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Nominalism and Immutability

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Can we do science without numbers? Quine (1961) says no. Numbers and other mathematical entities are indispensible. Field (1980) says yes. Numbers may be useful, but are not essential. With enough time and patience, we can do science without them.

Now consider a seemingly unrelated question: How much contingency is there? Spinoza says none. Whatever is true is necessary. Williamson (2013) says none with respect to what exists, but *some* with respect to how things are. The ordinary view, perhaps, is that there is some with respect to what exists, and some with respect to how things are.

These seemingly unrelated questions—one in the philosophy of math and science and the other in metaphysics—share an unexpectedly close connection. For as it turns out, a radical answer to the second leads to a breakthrough on the first.

The radical answer is immutabilism, a view first endorsed by Leibniz. Immutabilism says that there is some contingency with respect to what exists, but *none* with respect to how things are. Immutabilism is thus a sort of converse of necessitism, which is the view endorsed by Williamson.

The breakthrough is a new strategy for doing science without numbers. Field shows how to do science without numbers in classical mechanics. His strategy, though, requires the existence of spacetime points. This is fine, so far it goes. But there are reasons you might want an alternative: You might be a relationalist, and so reject the existence of spacetime points. You might be concerned that the strategy will not generalize, especially to theories formulated in terms of state space. Or you might simply wonder whether we can get by with less. I think that we can, and immutabilism is the way to do it.

1 Nominalism

We can distinguish two views about scientific theories. **Scientific nominalism** says that the best scientific theories include a nominalist theory. A theory is nominalist when

it quantifies over only concrete particulars.[1] Concrete particulars include things like particles.

Scientific Platonism, on the other hand, is the view that all of the best scientific theories are Platonist. A theory is Platonist when it is not nominalist. Thus, a Platonist theory will quantify over things like numbers or universals in addition to things like particles.[2]

Why might you be a scientific nominalist? Suppose that you are a **metaphysical nominalist**. You say that the only things that exists, fundamentally speaking, are concrete particulars. Suppose that you are also a **scientific fundamentalist**. You thus claim that fundamental reality is best described by at least one of the best scientific theories. In that case, scientific nominalism will follow.

Metaphysical nominalism, then, is one road to scientific nominalism. But there are others as well. For example, you might think that numbers exist, and that their existence is fundamental. Still, you might think that their role in science is merely *representational*. But if the role of numbers is merely representational, then we should be able to do science without them, given enough time and patience.[3]

Similarly, you might think that nominalist theories have certain virtues that Platonist theories lack. For example the most direct objects of scientific inquiry are concrete particulars—things like meter sticks and scales and particle accelerators. Nominalist theories are thus **intrinsic** in a way that Platonist theories are not. For nominalist theories explain the behavior of concrete particulars without appealing to anything *other* than concrete particulars. As such, you might think that nominalist theories are both more satisfying and more illuminating.[4] This in turn suggests that the best scientific theories may include nominalist theories, since they have a unique profile

1. This is only a rough characterization. What it means for a theory to be nominalist depends on the background ideology. For example, a theory stated in terms of a feature-placing language of the sort suggested by Quine (1971) has no quantification, and so thereby has no quantification over things like numbers or properties. But a theory written in a feature placing language could fail to be nominalist. It might, for example, place mathematical features (like being an integer) alongside physical features (like having mass). Thus, I prefer to adopt a "we know them when we see them" approach to identifying nominalist theories.

2. The views described here are *weak* scientific nominalism and *strong* scientific Platonism. Alternatively, you could be a *strong* scientific nominalist who thinks that all of the best physical theories are nominalist. You could also be a *weak* scientific Platonist who merely holds that some of the best physical theories are Platonist.

3. Field (1984) makes a similar point. He calls this view moderate Platonism.

4. For example, synthetic geometries, like those proposed by Tarski (1952), would seem to be both more satisfying and more illuminating than the corresponding analytic geometries.

when it comes to theoretical virtues.[5]

You can be a scientific nominalist, then, without being a metaphysical nominalist. The converse is also true: You can be a metaphysical nominalist without being a scientific nominalist.

For example, suppose you are a logical atomist, and so hold that they only fundamental facts are atomic facts. The best scientific theories, though, have strong and simple laws, which necessarily involve generality. Thus, scientific fundamentalism fails. But in that case, you could also think that the necessary generality requires mathematics, even if the the only fundamental things are concrete particulars. Thus, you could be a metaphysical nominalist without being a scientific nominalist.

For my own part, I am both a scientific nominalist and a metaphysical nominalist, though my commitment to the first is stronger than my commitment to the second.

Our focus in this paper will be scientific nominalism. What we want to know is: Can we build scientific theories while quantifying over nothing more than concrete particles? Can we do science without numbers? Thus, by nominalism, we will generally mean *scientific* nominalism. Likewise, by Platonism, we will generally mean *scientific* Platonism. Questions about metaphysical nominalism and metaphysical Platonism are also important, but will remain in the background, for the most part.

1.1 Quantities

The physical world is built using physical quantities like mass, charge, and distance. To fix on an example, suppose we perform a series of experiments and discover that the movement of particles is governed by Newton's laws. These laws require distance ratios between particles.[6] Thus, in order to state the laws, we need a language that can describe such things.

What are distance ratios? Suppose we use a meter stick to determine that a and b are two meters apart and c and d are one meter apart. Thus, a and b are twice as far apart as c and d. This is a distance ratio. Others include being three times as far apart, being half as far apart, and so on.

A Platonist can easily describe distance ratios. She could, for example, use a distance

5. That one might prefer nominalist theories because they are more intrinsic is a point originally made by Field (1984).

6. Here is a simple case: Suppose the world is Newtonian with gravity the only force. There are three particles a, b, and c with b between a and c. The particles are at rest relative to one another and a is as massive as b and c put together. In that case, by the law of universal gravitation, the collision of a and b will be simultaneous with the collision of b and c just in case the distance ratio of a and b to b and c is $\sqrt{2}$. But if there is no determinate distance ratio, the laws will fail to determine whether the collisions will be simultaneous. We thus get an unwanted failure of determinism.

ratio function from particles to real numbers. Thus, to say that a and b are twice as far apart as c and d, she would write:

$$\delta(a, b, c, d) = 2 \tag{1}$$

If you like, you might think of this as a definite description:

The distance ratio of a and b to c and $d = 2$.

Such descriptions let the Platonist describe particle configurations, apply the dynamical laws, and predict how particles move.

Suppose, though, that we are scientific nominalists. We are thus committed to theorizing about the physical world without using things like distance functions. In that case, how are we going to express facts involving distance ratios? This problem, as applied to physical quantities in general, is what Field (1984) calls **the problem of quantities**.

One strategy for solving the problem is **simple nominalism**. In the case of distance ratios, simple nominalism requires nothing more than particles and the relations of **congruence** and **betweenness**.

$\mathsf{Cong}(a, b, c, d)$	a and b are the same distance apart as c and d
$\mathsf{Bet}(a, b, c)$	b is on a straight line between a and c

In each case, the gloss on the right is merely intuitive. The congruence relation is naturally described by quantifying over distances and the between relations by quantifying over lines. But these are both just basic relations, so come with no commitment to the existence of things like distances or lines.

To see how a simple nominalism might account for distance ratios, consider a world with exactly four point particles arranged as follows:

The betweenness and congruence relations are as illustrated, with the particles a and b twice as far apart as b and c. This distance ratio is the one that we want to explain.

The strategy, in this case, is straightforward. The simple nominalist says that a and b are twice as far apart as b and c, in the sense that there is an x between a and b such that a and c are congruent with x and b, and x and b are congruent with b and c.

More generally, we can define the notion of an equally spaced line of particles using betweenness and congruence.[7] This gives us a defined polyadic predicate.

7. Particles a and b are *colocated* when there is some x such that $\mathsf{Cong}(a, b, x, x)$. The particles

$\mathsf{Line}(x_1, \dots, x_n) \quad x_1, \dots, x_n$ form an equally spaced line

We then express the claim that a and b are twice as far apart as b and c using:

$$\exists x\ \mathsf{Line}(a, x, b, c) \tag{2}$$

Looking at the previous diagram, you can see why this is adequate, at an intuitive level. Since we have an equally spaced line, the distance between each pair can be thought of as a unit. There are two units between a and b, but there is only one unit between b and c. So a and b are twice as far apart as b and c.

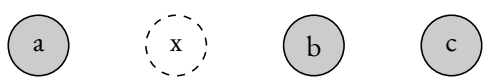
The problem is that simple nominalism only works if there happens to be enough particles and they happen to be in the right place. For consider a world just like the last, except that the second particle from the left has been deleted.

This is a world in which a and b are twice as far apart as b and c. There is, however, no way to fix this distance ratio using betweenness and congruence. After all, consider a world in which a and b are *half* as far apart as b and c instead.

These two worlds have exactly the same betweenness and congruence relations, but different distance ratios. Thus, there is no way to fix the relevant distance ratios using betweenness and congruence. Such worlds are **sparse**.

Field's solution is to accept the existence of spacetime points. Spacetime points, like particles, can stand in betweenness and congruence relations. Unlike particles, though, spacetime points are always numerous and well-organized. You can always count on them being exactly where they need to be.

For example, suppose that we have laws guaranteeing that whenever there are two things, there is a spacetime point halfway between them. Thus, in the first of our two sparse worlds, there will be a spacetime point x halfway between a and b.



But now, since there is once again *something* halfway between a and b, we can explain

$a_1, \dots, a_n$ form a *line* when no two of them are colocated and a_k is between a_j and a_h whenever $j \leq k \leq h$. The line is *equally spaced* if a_j and a_{j+1} are congruent with a_k and a_{k+1} for all j and k such that $0 \leq j < n$ and $0 \leq k < n$.

the necessary distance ratio the same way as before. That is, a and b are twice as far apart as b and c because:

$$\exists x\ \mathsf{Line}(a, x, b, c) \tag{3}$$

The only difference is that now, x is a spacetime point instead of a particle. But the basic structure of the explanation is exactly the same.[8]

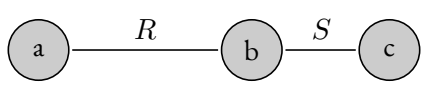
There is much to like about the substantivalist strategy. Spacetime points are concrete particulars, and so accepting their existence is consistent with nominalism. Fields are also naturally thought of as properties of spacetime points, and so there is some reason to think that we will need spacetime points *anyway* to account for fields. In that case, using spacetime points to explain distance ratios is no further cost.

On the other hand, there are also reasons to be wary. After all, spacetime faces its own slate of challenges, ranging from shift arguments to hole arguments.[9] There is also the concern that the strategy will not generalize. Some of our best science describes quantities using more exotic spaces like state space. But while it is reasonably clear that a nominalist can accept the points of *ordinary* spacetime as concrete particulars, it at best unclear whether a nominalist can accept the points of *state space* as concrete particulars. But in that case, how is a nominalist going to nominalize state space theories? Thus, we have good reason to wonder if there might be an alternative.[10]

Another strategy for expressing distance ratios is to accept distances rather than spacetime points. What are distances? Brent Mundy (1987) suggests a view on which distances are binary relations between particles. Particles thus *have* distances by *instantiating* them. Axioms governing distance relations are then given using second-order quantification into predicate position and a pair of second-order predicates.

$X \geq Y$	X is at least as great as Y
$\mathsf{Sum}(X, Y, Z)$	X and Y sum to Z

With appropriate axioms in place, we can then explain distance ratios, even in sparse worlds. For example, here is our three-particle world again.



8. Early axiomatizations of Euclidean space in terms of congruence and betweenness include (Veblen 1904) and (Pieri 1908). The project was later advanced by Alfred Tarski and his students, who gave increasingly simple axioms in (Tarski 1952), (Tarski 1959), and (Gupta 1965).

