Optionality, Scope, and Licensing: An Application of Partially Ordered Categories

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Abstract   This paper uses a partially ordered set of syntactic categories to accommodate optionality and licensing in natural language syntax. A complex but well-studied data set pertaining to the syntax of quantifier scope and negative polarity licensing in Hungarian is used to illustrate the proposal. The presentation is geared towards both linguists and logicians. The paper highlights that the main ideas can be implemented in different grammar formalisms, and discusses in detail an implementation where the partial ordering on categories is given by the derivability relation of a calculus with residuated and Galois-connected unary operators.

1 Introduction: The Problem and the Main Claims

Among the basic issues that all syntactic theories have to deal with are the following:

(1)   Often not only the broad categorial status of expressions but also their finer-grained subcategories are relevant for syntactic combination. E.g., while *will be hungry* combines with any noun phrase subject, *are hungry* requires one in the plural.

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(2)   Some expressions are optionally present, but they have fixed positions and are not iterable; e.g., numerals. Therefore *three black dogs* must be categorially distinct from *black dogs*. It is remarkable that despite this fact determiners like *those* apparently recognize that they are getting the desired complement, whether it is of the form *black dogs* or of the form *three black dogs*.

(3)   Some expressions that have sentential status are nevertheless ungrammatical as they stand and need a licensor for some of their components. Take *Mary drank any more wine*, which contains the negative polarity item *any*. *Drank* has found its arguments, but the addition of a decreasing operator is still required, as in *Not that Mary drank any more wine* and *whether Mary drank any more wine*.

A common way to approach (1) and (2) is to use Typed Feature Structures or other constraint-based grammar formalisms. See Kaplan and Bresnan (1982), Uszkoreit (1986), Carpenter (1992), Pollard and Sag (1994), Morrill (1994), Dörre and Manandhar (1997), Baldridge (2002), and others. The basic idea is that each syntactic category is a partially ordered set of subcategories (or, each expression is characterized by a feature structure and feature structures form partially ordered sets), and the combination of expressions requires subsumption, rather than identity, between the pertinent categories.[1]

Bernardi (2002) extended the same idea to the licensing problem in (3). Informally, assume the following partially ordered set of sentence types; the category labels are ad hoc speaking names:

$$\langle\{\text{Complete S, Incomplete S, Good-enough S}\}, \leq\rangle$$

with the following ordering relation:

(4)   Incomplete S   Complete S
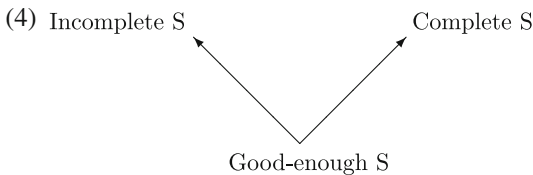
              Good-enough S

Crucially, *Incomplete S $\nleq$ Complete S*. The rest of the grammar will ensure that *Mary drank a glass of wine* is a Good-enough S, and *Mary drank any more wine* is an Incomplete S. *Whether* and *not that* select for an Incomplete S and yield a Complete S. They are free to combine with either one of our two sentences, given *Good-enough S $\leq$ Incomplete S*. On the other hand, whereas *Mary drank a glass of wine* is both Good-enough and Complete, *Mary drank any more wine* must combine with *whether* or *not that* to be part of a Complete S. *Whether* and *not that* act as licensors precisely because they can take a complement that is not Good-enough on its own and

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[1] Interestingly, Minimalism (Chomsky (1995) and subsequent literature) does not have a standard solution for partial ordering and optionality; see the discussion in Sect. 6.

turn it into a Complete S. This type of account generalizes to various other licensing relations.[2]

A technically novel feature of Bernardi's proposal is that the partially ordered set (poset) of categories consists of multiple smaller posets and systematic subsumption relations among their elements. To see how this is motivated by the linguistic problem at hand, consider the fact that expressions like *Mary drank a glass of wine* and *Mary drank any more wine* only differ in that the second one needs a licensor. In all other respects they are built in the same way, and moreover the former happily occurs in all the larger environments that contain a licensor for the latter. Thus the poset of subcategories for Good-enough expressions has to be duplicated, so to speak, by a poset of subcategories for Incomplete ones, with their elements pointwise ordered as S's are in (4). Natural languages exhibit a variety of different licensing relations, all of which call for a similar treatment. It is desirable, therefore, to have a formal device for "copying posets". Representing syntactic categories as formulae of some calculus, with the ordering relation provided by the derivability relation of that calculus, is well-suited to this purpose.

The need to "copy posets" constitutes a novel argument for the logical approach to partially ordered categories. A more standard logical undertaking is to formalize the ordering relation in the basic poset (the one which, in our terms, gets copied) and to fine-tune the rules of syntactic combination (e.g. Blackburn and Spaan (1993); Johnson and Bayer (1995)).

In sum, on the logical approach the exotic-looking (5) is traded for the more austere (6).

(5) $\langle \{\text{Complete S, Incomplete S, Good-enough S}\}, \leq \rangle$

(6) $\langle \{\psi_1, \psi_2, \psi_3\}, \longrightarrow \rangle$, where $\longrightarrow$ is the derivability relation of a particular calculus

Bernardi's proposal is developed in a type-logical setting, relying specifically on innovations in Kurtonina and Moortgat (1995). In type-logical grammar, categories are labeled with logical formulae. Then, each partially ordered set of subcategories is a set of formulae with a derivability relation on it.

Dörre et al. (1996) consider different ways to extend a Lambek grammar for syntactic combination with a calculus for subcategories (subtyping, features), using fibred and monolithic approaches. Bernardi (2002) can be seen as an example of the monolithic one, although not exactly the kind explored in Dörre et al. (1996). This paper presents two ways of subtyping that are equivalent from the point of view of the empirical data that we are going to work with. One follows Bernardi (2002) in all important respects; the other uses the fibred approach and introduces a calculus with just conjunction and disjunction. The two presentations will underscore that different grammar formalisms can adopt and implement the basic ideas.

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[2] The relation of *whether* to *Mary drank any more wine* is somewhat different from that of *what* to *did Mary see*. The reason is that the NPI-licensor does not care whether there is a NPI in its scope, whereas the fronted *what* must combine with a category that contains an object gap. Therefore the treatment of these two cases will not be identical.

We apply these ideas to the highly constrained scope interaction between quantifiers and negative operators in Hungarian. A smaller set of English quantifier scope data was treated in this way in Bernardi (2002), based on Beghelli and Stowell (1997) observations. Quantifier scope is an empirically new domain as compared to those traditionally treated in the literature using partially ordered sets of categories. The switch from English to Hungarian is motivated by the fact that Hungarian offers a richer set of data but, at the same time, a more transparent one. The surface left-to-right order of quantifier phrases in Hungarian largely mirrors their scope order; thus the language makes the syntax of scope directly observable. Another important property of Hungarian is that left-to-right order is determined by quantifier class (group denoting, distributive, counting, negative concord, etc. quantifiers), not by grammatical function, which makes the need for intricate subtyping obvious. Thirdly, the presence of those quantifier phrases is optional, and so the optionality problem described above arises on a large scale. In sum, Hungarian offers a good domain for illustrating the usefulness of partially ordered categories. It will also serve as a backdrop for the discussion of the monotonicity of licensing.

2 Plan

This paper has the following goals.

(I)   To present the basic ideas in a way accessible to linguists whose home theory is not type-logical grammar but either some Typed Feature Structure grammar or Minimalism. This is the task of Part I (Sects. 3 through 5).

(II) To illustrate and test the working of the theory with empirical data pertaining to Hungarian quantifiers, drawing from Szabolcsi (1997), Brody and Szabolcsi (2003), and other literature that is highly compatible with Beghelli and Stowell's approach to quantifier scope in English. This is the task of Part II (Sects. 6 through 10).

(III) To further elaborate and study the logic presented in Bernardi (2002). This is the task of Part III (Sect. 11 through 15).

3 Part I: The Basic Ideas

3.1 The Grammar

3.1.1 Proof Theoretical Approach

This paper presents the grammar in a proof theoretical format but strives to keep the basics as simple as possible. This section offers an informal introduction to some basic ideas and notations in Lambek grammar.

The proof theoretical approach to syntax presents syntax as a calculus, where the syntactic category labels assigned to lexical expressions are the axioms and the syntactic category labels derived for complex expressions—sentences among them—are the

theorems.[3] The idea of Lambek (1958) was to take syntactic category labels to be formulae of a propositional calculus with just material implication $\rightarrow$, notated in categories as $/$. The pertinent inference rules of this simple calculus are the ones corresponding to modus ponens (the elimination of $\rightarrow$) and conditionalization (the introduction of $\rightarrow$).

An expression of category y/x followed by an expression of category x forms an expression of category y. Compare modus ponens:

$$x \rightarrow y$$
$$x$$
$$\therefore \quad y$$

If an expression of category z and one of category x to its right form an expression of category y, then z derives y/x. Compare conditionalization:

$$\text{if} \quad \begin{matrix} z \\ x \\ \therefore \quad y \end{matrix} \quad \text{then} \quad \begin{matrix} z \\ \therefore \quad x \rightarrow y \end{matrix}$$

This calculus has a model theory with respect to which it is sound and complete (Kurtonina and Moortgat 1995). Linguists are most accustomed to logics in which the models contain individuals, events, worlds, etc. In those cases if p derives q, then every world in which p is true is also one in which q is true. In our case the models contain linguistic expressions, and the derivability relation between category labels says something about the syntactic behavior of the expressions, not about their meanings.[4]

$$A \longrightarrow B \text{ iff [expressions labeled } A] \subseteq [\text{expressions labeled } B]$$

That is, the derivability relation between category labels corresponds to a subset relation between the sets of expressions bearing those labels.

3.1.2 Functional Application, Scope, and Bridging

Recall problem (1) mentioned in the introduction: the verb phrase *will be hungry* combines with any noun phrase as its subject, but *are hungry* requires one in the plural, and *is hungry* requires one in the singular. This can be handled with the following assumptions:

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[3] Syntactic categories are sets of expressions: those expressions that belong to the given category. VP, $e\backslash t$, etc. are category labels: names for such sets. This distinction is important to bear in mind when one talks about categories as formulae, although the literature is often sloppy about it, and we also take the liberty to sometimes use the term "category" to refer to a category label.

