnapism against Putnam and Quine

Let me first point out that this very same passage of Quine's so-called historical argument has been of central concern to Friedman, as he wants to criticize this very picture, in particular Quine's web-of-belief, where all sentences are on a par. For Friedman, this is a central point in the development of his own version of a relative notion of the *a priori*. Friedman has argued that Carnap's picture as given in *Logical Syntax* may in spirit be correct, but that he has exposed himself to Quine's critique because of the particular way he has built up his linguistic frameworks, relying on a dichotomy between analytic and synthetic sentences relative a language *L*. Friedman concedes that Quine's arguments against Carnap are strong: "I have no desire to defend Carnap's particular way of articulating this distinction here",⁸⁵ and Putnam writes in 1990 that "Quine is surely right that the old notion of analyticity has collapsed, and I see no point in reviving it".⁸⁶

I think that even Friedman exaggerates the importance of Quine's critique. It is not clear, given Quine's arguments, that one has to abandon Carnap's general picture, although of course these arguments were important at the time to clarify the many problems of using the old Kantian "analytic" and "synthetic" for the picture that Carnap wanted to suggest. I sympathize with Burgess' formulation of a key difference between the Carnap of *Logical Syntax* and Quine as whether a question like "If I have as many fingers as toes, is the number of my fingers equal to the number of my toes?" can arise in more than one sense. For Carnap, there are two senses to the question, one of which is internal and the other which is external to the linguistic framework of (the concept of) number, whereas for Quine there is only one question here. Burgess concludes that if we take the concept of number for granted (i.e., with respect to such a linguistic framework), then we obtain an analytic statement: "if one has as many fingers as toes, then the number of one's fingers is the same as the number of one's toes", since it comes with the number concept. In the external sense, we can always ask whether the number concept should be accepted, and Quine cannot (says Burgess) recognize any distinction between these two questions, which "lays Quine open to the objection, raised especially by Charles Parsons, that his account of matters cannot do justice to the felt obviousness of elementary mathematics".⁸⁷

One major mistake in Quine's picture, according to Friedman, is that he views the holistic theory as consisting of statements that may all equally well be refuted, in the logical sense that if we arrive at a contradiction within a theory, then strictly speaking, any of the statements in the theory may be false, although the mathematical and logical statements certainly will have to be given up last, if

⁸⁵Friedman 2001, p. 33.

⁸⁶Putnam (1990)[1994], pp. 251–252.

⁸⁷Burgess (2004), pp. 47–48.

at all. But at the end of the day what saves these mathematical and logical parts of a theory is only pragmatic principles, or a behavioristic reluctance to give them up. For Friedman, the constitutive part of a theory, such as the calculus within Newton's mechanics,

is not happily viewed [...] as a conjunction of elements symmetrically contributing to a single total result. For one element of a conjunction can always be dropped while the second remains with its meaning and truth-value intact. In this case however, the mathematics of the calculus does not function simply as one more element in a larger conjunction, but rather as a necessary presupposition without which the rest of the putative conjunction has no meaning or truth-value at all. The mathematical part of Newton's theory therefore supplies elements of the language or conceptual framework, we might say, within which the rest of the theory is then formulated.⁸⁸

Hence, on Friedman's view, the mathematical (or other theoretical parts of a physical theory) may have an *a priori* status in the sense that this constitutive part "supplies elements of the language of conceptual framework" in which the empirical part can be formulated.

In general terms, Friedman is quite correct. His criticism also hits Putnam, who certainly has an epistemological view in the sense that Quine does, that is, a view in which all statements, *qua* statements, have the same status.

Friedman writes:

Quine is correct that pure formal logic is insufficient to characterize the relativized and dynamical, yet still constitutive notion of a priori principles Carnap was aiming at. [...] Yet [...] careful attention to the actual historical development of science, and, in particular, to the profound conceptual revolutions that have led to our current philosophical predicament, shows that relativized a priori principles of just the kind Carnap was aiming at are central to scientific theories. Although Carnap may have failed in giving a precise logical characterization or explication of such principles, it does not follow that the *phenomenon* he was attempting to characterize does not exist.⁸⁹

Friedman continues to argue that support for Carnap's attempt to formulate a theory of science through his linguistic frameworks, is provided by Thomas Kuhn's study of scientific revolutions, since the change in paradigm corresponds to change of language or linguistic framework in Carnap's distinction between

⁸⁸Friedman (2001), pp. 35–35.

⁸⁹Friedman (2001), p. 41.

external and internal questions, the latter questions being asked and (sometimes) answered within what Kuhn describes as *normal science*.⁹⁰

Carnap, however, thinks that one should use an artificial language to formulate the frameworks of science, whereas Friedman prefers “natural language” formulations and bases his views on historical investigations of the constitutive functions of parts of physical theories. Putnam has described the modernistic tendency in Carnap’s work to invoke artificial languages:

Carnap makes no bones about the fact that he regards planned language and planned society as clearly superior to unplanned language and unplanned society, just because they are *planned*. The formal systems that he talks about are seen by Carnap not as mere rational reconstructions of the language scientists use, but as forerunners of a future symbolic language that scientists will employ *instead of* unformalized language. Logical empiricism, at least in Carnap’s hands, turns into a sort of futurist intellectual architecture.⁹¹

Even with a somewhat lower ambition of connecting philosophy of language to these issues of conventional and constitutive principles, I think that Putnam would hold that Friedman is trying to escape the difficult problem of trying to formulate a conventional part of a scientific theory. Furthermore if one wants to suggest that there are conventional parts of a theory, then one should not evade the difficulties in the *philosophy of language* that have surrounded such suggestions throughout 20th century philosophy.

Before the “linguistic turn”, the ideas connected with the synthetic *a priori* as well as the emerging critique of this notion in the 1890s began to look quite obscure, since the terms involved in an explanation of a statement such as “all bachelors are unmarried” were obscure; one would say that the “conception” of the predicate (being unmarried) is contained in the idea or conception of the subject (bachelor), whereas the arithmetic truths such as $5 + 7 = 12$ were synthetic *a priori*, following Kant. To escape this fuzzy discussion, Putnam writes that philosophers of otherwise very different strands were in agreement that an investigation of the way humans understand and use language is of central importance.⁹² Putnam certainly views Carnap, as well as Quine and Wittgenstein, as part of this project of clarification. Although Putnam would say that historical investigations are important to provide examples, he would not think that this in itself is enough in order to get a clear picture of what a conventional statement is, even in a scientific theory.

⁹⁰Ibid., p. 41.

⁹¹Putnam (1981a)[1983], p. 172.

⁹²Ibid., pp. 170–171.

Friedman needs to give an account of “meaning” (or some idea of how to replace this notion), since he obviously thinks that the relative *a priori* principles are constitutive in the sense that empirically testable hypotheses cannot be meaningfully formulated without them; this is essential to his criticism of Quine’s holism. Without an account of *how* the *a priori* principles constitute meaning, Friedman’s description of scientific theories remains merely historical. But Friedman clearly retains some of Carnap’s ambition to explain the different functions of any scientific (physical) theory about the world, even if Carnap’s ambition was slightly different in the sense that he wanted to reconstruct science.

CHAPTER 2

Putnam and New Wittgenstein

Putnam ends his 1990 paper (I use the standard 1994 publication), *Revisiting Mathematical Necessity*, now influenced by Wittgenstein, by saying that naturalized epistemology seems most obscure when applied to mathematics and logic. A few sentences earlier he says that “[t]rying to justify mathematics is like trying to say that whereof one cannot speak one must be silent; in both cases, it only looks as if something is being ruled out or avoided.”¹

The main new thing in the 1990 paper is that Putnam has been influenced by the New Wittgenstein interpretation of the *Tractatus* (and the later Wittgenstein for that matter), which is the influence that has led him to hold that the very question of the revision of logic and mathematics is *unintelligible*.

I will argue that although he finds the earlier epistemological view to some extent unintelligible, he has little or nothing to replace the old picture with; there is still no explanation of the different functions of different parts of a theory, and mathematical statements are essentially still possible to overturn in the light of new findings in the sciences, a deeply problematic view, or so I will argue.

1. Putnam and Conant

Putnam aims to find a different way than Carnap’s of “stripping away the transcendental baggage” in a line of thought to which he is sympathetic, namely, a certain line of thinking that goes back to Kant and Frege, and which “has features in common with the philosophy of the later Wittgenstein rather than that of Carnap”.²

What Putnam finds attractive in the thought of Kant, and which he claims has helped him to understand Wittgenstein’s position in the *Tractatus* that the truths of logic are “tautologies”, or *sinnlos*, is Kant’s view that the logical truths are not descriptions of facts, not even in the sense that logic describes what may hold in “metaphysically possible worlds”. It is rather the form of coherent thought—to explain anything presupposes logic, which has no metaphysical presuppositions at all.³ This reading of Kant convinced Putnam that there is a

¹Putnam (1990)[1994], p. 260.

²Ibid., p. 246.

³Ibid., p. 247.

way of conceiving the logical laws as *sinnlos* in the *Tractatus*' sense without subscribing to the conventionalism of Carnap. In fact, Putnam believes that this view was by and large Frege's view as well, although it was the "waffling" on this issue (Frege sometimes held that the laws of logic were the most general natural laws) that led Wittgenstein to his own version of Kant's view.⁴

Carnap's conventionalism, as interpreted by Quine in 'Truth by Convention' [...], was an *explanation* of the origin of logical necessity in human stipulation; but the whole point of the Kantian line is that logical necessity neither requires nor can intelligibly possess any 'explanation'.⁵

Putnam has been influenced by Wittgensteinian ideas in a certain direction since 1987, when he apparently read Cora Diamond's *The Face of Necessity*, and somewhat later he was influenced by James Conant in the same direction, namely, in the direction of the so-called resolute reading of the *Tractatus* (the New Wittgenstein interpretation). The 1990 paper was in fact re-published in the volume *The New Wittgenstein* (2000), and the ideas of Putnam's paper have been extensively worked out in this direction by Conant in his *The Search for Logically Alien Thought: Descartes, Kant, Frege, and the Tractatus* (1991). Conant embraces Putnam's ideas, and writes:

[T]he story that emerges is one which I find myself wanting to tell. I will argue at the end of the paper that this story sheds a helpful light on why the text of the *Tractatus* assumes the *form* that it does—one of having the reader climb up a ladder which he is then asked to throw away.⁶

Conant identifies the following passage in Putnam's 1990 paper (part of which I quoted in the beginning of this section) with the New Wittgenstein *reading* to which he himself has contributed.

If it makes no sense to say or think that we have discovered that [...] [logic] is *wrong*, then it also makes no sense to offer a reason for thinking it is not wrong. A reason for thinking [...] [logic] is not wrong is a reason which excludes nothing. Trying to justify [...] [logic] is like trying to say that whereof one cannot speak one must be silent; in both cases it looks as if something is being ruled out or avoided.⁷

The point of Conant's comment is that Putnam now conforms to Conant's own interpretation of the final line of the *Tractatus*; that we are faced with a silence,

⁴Ibid., p. 248.

⁵Ibid., p. 248.

⁶Conant (1991), p. 128.

⁷Conant (1991), p. 156; Putnam (1990)[1994].

and that nothing has been said; it is not a question of deep nonsense, just “*einfa^{ch} Unsinn—simply nonsense*”. Conant continues by turning to §374 of the *Philosophical Investigations*, where “Wittgenstein formulates the task of philosophy as follows: ‘The great difficulty here is not to represent the matter as if there were something one couldn’t do.’”⁸

This is in effect a version of Putnam’s main objections to Quine’s view of the revisions in science, including mathematics and logic. Putnam suggests that the “cans” in the following classical lines by Quine are not intelligible “cans”.

Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system. Even a statement very close to the periphery can be held true in the face of recalcitrant experience by pleading hallucination or by amending certain statements of the kind called logical laws. Conversely, by the same token, no statement is immune from revision.⁹

Conant tries to illustrate his common position with Putnam through this particular reading of the *Tractatus* by means of an analogy with the Scholastic question of whether God could alter the logical truths. St Thomas Aquinas held that that not even God could alter the laws of logic, whereas Descartes thought such ideas were blasphemous since indeed God is capable of anything—He can even make contradictions true together! But how can Descartes know all this, if we are finite beings created to believe and not doubt the laws of logic, we who cannot comprehend all these alternatives that God could bring about? The argument here is that although we cannot of course *comprehend* these alternatives, we can *apprehend* a world in which contradictions were true; we cannot *embrace* the contingency of these (logical) truths, but we can *touch* it in our thought.¹⁰ From this brief summary of how Conant argues, it is clear that Descartes’ attempt here to explain the ultimate possibility of revising the “laws of logic” resembles Quine’s predicament. Conant explains:

We want to frame a thought (about that which we cannot be thought) but we run up against the problem that the thought we want to frame lies in its very nature beyond our grasp. [...] We need a way to think right up close to the edge of the limit of thought, close enough to get a glimpse of the other side. Descartes’s distinction between what we can embrace in thought and what we can only touch in thought is an attempt to characterize what is involved in trying to think both sides of the limit.¹¹

⁸Conant (1991), p. 156.

⁹Quine (1951)[1953]; Putnam (1990)[1994], p. 256.

¹⁰Conant (1991), 119–120.

¹¹*Ibid.*, p. 121.

As we will see, it becomes an important part of Putnam's later philosophy to embrace a view of the unintelligibility of asking certain questions, and in particular to assume that we can, in Quine's spirit, intelligibly say that we may revise our beliefs, or, in the spirit of Carnap, that pictures such as "logically possible worlds" make sense in a philosophical discourse. This is a part of the New Wittgenstein interpretation, which is an important influence on Putnam's thought, from 1990 around on, including the late *Ethics without Ontology* (2004). However, in a recent reply to Sami Pihlström (2006), Putnam delivers criticism of the New Wittgenstein interpretation (the resolute reading), in particular of the consequences of this "reading" drawn by its exponents, namely their view that philosophical "confusions" arise only from "failing to see that some piece of philosophical prose is *literally* 'nonsense'."¹² Putnam emphasizes there that he believes that they (James Conant, Cora Diamond, as well as the Norwich school, I presume) have understood Wittgenstein's thought well (at least in many respects) but not his philosophical prose. For Putnam (and his Wittgenstein, I would add), philosophical problems are not literally nonsense, nor or are they confusions of a *linguistic* nature.¹³

2. The analytic-synthetic distinction

Putnam writes that Quine's view on the traditional distinction analytic-synthetic should perhaps be viewed as continuum, and that it would perhaps be only from "behavioristic reluctance" to give up the traditional laws in classical logic that we would in the end retain these.

Putnam thinks that Quine is right about giving up the old analytic-synthetic dichotomy, but he does not think that this implies that we should replace it with a continuum as regarding the kind of truth (empirical versus non-empirical) between the three statements:

(1) It is not the case that the Eiffel Tower vanished mysteriously last night and in its place there has appeared a log cabin.

(2) It is not the case that the interior of the moon consists of Roquefort cheese.

(3) For all statements p , $\neg(p \wedge \neg p)$.

For Putnam, there is a matter of continuity between the statements (1) and (2) in this respect, in that perhaps he can be convinced that it is a mere "psychological fact" about *him* that the falsity of (2) seems "harder to imagine" than the falsity of (1). But to convince him of the falsity of (3), one

¹²Putnam (2006a), p. 71.

¹³Ibid., p. 71.

would have to put an alternative logic in the field” – “and *that* seems a fact of methodological significance”.¹⁴

For Putnam, what is necessary and what is empirical is conditioned on some conceptual scheme; there is no absolute division into empirical and non-empirical statements. The examples (1) and (2) above should be viewed as empirical and (3) as necessary, *relative our conceptual scheme*. However, as Putnam remarks in reference to Wittgenstein’s *On Certainty*, the statement *Water has boiled in the past* (water has boiled on at least one occasion) may look like a paradigmatic empirical statement, but then again: what would falsify or disconfirm it?

Given their agreement that there are no sharp boundaries between empirical and non-empirical statements, one would think that there is no essential difference between Quine and Putnam regarding necessity: since there are no sharp boundaries between what is empirical and what is not, it would seem to follow that any statement may be revised given new empirical information.

The radical difference between his own position and that of Quine’s consists for Putnam now in the challenge of the very intelligibility of Quine’s slogan that “no statement is immune to revision”.¹⁵ The question for Putnam is again not only whether we can revise a statement such as “Water has sometimes boiled”, but what sense “can” has here. In our present conceptual scheme, logical truths are statements whose negations we presently do not understand. It is not the case that they are unrevisable,

[...] it is that the question ‘Are they revisable?’ is one which we have not yet succeeded in giving a sense.¹⁶

3. Putnam on Quine vs. (the New) Wittgenstein

Putnam recalls Quine’s view that *translation practices* may come down to nothing more than retaining the same logical laws; otherwise we should question the translation manual. A revision of the logical laws may hence just come down to a *change of meaning* of the logical particles.

The point Putnam wants to raise in connection with the *unintelligibility* of revising logic and mathematics is that although Quine “rejects talk of ‘meaning’ and ‘synonymy’ [...] when fundamental metaphysical issues are at stake [...]”,¹⁷ Quine still opens the door for *allowing questions* such as whether we can

¹⁴Putnam (1990)[1994], p. 250.

¹⁵Ibid., p. 253.

¹⁶Ibid., p. 256.

¹⁷Ibid., p. 253.

[...] conceive of a community of of speakers whom (1) we would interpret and understand, and who (2) assent to a sentence *which we would translate as* ‘ $7 + 5 = 13$ ’.¹⁸

The reason is that, in *Word and Object* and elsewhere, Quine thinks that the only way to make sense of revising the *laws of classical logic* is through changing the “meaning” of the logical particles (or that one just “changes the subject”). In Quine’s *Carnap and Logical Truth*, Putnam finds support for this view:

Deductively irresolvable disagreement as to logical truth is evidence of deviation in usage (or meanings) of words.¹⁹

And Putnam finds in *Word and Object* that Quine argues that

it is part of translation practice to translate others so that their truth functions come out the same as ours (otherwise, one simply does not attribute truth functions to them) and so that ‘stimulus analytic’ sentences have stimulus analytic translations.²⁰

That is, Quine still holds that it may be very difficult to make any sense of revising sentences in classical logic, other than having a different stimulus-meaning for these; for Quine it is, of course, a change of “meaning” only in a naturalistic/behavioristic sense. Quine’s reason is that otherwise we would not trust our translation manuals, but it only comes down to this. For Putnam, this is evidence enough to conclude that for Quine it is possible to *raise the question* whether “ $7 + 5 = 13$ ” in a culture, whose language we may be able to translate.

Putnam now wants to challenge this general idea which he identifies in a broad spectrum of analytic philosophy. He proposes to retain the following idea from *It Ain’t Necessarily So*: We are presently not able to attach a clear *sense* to “*B* can be revised” if we cannot *describe* circumstances under which a belief *B* would be falsified.²¹ Furthermore, he combines this view with his reading of Wittgenstein to conclude:

In such a case we cannot, I grant, say that *B* is ‘unrevisable’, but neither can we intelligibly say ‘*B* can be revised’.²²

Hence, $5 + 7 \neq 12$ is (at present) unintelligible.

¹⁸Ibid., p. 253.

¹⁹Ibid., p. 261n8.

²⁰Ibid., pp. 261–262n8.

²¹Ibid., p. 253.

²²Ibid., pp. 253–254.

4. Riddles, meaning and sense

In order now for Putnam to *explain* the unintelligibility of, for instance, a challenge to Newton's physics before Einstein, but at the same time to explain how something we earlier could not doubt indeed turned out to be a false theory, Putnam turns to a metaphor used by Cora Diamond. It is her use of Wittgenstein's likening of difficult mathematical problems ("interesting mathematical questions") to riddles, in which the answer is unexpected, but makes sense when we know the solution, and also gives sense to the riddle or to the mathematical problem. Wittgenstein says that the difficulty facing the mathematician without a method of solution is

like the problem set by the king in the fairy tale who told the princess to come neither naked nor dressed, and she came wearing a fishnet. That might have been called not naked and yet not dressed either. He didn't really know what he wanted her to do, but when she came thus he was forced to accept it. It was of the form 'Do something which I shall be inclined to call "neither naked nor dressed".' It's the same with the mathematical problem. 'Do something which I shall be inclined to accept as a solution, though I don't know now what it will be like'.²³

Putnam claims that the words "naked" and "dressed" would not have to change their dictionary meanings; knowing the *sense of a statement* or question is knowing how the words are used in a particular context. I may use the literal meaning of words, but not understand what is meant by the use of those words.²⁴

What Putnam is after is this: we may use a "translation manual", thereby obtaining a literal translation, but this would perhaps not give us the sense of the statement, that is, how the words are used in a particular context.²⁵

One key example Putnam gives us is from the history of science.

'Momentum is not the product of mass and velocity' once had no sense; but it is part of Einstein's achievement that the sense he gave those words seems now *inevitable*. We 'translate' (or read) old physics texts homophonically, for the most part; certainly we 'translate' *momentum* homophonically. We do not say that the word 'momentum' used not to refer, or used to refer to a quantity that was not conserved; rather we say that the old theory was wrong in thinking that momentum was exactly *mv*. And we believe that wise proponents of the old theory *would* have accepted our correction had they known what we know. So this is not a case of giving a word a new meaning [...]. But

²³Diamond (1991), 267.

²⁴Putnam (1990)[1994], 256.

²⁵Ibid., 256.

that does not alter the fact that the sense we have given those words (or the use we have put them to) was not available before Einstein.²⁶

The translation idea using “homophonic practice” is still an adaptation to Quine’s way of looking at the problem; the “meaning” (stimulus-meaning for Quine) of the word “momentum” has not changed (in a significant way)—it is the *theory* that has changed. So far, this is the same picture as Putnam gives in *The Analytic and the Synthetic* and which is retained in Quine’s *Word and Object*. The Wittgensteinian point Putnam wants to add to this picture is that the senses of the words have changed. Putnam indirectly argues against verificationist philosophers such as Dummett, who would hold that the sense and meaning here is the same thing and that sense=meaning now has changed, so that in particular the sentences involving the old and new meaning of the word momentum are different statements. For Putnam, the sense of the statement (or of the words involved) has changed, since we may use it in a new context, which is not precluded in the “literary meaning” of the word. To recapitulate the discussion of kinetic energy from *The Analytic and the Synthetic*, Putnam’s view in 1990 is the same as it was in 1957: kinetic energy still means the same thing (it has the same extension), since the words “kinetic” and “energy” retain their meanings. They literally mean the same thing as before, but the sense has changed; we use the words differently and within a new context.

Putnam point is that a change of sense (connected to the use of a word) does not entail a change in the meaning of the words in any way close to the picture held by “logical positivists”, so that, for example, “all bachelors are married” is no longer be true, because we have allowed married people to be bachelors as well.

One might here recoil at Putnam’s historical insensitivity in claiming that “wise proponents of the old theory *would* have accepted our correction had they known what we know.” One interpretation would be that Putnam endorses a “strong theory of meaning” (or perhaps also a belief in the rational progress of science), in the sense that if we just explicate the words of our present physical theories, these would be accepted by Newton. However, I think that this is far from what Putnam has in mind. He rather thinks that if we provide Newton with our present context of knowledge and other contexts (such as perhaps current secular values in the scientific community), then Newton would accept Einstein’s theory, quantum mechanics, etc. (This is of course not far from saying nothing at all.)

Putnam now wants to move one step further, and apply to mathematics the basic idea here that scientific principles, such as, for instance, a formula for momentum, may have to change if further information is provided, although it

²⁶Ibid., 257.

was not intelligible to suggest this before new circumstances in the form of a new theory was available. More precisely, Putnam suggests that in principle the same type of change could happen for an example such as $5 + 7 = 12$:

But are we not in the same position with respect to a sentence like ‘In the year 2010 scientists discovered that 7 electrons and 5 electrons sometimes make 13 electrons’? Or with respect to ‘In the year 2010 scientists discovered that there are exceptions to $5 + 7 = 12$ in quantum mechanics’? If this is right, and I think it is, then perhaps we can see how to save something that is right in the Kant–Frege–early Wittgenstein line [...].²⁷

I find this claim that we may perhaps revise an arithmetic calculation such as $5 + 7 = 12$ given new information from the sciences rather confusing. Putnam’s point is that it is not intelligible to question $5 + 7 = 12$, but, within a now completely unknown theory, it would perhaps make sense to have examples where $5 + 7$ is not 12. As we will see, this position is retained in *Ethics without Ontology* (2004), where $5 + 7 = 12$ is an example of a *conceptual truth*.

There are several connections of this suggestion to other parts of Putnam’s writings. One connection we can make is to his earlier work *It Ain’t Necessarily So*. In this work Putnam suggests that we are not changing the meaning of “straight” or “straight line” line as we change the “truth value” of the statement that “we can return to the same place by traveling in a straight line (in the universe)”. The important feature is that we now have access to a context previously unknown to us which makes it possible for us to think that we may actually return to the same place, due to the relativistic properties of physical space. The insight is that we should not think of this universe as imbedded in an even larger container (perhaps) of a Euclidean super-space, which would be very much like applying non-Euclidean geometry on a sphere contained in Euclidean space. By doing the latter, we may of course consider the angle sum of a triangle on the sphere (to be greater than 180°), but we may not view the triangle on the sphere as a “real” triangle. Putnam’s earlier point in *It Ain’t Necessarily So* was that the new systems of axioms were not enough to convince us of the possibility of returning to the origin as we travel in a straight line in space, but that we in the light of the theory of relativity have to accept this new property of a straight line in this previously unknown context, just as, in an analogous sense, we may accept a triangle consisting of edges which are geodesics on the sphere as a *triangle* with the property that the sum of the angles are now different from 180° .

This analogy is perhaps the light of which we should understand Putnam’s “proposal” of the possible revision of $5 + 7 = 12$, preserving the ordinary addition for the cases we are used to now, but in a new and wider and presently unknown

²⁷Ibid., p. 254.

context, perhaps including a new way of adding the number of electrons in the vicinity of an atomic nucleus, where previously unknown things may occur, perhaps due to surprising properties of electrons not behaving as medium-sized enduring objects, in a way analogous to the surprises that the gravitation of the sun “has for our straight lines” as part of relativistic physics.

There is also another way we can take in order to try to understand the possible revision of $5 + 7 = 12$. Even if we now regard “all bachelors are unmarried” as an analytic statement, in particular in view of *The Analytic and the Synthetic*, Putnam claims that the statement itself may suddenly have a new use in science. In *Meaning Holism*, Putnam writes regarding a similar statement, “all vixens are foxes”, the following:

What of less scientific examples? As long as being unmarried (or never-been-married) male adult person is the only known and generally employed criterion for being a bachelor, then the word “bachelor” will continue to function (in purely referential contexts) as virtually an abbreviation of the longer phrase “male adult person who has never been married”. And similarly, unless “vixen” becomes an important notion in scientific theory, and various important laws about vixens are discovered, that word will continue to be used virtually as an abbreviation for “female fox”. [In a footnote Putnam writes: ‘Suppose for example we discovered that vixens are telepathic. If we thought that they were the only telepathic animals, then “vixens are telepathic animals” might come to be even more central than “vixens are female foxes”. And if we then discovered a male telepathic fox, we might well say “a few male foxes are vixens”.’] But either or both of these situations may change *as a result of empirical discovery*, with no stipulative redefinition of these words, and no unmotivated linguistic drift, being involved.²⁸

The question is if we would regard “all vixens are foxes”, or “all vixens are female foxes” as false, or “ $5 + 7 = 12$ ” as false, given new empirical discoveries, “with no stipulative redefinition of these words, and no unmotivated linguistic drift”?