9. See for example Dasgupta (2016).

10. This objection was originally raised by Malament (1982).

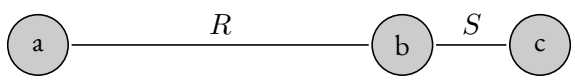
The particles a and b are then twice as far apart as b and c because:

$$R(a,b) \wedge S(b,c) \wedge \mathsf{Sum}(S,S,R) \tag{4}$$

This says that a and b have distance R and b and c have distance S. Moreover, S and S sums to R. This solution, though, is not available to nominalists, since it requires higher-order quantification over relations.[11]

Mundy can explain distance ratios in sparse worlds because he is a Platonist. Thus, even if the particles are sparse, he can still explain distance ratios because his *distance relations* are plentiful.[12] Like spacetime points, they are numerous and well-organized. You can always count on them being where they need to be.

On the other hand, if there are distance relations, but they are not plentiful, we will once again find ourselves unable to express the needed distance ratios. For example, suppose you are an Aristotelian. You thus deny that distance relations exist when not instantiated. Now consider the following sparse world. There are exactly three particles. This time, the particles a and b are *three* times as far apart as b and c.



Mundy will say that the distance ratio can be expressed with:

$$\exists X(R(a,b) \wedge S(b,c) \wedge \mathsf{Sum}(S,S,X) \wedge \mathsf{Sum}(S,X,R)) \tag{5}$$

This says that a and b stand in distance relation R and b and c stand in distance relation S. Moreover, there is a distance relation X such that S and S sum to X, and S and X sum to R. The problem for the Aristotelian is that this requires the existence of a distance relation X that is the sum of S and S. But if the only distance relations that exist are the ones that are instantiated, there is no such X. So the necessary distance ratio is once again inexpressible.

1.2 Modality and Uniqueness

We can give a theory of distance ratios without numbers, then, if we quantify over spacetime points or distance relations. But are there other options?

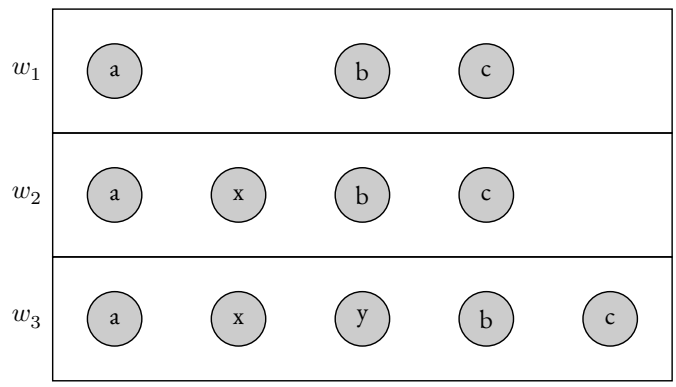
A natural thought is that perhaps we could give a theory of distance ratios using

11. A view that you might call *easygoing nominalism* accepts higher-order quantification into predicate position while rejecting first-order quantification over universals. In contrast, *serious nominalism* rejects both sorts of quantification. There is an ongoing dispute about whether easygoing nominalism is compatible with nominalism. Arthur Prior (1971) says yes. I am inclined to say no. For present purposes, we can just stipulate that our interest is in the question of whether we can do science as serious nominalists.

12. In particular, Mundy's axioms guarantee the distance relations form an *ordered semigroup*.

modality. For example, you might think that a and b are twice as far apart as b and c in a sparse world, not because there is actually a particle x halfway between a and b, but because there *could* have been.

As natural as this thought might be, modal strategies face a serious challenge, which we will call the **problem of uniqueness**. For suppose that the actual world w_1 is as illustrated below. The particles a and b are twice as far apart as b and c, and so this is the distance ratio we want to explain. The modal proposal, then, is that this is because there



is a possible world w_2 in which a, b, and c are exactly the same, but in which there is an additional particle x halfway between a an b, with the resulting four particles standing in the illustrated betweenness and congruence relations.

So far so good. The question is: What does it mean for a, b, and c to be *exactly the same* in worlds w_1 and w_2?

We could say that particles are exactly the same when they are *intrinsically* the same. Thus, the three particles in w_1 are exactly the same in w_2 because they stand in the same betweenness and congruence relations in w_2 that they do in w_1.

But now observe: World w_3 is possible. Moreover, the particles in w_1 stand in the same betweenness and congruence relations with each other in w_3 that they do in w_1. Thus, they are intrinsically the same and so, on the present proposal, *exactly* the same. The problem is that a and b are three times as far apart as b and c in w_3. So if the possibility of w_2 is enough to make a and b twice as far apart as b and c in w_1 then, by parallel reasoning, the possibility of w_3 is enough to make a and b three times as far apart as b and c in w_1. But in that case, the *very same particles* in the *very same world* are both twice as far apart and three times as far apart. This is absurd. So the proposal fails.

For the modal strategy to work, there has to be a relevant difference between world w_2 and world w_3. But if we only have congruence and betweenness relations *within* worlds, there is no relevant difference. This because the three particles from world w_1 stand in exactly the same betweenness and congruence within each world.

This leads to a natural suggestion. Maybe the worlds can be distinguished if we

allow for relations not only *within* worlds, but *across* worlds.

Looking back at the above diagram, suppose that the congruence relations across worlds are as they appear. Thus, a and b at w_1 are congruent with a and b at w_2, but not congruent with a and b at w_3. In that case, we *do* have a relevant difference between w_2 and w_3. Thus, congruence comparison across worlds give us a natural strategy for solving the problem.

The question is: Can a nominalist appeal to such comparisons across worlds? This will be the topic of the next section.

1.3 Cross-Modal Comparisons

We ordinarily think that Socrates could have been taller than he is and that the Athenians could have been happier than they are. In the first case, we are saying that there is a possible world in which Socrates is taller than he is in the actual world. And in the second, we are saying that there is a possible world in which the Athenians are happier than they are in the actual world. Such comparisons are **cross-modal comparisons**.

How should a nominalist understand cross-modal comparisons? There are a variety of approaches to the metaphysics of modality. For our purposes, we can focus on two, which will be used to illustrate the general challenge.

The **modal realist** says that modal facts are ultimately explained in terms of quantification over a pluriverse of island universes that are just as real and concrete as our own. The **modalist**, on the other hand, says that modal facts are basic and should be understood in terms of modal operators. The modalist, unlike the modal realist, is at best agnostic about whether there are universes other than our own.

Now consider how each might express cross-modal comparisons. Suppose that our universe contains a single particle named Alice. What we would like to say is:

Alice could have been more massive than she actually is. (6)

A modal realist can do this using a pair of relations $\mathsf{Ctp}(x, y)$ and $\mathsf{Mass}(x, y)$. The first says that x is a counterpart of y. The second says that x is more massive than y. The target sentence is then expressed with:

$$\exists x(\mathsf{Ctp}(x, a) \wedge \mathsf{Mass}(x, a)) \tag{7}$$

Thus, according to the modal realist, Alice could have been more massive than she actually is because there is a particle, somewhere in the pluriverse, that is both a counterpart of Alice and more massive than Alice.

The question is whether the modalist can do the same. She could try expressing (6) by writing:

$$\Diamond(\mathsf{Mass}(a, a)) \tag{8}$$

But this says that it could have been that Alice was more massive than herself, not that

it could have been that Alice was more massive than she actually is. A modalist could try adding an actuality operator, which would let her write:

$$\Diamond @(\mathsf{Mass}(a, a)) \tag{9}$$

But this says that Alice could have *actually* been more massive than herself, which is also incorrect. These exhaust the most obvious syntactic possibilities. So the modalist has no clear strategy for expressing cross-modal comparisons. Call this the **problem of cross-modal comparisons**.[13]

The standard solution is to use quantification over universals. In the case of mass comparisons, this means quantifying over mass properties. On this approach, a modalist will express (6) with:

$$\exists X(X(s) \wedge \Diamond(\exists Y(Y(s) \wedge Y > X))) \tag{10}$$

This says that there is a mass that Alice has, and it could have been that there was a mass that Alice had that was greater than the one she actually has.[14]

Now return to the question of nominalism. The nominalist would like to give a theory of distance ratios by using cross-modal comparisons. She could try being a modalist. But the standard solution for expressing cross-modal comparisons as a modalist is to use quantification over universals, which is not available to a nominalist.[15]

Could a nominalist be a modal realist? This question deserves some care. Suppose you are a nominalist and one day, you visit the oracle. You ask her whether there is a pluriverse. She says yes. Moreover, she tells you that the pluriverse is built entirely out of concrete particulars. There are no numbers, universals, or any other such things.

Should you give up your nominalism? Clearly not, or so it seems to me. How could learning that there are *more* concrete particulars be inconsistent with the view that everything is a concrete particular? Giving up nominalism, on the grounds that there are many concrete universes, would be like giving up theism, on the grounds that there are many gods.

Nominalism and modal realism, then, are broadly consistent. The question is whether there are coherent *grounds* for being both a nominalist and a modal realist, given our actual evidence. I say no. For as Lewis points out, insofar as we have reasons to

13. For further discussion, see Williams (1984), Wehmeier (2012), Mackay (2013), Kocurek (2016) and Berntson (2019).

14. Variations of the standard solution have been endorsed by a number of authors, including Morton (1984), Cresswell (1990), Milne (1992), and Kemp (2000).

15. If the nominalist can be an *easygoing* nominalist, then she can use the standard solution. Whether easygoing nominalism is compatible with nominalism is an interesting question. As in footnote 11, though, we can just stipulate that our interest here is in solutions that do not use higher-order quantification.

accept the pluriverse, those reasons are broadly similar to the ones we have for accepting numbers.[16] But the nominalist denies that we have good reason to accept numbers. Thus, she must deny that we have good reasons to accept the pluriverse. Thus, a nominalist cannot coherently be a modal realist, given our actual evidence.

2 Compossible Immutabilism

We saw in §1 that, in order to do science without numbers, the nominalist needs to give a theory of quantities. One natural strategy is to use modality. But for this to work, she needs to ensure that distance ratios are unique. This can be done if the nominalist has cross-modal comparisons. The problem, though, is that there is no clear strategy for expressing cross-modal comparisons as a nominalist.

In this section, we are going to describe a new view about modality called compossible immutabilism. This new view will let nominalists make cross-modal comparisons which, in turn, will give them a new strategy for doing science without numbers.

2.1 Immutability

We ordinarily think that things are **mutable**, in the sense that they could have had different properties and could have stood in different relations. For example: My coffee mug is blue and sitting on my desk. But while that may be, it could have been a different color. It could have been green instead of blue. It also could have had stood in different spatial relations. Instead of being on my right, for example, it could have been on my left. Thus, my coffee mug would seem to be mutable.

Things that are not mutable are **immutable**. Whether there are any immutable things is a matter of controversy, so there are no uncontroversial examples. Maybe God is immutable. Maybe numbers are.

Immutabalism is the view that necessarily, everything is immutable. Thus, while there may be contingency with respect to what exists, there is *no* contingency with respect to how things are. When things have properties and stand in relations, they have those properties and stand in those relations necessarily. The opposing view is **mutabilism**, which says that possibly, something is mutable.

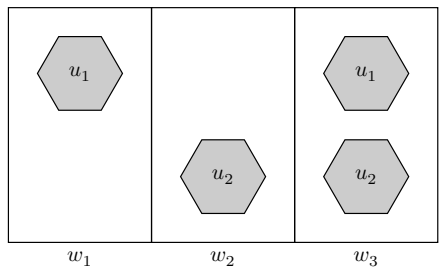
Immutabilism has some precedence in the history of philosophy. Leibniz held that necessarily, everything is worldbound. An individual is worldbound when necessarily, had anything been any different, that individual would not have existed.[17]

16. See (Lewis 2001, Chapter 1).

17. Whether Leibniz himself accepted worldbound individuals is controversial. The standard view, though, is that he did. See Mates (1989), Adams (1994), Cover and Hawthorne (1990, 1999), and Garber (2009).

One need not be a Leibnizian immutabilist, though, to be an immutabilist. In what follows, we are going to present a new view called **compossible immutabilism**. Unlike Leibnizian immutabilism, compossible immutabilism rejects the idea that as a matter of necessity, individuals are always worldbound.

Suppose that we have two possible worlds w_1 and w_2 as illustrated below. Each world contains a single universe, and each universes contain a number of particles. The



compossible immutabilist says that the particles in each universe have all of their basic properties necessarily. They also stand in basic relations necessarily. On this much, she agrees with the Leibnizian.