[4] An overview of the framework more suitable to linguists is Moortgat (2002), whereas logicians are referred to Moortgat (1997).

she ∈ Singular Noun Phrase, *they* ∈ Plural Noun Phrase
will be hungry wants an argument of category Noun Phrase
are hungry wants an argument of category Plural Noun Phrase
is hungry wants an argument of category Singular Noun Phrase
Singular Noun Phrase ≤ Noun Phrase
Plural Noun Phrase ≤ Noun Phrase

The reason why *will be hungry* can take either *she* or *they* as an argument is that although the latter are directly labeled as Singular/Plural Noun Phrases in the lexicon, the ordering relation tells us that every Singular/Plural Noun Phrase is a Noun Phrase. In proof theoretic terms, both lexical category labels derive the label Noun Phrase. In general,

(7) An expression of category *A/C* combines with an expression of category *B* as an argument iff *B* derives *C*.

Another way of saying this is that a functor category is always order reversing with respect to the category selection of its argument. If $B \leq C$, i.e. every expression in *B* is also in *C* then, if a functor category *F* combines with elements of *C* as an argument, it also combines with elements of *B*.

From our perspective scope taking can be reduced to functional application. A quantifier phrase like *every man* that denotes a generalized quantifier of type $\langle\langle e, t\rangle, t\rangle$ syntactically combines with its scope by Montague's Quantifying-in rule or some reincarnation thereof. Its interaction with other operators is determined by what sentential subcategory it can be quantified into (its argument category), and what sentential subcategory it feeds to higher operators (its value category). The fact that a quantifier binds a variable, whereas *not* does not, is immaterial from this perspective. The ordering directly pertains to just the sentential categories and is inherited by the generalized quantifier categories. The main body of this paper abstracts away from how atomic sentences are built (although a sample derivation of a full sentence is provided in Sect. 8). In this simplified context we schematically talk of the category of operators, including quantifier phrases, as s_{val}/s_{arg}.[5] As will be seen below, this is especially straightforward in Hungarian, where quantifier phrases in the preverbal field line up in their scopal order, rather than stay in subject, object, etc. position as in English. It is also straightforward in theories of scope like Bernardi and Moortgat (2007) and Barker and Shan (2006), which allow *s/s* to "migrate" to the level where the quantifier takes scope, "leaving behind" *dp*.[6]

The generalization in (7) simply extends to quantification, as in (8).

(8) A quantifier phrase (operator) of category s_{val}/s_{arg} takes immediate scope over a syntactic domain of category s_a iff s_a derives s_{arg}.

The ordering relation among functor categories is not given directly; it follows from the ordering among the subcategories that label their arguments and values.

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[5] The reader interested in an in-depth formal presentation of the treatment of QPs in categorial type logic is referred to Moortgat (1997), Bernardi and Moortgat (2007) and Barker and Shan (2006).

[6] Determiner Phrase, *dp* is the category label of expressions of type *e* or some lifting thereof.

Crucially to the solution of problems (2) and (3), recognizing the derivability (inclusion) relations among subcategories offers an account of when the intervention of some sentential operator OP_b between OP_a and OP_c is optional or obligatory.[7]

Given the total ordering and the category assignment as in (9), the left-to-right order of the operators follows:

(9) $s1 \longrightarrow s2 \longrightarrow s3$

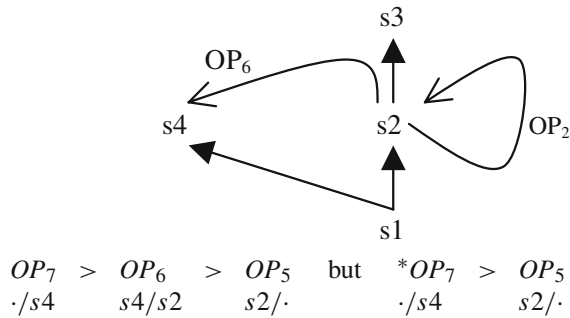$$\begin{array}{ccccc} OP_3 & > & OP_2 & > & OP_1 \\ s3/s3 & & s2/s2 & & s1/s1 \end{array}$$

By transitivity, $s1 \longrightarrow s3$, viz. OP_3 can also scope over OP_1 directly. The presence of OP_2 is optional.

$$\begin{array}{ccc} OP_3 & > & OP_1 \\ s3/s3 & & s1/s1 \end{array}$$

If the ordering relation is not total but partial, as in (10), then OP_7 may only scope over OP_5 if OP_6 intervenes and bridges between OP_5's value category and OP_7's argument category. For instance, given the derivability relations in (10) and OP_7 and OP_5 of category $\cdot/s4$ and $s2/\cdot$, respectively, OP_7 can precede OP_5 only if operator OP_6 of category $s4/s2$ mediates, because $s2 \not\longrightarrow s4$.[8] Linguistic operators are notated as curved arrows pointing from the argument category to the value category.

(10)   Partial ordering on categories

$$\begin{array}{ccccccccc} OP_7 & > & OP_6 & > & OP_5 & \text{but} & {}^*OP_7 & > & OP_5 \\ \cdot/s4 & & s4/s2 & & s2/\cdot & & \cdot/s4 & & s2/\cdot \end{array}$$

Bridging between two categories that the ordering relation does not relate to each other is crucial for the account of many phenomena, NPI-licensing among them.

3.1.3 Salvaging Incomplete Categories

Bernardi (2002) develops a proposal for negative polarity item (NPI) licensing, which we take to be a representative of licensing relations in general. From our perspective

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[7] In what follows, $X > Y$ notates "X precedes and/or scopes over Y", the longarrow $X \longrightarrow Y$ notates "X derives Y". We reserve the $\leq$ notation for the pretheoretical, informal notion of an ordering relation.

[8] In $\cdot/s4$ the dot acts as a placeholder for an arbitrary category.

the following two structures are alike, and whatever we say about NPI-licensing carries over to the licensing of subject/auxiliary inversion:

licensor	**licensee**
never	saw anything
never	would I do that

Let us flesh out the argument presented in Sect. 1. The need for the NPI to be licensed means that a structure containing the NPI is Incomplete unless it is within the scope of an appropriate operator. Let the category of the NPI be val_{NPI}/arg. Category val_{NPI} does not derive that of Complete sentences: it is Incomplete. If the NPI has widest scope within a structure, that structure inherits its value category, and thus inherits Incompleteness from the NPI. Being within the immediate scope of an expression whose value category is Good-enough or outright Complete salvages the structure; such an expression is a licensor. Its category is notated as val/arg_{lic}. For example:

$$
\begin{array}{ccc}
\text{seldom} & > \text{anything} & > \text{saw} \\
val/arg_{lic} & val_{NPI}/arg & arg
\end{array}
$$

The fact that the licensor is capable of scoping immediately above the structure containing the NPI shows that val_{NPI} derives arg_{lic}. Since an Incomplete category by definition cannot derive a Complete one, this means that arg_{lic} itself is an Incomplete category! On the other hand, the licensor does not require for there to be an NPI within its immediate scope. This means, in turn, that arg_{lic} is also derived by various Good-enough categories:

$$
\begin{array}{ccc}
\text{seldom} & > \text{everything} & > \text{saw} \\
val/arg_{lic} & val_{dist}/arg & arg
\end{array}
$$

So, we have (11), where s_n stands for an arbitrary subcategory of s.

(11)
$$val_{NPI}(\text{Incomplete}) \not\longrightarrow s_n (\text{Good-Enough})$$
$$arg_{lic}(\text{Incomplete}) \not\longrightarrow s_n (\text{Good-Enough})$$
$$val_{NPI}(\text{Incomplete}) \longrightarrow arg_{lic} (\text{Incomplete})$$
$$val_{dist}(\text{Good-Enough}) \longrightarrow arg_{lic} (\text{Incomplete})$$

It is in principle possible to set up the poset of syntactic categories in such a way that Incomplete categories reside in a "blind alley" that satisfies the requirements in (11). Given however the variety of licensing relations the grammar has to accommodate, this solution would probably be ad hoc and unable to capture finer patterns.

4 Division of Labor Between Logics

As is widely observed, a "grammar logic" needs two components: a logic of concatenation and a logic of feature descriptions, where the task of the latter is to check

subtype orderings. Dörre et al. (1996) show that the proof theoretical and model theoretical combination of these two components can be obtained in two ways: combination by augmentation or combination by replacement. The former is referred to as "fibred semantics" and proof theoretically it amounts to using combined proof systems. Combination by replacement corresponds to what Dörre et al. call a "monolithic semantics", which at the proof theoretical level amounts to having one derivability relation to reason both on linguistic structures and on features.

Dörre and Manandhar (1997) employ a fibred semantics by layering a Lambek proof system over a feature logic, where the former handles concatenation and the latter checks the derivability relation between basic categories. A string of words $(w_1 \ldots w_n)$ whose categories are labeled with the formulae $A_1 \ldots A_n$, respectively, is proved to be of category s by means of a Lambek calculus $(\longrightarrow_L)$ where the standard axiom schema $a \longrightarrow_L a$ is replaced by the following. $\longrightarrow_F$ is the derivability relation of the Feature Logic.

$$a_1 \longrightarrow_L a_2 \quad \text{if} \quad a_1 \longrightarrow_F a_2$$

In this section we layer the Lambek calculus over a propositional logic with just conjunction and disjunction. This proposal is formally similar to Johnson and Bayer (1995), although the linguistic application of conjunction and disjunction is rather different from theirs. Section 5 will then introduce the basic ideas of the "monolithic semantics" paradigm and present an alternative solution of the scope and licensing problems by means of a different extended Lambek system.

4.1 Subcategories of Good-Enough Sentences

Suppose that the empirical data require the distinction of five subcategories of Good-enough sentences with some particular ordering. The elements of such a set can be mechanically labeled using conjunctions of atomic formulae. Conjunction is commutative and it is notated with the dot for readability. We thus obtain the set $\langle \{A, A.B, A.C, A.B.D, A.B.C.D\}, \leq \rangle$, where $\leq$ is given by p.q $\longrightarrow$ p. What might be the linguistic interpretation of the atomic formulae? With an eye on the application to Hungarian, let each proposition state that an expression whose category is characterized by that proposition could be immediately preceded by some particular operator type. (Because all the elements of this set are sentential subcategories, "I am a sentence" is not represented as a conjunct.) This is illustrated in Fig. 1. Linguistic operators are added to the poset in the form of curved arrows.