I am not completely sure of what Putnam means by “no unmotivated linguistic drift”. It seems to me that the case he describes above is a form of linguistic drift, which applied to a case such as “all bachelors are unmarried” would mean that we do not change the meaning of the word bachelor so that we immediately get a counterexample to the statement. The whole point should rather be that there is a linguistic drift of some sort, that over time leads to completely new uses of our words, so that “all bachelors are unmarried” no longer has the same place in our language as it has now. For instance, the word “bachelor” may be much more interesting to associate with other things than the marital status of

²⁸Putnam (1986)[1990a], pp. 290, 337n.

people, which may function in a very different way. The new bachelors may perhaps start to marry each other, and perhaps people still think that they are bachelors in some sense.

We saw in Chapter 1 that Putnam has argued that “all bachelors are unmarried” is an analytic statement. We also saw that it was an essential part of his analysis that “bachelor” is not a law-cluster concept. Instead there was an exceptionless law that someone is bachelor if and only if he has never been married. This was not the case for the concept of energy, which was a law-cluster concept. But recall that although Putnam saved some analytic statements from Quine’s critique of Carnap, he still thinks that Quine was right regarding *a priori* truth (although Quine formulated his argument as yet another argument against analyticity): there are no absolute *a priori* truths, according to Putnam. The point is that “all bachelors are unmarried” and “all vixens are foxes” may be analyzed as analytic statements now, i.e., they may be analyzed as exceptionless laws. But there may be a “linguistic drift” (I think this is a fair description) to the effect that we may start using the words in other ways than before. Quine does not speak about statements or propositions (one could question whether it is the same statement or proposition if “all bachelors are unmarried” is no longer analytic), but rather about sentences, and in some sense this seems more appropriate. But, as we saw in Section 2, not even Quine thinks that we can have a different answer to $5 + 7$ than 12 unless “we change the subject”, i.e., we essentially *do something else*.

Mühlhölzer (2009) discusses the example of the change of sense of the concept “momentum”, and approves of the usage of a sense-meaning distinction in the case of the empirical sciences, but he is skeptical about its applicability in mathematics.

I think that the distinction between ‘meaning’ and ‘sense’, which Putnam presents in ‘Rethinking Mathematical Necessity’, is a very important one, and Wittgensteinians, who strongly tend to be concerned with ‘sense’ alone at the expense of ‘meaning’, should come to grips with it. At the same time, however, it seems to me that this distinction has got substance only in the case of empirical science and not in the case of mathematics.²⁹

According to Mühlhölzer, mathematical necessity is of a different kind, since it is in fact impossible to imagine the negation of “ $5 + 7 = 12$ ”; if we can imagine such a negation, then we are no longer adding. He writes, by comparing the mathematical rule of addition with an empirical experiment, the following.

²⁹Mühlhölzer (2009), p. 17.

But the process of *adding*, when applied to 5 and 7, cannot produce this or that result and remain *this* process: It is the process of *adding* only if it produces the result 12. That is the way we use the word “add” and the corresponding symbol “+”, and in precisely this use lies the mathematical necessity of “ $5 + 7 = 12$ ”.³⁰

One cannot but agree to this comment. Mühlhölzer also goes on to refer to Wittgenstein’s view, as expressed in *Remarks on the Foundations of Mathematics*, VI §8, that another outcome than 12 would show that we have not added 5 and 7.³¹

Mühlhölzer points out that Putnam’s interpretation of Wittgenstein goes wrong in many places, although he is also sympathetic to some of Putnam’s ways of reading Wittgenstein. In particular, Mühlhölzer is critical of Putnam’s view of mathematical necessity, as the latter tends to connect this necessity to what he in the 1990 paper refers to as a *conceptual scheme*. Although Mühlhölzer never mentions the “conceptual schemes”, he is critical of Putnam’s way of describing the necessity of mathematics as relative in a sense that we are unable to “imagine” any alternatives. Instead, Mühlhölzer argues that the necessity of $5 + 7 = 12$ should be viewed as a stipulated necessity, in very much the same sense as it is for Carnap.³²

In Chapter 4, I will argue that $5 + 7$ is a calculation and not a statement that is either true or false. In some sense one can view the “exceptionless laws” as analogous to calculations. The rule of addition should be exceptionless, since otherwise we are not following the rule of addition. One important argument in Chapter 4 will be to explain that $5 + 7 = 13$ is not analogous to the truth of “there is a triangle with an angle sum greater than 180° ”. Putnam suggests that it was not intelligible before the development of non-Euclidean geometry to say that such a triangle could exist. One might ask, is there a corresponding theory which will give us a “richer theory” for addition? We could perhaps have $5 + 7 = 12$ in the old familiar cases just as we still have a Euclidean geometry? It is important to explain the difference between asking whether there is a triangle with an angle sum greater than 180° and whether there are counterexamples to $5 + 7 = 12$. I will argue that $5 + 7 = 12$ should be viewed as a calculation, whereas the truth of “there is a triangle with an angle sum greater than 180° ” *depends* on a calculus, but the calculus itself is not a statement or consisting of several statements which are either true or false.

I believe that Putnam was right in his 1962 analysis of the historical background of the events that led to the rejection of the Euclidean world view. It was

³⁰Ibid., p. 15.

³¹Ibid., p. 15.

³²Ibid., p. 18.

not the case that people just had conflated geometrical and physical principles; it was an overthrow of a world view that took place in 1815–1915. This does not imply that there is not a well-functioning mathematical theory of Euclidean geometry, one which furthermore *cannot* be overthrown in light of new findings in physics. Einstein's theory of general relativity does not threaten a single theorem of Euclidean geometry.

Shanker writes that in a discussion with Waismann and Schlick regarding Einstein's new physics, Wittgenstein "emphasized that what Einstein had effectively demonstrated is that *geometry is syntax*: a system of logical rules which lay down the grammar for describing phenomena"; there is no sense in which physical straight lines are really geodesics—such an argument merely confuses the distinction between *rule* and *application* for a distinction between *pure* and *physical* space.³³

One might also wonder if Putnam does not violate his own New Wittgenstein principles, which seem to deny that we can make sense of the idea of the limit of language, much less the other side of of it, when he entertains the possibility of overthrowing $5 + 7 = 12$.

I think, however, that the serious issue is that Putnam (and Quine) cannot separate the two questions that Burgess highlights as good Carnapian principles: that we separate the question of the choice of our rules from the question of their application.

³³Shanker (1987), 270–271.

CHAPTER 3

Meaning, conceptual relativity and pluralism

In this chapter, I will describe Putnam's views of *meaning*, from the publication of *The Meaning of 'Meaning'* (1975) up to *Ethics without Ontology* (2004). In 1975, Putnam argued that meaning is external and he argued against any notion of meaning that arises "in our heads". He is critical of Carnap and Frege for not being sufficiently critical of psychologism. In particular, he is critical of Carnap's notion of *intension*, essentially the idea that there is something besides extension to the notion of meaning. Such a view was earlier held by Frege in his distinction between *Sinn* (roughly the same notion as intension) and *Bedeutung* (extension). The extensional view of meaning eventually leads Putnam to his internal realism, since such a view of meaning comes into conflict with both a radically skeptical point of view (that we are "brains in a vat"), as well as metaphysical realism, which Putnam now denounces.

As part of his internal realism, Putnam develops a verificationist conception of truth as well as a notion of *conceptual relativity*. Blackburn and Davidson and others have challenged Putnam's notion of conceptual relativity, which has survived in his later work, for instance in *Ethics without Ontology*. I will essentially defend Putnam on this point, and I will describe his development regarding his conception of meaning, which is connected to his conceptual relativism and later also to his notion of conceptual pluralism. This discussion will also be connected to Putnam's problem with the relation between language and reality, deeply connected to his view of meaning, which finally leads him to give up the language/reality dichotomy.

1. The meaning of 'meaning'

Putnam's most famous and most cited paper is *The Meaning of 'Meaning'* (1975)¹, which has influenced even philosophers hostile to many of Putnam's other works. In this paper, Putnam gives a comprehensive account of his earlier latent criticism of the view that as science progresses, and our beliefs change, so do the meanings and referents of our terms. This is the view of philosophers as diverse

¹Re-published in Putnam (1975)[1979b], to which all page numbers refer.

as Carnap, Kuhn and Malcolm.² The primary aim in the paper is to refute a number of philosophical dogmas shared by, for instance, Frege and Carnap. Putnam summarizes his own view in the slogan: “meanings just ain’t in the head”.

Putnam stresses that meanings “don’t exist in quite the way we tend to think that they do. But electrons don’t quite exist in the way Bohr thought they did, either. There is all the distance in the world between this assertion and the assertion that meanings (or electrons) ‘don’t exist’.”³ As we saw in Chapter 1, Putnam has tried to rehabilitate some of the notions attacked by Quine in *Two Dogmas of Empiricism*, for instance, the analyticity of “all bachelors are unmarried”. Perhaps more importantly, in *The Meaning of ‘Meaning’*, Putnam tries to save a notion of meaning from Quine’s conclusion that we should abandon meaning-talk. But in the course of elucidating what meanings really are, we are asked by Putnam not to assume anything we think we already know about “meaning”.

What Putnam now attacks is the notion of intension and the role it has been given in a certain influential tradition of analytical philosophy, which has not been radical enough in expelling psychologism. Extension and intension (or *Bedeutung* and *Sinn*) can be explained in the following way. The extension of rabbit is simply the set of rabbits; similarly, the extension of ‘creature with a heart’ and the extension of ‘creature with a kidney’ is the same if we assume that every creature with a heart possesses a kidney and vice versa. But one may think that these two terms differ in meaning, in a different sense than is captured by the extension. Putnam describes how, in one sense, meaning has become extension, and, in another sense, “meaning” means *meaning*. One such attempt (by Carnap) is to identify this latter meaning, this something other than extension, with *intension*.

Putnam now attempts to show that Frege and Carnap did not go far enough in their critique of psychologism, i.e., of concepts as something *mental*.

Frege and more recently Carnap and his followers, however, rebelled against this ‘psychologism’, as they termed it. Feeling that meanings are *public* property—that the *same* meaning can be ‘grasped’ by more than one person and by persons at different times—they identified concepts (and hence ‘intensions’ or meanings) with abstract entities rather than mental entities. However, ‘grasping’ these abstract entities was still an individual psychological act. None of these philosophers doubted that understanding a word (knowing its intension) was just a matter of being in a certain psychological state [...]. [T]he time-worn example of the two terms ‘creature with a kidney’ and ‘creature with a heart’ does show that two terms can have the same extension

²Floyd (2006), p. 17.

³Putnam (1975), p. 216.

and yet differ in intension. But it was taken to be obvious that the reverse is impossible: two terms cannot differ in extension and have the same intension. [...] For philosophers like Carnap, who accepted the verifiability theory of meaning, the concept corresponding to a term provided (in the ideal case, where the term had 'complete meaning') a *criterion* for belonging to the extension (not just in the sense of being a 'necessary and sufficient condition', but in the strong sense of *way of recognizing* if a given thing falls into the extension or not). [...] So the theory of meaning came to rest on two unchallenged assumptions:

(I) That knowing the meaning of a term is just a matter of being in a psychological state (in the sense of a 'psychological state', in which states of memory and psychological dispositions are 'psychological states'; no one thought that knowing the meaning of a word was a continuous state of consciousness, of course).

(II) That the meaning of a term (in the sense of 'intension') determines its extension (in the sense that sameness of intension entails sameness of extension).⁴

Putnam will now argue that "these two assumptions are not satisfied by *any* notion, let alone any notion of meaning". What has been called psychological or mental states in traditional philosophy hinges on what Putnam calls a methodological solipsism, which assumes that no psychological state presupposes any individual other than the subject who has the ascribed state. Being jealous, for instance, may involve other beings, so we now assume that we talk about psychological states in a narrow sense, with "a significant degree of causal closure". (Putnam does not believe in this procedure, but argues as if it were possible.) Let now *A* and *B* be two terms that differ in extension. We know from (II) above that the meanings = intensions of *A* and *B* are then different. From (I) we have that *knowing the meanings of A* and *B* are psychological states in the narrow sense, and these states must now determine the extensions of the terms *A* and *B*.⁵

[E]ven if meanings are 'Platonic' entities rather than 'mental' entities on the Frege-Carnap view, 'grasping' those entities is presumably a psychological state (in the narrow sense). Moreover, the psychological state uniquely determines the 'Platonic' entity. So whether one takes the 'Platonic' entity or the psychological state as the 'meaning' would appear to be somewhat a matter of convention. And taking the psychological state to be the meaning would hardly have the consequence that Frege feared, that meanings would cease to be public. For psychological states are 'public' in the sense that different people (and people in

⁴Ibid., pp. 218–219.

⁵Ibid., p. 221.

different epochs) can be in the *same* psychological state. Indeed, Frege's argument against psychologism is only an argument against identifying concepts with mental particulars, not with mental entities in general.⁶

Thus neither Carnap nor Frege goes far enough in their repudiation of psychologism—they still play with notions that can be attached to a psychologistic way of defining meaning through concepts or intensions, since nothing stops us from regarding these notions as psychological states. Frege's argument that psychological states are not public fails, as does Carnap's attempt to water down talk of intensions as abstract entities, through the analytic-synthetic distinction and by his later use of "meaning-postulates".⁷

One could perhaps read Putnam's ironic comment about Carnap ("whether one takes the 'Platonic' entity or the psychological state as the 'meaning' would appear to be somewhat a matter of convention") as saying that he could view the meaning-postulates *as-if* they were abstract entities, or mental entities in the narrow sense, but there is no way to distinguish one from the other; it is a "matter of convention".

An important point Putnam now makes is that no one has ever suggested, regarding *indexical* words like "I", "now", "this", "here", etc, that "intension determines extension". Everyone thinks and talks about himself or herself as "I [...]]", but the referent is obviously not the same. Putnam now argues that indexicality extends beyond the *obviously* indexical words. When we say "water", we could just as well say "water *around here*", or "water on this planet", or something else, but there is no useful abstract concept of water.

We now come to (the simplest case of) Putnam's argument that a psychological state does not determine extension. The story is the well-known science-fiction tale in which someone has a Doppelgänger on *Twin Earth*, where everything is the same except that Twin Earth is not filled with water, but with another

⁶Ibid., p. 222.

⁷See Floyd (2006), p. 28. In particular, I refer to her remarks on Frege's attempt to get rid of mental entities. Floyd writes about Frege's inability to tell us what *thoughts* are ultimately made of as follows:

Because of the primacy of logic in framing Frege's notion of *sense*, there are reasons to suppose that this silence is intrinsic to his conception: Frege had no clear stance from which to rule in or rule out any distinctive ontological category for thoughts beyond their being non-cognition-dependent, causally inert, nonspatial, and nontemporal. Thus for all we know, Fregean thoughts are ('mental') Ideas in a Platonic or Absolutely Idealist sense! (p. 29.)

substance, a liquid with a complicated chemical formula, abbreviated XYZ, instead of H₂O. Let us assume that it is very hard to detect any difference between water on Earth and the substance likewise called water on Twin Earth. We may assume that Oscar₁ lives on Earth around 1750, before anyone knew that water has the chemical formula H₂O. We assume more generally that chemistry was not developed either on Earth or on Twin Earth. The Doppelgänger Oscar₂ on Twin Earth is an exact duplicate of Oscar₁, and has the same appearance, feelings, thoughts, etc.

Now the extension of "water" on Earth consists of H₂O molecules, but the extension of "water" on Twin Earth consists of XYZ molecules. We assume that Oscar₁ and Oscar₂ have the same beliefs about what they both call "water", but the referents are not the same: Oscar₂ does not refer to water as we know it on Earth, but to another substance. This shows that the mental state in isolation does not fix the reference. In 1950, the difference between water and the other substance may be apparent both on Earth and on Twin Earth, but the extension of water and the other liquid has not changed: "Thus the extension of the term 'water' (and in fact its 'meaning' in the preanalytic usage of that term) is *not* a function of the psychological state of the speaker itself."⁸

In Putnam's positive account of linguistic meaning, the criterion (II) above is retained, that is, meaning determines extension, but (I) is given up; the psychological state of the individual does not determine what he means.⁹ Putnam proposes a meaning vector for a word. Such a vector should include at least

- (1) the syntactic markers that apply to the word (e.g., noun),
- (2) the semantic markers that apply to the word (e.g., liquid),
- (3) stereotype (e.g., colorless, transparent, tasteless, etc.),
- (4) a description of the extension (H₂O, give or take impurities).¹⁰

We will in addition have to rely on a certain linguistic division of labor: not everyone will be able to verify the difference between a beech and an elm, for instance, although we may have different meanings "in mind" when we use the words. The division of labor is an important part of the social dimension of cognition, Putnam argues, just as what he calls indexicality acknowledges the

⁸Putnam (1975), p. 224.

⁹Ibid., p. 270.

¹⁰Ibid., p. 269.

important contribution made by the environment: we talk of water *here*, not as an abstract entity.¹¹

2. A critique of Carnap

Before I turn to the implications of *The Meaning of Meaning* for Putnam's period of internal realism, I will examine an aspect of Putnam's critique in this paper of Carnap that continues a discussion I began in Chapter 1, Section 3. In particular, I will describe Putnam's criticism of Carnap's view of *intension*, which is also discussed in the second chapter of *Reason, Truth and History* (1981).

We presuppose the notion of a "possible world". The intension of "cat" is a function f_{cat} defined on the space of all possible worlds and whose value for a possible world x . i.e., $f_{\text{cat}}(x)$ is the set of cats in x . More generally and more precisely, a term T has meaning for the speaker X if X associates T with an intension $f = f_T$. The term T is true of an entity e in a possible world x if and only if e belongs to the set $f(x)$. According to Putnam,¹² Carnap spoke of "grasping" intensions rather than "associating", but clearly, Putnam writes, Carnap must have intended that we grasp *that* f is the intension of T , that we *associate* f with T in some way.¹³

Carnap's point (or the point of what Putnam more generally refers to as the "California semanticists") is that the language under consideration is an *ideal* language. A term in ordinary language does not have a single precise intension. Putnam objects here that the notion of *grasping* an intension is totally unexplained, as is of course the reformulation using "associating an intension". The mathematical precision Carnap attains in his theory does not make it easy to understand what it would mean to have an intension in one's mind, or to think about one, or to grasp one, or to associate an intension with anything.¹⁴

Putnam speculates that Carnap may not have noticed this difficulty because of his *verificationism*. The early Carnap thought that understanding a term is the ability to verify whether or not any given entity falls within the extension of the term.¹⁵ In terms of grasping an intension, this would amount to the ability to verify if an entity e in any possible world x belongs to $f(x)$ or not. Modifying his view in the light of Quine's insistence that sentences face the tribunal of experience collectively and not individually, Carnap may have restricted the same argument to "observation terms". At any rate, Putnam writes:

¹¹Ibid., p. 271.

¹²Ibid., p. 263.

¹³Ibid., p. 263.

¹⁴Ibid., p. 263.

¹⁵Ibid., p. 264.

[I]f one is *not* a verificationist, then it is hard to see California semantics as a theory at all, since the notion of *grasping* an intension has been left totally unexplained.¹⁶

Thus either Carnap has to rely on verificationism in order to make “grasping an intension” intelligible, or he has to give up on intension as meaning.

One important question that we have met earlier¹⁷ in connection with Goldfarb and Ricketts’ recent elucidations of Carnap’s *Logical Syntax* is whether one could here regard Carnap’s attitude to verificationism as merely a proposal, and perhaps if empiricism in general could be viewed as merely a pragmatic proposal (and then formalized within one or several linguistic frameworks). Ricketts and others suggest that this is the correct interpretation, whereas Putnam regards the principle of tolerance as dependent on the verification principle, since, as we saw in Chapter 1, the questions external to the linguistic frameworks and their well-defined rules of inference should be regarded as cognitively empty and senseless. This is the verification principle, according to Putnam.

We have here another argument for the interpretation of Carnap that suggests that the verification principle is presupposed by his scheme, rather than following optionally as a choice made in virtue of the principle of tolerance. In Chapter 5 of *Reason, Truth and History*, Putnam also argues that there is no justification for the verification principle from the point of view of the verification principle itself.

3. Internal realism

In 1976, Putnam starts to realize that his own realism has been left as an unexplained silent presupposition, not itself analyzed in any great detail. In fact, he begins to be uncomfortable with a certain representational realism, where correspondence to a given reality is seen to solve all problems of reference. It is not clear if Putnam ever saw himself as a “metaphysical realist”, but this is a position he now attacks, together with a relativistic or skeptical position that can be seen as similar to metaphysical realism, as will now be explained.

In a thought experiment in Chapter 1 of *Reason, Truth and History* (1981), Putnam asks us to conceive of the possibility that we are in fact *brains in vat*, connected to each other by our nerve endings, forming an automatic machinery that is programmed to create a collective hallucination. So when I am talking to you, we are all under the illusion that I use my mouth to speak and you hear things via your ears, but there is no mouth and there is no ear. I am not mistaken about your real existence, but I am mistaken about the existence of your body

¹⁶Ibid., p. 264.

¹⁷Chapter 1, Section 3.

and the external world, except for brains in a vat. Putnam now asks us: “Could we, if we were brains in a vat in this way, *say* or *think* that we were?”¹⁸

The conclusion is that we couldn’t, according to Putnam. Although it violates no physical law, that “we are brains in a vat” is self-refuting. Putnam scorns the view of some philosophers (presumably Carnap, Tarski and others) that we could treat the scenario as a *possible world*, where all of us are brains in a vat, as if this were a place. But he does not merely scorn the acceptance of formulations involving possible worlds. Using the insights from *The Meaning of Meaning*’ regarding reference, Putnam shows that the brains-in-a-vat scenario is not a possible world, and hence that we are not brains in a vat. The argument against that we are brains is that the brains in a vat cannot *refer* to what we can refer to, even if they can “say” what we can say; in particular, “they cannot think or say that they are brains in a vat (*even by thinking ‘we are brains in a vat’*).¹⁹ Meaning is external: were we brains in a vat, we could not express this in any sense that could refer to the world.

Putnam credits Wittgenstein for being the first philosopher who grasped “the enormous significance” of the insight that *mental images* (and perhaps more importantly, “concepts”) do not intrinsically represent what they are about, and this goes for words and texts as well. Suppose we have dropped a picture of a tree from our spaceship on a planet with humans who are precisely like us, except that they have never seen a tree. Since there are no trees on their planet, the image someone from this planet makes from the picture does not refer to a tree. They look at the picture and have no idea what it can be: perhaps an animal of some sort? Perhaps there is a strange causal chain linking actual trees to mental images. But then the problem arises if the picture we dropped was not really a picture of a tree but was just some spilled paint that accidentally looked like a tree (to us). It does not matter if we consider texts, pictures, or other images (a “sense-datum” perhaps); they do not have a necessary connection with what is allegedly depicted. Monkeys can write *Hamlet* with a certain very low probability by typing letters at random, but it is not the case that such a physical realization of a text would have an intrinsic, built-in, magical connection to what it represents, a connection independent of how it was caused. The examples of monkeys typing *Hamlet* and ants tracing a curve in the sand that looks like Winston Churchill are meant to show that even if such events could happen, even with the tiniest probability, *the ant* has not made a picture of Winston Churchill. If Churchill hadn’t existed, the same curve could have been traced out anyway. If, according to a thought experiment, a computer is programmed to fool us that it is an intelligent being, then it would at least become obsolete if trees stopped

¹⁸Putnam (1981), p. 7.

¹⁹Ibid., p. 8.

to exist on our planet, since it would talk about something we could perhaps not even identify, and the obsolescence of the computer's conception of our world would probably be evident much sooner than that.²⁰

Similarly, brains in a vat cannot refer to trees, and hence not to an external world at all, in particular not to brains in a vat. That would presuppose that there is a God's eye perspective (truth independent of observers) of the world, from which the story could be told. Putnam concludes (or supplies the additional argument) that "Internalist philosophers dismiss the 'Brain in a Vat hypothesis'. For us, the 'Brain in a Vat World' is only a *story*, a mere linguistic construction, and not a possible world at all."²¹

4. Internal realism and truth

In this section, I will describe Putnam's conception of *truth* during his internal realism period.²² The conception of truth he put forward at the end of the 1970s and the beginning of the 1980s is one of the two pillars of Putnam's mid-period position of internal realism. The other pillar is the notion of *conceptual relativity*, a position which he continues to defend in his later period, including in his book *Ethics without Ontology* (2004).

In a 1990 reply²³ to Simon Blackburn's *Enchanting Views*²⁴, Putnam rejects the conception of truth espoused in his earlier period of internal realism, but retains his conceptual relativism.²⁵ In his reply to Blackburn, Putnam characterizes his earlier conception of truth in the following way.

[A] statement is true just in case a competent speaker fully acquainted with the use of the words would be fully rationally warranted in using those words to make the assertion in question, provided she or he were in a sufficiently good epistemic position.²⁶

In his major internal realist work, *Reason, Truth and History* (1981), Putnam makes a comparison between an *epistemic ideal condition* with an idealization such as a frictionless plane, and says that although we cannot really attain such

²⁰Ibid., pp. 1–12.

²¹Ibid., p. 50.

²²There is a confusion about the terminology of the position of *internal realism*. In the 1976 address *Realism and Reason*, Putnam referred to internal realism as the position he had held in *The Meaning of Meaning* (1975) and even in his earlier writings on functionalism. Hence it was not a term for Putnam's new position, but careless readers started to refer to Putnam's new view as internal realism. In *Reason, Truth and History* (1981), Putnam "capitulated" to this terminology. (Putnam (1999), p. 182n.)

²³Published in *Reading Putnam* (1994).

²⁴Published in the same volume.

²⁵Clark and Hale, (1994), pp. 242–243.

²⁶Ibid., p. 242.

ideals, there is nonetheless “cash value” in talk of frictionless planes, since these may certainly be approximated well, in practice granting the sufficiency in “sufficiently good epistemic condition”. Putnam’s aim in saying this is not to give a formal definition of truth (he thinks that truth is not such a clear notion, aside from the “stock examples” such as “snow is white”, which have occupied philosophers), but rather to make an informal elucidation. There are two key ideas:

(1) Truth is independent of justification here and now, but not independent of *all* justification. To claim that a statement is true is to claim that it could be justified.

(2) Truth is expected to be stable or “convergent”; if both a statement and its negation are justified under as ideal conditions as possible, there is no sense in thinking of the statement as *having* a truth-value.²⁷

The first characterization of truth, i.e., (1), certainly bears the mark of realism, whereas (2) resembles Dummett’s anti-realist position that was inspired by intuitionism in mathematics, and the rejection of the principle of bivalence there, i.e., the position that one cannot say of a given statement that it is either true or false, sometimes viewed as a central tenet of any realistic position.