The compossible immutabilist, though, parts ways with the Leibnizian when it comes to compossibility. For the compossible immutabilist claims that pairs of possible worlds can always be composed. In the present case, this means that given that we have worlds w_1 and w_2, there is a third world w_3 in which everything from world w_1 exists and everything from world w_2 exists. A Leibnizian, though, denies that there is any such world w_3, on the grounds that individuals are necessarily worldbound.

What we are going to do now is make all of this more precise. Thus, we will describe the language of immutabilism in §2.2. We will then say more about various principles that the compossible immutabilist might accept in §2.3.

2.2 Language

We are going to think of the immutabilist as having a fundamental language $\mathcal{L}_I$. This language is a first-order plural language, which includes:

Singular variables	$x, y, z, \ldots$
Plural variables	$xx, yy, zz, \ldots$
Singular quantifiers	$\exists x, \forall x$
Plural quantifiers	$\exists xx, \forall xx$
Truth-functional operators	$\wedge, \vee, \supset, \neg$
Modal operators	$\Box, \Diamond$

Singular variables range over **individuals**. Plural variables range over **pluralities**.

What are pluralities? A plurality is not a *thing*, like a particle or a spacetime point.

A plurality is simply *things* taken together.

To borrow an example from George Boolos (1984), suppose you have a bowl of Cheerios. You consider each individually: Here is one Cheerio. Here is another. Besides each individual Cheerio, though, you might also consider the *Cheerios* taken together. For the Cheerios have properties that are not had by any Cheerio. The Cheerios fill the bowl, but no Cheerio fills the bowl. The Cheerios weigh 50 grams, but no Cheerio weighs 50 grams.

You might then wonder, what are the Cheerios? Maybe the Cheerios are a set. Maybe the Cheerios are a fusion.

Maybe. But many of us think that Cheerios taken together are *thin* in a way that sets and fusions are not. After all, if you accept that there is a set of Cheerios, then you are ontologically committed to sets. And if you accept that there is a fusion of Cheerios, then you are ontologically committed to fusions. But if you accept that there are Cheerios taken together, you are ontologically committed to nothing more than each of the Cheerios, taken individually. The Cheerios taken together are what we are calling a plurality.

This point is worth belaboring because, in the present context, we want to ensure that immutabilism is consistent with nominalism. Thus, it is important that pluralities are not abstract things, like sets.

Predicates in the immutabilist language are sorted. Thus, each argument place takes either terms for individuals or terms for pluralities, but not both.[18] Predicates are allowed to have any finite arity. The logical predicates are:

$x =_s y$	x is identical y
$xx =_p yy$	the xx are identical to the yy
$x \prec yy$	x is among the yy
$x \sim y$	x is connected to y

The first is a singular identity predicate. The second is a plural identity predicate.[19] The third is the **among** predicate. This lets us say which individuals are among with pluralities. The fourth is a **connectedness** predicate. Two things are connected, intuitively speaking, when they are in the same universe.

Once the immutabilist has a language, there is the question of what logic she should accept. There are many systems that could be built. Our focus here will be on a system called **CI**, which is the system that I in fact accept. It includes the core principles of

18. That is, there are no complex sorts, including disjunctive sorts. Complex sorts, though, could easily be added.

19. Context will generally determine which identity predicate we have in mind and, so, we will generally drop the subscripts.

compossible immutabilism, along with others that help to fill out the view.[20]

CI includes a positive free logic, along with the propositional modal logic **S5**. That **CI** is based on a free logic ensures individuals and pluralities can both exist contingently. Names are only assigned to individual and pluralities that exist. There are also axioms guaranteeing that pluralities are rigid and that connectedness is an equivalence relation.

CI is a plural logic, and so there is the question of which things form pluralities. Compossible immutabilism *as such* is compatible with a wide variety of answers. Our system **CI**, though, will include all the instances of Plural Comprehension.[21]

Plural Comprehension $\exists xx \forall y(y \prec xx \equiv \phi(y))$

This is a schema, which tells us that for any condition, there is a plurality consisting of precisely those things which satisfy that condition. Plural Comprehension also entails two further principles:

Everything $\exists xx \forall y(y \prec xx)$
Nothing $\exists xx \forall y(y \not\prec xx)$

Everything says that there are things such that everything is among them. Nothing says that there are things such that nothing is among them. Thus, Nothing guarantees the existence of an empty plurality.[22]

We can now define several useful notions. A plurality xx is **closed under conectedness** when for all y and z, if y is among the xx and y is connected to z, then z is among the xx. A plurality xx is a **subplurality** of another plurality yy when every x that is among the xx is also among the yy. A **universe** is a non-empty plurality that is closed under connectedness and in which every pair of individuals is connected. A **multiverse** is a plurality that is closed under connectedness in which some pair of individuals is *not* connected.

In what follows, it will be helpful to talk about pluralities of universes. This, however, raises an immediate complication. For universes themselves are pluralities of individuals, so pluralities of universes would have to be pluralities of pluralities. That is, pluralities of universes would have to be *plupluralities*. But our immutabilist language

20. A full model-theoretic description of **CI** is included in the appendix.

21. The formula ϕ is allowed to be an open formula with with both individual and plural free variables, though xx is not allowed to occur free in ϕ.

22. **CI** does not include full plural logic. This is because accepting full plural logic is inconsistent with some of my own reasons for being a nominalist. For example, one reason that I am a nominalist is that I deny that there is any objective fact of the matter regarding the size of the continuum. But if we have full plural logic, only one size of the continuum is logically consistent. Thus, there will be an objective fact of the matter regarding the size the continuum *would* have had, *had* there been a continuum. **CI** does not include full plural logic, so avoids this sort of unwanted objectivity.

has no syntax for talking about plupluralities. So it would seem that we have no way to talk about pluralities of universes.

Our solution will be to identify pluralities of universes with the plurality of individuals in those universes. Thus, suppose that we have two universes u_1 and u_2. What we are going to call the plurality of u_1 and u_2 is just the plurality of individuals that are in either u_1 or u_2. Or putting it another way, what we are going to call a **plurality of universes** is just a plurality of individuals closed under connectedness. We will then say that a universe xx is *among* a plurality yy of universes when the xx are a subplurality of the yy.

2.3 Principles

There are many ways to be an immutabilist. What all immutabilist have in common, though, is a commitment to the idea that necessarily, everything is immutable. This can be expressed with a the following schema:

Immutability $\Box\forall x_1 \ldots \Box\forall x_n \Box(R(x_1, \ldots, x_n) \supset \Box(\mathsf{Exists}(x_1) \wedge \ldots \wedge \mathsf{Exists}(x_n) \supset R(x_1, \ldots, x_n)))$

Immutability says that necessarily, if individuals stand in a relation then, necessarily, they stand in that relation, so long as they all exist.

There is a sense in which Immutability, on its own, does not capture the full immutabilist picture. For the immutabilist says that there is no contingency with respect to how things are, only contingency with respect to what exists. The problem is that while Immutability guarantees that existent things have the same properties and stand in the same relations across worlds, it does not guarantee the same for non-existent things.

My own preference is to solve the problem by accepting serious actualism. This is the view that things have properties and stand in relations only when they exist.[23]

Actuality $\Box\forall x_1 \ldots \Box\forall x_n \Box(R(x_1, \ldots, x_n) \supset \mathsf{Exists}(x_1) \wedge \ldots \wedge \mathsf{Exists}(x_n))$

Actuality guarantees that non-existing things never have properties nor stand in relation. Thus, non-existent things cannot have different properties or stand in different relations

23. We are going to restrict the substitution instances of R to *non-logical* predicates. Thus, things are allowed to stand in logical relations, even when they fail to exist. The view that things fail to stand in logical relations when they fail to exists is a view that you might call *very* serious actualism. I have no opposition to very serious actualism, but adopting it here would add unnecessary complexity that might distract from the main discussion.

in different worlds.

For those who are not serious actualists, another approach would be to accept Strong Immutability in place of Immutability.

Strong Immutability $\Box\forall x_1 \ldots \Box\forall x_n \Box(R(x_1, \ldots, x_n) \supset \Box(R(x_1, \ldots, x_n)))$

Strong Immutability says that if things stand in a relation, then necessarily, they stand in that relation *regardless* of whether they exist. In contrast, Immutability merely claims that if things stand in a relation then, necessarily, they stand in that relation *assuming* that they exist.

As noted earlier, a Leibnizian immutabilist accepts not only Immutability, but the further claim that individuals are always worldbound.

Worldbound $\Box\forall x_1 \ldots \Box\forall x_n(\phi(x_1, \ldots, x_n) \supset \Box(\mathsf{Exists}(x_n) \supset \phi(x_1, \ldots, x_n)))$

This can be read as saying that necessarily, every individual is such that necessarily, had anything been different, it would not have existed. Worldbound clearly entails Immutability. Thus, Leibnizian immutabilism is a genuine form of immutabilism.

What we are going to call **compossible immutabilism** is another form of immutabilism. Like Leibnizian immutabilism, it accepts Immutability. However, unlike Leibnizian immutability, it accepts Compossibility in place of Worldbound.

Compossibility $\Box\forall xx \Box\forall yy \Diamond(\mathsf{Exists}(xx) \wedge \mathsf{Exists}(yy))$

Compossibility says that given any two possible pluralities, it could have been that those pluralities existed together. Given that that Everything is not just true, but necessary, it follows that given any two possible worlds, there is a third possible world in which everything from the first two exists together. Compossibility and Worldbound are inconsistent, so long as there are distinct possible worlds that are non-empty.[24]

Immutability is a schema. Thus, R can be replaced by any basic predicate in the language. These predicates may include predicates with argument places taking plural terms. In that case, the singular variables and singular quantifiers will need to be replaced with plural variables and plural quantifiers.[25]

The restriction of Immutability to basic predicates is important. What this restriction means is that the immutabilist is *not* committed to the view that things are immutable with respect to arbitrary conditions or arbitrary complex predicates. Rather,

24. That is, so long as $\Diamond\exists x \Diamond\exists y(\neg\mathsf{Exists}(x))$.
25. The same will apply for any other schemas given throughout the rest of this paper.

she only committed to things being immutable with respect to a sparse collection of basic predicates.

To illustrate the importance of the restriction to basic predicates, suppose that we were to add lambda abstraction to our language. Thus, for every condition ϕ, there is a corresponding complex predicate $\lambda x(\phi)$. Suppose furthermore that we allow these complex predicates as substitution instances for Immutability. In that case, Immutability will entails Worldbound. But in that case, Immutability will entail that Compossibility fails, so long as their are distinct non-empty possible worlds.

There are various apparent counterexample to Compossibility. For example, Fritz and Goodman (2017) consider the following: Suppose there is a knife factory in which there is a single handle h and two blades b_1 and b_2. It could have been that the handle and the first blade were put together and, in that case, there would have been a knife a. It also could have been that the handle and the second blade were put together and, in that case, there would have been a knife b. But now, assuming that as a matter of necessity, a knife exists only when its blade and handle are put together, it follows that a and b could not have existed together. Thus, Compossibility fails.

Such apparent counterexamples, though, depend on the denial of Immutability. For suppose that we treat 'put together' as a basic predicate. In that case, Immutability and the background modal logic will entail that h and b_1 could have been put together only if they are actually put together. Likewise for h and b_2. But h is actually put together with at most one blade. So there are not two such possible knives a and b and, therefore, we do not have a counterexample to Compossibility.

Compossibility tells us that two possible worlds can always be pasted together. There is still the question, though, of when worlds can be cut apart. For my own part, I accept the following two principles:

Coexistence $\Box\forall x\Box\forall y\Box(x \sim y \equiv \Box(\mathsf{Exists}(x) \equiv \mathsf{Exists}(y)))$
Separability $\Box\forall xx\Diamond(\mathsf{Exists}(xx) \wedge \forall y\exists x(x \prec xx \ \wedge x \sim y))$

Coexistence says that necessarily, if things are in the same universe, then they are necessarily coexistent. Thus, existence is a holistic matter. Entire universes stand or fall together. Given Coexistence, Separability tells us that necessarily, for any plurality of universes, those universes could have existed without any others.