4.2 Subcategories for Incomplete Sentences

The two important desiderata for labelling Incomplete sentences are now as follows. First, each category in the Good-enough set should unidirectionally derive one within the Incomplete set (of the kind under consideration). Second, the poset of Incomplete categories should be isomorphic to the poset of Good-enough ones. This is because the internal composition and external behavior of sentences with and without licensees is
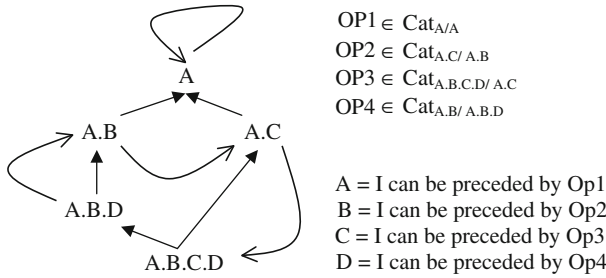
$$OP1 \in Cat_{A/A}$$
$$OP2 \in Cat_{A.C/\,A.B}$$
$$OP3 \in Cat_{A.B.C.D/\,A.C}$$
$$OP4 \in Cat_{A.B/\,A.B.D}$$

A = I can be preceded by Op1
B = I can be preceded by Op2
C = I can be preceded by Op3
D = I can be preceded by Op4

Fig. 1 Conjunctive labels and linguistic operators
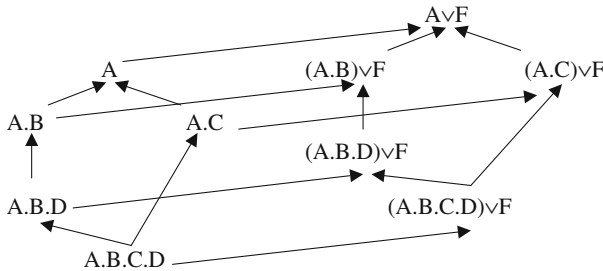

Fig. 2 Disjunction used to copy a poset

the same in all other respects, but Incompleteness must not leak back into the Complete set. One simple way to achieve these is to use disjunction to form Incomplete categories and to add $p \longrightarrow p \vee r$ to the conjunctive logic in Sect. 4.1. In Fig. 2, F = "I need an X- licensor", for some grammatical property X.

As was pointed out above, linguistic expressions have many different licensing needs. These may be independent of one another, in which case multiple posets of Incomplete categories can be created using derivationally independent formulae F_i. Alternatively, the elements of different posets of Incomplete categories may stand in the derivability relation.

Disjunction is appropriate for our purposes, because the following hold:

(12) a. $p \longrightarrow p \vee q$ and b. $p \vee q \not\longrightarrow p$
(13) $(p_1 \wedge \ldots \wedge p_n) \longrightarrow p_i \wedge \ldots \wedge p_j$ iff
$(p_1 \wedge \ldots \wedge p_n) \vee r \longrightarrow (p_i \wedge \ldots \wedge p_j) \vee r$

Disjunction does not create isomorphic copies in general: (14)a is true but (14)b is not.[9]

(14) a. If $a \longrightarrow b$ then $(a \vee c) \longrightarrow (b \vee c)$
b. If $(a \vee c) \longrightarrow (b \vee c)$ then $a \longrightarrow b$

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[9] Note that $\cdot \vee c$ is a closure operator: $a \longrightarrow a \vee c$, $a \vee c \longrightarrow b \vee c$ if $a \longrightarrow b$, and $(a \vee c) \vee c \longrightarrow a \vee c$. Due to these properties (14)b does not hold.

Let a and b be $d \vee c$ and d respectively. Then $(d \vee c) \vee c \longrightarrow d \vee c$, but $(d \vee c) \not\longrightarrow d$. However, in our case a and b will be of a specific shape, namely they are either conjunctions of atoms, or conjunctions of atoms followed by the disjunct c_i, and the number of disjunctions in a and b is the same. This fact guarantees (13), i.e. isomorphic copies.

Disjunction has the right logical properties, but is there an intuition behind its use for labeling expressions that may or may not require licensing? $p \vee r$ entails $\neg r \rightarrow p$, which can be read as saying, "If I do not need a licensor (either because I contain no NPI or because I am within the scope of a licensor), then I am of category p."

5 Division of Labor Within One Logic

As explained in Dörre et al. (1996), an alternative solution to the "fibred semantics" approach discussed above is to employ a "monolithic semantics" method, which replaces the basic categories with "richer" ones and uses the same derivability relation to check concatenation as well as feature correctness. In particular, Dörre et al. explore the possibility of replacing basic categories by feature terms which, like the basic categories, are modal propositions, i.e., denote sets of worlds. In this setting, a basic category would be of the format $f : T$ where f is a feature (i.e., attribute) and T is a sort (i.e., value) and $f : T$ describes a node with an outgoing f-arc leading to a node matching the description T. Boolean combinations of sorts with $\wedge$, $\vee$ and $\neg$ are allowed and $f : T$ is taken to act as $\Diamond T$ over the f-accessibility relation. Following this line, we are now going to present an alternative solution to the problems discussed so far by taking advantage of the extension of the Lambek calculus proposed in Kurtonina and Moortgat (1995) and in Areces and Bernardi (2004).

Kurtonina and Moortgat extend the language of the Lambek calculus with residuated unary operators and prove that the extended language is complete with respect to Kripke semantics for all frames and valuations, with sets of linguistic structures for worlds (cf. the sets of propositions in modal logic). Areces et al. (2003) further extend the calculus with Galois-connected operators and again prove its completeness with respect to Kripke models. Hence, we have at our disposal a richer logical vocabulary to build syntactic categories. This allows for a clear division of labor, but this time it is between components of the same logic. As before, binary residuated operators account for concatenation, but now we employ the unary residuated operators to check the relation among the Good-enough sentential categories, and the Galois-connected ones to generate copies of it for Incomplete sentential categories. All these components belong to the same logic: the proof system incorporating them is complete with respect to one and the same class of models.

The relation between feature logic and modal logic has been deeply studied, e.g., Blackburn and Spaan (1993). It is beyond the scope of this paper to compare our proposal with this work. It is important however to underline the fact that we use residuated operators rather than dual ones, and that our formulae do not reflect the articulation of attribute value matrices.

5.1 Residuated Pairs of Connectives

Our framework is a Categorial Grammar (CG) known as Categorial Type Logic (CTL).[10] It consists of (i) the logical rules of binary operators[11] and (ii) the logical rules of unary operators.

5.1.1 Residuated Binary Connectives

The rules of the binary operator / are the same as the introduction and elimination rules of the propositional calculus $\rightarrow$, see the standard deduction theorem:

$$\Gamma \bullet p \longrightarrow q \text{ iff } \Gamma \longrightarrow q/p$$

In words: Γ concatenated with p belongs to the category q if and only if Γ belongs to the category q/p. The $\bullet$ indicates the concatenation of structures.

The relation above between the $\bullet$ and the / is known in algebra as the residuation principle. The $\bullet$ and the / form a residuated pair in the same way as addition and subtraction, or multiplication and division do. Recall how one solves an algebraic equation like $3 \times x \leq 5$ by isolating the unknown x using the law connecting $(\times, \div)$ and producing the solution $x \leq \frac{5}{3}$. The law connecting these two binary (residuated) operators says:

$$x \times y \leq z \text{ iff } x \leq \frac{z}{y}$$

In CTL, such a pair of operators is used to put together and take apart linguistic expressions as sketched in Sect. 3.1.1.

As it was highlighted in (7) above, it follows from residuation that A/C is order reversing (with respect to category selection) in its argument position (C), and order preserving in its value position (A). If B derives C, then A/C is also happy with B as an argument; if A derives D, then A/B also counts as D/B. This is formally represented by the inferences below.

(15) $$\frac{B \longrightarrow C}{A/C \longrightarrow A/B} \qquad \frac{A \longrightarrow D}{A/B \longrightarrow D/B}$$

5.1.2 Residuated Unary Operators

The residuated unary operators, to which we now turn, will serve to create a fine-grained partial order of categories. We show that the partial order among the sentential subcategories required to control scope and word order can be encoded as the derivability relation driven by residuated unary operators.

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[10] Alternative names are Type Logical Grammar, see for instance Morrill (1994), and Multi-modal Categorial Grammar (Moortgat and Oehrle 1994).

[11] The binary operators are \, $\bullet$, /. For ease of exposition we will focus only on / and $\bullet$.

Kurtonina and Moortgat (1995) further explored the space of the Lambek calculus by exploiting unary operators inspired by tense logic. The idea of this line of research is to take the *minimum logic*, i.e. a logic characterized by those properties that are at the core of any logic (namely, transitivity of the derivability relation, upward/downward monotonicity of operators and their compositional behavior) as a starting point to analyze linguistic universals, and then extend its language so as to increase its expressivity and analyze linguistic structures and cross-linguistic variation.[12]

To give an intuitive example, the past possibility and future necessity operators of tensed modal logic have just the core properties. That is, they obey the algebraic principle of residuation introduced above:

$$PastPoss\,A \longrightarrow B \ \text{ iff } \ A \longrightarrow FutNec\,B$$

Following Kurtonina and Moortgat (1995), we fashion our residuated pair of unary operators after these and notate them as $\Diamond$ and $\Box$. These symbols are hijacked for typographical convenience and must not be confused with the standard modal operators, which form a pair of duals and not a pair of residuals.[13] Thus, in the notation to be used below:[14]

(16) $\Diamond A \longrightarrow B$ iff $A \longrightarrow \Box B$

The properties below follow from (16); see details in Sect. 12.

1. $\Diamond\Box A \longrightarrow A$ [Unit]
2. $A \longrightarrow \Box\Diamond A$ [Co-unit]
3. $\Diamond A \longrightarrow \Diamond B$, if $A \longrightarrow B$ [$\Diamond$ upward monotonic]
4. $\Box A \longrightarrow \Box B$, if $A \longrightarrow B$ [$\Box$ upward monotonic]
5. $\Diamond\Box A \longrightarrow \Diamond\Box B$, if $A \longrightarrow B$ [$\Diamond\Box$ upward monotonic]
6. $\Box\Diamond A \longrightarrow \Box\Diamond B$, if $A \longrightarrow B$ [$\Box\Diamond$ upward monotonic]

In this paper, we use the $\Diamond$ and $\Box$ operators as *decorations on sentential categories*. The derivability relation among decorated categories defines a partial order. As in Bernardi (2002), that partial order will be used to express the fine-grained partial ordering among sentential categories that is necessary to capture the differential scoping abilities of quantifier phrases. Section 7 will illustrate this with Hungarian material.

Figure 3 illustrates the derivability relations within one small set of decorated categories. It exhibits all the derivability relations that exist within the given set of categories (although of course these categories are derivable from infinitely many others and derive infinitely many others). The reader is invited to consult Sect. 12 for details.

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[12] Of course a more basic question is the identification of **the** minimum logic.