In *Reason, Truth and History*, this view of truth was an important part of the rejection of metaphysical realism, in that the view that there is one unique description of the world. Putnam writes

The perspective I shall defend [...] I shall refer to as the *internalist* perspective, because it is characteristic of this view to hold that *what objects does the world consist of?* is a question that it only makes sense to ask *within* a theory of description. Many ‘internalist’ philosophers, [...] hold further that there is more than one ‘true’ theory or description of the world. ‘Truth’, in an internalist view, is some sort of (idealized) rational acceptability—some sort of ideal coherence of our beliefs with each other and with our experiences *as those experiences are themselves represented in our belief system*—and not correspondence with mind-independent or discourse-independent ‘states of affairs’.²⁸

As we will see later in this chapter, the gist of this rejection of metaphysical realism will be continued in a slightly different manner, involving in the end a rejection of a language/reality dichotomy that renders obsolete his “anti-realist” conception of truth from the early 1980s.

In *Reason, Truth and History*, Putnam contrasts his conception of truth with Dummett’s by saying that we cannot identify truth with rational acceptability,

²⁷Putnam (1981), pp. 55–56.

²⁸Ibid., p. 49–50.

because it is essential that *truth is not lost*, whereas justification may be lost. This is why Putnam still views himself as a realist (and explains perhaps why he used the term internal realism). Putnam exemplifies this claim with the statement that “the earth is flat”, which was probably rationally acceptable 3000 years ago, but not now. It was not, however, *true*, even 3000 years ago, because this would mean that the earth has changed its shape since then. For Putnam, truth here (1981) was an *idealization* of rational acceptability (if we find ourselves in “sufficiently good epistemic conditions”).²⁹ Unless we impose such an ideal epistemic condition for truth, we could lose truth in the sense that what was earlier true (e.g., “the earth is flat”) is no longer true.

Davidson has criticized both Dummett’s anti-realist view and Putnam’s internal realist view on truth, respectively. In *Epistemology and Truth* (1988)[2001], Davidson has difficulties keeping the two distinct, although he cites Putnam’s declaration that they differ insofar as Putnam claims that he has a notion of *idealized* justified assertability in contrast to Dummett’s justified assertability. Davidson is somewhat surprised to find that Putnam interprets Dummett in this way, since he thinks that, on a close reading, Dummett would be seen as subscribing to the same view as Putnam. On the other hand, if Putnam’s description of Dummett is correct, then Putnam is certainly right in pointing out that with Dummett’s view, “truth can be lost”, and Davidson shares Putnam’s view that this is an unacceptable consequence. But Davidson is also unhappy with the main idea, i.e. that of truth as idealized justified assertability:

One suspects that if the conditions under which someone is ideally justified in asserting something were spelled out, it would be apparent that they would either allow the possibility of error, or that they are so ideal as to make no use of the intended connection with human abilities.³⁰

I think that Davidson is right that the “sufficiently good epistemic position” Putnam talks about is a rather empty idea and seems to collapse as an idea of truth under an ideal epistemic position, since we cannot be sure that the epistemic conditions are good enough. It is easy then to slide into the radical pragmatist camp, that is, to identify our best researched claims and most successful beliefs with the true ones. In a critical paper published in *Rorty and His Critics*, Davidson is critical not only of connecting truth with justified assertability (in any sense), but also with the pragmatist retreat from the objectivity of truth. Davidson agrees with the pragmatists that one cannot both have objective truth and truth as a goal for our investigations. For Davidson, truth is pointless as a goal, a norm

²⁹Ibid., p. 55.

³⁰Davidson (1988)[2001b], p. 187.

to strive for.³¹ This sounds like a strange reason to preserve an objective notion of truth. Nevertheless, Davidson's main argument is that without the requirement of an ideal epistemic position, the notion of truth proposed by Putnam is subjective. According to Davidson, the resulting account of truth thus becomes circular, since truth becomes guaranteed by the possibility of ideal conditions being met.³²

What Davidson is alluding to when he speaks of "the intended connection with human abilities" in the quote above, is Putnam's way of proposing "a *human* kind of realism, a belief that there is a fact of the matter as to what is rightly assertable *for us*, as opposed to what is rightly assertable from the God's eye view so dear to the classical metaphysical realist".³³ But Davidson thinks that there are no arguments for Putnam's position; Putnam has no argument for why there cannot be another position than his own or the metaphysical realism he describes. Davidson's criticism takes aim at what he sees as a too strong *immanence of truth* (the reliance of the truth *for us*), not in the "trivial" way that "truth of sentences or utterances is relative to a language" (Putnam would say that this is the alternative of changing the "linguistic meaning" of the terms involved), but such that "a sentence of yours and a sentence of mine may contradict each other, and yet each be true 'for the speaker'." Davidson finds it difficult to "think in what language this point can be persuasively stated".³⁴ It would seem difficult to defend conceptual relativism from this criticism; we will now turn to this difficulty.

5. Conceptual relativism and conventions

We will now treat Putnam's *conceptual relativism*, a position that Putnam has maintained since he gave up on internal realism, which was connected to the conception of truth we discussed briefly in the previous section. This conceptual relativism depends on the arguments against metaphysical realism, that is, the perspective of some fixed totality of mind-independent objects together with an idea that there is one true and complete description of the way the world is.³⁵

In *Equivalence* (1983b), Putnam describes what calls "cognitive equivalence" through examples from physics and a practice among physicists which he claims has been around since the end of the 19th century, that of working with equivalent descriptions. One obtains an equivalent description of a theory T_1 if one changes "pressure" to the "cube root of pressure"; the new theory T_2 will then

³¹Davidson[Brandom] (2000), p. 67.

³²Ibid., pp. 67–68.

³³Davidson (1988)[2001b], p. 187; Putnam (1983), p. xviii.

³⁴Davidson (1988)[2001b], p. 187.

³⁵Putnam (1981), p. 49.

trivially be equivalent with T_1 . This is not a philosophically important phenomenon of equivalence of theories. Rather, equivalence is important when “it does not seem that anyone has *altered* the ordinary meaning of any expression and yet, for factual reasons, apparently incompatible bodies of theory turn out to be equivalent”.³⁶

The following recurrent example from Putnam’s writings is illustrative:

Story 1. Space-time consists of objects called points (point-events). These have no extension, and extended space-time is built up out of them just as, in classical Euclidean geometry, the extended line, plane, and solid bodies, are built up out of unextended spatial points.

Story 2. Space-time consists of extended space-time neighborhoods. All parts of space-time have extension. This corresponds to the theory (advanced by Whitehead), that classical Euclidean space consists of extended spatial neighborhoods. On Whitehead’s view, ‘points’ are mere logical constructions and not real spatial objects: a point is (identified with) a convergent set of solid spheres (i.e., spheres together with their interiors).³⁷

Story 1 and story 2 are equivalent descriptions, since it makes no difference to physical explanation whether we assume the existence of real space-time points or see them as mere logical constructions.

A ‘hard-core’ realist might claim that there is a fact of the matter as to which is true, story 1 or story 2. [...] [I]t leads either to skepticism or a revival of metaphysics of the kind Kant persuaded us to abandon. It leads to the former – skepticism – if we say that there is a fact of the matter as to which story is true, but that we can never *know* that fact. [...] It leads to metaphysics in the bad sense – the kind that claims to *a priori* knowledge about noumenal realities – if we claim that we know on extra-scientific grounds either that story 1 is true or that story 2 is.³⁸

Story 1 can now be interpreted in story 2 in many ways, since we may assume that “points” are sets of spheres whose radii are $1/2^n$, or we may assume that that they are sets of spheres of radii $1/3^n$, but there is no fact of the matter for a scientific realist, Putnam says, which is the correct translation; they are all correct, but this means that we will have to give up the idea of terms in our theories as images of real objects (noumenal objects). Furthermore, we will have to give up the idea of theory-independent objects: “Any sentence that changes truth value on passing from one correct theory to another correct theory – an

³⁶Putnam (1983b), p. 41.

³⁷Ibid., p. 42–43.

³⁸Ibid., p. 43.

equivalent description – will express only a *theory-relative* property of the world. [...] [I]f we concede that story 1 and story 2 are equivalent descriptions, then the property *being an object* (as opposed to a class or set of things) will be theory relative”.³⁹

This example shows well what Putnam wants to illustrate with his *cognitive equivalence*, an essential part of the notion of conceptual relativity: it does not matter which formalization we use. As we will see, conceptual relativity is part of a wider notion of *plurality* in Putnam’s later philosophy, but for a long time the examples describing “tables and chairs” were mixed together with the scientific examples involving cognitively equivalent formalizations as in *Story 1* and *Story 2*. Putnam later made a sharp distinction between different descriptions in the latter (narrow) sense with the broader use of “different descriptions”, let’s say of the content of a room, one in terms of fundamental physics and one using our ordinary vocabulary of tables and chairs, both purportedly describing the same state of affairs. Such different descriptions are not *cognitively* equivalent. They contain very different information; the scientific description is not at all correlated to the ordinary descriptions of tables and chairs. Putnam’s notion of plurality, to which I will return later in this chapter, involves a critique of Quine’s (and Carnap’s) idea of a “first class conceptual scheme”, i.e., the conceptual scheme of science serving us with the best descriptions of our world. For Putnam it is a deep insight of the American pragmatists that we may have very different, but equally right, descriptions of phenomena in our world. Different descriptions may serve entirely different purposes.

As a rejection of metaphysical realism, Putnam’s position has certain similarities with Nelson Goodman’s “irrealism” (as presented in his *Ways of World-making*), described by Putnam at length in *Renewing Philosophy* (1992), in order to explain their important differences. Both Goodman and Putnam stress the plurality of descriptions and that there cannot be one right version of the world: there is no unique true description of reality.⁴⁰ In fact, this idea is an old one, as Putnam remarks, going back at least to Hertz’ attempt to describing equally adequate “world pictures”. But Goodman and Putnam differ radically, because Goodman not only attacks the idea that our conceptual schemes are just different descriptions but also the idea that these descriptions are of “the same facts”. For Goodman, it makes no difference to say that there are different descriptions of the world, different versions of the world, or *many worlds*, or perhaps *no world*; either there are many, or world-talk is nonsense. In Putnam’s reading of Goodman, there cannot be two descriptions of the same world such as in the example

³⁹Ibid., p. 44.

⁴⁰Putnam (1992), p. 110.

above, where we talk about space-time points as individuals or merely as abstractions; incompatible versions refer to different worlds, since otherwise we have no worlds, just versions. These worlds are furthermore made by us, out of previous worlds or versions.⁴¹

It is clear that in Putnam's presentation, Goodman cannot accept that the different descriptions are contradictory only at face value, or with respect to their surface grammar. It is one of Putnam's ambitions to show that we can make sense of having different (but on the surface incompatible) equivalent descriptions of the same world, but it requires that we are sensitive to notions of "meaning" and do not just transfer the meaning of words and "sentences" from ordinary talk of tables and chairs to areas where such a notion may not apply, or at least not in the same way.

In what follows, I will present Putnam's journey to a position in which he can reconcile his ambitions to justify realism with his conceptual relativity and pluralism, involving in particular his further considerations of "meaning" and "truth". For the time being, suffice it to say that Putnam thinks that there are cognitively equivalent descriptions, which may be contradictory only in a superficial way, in that we may formalize a physical theory by regarding points as individuals or as limits (the "contradiction" being that we assume that points are individuals in one formalization but not in the other).

One important question here then is whether the choice of either of these descriptions is a merely one of convention, perhaps as in Carnap's sense. For Goodman, as we have seen, this is impossible, since different descriptions give us "different worlds". As we will see Putnam will come quite close to Carnap, in the sense of allowing different systems of formalizations, but there are important differences. Putnam rejects any idea of convention that challenges fallibilism in the sense that future considerations may force us to give up even the choice of formalization between space-time points as individuals or as limits of spheres. Although he later finds it "silly"⁴² to assume that it would make any difference if we were to formalize geometry from point individuals and points defined through limits, we have the following characteristic statement in the other direction.

An example (my own, not Quine's) is the conventional character of any answer to the question "Is a point identical with a series of spheres that converges to it?" [...] [W]hat Quine pointed out (as applied to this case) is that when I say, 'We can do either', I am assuming a diffuse background of empirical facts. Fundamental changes in the way we do physical geometry could alter the whole picture. The fact that a truth

⁴¹Ibid., pp. 110–111.

⁴²Putnam (2004), p. 46. In the same work he says that "I am not claiming that conventions of the kind I am describing might never have to be given up for presently unforeseeable reasons." (Ibid., p. 44.)

is toward the “conventional” end of the convention-fact continuum does not mean that it is *absolutely* conventional—a truth by stipulation, free of every element of fact.⁴³

This analogy of the analytic-synthetic continuum, but now one of a continuity between fact and convention, is for Putnam a presupposition for the conceptual relativity he argues for. Jennifer Case even thinks that the terminology of conceptual relativism should be replaced by “the doctrine of the interpenetration of fact and convention”,⁴⁴ since we make up uses of words, none of which is copied from the intrinsic nature of reality itself—they rather help us to define the world, although some of our sentences state facts, the truths of which we do not make up.⁴⁵ This is an explanation of Putnam’s “Hegelian metaphor”: “The mind and the world jointly make up the mind and the world.”⁴⁶ In his *Reply to Jennifer Case* (2001), Putnam corrects Case’s identification of the interpenetration of fact and convention with his notion of conceptual relativity, since conceptual relativity implies interpenetration of fact and convention, but we have many sorts of conventions, not only those involving “cognitively equivalent but seemingly incompatible descriptions”.⁴⁷

Let us now return to Goodman, for an example of the interpenetration between fact and convention. Goodman argues that not only the descriptions but also the facts or worlds change. Since (according to Goodman) these worlds are made by us, we could ask the question: “Did we then make the stars?”. Goodman would say that there is a sense in which we did.⁴⁸ Putnam asks us to consider the following line of thought. Think of the constellation the Big Dipper. Of course we didn’t make this constellation as a carpenter makes a table, but perhaps we did make the Big Dipper a constellation, and, in that respect, made the Big Dipper? Now, many of us would perhaps concede that there is something to this, and we could perhaps say that although we didn’t make the Big Dipper as a carpenter makes a table, we made it “by constructing a version in which that group of stars is seen to exhibit a dipper shape, and by giving it a name, thus, as it were, institutionalizing the fact that that group of stars is metaphorically a big dipper [...] Stars are a ‘natural kind’, whereas constellations are an ‘artificial kind’.”⁴⁹

⁴³Putnam (1989), pp. 112–113.

⁴⁴Case (2001), p. 429.

⁴⁵Ibid., p. 429.

⁴⁶Ibid., p. 429; Putnam (1981), p. xi.

⁴⁷Putnam (2001b), p. 437.

⁴⁸According to Putnam, it was indeed a response to a question asked by Israel Scheffler at an APA conference in 1979. (Putnam (1992), p. 111.)

⁴⁹Ibid., p. 112.

However, Goodman will now go further and say that this difference we are now assuming between a “natural kind” such as stars and our own “constructions” is chimerical, since stars are clouds of glowing gas, but not every such cloud of glowing gas with thermonuclear reactions is considered to be a star (they fall into other astronomical categories) and some stars do not glow: didn’t *we* group all these different objects into a single category *star*?⁵⁰

Putnam finds Goodman’s arguments easy to refute. The extension of the term Big Dipper is fixed by linguistic convention: the “Big Dipper” is a *proper name* of a finite list of stars. This is not to say that the convention to name a group of stars expresses an “analytic” truth. We may still name a certain constellation the Big Dipper, even if for some reason one of the stars “went out”. It is also unclear what we would do if another star turned up in the vicinity of the Big Dipper—it would depend on linguistic practice that would develop. In any case, “star” has an extension that cannot be fixed by giving a list; and no star is in the extension because it is *called* a star.⁵¹ Putnam writes:

[W]e didn’t make Sirius a star. Not only didn’t we make Sirius a star in the sense a carpenter makes a table, *we didn’t make it a star*. Our ancestors and our contemporaries (including astrophysicists), in shaping and creating our language, created the concept *star*, with its partly conventional boundaries, with its partly indeterminate boundaries, and so on. And that concept *applies to Sirius*. The fact that the concept *star* has conventional elements doesn’t mean that *we* make it the case that that concept applies to any particular thing, in the way in which we made it the case that the concept “Big Dipper” applies to a particular group of stars.⁵²

Here we see an example of the interpenetration of fact and convention. The existence of the Big Dipper is more on the conventional end of the scale, the existence of the stars more on the factual end, and, as Putnam suggests, the existence of constellations maybe somewhere in between.

But a problem arises when we take an example such as how we should formalize geometry. In several places, Putnam is clear about that the different geometrical systems are “cognitively equivalent”,⁵³ but at the same time this equivalence should not be viewed as implying “unrevisability”. There is a tension (that prevails throughout Putnam’s work) between a Quinean view that we may have to revise what is a convention in the light of a new theory, or new empirical

⁵⁰Ibid., p. 112.

⁵¹Ibid., pp. 113–114.

⁵²Ibid., pp. 114.

⁵³See for instance *Equivalence* (1983b), *Reply to Jennifer Chase* (2001), *Ethics without Ontology* (2004).

circumstances, and the view expressed that it is “silly” to think that a particular formalization of geometry matters as to which is really “true”.

In a reply to Simon Blackburn’s paper on Putnam’s internal realism *Enchanting Views* (1994), Putnam writes that his position resembles that of Carnap’s in *Empiricism, Semantics, and Ontology*:

inasmuch as I hold that differences in ontology *sometimes* amount to no more than differences in how we use words. But unlike Carnap, I do not rest the distinction between questions which have to do with the choice of a linguistic framework and empirical questions on an absolute analytic-synthetic distinction. Whether something is or is not ‘conventional’, i.e. whether what is at stake is no more than a question of how to talk, is itself something to which empirical facts are relevant. There is a continuum stretching from choices which, by our present lights, are just choices of a way of talking to questions of what are plainly empirical fact, but there is nothing here which is *guaranteed* to be true no matter what the facts turn out to be. What I would like to criticize Quine for is the suggestion that a distinction between fact and language-choice which is not absolute, not drawn once and for all unrevisably, is of no use.⁵⁴

We see here one of many examples where Putnam stresses that we may have a use for conventions, but that we cannot have a distinction drawn once and for all between conventions and facts. The untenability of such a distinction is one version of Quine’s thesis that there are no *a priori* truths.

Returning to the notion of conceptual relativity proper, Goodman has also something to say about the earlier example we discussed above concerning space-time points, that is, story 1 and 2 concerning conceptual relativity. Goodman regards these descriptions as incompatible, but still both right, they are true of different worlds, if any at all. Putnam now compares Goodman’s picture with that of Donald Davidson’s (and Quine’s in this respect). Davidson finds the two versions incompatible, but draws the conclusion that since incompatible versions cannot both be true by standard logic, it is unintelligible to hold that both versions are true. Quine says that one may pick one of the versions some of the time and the other at other times, but I may only say that one of them is true at a given point in time, since I would otherwise contradict myself. Quine also rejects Goodman’s alternative of many worlds, designed to meet an objection like this, since it violates the “principle of parsimony”. Putnam continues:

Goodman and Davidson seems to me to be making the same mistake—although, as so often happens in philosophy, it leads them into opposite camps. Davidson and Goodman both accept without question the

⁵⁴Putnam (1994), p. 247–248.

idea that statements which appear to be incompatible, taken according to their surface grammar, really are incompatible, even in cases like these. If the sentence, ‘points are mere limits’ is a contrary of the sentence ‘points are not limits put parts of space’, even when the first sentence occurs in a systematic scheme for describing physical reality and the second occurs in another systematic scheme for describing physical reality *even though the two schemes are in practice thoroughly equivalent*, then we are in trouble indeed.⁵⁵

Putnam does not say that the two schemes are equivalent just because they lead to the same predictions, i.e., because they are “empirically equivalent”, but rather because one of the sentences in question (for instance, “points are part of space-time”) may be correlated in an effective way with a “translation” in the other scheme, and this translation will describe the same state of affairs, not by introducing a transcendent ontology of states of affairs, but by using a language already in place.⁵⁶

The statement “between any two distinct points on a line there is a third point” is true in the version where we allow points to exist, but we can translate it into a version that assumes points to be limits of concentric circles in order to obtain a true statement, even if we do not assume the existence of points anymore, since the existence of such concentric circles between two families of concentric circles is guaranteed. This is not to say that the statements have the same “meaning”—the ordinary practices to which the ordinary-language notion of “meaning” was never designed to do this job. Putnam writes: “the ordinary practices [...] crumble when confronted with such cases [...] we can [instead] say that the words ‘point’, ‘limit’, and so forth have different ‘uses’ in these two versions.”⁵⁷

6. The example of mereology

In *Truth and Convention* (1987), Putnam tries to clarify some issues related to his so-called *internal realism* from *Reason, Truth, and History* (1981). In particular, his aim is to explain his conceptual relativism in response to criticism by Davidson and others.

Suppose we have a universe with three individuals x_1, x_2, x_3 , a world “à la Carnap”⁵⁸. For someone who subscribes to *mereology*, that is, a theory in which we may always “add” any components of our universe (my nose and the Eiffel

⁵⁵Putnam (1992), p. 116.

⁵⁶Ibid., p. 117.

⁵⁷Ibid., p. 119.

⁵⁸This is called so not because Carnap thought that the world has a particular number of objects, but because he used to work with such model-examples, in fact together with the young Putnam, as he was exploring inductive logic.

Tower, for instance), the world now consists of seven objects (disregarding a “null object”), namely

$$x_1, x_2, x_3, x_1 + x_2, x_1 + x_3, x_2 + x_3, x_1 + x_2 + x_3.$$

Such a theory of mereology was suggested by the Polish logician Lezniewski, and Putnam refers to this as a world “à la Polish logician”.⁵⁹ For “Carnap” there are three objects in this world, whereas for the “Polish logician”, there are seven objects. Putnam refers to this phenomenon of a conventional use of “exists” as an example of conceptual relativity, and exemplifies his way of finding an intermediate way between relativism and metaphysical realism. Once we have determined a use of “exist”, for example, the answer to the question, “How many objects are there?”, is no longer a matter of convention. But the question “How many objects are there *really*?” does not make sense.⁶⁰

One important point Putnam now wishes to make concerning *conceptual relativity* is that it is wrong to think that either we use a different *meaning* of “exist” in the narrow linguistic sense, or “Lezniewski” and “Carnap” contradict each other when they say how many objects there exist in this small universe. Putnam claims that his critics just say that his “conceptual relativity is merely an example of the possibility of using ‘exist’ in a more inclusive or less inclusive way”.⁶¹

The reasons for Putnam not to accept this objection are two:

(A) we cannot carve up reality in prescribed pieces (this is metaphysical realism’s own cookie cutter metaphor), and

(B) the logical primitives themselves, “object” and “existence” in particular, have a multitude of uses and no absolute “meaning”.⁶²

Consider now the following two sentences:

(1) There is an object which is partly red and partly black.

(2) There is an object which is red and an object which is black.

The sentence (2) is true both for “Carnap” and the for the Polish logician if x_1 is red and x_2 is black. (1) is true in the Polish logician’s version and Putnam now asks what status it has in the “Carnapian” version.

⁵⁹Putnam (1987), pp. 96–97.

⁶⁰Ibid., p. 98.

⁶¹Putnam (2004), p. 40.

⁶²Putnam (1987), p. 97.

Putnam lets an imagined critic of his ideas (a metaphysical realist taking a certain position as to what objects there are) argue that there is no object of which x_1 and x_2 are parts—hence (1) is simply false. If there was such an object as (1) says there is, this object would have to be distinct from x_1 and x_2 , namely x_3 , but this is not an object that “overlaps” with x_1 and x_2 . In particular, there is no object $x_1 + x_2$.⁶³ This is basically the metaphysical realist’s alternative, which says that we may carve up reality in one definite way, and s/he also maintains that there is a correct meaning of “object”.

But there are also other possible ways of seeing the the relation between (1) and (2), all of which assume that there is a *fact of the matter* as to the correct relation between (1) and (2), something Putnam repudiates.⁶⁴ I will concern myself mainly with the positions of Quine and Davidson, as these are described by Putnam, but I should also mention that other positions that Putnam rejects as definite solutions are that (1) and (2) are mathematically and/or logically equivalent. Putnam’s view is that these notions are not well-defined enough.

Davidson would deny that (1) and (2) are alike in meaning, and Quine would say that it is possible to talk as if (1) is true (which may be practical since we say things in fewer words in the “richer” language), but that we may reduce talk of mereology to “Carnap’s” language. It was not in the context of mereology that Quine held such a view, but regarding numbers: he saw talk of numbers as a *façon de parler*. We should really be talking of sets, in line with Frege and Russell’s interpretation of number theory, if pressed about what we are talking of when we say that numbers exist, for example, when we say that there exist (or there are) prime numbers greater than a million. This is Quine’s basic idea in *On What There Is* (1948)[1953]; talk of numbers should be explained and not just be dismissed, and set theory replaces our “manner of speaking” without cheating.

A metaphysical realist would approve of this scheme since it would allow us to talk *as if* mereological sums exist, even it is merely a *façon de parler*. Putnam and this metaphysical realist would agree that they translate (1) by a reductive, or relative, translation into (2), not by identifying individual words of (1) with individual words in (2), but rather we identify (1) with (2). The reasoning goes as follows: a mereological sum is partly red if and only if it contains a red atom, and similarly it is partly black if and only if it contains a black atom, so in this sense we may translate (1) into (2) so that, and this will now be emphasized by the metaphysical realist, the mereological sums are not identified with anything; the phrase “object which is partly red and partly black” has “no translation by itself”.⁶⁵ What Quine was up to was rather to give a possibility of reducing talk of numbers and sets to sets only; we should not assume the existence of numbers

⁶³Ibid., p. 98–99.

⁶⁴Ibid., p. 103.