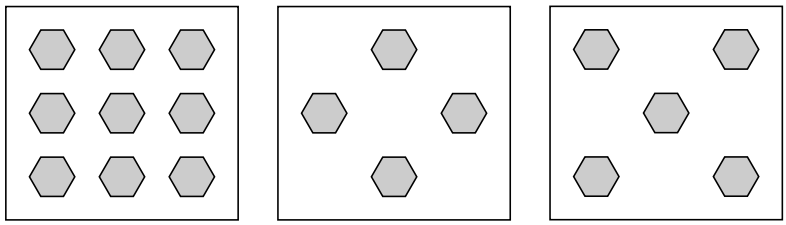
Note that given Coexistence, the immutabilist has the option of dropping connectedness as a basic predicate. This because she can define connectedness with:

$$x \sim y \equiv_{def} \Box(\mathsf{Exists}(x) \equiv \mathsf{Exists}(y)) \tag{11}$$

On this approach, *what it is* for two things to be in the same universe is for them to be necessarily coexistent. Defining connectedness in terms of coexistence has the further benefit of simplifying the immutabilist logic. If connectedness is taken as basic,

the immutabilist needs basic axioms ensuring that it is an equivalence relation. But if connectedness is defined in terms of coexistence, this can be derived, given that the background modal logic is normal.

The full system **CI** that I accept includes Immutability, Compossibility, Actuality, Coexistence, and Separability. It represents what you might think of as a *cut-and-paste* theory of modality. For example, suppose that we start with world w_1 below, with the hexes being universes. Coexistence and Separability then tells us that we can cut w_1



w_1 w_2 w_3

apart, so long as we are careful to cut around universes, and not through them. Thus, the possibility of w_1 entails the possibility of w_2 and w_3. On the other hand, suppose that we start with w_2 and w_3. Compossibility then tells us that we can paste these worlds together to get w_1. Immutability and Coexistence ensure that when worlds are cut apart or pasted together, universes remain the same intrinsically. Actuality ensures that there are no changes with respect to non-existing universes. Finally, the background modal logic ensures that the *modal* properties of universes remain the same after worlds are cut apart or pasted together. Thus, the only differences across worlds are with respect to which universes exist.

Compossibility says that two worlds can always be pasted together. Alternatively, though, you could accept something stronger. Namely:

Possible Pluriverse $\Diamond\exists xx\Box\forall x(x \prec xx)$

This says that there could have been a pluriverse, where a pluriverse is a plurality that includes every possible individual. Thus, where Compossibility says that two worlds can always be pasted together, Possible Pluriverse says that all the worlds *whatsoever* can be pasted together.

I am undecided about Possible Pluriverse, but am somewhat inclined to deny it. Denying Possible Pluriverse is equivalent to accepting:

Extensibility $\Box\forall xx\Diamond\exists x(x \not\prec xx)$

This says that necessarily, given any plurality of things, there could have been something that was not among them. Thus, the domain of possible individuals turns out to be indefinitely extensible.

Now that we have immutabilism fully on the table, it can be distinguished from

two other views in the literature.

Necessitarianism $\Box\forall x_1 \ldots \Box\forall x_n \Box(\phi(x_1, \ldots, x_n) \supset \Box(\phi(x_1, \ldots, x_n)))$
Necessitism $\Box\forall x \Box(\mathsf{Exists}(x))$

The first is necessitarianism. This says that necessarily, whenever a condition is satisfied, it is necessarily satisfied. There is thus no contingency whatsoever. The second is necessitism. This says that necessarily, everything necessarily exists. There is thus no contingency with respect to what exists, though there may be contingency with respect to how things are.

An immutabilist can accept that there is contingency with respect to what exists. Thus, an immutabilist need not be a necessitist. But necessitarianism is equivalent to the conjunction of necessitism and immutabilism. Thus, an immutabilist need not be a necessitarian either.

In fact, our own system **CI** is committed to a maximally strong form of contingentism. After all, Plural Comprehension entails Nothing. But in that case, given Separability, there could have been nothing. Given the rest of the background logic, we then have:

Strong Contingentism $\Box\forall x \Diamond(\neg\mathsf{Exists}(x))$

Necessarily, everything is contingent. Thus, there are no necessarily existing things, nor could there have been.

2.4 De Re Contingency

The most obvious objection to immutabilism is that it simply gets the modal facts wrong. The mug on my desk is blue, but could have been a different color. Socrates is a philosopher, but could have been a mathematician. Thus, immutabilism fails because it is simply false that things always have their properties and relations necessarily.

There are various responses an immutabilist could make. One would be to restrict her immutabilism to fundamental things. On this approach, the view is that necessarily, *fundamental* things are immutable with respect to *fundamental* properties and *fundamental* relations. Thus, the world is built on top of an immutabilist base. This leaves open the question of whether ordinary things, like coffee mugs, are immutable with respect to ordinary properties, like being blue.

You might object that this still gets the modal facts wrong. For example, suppose that particles are fundamental individuals and that spatial relations are fundamental relations. Immutabilism will thus entail that if particles stand in certain spatial relations, they necessarily stand in those spatial relations. But the very same particles could have stood in different spatial relations. Thus, immutabilism fails, even when restricted to

the fundamental.

We do, perhaps, ordinarily think that particles stand in spatial relations contingently. But there is no reason to think that common sense is reliable with respect to such matters. The history of science, after all, is the history of common sense failing as a guide to fundamental physical reality. Common sense would have us believe that every particle has both a determinate location and a determinate momentum. But in fact, we know that no particle can have both a determinate location and a determinate momentum, given Heisenberg uncertainty. Similarly, common sense would have us believe that events sometimes happen simultaneously. But if general relativity is to be believed, there is no such thing as events happening simultaneously. Thus, if particles are in fact immutable, this is just one more way in which common sense fails as a guide to fundamental physical reality.

More broadly: Fundamental theories should be judged on the basis of various theoretical virtues, like simplicity and explanatory power. Whether a fundamental theory conforms to common sense counts for little. But in that case immutabilism—as a theory of the fundamental—cannot be rejected on the basis that it conflicts with common sense. Its success will have to be judged holistically on the basis of its ability to secure various theoretical virtues.

A second kind of immutabilist response would be to distinguish between two kinds of modality. On the one hand, there is fundamental modality, which is what the immutabilist uses when doing physics. On the other hand, there is ordinary modality, which is the stuff of common sense.

The immutabilist, then, might claim that her immutabilism is restricted to fundamental modality. Thus, her view is that necessarily (in the fundamental sense), if individuals stand in a relation, it is necessary (in the fundamental sense) that they stand in that relation, assuming they exist. But this is compatible with the view that there are individuals that could (in the ordinary sense) have stood in different relations.

The challenge for the immutabilist, then, is to show that she can explain the ordinary modal facts, which are mutabilist, in terms of the fundamental modal facts, which are immutabilist. One way to do this would be to use counterpart theory. Giving a full immutabilist counterpart theory is somewhat involved, and so this project will be left for another time.[26] That said, the role that counterpart theory might play in explaining ordinary de re contingency will be illustrated in the next section.

26. See my (2021b).

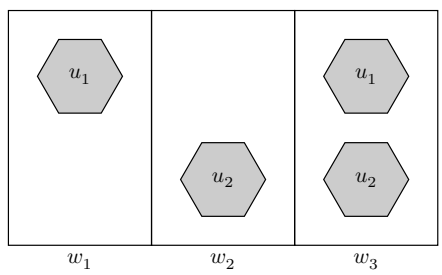
3 Immutabilist Cross-Modal Comparisons

We are now going to present a compossible immutabilist strategy for expressing cross-modal comparisons.

Suppose that as before, the actual world contains exactly one universe, and that this universe contains exactly one particle named Alice. What we want to show is that an immutabilist can express the claim that Alice could have been more massive than she actually is.

To do this, we are going to use an immutabilist language that has two non-logical predicates $\mathsf{Cpt}(x, y)$ and $\mathsf{Mass}(x, y)$. These are the same predicates that were used by the modal realist in §1.3.

Thinking in terms of worlds, here is the basic strategy. Suppose that the actual world is w_1, which is illustrated below. The only universe in this world is u_1 and the only particle in that universe is Alice. What the immutabilist would like to say is that Alice



could have been taller than she actually is because there is a world w_2 with a universe u_2 that contains a particle (call her Margaux) such that Margaux at w_2 is a counterpart of Alice at w_1 and Margaux at w_2 is more massive than Alice at w_1.

The problem, of course, is that in order to do this, an immutabilist needs relations across worlds. But how are such relations to be understood?

Our solution will be to define relations across worlds in terms of relations *within* worlds. In the present case, we claim that *what it is* for Margaux in w_2 to be more massive than Alice in w_1 is for there to be some w_3 such that Margaux in w_3 is more massive than Alice in w_3. Similarly, *what it is* for Margaux in w_2 to be a counterpart of Alice in w_1 is for there to be some w_3 such that Margaux in w_3 is a counterpart of Alice in w_3. Since the relevant relations within worlds can be expressed, the relevant relations across worlds can also be expressed.

Putting this into the official immutabilist language, suppose that we want to express (6). This, the immutabilist claims, can be done with:

$$\Diamond(\exists x(\mathsf{Cpt}(\mathsf{x}, \mathsf{a}) \wedge \mathsf{Mass}(\mathsf{x}, \mathsf{a}))) \tag{12}$$

This says that it could have been that there was a particle that was both a counterpart

of Alice and more massive than Alice.

Now hold on, you might say. This definition of the mass relation across w_1 and w_2 in terms of the mass relation within w_3 only works if Alice and Margaux each have the *same* mass across worlds. But they could have had different masses across worlds and, in that case, the proposed definition fails.

For example: Suppose that Alice in w_1 is two grams, Margaux in w_2 is one gram, Alice in w_3 is two grams, and Margaux in w_3 is three grams. In that case, Margaux in w_3 is more massive than Alice in w_3. But Margaux in w_2 is not more massive than Alice in w_1. Thus, the proposed definition fails.

The first observation is that our official immutabilist language has only a single comparative mass predicate. Thus, there is no way to say that Alice in w_1 is two grams, that Margaux in w_2 is one gram, and so on. Strictly speaking, then, there is no way to state the counterexample. But of course, this can be easily fixed: We can just add an appropriate family of basic monadic predicates. Thus, for all rational numbers r, our language will have a basic monadic predicate saying that x is exactly r grams.

Now that we can properly state the counterexample, the immutabilist has a ready response. Since being two grams is a basic predicate, it follows by Immutability that if Margaux is two grams in w_2, she is also two grams in w_3. But as a matter of necessity, nothing is both two grams and three grams. Thus, since Margaux is two grams in w_3, she is not three grams in w_3, as the counterexample requires. So the objection fails.

Alternatively, the objection could be raise from what you might think of as the external perspective. From the external perspective, we start with a language that can directly express relations across worlds.[27] We then consider whether relations within worlds are enough to fix the relevant relations across worlds.

The answer would seem to be no. For suppose that Margaux in w_3 is more massive than Alice in w_3. Still, there are two logical possibilities. One is that Margaux in w_2 is more massive than Alice in w_1. The other is that Alice in w_1 is more massive than Margaux in w_2. Thus, settling the mass relations within w_3 is not enough to settle the mass relations across w_1 and w_2.

The immutabilist, though, will insist that her proposal does settle the relevant mass relations across worlds. After all, her view is that *by definition* Margaux in w_2 is more massive than Alice in w_1 iff there is some world w_3 in which Margaux is more massive than Alice. So, given that Margaux is more massive than Alice in world w_3, that settles

27. This might be a two-sorted language with a sort for worlds and a sort for individuals. Predicate would then be indexed with an appropriate number of argument places for worlds. Thus, we would be able to write $\mathsf{Mass}(m, w_2, a, w_1)$ to say that Margaux at w_2 is more massive than Alice at w_1. For more on this sort of world-indexed language, see my Berntson (2019).

the question of whether Margaux in w_2 is more massive than Alice in w_1.

By way of analogy, suppose that you are a Platonist. You say that Margaux at w_2 is more massive than Alice at w_1 because Margaux has the property of being two grams at w_2 and Alice has the property of being one gram at w_1. Moreover, at every world, being two grams is greater than being one gram.