[13] Past possibility and past necessity (as well as future possibility and future necessity) are duals, whereas, past possibility and future necessity are residuals. In the latter case the accessibility relation for the $\Diamond$ is inverted for the $\Box$, which is sometimes indicated by superscripting the $\Box$ with a down-arrow. We do not follow this cumbersome notation here, because this paper uses the unary operators unambiguously, only in the residuated sense.

[14] Roughly, the $\Diamond$ is a unary $\bullet$ and the $\Box$ is a unary implication. That is, take "$\Diamond \cdot$" to be "$\cdot \bullet p$" and "$\Box \cdot$" to be "$\cdot / p$", where the unary operator is obtained by fixing one argument of the binary operator as p.

s9
□□□◊◊**p**

s8
□□◊◊**p**

s7
□□◊◊◊◊□**p**

s3
□◊**p**

s10
◊□□□◊◊**p**

s4
□◊◊□**p**

s2
p

s5
◊□□◊**p**

s1
◊□**p**

s6
◊□□◊◊□**p**
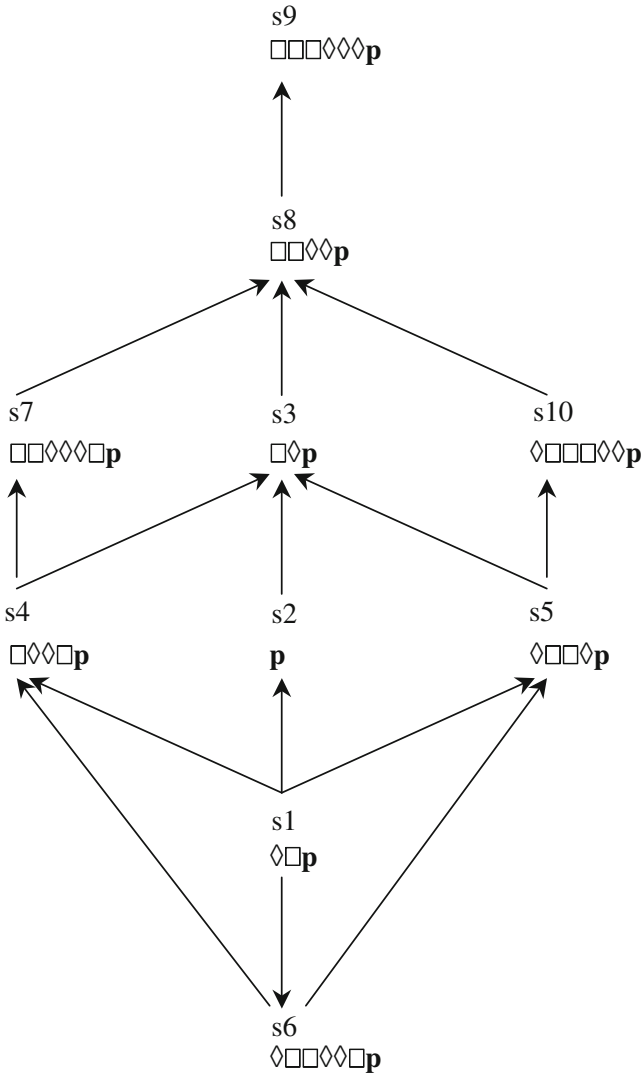
Fig. 3 Derivability relations among a few operators

5.1.3 Multiple Modes for Unary Operators

The last feature of the calculus of residuation to be introduced here is the availability of multiple modes for the unary operators. There are various linguistic applications of multimodality in CTL, some of them quite different from our own application.[15] Suppose we have just two modes, one notated with empty ◊, □ and another notated with

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[15] See Heylen (1999) for a detailed study of the use of unary operators to encode feature structure information.

s9
□□□◊◊◊p

s8
□□◊◊p

s7~   s7   s3   s10   s10~
□□◊◊♦■p   □□◊◊◊□p   □◊p   ◊□□□◊◊p   ♦■□□◊◊p

s4~   s4   s2   s5   s5~
□◊♦■p   □◊◊□p   p   ◊□□p   ♦■□◊p

s1
◊□p

s6
◊□□◊◊□p

s1~
♦■p

Fig. 4 Derivability in the space of Fig. 3 with two modes. Circles indicate irrelevantly crossing lines

filled ♦, ■. The two modes will add further flexibility to the logic whose derivability relation formalizes the partial ordering of sentential categories.

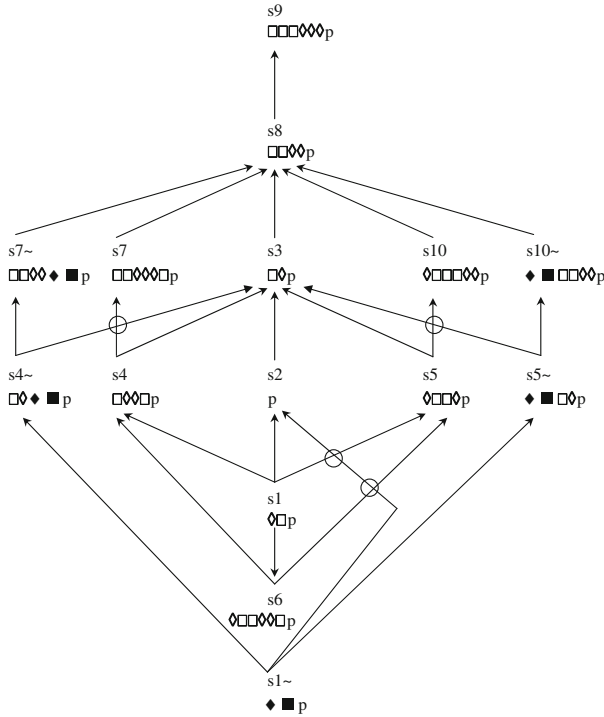The consequences of residuation listed above hold for unary operators of the same mode. Distinct modes do not mix, i.e. there is no law that derives anything from $\Diamond \blacksquare A$. On the other hand, the same Co-unit property that gives $\Diamond \Box s \longrightarrow \Box \Diamond \Diamond \Box s$ also derives $\blacklozenge \blacksquare s \longrightarrow \Box \Diamond \blacklozenge \blacksquare s$. Likewise, the Unit property that gives $\Diamond \Box \Box \Diamond s \longrightarrow \Box \Diamond s$ also produces $\blacklozenge \blacksquare \Box \Diamond s \longrightarrow \Box \Diamond s$. This means that several alternative paths may be constructed from one element of the partially ordered set to another: one involving only operators in the empty mode, another involving both empty and filled ones, etc. Figure 4 illustrates one way of adding operators in the filled mode to the set in Fig. 3. The linguistic role of the two modes is discussed in Sect. 7.

5.2 Galois-Connected Unary Operators

Bernardi (2002) proposes a systematic way to encode the kind of derivability relations described in (11) using unary Galois operators. These were first introduced into CTL in Areces and Bernardi (2004) inspired by Dunn (1991), Goré (1998). The completeness and decidability of the system is proved in Areces et al. (2003). These authors show that the realm of minimum logic (i.e. the logic characterized by just the core properties of the transitivity of derivability, the monotonicity of the logical operators and their

compositional behaviour) has space for operators that reverse the derivability relation among formulae. Recall

$$\Diamond A \longrightarrow B \ \text{iff} \ A \longrightarrow \Box B$$

Let $^0\cdot$ and $\cdot^0$ be two unary operators. They are said to be Galois-connected if they obey the definition below.

$$B \longrightarrow {}^0A \ \text{iff} \ A \longrightarrow B^0$$

These two operators behave exactly like $\Diamond$ and $\Box$, except that they are downward monotonic, cf. the fact that B occurs on the righthand side of the arrow in $\Diamond A \longrightarrow B$ but on the lefthand side in $B \longrightarrow^0 A$. The algebraic analogy now involves reciprocals: the greater a number, the smaller its reciprocal:[16]

$$x \times y \leq 1 \ \text{iff} \ x \leq \frac{1}{y}$$

As in the case of $\Diamond$ and $\Box$, the properties regarding the composition and the monotonicity behavior of the Galois operators follow, namely:

1. $A \longrightarrow {}^0(A^0)$
2. $A \longrightarrow ({}^0A)^0$
3. $^0A \longrightarrow {}^0B$, if $B \longrightarrow A$ $[^0\cdot$ downward monotonic$]$
4. $A^0 \longrightarrow B^0$, if $B \longrightarrow A$ $[\cdot^0$ downward monotonic$]$
5. $^0(A^0) \longrightarrow^0 (B^0)$, if $A \longrightarrow B$ $[^0(\cdot^0)$ upward monotonic$]$
6. $(^0A)^0 \longrightarrow (^0B)^0$, if $A \longrightarrow B$ $[(^0\cdot)^0$ upward monotonic$]$

Notice that since the composition of two downward monotonic operators is upward monotonic, $^0(A^0)$ and $(^0A)^0$ are upward monotonic in A. In what follows we will only use them in pairs, i.e. as (composite) upward monotonic operators.

Double-Galois operators can be used to create additional copies of the poset given by $\Box$ and $\Diamond$. As (17) indicates, each s_a derives $^0(s_a^0)$, and if $s_a \longrightarrow s_b$, $^0(s_a^0) \longrightarrow {}^0(s_b^0)$. This means that the derivability relations within each double-Galois copy are the same as those within the $\Box$ and $\Diamond$ segment of the poset. However, the paths are unidirectional: double-Galois operators can only be added, not removed. This means

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[16] As in the case of the unary residuated operators $\Diamond$ and $\Box$, the Galois-connected unary operators can be seen as binary operators with a fixed argument:

$$x \times 2 \leq z \ \text{iff} \ x \leq \frac{z}{2} \quad \text{let} \ \Diamond \cdot = \cdot \times 2 \text{ and } \Box = \frac{\cdot}{2}$$

$$y \leq \frac{2}{x} \ \text{iff} \ x \leq \frac{2}{y} \quad \text{let} \ {}^0\cdot = \frac{2}{\cdot} = \cdot^0$$

If we take two directional implications \ and / instead of undirectional reciprocal $\frac{.}{.}$, we obtain $^0\cdot \neq \cdot^0$

that Good-Enough categories derive Incomplete ones, but no Incomplete category derives a Good-Enough one. This is precisely what Incompleteness is.[17]

(17)

Good-Enough		**Incomplete**		**Incomplete**
s_b	$\longrightarrow$	$^0(s_b^0)$	$\longrightarrow$	$(^0(^0(s_b^0)))^0$
$\uparrow$		$\uparrow$		$\uparrow$
s_a	$\longrightarrow$	$^0(s_a^0)$	$\longrightarrow$	$(^0(^0(s_a^0)))^0$

An advantage of using the double-Galois operators to encode Incompleteness is that we now have a systematic solution, rather than an ad hoc "blind alley" for Incomplete categories.