⁶⁵Ibid., pp. 102–103.

other than as sets. Using the same argument as from *On What There Is*, the metaphysical realist will be able to draw a somewhat stronger conclusion (one that Putnam does not accept) in the case of mereology, since the mereological sums just disappear in the translation scheme: they do not exist at all, as anything.⁶⁶ The problem Putnam identifies with Quine's approach is not only the way it may be used by a "metaphysical realist". Putnam says that he does not think that Quine would regard every formal reduction by a translation scheme as evidence for the non-existence of certain entities. But there is certainly a way of reasoning in terms of existence that when the reductions do not go both ways, there is evidence for denying the real existence of, for instance, numbers.⁶⁷

Here one might raise the issue of the so-called Quine–Putnam indispensability argument, which in "popular" terms says that we need to introduce abstract entities in order to save "truth" in mathematics. That is, one introduces entities ("posits") as truth-makers, instead of relying on, for instance, "truth by convention". In the *Ethics without Ontology* (2004), Putnam states that all talk of existence in mathematics could be replaced by an interpretation within modal logic. It is clear from *Ethics without Ontology* that Putnam wants us "to safeguard the cognitive credentials" of mathematics, without assuming that propositions such as " $2 + 2 = 4$ " describe their own state of affairs, in analogy with empirical propositions.⁶⁸

Every statement about the 'existence' of any mathematical entities is equivalent (equivalent mathematically, and equivalent from the point of view of applications as well) with a statement that doesn't assert the actual existence of any mathematical objects at all, but only asserts the *mathematical possibility* of certain structures.⁶⁹

Putnam furthermore characterizes Quine as a reluctant Platonist in the same work, in that Quine clearly admitted sets as part of Ontology, i.e., ontology as onto-theology, as Putnam in the spirit of Heidegger characterizes this view.⁷⁰ Quine apparently became aware late in life of the possibility that one can argue as Putnam does, i.e., "by formalizing mathematics in a *modal logical language*, one that takes as primitive (mathematical) *possibility and necessity*",⁷¹ but thought that it made the ontological commitments unclear. Putnam writes: "The very idea

⁶⁶Ibid., pp. 101–102.

⁶⁷Ibid., p. 102.

⁶⁸Putnam (2004), p. 54. This is in fact Conant's interpretation of Putnam, but one that according Putnam describes his own position.

⁶⁹Ibid., p. 67.

⁷⁰Ibid., p. 79–80.

⁷¹Ibid., p. 82.

that the modalities have (or may have) *hidden* ‘ontological commitments’ shows just how deep the Platonist bug had bitten Quine by this time.⁷²

In *Metaphysics without Ontology* (2006), Claudine Tiercelin argues that when Putnam rejects the Quine–Putnam indispensability argument, he may be relying on yet a foundationalist metaphysics, as he is taking modalities as primitive. She also thinks that Putnam has brought us close to a pluralistic nominalism, rather than any sort of realism.⁷³ Putnam’s answer to Tiercelin is quite illuminating here. In a section called “The Quine indispensability thesis and my indispensability thesis”, he writes as follows:

Tiercelin speaks of my ‘rejection of the Quine–Putnam indispensability argument’, but this is misleading. In ‘Mathematical Truth’ I argued that the internal success and coherence of mathematics is evidence that it is true under *some* interpretation, and that its indispensability for physics is evidence that it is true under a *realist* interpretation – the antirealist interpretation I considered there was Intuitionism. This is a distinction that Quine nowhere draws. In *Philosophy of Logic* I argued that at least some set theory is indispensable in logic *and* in physics. (But I had already, in ‘Mathematics Without Foundations’, said that set theory and modal logical mathematics were ‘equivalent descriptions’, so that was in no way an argument for realism about sets *as opposed to* realism about modalities.) In sum, my ‘indispensability’ argument was an argument for the *objectivity of mathematics in a realist sense* – i.e., for the idea that mathematical truth must not be identified with provability.⁷⁴

The rejection of an identification of Quine’s and Putnam’s “indispensability” arguments resurfaces in *Was Wittgenstein Really an Anti-realist About Mathematics?* (2001), in which Putnam argues for what he calls a “logicist insight” that empirical science contains “mixed statements”, empirical statements that speak of “functions and their derivatives as well as of physical entities”.⁷⁵ He argues that Newton’s law of gravitation, for example, presupposes the existence of real numbers (or at the very least rational numbers).⁷⁶ In mathematical physics, differential equations such as the wave equation may have solutions that are not recursively calculable, but there must be an answer to the question “what the solution of the differential equation is to such-and-such a number of decimal

⁷²Ibid., p. 82.

⁷³Tiercelin (2006), pp. 59–60.

⁷⁴Putnam (2006), pp. 92–93.

⁷⁵Putnam (2001), p. 157.

⁷⁶Ibid., p. 158.

places”.⁷⁷ This is part of a *scientific realism* that Putnam still endorses.⁷⁸ I think Putnam is here still grappling with a problem that he inherits from Quine: that there is no such thing as a pure mathematics, that mathematics is *justified* by its scientific use, as an integrated part of science. I think this makes it difficult for Putnam to view simple arithmetic “statements” such as “ $5 + 7 = 12$ ” as rules, and not as descriptions, or so I have argued in Chapter 2. A continuation of this critique will follow in Chapter 4.

What is most interesting in Putnam’s reply to Tiercelin, cited above, is that he emphasizes that set theory and modal logical mathematics are equivalent descriptions. Sami Pihlström has noted that Putnam does not, in contrast to what Tiercelin writes, think of modal logic as a new foundationalist metaphysics for mathematics; the mathematical possibility Putnam is talking about should not be “conflated with a metaphysical kind of possibility more basic than mathematics itself”.⁷⁹ Both moral and mathematical objectivity can stand on their own and they have no need of truth-makers. It is part of conceptual relativity that we have equivalent descriptions within mathematics, and this is certainly not to be regarded as support of ontological commitments; on the contrary, it shows that we should free ourselves of such commitments. In particular, the alleged “truth-makers” have no clear identity-relations; we are free to stipulate these relations, an *instance of conceptual relativity*, i.e., “whether functions are a kind of set, or sets a kind of function; whether numbers are sets or not, and if they are sets, *which* sets they are”.⁸⁰

Returning to the example of mereology, Putnam’s notion conceptual relativity means that we may choose either the language of “Carnap”, allowing a statement such as (2), “there is an object which is red and an object which is black”, or we may choose the mereological language of the Polish logician and say (1), “there is an object which is partly red and partly black”. It is important to understand here that “there is no such thing as the ‘proposition’ which one of these sentences affirms and the other denies”.⁸¹ One such candidate which Putnam discusses in a reply to Samuel Blackburn⁸² would be

⁷⁷Ibid., p. 159.

⁷⁸In his reply to Tiercelin, he writes that this means a rejection of operationalism and instrumentalism. (Putnam (2006), p. 93.)

⁷⁹Pihlström (2006), p. 5; Putnam (2004), p. 67.

⁸⁰Putnam (2004), p. 66.

⁸¹Putnam (1991), p. 404. In his reply to Throop and Doran, Putnam says this with respect to the two sentences, “there are three object on the table” and “there are seven object on the table”, using “Carnap’s” and the Polish logician’s language, respectively, but the point is the same.

⁸²Clark and Hale (1994), p. 245.

(3) There is no object which is partly red and partly black.

This would apparently contradict (1), but Putnam argues against Blackburn that there is supposed to be a genuine inconsistency here. Putnam's reason for rejecting the inconsistency between (1) and (3) is that the words "object" and "exist" are used differently in the two languages—and this blocks Blackburn's argument.⁸³

The translation scheme that Putnam proposes in fact translates (1) into (2); we interpret that we have an object which is partly black and partly red as having an object which is red and an object which is black, since this is the only possible solution. Since (2) and (3) are consistent (this would also be true in the case of examples with non-unique solutions) and *phrased in the same language*, there is no problem. Blackburn seems to anticipate this reply and argues that then we have a difference in meaning of "object" and "exist". Putnam's recurrent response to this is that then we are not talking about "meaning" in any way that an ordinary speaker would recognize as meaning. We will discuss the type of change Putnam suggests is made when we *use* "object" and "exist" in different ways. The difference in use of words like "object" and "exist" that Putnam suggests is finally developed into an embracement of Wittgenstein's "meaning as use", as will see from the *Dewey Lectures* and the *Ethics without Ontology*.

Putnam's view is quite similar to Carnap's (the real Carnap) in many ways. In Putnam's eyes, Carnap was a conceptual relativist. Using his principle of tolerance, Carnap would be happy to say that we can make (1) false by using "Carnap's" language, or we can make (1) true by using the language of the Polish logician. There is certainly no evidence against numbers or mereology; we are free to choose languages containing either.

But these arguments may not convince philosophers who pursue "meaning theory", such as Davidson and his followers, as Putnam writes. The objection from such a camp is for example that sentence (2) is not a sentence one would ordinarily offer as an explanation of the truth-conditions of sentence (1).⁸⁴ Mathematical or logical equivalence (whatever that is) is not enough for these critics; we have to have something called a correct "translation practice".

Davidson regards "truth" as primitive and tries to get at meaning⁸⁵ by applying Tarski's definition of truth. For example, we have the truth-condition "snow is white" is true if and only if snow is white. This kind of theory of meaning is sometimes referred to as a truth-conditional theory of meaning; the meaning of "snow is white" is given by the truth condition.

⁸³Ibid., p. 245.

⁸⁴Putnam (1987), p. 100.

⁸⁵Davidson (2001a), p. xvi.

[I]t is not an accidental fact about that English sentence, but a fact that *interprets* the sentence. Once the point of putting things this way is clear, I see no harm in rephrasing what the interpreter knows in this case in a more familiar vein: he knows that ‘Snow is white’ in English *means that* snow is white.⁸⁶

The problem with giving a translation of (1) into (2) given such a truth-conditional semantics is that if “there is an object which is partly red and partly black” is true, then there is an object that is partly red and partly black. Given our “ordinary translation practice” it becomes difficult for Davidson-inspired philosophers to regard this meaning of (1) also as the meaning of (2), i.e., that there is an object that is red and an object that is black. The meaning of the word “object” is fixed by the home language into which we make our translations. Our home language becomes the fixed coordinate system to which we try to relate (1) and (2). It is not enough in a Davidsonian view that we can make the relative translation as Putnam suggests above, correlating (1) and (2), by making the specific interpretation of (1) in “Carnap’s” language. Putnam writes the following about Davidson and “meaning theory”:

[A] ‘meaning theory’ it is said, must not correlate *any* extensionally or even mathematically correct truth-conditions with the sentences of the language the theory describes; the sentence used to state a truth condition for a sentence must be one that might be correlated with that sentence by ‘translation practice’.⁸⁷

Putnam argues that Davidson has what just appears to be an innocent route to his view of translation. If we apply Davidson’s point of view to an “alien language”, and I regard a sentence as meaningful, then I must also be able to give a *truth-condition* for that sentence *in my own language*. This is a key idea in Davidson’s work: there has to be a common reference point, a system of coordinates to which conceptual schemes are relative, and this is supposed to falsify the whole idea of conceptual relativity.⁸⁸ The idea of uninterpretable languages does not make sense to Davidson. Our point of reference must be a “home language” into which other languages are interpretable, or they are simply not languages.

If my ‘own’ language is Carnap’s, and we accept no ‘truth-condition’ for (1) statable in Carnap’s language will satisfy the constraints on translation practice any better than (2) did, then the conclusion is forced: the Polish logician’s language is *meaningless*. Of course, we might simply adopt the Polish logician’s language as our own to begin

⁸⁶Ibid., p. 175.

⁸⁷Putnam (1987), p. 100.

⁸⁸Davidson (2001b), p. 39.

with. But what we cannot do, according to Davidson, is to regard both choices as genuinely open.⁸⁹

Natural languages are furthermore viewed as being translated into predicate calculus, or, rather, natural languages are viewed as having the predicate calculus built into them in a hidden way from the very beginning. One claims to be making the hidden predicate calculus structure of the natural language explicit, rather than translating from one linguistic structure to another.

A typical criticism of Putnam's view along this line is leveled by Matti Eklund in *Putnam's Ontology* (2008) and which is a good example of how interpreters of Putnam's conceptual relativity go astray. Eklund suggests that the following passage constitutes a proof of the falsity of the doctrine of conceptual relativity, with particular reference to Putnam:

The idea behind the thesis of conceptual relativity is that 'exists' and what we may call other ontological expressions of English ('there are', 'object', 'some', ...) are somehow indeterminate in meaning. This would appear to mean that there are two possible languages English₁ and English₂ we could speak, with the ontological expressions of English being relatively precisified in one way in English₁ and in another way in English₂. To stick with the example Putnam employs, we can imagine that in English₁ 'there are mereological sums' comes out true, and in English₂ 'there are mereological sums' comes out false. I think we can reduce this claim to absurdity. Here goes.

In English₁ there can be a singular term '*t*', purporting to refer to some mereological sum, such that there are some true atomic sentences of the form "*F(t)*" of English₁. Now, what should be said about the sentence "*F(t)*" of English₁ in English₂ (or in English, for that matter)? It seems clear that the correct thing to say is that it is true. (Indeed, I just said in English that it is true.) But for an atomic sentence, of any language, to be true, the singular terms in that sentence must refer. This is a fact we can surely give expression to in English and in English₂. So we can conclude in English and in English₂ that '*t*' refers. But for '*t*' to refer, there must be a referent for '*t*'. In English₂ we can conclude that '*t*' refers. This, in conjunction with the fact that the referent of '*t*', if any, is a mereological sum contradicts the supposition that in English₂ 'there are mereological sums' is not true. Hence the thesis of conceptual relativity is false.⁹⁰

Let us now consider what Eklund says by letting "*F(t)*" be the statement in English₁ that "*t* is partly red and partly black". Eklund suggests that the term "*t*", which could be the mereological sum $x_1 + x_2$, where x_1 is a red object and

⁸⁹Putnam (1987), p. 104.

⁹⁰Eklund (2008), p. 2 (according to online preprint version).

x_2 is a black object, is a singular term in English₂, but this conclusion does not follow at all.

Eklund says that if “ $F(t)$ ” is true in English₁ it is also true in English₂ (which is highly questionable); Eklund presupposes that both English₁ and English₂ (the Polish logician’s and “Carnap’s” languages, respectively) are part of English, which translates one formulation of the sublanguage English₁ into the same formulation, i.e., “ $F(t)$ ”, in English₂, and particularly keeping the notion of “singular terms” invariant under translation.

Eklund argues that since “ t ” is a singular term in English₁ it must also be a singular term in English₂, and, since “ $F(t)$ ” is true in English₂, then “ t ” has to refer and then there has to be a mereological sum even in “Carnap’s” language, “contradiction”. But this deduction is false, since it presupposes that “ t ” is a singular term in English₂, something that is not true in our example of mereology. I think it is more appropriate to translate “ $F(t)$ ” in English₁ into something like “ $G(\alpha)$ ” in English₂, where α is not a singular term.

7. Jennifer Case’s “clarification”: a problem of meaning

At the end of the 1990s and in the beginning of the 2000s, Jennifer Case wrote a couple of papers, *On the Right Idea of a Conceptual Scheme* (1997) and *The Heart of Putnam’s Pluralistic Realism* (2001), that had great significance for Putnam’s work. Putnam acknowledges this in a reply to the second paper (which also treats her first), in *Reply to Jennifer Case* (2001b).

In her first paper, Case tries to show that Donald Davidson’s arguments against “conceptual schemes” (in *On the Very Idea of a Conceptual Scheme* (1974)⁹¹) does not threaten Putnam’s use of that notion. She proposes, however, a certain modification of Putnam’s terminology and suggests a number of clarifications in Putnam’s own thought. In particular, Case argues that in translation practices we rely on “sameness of meaning” between “natural languages”, whereas different conceptual schemes (in the sciences for example) may even be equivalent without us being able to identify any “sameness of meaning”, as when we say “there are three objects” in different “natural languages” such as English or Czech.⁹² It is one thing to translate between natural languages, and a completely different thing (using relative translation) to translate between different *optional languages* as “Carnap’s” language and the one of the Polish logician, supplying us with different answers as to how many objects there are, three or seven, for example.

Davidson argues that if there are multiple nontrivial conceptual schemes, then there are uninterpretable languages, and, since such languages cannot exist,

⁹¹Re-published in Davidson (2001a), pp. 183–198.

⁹²Case (1997), 12–13.

there cannot exist multiple conceptual scheme. But then there cannot even exist one conceptual scheme, since the notion does not make sense.⁹³ Case writes that both Putnam and Davidson reject what the latter calls the "scheme-content dualism", the third dogma of empiricism, that is, a conceptual dualism between organization and something waiting to be organized.⁹⁴ However, according to Case, Davidson "throws out the very idea of a conceptual scheme", when he shows that this scheme-content dualism cannot be relied on when we interpret languages using the notion of "sameness of meaning".⁹⁵ But she finds Davidson's view of language too narrow, since we could apply relative interpretation to parts of language which do not belong to our natural language in this way. She finds that Putnam's view of language is a much more realistic one in this sense, but that Davidson is correct regarding the preservation of meaning across natural languages, implying that there are no conceptual schemes involved in saying that there is a computer on my desk.⁹⁶

Case does not think that Putnam has been explicit enough regarding the distinctions she suggests, but that he has in fact a somewhat unclear version of the move she proposes. She argues (and Putnam affirms) that her distinction was in a sense anticipated in the last passage of Putnam's *Truth and Convention* (1987) in a terse way as follows, where we have added Case's qualifying subscripts 'o' and 'n' for optional and natural languages, respectively:

It seems to me that the very assumption that there is such a thing as the radical interpreter's 'own' language_[o]—one language_[o] in which he can give the truth-conditions for *every* sentence in every language_[o,n] he claims to be able to understand—is what forces the conclusion. As long as one operates with this assumption, conceptual relativism [i.e. relativism – Case's remark] will seem unintelligible (as it does to Davidson). But if one recognizes that the radical interpreter himself may have more than one 'home' conceptual scheme [i.e. language_[o] – Case's remark] and that 'translation practice' may be governed by more than one set of constraints, then one sees that conceptual relativity does not disappear when we inquire into the 'meanings' of the various conceptual alternatives [...].⁹⁷

Case explains Putnam's arguments from *Truth and Convention* as follows. We may translate "det finns sju objekt" (i.e., "there are seven objects" in Swedish, formulated in the *optional language* of the Polish logician) into English either by using Carnap's *optional language*, giving "there are three objects", or we may

⁹³Ibid., p. 13.

⁹⁴Ibid., p. 13.

⁹⁵Ibid., p. 13.

⁹⁶Ibid., p. 13.

⁹⁷Ibid., p. 12; Putnam (1987), p. 104.

use the Polish logician's *optional language*, giving us "there are seven objects". We could of course also have obtained the result "there are 127 objects" in the optional language of the Polish logician, if the original sentence were formulated in the optional language of "Carnap": "The choice of translation will depend upon the context and also upon the interpreter's interests."⁹⁸ Putnam accepts Case's clarification:

The most common misunderstandings are (1) that by a 'conceptual scheme' I meant any body of thought and talk at all, including our ordinary talk of tables and chairs; and (2) that by 'conceptual relativity' I meant a doctrine which implies that every conceptual scheme in *this* sense, *every* body of thought and talk, has an alternative which is incompatible with it [...] but equally true.⁹⁹

Putnam is referring to statements such as "there is a computer on my desk", which he says is certainly not true or false depending on the choice of a conceptual scheme. But I think we should be careful about how we interpret Putnam on this point. Many different philosophies of "truth" and "meaning" would accept that the statement "there is a computer on my desk" is true if there is a computer on my desk, but this does not entail that Putnam has surrendered to a Davidsonian theory of truth and meaning.¹⁰⁰ What Putnam wants to underline in his reply to Case is rather that one cannot always use a theory of meaning resting on simple translation between natural languages such as English and Swedish in the sense that we accept that tables come out as tables and chairs as chairs.

As we will see *this is too simplified a picture* of what is going on even in such translations; in a sense, one can say that this view of meaning as translation contains no information. It is simply redundant to say that such everyday words come out the same in two languages that largely share the same culture and history.

In addition, by developing a *general* theory of meaning from this picture means relying on such a simplified picture of sameness of meaning, that every change in the use of "object" and "exist" renders a "new meaning" that Davidson and Blackburn have to make sense of, perhaps by analogy with words like "table" and "chair". But as Putnam repeatedly argues, to do so "is to try to force the ordinary-language notion of meaning to do a job for which it was never designed".¹⁰¹ In a reply to Blackburn, Putnam writes:

⁹⁸Case (1997), p. 12.

⁹⁹Putnam (2001b), p. 431.

¹⁰⁰This is certainly not true in his earlier *Dewey Lectures* and his later *Ethics without Ontology*, but Putnam does not seem to want to expand on this issue in his short reply to Jennifer Case.

¹⁰¹Putnam (1992), p. 118.

'Which translation scheme, if any, preserves the *meanings* of the sentences being translated?' is a bad question. The ordinary notion of 'meaning' was simply not invented for this kind of case. In contrast a metaphysical realist would say 'Well all that is involved is a difference in meaning of a perfectly ordinary kind. The fact is just that "object" sometimes includes mereological sums and sometimes excludes them' [...]; but this reply assumes that mereological sums are objects (otherwise one couldn't include them in one's ontology), and the choice of different 'meanings' of the word 'object' is just a subclass from a universe, or fixed totality, of all objects. Here I part company with Blackburn. I believe that there are different uses of 'object' and no metaphysically privileged use (and thus no sense to the notion of a totality of all objects fixed once and for all).¹⁰²

"Meaning" as used in ordinary language is not "well-defined" enough to explain differences of "exist" when we speak in "Carnap's" language or in that of the Polish logician.

The picture of meaning from translation schemes between different natural languages as we translate "tables" and "chairs" into tables and chairs respectively may thus lead us to draw the conclusion that we cannot accept two different descriptions in science as both true, even if the formalizations are "obviously" equivalent (at least from our present point of view, if are going to be faithful to Putnam's own picture). For instance, we may rely on either one of the different formalizations of a space-time point, i.e., as either an individual or as a limit, for example, as giving rise to the true description that "there is a third point on a line between any two points". For Putnam, the truth of this assertion does not depend on the particular formalization of a point. We should not draw an analogy between "point as an individual" and "point as a limit" and, say, "table" and "chair", respectively, and then surmise that because these words (table and chair) do not have the same meaning in the ordinary sense, we have two different meanings of the two different formalizations of a point. It is this analogy that gives rise to the artificial question whether a statement such as "there is a third point on a line between any two points" is true for both formalizations of a "point", the one with points as individuals and the one with point as limits. It is not because the two statements have the same "meaning", in the sense of ordinary translation between languages, that the statement "there is a third point on a line between any two points" is true in both versions.

Case invokes Putnam's discussion of the contrast between the *meaning* of words and statements (relying on the use of a "translation manual" to obtain

¹⁰²Clark and Hale (1994), p. 246.

what he calls a “literal translation”) and *sense*¹⁰³ from *Reply to James Conant* (1992).¹⁰⁴ As we saw in Chapter 2, Felix Mühlhölzer considers Putnam’s distinction between meaning and sense in *Putnam, Wittgenstein and the Objectivity of Mathematics* (1999). He writes:

If I understand it correctly, *meaning*, as Putnam uses this notion here, is essentially conceived as an invariant under translation. To take an example which figures prominently in his work: According to our actual translation practice, the term ‘momentum’, as used by Newton, did *not* alter its meaning when used by Einstein, despite the fact that Einstein negated Newton’s assertion

(1) Momentum is the product of mass and velocity; a negation which Newton himself could not give any sense.¹⁰⁵

The sense of “momentum” has now changed, although the meaning of the word is the same as before. Recall now (Chapter 2, Section 4) that Mühlhölzer thinks that “Wittgensteinians, who strongly tend to be concerned with ‘sense’ alone at the expense of ‘meaning’, should come to grips with it”. In the same quote, however, Mühlhölzer stresses that he thinks that one should also recognize that this notion of lexical meaning has substance only in empirical science and not in mathematics. This is why Mühlhölzer thinks that Putnam makes a mistake when he draws a parallel to mathematics from his insight that the negation of “momentum is the product of mass and velocity” once had no sense, but when we are presented with a theory in which such a sense can be made (such as Einstein’s), then sense can be made of a different formula for momentum, but the meaning of the word “momentum” is retained; “momentum” has an *invariant meaning*.¹⁰⁶ But when we apply this scheme to (pure) mathematics, we are quite mistaken to think that there is an *invariance of meaning* of “triangle” as we move from Euclidean to non-Euclidean geometry. By accepting the intelligible use of the negation of “a triangle has at most one angle that is a right angle”, in view

¹⁰³The same distinction as discussed in *Rethinking Mathematical Necessity* (1990)[1994] borrowed from Cora Diamond and with the example of “momentum” in Chapter 2, where momentum was viewed by Putnam (1990)[1994] to retain its meaning (in that we “translate” old texts “homophonically”) in Einstein’s and Newton’s physical systems, but having different senses in these two theories.

¹⁰⁴The reply to Conant is to the latter’s paper *The Search for Logically Alien Thought: Descartes, Kant, Frege and the Tractatus* which in turn was developed from considerations Putnam made in *Rethinking Mathematical Necessity*.

¹⁰⁵Mühlhölzer (2009), pp. 16–17.

¹⁰⁶To be more precise: Mühlhölzer mentions this *invariance of meaning* in the context of physical geometry, since we have concrete constellations of mountain tops to represent physical triangles, etc., but he does say that the notion of meaning he seems to find in Putnam’s writings here, meaning as invariant under translation, works in this way in empirical science. (Ibid., pp. 17–18.)

of non-Euclidean geometry, there is no invariance of the meaning of the word triangle, since there are no external reference points to pure mathematics (pure geometry). Thus Mühlhölzer arrives at the conclusion that meaning and sense coincide in mathematics. In fact, we have only "sense" in mathematics. According to Mühlhölzer, we stipulate a new meaning (or sense) for "triangle" within non-Euclidean geometry, contrary to Putnam's view.¹⁰⁷

Mühlhölzer's analysis of Putnam's view of pure mathematics is correct in an important respect. Putnam says that it is not intelligible to negate " $5 + 7 = 12$ ", while at the same time he is using analogy arguments to open up for the possibility of changes to such expressions in a future, within some new and now not available theory. Putnam often uses an analogy with the overturn of Euclidean geometry in order to make this point.¹⁰⁸ Mühlhölzer argues that Putnam is quite mistaken in thinking that the necessity of " $5 + 7 = 12$ " could be viewed in this way:

In order to treat ' $5 + 7 = 12$ ' as necessary, it is enough that—irrespective of what we can *imagine*—when we *actually* count from 1 to 5 and then add further seven steps of counting, we normally get 12; and that we normally get 12 when we unite 5 and 7 objects (in reality and thought). These familiar experiences are then 'hardened [by us] into a rule', as Wittgenstein says in RFM VI §§22-23, and it is exactly this process of transforming them into a rule that brings about the necessity of the statement ' $5 + 7 = 12$ '.¹⁰⁹

At the same time, I think Mühlhölzer misrepresents Putnam in attributing to him a theory of meaning as invariant under translation. I also think that Mühlhölzer is mistaken in thinking that such a theory of meaning is valuable in empirical sciences, *in contrast* to the use-based notion of meaning, i.e., *sense*. In his middle and late periods, Putnam is hostile to the ordinary use of meaning (usually connected to the extension of a word such as "rabbit") in the context of science. In fact, he seems to think that the notion of meaning in play when people talk about literal translation comes down to little more than identifying a name or a sound, such as the word "momentum". Putnam's realism regarding our choice to retain the word consists in that it indeed explains the same phenomena. But from at least 1981 onwards, there is no *correspondence theory* of truth in Putnam's philosophy that we may rely on in construing a relation between language and the world.¹¹⁰

¹⁰⁷Ibid., pp. 17–18.

¹⁰⁸See, for example, Putnam, *Reply to James Conant* (1992b), p. 375.