While most consider this sort of Platonist view to be fully adequate, we can raise the same sort of objections from the external perspective. For even if two grams is greater than one gram *within* every world, it does not follow—from the external perspective—that two grams at w_2 is greater than one gram at w_1. It could be that one gram at w_1 is greater than two grams at w_2. But in that case, it could be that Alice at w_1 is more massive than Margaux at w_2, even though Alice at w_1 is one gram and Margaux at w_2 is two grams.

The lesson, I think, is not that the Platonist cannot account for mass relations across worlds. The lesson is that objections posed from the external perspective are not legitimate. From the Platonist perspective, *what it is* for two grams at w_2 to be greater than one gram at w_1 is for two grams to be greater than one gram within every world. The fact that higher-order comparisons of masses within worlds do not fix higher-order comparisons of masses across worlds, from the external perspective, is neither here nor there. The same, though, goes for immutabilism.

Thus, if we are going to raise objections to the immutabilist proposal, they should be from the *internal* perspective. From the internal perspective, we grant that the immutabilist can fix relations across worlds in the way proposed. Still, we might object that the resulting relations across worlds are incoherent.

For example: Suppose there is a world w_3 in which Margaux is more massive than Alice. Suppose there is also a world w_4 in which Alice is more massive than Margaux. In that case, given the proposed reduction of mass relations across worlds, it follows that Alice in w_1 is more massive than Margaux in w_2, and that Margaux in w_2 is more massive than Alice in w_1. Thus, we have a failure of anti-symmetry.

This sort of counterexample, though, is ruled out by Immutability. For suppose that the *more massive than* is necessarily anti-symmetric. In that case, since Margaux is more massive than Alice in w_3, Alice is *not* more massive than Margaux in w_3. But then by Immutability and the background modal logic, Alice is not more massive than Margaux in w_4.

Another concern you might have is that while the immutabilist can express some cross-modal comparisons, she cannot express those in which the existence of a multiverse is explicitly denied. For example:

> Alice could have been more massive than she actually is *without having been in a multiverse*. (13)

An immutabilist, though, can express such claims by using an additional possibility

operator:

$$\Diamond\exists x(\forall y(x \sim y) \wedge \Diamond(\mathsf{Cpt}(x,s) \wedge \mathsf{Mass}(x,s))) \tag{14}$$

Thus, (13) is expressed by saying that there could have been someone, who was not in a multiverse, who could have been both a counterpart of Alice and more massive than Alice.

Finally, you might object to our immutabilist theory of cross-modal comparisons on the grounds that counterpart relations across worlds cannot be reduced to counterpart relations within worlds. Maybe Margaux in w_3 can be a counterpart of Alice in w_3 without Margaux in w_2 being a counterpart of Alice in w_1.

For present purposes, we are going to set these concerns aside. There are three reasons for this. First, in the present context, we can use a liberal counterpart relation such that every particle in every world is a counterpart of every particle in every world. Maybe there are other counterpart relations that are useful for various purposes, but they are not needed for *present* purposes.

Second, we will give an immutabilist theory of distance ratios in §5.2 without using a counterpart relation. Thus, while counterpart theory may be useful for reconciling immutabilism with ordinary language and common sense, it is not needed for doing science without numbers.

Third, as mentioned at the end of the last section, giving a fully general immutabilist counterpart theory is somewhat involve. Thus, this is a project that we will pursue elsewhere.[28]

4 Fictionalism

We described compossible immutabilism in §2. This lets the nominalist solve the problem of cross-modal comparisons, which was posed in §1.3. We are now going to move towards solving the problem of quantities in §5.

Doing that will be easier if we have an additional tool. That tool is **pluriverse fictionalism**. According to the pluriverse fiction, there is vast plurality of concrete universes that includes every possible universe. Because we are interested in nominalism, our pluriverse fiction will be a *nominalist* pluriverse fiction. Thus, according to the fiction, there are no numbers or universals or other things of that sort.

What we are going to show in the appendix is that fictionalist talk about the pluriverse is structurally equivalent to immutabilist talk in terms of modal operators.

28. Namely, in my (2021b).

Thus, any fictionalist theory T_F can be directly translated into an immutabilist theory T_I with the same logical structure.

This suggests a natural approach to building immutabilist scientific theories. First, we build a physical theory T_F in the pluriverse fiction. To show that this theory is empirically adequate, we expand the fiction to include set theory. This gives us a combined fiction in which we can prove the necessary representation and conservativity theorems. This establishes that T_F is empirically adequate. We then translate the fictionalist theory T_F into the immutabilist theory T_I. T_I is structurally equivalent to T_F. Structural equivalence preserves empirical adequacy. Thus, since T_F is empirically adequate, so is T_I.

The advantage of this procedure is that as an immutabilist, you can do science within the pluriverse fiction. You can also build immutabilist theory directly for my own part, I find it easier to work in the pluriverse fiction, and then translate the results. Thus, in the rest of this section, we are going to build a fictionalist language and logic that an immutabilist can use to build scientific theories.

4.1 Language

We are going to think of the fictionalist as having a plural language $\mathcal{L}_F$. The syntax is similar to that of the immutabilist language from §2.2.

Singular variables	$x, y, z, \ldots$
Plural variables	$xx, yy, zz, \ldots$
Singular quantifiers	$\exists x, \forall x$
Plural quantifiers	$\exists xx, \forall xx$
Truth-functional operators	$\wedge, \vee, \supset, \neg$
Logical predicates	$=_s, =_p, \prec, \sim$

There are also important differences. Unlike the immutabilist language, the fictionalist language does not have modal operators. Instead, it has a singular actuality predicate, which can be used to define a plural actuality predicate.[29]

$\mathsf{Act}(x)$	x is actual
$\mathsf{Act}(xx)$	the xx are actual

Thus, the fictionalist draws a basic distinction between actual and non-actual individuals. The immutabilist, on the other hand, has no need for such distinctions, since her

29. $\mathsf{Act}(xx) \equiv_{def} \forall x(x \prec xx \supset \mathsf{Act}(x))$

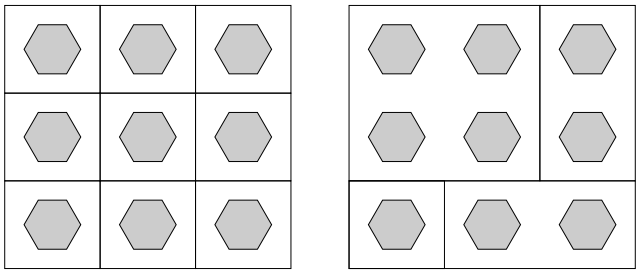
quantifiers only range over actual individuals.

4.2 Pluralism about Worlds

We have a fictionalist language then. That language has the resources for talking about a vast pluriverse of island universes. The question is: What are possible worlds?

The most common view is **singularism**, which says that only individual universes are world. This is the view endorsed by Lewis (2001). The competing view is **pluralism**. The pluralist says that a world need not be a single universe. Rather, a world is any *plurality* of universes.

In visual terms, consider the diagram below. The singularist says that the pluriverse is as illustrated on the left. The hexes are universes and the boxes are worlds. The



pluralist says that the pluriverse is as illustrated on the right. For the pluralist, any plurality of universes *whatsoever* counts as a world. Since worlds can overlap, this is hard to illustrate. So we have included only a selection of non-overlapping worlds. Some have only one universe. Some have many.

Following Lewis, fictionalists and modal realists have generally been singularists. But singularism faces two serious problems. The first is that there is no world in which there is a multiverse. The second is that there is no world in which there is nothing. But there *could* have been a multiverse and there *could* have been nothing. So singularism would seem to be materially inadequate.[30]

Pluralism solves both problems. There can be pluralities of universes that contain many universes. Hence, there can be worlds in which there is a multiverse. Given Nothing, there is an empty plurality. The empty plurality is trivially closed under connectedness. So there is a world in which there is nothing.

Our fictionalism will adopt pluralism about worlds. This means that it will be useful

30. Lewis raises both objections himself on pp. 72-3 of his (2001). For further discussion, and other possible solutions, see Yablo (1999), Sider (2003), and Parsons (2007).

to have a defined *plural* world predicate.[31]

$\mathsf{World}(xx)$ the xx are a possible world

Thus, when the fictionalist says that there is a possible world, what she really means is that there are things that together form a possible world.

4.3 Principles

We have both a fictionalist language and a view about possible worlds. What we want to do now is find principles that guarantee that fictionalist talk can always be translated as immutabilist talk, and visa-versa.

More precisely: We want to identify a minimal immutabilist system **I** and a minimal fictionalist system **F** such that any sentence in the immutabilist language $\mathcal{L}_I$ can be translated as a sentence of the fictionalist language $\mathcal{L}_F$, and visa-versa, with these translations preserving logical entailment. Two such systems are fully specified in the appendix. It will be useful, though, to say bit about how these systems work in general terms.

The immutabilist system **I** is strictly weaker than the system **CI** we built earlier. As compared to that system, there are two important differences: One is that Separability and Compossibility are dropped. The other is that Plural Comprehension is replaced with Everything. Thus, as a plural logic, **I** is quite weak. The only plurality whose existence it guarantees is the universal plurality.

The matching fictionalist system **F** has classical rules for the quantifiers and the truth functional connectives. It also has axioms guaranteeing the connectedness is an equivalence relation. Other principles include:

Actual World	$\exists xx \forall y(y \prec xx \equiv \mathsf{Act}(y))$
Actual Closure	$\forall x \forall y(\mathsf{Act}(x) \wedge x \sim y \supset \mathsf{Act}(y))$
Actual Names	$\mathsf{Act}(t)$ when t is a name

The first says that there is a plurality of everything actual. The second says that if something in a universe is actual, then everything in that universe is actual. The third says that names are only assigned to individuals and pluralities that are actual.

Importantly, **F** includes a basic principle governing how relations interact with worlds. This is easiest to state if we have a defined subplurality predicate.[32]

$xx \subset yy$ the xx are a subplurality of the yy

31. $\mathsf{World}(xx) \equiv_{def} \forall x(x \prec xx \supset \forall y(x \sim y \supset y \prec xx)$
32. $xx \subset yy \equiv_{def} \forall z(z \prec xx \supset z \prec yy)$

When thinking in terms of individuals, the principle says that whenever things stand in basic relations, there is some possible world in which they all exist.

Relation World $\forall x_1 \ldots \forall x_n (R(x_1, \ldots, x_n) \supset \exists yy(\mathsf{World}(yy) \wedge (x_1 \prec yy \wedge \cdots \wedge x_n \prec yy))$

Predicates with plural arguments are allowed as substitution instances. When substituting such predicates, the corresponding variables and quantifiers are replaced with plural variables and plural quantifiers. The corresponding $\prec$ predicates are replaced with $\subset$ predicates.

Two further principles follow from Relation World. Individuals and pluralities always stand in identity relations to themselves. Thus, Relation World gives us :

Possible Individuals $\forall x \exists yy(\mathsf{World}(yy) \wedge x \prec yy)$
Possible Pluralities $\forall xx \exists yy(\mathsf{World}(yy) \wedge xx \subset yy))$

The first says that every possible individual is in a world. The second says that every possible plurality is in a world.

This gives us the systems we wanted. Any fictionalist theory in **F** can be translated as an immutabilist theory in **I**, and visa-versa. The translations are provided in the appendix. The basic idea, though, is that fictional quantification over the pluriverse is translated as modalized quantification.

$$\exists x(\phi) \mapsto \Diamond \exists x(\phi)$$
$$\forall x(\phi) \mapsto \Box \forall x(\phi)$$

Going the other way, modal operators are translated using plural quantifiers.

$$\Diamond(\phi) \mapsto \exists xx(\mathsf{World}(xx) \wedge \phi)$$
$$\Box(\phi) \mapsto \forall xx(\mathsf{World}(xx) \supset \phi)$$

Quantifiers are then translated as restricted quantifiers.