A similar effect could be achieved with different modes instead of double-Galois operators. Instead of using $A \longrightarrow {}^0(A^0)$, one could create, for every Good-Enough category A, a corresponding NPI-containing category, using the Co-Unit property with some designated modality:

$$A \longrightarrow \Box_n \Diamond_n A$$

If one needs more Incomplete copies, then instead of iterating different Galois-connected pairs, iterations of $\Box \Diamond$ in different modes can be used.

There are two reasons why using double-Galois operators is neater. The weaker one is that it does not require introducing newer and newer modes. A stronger reason is that the Galois-connected operators only have (the analog of) the Co-Unit property, whereas the residuated ones have both Co-Unit and Unit. Therefore they produce many more derivations and the logical space becomes much richer. In this case, increase in richness may be undesirable. If however the number of categories in a given linguistic application is relatively small, using just residuated operators is logically and conceptually simpler.

6 Part II: Linguistic Applications

6.1 Quantifier Order and Scope in Hungarian

6.1.1 A Bird's Eye View

The syntax of scope in Hungarian will serve as our testing ground. Our interest is not in the Hungarian operators per se, but rather in the fact that (i) they illustrate a case where the surface syntactic distribution of expressions depends on their interpretable features, and (ii) they are numerous enough to give rise to rather complex interactions.

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[17] The pairs $^0(.^0)$ and $(^0.)^0$ are closure operators, therefore the iteration of the same pairs of Galois produces equalities, viz. $(^0(^0A)^0)^0 \longleftrightarrow (^0A)^0$ and similarly for the other pair. On the other hand, the iteration of different pairs, i.e. $^0(.^0)$ followed by $(^0.)^0$ and conversely $(^0.)^0$ followed by $^0(.^0)$ produces inequalities, $(^0A)^0 \longrightarrow {}^0(((^0A)^0)^0)$ but $^0(((^0A)^0)^0) \not\longrightarrow (^0A)^0$ and similarly for the other combination (see more details in Sect. 13). Turning back to our application, the iterations of different pairs of Galois give us the possibility to express many "Incomplete" sentential posets.

To a significant extent, the syntax of scope is **the** syntax of Hungarian: the left-to-right order of operators in the preverbal field unambiguously determines their scopal order. Another remarkable property is that the possible orders are determined by quantifier class and not by grammatical function. Thus, the examples in (18) illustrate the fact that a distributive universal must precede a counting quantifier with *kevés* 'few', irrespective of which is the subject and which is the direct object and the fact that, given their order, the former inescapably outscopes the latter.[18]
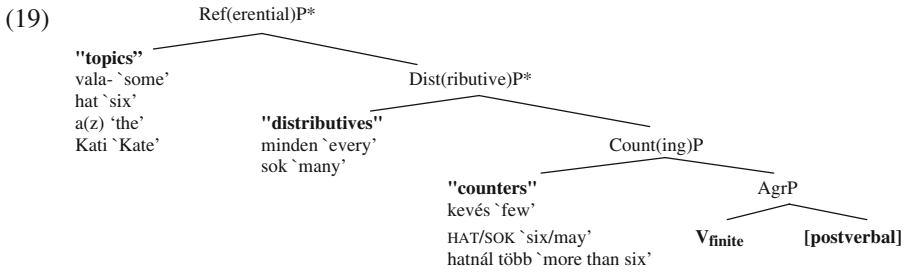
(18) a. Minden doktor   kevés filmet   látott.
        every   doctor-nom few   film-acc saw
        'Every doctor saw few films', viz. $every_{Subject} > few_{Object}$

     b. Minden filmet   kevés doktor   látott.
        every   film-acc few   doctor-nom saw
        'Few doctors saw every film', viz. $every_{Object} > few_{Subject}$

     c. *Kevés doktor   minden filmet   látott.
        few   doctor-nom every   film-acc saw
     d. *Kevés filmet   minden doktor   látott.
        few   film-acc every   doctor-nom saw

A common way to capture these facts has been to assume that operators move into designated positions in the manner of wh-movement, and their left-to-right order translates into a quantifying-in hierarchy. This assumption differs from Fox-Reinhart style Interface Economy, according to which quantifier scope is assigned by Quantifier Raising, an adjunction operation that applies only when it makes a truth conditional difference (Fox 1999; Reinhart 2006). Both Fox and Reinhart concern themselves with scope assignment that has no effect on surface constituent order: covert scope shifting in English. The case of Hungarian is different: quantifier phrases in Hungarian occur in the positions to be discussed below irrespective of whether this has a disambiguating effect. Even if one were to ignore the fact that left-to-right order determines interpretive order, the syntax of Hungarian would have to account for the fact that certain word orders are grammatical and others are not.

The following diagram illustrates three of the relevant positions with their characteristic inhabitants. For space reasons only the determiners are included. Only definite/specific DPs that denote pluralities occur in topic (RefP). Quantifiers in the distributive position (DistP) do not support collective readings. Both these positions host only upward monotonic expressions. Quantifiers in the counter position (CountP) may belong to any monotonicity type and are interpreted as performing a counting operation. See Hackl (2006) for experimental psycholinguistic evidence for counting as a distinct verification strategy. Some though not all quantifier phrases may occur in more than one position and their interpretations vary accordingly. An example in (19) is *sok* 'many'. When *sok ember* 'many men' occurs in the counter position, which is the only possible position for *hatnál több ember* 'more than six men', it supports both distributive and collective readings, but when it occurs in the distributive position, which

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[18] We draw directly from the results of Szabolcsi (1981, 1997), Brody and Szabolcsi (2003), Kiss (1987, 1991, 1998, 2002, Puskas (2000), Horvath (2000, 2006), Hunyadi (1999), and Surányi (2003).

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is the only possible position for *minden ember* 'every man', the collective interpretation is not available. Such semantic matters are discussed in detail in Szabolcsi (1997).

(19)

```
                    Ref(erential)P*
                  ╱              ╲
         "topics"                 ╲
         vala- `some'              ╲
         hat `six'             Dist(ributive)P*
         a(z) 'the'              ╱         ╲
         Kati `Kate'            ╱           ╲
                       "distributives"       ╲
                       minden `every'     Count(ing)P
                       sok `many'          ╱        ╲
                                          ╱          ╲
                                 "counters"           AgrP
                                 kevés `few'          ╱    ╲
                                 HAT/SOK `six/may'   ╱      ╲
                                 hatnál több `more than six'
                                              V_finite   [postverbal]
```

The filling of each of these positions is optional; however, all the positions can be filled simultaneously. RefP and DistP are recursive (cf. the Kleene stars), subject to the same "left-to-right order determines scope" rule. Preverbal operators normally outscope all postverbal ones. Therefore, a counter gets a chance to outscope a distributive quantifier if the latter occupies a postverbal position.[19]

Postverbal quantifier order is virtually free. Kiss (1998) and Brody and Szabolcsi (2003) argue however that the sequence of operator positions observed preverbally reiterates itself in the postverbal field. The impression of postverbal order freedom is due to the fact that of the inflectional heads that separate the operator sequences— Agr(eement), T(ense), etc.—only the highest is visible: the one that hosts the finite verb. Therefore two adjacent operators in the postverbal field need not belong to the same operator sequence and need not conform to the sequence-internal hierarchy. The overt or covert inflectional heads play the kind of beneficial bridging role that was described in Sect. 3.1.2.[20]

6.1.2 Total Order?

An important fact about the operators reviewed above is that they can all co-occur. Adding focus, negation, and question words to the mix raises new questions about how expressions, and their categories, can be ordered.

First consider focus. Hungarian is one of those languages that have a reflex of focussing in surface syntax. Counting quantifiers and foci (emphatic focus, identificational focus, and phrases modified by *csak* 'only') are complementary in the immediately

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[19] Inverse scope, i.e. one that does not match left-to-right order and where, specifically, a postverbal operator outscopes preverbal ones, is possible in two main cases: (i) with a postverbal specific indefinite, and (ii) with a postverbal distributive that bears primary stress. Neither of these is assumed to involve overt or covert operator movement and will not be further discussed in this paper. The wide existential scope of indefinites may be attributed to existential closure over choice functions à la Reinhart (1997). As regards primary stressed postverbal distributives, both Kiss (1998) and Brody and Szabolcsi (2003) argue in detail that they effectively occur in the highest DistP projection and their postverbal ordering is obtained using permutation rules that do not affect c-command and scope relations.

[20] The analysis of the postverbal field is a matter of some disagreement, see Surányi (2003). The postverbal facts will play little role in this paper; they are mentioned only to enable us to provide a concrete sample derivation in Sect. 8.

preverbal position. As they never co-occur, no left-to-right ordering can be established between them:

(20)

$$\text{topic} \ > \ \text{distributive} \ > \ \begin{array}{c} \text{counter} \\ \text{focus} \end{array} \ > \ \text{verb} \dots$$

Next consider negation. The preverbal field may contain two distinct instances of sentential negation (*nem*), to be dubbed as hi-neg and lo-neg when the distinction is necessary. The two happily co-occur and, naturally, do not cancel out, when an appropriate third party intervenes. The postverbal field houses no negation. See Koopman and Szabolcsi (2000, Appendix B).

Even putting aside negative polarity items, operators come in different flavors as regards their ordering constraints with respect to negation. Negation may follow a focus or a counter, and it may precede a focus, though not a counter:

(21)   (*hi-neg) > counter > lo-neg > verb . . .
       hi-neg > focus > lo-neg > verb . . .

Distributive universals cannot scope immediately above negation, nor for that matter immediately below it,[21]

(22)   (*neg) > distributive-∀ > (*neg)

whereas negative concord (NC) universals such as *senki* 'no one' come with something like the opposite restriction:[22]

(23)   *NC-∀ > *OP*,   unless *OP* = neg or *nem_minden* 'not_every' or NC-∀

In contrast, topics and distributive existentials do not care whether or not there is negation in their immediate scope:

(24)   topic > (neg)
       distributive-∃ > (neg)

It follows from (21) and (23) that counters and NC universals cannot co-occur in the preverbal field. Likewise, it follows from (22) and (23) that distributive and NC universals cannot co-occur in the preverbal field. Therefore there is no basis for ordering

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[21] Examples with *nem minden fiú* 'not every boy' contain phrase internal negation and not a hi-neg preceding the quantifier *minden fiú* in one of its otherwise legitimate positions. The critical data that show this involve order interaction with verbal particles. The verbal particle (*fel* 'up', etc.) precedes the verb unless the next element to the left is negation, or a focus, or a counter. With non-negated *minden*-phrases the only possibility is (i). However, *nem minden*-phrases require the verbal particle to follow the verb, as in (ii). Thus *nem minden*-phrases represent a separate quantifier class, cf. also (23).