¹⁰⁹Mühlhölzer (2009), p. 13.

¹¹⁰Putnam (1999), p. 183n.

I think Mühlhölzer is mistaken when he argues that we may retain the words “momentum” and “triangle” in physical geometry just because there are concrete constellations that are preserved over time, such as mountain tops. Surely there are pragmatic reasons to retain these words, important scientific contexts as well as historical reasons, not to mention that we want to preserve a continuity of our investigations, since we often take ourselves to be explaining the same phenomena. And I believe that the reasons for using the word “triangle” for an object on a sphere which has an “angle sum” greater than two right angles (and hence was certainly not viewed as a triangle by Euclid) are very much the same reasons as in physical science: we have now geodesics that play the same role as Euclidean straight lines, and this justifies that we extend the concept of a triangle. We accept and understand an “object” as a triangle. Putnam opposes the idea that we have stipulated a new meaning of “triangle”,¹¹¹ and I think that what he intends by this rejection is that we do not arbitrarily stipulate that a new geometrical object is called a “triangle”.

We *understand* these new objects as triangles, by virtue of a change of the sense of the word triangle provided by a new geometrical context. But although I agree with Putnam that there is a difference between accepting something as a triangle rather than just making up what we “mean” by it, I think that we could also have chosen to think that this new object on a sphere is not a triangle. There is nothing intrinsic to non-Euclidean geometry that requires its objects to be related to the Euclidean objects in the way that we have triangles on spheres, etc. It is rather that there is an important sense in which we may understand them as triangles, although one could still argue that there are other ways in which they are not.

Putnam is in fact quite critical of a definition of meaning by translation practices. Within such theories, there are vocabularies in which words have meanings that do not depend on the use of these words, and it is this lexical notion of meaning to which Putnam is referring when he says that we may know the lexical meaning of a word without knowing what is being said. Mühlhölzer is right in his claim that Putnam retains such a position, but instead of affirming it in the context of empirical science (as Mühlhölzer does), one should rather be quite critical of this distinction between meaning and sense. As we have seen, Putnam is rather critical of the power of lexical or linguistic meaning, i.e. meaning defined through a Davidsonian translation practice. However, he hangs on to this distinction between lexical meaning and sense as a use-based notion of meaning (connected to understanding) in this transition period, before he lets go of the language/reality dichotomy altogether.

¹¹¹Putnam (1992b), p. 375.

In a sense I have gone too far in my defense of Putnam against Mühlhölzer's criticism, since, as we saw in connection with Putnam's treatment of the change of sense of the word "momentum" and the preservation of a lexical meaning, Putnam certainly accepts some part of the Davidsonian (or perhaps Quinean) picture. But even during this transition period (at least in his 1992 response to Conant), Putnam is clearly doubtful of the employment of such notions, since his basic criticism is that this sort of meaning is not connected to how we understand language. In *Reply to James Conant*, he writes:

What I argue [...] is that the word 'sense' in questions like 'In what sense do you mean that?' is much more flexible than the word 'meaning' as used in philosophers' talk of 'translation manuals' and 'recursive specifications of meaning'. To use an example of Charles Travis, suppose someone paints the leaves on my Japanese ornamental tree (which has copper-colored leaves) green. If someone who doesn't know what happened remarks that my tree has 'green leaves', is that right or wrong? We may reply that it all depends on what sense we give to 'green leaves'; but I don't think that this shows that either 'green' or 'leaves' has two *meanings*. Rather, it shows that even given the (dictionary) meanings of the words, we do not always know what a particular sentence *says* (if anything). The content of a token sentence depends on the meaning of its words in the language, but it also depends on a multitude of features of the context.¹¹²

This is clearly a rebuttal of other philosophers' reliance on "translation manuals", designed after an empirical investigation of what words in one natural language "mean" in another, since even if dictionaries may be quite detailed in their explanations of words in another language (after a thorough empirical investigation), they may still fail to give us any understanding of what is being said or written, i.e., "what is meant". It may require a quite artificial example (as that of the Japanese ornamental tree given above) to illustrate this point, since the effectiveness of translation manuals (and translation programs) is relatively good in many cases, but this does not prove that this empirically grounded translation notion of meaning is what "meaning is". There is an implicit assumption of a language/reality dichotomy at work here when Putnam says that "the content of a token sentence depends on the meaning of its words in the language, but it also depends on a multitude of features of the context". How else to explain the "also" here? Putnam seems to suggest that there is a lexical meaning and *also* a dependence of the context. One would presume that the lexical fixation of the meaning of a word is empirically investigated with respect to its uses in many contexts, but that we may also find a new context where the empirically

¹¹²Putnam (1992b), p. 375.

based lexical meaning is of no help. This rather shows us that there should be no dichotomy between the meaning of a word and its use in a context.

Case writes that the difference of *sense* between distinct uses of a word is a function of the extending of a single concept expressed by that word, whereas the difference of *meaning* between distinct words is a function of the expression of separate concepts by those words.¹¹³ She concludes that translation between two conceptual schemes (or optional languages) is very different from translation between two natural languages, since we are indeed translating “there are three objects” from one conceptual scheme into “there are seven objects” in another scheme. Case also remarks that translation between optional languages alters sense while preserving meaning, whereas translation between natural languages preserves both.¹¹⁴

It is interesting here to compare Case’s view of the relation between meaning and sense with Mühlhölzer’s, who equates meaning and sense in pure mathematics, since he finds no concrete constellations preserving meaning there.¹¹⁵ Case, on the other hand, equates meaning and sense for “natural languages”. Now, if Case were intending to suggest by this that meaning is really sense (meaning as use), I think she would be right. But I rather think that she wants to suggest that all there is to sense as applied to natural languages is meaning as given by “translation practice” (in Davidson’s sense).

Case’s particular employment of a distinction between meaning and sense does not seem to have any effect on Putnam since, for him, meaning eventually becomes identified with sense, a workable notion for all of language, including the optional languages we use. However, although Putnam does not highlight this potential difference between himself and Case, we will see that the translation attending “optional languages” may have counterparts in “natural languages”. For the later Putnam, the difference between these two types of languages does not really depend on a different conception of meaning at work (and in this sense there are not different types of languages), but rather on that the employment of technical vocabulary of science, such as the sets and functions of mathematics, is optional, whereas the employment of words such as “tables” and “chairs” may not be as optional. As we shall see, however, there is no clear-cut boundary between objects we *have to* quantify over and those which we do not have to quantify over.

¹¹³Case (2001), p. 428.

¹¹⁴Ibid., p. 428.

¹¹⁵I am not sure whether this would mean that Mühlhölzer is in fact presupposing a language/reality dichotomy to be discussed in Chapter 4. Wittgenstein’s employment of “meaning as use” is a way of avoiding of having fixed meanings to words in a vocabulary, meanings that rely on “empirical observations”. What Mühlhölzer says could be viewed as a defense for such a view.

8. Meaning as use and pluralism

Wittgenstein said that “meaning is use”:

For a *large* class of cases—though not for all—in which we employ the word ‘meaning’ it can be defined thus: the meaning of a word is its use in the language.¹¹⁶

But he never endorsed a meaning-sense dichotomy as Putnam suggested in *Rethinking Mathematical Necessity* or in Mühlhölzer’s or Case’s interpretations of Putnam’s work. Here, in §§531–532 of the *Philosophical Investigations*, he rather says that we may sometimes insist on understanding a sentence only by having it repeated, but this is a rather special way, such as when we have a line of poetry or piece of music repeated.

We speak of understanding a sentence in the sense in which it can be replaced by another which says the same; but also in the sense in which it cannot be replaced by any other. (Any more than one musical theme can be replaced by another.)

In the one case the thought in the sentence is something common to different sentences, in the other, something that is expressed only by these words in these positions. (Understanding a poem.)

Then has ‘understanding’ two different meanings here?—I would rather say that these kinds of use of ‘understanding’ make up its meaning, make up the *concept* of understanding.

For I *want* to apply the word ‘understanding’ to all this.

As we will see from Putnam’s meditations in the *Dewey Lectures* presented in Chapter 4, it does not help to speak of meaning “as use”, if we have not qualified what we intend by “use”. But for now, we may think of it as sense, as understanding an expression, which is dependent on context.

In *Ethics without Ontology* (2004), Putnam drops the notion of sense and explicitly says that he wants to use what he views as Wittgenstein’s “looser” notion of “meaning as use” instead of a narrow notion of linguistic meaning, the notion of meaning that concerns Donald Davidson, for which the criterion for two expressions to have the same meaning is given by translation practice. Putnam now says that he agrees with his critics that there is a difference in meaning of “exist” between the Polish logician and “Carnap”, but that this difference should be understood in the Wittgensteinian sense of meaning as use, a difference that does not have to be described in a way that presupposes the existence of mereological sums or not:

¹¹⁶Wittgenstein (1953), §43.

[T]he Polish logician speaks as if, corresponding to any set of (more than one) individuals in a ‘Carnapian’ universe, there is a further individual which has as parts the members of that set. As a spatial location, the Polish logician assigns to this supposed (or pretended) individual the spatial region which is the geometrical sum of the regions [...] occupied by the Carnapian individuals in that set. *This* description is neutral as to whether these supposed or pretended individuals are ‘real’ individuals or mere logical constructions.¹¹⁷

The point Putnam wishes to make is that this explanation of the Polish logician’s use of his language does not employ any *translation* of the words “exist” and “object”, and hence the neutrality of whether the objects are “real” or logical constructions is not part of Davidson’s theory of meaning at all.

[I]t is, rather, a *manual of instructions* for talking the way the Polish logician talks. But it describes the difference between the way the Polish logician *uses* her language and the way the Carnapian *uses* her language. In the wide sense of the term ‘meaning’, meaning as use, there is a difference in ‘meaning’ here. But it is not *trivial*, because it is not the case that the person who gives this description of the Polish logician’s language has to agree that what the Polish logician says is *true*, or that the disagreement between the Carnapian logician and the Polish logician is ‘only apparent’. The neutral description allows for the possibility that someone might think that there *aren’t* any such things as ‘mereological sums’, that the whole idea of ‘mereological sums’ is crazy. [Nonetheless] I can *reinterpret* what the Polish logician says so that it comes out true.¹¹⁸

The conceptual relativity that Putnam defends implies that our natural language leaves completely open the question of which is the right way of using words such as “object”, “exist”, etc. Set theory and mereology are both examples of *optional languages*, in Case’s terminology:

The optional language of set theory and the optional language of mereology represent possible *extensions* of our ordinary ways of speaking. If we adopt mereology, or if we adopt both mereology and set theory, then of course we will say that there exist mereological sums. If we adopt set theory but reject mereology as unnecessary or useless, then we will say that mereological sums do not exist [...]. [T]he question whether mereological sums “really exist” is a silly question. It is *literally* a matter of convention whether we decide to say that they exist.¹¹⁹

¹¹⁷Putnam (2004), pp. 41–42.

¹¹⁸Ibid., p. 42.

¹¹⁹Ibid., p. 43.

Putnam does not say that he proposes conventional truths in Carnap's sense, but rather after David Lewis as a certain kind convention that is a "*solution to a certain kind of coordination problem*".¹²⁰ Examples of such conventions are choices on which road the cars should drive in order to solve a certain traffic problem. It is literally a matter of convention on which road the cars drive on, the left side or the right side, and this type of convention does not involve the Carnapian metaphysical notions of "analyticity" or "apriority" or "unrevisability".¹²¹ And

[i]n the same sense of convention, I claim, it is a matter of convention whether one decides, in a given formal context, to accept the axioms of mereology.¹²²

Instead of asking about the existence of points, etc., we should treat such "questions" by supplying a *conventional* choice of formalization within a scientific optional language, assuming either the existence of individual points or not, and both choices leave all causal explanations unaffected. That is, it does not matter to the physical theory whether we assume the existence of points or not. A question whether there *really* are points is silly.¹²³ Putnam writes:

To suppose that 'points are really individuals' has an unknown truth value would be to suppose that 'individual' has somehow its meaning fixed apart from its use, [...].¹²⁴

The conclusion we should draw from this is that there is no "right sense" to be dictated for words such as "individual", "object", "exist", etc.¹²⁵

In many ways Putnam just re-casts his insights from the internal (or perhaps pragmatic) realist period of conceptual relativity, in the terminology of "meaning as use" and Case's notion of "optional languages". What he is really aiming at in *Ethics without Ontology* is something more than such reformulations and an embrace of a liberal notion of "convention". I think that the bigger issue is that Putnam wants to come to terms with pluralism within language as a whole. He adheres to Jennifer Case's observation that conceptual relativity is a special

¹²⁰Ibid., p. 44.

¹²¹Ibid., p. 44. Notice here, that Putnam's formulation does not preclude *revisability*, which is problematic. One gets the impression that Putnam is too concerned with distancing himself from Carnap. I think that Friedman's attitude towards Carnap (discussed in Chapter 1) is much less strained, in the sense that he thinks that the particular way in which Carnap formulated his scheme with conventional and factual statements was based too much on the analytic-synthetic dichotomy, vulnerable to Quine's critique, but that it is important for *functional* purposes to distinguish between different types of statements, those which just set the stage for what is possible to formulate and those which purport to describe factual circumstances.

¹²²Ibid., p. 44.

¹²³Ibid., p. 47.

¹²⁴Ibid., p. 47.

¹²⁵Ibid., p. 47.

case of the wider notion of *pluralism*. In *Reply to Jennifer Case*, Putnam for the first time changes his mind about whether the notion of conceptual relativity includes cases such as two different descriptions of a room, a scientific one involving particles and fields, i.e., using an optional language, and another involving our ordinary vocabulary of tables and chairs.¹²⁶ The reasons that we do not have an instance of conceptual relativity here is that

- (a) such descriptions are not incompatible at face value (or with respect to the surface grammar of the expressions);
- (b) they are not cognitively equivalent—the information that the two descriptions supply are quite different.

The doctrine of *pluralism* allows either one of the descriptions of the room, that is, the scientific one or the one describing tables and chairs; i.e., it allows “both of these schemes without being required to reduce one or both of them to some single fundamental and universal ontology”,¹²⁷ and this doctrine contains the notion of conceptual relativity. In *Ethics without Ontology*, Putnam coins the term *conceptual pluralism* for this wider notion of plurality, but he adds important features to it, which one can see in the light of his Wittgensteinian turn.

As I noted in the previous section, I think that Case’s simplified analysis of meaning and sense as applied to natural languages and optional languages, coinciding in the former case and being different in the latter case, leads her to the conclusion that it is within the optional languages we obtain extensions of concepts such as “object” and “exist”, thus explaining conceptual relativities in the sciences. At the same time she defends Davidson’s point of view regarding natural languages. Putnam is not content with this, as is clear from *Ethics without Ontology*, since, as we have seen, he explicitly challenges Davidson’s notion of meaning in a way that does not make Davidson’s notion important to what Putnam has to say about the different descriptions—meaning as lexical translation is now viewed as deeply problematic.

What Putnam now suggests is that the same phenomenon may occur for so-called “natural languages” as for our optional sublanguages. In fact, as Putnam discusses the nature of words used in the non-optional languages that we use daily in our communication with others, Putnam is critical towards Davidson’s *On the Very Idea of a Conceptual Scheme*. In particular, Davidson never delved into the examples of Whorf¹²⁸ in his criticism of Whorf’s work, in which, for

¹²⁶A view which he endorses as late as in *Renewing Philosophy* (1992).

¹²⁷Putnam (2001b), p. 437.

¹²⁸Putnam (2004), pp. 50, 139n22.

example, Whorf pointed out that the two very different sentences in English

(1) I have an extra toe on my foot.

(2) I pull the branch aside.

are expressed in Shawnee by two very similar sentences morphologically and grammatically as, respectively,

(3) I fork-tree on-toes (have).

(4) I fork-tree by-hand-cause.

The point of this example is that there is a morpheme in Shawnee that is translated as “fork-tree” (or “fork-shaped pattern”) and that what appears to be *at least for us* two very different assertions render almost identical sentences in the morpheme-by-morpheme translations. What Putnam briefly says here (arguing against Davidson) is that the meanings of Shawnee sentences may not come out correctly as we make a translation between Shawnee and English; in fact, we may not really understand (or translate) their way of thinking at all, due to their completely different set-up of things, properties and situations. This is illustrated well by looking at this very different culture, but, as Putnam points out, can also be seen from the notorious difficulty of translating the English word *mind* into French (*esprit*) and German (*Geist*) without loss or corruption of meaning.¹²⁹

Davidson has famously argued against Whorf that the very fact that Whorf could translate Shawnee into English at all shows that there is no difference in ‘conceptual scheme’ between the two languages, and the same argument is commonplace today in papers and courses on psycholinguistics. However, this argument assumes that English already had *that* notion of a ‘fork-shaped pattern’ (or ‘fork-tree’) *before* Whorf wrote his paper. In fact, the whole argument of Davidson’s ‘The Very Concept of a Conceptual Scheme’ assumes that *translation leaves the language into which we translate unaffected*. I deny both of these premises. I think Shawnee has an “ontology” of patterns that (normal) English lacks, although we could, of course, *add* it to English; and I think that the conceptual scheme of English is constantly being enriched by interactions with other languages, as well as by scientific, artistic, etc., creations.¹³⁰

¹²⁹Ibid., p. 50.

¹³⁰Ibid., p. 50.

Someone speaking Shawnee may adopt certain English ways of thinking and hence corroborate the translations. The point is that Davidson *takes it for granted* that no such problems arise when we translate.

Although Shawnee, French and English are not optional languages, they certainly show that there is no single way in which we “quantify” when we describe very simple things in our language. Our natural or national languages cannot simply be made to conform to a simplified predicate calculus in the sense that it is clear what objects there are and how we use words, which from a predicate calculus point of view looks innocent, such as “there exists”. The world does not dictate a unique or true “way of dividing the world into objects, situations, properties, etc.” The idea that there is, for Putnam, is “a piece of philosophical parochialism”, which is behind the subject of Ontology, or “onto-theological” projects made respectable in analytic philosophy by Quine.

For the 2004 Putnam, when we follow these Ontological projects, assuming that there is a single real literal meaning of exist, identity, etc., “one that is cast in marble, and cannot be either contradicted or expanded without defiling the statue of the god, we are already in Cloud Cuckoo Land”.¹³¹ The problem for Quine was that he did not want to allow for “endless possibilities of extending our notions of ‘existence’ (Conceptual Pluralism)”,¹³² and saw these different ways of speaking as just loose talk and secondary conceptual systems, in comparison to our first grade conceptual system, science.¹³³ Putnam has made a long journey from himself idolizing the scientific descriptions as the prime version of our world, now urging us to adopt his conceptual pluralism, i.e., to respect different language games.

In place of Ontology (note the capital “O”), I shall be defending what one might call *pragmatic pluralism*, the recognition that it is no accident that in everyday language we employ many different kinds of discourses, discourses subject to different standards and possessing different sorts of applications, with different logical and grammatical features—different “language games” in Wittgenstein’s sense—no accident because it is an illusion that there could be just one sort of language game which could be sufficient for the description of all of reality!¹³⁴

¹³¹Ibid., p. 85.

¹³²Ibid., p. 85.

¹³³Ibid., p. 85.

¹³⁴Ibid., pp. 21–22.

CHAPTER 4

Meaning, Truth and Commonsense Realism

1. Introduction

In the *Dewey Lectures* held in March 1994¹ at Columbia University, Putnam develops an alternative picture to that of his internal (or pragmatic) realism, although he continues to defend a central tenet of this earlier position, that of *conceptual relativity*. Putnam's natural realism, or common sense realism, has not been discussed as much as his earlier internal realism, but I think that the new form of realism Putnam advocates here constitutes an important development of his earlier position, in that his view of meaning becomes consistent with a refutation of idealism and metaphysical realism in a way that preserves both what William James called the "natural reactions of the common man", and a notion of conceptual relativity inspired by Wittgenstein's thoughts on meaning as use. Putnam dismisses his earlier internal realism, since its conception of truth hinged on a picture of an interface between language and reality, an interface that has been common in philosophy since the 17th century. However we describe such an interface (for instance, as "sense data" *caused* by a real world to which we have no direct access), cognitive contact with our world is lost, and we get a language/reality dichotomy, which has been the source of a distorted philosophy of language within analytic philosophy.

I will not discuss Putnam's account of perception at any length; I have concentrated on the implications of Putnam's reasoning on conception, or what Putnam in the third and last part of the *Dewey Lectures* calls "the face of conception", since I will primarily follow what Putnam has to say on questions relevant to the philosophy of mathematics. But I will also focus on Putnam's attempt to get beyond the language/reality dichotomy with the aid of Wittgenstein, in particular by an employment of Wittgenstein's "meaning as use". Putnam admits that his earlier thoughts about *use* in this context were connected to a picture of the brain as a kind of computer (a cognitive science view), with inputs in the form of sense data. Putnam calls such a picture "Cartesianism cum materialism":

¹These were then published twice: in *Journal of Philosophy* (1994), No. 9, as "Sense, Nonsense, and the Senses", and as Part I of *The Threefold Cord* (1999), Columbia University Press.

[I]t is a profound mistake to equate serious science with the Cartesianism cum materialism that has for three centuries tried to wrap itself in the mantle of science. Today that attempt often takes the form of empty talk about ‘the conceptual structure of the mind’—talk that simply takes for granted the idea that thinking *is* syntactic manipulation of symbols. Nothing in the successes of serious psychology or linguistics endows that view with content. Instead, such talk frequently lowers the level of philosophical discussion to that of popular ‘scientific’ journalism.²

It is a mistake to view language as simply “marks and noises”, into which a significance has to be read—this is not the right way of countering the equally false view that words and sentences in language have their meanings intrinsically. To view these two alternatives as the only available options is essentially Dummett’s way of structuring the realism/anti-realism debate, where Wittgenstein is erroneously placed in the anti-realist camp. In fact, Dummett views Wittgenstein as rather extreme in the sense that he sees Wittgenstein as defending a “full-blooded conventionalism” when the latter remarks that language is fine as it is (that language is already in place), and that rules such as $2 + 2 = 4$ are in no need of justification.

Dummett’s view rests on a view of language that Putnam denounces in the *Dewey Lectures*. While he does not attribute to Dummett an explicit “cognitive science” standpoint, Putnam suspects that Dummett fails to recognize that he starts off from underlying assumptions similar to the ones lurking behind much of cognitive science:

[Because] his emphasis on *formal proof* as a model for verification, and his insistence that the goal of philosophy of language should be to *specify recursively* how the sentences of the language can be verified, suggest to me that his picture of language is closer to the ‘cognitive scientific’ version of the Cartesian cum materialism picture than he himself may realize. [...] Dummett sees no alternative to the picture [...] except to postulate mysterious mental acts; and that is because he has from the beginning felt obliged to regard his own thoughts as if they were syntactic objects that require rules of manipulation.³

Putnam’s condemnation of the postulation of “mysterious mental acts” is part of his project to undermine the view of the relationship between language and reality that has caused analytic philosophy to produce all sorts of metaphysical doctrines in the attempt to extricate itself from the problems stemming from its own unexamined presuppositions. In Chapter 3 we encountered some examples

²Putnam (1999), pp. 48–49.

³Ibid., pp. 58–59.

of such doctrines. One example is Quine's postulation of abstract entities such as sets; another is Davidson's assumptions of a home language that contains all possible variations of how we can intelligibly mean something. In this chapter, Dummett will provide a third example.

During one period, beginning in the mid-70s, Putnam was strongly influenced by Dummett; I suspect that the harsh criticism leveled at Dummett in the quote above is at least as much directed towards the author of *Reason, Truth and History* (1981), Putnam's major work during his internal realist period. Putnam always had his own particular (holistic) version of verificationism, connected to a view of truth that he insisted made him a realist. As Davidson has noted, the difference between Dummett and Putnam on this point has been slight.⁴ Putnam's verificationistic position on truth was a result of what he saw as an "antinomy of realism" that seemed to follow from his model theoretic considerations, applying the Skolem–Löwenheim theorem to everyday language within the Cartesianism cum materialism picture. He drew the conclusion that everything that happens within the sphere of cognition would leave the "objective reference of our terms" undetermined, but this contradicted his more Wittgensteinian view that "either the use of language *already* fixes the interpretation [of our words] or *nothing* can".⁵ For Putnam, it seemed almost magical that the world interprets the words for us with "noetic rays" stretching from the outside into our heads.⁶ The solution that he arrived at was his own form of "anti-realism" (argued at the time as a species of "realism"), a verificationist notion of truth, with the qualification (constituting the alleged difference with respect to Dummett) that truth is identified with verification to a "sufficient degree to warrant acceptance under sufficiently good epistemic conditions" (the last part being the realist strain, a "world-involving notion").⁷ As we saw in Chapter 3, Davidson became critical of Putnam's notion of truth as an alleged improvement upon Dummett, and Putnam later conceded to this criticism, adding that it certainly was as magical as the metaphysical realism he had already rejected.

When Putnam follows John McDowell in undercutting the whole picture of a language/reality dichotomy, he also embraces some of Wittgenstein's writings. In particular he sides with the nowadays common Wittgensteinian view (but defended with slightly different arguments) that, although there are no "mental tracks" that support us in the following of rules, we are in no need of a community or an external practice either (the communitarian anti-realist interpretation). When we express ourselves *in our language* (which is neither a formalized language, nor mere marks and noises), we have no problem following

⁴Davidson (1988)[2001b], p. 187.

⁵Putnam (1999), pp. 16–17.

⁶Ibid., p. 17.

⁷Ibid., p. 17.

simple rules such as addition, and we know exactly what is meant by saying “my brother is giving a concert in New York”, without any assertability conditions.

When we say that our “language is (already) in place”, it may seem as if we are cheating, but Putnam’s point is that words such as “truth” and “proposition” are used in a way in analytic philosophy that gives rise to many problems, including the realist/anti-realist debate. One often wants to define one of these notions (truth or proposition) with respect to the other (using mathematical logic as a model), which is left as primitive (such as Davidson’s notion of truth). This gives a certain “anti-realist” (in particular, a deflationist) interpretation to Wittgenstein’s formulation that to say that ‘ p ’ is true, is just to say p . It is not the case that Wittgenstein here endorses a deflationist definition of truth in the sense of mathematical logic, with p being a variable proposition. The German word Wittgenstein uses for “proposition” (as translated by Anscombe) in the *Philosophical Investigation* is *Satz*. To assume that a *Satz* is either true or false is not to assume a metaphysical doctrine such as the principle of bivalence (that a proposition in the mathematical logic sense is either true or false). The point is that a *Satz* is true or false *in our language* and not in an ideal language. Even the language of mathematics should in principle be treated as *our language*. We say, for instance, that the twin prime conjecture is either true or false. In our mathematical language, there are no other possibilities, although a proof may itself be surprising, and lead to surprising connections. Conversely, a grammatical string of sounds and noises is not automatically a *Satz*.