Once we have our translations, there are certain striking connections between the principle we might accept on the fictionalist side and the principles we might accept on the immutabilist side. For example, suppose we accept full Plural Comprehension on the fictionalist side. When translated, this corresponds to accepting not just Plural Comprehension, but also Possible Pluriverse and Separability on the immutabilist side. Going the other way, suppose we accept Possible Pluriverse on the immutabilist side. This corresponds to accepting Everything on the fictionalist side.

Finally, it should be pointed that the variety of modal fictionalism that I support is rather different than the modal fictionalism suggested by Rosen (1990). Rosen aims to reduce modal facts to categorical facts about what is true in the fiction. It could have been that ϕ because, according to the fiction, there is a possible world at which ϕ. My

own view is exactly the reverse: What is true in the fiction is true in the fiction because it appropriately represents the modal facts.[33] Thus, I think of the pluriverse fiction as a useful tool for reasoning about the modal facts, not a strategy for reducing or otherwise eliminating them.

5 Distance Ratios

We are now going to give a compossible immutabilist theory of distance ratios. The general strategy generalizes to other quantities, and so solves the problem of quantities raised in §1.

Our basic approach will be the one suggested at the beginning of §4. That is, we will start by sketching a fictionalist theory of distance ratios in §5.1. We will then translate this fictionalist theory as an immutabilist theory in §5.2.

5.1 Fictionalist Distance Ratios

Our fictionalist theory will use the fictionalist language from §4.1, extended to include betweenness and congruence predicates.[34] There are different ways to go here, but we will think of the fictionalist as accepting a pluriverse of all *physically* possible universes, rather than a pluriverse of all metaphysically possible universes.

The general strategy will be to give a theory of distance ratios using congruence relations across universes. But this means that our fictionalist will be departing from Lewis's modal realism in yet another important respect.

Lewis says two things about spatiotemporal relations and universes. First, he says that things are in the same universe when there is *some distance* between them. Second, he says that things are in the same universe when there are *spatiotemporal relations* between them.[35] The second condition is problematic because it rules out congruence relations across universes.

Lewis treats his two conditions as equivalent. But in fact, the second is much stronger than the first. Our strategy, then, will be to accept the first while rejecting the second. Thus, two things are in the same universe when there is some distance between them. But two things can stand in spatiotemporal relations without being in

33. Thus, the brand of fictionalism I support has more in common with the non-proxy reduction strategy from Fine (2005) or the ersatz pluriverse from Sider (2002).

34. In fact, to give a full theory of distance ratios, we need some sort of device for saying *there are finitely many*. My preferred approach is to use a monadic plural predicate $\mathsf{Fin}(xx)$ saying that the xx are finite. This predicate is then treated as logical. For present purposes, this complication can be ignored.

35. Lewis (2001, p.2).

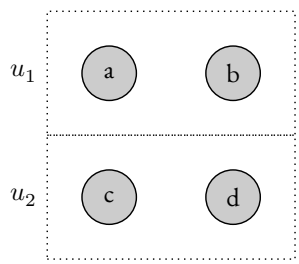
the same universe. This will allow for congruence relations across universes.

Filling in the details, say that a and b are **self-congruent** when a and b are congruent with a and b. We then claim that:

$$\forall a \forall b (a \sim b \equiv \mathsf{Cong}(a, b, a, b)) \tag{15}$$

There is some distance between two things if and only if they are self-congruent. So (15) can be read as saying that two things are in the same universe if and only if there is some distance between them.

This allows for congruence relations across universes. For example, consider the two universes illustrated below. Particles a and b are in the same universe because they are self-congruent. Likewise for c and d. Particles a and b are also congruent with c and d. We can thus use congruence to compare two particles in the first universe with two particles in the second. What we cannot do is use congruence to relate two particles



in *different* universes with any other two particles. For example, a and c cannot be congruent with b and d, nor can a and c be congruent with c and d.[36] Thus, we are allowing certain kinds of congruence relations across worlds, but not others.

Next, we want to rule out betweenness relations across universes. These are not needed and, moreover, would be problematic.[37]

$$\forall a \forall b \forall c (\mathsf{Bet}(a, b, c) \supset \mathsf{Cong}(a, b, a, b) \wedge \mathsf{Cong}(b, c, b, c)) \tag{16}$$

This says that b is between a and c only if there is some distance between a and b and some distance between b and c. Given (15), this entails that if b is between a and c, then

36. Suppose for reduction that a and b are in different universes, but that a and b are congruent with some x and y. By Cong-Symmetry and Cong-Transitivity (which are given below), a and b are then congruent with a and b. But then by (15), a and b are in the same universe, which is contrary to assumption.

37. This because they would leave the fictionalist vulnerable to shift arguments. For example, suppose that u_1 has exactly two particles a and b and that these particles are one foot apart. If we have cross-universe betweenness relations, we can define cross-universe colocation relations. We can thus describe one universe u_2 that contains exactly one particle c and another universe u_3 that contains exactly one particle d, with c colocated with a and d colocated with b. Thus, u_2 is just like u_3, except that the location of everything is shifted by a foot.

all three particles are in the same universe. There are thus no betweenness relations across universes.

Putting these things together, the basic picture is one on which there is a pluriverse in which universes are pluralities of spatiotemporally connected particles. For the most part, there are no basic relations across universes. The only exceptions are certain kinds of congruence relations.

When giving a substantivalist theory of distance ratios, there are two kinds of axioms. There are **existence axioms**, which entail that certain spacetime points exist. There are also **restriction axioms**, which restrict how spacetime points can be arranged.

Our fictionalist theory will also have both existence and restriction axioms. The difference is that where a substantivalist uses existence axioms to fill spacetime with points, a fictionalist uses existence axioms to fill the pluriverse with universes.

We are not going to give a full slate of axioms here.[38] What we are going to do, though, is show that a fictionalist can solve the uniqueness problem. And for that, we need three restriction axioms:

Cong-Symmetry:	$\forall a \forall b \forall c \forall d(\mathsf{Cong}(a,b,c,d) \supset \mathsf{Cong}(c,d,a,b)$
Cong-Transitivity:	$\forall a \forall b \forall c \forall d \forall e \forall f(\mathsf{Cong}(a,b,c,d) \wedge \mathsf{Cong}(c,d,e,f)$ $\supset \mathsf{Cong}(a,b,e,f))$
Three-Segment:	$\forall a \forall b \forall c \forall d \forall e \forall f(\mathsf{Cong}(a,b,d,e) \wedge \mathsf{Bet}(a,b,c) \wedge$ $\mathsf{Bet}(d,e,f) \supset (\mathsf{Cong}(a,c,d,f) \equiv \mathsf{Cong}(b,c,e,f)))$

The first says that congruence is symmetric. The second says that congruence is transitive. The third is a simplified version of what is sometimes called the five-segment axiom.

As an illustration of how existence axiom work, suppose that the substantivalist has an axiom saying that for any two spacetime points, there is a third between them.

$$\forall a \forall b \exists x(\mathsf{Bet}(a,x,b)) \tag{17}$$

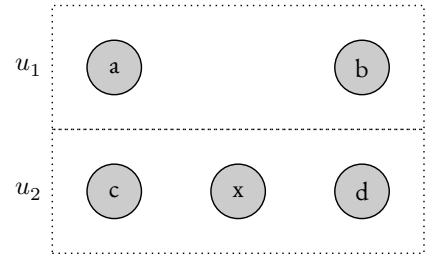
For a fictionalist, the corresponding axiom says that for any pair of particles, there is a pair of duplicates, somewhere in the pluriverse, that have a third between them.

$$\forall a \forall b \exists c \exists d \exists x(\mathsf{Cong}(a,b,c,d) \wedge \mathsf{Bet}(c,x,d)) \tag{18}$$

Now suppose that there is a universe u_1 with exactly two particles, as illustrated on the next page. Since the duplicates required by (18) do not exist in u_1, the result will be a second universe u_2 in which they do exist. Thus, existence axioms, like this one, are used to fill out the pluriverse.

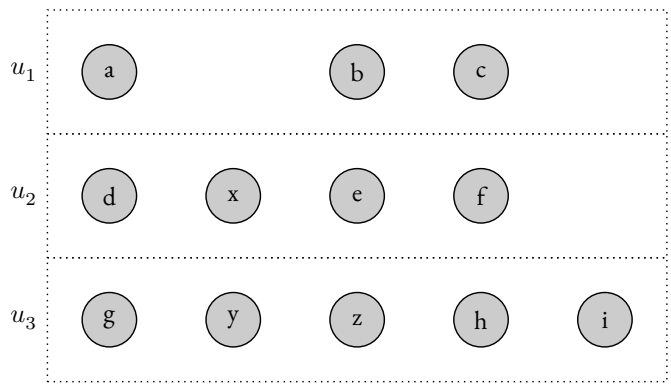
Now that we have axioms, we want to show that the fictionalist can solve the

38. This is done in my Berntson (2021a).



problem of uniqueness. To simplify a bit, we can suppose that we only need to show that if a and b are twice as far apart as b and c, then they are not also three times as far apart.

Consider a pluriverse with exactly three universes, as pictured below. Our universe is u_1 and has exactly three particles. We can suppose that a and b are twice as far apart as b and c, and so this is the distance ratio that we want to explain. The fictionalist says



that this distance ratio is explained by the existence of a universe like u_2. That is, a and b are twice as far apart as b and c in u_1 because there is a universe u_2 in which some d, x, e, and f form an equally spaced line. Moreover, a and b are congruent with d and e, and b and c are congruent with e and f.

Now suppose for reductio that a and b are also three times as far apart as b and c. For the fictionalist, this means that there is a universe u_3 with particles g, y, z, h, and i forming an equally spaced line. Moreover, a and b are congruent with g and h, and b and c are congruent with h and i.

We can now prove a contradiction. First, we use symmetry and transitivity to show that d, x, e, and f are pairwise congruent with y, z, h, and i. By two applications of the three-segment axiom, we then have $\mathsf{Cong}(d, f, y, i)$. Since $\mathsf{Cong}(d, f, g, i)$ by symmetry and transitivity, another application of the three-segment axiom gives us $\mathsf{Cong}(d, d, g, y)$. So g and y are colocated. But g and y are part of an equally spaced line, so are *not* colocated. Thus, we have a contradiction. So the very same particles in the very same world cannot be both twice as far apart and three times as far apart.

5.2 Immutabilist Distance Ratios

We are now going to translate the fictionalist theory from the last section into an immutabilist theory of distance ratios. That that theory will be given using an immutabilist language with betweenness and congruence predicates. The translation procedure is given in the appendix, but was also briefly described at the end of §4.3.

To translate our fictionalist theory, we will start with the restriction axioms. The fictionalist claims, for example, that congruence is symmetric. Applying the immutabilist translation scheme gives us:

$$\Box\forall a\Box\forall b\Box\forall c\Box\forall d(\mathsf{Cong}(a,b,c,d) \supset \mathsf{Cong}(c,d,a,b)) \tag{19}$$

Given Actuality, this is equivalent to:

$$\Box(\forall a\forall b\forall c\forall d(\mathsf{Cong}(a,b,c,d) \supset \mathsf{Cong}(c,d,a,b)) \tag{20}$$

Thus, where the fictionalist says that congruence is symmetric, the immutabilist says that congruence is *necessarily* symmetric. Likewise for other restriction axioms.