(i)   Minden fiú fel-ébredt.
      every boy up-woke

(ii) Nem minden fiú ébredt fel.
      not every boy woke up

For simplicity's sake the mini-grammar to be presented in Sect. 7 does not include verbal particles.

[22] Hungarian is a so-called strict negative concord language. Negative concord items (NC) are interpreted as universals, following Szabolcsi (1981), Giannakidou (2000), Puskas (2000).

NC universals with respect to either counters or distributive universals within the same operator sequence.

Finally, question words such as *ki* 'who' occur in the preverbal position. The received wisdom is that they are foci; but unlike other foci they can only be preceded by topics.

(25)   (*distributive) > wh
        (*neg) > wh

These observations present a challenge for any theory of syntax that assumes that the ordering of functional heads (Neg, Dist, etc.) is total. Minimalism is such a theory. Could (19) be extended to the additional data, maintaining a total order? This question has not been addressed in the Minimalist literature, so we outline an answer from scratch. A total ordering could be maintained if the proposal is supplemented with further assumptions or constraints. One contender would be as follows (only the head categories are listed):

(26)   Ref* > Dist* > Hi-Neg > Pred > Lo-Neg > AgrS . . .

One supplementary assumption would be that counters and foci are not two distinct categories in complementary distribution; instead both carry a [pred] feature and compete for the specifier position of a Pred head. This analysis follows the "focussing as predication" view recently advocated by Kiss (2001, 2006). The view is not uncontroversial (see Horvath (2000, 2006) for another view), but for present purposes it suffices that such a unification is imaginable. The fact that identificational foci and *csak* 'only' phrases can be preceded by Hi-Neg, but counters cannot (unless they have a contrastive component) is one argument for the distinct categories analysis. However, it could be accommodated in (26) by adding that Hi-Neg requires its complement to carry the feature [contrast], and not all [pred] phrases have [contrast]. *Ki* 'who' will have [pred] but the PredP dominating it will be specified not to be the complement of a head with [neg] or [dist].

Another supplementary assumption would be that distributive universals, distributive existentials, and NC universals all have a [dist] feature and are thus headed for the specifier of a Dist head, but they are marked differently as to what features the complement of Dist should carry. NC universals require that the closest head below have a [neg] feature; distributive universals require that the same head not have [neg]; distributive existentials and expressions with [topic] are not marked in this regard. Not only *nem* 'not' has [neg], but also *nem mindenki* 'not everyone', and *senki* 'no one, NC' come with a [neg] feature that they transmit by specifier-head agreement. The treatment of *nem mindenki* itself remains difficult. One might say that it has [dist] and [neg] features and additionally requires that the complement of the Dist head have [neg] or [contrast] or [bare agr], where [bare agr] is an ad hoc feature to pick out the verb separated from its particle.

This will suffice to show both that the total order in (26) could be maintained and what kind of cost this would incur. A description using a total order of categories is possible, but the result does not look very Minimalist. Put in general terms, this description preserves the illusion of a total order by not assigning a status to the featural restrictions within the theory of syntax. This might be fine if all the restrictions

follow from the semantics of the expressions involved. The restrictions on question words probably do, but it is not obvious that the same holds for all the other restrictions.

6.1.3 Optionality

The Hungarian data highlight another fundamental question. As was noted in 6.1.1, the presence of all the operators discussed in this section is optional. Consider:

(27) Tudom, hogy [$_{\text{RefP}}$ az emberek [$_{\text{AgrSP}}$ láttak]].
 know-1sg that the men saw-3pl.1sg
 'I know that the men saw me'

(28) Tudom, hogy [$_{\text{DistP}}$ minden ember [$_{\text{AgrSP}}$ látott]].
 know-1sg that every man saw-3sg.1sg
 'I know that every man saw me'

(29) Tudom, hogy [$_{\text{AgrSP}}$ láttál]].
 know-1sg that saw-2sg.1sg
 'I know that you saw me'

These examples raise the optionality problem (2) of Sect. 1. The complementizer head *hogy* 'that' is apparently equally happy to recognize RefP, DistP, and AgrSP as suitable arguments. Likewise, Ref selects for DistP, but it is equally happy with AgrSP, and so is Dist, which selects for PredP. How are the complement selection requirements of these heads satisfied?

The optionality problem is by no means specific for Hungarian; Hungarian just illustrates it on a large scale. Although it is no novelty in formalisms using partially ordered sets of categories or features, it does not seem to have received much attention in the Minimalist literature and we are not aware of a standard solution. In line with the influential proposal in Cinque (1999) that the sequence of functional heads is invariant and universal, one hypothesis might be that whereas the full sequence of categories in (26) is always present, the individual categories need not host lexical items in every sentence. This hypothesis would have been easy to accommodate in earlier, phrase structure rule based versions of generative syntax, but it is not in current Minimalism. The problem is that in Minimalist Theory categorial structure is projected from the lexical items that make up the sentence: no lexical item, no category. So, one would need to postulate that for each optional head category there exists a "dummy lexical item", which has no ability to attract a phrase to its specifier but suffices to project the phrasal category that satisfies the complement selection requirements of the head above it. Moreover, to accommodate the constraints discussed in connection with the total order (26), one would need to ensure that phrasal categories headed by dummies inherit the features of the next phrasal category below them that is headed by a real lexical item.

The conclusion is the same as that of the previous subsection: a solution involving an invariant sequence of categories is in principle possible, but it does not look very Minimalist.

6.1.4 Partial Order and Derivability/Inclusion Relations: Two Birds with One Stone

As was pointed out in Sects. 1 and 3, the optionality problem receives a natural solution if expressions are not thought to have a unique category label but derivability/inclusion relations among categories are recognized. If the grammar recognizes the DistP $\longrightarrow$ RefP (DistP $\subseteq$ RefP) relation, and the complementizer head *hogy* 'that' does not look for a complement specifically labeled as RefP but accepts any category that derives RefP, then the grammaticality of (28) no longer comes as a surprise; and similarly for the other examples.

Thus the solution to the optionality problem points to a partially, not totally, ordered set of categories. This suggests that the effort to create a total order in (26) and supplement it with an extra-theoretical device, a set of featural constraints as detailed above, is unnecessary. Those same constraints can be expressed as finer details of the partial ordering. The next section demonstrates how this works in CTL or in conjunctive logic.

The same conclusion that syntactic categories should be partially, rather than totally, ordered was reached in Nilsen (2002, 2004) within Minimalist syntax, based on somewhat different data. Nilsen's empirical arguments come from the distribution of adverbs in Norwegian. He observes, contra Cinque (1999), that Norwegian adverbs by default occur in any order; whatever ordering restrictions one finds follow from the fact that the individual abverbs are often ordered with respect to negation.

7 A Mini-Grammar of Operators in Hungarian

7.1 Using Formulae Decorated with Unary Residuated Operators

Within the multi-modal categorial type logic introduced in Sect. 5, capturing the partial order of the most important Hungarian operators requires the use of a portion of the logical space exhibited in Fig. 4. For easier reference each sentential category is given a number. The category $s1$ is assigned to sentences whose initial element is an inflected verb.

Prior to locating the Hungarian operators in this space, we draw attention to the two modes in Fig. 4. The basic mode is represented with empty boxes and diamonds. The filled boxes and diamonds can be seen to add an alternative dimension to some parts of the system; to highlight this, the sentential categories that involve filled modes have the numbers of the corresponding categories in the empty mode plus a tilde. So, for example, parallel to $\Diamond\Box p$ (= $s1$) is $\blacklozenge\blacksquare p$ (= $s1^\sim$) and parallel to $\Box\Diamond\Diamond\Box p$ (= $s4$) is $\Box\Diamond\blacklozenge\blacksquare p$ (= $s4^\sim$). The two alternative dimensions merge where s_n and $s_n^\sim$ derive the same category. $s1$ and $s1^\sim$ both derive p (= $s2$), and $s4$ and $s4^\sim$ both derive $\Box\Diamond p$ (= $s3$), etc. The categories based on the filled mode will be used to capture the behavior of those operators—negative concord items—that must scope immediately above negation or another negative concord item. So $s1$ is the category of basic affirmative sentences and $s1^\sim$ the category of basic negative sentences. The argument category of lo-neg is $s1$ and its value category is $s1^\sim$, yielding the functor category $s1^\sim/s1$.
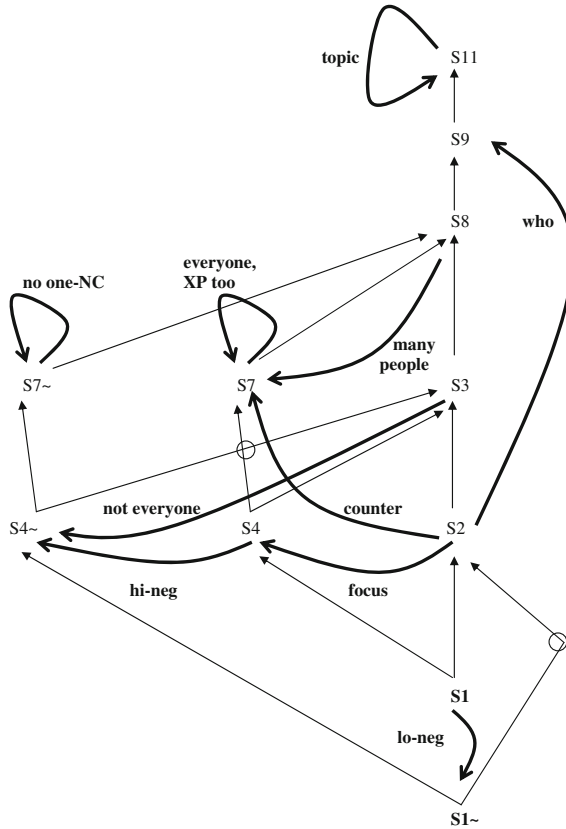
Fig. 5 Hungarian operators in the space of Fig. 4

Figure 5 adds operator expressions to this diagram, but to reduce clutter, those categories that are not immediately relevant are trimmed off, and the decorated categories are removed. Operator expressions have the category val/arg and are represented in Fig. 5 as curved arrows pointing from the argument category to the value category. The curved arrows are labelled either with the informal names of the classes (topic, counting quantifier, focussed XP, hi-neg, lo-neg) or with a representative member (*who*, *no one-NC*, *everyone*, *XP too*, *many people*, *not everyone*).[23]

Expressions that are neutral as to scoping directly above negation have argument categories on the $s2 - s3 - s8 - s9 - s11$ track; those that must not scope directly above negation, on the $s1 - s4 - s7$ track; and negative concord items that must scope directly above negation or one of their own kind, on the $s1^\sim - s4^\sim - s7^\sim$ track.