Thus, Putnam’s conception of language has become very different from what one ordinarily finds in analytic philosophy. In particular, it is interesting to note that he thinks that we have overemphasized the difference between human and animal cognition:

[A] wolf could expect to find a deer on a meadow, and its ability to expect that is a primitive form of our ability to expect to find a deer on the meadow. Our highly developed and highly discriminating abilities to think about situations that we are not observing are developments of powers that we share with other animals. But, at the same time, one must not make the mistake of supposing that language is merely a ‘code’ that we must use to transcribe thoughts we could perfectly well have without the ‘code’. This is a mistake, not only because the simplest thought is altered [...] by being expressed in language but because language alters the range of *experiences* we can have.⁸

I generally side with Putnam on his new view of language, in particular his denouncement of a language/reality dichotomy, but I will also argue that Putnam

⁸Ibid., p. 48.

has not really connected this insight to his analysis of the role of arithmetic “statements” such as $5 + 7 = 12$.

The problem, I think, is that he does not separate “calculus” and “prose” in Wittgenstein’s philosophy of mathematics, and this leads him to view $5 + 7 = 12$ as a “proposition” with some content. Although he is certainly combating the idea that we have here a description of a particular reality, his account amounts to a pragmatic motivation for these arithmetic assertions: we “cannot do without them”. Putnam’s problem with prose/calculus is also expressed in his analysis of the later Wittgenstein’s view that one and the “same” mathematical theorem may have more than one proof, in the sense that an ancient geometric proof and a modern algebraic proof may both prove the “same” theorem.

This has been contested in the historiography of mathematics, sometimes rightly so, since defenders of such a view (notably André Weil) have gone so far as to say that the Greeks had an algebra. I will not discuss the historiographical question in any depth, but I think that it is nonsense to say that the Greeks had an algebra in geometrical disguise. Yet, there is a sense in which they proved the same theorem as we do using modern algebra. And this sense is connected to Putnam’s characterization of mathematical necessity, inspired by Cora Diamond’s reading of Wittgenstein, and, in particular, her notions of “the face of meaning” and “the face of necessity”.

Here we need to distinguish once again between the necessity expressed through a calculus and the necessity whereby, for example, we call a triangle in Riemannian geometry a “triangle”. There is no mathematical necessity involved in this naming, although Putnam is at least close to claiming that there is, something that goes back to *It Ain’t Necessarily So* (1962). Putnam is certainly right that we do not simply stipulate that something (anything) is a triangle; more is involved, namely, that we can *understand* the new mathematical “object” as a triangle. But I think that Putnam does not properly analyze the difference between the “necessity” of $5 + 7 = 12$ and the “reasons” behind the extension of our concept of a triangle. Although we may be compelled to call this new mathematical “object” a triangle, we could in principle have called it something else, refusing to call anything with an angle sum greater than 180° a “triangle”. For Putnam, early and late, there is an important analogy between this particular example (that we eventually found a triangle with an angle sum greater than 180°), a result of the development of non-Euclidean geometry, and every other mathematically necessary “assertion”, supporting the Quinean idea that there is no such thing as an unrevisable “statement”.

But there is a crucial difference between a suggestion (unintelligible as it is at present in Putnam’s view) that we could revise $5 + 7 = 12$, in analogy with the example of the triangle, and to *say* that there is a *triangle* with an angle sum greater than 180° . The analogy is a false one. Non-Euclidean geometry provided

a new calculus with geometrical objects, some of which were new, but which may be regarded as “natural” extensions of older ones. But there is no necessity built into this new mathematical context that we regard the new “triangles” (on spheres, for example) as *triangles* (in the old sense). On the contrary, the purported necessity here is one that is suggested from the “prose” and not one that is part of the necessity of a calculus. Putnam uses Cora Diamond’s metaphor of a picture face to include both the mathematical necessity in the rules of a calculus and that of the “prose”, the latter which is not really necessity at all, but a way of making and recognizing sense. I do not think that the face-metaphor is particularly helpful as an aid for identifying the necessity of $5 + 7 = 12$ in Wittgenstein’s philosophy, or for accounting for it. Thus, there is a remaining problem within Putnam’s philosophy, but one which could be modified. I think that Putnam has the resources to come to terms with the remaining problem of necessity, once he is less chained to Diamond’s and Conant’s readings of Wittgenstein.

2. Perception, meaning as use, and concepts

Putnam links a certain language/reality dichotomy⁹ to the structuring of the realism/anti-realism debate in analytic philosophy. This structuring of the debate precludes at the outset a number of ways of addressing profound philosophical problems, notably those of Wittgenstein. Wittgenstein is usually placed squarely in the anti-realist camp, and even considered to be quite an extreme anti-realist. In this section, I will briefly try to explain the connection between the realism/anti-realism debate and the problem of language and reality with respect to Putnam’s views on perception. He argues that the repression of the problems of perception has obstructed “the possibility of progress with respect to broader epistemological and metaphysical issues” since the 1960s.¹⁰

Putnam praises Moore and Russell for their attention to the problem of perception, and also Strawson to some extent, but regrets that Wittgenstein has not had much influence on this topic. Neither has Austin’s *Sense and Sensibilia*, published in 1962, two years after Austin’s death. Austin’s work is Putnam’s main inspiration regarding the problem of perception, and Putnam devotes the second of the three *Dewey Lectures* to Austin’s view of perception. I will not say much about Austin or Putnam’s use of him, but it is important to make the connection with what I have to say about the realism/anti-realism debate, which has implications for my particular study of Putnam on *meaning* and *necessity*, including my own thoughts on mathematical necessity.

Influenced by John McDowell’s *Mind and World*, Putnam argues against the picture of an *interface* between our cognitive powers and the external world

⁹This is my terminology.

¹⁰Putnam (1999), p. 11.

which makes it seem as if our cognitive powers cannot reach the objects themselves.¹¹ Putnam chooses not to use the label direct realism,¹² which he identifies with a mere linguistic “reform” (in the hands of e.g. Searle) the upshot of which is to get rid of the notion of sense data of visual experience a little too easy by *saying* that we do not perceive visual experiences, we *have* them.¹³ Putnam thinks that this move, which allows us to drop the problem of perception, is a bit too facile. In the spirit of William James in honoring the “natural realism of the common man”, Putnam introduces the term *natural realism* for the position he now wishes to defend, using arguments from the later Wittgenstein and Austin in particular.

Putnam sees himself as arguing against a long philosophical tradition, going back to the 17th century, of viewing immediate objects of perception as mental nonphysical objects. This tradition has prevailed in many disguises, using different terminologies, including “Cartesian ‘ideas’ or Humean ‘impressions’ or Machian ‘sensations’ or Russellian ‘sense data’.”¹⁴ Putnam does not think that this view belongs to a historical past. On the contrary, he sees similar views in current philosophy—including his own internal realistic period—as deeply problematic. In analytic philosophy, he identifies the view with cognitive science, a view that emphasizes the brain as a computer that calculates with *representations*. This view has precursors throughout the history of philosophy, even if one now identifies the mind with the brain. In fact, this way of thinking often leads to a picture in which the “representations” in the cerebral computer become analogous to the classical theorist’s “impressions”. These “representations” are thought to be connected causally but not cognitively to the organism’s environment.¹⁵ Hence the interface: the impossibility of being in cognitive contact with the world. Putnam claims that it is indeed difficult to see how thought and language can hook onto the world without seriously considering the problem of perception. In fact, he believes that the present views of the possible alternatives in philosophy “depends precisely on a broad, if vague, consensus on the nature of perception”.¹⁶ I take him to be alluding here to dogmatic realistic or anti-realistic positions. Putnam is in particular self-critical of the way he treated the “realism issue” at the end of the 1970s (and presumably including the 1981 publication *Reason, Truth and History*):

¹¹Ibid., p. 10.

¹²Austin delivered a critique of this terminology, because of the traditional epistemologist’s question begging use of *direct* and *indirect*, Ibid, p. 180 n. 20.

¹³Ibid., p. 10.

¹⁴Ibid., p. 180n16.

¹⁵Ibid., p. 9–10.

¹⁶Ibid., p. 12.

[...] I did not see that issue as closely involved with issues about perception, or with a particular set of assumptions about the powers of the human mind, and, if I had, I would not have been content with the appeals to what I called ‘verificationist semantics’ in those essays. At the time I argued that our understanding of our language must consist in the mastery of its use. And I went on to say, ‘To speak as if *this* were my problem, I know how to use my language, but now, how shall I single out an interpretation? is nonsense. Either the use of the language *already* fixes the “interpretation” or nothing can.’ I still agree with those *words*. But I would say them in a different spirit now. The difference has to do with how one hears what is involved in an appeal to ‘use’. The notion of use that I employed then was a ‘cognitive scientific’ notion, that is, use was to be described largely in terms of computer programs in the brain.¹⁷

I find this to be a revealing description of his earlier self, which is very difficult to detect from reading the first chapter of *Reason, Truth and History*, that is, the “Brains in a vat” chapter, where he gives a Wittgensteinian analysis of what a *concept* is. Recall from Chapter 3 Putnam’s science fiction argument of the paint-splash qualitatively similar to “visual images of trees”, “but unaccompanied by any *concept* of a tree”.¹⁸ The argument was employed to show that images do not necessarily refer. In case a mental representation refers necessarily to an external object, Putnam continues, then they must be *concepts*, not images; but “what are *concepts*”?¹⁹ In *Reason, Truth and History*, Putnam gave the following answer:

Concepts are not mental presentations that intrinsically refer to external objects for the very decisive reason that they are not mental presentations at all. Concepts are signs used in a certain way; the signs may be public or private, mental entities or physical entities, but even when the signs are ‘mental’ or ‘private’, the sign itself apart from its use is not the concept. And signs do not themselves intrinsically refer. [...] A man may have all the images you please, and still be completely at loss when one says to him ‘point to a tree’, even if there are a lot of trees present. He may even have the image of what he is supposed to do, and still not know what he is supposed to do. For the image, if not accompanied by the ability to act in a certain way, is just a *picture*, and acting in accordance with a picture is itself an ability that one may or may not have. [...] He would still not know that he was supposed to point to a tree, and he would still not *understand* ‘point to a tree’. [...] [N]o matter what sort of inner phenomena we allow as possible *expressions* of thought, [...] it is not the phenomena themselves that

¹⁷Ibid., p. 13–14.

¹⁸Putnam (1981), p. 17.

¹⁹Ibid., p. 17.

constitute understanding, but rather the ability of the thinker to *employ* these phenomena, to produce the right phenomena in the right circumstances [...]. [T]he attempt to understand thought by what is called ‘phenomenological’ investigation is fundamentally misguided; for what the phenomenologists fail to see is that what they are describing is the inner *expression* of thought, but that the *understanding* of that expression—one’s understanding of one’s own thoughts—is not an *occurrence* but an *ability*.²⁰

Thus concepts are signs that are used in a certain way. The word “triangle”, for instance, is associated with the concept of a triangle in that this word is a sign which is appointed a certain role, a certain use, within a calculus, one could say. We should not regard the concept of a triangle as an entity in our mind, or as a freestanding entity anywhere else for that matter, but rather as a sign that is intimately connected to our ability to use it in such-and-such a way.

This sounds very much like the Wittgenstein of the *Philosophical Investigations*, which Putnam acknowledges. The applications of the arguments run similarly. Putnam says, for instance, that a mathematician may associate the proof of the prime number theorem with a blue flash, but of course this does not mean that anyone with “a blue flash in his mind” could unpack such a “blue flash” as the prime number theorem. But although Putnam in his middle period follows Wittgenstein in his analysis of mental images, there is at the same time a problematic view of “inner” and “outer” expressions in the passage cited above.

There is nothing on the surface in these passages that points to what Putnam later sees as a problem with the employment of “use”. But as we shall see shortly, the image of an interface between our cognitive powers, language and thought, on the one hand, and the external world, on the other, draws us into a certain predicament, namely, the arguments involved in the realism/anti-realism debate, and further, limits the possibility of understanding “use” in an adequate way. Although Putnam characterizes his earlier internal realist view of “use” as a cognitive scientific view, closely related to a view of “use” in the computer program sense, even then he thought that one will have to specify an environment for the language user. But this environment is viewed as merely contributing with the external *causes* of the language user’s words, a view that was manifest in Putnam’s internal realist period, emanating from *The Meaning of Meaning*’.

In the *Dewey Lectures*, Putnam claims to adopt Wittgenstein’s notion of “use” faithfully, which requires that one realizes that the use of words in a *language game* are to be explained (“in most cases”, Putnam adds) by employing the vocabulary of that game, or one internally related to this vocabulary. An understanding of the statement “there is a coffee table in front of me”, or a description

²⁰Ibid., p. 18–20.

of its use, presupposes that we perceive coffee tables. By “perceive”, Putnam does not mean merely “see” or “feel” (i.e., including cases when one does not know what a coffee table is), but rather in “the full achievement sense, the sense in which to see a coffee table is to see that it is a coffee table that is in front of one”.²¹

The *slogan* “meaning is use” will not be of much help, Putnam argues, even if we are now hinting at a non-scientific employment of use, in contrast to the “Cartesian cum materialist picture” that serves a scientific version well, unless we qualify what we mean by it in the context of the problems it is intended to help us resolve. For Putnam, “meaning is use” can be rephrased as “understanding is having the abilities that one exercises when and in using language”.²² As we will see below, this view is connected to Cora Diamond’s employment of “sense”, as discussed earlier.

Thus Putnam’s rejection of the language/reality dichotomy in the *Dewey Lectures* is a key to understanding his criticism of the realism/anti-realism debate.

3. The “face of cognition” and Putnam’s Wittgenstein

In the last section of the *Dewey Lectures*, “The Face of Cognition”, Putnam connects what he sees as the right way of understanding “visual experiences”, as well as thinking and remembering, with a fundamental aspect of our language. The point of the example of the duck–rabbit drawing of the *Philosophical Investigations* is not only to undermine the notion of a “sense datum”, i.e., the idea that there is a second image inside our head. Wittgenstein also wants to show that our “visual experience” of the duck–rabbit is not like the physical pictures we draw (this is why we “see” a duck or a rabbit; it is hardly possible to “see” both at the same time). An image on paper can be interpreted one way or another. But there are no such images in the mind similarly awaiting interpretation. Putnam remarks that Wittgenstein makes the same point about words and sentences in thinking.²³

When we know and use a language well, when it becomes the vehicle of our own thinking and not something we have to mentally translate into some more familiar language, we do not pace Richard Rorty, experience its words and sentences as ‘marks and noises’ into which a significance has to be read. When we hear a sentence in a language we

²¹Putnam (1999), p. 14.

²²Ibid., p. 15.

²³Ibid., pp. 45–46. However, Putnam thinks that we need to qualify “sentence” here, since Wittgenstein’s use of *Satz*, translated by Anscombe as *proposition* and usually understood by analytic philosophers as *sentence*, rather involves a rejection by Wittgenstein of the usual sentence/proposition dichotomy. (Ibid., p. 191n8.) Putnam here also refers to the *Philosophical Investigations* §503–, and Charles Travis’s *The Uses of Sense*, Oxford University Press, 1989.

understand, we do not associate a sense with a sign design; we perceive the sense *in* the sign design. Sentences that I think, and even sentences that I hear or read, simply do refer to whatever they are about—not because the ‘marks and noises’ that I see and hear (or hear ‘in my head’, in the case of my own thoughts) intrinsically have the meanings they have but because the sentence in use is not just a bunch of ‘marks and noises’.²⁴

Rorty’s way of referring to language users as producers of “marks and noises”,²⁵ puts him in the same camp as Quine and others who reject the idea of sentences as bearers of meaning, which is indeterminate until an interpretation has been made. In the *Philosophical Investigations* §§503–, Wittgenstein undermines both the idea that the meanings come with the words or sentences themselves, somehow floating above them and connecting them with reality, as Dummett would caricature realism, but also the description of our language as mere “marks and noises” requiring interpretation, or, in Dummett’s view, that sentences only have assertability conditions. But to assume at the outset that these are the only possible alternatives is to accept uncritically Dummett’s analysis. One of Putnam’s major programs in his later philosophy is to employ insights from the *Philosophical Investigations* in particular to argue against this way of structuring the debate.

Putnam cites an example from the third “Lecture on Religious Belief”, in Wittgenstein’s *Lectures and Conversations on Aesthetics, Psychology, and Religious Belief*: there is a “technique of usage” in place through which Wittgenstein can think the thought “my brother is giving a concert in New York”, while his brother is in fact giving a concert in New York. But this has nothing to do with Dummett’s “assertability conditions”. Wittgenstein’s thought about his brother is not an object to which we need to add an interpretation in terms of a method of verification such as assertability conditions, i.e., that he can think “my brother is in New York” if his brother has told him he is going to give a concert on a particular date. Putnam cites Wittgenstein: “when we say, and *mean*, that such-and-such is the case, we—and our meaning—do not stop anywhere short of the fact; but we mean *this—is—so*”.²⁶

This is exactly the place where Putnam’s analysis of the rule-following paragraphs (§§185–242) of the *Philosophical Investigations* becomes important. The rule-following problem begins with the example of “adding 2”, when a child after 1000 continues the sequence with 1004, 1008, 1012, etc. Some interpreters

²⁴Ibid., p. 46.

²⁵Ibid., p. 191n9.

²⁶Ibid., pp. 47–48.

(e.g., Dummett, Kripke and Crispin Wright) have claimed that since Wittgenstein holds that there is nothing “out there” that guarantees that the rule cannot be followed as 1004, 1008, 1012, . . . , after 1000, we are just left with the conclusion that Wittgenstein holds what Dummett has referred to as full-blooded conventionalism, that is, the view that a decision may have to be made at any point in the continuation of following the rule.

4. Putnam on rule-following

There is an immediate connection between the *Dewey Lectures* and Putnam’s treatment of the rule-following considerations in *Was Wittgenstein Really an Anti-realist about Mathematics?* (2001). In the *Dewey Lectures*, Putnam briefly comments on the rule-following paragraphs when criticizing anti-realism, which he thinks has certain things in common with a Platonist view, namely, that “thinking about something can be a *freestanding* activity, unsupported by many other activities, linguistic and nonlinguistic”.²⁷ Putnam takes the example that we imagine seeing Eisenhower receive the German surrender in 1945, which is intended to illustrate that a “person must possess a whole range of abilities, intellectual and practical” in order to understand what is going on. He adds that “one might say that *thinking* is not something only one person could do, and then only once”.²⁸

In *Was Wittgenstein Really an Anti-realist about Mathematics?*, Putnam takes up this last claim, that thinking is not something one person could do and then only once, to argue that it cannot lend support to an anti-realist interpretation of Wittgenstein, such as has been suggested by Kripke. Putnam argues against Kripke’s “communitarian” interpretation of Wittgenstein that the point is that it would not be possible for one individual to follow a rule in isolation from a linguistic community, unless we in our imagination take him into our community and apply our notions of rule-following to him.²⁹ Putnam observes that the closest Wittgenstein ever comes to suggesting anything like this is in §199:

Is what we call ‘obeying a rule’ something it would be possible for only *one* person to do, and to do only *once* in his life? [...] It is not possible that there should have been only one occasion on which a person obeyed a rule. It is not possible that there should have been only one occasion on which a report was made, an order given or understood, and so on.—To obey a rule, to make a report, to give an order, to play a game of chess are *customs* (uses, institutions).³⁰

²⁷Ibid., p. 47.

²⁸Ibid., p. 47.

²⁹Putnam (2001), p. 144.

³⁰Wittgenstein (1953); Putnam (2001), p. 144. Putnam has retranslated the German original.

Putnam suggests that the word “institution” is what has lent support to Kripke’s skeptical “community standards interpretation”, but he believes that Kripke has failed to notice how much weaker the statement of §199 is in comparison to his own interpretation of the senselessness of one person following a rule in isolation. Putnam does not find it strange that one person in isolation could follow a rule; the point of §199 seems rather to be that it should have happened exactly *once* that a rule is obeyed, an order is given, etc. Putnam stresses that Wittgenstein’s reference to institutions is not a sociological observation.

Against anti-realist interpreters such as Kripke and Horwich, Putnam argues that rule-following in a certain way does presuppose the existence of regularities in the world and also the existence of communal practices; but it does not follow that the actual following of a particular rule depends on communal acceptance, nor does it depend on a direct correspondence with regularities in the world (or in the mind).

Putnam now argues that if the rule-following paragraphs in the *Philosophical Investigations* express skepticism, it is *directed at philosophical accounts of rule-following*.³¹

What readers like Kripke and Horwich have done is to take Wittgenstein to oppose not only *metaphysical* realism about rule-following but also our commonsense realism about rule following, when what Wittgenstein actually doubts is the need for and the possibility of a philosophical *explanation* of the rule-following that will justify the common sense things we say [...].³²

There is a way of thinking about practices and regularities that allows for even an individual to make it a custom to do something regardless of whether this is done or not done by his community (Putnam notes that Wittgenstein uses the German word *Gepflogenheit*, which does not refer to communal practice, as does the English word “custom”). This does not mean that we should define “rule” in terms of “regularity” or reduce the notion of rule to the notion of regularity. What Wittgenstein does, on Putnam’s reading, is show us that it is senseless to think of “a world in which for one moment there was a rule even though none of the regularities in linguistic and extra-linguistic behavior which give content to the ‘rule’ talk were ever in place.”³³

This grammatical point does not hinge on a circular argument (the notion of regularity, *Regelmässigkeit*, already presupposing the notion of a rule, *Regel*). No circle is involved here, according to Putnam, since, as Wittgenstein says in §208,

³¹Putnam (2001), pp. 146–147.

³²Ibid., p. 147.

³³Ibid., p. 145.

none of the words (regular, uniform sameness) is explained by means of itself. Instead, we rely on how these words are explained to anyone:

I shall explain these words to someone who, say, only speaks French by means of the corresponding French words. But if a person has not yet got the *concepts*, I shall teach him to use the words by means of *examples* and by *practice*.—And when I do this I do not communicate less to him than I know myself.

In the course of this teaching I shall show him the same colors, the same lengths, the same shape, I shall make him find them and produce them, and so on. I shall, for instance, get him to continue an ornamental pattern uniformly when told to do so.—And also to continue progressions. [...] Imagine witnessing such teaching. None of the words would be explained by means of itself; *there would be no logical circle* [emphasis added by Putnam].³⁴

Putnam observes two things here:³⁵

(1) We all know how to explain these notions in our usual *ordinary* ways, not philosophical ways that presuppose “Platonism” or “mentalism” or some other philosophical account.

(2) “And when I do this I do not communicate less to him than I know myself.” That is, there is no other kind of explanation of the notions involved in following a rule which one is in possession of either by virtue of being a philosopher or by virtue of having direct acquaintance with something ineffable.

One of Wittgenstein’s antagonists in the rule-following paragraphs is Ramsey, who claims that following a rule consists in following “psychological laws”,³⁶ even if no *explanation* can be provided for the rule following. According to both Putnam and Diamond, Ramsey combines a certain mentalism about rule-following with an idea of the incommunicability of knowledge. Both these issues are attacked in the *Philosophical Investigations*. As we have seen, there have been “skeptical” interpretations of Wittgenstein’s rule-following paragraphs in the wake of Wittgenstein’s criticism of such mentalistic (or Platonic) ideas. It is now widely held among Wittgenstein inspired philosophers that Wittgenstein’s aim is to show that the ordinary cases of rule-following require no explanation in terms of something else, mental entities, or in something “out there”.

³⁴Wittgenstein (1953); Putnam (2001), pp. 146–147.

³⁵Putnam (2001), p. 146.

³⁶Ibid., p. 146. Putnam credits Cora Diamond with this observation.

As an important part of developing the new position of common sense realism, Putnam uses Wittgenstein's rule-following paragraphs to demonstrate that no further explanation than our *ordinary* ways of explaining rule-following to each other is needed for the rule-following to be in place. Inspired by Stanley Cavell, Putnam distinguishes between *ordinary* uses and *philosophical* explanations. Take, for instance, the twin prime conjecture ("there are infinitely many primes with the mutual difference 2"). Philosophers, such as Dummett, may say of such a proposition that we cannot say that it is either true or false, since it would be an instance of the metaphysical principle of bivalence. But for the working mathematician, the question "is the conjecture true or not?", plays an entirely different role, since for him no other *working* alternatives exist. This is how we should understand a reference to the *ordinary* (practices).

For Dummett, truth becomes a complete mystery if it cannot be "recognized"; thus he is critical of recourse to the mathematician's practice. In *Reply to James W. Allard* (2007), following Allard's exposition of some of Dummett's anti-realists ideas, Dummett responds as follows (and I think that one may view this as a kind of criticism of both Putnam and Quine):

Consider a holist view of mathematics. The holist starts out by agreeing with the intuitionist that a mathematical statement is true only if there is a proof of it. But it quickly turns out that they understand the word "proof" in very different ways. The holist understands it as covering any proof that the mathematical community at large (which does not include constructivist mathematicians) agrees to be valid. The intuitionist takes it as covering a very restricted type of proof that he calls 'canonical'.³⁷

Dummett insists on a compositional theory of understanding (or meaning): the understanding (or meaning) of any expression depends only on the understanding (or meaning) of simpler expressions, including those expressions of which it is composed, and the "canonical" proofs in mathematics presuppose such an understanding. Dummett claims that holism does not provide a defense of realism, but rather "presents an obstacle to anti-realism". According to Dummett, it relies on authority (the mathematician, the mathematical community). The holist cannot answer the question of what the meaning of a particular statement consists in, he instead relies on the standard view among working mathematicians that in order to understand Fermat's last theorem, we need "only" a knowledge of the mathematics involved to follow Wiles's proof (and the proofs of results it uses).³⁸ Thus Dummett views Quine's holism, and presumably also Putnam's position,

³⁷Dummett (2007), p. 151.

³⁸Ibid., pp. 151–152.

to be ultimately *unphilosophical*, since they rely on authority, and assimilate the mathematician's unsophisticated perspective of his own practice.

For Dummett, my understanding of the twin prime conjecture cannot rely on current mathematical practice, the mathematician's way of thinking of such a conjecture of being either true or false, since my understanding of a statement such as "there are infinitely many prime numbers with difference 2" consists in my ability to *recognize* if this statement is verified.³⁹ The negation of the statement is understood in a similar way, but both statements may lack the property of *being verified*. Putnam says "classical truth" may hence not be possessed by either alternative of, for example, the twin prime conjecture or its negation, or of the statement that 'Lizzie Borden killed her parents with an axe' or its negation. This is why Dummett thinks that truth is either a useless property, or we should drop the idea of truth as a the bivalent property, i.e., the claim that statements like the ones above are either true or false.⁴⁰

The reason why Dummett thinks that those who do not subscribe to his own theory of proof simply rely on authority is very much the same reason he gives for his rejection of the principle of bivalence; it is another variant of the anti-realist interpretation of Wittgenstein on rule-following, that for Wittgenstein we are in need of a community in order to follow our rules. The problem is that Dummett does not think that mathematical language takes care of itself, and Putnam's ambition is to try do justice to the mathematician's *ordinary* ways of reasoning, within his own language. In Section 5, I will discuss Putnam's reliance on Wittgenstein's arguments that when we use "proposition" and "truth", etc., *in our language*, we do not commit ourselves to any errors when we say either "Either Lizzie borden killed her parents with an axe or she did not". Dummett uses words such as "proposition", etc., in a formalized way, as if language were an ideal calculus. But then, in what way is the mathematician right when he speaks of "either the twin-prime conjecture is true or it is false"? Mathematical language is, in this respect, not very different from everyday language, but the point is that the *ordinary* language the mathematician is using here is the *prose* of mathematics, and not the calculus itself. This will be the theme of Section 9.