Now for the existence axioms. Suppose that the fictionalist accepts an existence axiom saying that, for any pair of particles, there is a duplicate pair of particles with a third somewhere between them. Our translation scheme gives:

$$\Box\forall a\Box\forall b\Diamond\exists c\Diamond\exists d\Diamond\exists x(\mathsf{Cong}(a,b,c,d) \wedge \mathsf{Bet}(c,x,d)) \tag{21}$$

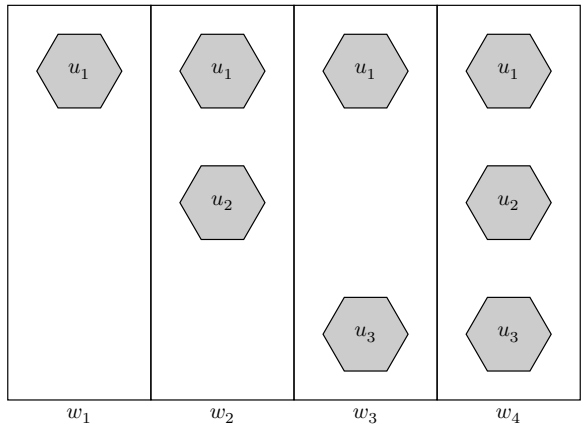
By Actuality and Immutability, this is equivalent to:

$$\Box\forall a\forall b\Diamond\exists c\exists d\exists x(\mathsf{Cong}(a,b,c,d) \wedge \mathsf{Bet}(c,x,d)) \tag{22}$$

Thus, where the fictionalist says that for any pair of particles, *there is* a duplicate pair with a third somewhere between them, the immutabilist says that necessarily, for any pair of particles, there *could have been* a duplicate pair with a third somewhere between them, somewhere in the multiverse. Thus, where the substantivalist uses existence axioms to fill out spacetime, and the fictionalist uses them to fill out the pluriverse, the immutabilist uses them to fill out how things could have been.

The immutabilist, like the fictionalist, can now solve the problem of uniqueness. As before, we will illustrate the basic strategy by showing that the very same particles in the very same world cannot be both twice as far apart and three times as far apart.

Suppose that we have three universes u_1, u_2, and u_3. The betweenness and congruence relations within those universes are as illustrated on the previous page. The actual world is w_1 and has exactly one universe, which is u_1. This is illustrated on the following page. Now suppose that a and b are twice as far apart as b and c in w_1. For the immutabilist, this is explained by the fact that there is a possible world w_2 that contains



both u_1 and u_2. The particles in these universes stand in the cross-universe congruence relations illustrated on page 32.

So far so good. Now suppose for reductio that besides being twice as far apart, a and b are also three times as far apart as b and c at w_1. For the immutabilist, this means that there is a possible world w_3 that contains u_1 and u_3. The congruence relations between those universes at w_3 are not, however, as illustrated on page 32. Rather, a and b are congruent with g and h, and b and c are congruent with h and i.

We can now derive a contradiction. First, we note that since w_2 and w_3 are both possible, there is a fourth possible world w_4 in which u_1, u_2, and u_3 all exist by Compossibility. Immutability entails that the congruence and betweenness relations from w_2 and w_3 carry over to w_4. But given our translated restriction axioms, w_4 is in fact impossible, since we can prove a contradiction the same way we did for the fictionalist. Thus, the very same particles in the very same world cannot be both twice as far apart and three times as far apart.

This gives the nominalist a solution to the problem of uniqueness. With a bit more work, she can give a full slate of axioms and full theory of distance ratios.[39] Since the basic strategy generalizes to other quantities, we also have a solution to the problem of quantities. We also have a new approach to doing science without numbers.

6 State Space

We saw how to use immutabilism to give a theory of distance ratios in §5. One of the main advantages of the immutabilist approach, as compared to the existing substantivalist approach from Field (1980), is that it gives the nominalist a natural

39. See my Berntson (2021a).

strategy for understanding state space. Thus, in this section, we are going to briefly sketch the problem, and show what an immutabilist theory of state space might look like, in very general terms. Most of the interesting details will have to be left for another time.

In his review of *Science Without Numbers*, David Malament (1982) raises what has become a serious challenge to nominalism.

The challenge goes like this: In classical physics, the dynamical laws are given using mathematical Euclidean spacetime.[40] This mathematical spacetime is just a mathematical object with certain formal features. Nominalists, of course, deny that there are such things. But in that case, how is a nominalist going to state dynamical laws?

Field's solution is to trade mathematical spacetime for *concrete* spacetime. This concrete spacetime is built out of spacetime points, thought of as concrete material particulars. Thus, Field's solution is to be a substantivalist.

The problem, Malament points out, is that modern physics is often formulated in terms of not just mathematical spacetime, but also mathematical **state spaces**. A nominalist cannot accept mathematical state spaces, and so needs to find a concrete replacement. But what is that concrete replacement going to be? Call this **Malament's challenge**.

To illustrate the basic problem, Malament uses the case of Hamiltonian mechanics. His discussion is worth quoting at length:

> [I]t is simplest to identify Hamiltonian mechanics by its determination of a class of mathematical models. Each model is of form $\langle M, \Omega_{ab}, H\rangle$ where M is an even-dimensional manifold, Ω_{ab} is a symplectic form on M, and H is a smooth, real-valued ("Hamiltonian") scalar field on M. The points of M represent "possible dynamical states" of a given mechanical system. (Ω_{ab} and H jointly determine a "Hamiltonian vector field" which characterizes the dynamic evolution of the system.) Now Field can certainly try to trade Ω_{ab} and H in favor of "qualitative relations" they induce on M. If successful, he can reformulate the theory so that its subject matter is the set of "possible dynamical states" (of particular physical systems) and various relations into which they enter. But this is no victory at all! Even a generous nominalist like Field cannot feel entitled to quantify over *possible dynamical states*.

Hamiltonian mechanics can be thought of as a class of mathematical state spaces. Each mathematical state space is built using mathematical states, which are just points

40. Or Galilean spacetime or Maxwellean spacetime.

in a mathematical structure. These points are characterized using things like sets, functions, and real numbers.

This means that in order to give a corresponding nominalist theory, we need to do two things. First, we need to replace the sets, functions, and real numbers used to characterize mathematical states with intrinsic relations between them. Second, she needs to replace mathematical states with something concrete.

If we were Platonists, we could replace mathematical states with universals. We could, for example, replace mathematical states with **state relations**. These are binary relations between universes and times. Each such relation is a maximally specific way for a universe to be at a time. A universe then evolves over time because it stands in different state relations to different times. But nothing like this strategy is available to the nominalist.

One approach to the problem would be to look for empirically equivalent theories that use something like a concrete four-dimensional spacetime in place of mathematical state space. Perhaps this can be done, and the project deserves more attention.[41] But while this may work for some theories, it may not work in others. Thus, the nominalist has reason to want a more direct—and general—approach to the metaphysics of state space.

This is not the place to show that we can nominalize any state space theory that might come along. Nor is it the place to show, in any detail, that we can nominalize even a relatively simple state space theory like Hamiltonian mechanics. What we want to do, though, is motivate the idea that a immutabilist could, with some time and effort, give a nominalist theory of Hamiltonian mechanics.

The basic strategy is the same as before. First, we give a theory of Hamiltonian mechanics within the pluriverse fiction. Next, we use the combined fiction to prove empirical adequacy. Once we have that, we can directly translate the original fictionalist theory into an immutabilist theory. That translation procedure preserves empirical adequacy. So the resulting immutabilist theory is also empirically adequate.

To start, then, we will have a fictional pluriverse populated with **slices**. These are what you might think of as time-slices of universes. Slices are much larger than ordinary concrete particulars like cars, trees, and coffee mugs. Their existence is also much briefer. Still, neither of these are serious grounds for denying that slices are concrete particulars. Thus, the nominalist can use them to build a fictional pluriverse.

The character of each slice is fixed by what we will call **structural relations**. So for example, we might fix the number of particles in each slice by using an *at least as many particles* relation between slices. Slices with zero particles are slices such that there are

41. See for example Chen (2017).

no slices with strictly fewer particles. Slices with exactly one particle are slices such that the only slices with strictly fewer particles are slices with zero particles. And so on.

Suppose that after some time, we have selected enough structural relations to fix the positions of particles, along with their momentum.[42] We then need to add temporal relations to our fictional pluriverse. For these, we could use temporal betweenness and temporal congruence.

$\mathsf{TempBet}(\mathsf{a}, \mathsf{b}, \mathsf{c})$
$\mathsf{TempCong}(\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{d})$

The first says that b is temporally between a and c. The second is temporal congruence. It says that the duration between a and b is the same as the duration between c and d. These can be used to fix duration ratios between slices in much the same way that spatial betweenness and congruence can be used to fix distance ratios between particles.

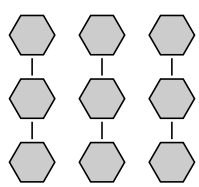
Once we have both structural relations and temporal relations, we need universes. That is, we need to say which slices are connected. One answer is that they are connected when they are temporally self-congruent.

$$a \sim b \equiv \mathsf{TempCong}(a, b, a, b) \tag{23}$$

Or perhaps more intuitively, two slices are in the same universe when there is some temporal duration between them.

Once we have our basic relations between states, we need laws governing them. There will be laws governing the structural relations. There will also be laws governing the temporal relations. Importantly, there will also be dynamical laws telling us how temporal relations are determined, given the structural relations. Thus, the dynamical laws will tell us which slices are connected, and so which slices form universes.

This gives us a pluriverse like the one illustrated below. Hexes represent slices. There are lines between slices that are temporally connected. This gives us three strands of



slices. Each strand is a universe. The dynamical laws determine which slices stand in temporal relations and, so, which slices form universes. Slices that are not connected

42. To see one way in which this might be done, see Schroeren (2020).

with lines do not form universes. Thus, take the three slices on either diagonal. These slices are not temporally connected, and so do not form a universe

7 Objections

The success of our new strategy for doing science without numbers depends on the possibility of a multiverse. This is a substantial claim about the physical world. Thus, you might object on the grounds that a multiverse is physically impossible.

In response: I prefer immutabilist theories formulated in terms of physical possibility. But the immutabilist strategy itself is quite general, and compatible with other views. Thus, if you think that a multiverse is physically impossible, you could use a broader notion of possibility (like metaphysical possibility) instead.

That said, I maintain that a multiverse is physically possible. By way of analogy: Is it physically possible for the stars to have spelled out a Shakespearean sonnet? Surely yes. But why think so? The reason, it seems to me, is that such possibilities are consistent with the laws, and we have no other reason to reject them.

The same goes for the physical possibility of various multiverses. The existence of a multiverse is consistent with the laws. Moreover, we have no reason to reject such possibilities. In fact, quite the opposite: Accepting the possibility of various multiverses gives us a powerful tool for doing science without numbers.

This brings us to a second objection. Immutabilism describes the world using modal operators. Thus, immutabilist theories would seem to be committed to fundamental modality. But modality is *not* fundamental. Fundamental reality is not filled with threats and promises, or so you might say.

In response: We started by distinguishing scientific nominalism from metaphysical nominalism. We then set out to defend *scientific* nominalism. We did this by building theories using a language with modal operators, which were not defined in other terms. That much is true.

But all of this is compatible with a wide range of views about fundamental reality. You could deny scientific fundamentalism, and so hold that fundamental reality is not characterized by any of the best scientific theories. Or you could hold that it is equally well characterized by several scientific theories, some of which have basic modality, and some of which do not. On this sort of view, fundamental reality is simply *ambivalent* about modality. You could think that modal facts are reducible to other facts, like those involving the notion of essence. Or you could deny that there is any such thing as fundamental reality altogether.

For my own part, I accept fundamental modality. This is in part because I am an anti-Humean. The fundamental laws, I say, are fundamental physical necessities. But in that case, I am *already* committed to fundamental modal notions. Thus, using those

modal notions to also do science without numbers is no additional cost.

Fully defending the use of modal notions in fundamental theorizing would take us beyond the scope of this paper. But suffice it so say, the notions of physical possibility and necessity strike me as being in perfectly good working order. We have precise formal systems for reasoning about them. And we have robust scientific practice for determining what is physically possible and what is physically necessary. Thus, I can see no reason to avoid such notions in fundamental theorizing.

8 Conclusion

We started in §1 by describing scientific nominalism and one of the main challenges it faces, which is the problem of quantities. One natural solution to the problem appeals to modality. That solution, though, would seem to require cross-modal comparisons. But there is no obvious strategy for expressing such comparisons as a nominalist.

Our response was to introduce a new view about modality called compossible immutabilism in §2. We then showed that a nominalist can use compossible immutabilism to express cross-modal comparisons in §3.