To see how Fig. 5 captures other data reviewed in Sect. 6.1.1, recall that counters ($s7/s2$) and foci ($s4/s2$) do not co-occur and are therefore not ordered with respect

---

[23] Because question words can only be preceded by topics, a distinction between $s9$ and $s11$ is necessary. These categories are added only to this diagram.

to each other. Notice that neither $s4$ nor $s7$ derives $s2$. The reader is invited to find other examples.

Whenever the value category of an expression derives the argument category of another, the predicted results are grammatical, although more than five or six operators preceding the verb may sound crowded. Consider just one example:

Kati	hat napon át	mindenkivel	sok újságot
Kate	for six days	with everyone	many pieces of news.acc
$s11/s11$	$s11/s11$	$s7/s7$	$s7/s8$

rosszindulatból	nem	közölt.
out of malice	not	shared.3sg
$s4/s2$	$s1^\sim/s1$	$s1/\ldots$

'For six days, for every person there were many pieces of news such that it was out of malice that Kate did not share those pieces of news with that person'

The standard assumption is that like categories coordinate. In the present context this can be implemented in two ways. The default version would be that categories A and B may conjoin iff they derive the same category C. But in fact operators belonging to different classes do not conjoin. This judgment is especially clear in the preverbal domain with verbs that have particles like *ki* 'out'. (Postverbal conjunctions are less diagnostic, because they may involve gapping.)

*Minden lány	és	hatnál több fiú	kiment/ment ki.
every girl	and	more than six boys	out-went/went out

*Minden fiú	és	Kati	kiment.
every boy	and	Kate	out-went

The default version does not predict these data. For example, both $s7/s7$ and $s11/s11$ derive $s11/s7$. Thus a stricter formulation may be warranted: either A should derive B or vice versa, but invoking a third category C is not allowed.

7.2 Using Boolean Formulae

It goes without saying that the small poset in Fig. 4 can also be obtained using conjunctive formulae: see Fig. 6. The only question is whether the atomic propositions in Fig. 6 can be given a straightforward linguistic interpretation. The answer is Yes. We continue to assume that the categories of Hungarian operators are as the curved arrows in Figs. 5 and 8 indicate. ($s10$, $s10^\sim$, $s5$, $s5^\sim$, and $s6$ were not part of the actual Hungarian category inventory, so G and H are not given a realistic interpretation.)

A = A topic can immediately precede me
B = A distributive existential can immediately precede me
C = A negated universal can immediately precede me
D = A negative concord item can immediately precede me
E = A distributive universal can immediately precede me
F = A counter or a focus can immediately precede me
I = Lo-negation can immediately precede me
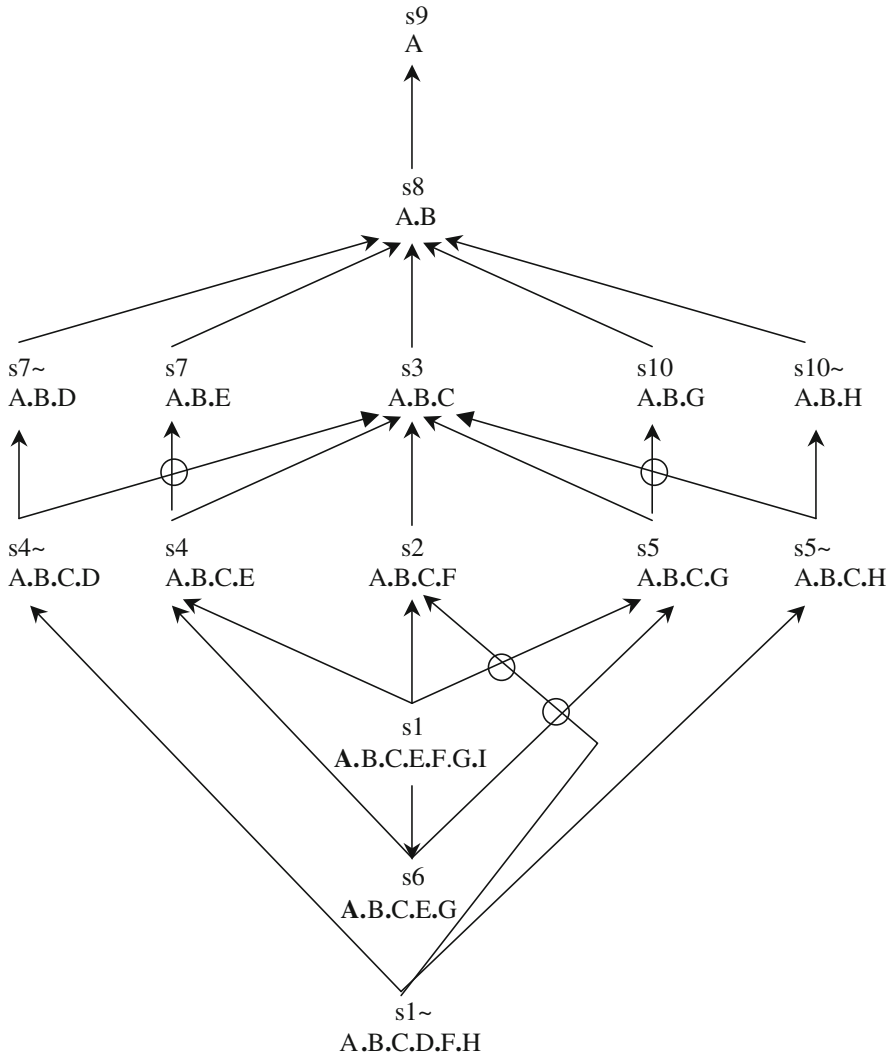
Fig. 6 The partially ordered set in Fig. 4 with conjunctive formulae (conjunction notated with a dot for readability)

For example,

s4 = A.B.C.E = A topic, a distributive existential, a negated universal, and a distributive universal each can immediately precede me.

s4~ = A.B.C.D = A topic, a distributive existential, a negated universal, and a negative concord item each can immediately precede me.

To wit, s4 derives the argument categories of topics (s11), distributive existentials (s8), negated universals (s3), and distributive universals (s7). So the propositions A through I express exactly the kind of information that we used to model the behavior

of Hungarian operators earlier in this paper. Even the generalizations that emerge are the same. Members of the $s1 - s4 - s7$ track share $E =$ "A distributive universal can immediately precede me"; members of the $s1^{\sim} - s4^{\sim} - s7^{\sim}$ track share $D =$ "A negative concord item can immediately precede me", and the conjunct that only $s1$ has is $I =$ "Lo-negation can immediately precede me".

As was explained in Sect. 4.2, categories for polarity items and licensors can be defined using disjunction. Although different NPIs may require different licensors (see Sect. 9 for detailed discussion) adding a single new feature could suffice. Let J be "I need a polarity licensor". The argument category of hi-neg, a strong NPI-licensor is written as $^0(s4^0)$ using Galois operators. Using Boolean formulae it will be as follows:

$^0(s4^0) = (A.B.C.D) \vee J =$ A topic, a distributive existential, a negated universal, and a distributive universal each can immediately precede me, or I need a polarity licensor.

J by itself creates a single copy of the poset in Fig. 6. Further licensing needs K in the language may either be independent of J and/or each other, in which case they will appear as independent disjuncts, or may exhibit derivability relations, which can again be captured using conjunction and disjunction, replicating the multiplicity of copies created by productive iterations of pairs of Galois operators.

The choice between modal and Boolean operators in the definition of subcategories may depend on the linguistic application. The use of residuated and Galois-connected operators fits seamlessly into a grammatical framework that already exploits such operators. See Sect. 8 for a brief demonstration. On the other hand, the Boolean definitions make the descriptive content of the subcategories more transparent.

As Suresh Manandhar (p.c.) has pointed out to us, the same atomic propositions could also be used to define a typed feature structure, [scope: Sco], where scope is a feature name and Sco is the type restriction on the value. By defining a multiple inheritance hierarchy, where a given type can inherit constraints from two super-types neither of which subsumes the other, one can postulate the relations among types obtained by means of conjunction and disjunction. Let PosSco0 $\leq$ Sco and LicSco0 $\leq$ Sco, with PosSco2 $\leq$ PosSco1 $\leq$ PosSco0 and LicSco2 $\leq$ LicSco1 $\leq$ LicSco0, etc. Thinking of scopal operators as s/s, the following kind of functions can be expressed:

$s/(s \wedge [\text{scope} : \text{PosSco} \vee \text{LicSco}])$: can license an NPI

$s/(s \wedge [\text{scope} : \text{PosSco}])$: cannot license an NPI

For concreteness, assume that the values PosSco come from a typed feature hierarchy that replicates the derivability relations of Fig. 6. Thus the general scope possibilities depend on the specific type PosSco and its position within the hierarchy. One of the values LicSco can be "I need a polarity licensor", as in J above.

8 A Sample Natural Deduction Style Derivation

Although this paper is concerned only with the operator categories (those of quantifiers and negation), for concreteness we spell out the complete Natural Deduction style derivation of a simple sentence, including inflectional categories. This shows how the optional operators co-exist with obligatory elements in a grammar, and demonstrates how information is passed on using modal decorations and structural rules. The present section presupposes familiarity with Moortgat (1997) and only adds brief comments pertaining to our innovations in the treatment of syntactic phenomena.

(30) hogy mi nem lát-t-unk   mindenkit.
　　 that   we not   see-past-1pl everyone-acc
　　 '. . .that we did not see everyone'

The analytical assumptions in Fig. 7 follow strictly those argued for in Brody and Szabolcsi (2003) and merely recapture them in a different framework. These assumptions are as follows. Inflectional heads are obligatory; operator expressions are optional. Figure 7 contains three inflectional heads, C (complementizer, see *hogy* 'that'), Agr (agreement, see *unk* '1pl'), and T (tense, see *t* 'past'); for transparency, we write C, Agr and T in the derivation. Morphology spells out the finite verb in Agr but the verb does not move there in syntax. The sequence of operator heads is reiterated above T and Agr. Negation occurs only in the operator sequence above Agr; topics and distributives occur in any of the operator sequences. Although within a single sequence the universal is ordered before negation, negation is capable of scoping over it in (30) because the universal occurs in a lower sequence. The intervening Agr head bridges between the two sequences, in much the same way as NPI-licensors bridge between Incomplete and Good-Enough categories. The topic *mi* 'we' occurs in the Agr-sequence. C closes off the clause and is not preceded by any operators.