Putnam also considers what he calls the deflationist view of understanding, represented, for example, by Horwich in *Truth* (1990) and Carnap (somewhat anachronistically) in *Truth and Confirmation* (1949):

These philosophers agree with Dummett in thinking of our understanding of our sentences as *consisting in* our knowledge of the conditions under which they are verified, although they reject Dummett's notion of 'conclusive verification', replacing that notion with a notion

³⁹Putnam (1999), p. 50.

⁴⁰Ibid., p. 51.

of degrees of verification. They also reject Dummett's claim that we must not think of truth as a bivalent property, although they agree that it is not a 'substantive property' about which some metaphysical story needs to be told; rather they claim that rejecting that metaphysical picture of what truth is does not require us to give up the Law of the Excluded Middle, ' $p \vee \neg p$ '.⁴¹

The deflationist allows us to accept the principle of bivalence: "either p is true or its negation is true" for any declarative sentence p , but according to Putnam they view this acceptance merely as a "linguistic practice", a linguistic convention in Carnap's terminology.

Putnam now defends the *common sense realism* he finds in Wittgenstein, by suggesting that the *metaphysical realist* is right in thinking that there is something more to "Either Lizzie Borden killed her parents with an axe or Lizzie Borden did not kill her parents with an axe" than a mere linguistic convention of claiming this. What is correct in deflationism, in Putnam's view, is that ' p ' is true may just be replaced by p . There is no *substantive property* underlying our language games that is needed to make a statement true (that underwrites *true*); empirical statements already make claims about the world, whether or not they include the word "true". But Putnam also writes:

What is wrong in deflationism is that it cannot properly accommodate the triism that certain claims about the world are (not merely assertable or verifiable but) *true*.⁴²

5. Putnam on Wittgenstein and truth

One of Putnam's key points in the *Dewey Lectures* is his emphasis on Wittgenstein's view that truth cannot be a *freestanding* property, so that if we find out what this property is, "we will know what the *nature* of propositions is and what the *nature* of their correspondence to reality is."⁴³ In this section, I will unpack the common sense view of truth that Putnam attributes to Wittgenstein, with particular emphasis on how this view may be applied to mathematics.

Putnam is sympathetic to what he sees as an insight of both Tarski and Wittgenstein, that there is a close connection between understanding a "sentence" and understanding that the "sentence" is true. The example "Lizzie Borden killed her parents with an axe" is normally not viewed as lacking truth value, nor is our ordinary belief that there may be killers who cannot be detected, say in a criminal investigation, dependent on magical powers of the mind. Different

⁴¹Ibid., p. 51.

⁴²Ibid., p. 56.

⁴³Ibid., p. 68.

sorts of positions in analytic philosophy, whether anti-realist or realist in a metaphysical sense presuppose notions such as “truth” or “proposition”, for instance in the sense that we may explain the notion of a proposition in terms of “truth”: “a proposition is something that is either true or not true”.

Putnam agrees with Wittgenstein’s holistic view that “truth” and “proposition” (*Satz*) are dependent on each other, not as is usually explained, but rather in a way that makes “the notion of truth and the notion of proposition mesh together like a pair of gears in a machine”.⁴⁴ The understanding of a proposition relies on the mastering of a language-game, which includes language together with “the actions into which it is woven”.⁴⁵ The notion of truth depends on the use of the signs in our language game in an analogous sense as the notion of checking in chess depends on how we move the pieces in chess—this is the holism which Putnam attributes to Wittgenstein.⁴⁶

In order to elucidate what Wittgenstein has in mind, Putnam cites the full §136 of the *Philosophical Investigations* in full as we do now.

At bottom, giving ‘This is how things are’ as the general form of proposition is the same as giving the definition: a proposition is whatever can be true or false. For, instead of ‘This is how things are’, I could have said ‘This is true’. (Or again, ‘This is false’.) But we have

‘*p*’ is true = *p*

‘*p*’ is false = not-*p*

And to say that a proposition is whatever can be true or false amounts to saying: we call something a proposition when *in our language* we apply the calculus of truth-functions to it.

Now it looks as if the definition—a proposition is whatever can be true or false—determined what a proposition was, by saying: what fits the concept ‘true’ or whatever the concept ‘true’ fits, is a proposition. So it is as if we had a concept of true and false which we could use to determine what is and what is not a proposition. What *engages* with the concept of truth (as with a cogwheel) is a proposition.

But this is a bad picture. It is as if one were to say ‘The king in chess is *the* piece that one can check’. But this can mean no more than that in our game of chess we only check the king. Just as the proposition that only a *proposition* can be true or false can say no more than that we can only predicate ‘true’ and ‘false’ of what we can call a proposition. And what a proposition is is in one sense determined by the rules of sentence formation (in English, for example), and in another sense by the use of the sign in a language-game. And the use of the words ‘true’ and ‘false’ may be among the constituent parts of

⁴⁴Ibid., p. 67.

⁴⁵Ibid., p. 67. From the *Philosophical Investigations* §7.

⁴⁶Ibid., p. 67.

the game; and if so it *belongs* to our concept ‘proposition’ but does not ‘fit’ it. As we might also say, check *belongs* to our concept of the king in chess (as so to speak a constituent part of it). To say that check did not *fit* our concept of the pawns, would mean that a game in which the pawns were checked, in which, say, the players who lost their pawns lost, would be uninteresting or stupid or too complicated or something of the kind.⁴⁷

Putnam draws three main conclusions from the paragraph cited, which are all directed against the view that when Wittgenstein says that ‘*p*’ is true = *p*, he is a deflationist in the sense we described this position at the end of Section 4.⁴⁸ Putnam has three arguments against the label “deflationism” regarding §136:

(1) Wittgenstein did not oppose the idea that empirical propositions “correspond to reality”. What he says in §136 is this: something is not a proposition in virtue of fitting a freestanding concept “truth”, but neither can we explain truth by saying that for any proposition *p*, ‘*p*’ is true = *p*; neither “truth” or “proposition” (*Satz*) is a foundation on which the other rests.

(2) A genuine *Satz* is characterized by being regarded as either true or false *in our language*.

(3) A grammatical string of sounds or marks that is neither true or false is not a *Satz*.⁴⁹

Hence the difference between Wittgenstein’s analysis and the usual one in analytic philosophy which emphasizes the role of propositions as being either true or false (as having a “meaning”) is that “truth” and “proposition”, etc., are used as if the language were an “ideal language” like mathematical logic. The point is not that our language is a mere profane and less complicated approximation of such an “ideal”—it is *more* complicated without the assumption that a freestanding truth and its consequences often paired with a metaphysical realism lead to. The *commonsense realism* that Putnam credits Wittgenstein with does not make such metaphysical assumptions, such as that of a freestanding notion of truth. But it is an equally important feature of this commonsense realism that it allows for knowledge claims to be responsible to reality in different ways, where correspondence may be one way (“this chair is blue” may correspond to the fact that

⁴⁷Wittgenstein (1953), §136.

⁴⁸Putnam mentions Kripke, who interprets §136 as a “clear expression of deflationism” (Putnam (1999), p. 67). Mühlhölzer refers to §136 as a “notorious deflationary remark” (Mühlhölzer (2009), p. 5).

⁴⁹Putnam (1999), pp. 67–68.

this particular chair is blue). The metaphysical realist assumption that there is just *one* way rests on a mysterious “correspondence” relation that “underwrites the very possibility of there being knowledge claims”.⁵⁰

On the other hand, marks and noises may certainly not always give rise to a “face of meaning”,⁵¹ an allusion to the way meaning is treated by Cora Diamond (using the metaphor that we recognize a face without relying on the precise shape of its parts, although these parts *do* build up the picture-face) in *The Face of Necessity*. In its *Appendix*, Diamond concludes from her considerations of Dummett’s interpretation of Wittgenstein that mere marks and noises certainly do not produce anything we would recognize as meaning.⁵² Her argument here is reminiscent of Putnam’s argument in the *Brains in a vat* chapter of *Reason, Truth and History*, that an ant certainly has not made a picture of Winston Churchill, even if we see the pattern in the sand traced out by the ant as Winston Churchill. Diamond also connects this insight to the nature of language as something other than marks and noises. It is an important insight that language gets its meaning, not because words, sentences or statements (whatever they are) have an intrinsic meaning, nor do words or sentences get their meanings from assertability conditions or truth conditions relying on a freestanding notion of truth. Dummett’s and Davidson’s views are built on a distorted use of “proposition” and “truth”. The anti-realist (including a deflationist) interpretation of Wittgenstein relies in Putnam’s eyes on the use of these terms within analytical philosophy, and mathematical logic in particular. One of Wittgenstein’s main purposes was to help us distance ourselves from playing with these notions as if they were freestanding, without taking their interdependence in our language games into consideration.

Instead of a freestanding concept of truth which gives us the nature of a proposition and its correspondence to reality, Wittgenstein wants us to look at ethical language, mathematical language and even at imprecise language, which may be “clear” in context. However, as Putnam remarks,⁵³ it is not because the context makes imprecise language exact or precise that it is clear, but because exactness is out of place. Putnam alludes to §186 of the *Philosophical Investigations*, where Wittgenstein suggests that “stand roughly here” may work as an explanation, and furthermore suggests (also in §69) that we are misled by any recourse to exactness—what is its definition?

6. The face of meaning and necessity

Dummett has famously interpreted Wittgenstein’s alleged conventionalism as “full-blooded”. Dummett argues that the necessity of $5 + 7 = 12$ has to build on

⁵⁰Ibid., p. 68.

⁵¹Ibid., p. 69.

⁵²Diamond (1991), p. 261.

⁵³Putnam (1999), p. 198n.

the correctness of counting. He is critical of Wittgenstein, who sees addition as a “new” rule. Dummett writes:

Wittgenstein goes in for a full-blooded conventionalism; for him the logical necessity of any statement is always the *direct* expression of a linguistic convention. That a given statement is necessary consists always in our having expressly decided to treat that very statement as unassailable; it cannot rest on our having adopted certain other conventions which are found to involve our treating it so. This account is applied alike to deep theorems and to elementary computations. To give an example of the latter, the criterion which we adopt in the first place for saying that there are n things of a certain kind is to be explained by describing a procedure of counting. But when we find that there are five boys and seven girls in a room, we say that there are twelve children altogether, without counting them all together. The fact that we are justified in doing this is not, as it were, implicit in the procedure of counting itself; rather, we have chosen to adapt a *new* criterion for saying that there are twelve children, different from the criterion of counting up all children together. It would seem that, if we have genuinely distinct criteria for the same statement, they may clash. But the necessity of ‘ $5 + 7 = 12$ ’ consists just in this, that we do not count anything as a clash; if we count the children all together and get eleven, we say, ‘We must have miscounted’.⁵⁴

Thus, if someone got a different answer than 12, then there must have been a miscount; if the person does not admit it to us, then he is not responsible “to the sense [meaning] we have already given to the words of which the statement is composed”.⁵⁵

Putnam’s argument against Dummett here has deep roots. Already in 1960, when Dummett and Putnam participated in the same conference, Putnam was critical of logical positivism’s “observational/theoretical” dichotomy, defended by Dummett at the time. Putnam launched an attack on the view that the meaning of the word “small”, for instance, changes when we talk about “things that are too small for us to see”, or that the meanings of the words “thing” and “see” change. Putnam discussed this in the context of a children’s tale (about people too small for us to see), but thought of the argument as analogous to looking at small things “too small for us to see” in a microscope. The point Putnam wanted to make in 1960 was that “so-called observation terms in science can typically be used to describe *un*observables as well as observables and that such a use does not involve any change of meaning [of, e.g., small],”⁵⁶ in contrast to Dummett’s

⁵⁴Dummett (1978), p. 170.

⁵⁵Ibid., p. 179

⁵⁶Putnam (1999), p. 59.

view. This was presumably an attempt to criticize the phenomenism of logical positivism (a critique in the spirit of Quine).

Putnam explains that he would now qualify his position to include only scientific talk that depends on the “provision of coherent explanatory detail” and not talk we cannot make sense of (such as talk of the metabolism in a fairytale of people too small for us to see). One could perhaps question, Putnam writes, whether we actually are able to conceive of quantum mechanical particles literally as “particles”, but here the problem is rather that nature has proven not to be particularly visualizable in the ultrasmall, and not because “things too small for us to see” has no application. Microbes are too small for us to see, but the meaning of “small” is not changed when we describe them in this way.⁵⁷ Putnam writes:

But does what we *see* have the same meaning have anything to do with what does have the same meaning? For Dummett (early and late) the answer is ‘no’. Our natural picture of what we are doing with our words and our thoughts has no philosophical *weight* in his eyes.⁵⁸

This reasoning is an echo of a sentiment at play in the more literary essay *Convention: a theme in philosophy* (1981)[1983], in which Putnam describes Carnap’s conventionalism as an “as-ifism”, and he goes on to contrast the *conventional* with the *natural*. “Two great analytic philosophers”, Quine and Wittgenstein, “reached the conclusion that convention is a relatively superficial thing,”⁵⁹ and conventionalism is a clinical form of philosophy that does not, in the end, take our natural reactions into consideration. Putnam mentions Quine’s rejection of conventionalism as empty make-believe (saying that mathematics and logic are conventions merely comes down to saying that we accept mathematics and logic) and Wittgenstein’s rule-following considerations as examples of philosophical argumentations that take “the natural” seriously.

Our *natural* reaction when we look in a microscope is that we think of “small” in the same way as we do when we think of larger (small) objects: “the problem with Dummett’s account is that it fails to properly describe who we are and the sense our practices have for *us*.”⁶⁰ Putnam refers to Cora Diamond’s discussion of how games are identified by their rules.

Diamond considers the example of playing chess with the rules just slightly changed so that we cannot move a pawn as a first move. What is the difference

⁵⁷Ibid., p. 60.

⁵⁸Ibid., p. 60.

⁵⁹Putnam (1981)[1983], p. 174.

⁶⁰Putnam (1999), p. 64.

between the two games? Here Diamond uses a notion of Wittgenstein's (discussed in her *Secondary Sense*, which was re-published in *The Realistic Spirit*). In *Philosophical Investigations* §282, Wittgenstein says:

When children play at trains their game is connected with their knowledge of trains. It would nevertheless be possible for the children of a tribe unacquainted with trains to learn this game from others, and to play it without knowing that it was copied from anything. One might say that the game did not make the same *sense* to them as to us.

Diamond then elaborates on the use of "sense" in these situations and concludes that it cannot be identified with the "rules for the use" or "rules of a game" or the psychological accompaniments to them.⁶¹ She refers to the difference people perceive between playing different games as the different *sense* the activity have for such players, and she compares the same sense for such activities to be like two faces having the same expression:

I can, for example, compare saying that two activities have the same sense to saying that two picture-faces have the same expression. This is not like saying that the mouths are the same length, the eyes the same distance apart: it is not that kind of description. But it is not a description of *something* else, the expression, distinct from that curved line, those two dots, and so on. Just as I can lead you to make comparisons of the expressions of picture-faces, I may be able to lead you to make comparisons of sense, e.g., by showing you obvious differences in sense.⁶²

For her main argument, Diamond uses a slightly simpler example than $5 + 7 = 12$, which is Dummett's example. She wants to avoid certain formulations of the problem (Dummett's use of "miscount", for example), which she finds unclear in Dummett's own description. Putnam likewise refers to Diamond's example in detail, but I don't find any difficulty in explaining the main point Putnam wants to make from the original example.

The main point of Putnam's elucidation of mathematical necessity is that when we add the numbers 5 and 7, *we* do not think that we have included an *arbitrary* rule of addition to the activity of first counting five boys and seven girls and then counting all the children together. The necessity of $5 + 7 = 12$ is found in our "natural reaction" of seeing "one face in another".⁶³ In fact, if we did not equate the results of the two activities (of counting and of adding), we would be *stupid*.⁶⁴ Putnam makes the following analogous point for the case of

⁶¹Diamond (1991), p. 248

⁶²Ibid., p. 249

⁶³Putnam (1999), p. 64.

⁶⁴Ibid., p. 64

someone who would attach a different meaning of “things too small for us to see” after the invention of a microscope.

[T]he problem with Dummett’s account is that it fails to properly describe who we are and the sense our practices have for *us*. It fails to capture the way in which we ‘see the face’ of the activity of seeing something with our eyes in seeing something with a microscope, and in which we ‘see the face’ of using a magnifying glass to look at something very small in using a microscope to look at something ‘too small to see with the naked eye’. And like Diamond, I am suggesting that the sameness of the ‘sense’ of *small* in these cases is not an identity of ‘rules’, nor yet a ‘description of *something* else’ than the way we use these words in these cases.⁶⁵

For Dummett, any change of the rules is a change of the meaning of the words, and he denies therefore that any new rule has been introduced (even in more complicated examples of mathematical necessity, such as the proof of theorems). On Dummett’s rendering, Wittgenstein thinks that a change in the rules also changes the meaning of the words, and (even the actual) Wittgenstein claims that a new rule has been introduced, leading to Dummett’s charge of full-blooded conventionalism, i.e., the meanings of the counting words have changed. But Putnam claims that the assumption Dummett makes about Wittgenstein is wrong. Putnam writes:

The analogy between seeing the same facial expressions in two different configurations of lines and dots and seeing a necessity common to different practices of counting and calculating is an illuminating one. Seeing an expression in the picture face is not just a matter of seeing the lines and dots; rather it is a matter of seeing something *in* the lines and dots—but this is not to say that it is a matter of seeing something *besides* the lines and dots. Both sides in the debate between ‘realists’ and ‘antirealists’ about mathematical necessity believe that we are confronted with a forced choice between saying either (1) that there is something *besides* our practices of calculation and deduction that underlies those practices and guarantees their results; or (2) that there is *nothing but* what we say and do, and the necessity we perceive in those practices is a mere illusion. Wittgenstein, here as elsewhere, wants to show us that it is a mistake to choose either the ‘something besides’ or the ‘nothing but’ horn of the dilemma. [...] [T]he ‘realist’ and the ‘antirealist’ [...] share the same picture of mathematical necessity, one according to which there must be *something* forcing us—the necessities that our rules reflect, conceived of as something external to our mathematical

⁶⁵Ibid., p. 64.

practices and the ways of thinking internal to them. [...] The philosophical task here lies in seeing that giving up on the picture which both the realist and the antirealist share is not the same thing as giving up on our ordinary logical and mathematical notion of necessity.⁶⁶

Wittgensteinian philosophers today generally argue against the view that realism and anti-realism are mutually exclusive positions. We have already seen this in Putnam's account of the rule following paragraphs of the *Philosophical Investigations*, and there is a good elucidation of the problem with labeling Wittgenstein an anti-realist in this context also in Goldfarb's recent paper (2009).

Both the realist and the anti-realist assume that there is *something* that accounts for the necessity of the results of our calculations (and other practices), something out there; if this something is missing, then the necessity is also gone. As we have seen, Putnam embraces Diamond's account of the *face of necessity*. On this analogy, two different pictures may display the same face without the mouths having the same length, without the eyes being the same distance apart: "it is not that kind of description, but it is not a description of *something* else". There is a "face of meaning" that we associate with this type of necessity, and I will argue that there is a problem with Putnam's argument when he tries to explain not only why two different proofs may prove the same theorem, but also the mathematical necessity of $2 + 2 = 4$ using this metaphor. I think that Diamond's face-metaphor makes sense in the former case, but not in the latter.

7. Conant on Putnam's Wittgenstein and pragmatism

James Conant has been in a longstanding dialogue with Putnam concerning the latter's way of using Wittgenstein to come to terms with problems on necessity, truth and objectivity in mathematics.

In *Ethics without Ontology*, Putnam bases some of his insights on Conant's *On Wittgenstein's Philosophy of Mathematics II*, which was the sister article to Putnam's *On Wittgenstein's Philosophy of Mathematics I*.⁶⁷ Putnam finds that Conant expresses his own views very accurately when he writes:

[E]thical and mathematical thought represent forms of reflection that are fully governed by norms of truth and validity as any other form of cognitive activity. But he [Putnam] is not friendly to the idea that, in order to safeguard the cognitive credentials of ethics or mathematics, one must therefore suppose that ethical or mathematical thought bears on reality *in the same way* as ordinary empirical thought; so that, in order to safeguard talk of the truth of propositions such as 'it is wrong to

⁶⁶Ibid., p. 63.

⁶⁷Putnam's paper was later expanded to the longer "version": *Was Wittgenstein Really and Anti-realist about Mathematics?*, which was influenced by Conant's paper/reply.

break a promise' or ' $2 + 2 = 4$ ', one must suppose that, like ordinary empirical propositions, such propositions, in each sort of case, 'describe' their own peculiar state of affairs.⁶⁸

Indeed it is an important part of Putnam's project to show that there is a way of understanding *objectivity* in mathematics and in ethics without relying on there being a philosophical account of the relation between language and reality that makes it possible to view mathematical and ethical statements as descriptions of reality. Putnam is equally hostile to what too many philosophers regard as the only other alternative, that is, a view that there is a (metaphysical) difference between those sentences that genuinely describe reality and those that merely purport to describe reality.⁶⁹

Putnam argues that being sensitive to the different functions of language is to acknowledge that we can talk of the *truth* (i.e., the objectivity of) $2 + 2 = 4$, without relying on some correspondence to a certain region of reality. With the aid of Conant's reading of Wittgenstein, Putnam has developed a kind of *pragmatic* view of the truth of statements such as $2 + 2 = 4$. I will now illustrate how they arrive at this view.

Putnam and Conant are both inspired by a reading of Wittgenstein's critique of the view that mathematical propositions refer to reality, found in his *Lectures on the Foundations of Mathematics*. Conant cites Putnam's comments on Lecture XXVI, in which Wittgenstein says that "Mathematical propositions do not treat of numbers. Whereas a proposition like 'There are three windows in this room' does treat of the number 3." Conant points out that it is important here to distinguish between how "mixed statements 'treat of' numbers (or functions and their derivatives) and the sense of how they 'treat of' physical entities".⁷⁰

Wittgenstein compares the use of mathematical words, such as "three", with how we use words such as "rain" in a sentence such as "it is not raining", that is, when a correspondence between the word and a reality is not obviously available. We have to get clear about different senses of "corresponds":⁷¹

Wittgenstein suggests that we say of such words that the reality which corresponds to them is our having a use for them; and he suggests that this is analogous to something that one might mean in talking of the reality which corresponds to a proposition of mathematics [...].

To say *A reality corresponds to* $2 + 2 = 4$ is like saying *A reality corresponds to* 2. It is like saying a reality corresponds to a rule, which

⁶⁸Conant (1997), p. 202.

⁶⁹Ibid., p. 202.

⁷⁰Ibid., p. 219.

⁷¹Ibid., p. 219.

would come to saying: *It is a useful rule, most useful—we couldn't do without it for a thousand reasons, not just one.*⁷²

Hence, Conant describes Wittgenstein's position in the *Lectures*, which Putnam endorses, as a pragmatic view on the way mathematics bears on reality in mixed statements. A mathematical rule does not have a correspondence to a reality "of the kind we first expect" as Wittgenstein writes at the end of Lecture XXV, "but rather will lie in the rule being of such a sort that it is rendered important and justified by all sorts of fact—facts about the world and about us—so that we shall not want (and perhaps may not even know what it would mean) to do without it."⁷³

Hence there is a pragmatic flavor to Putnam and Conant's way of understanding rules such as $2 + 2 = 4$. For Putnam, to say that " $2 + 2 = 4$ is true" is to say that it is inevitably used in our human activities, and we cannot do without it.

But then again, for Putnam, there is ultimately no guarantee that we will never make sense of $2 + 2 \neq 4$.⁷⁴ We will first recapitulate Putnam's arguments in *Ethics without Ontology* before proceeding to examine the pragmatic view of regarding $2 + 2 = 4$ as something we cannot do without.

8. Conceptual truths

Putnam continues to develop the idea of "statements whose negations we do not (presently) understand" from *Revisiting Mathematical Necessity* further in *Ethics without Ontology*, but now under the notion of *conceptual truths*. Putnam argues that this is no metaphysical category, although it

[...] *becomes* metaphysical if one supposes, as Quine and his opponents did, that *which* truths are conceptual truths is something we can know *incorrigibly*.⁷⁵

The problem with analytical truths according to Putnam, is that they were supposed to be an example of *unrevisable* knowledge. Further, he argues:

[T]here is an older view, one represented both by Hegelians and pragmatists [...], according to which conceptual truths are not 'analytic' in the way 'All bachelors are unmarried' is thought to be analytic – they are not 'trifling' truths, nor are they unrevisable. According to this tradition, we know that something is a conceptual truth by way of *interpretation*, and interpretation is by itself an essentially corrigible activity.⁷⁶

⁷²Ibid., p. 220.

⁷³Ibid., p. 220.

⁷⁴Putnam (2004), p. 63.

⁷⁵Ibid., pp. 60–61.

⁷⁶Ibid., p. 61.

For Putnam, a conceptual truth is a truth whose negation is an assertion that makes no relevant sense at present. And he thinks that this idea of a conceptual truth fits well with the “recognition that conceptual truth *and* empirical description interpenetrate”,⁷⁷ exemplified by what he calls *mixed statements*⁷⁸ (e.g., in physics). Newton’s law of gravitation or the formula for momentum, for instance, fall into this category: they are empirical descriptions and conceptual truths. And any given conceptual truth has this status only relative a body of beliefs and the conceptual connections that we accept; we may “*come to see how something that previously made no sense could be true*”.⁷⁹

I think that Putnam here has “pragmatically” modified his older Quinean picture of the revisability of all “sentences” (or “assertions”) only marginally, since we may change our interpretation of a particular “sentence” depending on the way conceptual relations interpenetrate with the facts. Putnam claims that he differs from Quine, but the alleged difference is that Quine does not recognize any differences among scientific truths. Putnam certainly thinks that it is unintelligible to question “conceptual truths”, but the qualification “unintelligible” does not add much, if we still allow for the conceptual truths to be revisable in principle, even if it may be unintelligible now, given the interpenetrations of fact and convention we are given at present.

The conception of conceptual truth that I defend [...], recognizes the interpretation of conceptual relations and facts, and it grants that there is an important sense in which knowledge of conceptual truths are corrigible. But unlike Quine’s conception, which scraps almost all distinctions among scientific truths (except for recognizing a small class of which Quine called ‘stimulus analytic’ truths), my conception regards it as a fact of great *methodological* (and not merely ‘psychological’) significance, a matter of how inquiry is structured, that there are assertions whose negations *make no sense* if taken as serious assertions [...]. And this is a methodological—as opposed to purely ‘psychological’—significance because the questions ‘How do you know that not-*p* isn’t the case?’ and ‘What *evidence* do you have that not-*p* isn’t the case?’ and ‘What *proof* do you have that not-*p* isn’t the case?’ are questions that can be raised and discussed only *if* we have succeeded in *making sense* of the ‘possibility that not-*p*.’ Conceptual truths are not ‘foundations of our knowledge’ in the old absolute sense, but they are foundations in the sense that Wittgenstein pointed to when he wrote in *On Certainty* that ‘one might say that these foundation walls are held up by the whole house’.⁸⁰

⁷⁷Ibid., p. 61.