This gave us a new strategy for doing science without numbers. The basic strategy is to use the fictionalist language from §4 to build scientific theories, which can then be translated into compossible immutabilist theories that make extensive use of cross-modal comparisons. To show how the strategy works—and to motivate the idea that can solve the problem of quantities—we showed how to give a theory of Euclidean distance ratios in §5. We also quickly sketched an immutabilist theory of state space in §6

Now of course, there is still much work to be done. What we would really like is not just a full immutabilist theory of classical mechanics, but a full immutabilist theory of general relativity, along with a full immutabilist theory of quantum mechanics. Those hard projects will obviously need to be left for another day. But for now, there is reason for optimism. My own conviction is that with enough time and patience, those hard projects can be completed.

Appendix: Models, Translations, and Structural Equivalence

In this appendix, we are going to give a model-theoretic specification of the systems **CI**, **I**, and **F** from the main text. We will then provide recursive translation from the sentence of $\mathcal{L}_I$ to the sentences of $\mathcal{L}_F$, and visa-versa, that preserve logical entailment, relative to the system **I** and **F**. Thus, given any immutabilist theory T_I in system **I**, there is a structurally equivalent fictionalist theory T_F in **F**, and visa-versa.

Definition 8.1: A **frame** is a tuple $\langle W, @, D, P\rangle$. This includes a set of worlds $W \subset P$, an actual world $@ \in W$, a singular domain D, and a plural domain $P \subset \mathcal{P}(D)$. Moreover, every frame also meets the following conditions:

For every $x \in D$, there is some $w \in W$ such that $x \in w$.
For every $x \in P$, there is some $w \in W$ such that $x \subseteq w$.

Each world is the set of individuals that exist at that world. Thus, each world is identified with its singular domain. Each world is also a set in the plural domain. Thus, each world is also a plurality. The two indented conditions tell us that every individual is in a world, and so every individual is a possible individual. Likewise, every plurality is in a world. Thus, every plurality is a possible plurality. Finally, given any frame, we can define a plural domain function $p : w \mapsto \mathcal{P}(w) \cap P$. This tells us which pluralities exist at which worlds.

As we go on, it will be helpful to establish a notational convention. Suppose that we have a world w and a predicate R. Predicates have sorted argument places. Worlds also have both a singular domain and a plural domain. Thus, it will be helpful to have notation pairing argument places with domains. For this, we will use δ_i^w, where $\delta_i^w = w$ when the i-th argument place of R is singular, and $\delta_i^w = p(w)$ when the i-th place of R is plural. When the superscript is dropped, δ_i will refer to the appropriate outer domain for each argument place. Thus, $\delta_i = W$ when the i-th argument place of R is singular, and $\delta_i = P$ when the i-th argument place is plural.

Definition 8.2: A **valuation** is a function $[\![\cdot]\!]$ assigning denotations to names and predicates. This is done in the following way:

$$\begin{array}{lcl} [\![c]\!] & \in & @ \\ [\![cc]\!] & \in & p(@) \\ [\![R]\!] & \subseteq & \delta_1 \times \cdots \times \delta_n \text{ when } R \text{ is a non-logical predicate} \end{array}$$

Definition 8.3: An **immutabilist model** $\mathcal{M}_I = \langle W, @, D, P, [\![\cdot]\!]^I\rangle$ is a frame together with a valuation function for the names and predicates in $\mathcal{L}_I$.

Definition 8.4: A **fictionalist model** $\mathcal{M}_F = \langle W, @, D, P, [\![\cdot]\!]^F\rangle$ is a frame together with a valuation function for the names and predicates in $\mathcal{L}_F$.

To simplify the semantic clauses below, we can assume that the only basic truth-

functional operators are $\wedge$ and $\neg$. The only basic quantifiers are $\forall x$ and $\forall xx$. For the immutabilist language, the only basic modal operator is $\Box$. The other operators and quantifiers are then defined in the usual way.

Definition 8.5: Let $\mathcal{M}_I$ be an immutabilist model. When $w \vDash_\sigma \phi$, we say that ϕ is true at world w relative to variable assignment σ. This relation is defined recursively:

$w \vDash_\sigma R(t_1, \dots, t_n)$	iff	$\langle \sigma(t_1), \dots, \sigma(t_n) \rangle \in [\![R]\!] \cap \delta_1^w \times \cdots \times \delta_n^w$
$w \vDash_\sigma t_1 \prec t_2$	iff	$\sigma(t_1) \in \sigma(t_2)$
$w \vDash_\sigma t_1 =_s t_2$	iff	$\sigma(t_1) = \sigma(t_2)$
$w \vDash_\sigma t_1 =_p t_2$	iff	$\sigma(t_1) = \sigma(t_2)$
$w \vDash_\sigma t_1 \sim t_2$	iff	$\sigma(t_1) \in w$ iff $\sigma(t_2) \in w$ for all $w \in W$
$w \vDash_\sigma \neg\phi$	iff	$w \nvDash_\sigma \phi$
$w \vDash_\sigma \phi \wedge \psi$	iff	$w \vDash_\sigma \phi$ and $w \vDash_\sigma \psi$
$w \vDash_\sigma \forall x\phi$	iff	$w \vDash_{\sigma^*} \phi$ for all σ^* such that $\sigma^*(x) \in w$
$w \vDash_\sigma \forall xx\phi$	iff	$w \vDash_{\sigma^*} \phi$ for all σ^* such that $\sigma^*(xx) \in p(w)$
$w \vDash_\sigma \Box\phi$	iff	$v \vDash_\sigma \phi$ for all $v \in W$

A sentence ϕ is true in $\mathcal{M}_I$ when true at the actual world relative to all variables assignments. When it is, we write $\mathcal{M}_I \vDash \phi$.

Definition 8.6: Let $\mathcal{M}_F$ be a fictionalist model. When $\vDash_\sigma \phi$, we say that ϕ is true relative to variable assignment σ. This relation is defined recursively:

$\vDash_\sigma R(t_1, \dots, t_n)$	iff	$\langle \sigma(t_1), \dots, \sigma(t_n) \rangle \in [\![R]\!]^I$
$\vDash_\sigma t_1 \prec t_2$	iff	$\sigma(t_1) \in \sigma(t_2)$
$\vDash_\sigma t_1 =_s t_2$	iff	$\sigma(t_1) = \sigma(t_2)$
$\vDash_\sigma t_1 =_p t_2$	iff	$\sigma(t_1) = \sigma(t_2)$
$\vDash_\sigma t_1 \sim t_2$	iff	$\sigma(t_1) \in w$ iff $\sigma(t_2) \in w$ for all $w \in W$
$\vDash_\sigma \neg\phi$	iff	$w \nvDash_\sigma \phi$
$\vDash_\sigma \phi \wedge \psi$	iff	$w \vDash_\sigma \phi$ and $w \vDash_\sigma \psi$
$\vDash_\sigma \forall x\phi$	iff	$w \vDash_{\sigma^*} \phi$ for all σ^*
$\vDash_\sigma \forall xx\phi$	iff	$w \vDash_{\sigma^*} \phi$ for all σ^*
$\vDash_\sigma \mathsf{Act}(t)$	iff	$\sigma(t) \in @$

A sentence ϕ is true in $\mathcal{M}_F$ when true relative to all variable assignments. When it is, we write $\mathcal{M}_F \vDash \phi$.

Definition 8.7: A model $\mathcal{M}$ is **regular** if for every basic non-logical predicate:

$\langle a_1, \dots, a_n \rangle \in [\![R]\!]$ only if there is some $w \in W$ such that $\langle a_1, \dots, a_n \rangle \in \delta_1^w \times \cdots \times \delta_n^w$.

Roughly speaking, a model is regular when individuals stand in non-logical relations

only when they exist together in some worlds. The above formulation generalizes this basic thought to include pluralities.

Definition 8.8: A frame is **compossible** when for every $w, v \in W$, there is a $u \in W$ such that $w \cup v \in u$.

Definition 8.9: A frame is a **separable** when for every $S \subset W$, if $\cup(S) \in P$, then $\cup(S) \in W$.

As you would expect, when a model is based on a compossible frame, we will say that the model is compossible and, when a model is based on a separable frame, we will say that the model is separable.

Observation 8.10: Every compossible model is regular.

Definition 8.11: A model is a **standard** when it is compossible and separable and such that every instance of Plural Comprehension is true at every world.

We can now characterize the three systems from the main text. **CI** consists of those sentences of $\mathcal{L}_I$ that are true in all standard models. **I** consists of those sentence of $\mathcal{L}_I$ that are true in all *regular* models. Finally, **F** consists of those sentences of $\mathcal{L}_F$ that are true in all regular models.

What we want to show now is that the languages $\mathcal{L}_I$ and $\mathcal{L}_F$ are structurally equivalent, modulo **I** and **F**.

We will say that an $\mathcal{L}_I$ language and $\mathcal{L}_F$ language **correspond** when they have the same names and non-logical predicates. Given any two such languages, two models $\mathcal{M}_I$ and $\mathcal{M}_F$ **correspond** when the valuation functions assign the same names and the same predicates the same denotations.

Theorem 8.12: *Let $\mathcal{L}_I$ and $\mathcal{L}_F$ be corresponding languages. There is then a recursive translation g from the sentences of $\mathcal{L}_I$ to the sentences of $\mathcal{L}_F$ such that $\mathcal{M}_I \vDash \phi$ iff $\mathcal{M}_F \vDash g(\phi)$ whenever $\mathcal{M}_I$ and $\mathcal{M}_F$ are corresponding models.*

Proof. We will first divide the plural variables of $\mathcal{L}_I$ into two infinite stocks xx_i and yy_i. Now take any sentence ϕ. We can suppose that none of the variables in the second stock appear in ϕ. For if they do, we can rewrite ϕ as ϕ^* using relettering, where ϕ^* does not include any such variables. We then translate ϕ as follows:

$$g[\phi] = \exists yy_0(\forall x(\mathsf{Act}(x) \equiv x \prec yy_0) \wedge g_0[\phi])$$
$$g_n[R(t_1, \dots, t_n)] = R(t_1, \dots, t_n)$$
$$g_n[\neg\phi] = \neg g_n[\phi]$$
$$g_n[\phi \wedge \psi] = g_n[\phi] \wedge g_n[\psi]$$
$$g_n[\forall x\phi] = \forall x(x \prec yy_n \supset \phi)$$

$$g_n[\forall xx\phi] = \forall xx(xx \prec yy_n \supset \phi)$$
$$g_n[\Box(\phi)] = \forall yy_{n+1}(\mathsf{World}(yy_{n+1}) \supset g_{n+1}[\phi])$$

This gives us our translation. We could then easily verify, by induction on complexity, that $\mathcal{M}_I \vDash \phi$ iff $\mathcal{M}_F \vDash g(\phi)$ whenever M_I and M_F are corresponding models. ■

Theorem 8.13: *Let $\mathcal{L}_F$ and $\mathcal{L}_I$ be corresponding languages. There is then a recursive translation f from the sentences of $\mathcal{L}_F$ to the sentences of $\mathcal{L}_I$ such that $\mathcal{M}_F \vDash \phi$ iff $\mathcal{M}_I \vDash f(\phi)$ whenever $\mathcal{M}_F$ and $\mathcal{M}_I$ are corresponding models.*

Proof. We start by dividing the plural variables of $\mathcal{L}_F$ into an infinite stock xx_i and a single yy. Now take any sentence ϕ of $\mathcal{L}_F$. We can suppose that the plural variable yy appears nowhere in ϕ. Because if it did, we could use relettering to derive a logically equivalent ϕ^*, and then use that for our translation. We then recursively translate ϕ as follows:

$$f[\phi] = \exists yy(f^*[\phi])$$
$$f^*[R(t_1, \dots, t_n)] = R(t_1, \dots, t_n)$$
$$f^*[\neg\phi] = \neg f^*[\phi]$$
$$f^*[\phi \wedge \psi] = f^*[\phi] \wedge f^*[\psi]$$
$$f^*[\forall x\phi] = \Box(\forall x\phi)$$
$$f^*[\forall xx\phi] = \Box(\forall xx\phi)$$
$$f^*[\mathsf{Act}(t)] = t \prec yy$$

As before, it remains to be shown that $\mathcal{M}_F \vDash \phi$ iff $\mathcal{M}_I \vDash f(\phi)$ whenever $\mathcal{M}_F$ and $\mathcal{M}_I$ are corresponding models. But once we have the translation scheme, this can easily be shown by induction. ■

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