⁷⁸See *Was Wittgenstein Really an Anti-Realist about Mathematics?*

⁷⁹Putnam (2004), p. 61.

⁸⁰Ibid., pp. 62–63.

As an example of a conceptual truth, Putnam mentions here the example “ $2 + 2 = 4$ ”.⁸¹ This is precisely the same picture we saw in *Rethinking Mathematical Necessity* an example of an assertion p (if taken as a serious assertion), where we cannot raise the questions of knowledge/evidence/proof of not- p , since we have made no sense of the possibility of not- p ; i.e., we have (at present) no sense of $2 + 2 \neq 4$, so there is no intelligible way to speak of a revision of $2 + 2 = 4$.

9. A problem that remains

We have seen that Putnam embraces the idea that arithmetical statements such $2 + 2 = 4$ are necessary in a pragmatic sense, as elaborated by Conant, but also that he is still under the influence of Quine: it is a conceptual truth (or at least we interpret it as such for our present applications), but there is no guarantee that we will not someday may have to accept that $2 + 2 = 4$ is not true in full generality.

Here I will try to challenge this idea, which I think is a remaining problem for Putnam in his analysis of mathematical necessity, in particular concerning simple arithmetic calculations. My criticism should be viewed as an attempt to improve a part of Putnam’s philosophy, a philosophy which I more or less endorse.

First, the pragmatic interpretation of Wittgenstein, that is, an interpretation that we cannot do without (“not even once”) arithmetic examples such as $2 + 2 = 4$, is problematic in itself. Wittgenstein expresses criticism of this idea in *Philosophical Grammar*. The following passage is from §133.

If I want to carve a block of wood into a particular shape any cut that gives it the right shape is a good one. But I don’t call an argument a good argument just because it has the consequences I want (Pragmatism). I may call a calculation wrong even if the actions based on this result have led to the desired end.⁸²

“Grammar is not accountable to any reality”,⁸³ Wittgenstein says in the same paragraph, and it is clear that, in his view, there is no way to account for the necessity of $2 + 2 = 4$ through any function such a calculation may have in connection with our application of it. In §134 he continues this line of thought:

I do not call rules of representations conventions if they can be justified by the fact that a representation made in accordance with them will agree with reality. [...] The rules of grammar cannot be justified

⁸¹Ibid., p. 63.

⁸²Wittgenstein (1974), §133.

⁸³Ibid., §133.

by shewing that their application makes a representation agree with reality.⁸⁴

There is a change, however, from this middle period, represented by the above passages of the *Philosophical Grammar*, to the late period, in

Wittgenstein's (re-)introduction of an extra-mathematical application criterion, which is used to distinguish mere 'sign-games' from mathematical language-games.⁸⁵

In the *Remarks of the Foundations of Mathematics*, Wittgenstein wrote:

I want to say: it is essential to mathematics that its signs are also employed in *mufti*.

It is the use outside mathematics, and so the *meaning* of the signs, that makes the sign-game into mathematics.⁸⁶

And, furthermore,

Concepts which occur in 'necessary' propositions must also occur and have a meaning in non-necessary ones.⁸⁷

Hence the meanings of mathematical "statements" (that is, what makes them statements or propositions) are given in applications of the rules. But we tend to confuse mathematical language with a kind of careless Platonism, as a consequence of the use of "prose" in mathematics.⁸⁸

One could also perhaps say that the "prosaic" talk in mathematics compels us to construct a language/reality dichotomy of a special kind, where the philosophical problem then becomes a search for the reality to which the statements are thought to correspond. But a calculation such as $2 + 2 = 4$ (according to the calculation rule *addition*) should not be viewed as a statement that corresponds to some facts "out there". Putnam has realized this, but ends up with an essentially pragmatic solution, that $2 + 2 = 4$ is something we cannot do without; but then again, whose truth-value may be altered in view of new interpretations of facts and conventions. Putnam's problem is that he insists on treating $2 + 2 = 4$ as a proposition, as we saw at the end of the last section, when we considered Putnam's conceptual truths in *Ethics without Ontology*. There Putnam talks of raising questions of not- p , where p may be " $2 + 2 = 4$ ". It is important to realize

⁸⁴Ibid., §134.

⁸⁵Rodych (2011), §3.5.

⁸⁶Wittgenstein (1956)[1978], Part V, §2 (1942-43).

⁸⁷Ibid., Part V, §41.

⁸⁸This is thematized by Wrigley (1977).

that the sign-games that occur *in* our mathematical calculations are not statements or propositions, but, when we are talking (or writing) about mathematics, we do use prosaic language. This prosaic talk connects mathematics with other activities, in particular with the applications of mathematics. But it causes philosophical problems.

What is especially misleading is the use of ordinary words such as “triangle”. The prosaic question, “is there a triangle with an angle sum greater than 180° ?”, is described by Putnam as literally unintelligible around 1700, but which became intelligible with the development of non-Euclidean geometry.⁸⁹ It is a mistake to use this as an analogy to claim that it is unintelligible (now) to negate “ $2 + 2 = 4$ ”, but that it may someday be possible. The confusion rests on the treatment of $2 + 2 = 4$ as a proposition, which it is not. “There is a triangle with an angle sum greater than 180° ”, on the other hand, is a proposition, but it is not a calculation. The affirmation, “yes, there is such a triangle” depends on a new calculus, within non-Euclidean geometry. Given this new calculus, it makes sense to say that there is such a triangle, but there is no mathematical *necessity* involved (as in a calculation of the type $2 + 2 = 4$) in saying that these new mathematical “objects” are *triangles*. I think that Cora Diamond’s metaphor of the “face of necessity” or the “face of meaning” are well suited to this case, i.e., we “recognize” a triangle within a new calculus, but it certainly not the kind of necessity we associate with the necessity of $2 + 2 = 4$ where another result would mean that we are no longer adding.

In *Was Wittgenstein Really an Anti-realist about Mathematics?*, Putnam considers the problem of whether we get the “same” proposition in classical geometry by means of an algebraic proof. Wittgenstein was attracted to the idea that the proposition changes with the proof, but Putnam argues that the late Wittgenstein held a contrary view. Putnam here argues against Juliet Floyd in her *On Saying What You Really Want To Say: Wittgenstein, Gödel and the Trisection of the Angle* (1995), where Floyd (in Putnam’s view) argues in favor of a verificationist reading of Wittgenstein. Putnam wants to show that the very passages she uses can be used to show that Wittgenstein argued against such a picture.

To this end, Putnam cites from Wittgenstein’s *Remarks on the Foundations of Mathematics*,⁹⁰ Part VII, §10 (1941). The argument may be seen as supporting Diamond’s account that we can sometimes see one picture-face in another.

Now how about this—ought I to say that the same sense can only have *one* proof? Or that when a proof is found the sense alters?

Of course some people would oppose this and say: ‘Then the proof of a proposition cannot ever be found, for if it has been found,

⁸⁹Putnam (2004), p. 63.

⁹⁰Putnam (2001), pp. 141–142n5; p. 156n30.

it is no longer the proof of *this* proposition.’ But to say this is so far to say nothing at all.—

It all depends *what* settles the sense of a proposition, what we choose to say settles its sense. The use of the signs must settle it; but what do we count as the use?—

That these proofs prove the same proposition means, e.g.: both demonstrate it as a suitable instrument for the same purpose.

Putnam argues that this provides ample evidence that in 1941 Wittgenstein had abandoned his earlier verificationism. He writes that the last sentence shows that Wittgenstein

explicitly allows that we *can* see two different proofs as proving the ‘same’ mathematical proposition, contrary to many interpretations of Wittgenstein.⁹¹

Here Putnam applies Cora Diamond’s metaphor in a reasonable way when he writes that “we sometimes ‘see the face’ of one mathematical language-game in another mathematical language-game”, and hence that “it is a fundamental feature of our mathematical lives that we do not experience every change in our mathematical language-games as a change in the very meaning of the sentences”.⁹²

But this is a very different application of the metaphor than to $2 + 2 = 4$. Putnam puts too much weight on Diamond’s face metaphor. It is supposed to give both an account of mathematical necessity, for instance of $2 + 2 = 4$, and also of how we obtain the same theorem in mathematics using different proofs. To group together the mathematical necessity of arithmetical calculations such as $2 + 2 = 4$ and that mathematical theorems may have different proofs using the same description in terms of the face metaphor seems dubious even for exegetical reasons, since Wittgenstein himself continued the cited passage above in the following way (omitted by Putnam):

And the purpose is an allusion to something outside mathematics.⁹³

It is thus important that we go *beyond* our rules of calculation when we consider two different proofs, say an ancient geometrical and a modern algebraical proof, as proving the *same theorem* in mathematics. The sameness here is not the same as the one obtaining between the left hand side and the right hand side of $2 + 2 = 4$.

⁹¹Ibid., p. 156n.

⁹²Ibid., pp. 155–156.

⁹³Wittgenstein (1956)[1978], Part VII, §10 (1941).

APPENDIX

Lakatos and mathematical fallibilism

A philosopher who has suggested that mathematics is revisable, but in a very special way is Imre Lakatos. He has even suggested that mathematical theorems are fallible in a sense which may remind us of Karl Popper's view of the development of the natural sciences. In the following I would like to show how Lakatos' emphasis on mathematical "prose" (without, of course, referring to such a notion), challenging the "calculus" part of mathematics, leads to a very strange position as regarding mathematical proofs. In the following I will challenge his position, using some of the arguments I have presented in connection with my study of Hilary Putnam. I have chosen to include the example of Lakatos' *Proofs and Refutations* to this thesis in order to illustrate the problems of revisability in mathematics. I am not suggesting that Putnam and Lakatos have the same position. I believe that Putnam's view is closer to that of mainstream mathematical logic, in that one treats well-formed formulas in formal logic as expressing propositions, and I think that this is problem that surfaces when he says that $2 + 2 = 4$ may in a future be revised, although he argues that to say this is in a certain sense not intelligible. Lakatos rather defends the view that it is a myth that there are infallible proofs in mathematics. He argues that it is a mistaken view of mathematical proof to think that counterexamples cannot emerge, and that this is an essential feature of mathematics. Proofs are not absolute in the sense that they cannot be criticized. But there is little to Lakatos' theory that explains how this can be, that is, that we can have counterexamples to proved theorems, except when a mistake has been made.

In the Author's Introduction to *Proofs and Refutations*, Lakatos is critical towards the reduction of the philosophy of mathematics to that of metamathematics, which makes us unable to characterize the "situational logic of mathematical problem-solving" related to the growth of informal ("inhaltliche") mathematics.¹ Lakatos writes:

I shall refer to the school of mathematical philosophy which tends to identify mathematics with its formal axiomatic abstraction (and the philosophy of mathematics with metamathematics) as the 'formalist' school. One of the clearest statements of the formalist position is to be

¹Lakatos (1976), p. 1.

found in Carnap [1937]. Carnap demands that (a) ‘philosophy is to be replaced by the logic of science [...]’, (b) ‘the logic of science is nothing other than the logical syntax of the language of science [...]’, (c) ‘metamathematics is the syntax of mathematical language’ (pp. xiii and 9). Or: philosophy of mathematics is to be replaced by metamathematics. Formalism disconnects the history of mathematics from philosophy of mathematics, since according to the formalist concept of mathematics, there is no history of mathematics proper. [...] Formalism denies the status of mathematics to most of what has been commonly understood to be mathematics, and can say nothing about its growth. [...] Under the present dominance of formalism, one is tempted to paraphrase Kant: the history of mathematics, lacking the guidance of philosophy, has become *blind*, while the philosophy of mathematics, turning its back on the most intriguing phenomena in the history of mathematics, has become *empty*. ‘Formalism’ is a bulwark of logical positivist philosophy. According to logical positivism, a statement is meaningful only if it is either ‘tautological’ or empirical. Since informal mathematics is neither ‘tautological’ nor empirical, it must be meaningless, sheer nonsense. The dogmas of logical positivism have been detrimental to the *history and philosophy of mathematics*.²

It is interesting to note that Lakatos is not only critical of the logical positivists in his introduction, but also mentions Quine (who gave Lakatos’ book a good review³), and who in his *Mathematical Logic* (1951), says that

this reflects the characteristic mathematical situation; the mathematician hits upon his proof by unregimented insight and good fortune, but afterwards other mathematicians can check his proof.⁴

Lakatos comments on this as follows:

But often the checking of an *ordinary* (informal) proof is a very delicate enterprise, and to hit on a ‘mistake’ requires as much insight and luck as to hit on its proof: the discovery of ‘mistakes’ in informal proofs may sometimes take decades—if not centuries.⁵

Lakatos ends his introduction by stating the goal of his work as elaborating

the point that informal, quasi-empirical, mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations.

²Ibid., pp. 1–3.

³Quine (1977), pp. 81–82.

⁴Lakatos (1976), p. 4.

⁵Ibid., p. 4.

Here, I will challenge Lakatos' view of mathematics as quasi-empirical, "in-haltliche" mathematics, as proofs are concerned. That is, I will give Quine right on the point that it is an essential feature of mathematics that mathematicians can check the proofs of other mathematicians, and that there are correct proofs that cannot be challenged by counterexamples, at least not in the way that Lakatos describes the situation. Here one can see that Lakatos defends a very different position from those of Quine's and Putnam's; I believe that Putnam would agree with Quine.

To this end, I will discuss Lakatos' second historical example in *Proofs and Refutations*, namely, that of the Cauchy sum theorem, that is, that "the limit of any series of continuous functions is itself continuous". Today this is a false theorem, since there are simple counterexamples, and in elementary calculus one learns that a sufficient condition for the conclusion (that the limit function is *continuous*) is that of assuming that the sequence of continuous functions (for instance, of partial sums of a series of continuous functions) is uniformly convergent to the limit function.

Cauchy's proof from 1821 in his book *Cours d'Analyse* was early on suspected of not being correct. One of the reasons behind this suspicion was due to a novelty introduced in Cauchy's own book, since he there made a new, and precise, definition of continuity, aimed at bringing mathematics into a more precise form, by transferring an intuitive picture of continuity into "arithmetical language". But the definition excluded the function series considered in Fourier's work *Mémoire sur la Propagation de la Chaleur*, published already in 1808. For us, as Lakatos points out,⁶ we find in Fourier's work a series that is a counterexample to Cauchy's theorem:

$$\cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x - \dots$$

A partial sum of this series consists of finitely many continuous functions and thus is itself a continuous function, but the limit function is not continuous. The graph of the limit function consists of lines parallel to the x -axis, alternately above and below the axis with a distance of $\frac{\pi}{4}$ between them.⁷ Lakatos observes that this type of function was not seen to be an example of a discontinuous function before Cauchy's new arithmetical definition. Cauchy's definition was

⁶Ibid., p. 128

⁷Ibid., p. 129.

accepted, as was also Fourier's series.⁸ In Lakatos' view, this leads to the following mystery: "how could a proved theorem be false, or 'suffer exceptions'?"⁹

In 1826 Abel made an attempt to clarify the situation, but Lakatos' describes his contribution rather negatively, since Abel merely restricts the theorem to a safe domain, without making a proper investigation of the conditions for which the theorem is valid. Abel restricted his attention to power series, in view of what he perceived as counterexamples to Cauchy's theorem, that is, the examples of the type Fourier had already given.¹⁰ Lakatos claims that it is wrong to think that the notion of uniform convergence was Abel's invention, because convergence for power series coincides with uniform convergence. More importantly, Lakatos finds Abel's reaction to the confusion about Cauchy's sum theorem as an example of the "heuristically sterile exception-barring method".¹¹

In Lakatos' story of the developments of Cauchy's theorem, he credits Seidel with the discovery in 1847 of a "hidden lemma" in Cauchy's proof,¹² of which a correct analysis led Seidel to the notion of uniform convergence. The main reasons for nothing to happen between 1821 and 1847 was, according to Lakatos, "the prevalence of Euclidean methodology":

Why did the leading mathematician's from 1821 to 1847 fail to find the simple flaw in Cauchy's proof and improve both the proof-analysis and the theorem? The first reply is that they did not know about the method of proofs and refutations. They did not know that after the discovery of a counterexample they had to analyse the proof carefully and identify the guilty lemma. They dealt with global counterexamples with the help of heuristically sterile exception-barring method. In fact, Seidel discovered the proof-generated concept of uniform convergence and the method of proofs and refutations at one blow.¹³

That is, Lakatos credits Seidel with both the particular discovery of the notion of uniform convergence¹⁴ and the methodology of proofs and refutations that Lakatos now defends, and which is in conflict with the sterile Euclidean methodology. Lakatos continues:

⁸Ibid., p. 131. Note that Lakatos says that the Fourier series were accepted, although it was certainly unclear whether the limit in Fourier's example should be viewed as a *function*, something to which I will return. Lakatos only mentions that it was unclear whether the series actually were convergent.

⁹Ibid., p. 131.

¹⁰Ibid., pp. 134–134. Abel suggested the counterexample $\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \dots$.

¹¹Ibid., p. 136.

¹²Ibid., p. 132.

¹³Ibid., p. 136.

¹⁴This has been challenged by Ivor Grattan-Guinness in Guinness (1986).

The Cauchy revolution of rigour was motivated by a conscious attempt to apply Euclidean methodology to the Calculus. He and his followers thought that this was how they could introduce light to dispel the ‘tremendous obscurity of analysis’. Cauchy proceeded in the spirit of Pascal’s rules: he first set out to define the obscure terms of analysis—like limit, convergence, continuity, etc.—in the perfectly familiar terms of arithmetic, and then he went on to prove everything that had not previously been proved, or that was not perfectly obvious.¹⁵

The Euclidean methodology that was introduced was seen by Cauchy and Abel as a necessary step away from the 18th century “inductive methods”.¹⁶ But this new methodology was inherently hostile to the notion of counterexamples, and Lakatos considers counterexamples to be of central importance to mathematical research. Counterexamples cannot really exist in view of Euclidean methodology, and, in Lakatos’ view, this puts strains on what we can accomplish in mathematics. Abel’s recourse to exception-barring is one such example of how the Euclidean methodologist treats counterexamples; they should be avoided, they do not contribute in any serious way to the development of mathematics; counterexamples are viewed as signs of error within “Euclidean methodology”. Lakatos cites Cauchy saying: “I make all uncertainty disappear.”¹⁷ Nevertheless, there were counterexamples to the Cauchy sum theorem. Lakatos writes that there were only two ways out:

[E]ither to revise the whole infallibilist philosophy of mathematics underlying the Euclidean method, or somehow hush up the problem. Let us first see what would be involved in revising the infallibilist approach. One would certainly have to give up the idea that all mathematics can be reduced to indubitably true trivialities, that there are statements about which our truth-intuition cannot possibly be mistaken. One had to give up the idea that our deductive, inferential intuition is infallible. Only these two admissions could open the way to the free development of the method of proofs and refutations and its application to the critical appraisal of deductive argument and to the problem of dealing with counterexamples.¹⁸

The editors of Lakatos’ posthumous *Proofs and Refutations* claim that it must be an error on Lakatos’ part to demand that we make the second of these admissions in order to arrive at the method of proofs and refutations, since they argue that “by a sufficiently good ‘proof analysis’ all the doubt can be thrown onto

¹⁵Lakatos (1976), p. 137.

¹⁶Ibid., 138.

¹⁷Ibid., p. 138.

¹⁸Ibid., p. 138.

the *axioms* (or antecedents of the theorem) leaving none on the proof itself”.¹⁹ To this they add that “first order logic has arrived at a characterization of the validity of an inference [...] which make valid inferences essentially infallible”.²⁰ That we do not have to give up the idea “that our deductive, inferential intuition is infallible” is certainly something Lakatos would have realized, they argue, and furthermore something he would have changed, since he certainly appreciated formal deductive logic. The editors suggest that the method of proofs and refutations may rather consist in an explication of all the assumptions that have to be made in order for a proof to be valid.²¹

I believe that this suggestion is wishful thinking on the editors’ part (a similar conclusion is drawn by Corfield (1997)), since Lakatos on the page immediately following the last quoted passage, continues to argue that the error of the Euclidean methodology is rather that mathematics has a “privileged infallible status”.²² He writes:

It was the infallibilist philosophical background of Euclidean method that bred the authoritarian traditional patterns in mathematics, that prevented publication and discussion of conjectures, that made impossible the rise of mathematical criticism. Literary criticism can exist because we can appreciate a poem without considering it to be perfect; mathematical or scientific criticism cannot exist while we only appreciate a mathematical or scientific result if it yields perfect truth. A proof is a proof only if it proves; and it either proves or it does not. The idea—expressed so clearly by Seidel—that a proof can be respectable without being flawless, was a revolutionary one in 1847, and, unfortunately, still sounds revolutionary today.

It is clear that Lakatos thinks that a mathematical proof does not have to be infallible, that it should not be a norm that a proof should be infallible. But he is quite mistaken to cite Seidel’s words to this end, since Seidel only argues, in connection with the Cauchy sum theorem, that if a theorem is not universally valid, we have to subject the proof to a more detailed analysis to discover a hidden hypothesis and to restore the theorem by assuming this hypothesis in the theorem.²³ Lakatos, on the other hand, views the search for counterexamples as a universal method in mathematics, because these always lure in the background to proved theorems. In fact, proofs and counterexamples make up a dialectics of mathematical research in Hegelian way. Applied to the Cauchy sum theorem, Lakatos describes a thesis–antithesis–synthesis development of how the theorem

¹⁹Ibid., 138–139n4 (*Editors note*.)

²⁰Ibid., p. 138n4.

²¹Ibid., p. 139n4.

²²This very formulation appears in note 1, p. 139.

²³Ibid., p. 136.

goes through different phases according to such a scheme:²⁴

Thesis: An application of Leibniz' principle of continuity ("what is true up to the limit is true for the limit"²⁵) gives that the limit of any sequence of continuous functions $\{f_n\}$ converges to a continuous function f .

Antithesis: Cauchy's definition of continuity legalizes Fourier's series as counterexamples.

Synthesis: "The guilty lemma to which the global counterexamples are also local ones is spotted, the proof improved, the conjecture improved. The characteristic constituents of the synthesis emerge; the *theorem* and with it the proof-generated concept of uniform convergence."

Much has been written about this story since Lakatos wrote his book. It seems now that Seidel made *one* of the solutions to the problem. There were also others involved, as is studied in Kajsia Bråting's *PhD thesis* (2009).

I am not suggesting that we should look at these other mathematicians for the "correct" or even the "earliest" solution, but it may cast some light on the problem of why it took such a long time for this problem to be solved, and in this respect I will challenge Lakatos' theory that there was a new method of proof (including a new view of proofs) called "the method of proofs and refutations" that were not invented until 1847.

More importantly, the alternative description I sketch below rather shows that it was a matter of setting up a larger background of definitions, not only of continuity but of *function*, *convergence*, etc., in order to construct a working and coherent *system* of mathematical terms. This observation I think justifies the conclusion that the problem between 1821 and 1847 (or 1853, the year when Cauchy revisited his theorem), was that one had not been able to set up a *calculus* for solving the problem. Bråting mentions other possible reconstructions of Cauchy's sum theorem, such as Laugwitz' reconstruction of Cauchy's theorem in terms of infinitesimals, but the point is that we do not know what Cauchy had "in mind" originally, and perhaps a good answer is that he did not know himself, since he lacked a system of well-defined mathematical terms (functions, etc.) to express an intuition he had about the convergence of function series. He was working in an excessive way on the level of mathematical "prose", although this was something he wanted to avoid. There is not one single right way of setting up an adequate mathematical system, but there was in 1821 no system of defined

²⁴Ibid., pp. 144–145.

²⁵Ibid., p. 128.

mathematical terms available to make a precise mathematical sense of Cauchy's "theorem".

Bråting, Domar and Grattan-Guinness (see the references in Braating (2007)), have all shown that the Swedish mathematician Björling was involved in this development as early as 1846, when he published a text in Latin regarding Cauchy's sum theorem. He distinguishes between convergence for a given value, and convergence for all values of a variable x . (I will mainly follow the exposition of Bråting in the following.) It is tempting to think that this distinction is the same as the distinction between pointwise and uniform convergence, and it may well be an attempt in that direction, but Björling was never able to connect the variables n and x in expressions like $f_n(x)$; he only "quantified" over the variable x . Another way of interpreting Björling's "convergence for all values of x " (as Bråting does) is to allow x to move, which could be regarded as equivalent to uniform convergence, in the sense that we for all n are allowed to choose a new x . But the situation is unclear, since we in modern terminology can give several interpretations. For instance, if x is not regarded as fixed, as real numbers are viewed today, it becomes unclear whether we have something that in modern terminology can be written as $f_n(x_m)$, and where it is not clear how we should regard the dependence between m and n ; should we for example have x_m converging to some x before we let " $n \rightarrow \infty$ ", or should we choose an n for every m ?

The sense moral of this discussion is that Cauchy's 1821 assumption that the sum theorem holds in the "vicinity of this particular value", is very difficult to interpret in modern terminology; in fact there may be no adequate modern formulation that captures what Cauchy had "in mind". This situation is further complicated by a floating function concept; Björling's theory reveals that a function is simply a variable expression, causing Björling to find properties of functions as he goes along. In one of Björling's expositions, a function may have two values, for instance, the derivative of the absolute value function has two values at 0. From this evidence, Bråting is able to conclude that the whole situation between 1821 and 1853 is very complicated from a *conceptual* point of view. Bråting asks: is the "limit function" one-valued? Today's concept of pointwise convergence *depends* on the modern function concept (it depends on the requirement that at most function value should be assigned). Bråting's conclusion is that during the period 1821–1853, i.e., between Cauchy's two versions of the theorem it was impossible to make a unique sense of $f_n(x)$ and what convergence of this expression means. These questions are all important in order to understand the long period of silence that occupies Lakatos in the example of Cauchy's sum theorem. Instead of thinking that mathematicians were prevented from raising critique of the style of "Euclidean" proof, which Lakatos sees Cauchy as partly responsible for, one can also think of the period of silence as caused by severe

difficulties to come to terms with some of the most fundamental concepts of analysis.

Mark Steiner has written a very good paper on Lakatos, *The Philosophy of Mathematics of Imre Lakatos* (1983), and I side with Steiner's interpretation of Lakatos, although he does not formulate himself in terms of mathematical prose and calculus. Steiner argues that Lakatos' view is not that mathematical knowledge is possible without formal proof, but rather (and he cites evidence for this, where Lakatos claims that we never know, we only guess, and that there is nothing wrong with an infinite regress of guesses in mathematics) that "for 'fallibilist' Lakatos, knowledge (meaning certainty) is impossible even in mathematics".²⁶

²⁶Steiner (1983), p. 505.

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