The topic and the universal bind traces of category dp. [/I] is interpreted as λ-abstraction, allowing operators to bind their traces.

Undecorated s serves as the category of uninflected sentences. The obligatoriness of inflectional heads is captured by assigning them categories decorated with an indexed box (see Moortgat (1999)) for a detailed description of this use of unary operators). T(ense) for example has the category $\Box_T(s1/s)$. [$\Box$E] moves the decoration over to T in the form of $\langle \ldots \rangle_T$, and the structural rules abbreviated as [Pxxx] pass it back to the whole chunk containing T, right before Agr should enter the picture. Agr now has the category $\Box_A(s1/\Box_T s11)$, which crucially differs from that of T in that the argument it seeks is not uninflected s but a sentence already containing T. [$\Box$I] allows $(everyone \circ (T \circ (see \circ \Diamond \Box dp)))$ to be recognized as such. The same holds for C requiring an argument that contains Agr.

The value categories of all operators derive $s11$. Both Agr and C have $s11$ as their argument categories. This allows any subset of the operator expressions to occur right below C and Agr. The categories of operators that freely occur in any sequence (i.e. either preverbally or postverbally) are not tagged for inflectional heads. Negation however occurs only in the preverbal field. To ensure this its argument category is decorated with $\Box_A$. The $\Box_A$ decoration on its whole functor category plays the same role as it does with Agr.

$$\frac{\text{see} \vdash (s/dp)/dp \qquad \diamond\square dp \vdash dp}{\text{(see} \circ \diamond\square dp) \vdash s/dp} \; [/E]$$

$$\frac{(\text{see} \circ \diamond\square dp) \vdash s/dp \qquad \diamond\square dp \vdash dp}{((\text{see} \circ \diamond\square dp) \circ \diamond\square dp) \vdash s} \; [/E]$$

$$\frac{T \vdash \square_T(s_1/s)}{\langle T\rangle_T \vdash s_1/s} \; [\square E]$$

$$\frac{\langle T\rangle_T \circ ((\text{see} \circ \diamond\square dp) \circ \diamond\square dp) \vdash s_1}{(\langle T\rangle_T \circ (\text{see} \circ \diamond\square dp)) \circ \diamond\square dp \vdash s_1} \; [2P_{assoc}]$$

$$\frac{(\langle T\rangle_T \circ (\text{see} \circ \diamond\square dp)) \circ \diamond\square dp \vdash s_1}{(\langle T\rangle_T \circ (\text{see} \circ \diamond\square dp)) \circ \diamond\square dp \vdash s_7} \; [s_1 \longrightarrow s_7]$$

$$\frac{(\langle T\rangle_T \circ (\text{see} \circ \diamond\square dp)) \circ \diamond\square dp \vdash s_7}{(\langle T\rangle_T \circ (\text{see} \circ \diamond\square dp)) \vdash s_7/\diamond\square dp} \; [/I]$$

$$\frac{\text{everyone} \vdash s_7/(s_7/\diamond\square dp)}{\text{everyone} \circ (\langle T\rangle_T \circ (\text{see} \circ \diamond\square dp)) \vdash s_7} \; [/E]$$

$$\frac{\langle\langle (\text{everyone} \circ (\langle T\rangle_T \circ (\text{see} \circ \diamond\square dp)))\rangle\rangle_T \vdash s_{11}}{(\text{everyone} \circ (T \circ (\text{see} \circ \diamond\square dp))) \vdash \square_T s_{11}} \; [s_7 \longrightarrow s_{11}] \; [P_{xxx}] \; [\square I] \; [/E]$$

$$\frac{\text{Agr} \vdash \square_A(s_1/\square_T s_{11})}{\langle \text{Agr}\rangle_A \vdash s_1/\square_T s_{11}} \; [\square E]$$

$$\frac{\langle \text{Agr}\rangle_A \circ (\text{everyone} \circ (T \circ \text{see})) \vdash s_1}{\langle\langle \text{Agr}\rangle_A \circ (\text{everyone} \circ (T \circ \text{see}))\rangle_A \vdash \tilde{s_1}} \; [2P_{assoc}]$$

$$\frac{\langle\langle \text{Agr}\rangle_A \circ (\text{everyone} \circ (T \circ \text{see}))\rangle_A \vdash \tilde{s_1}}{(\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see}))) \vdash \square_A s_1} \; [\tilde{s_1} \longrightarrow s_{11}] \; [/I]$$

$$\frac{\text{not} \vdash \square_A(\tilde{s_1}/\square_A s_1)}{\langle \text{not}\rangle_A \vdash \tilde{s_1}/\square_A s_1} \; [\square E]$$

$$\frac{\langle \text{not}\rangle_A \circ (\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see}))) \vdash s_{11}}{(\text{not} \circ (\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see})))) \vdash \square_A s_{11}} \; [P_{xxx}]$$

$$\frac{\langle (\text{not} \circ (\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see}))))\rangle_A \vdash \square_A s_{11}}{\dots} \; [\square I] \; [/E]$$

$$\frac{\text{we} \vdash s_{11}/(s_{11}/\diamond\square dp)}{\text{we} \circ (\text{not} \circ (\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see})))) \vdash s_{11}}$$

$$\frac{\langle \text{we} \circ (\text{not} \circ (\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see}))))\rangle_A \vdash s_{11}}{\text{we} \circ (\text{not} \circ (\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see})))) \vdash \square_A s_{11}} \; [P_{xxx}] \; [\square I] \; [/E]$$

$$\frac{C \vdash c/\square_A s_{11}}{C \circ (\text{we} \circ (\text{not} \circ (\text{Agr} \circ (\text{everyone} \circ (T \circ \text{see}))))) \vdash c}$$

Fig. 7 Sample derivation

In sum, our grammar involves several different cases of feature transmission:

(i) Gaps of category *dp* are inherited from expression to expression.
(ii) Tense marks the containing phrase with T, which Agr then requires on its argument.
(iii) Negation both requires its argument to be marked Agr and marks its container phrase with Agr (which in turn C requires on its argument).
(iv) Negative concord (NC) items both require their argument to be negative and mark their container phrases as a negative one.

Cases (i) through (iii) are not local. Their treatment relies on the "lock and key" properties of the unary residuated operators, aided by structural rules for associativity and commutativity. Gaps also require hypothetical reasoning. Case (iv) is strictly local: nothing can intervene between the NC items and negation. This can be handled by lexically decorating their categories with features (expressed using either unary operators or conjunctive formulae) and assigning a "loop" category to NC items.

9 The Monotonicity of Licensing

This section will examine the implementation of Bernardi's theory of licensing in a realistic setting. But an empirical property of NPI-licensing has to be introduced first.

Different negative polarity items require different licensors. Zwarts (1983) proposed that the relevant distinctions can be made in terms of the "negative strength" of the licensors, characterizable with how many of the de Morgan implications each bears out.

f is anti-morphic (AM)   iff $f(a \vee b) = fa \wedge fb$ and $f(a \wedge b) = fa \vee fb$
   e.g., *not*
f is anti-additive (AA)   iff $f(a \vee b) = fa \wedge fb$
   e.g., *never*, *nobody*
f is decreasing (DE)   iff $f(a \vee b) \longrightarrow fa \wedge fb$
   e.g., *seldom*, *at most five men*

Thus we have the following subset relations:

(31)   anti-morphic $\subseteq$ anti-additive $\subseteq$ decreasing

Van der Wouden (1997) provides a detailed discussion of the Dutch NPI-licensing data in these terms. To use examples from other languages, Nam (1994) argues that the Korean exceptive *pakkey* 'only' is an NPI that requires an antimorphic licensor. English *in weeks* requires an antiadditive one, and *ever* is satisfied with one that is (roughly) decreasing:

a. We haven't been there in weeks.
b. Nobody has been there in weeks.
c. *At most five men have been there in weeks.

a. We haven't ever been there.
b. Nobody has ever been there.
c. At most five men have ever been there.

These properties play a role in other licensing relations as well. Roughly decreasing adjuncts undergo negative inversion in English (Büring 2004):
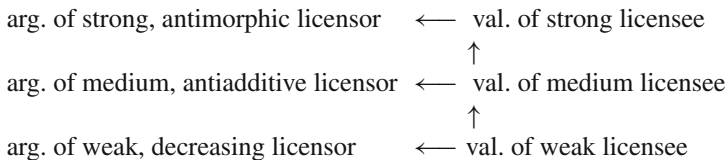
(32)   Under no / few / *some circumstances would I do this.

These data sets exhibit what we may call "the monotonicity of licensing":
    Monotonicity of Licensing:

1.   A weak NPI is licensed by an operator that is decreasing
     or stronger.
2.   A medium NPI is licensed by an operator that is anti-additive
     or stronger.
3.   Negative inversion involves adjuncts that are decreasing **or stronger**.

We expect the syntax of licensing to conform to this generalization (where it indeed holds). How could this be done? One possibility is for *nobody*, for instance, to be tagged separately as decreasing and as anti-additive. But one hopes that it is not necessary to resort to such brute force methods, and the monotonicity of licensing can be captured in the form of derivability (inclusion) relations.

At first blush one might think that this requires incorporating the inclusion relations in (31) into the syntax, but that is not the case.[24] It suffices if the following derivability relations hold between the categories:

$$
\begin{array}{ccc}
\text{arg. of strong, antimorphic licensor} & \longleftarrow & \text{val. of strong licensee} \\
& & \uparrow \\
\text{arg. of medium, antiadditive licensor} & \longleftarrow & \text{val. of medium licensee} \\
& & \uparrow \\
\text{arg. of weak, decreasing licensor} & \longleftarrow & \text{val. of weak licensee}
\end{array}
$$

It is easy to see that if these relations hold, a weak licensee like *ever* for example can be licensed by any of the three kinds of licensors—without the syntax incorporating any derivability relations between the categories of the licensors.

9.1 Is It Logically Viable?

Does our calculus in general and the logical space explored in Fig. 3 in particular make it possible to pick categories in the desired way? The following assignment of value categories to licensees and argument categories to licensors will do. No arrow between two categories means no derivability.[25]

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[24] In Sect. 9.3 we come back to the question whether incorporating (31) into the syntax would be possible at all.

[25] The downward monotonic nature of both the licensors of NPI and the Galois operators is a pure coincidence. Notice that we use Galois operators always in a pair, i.e. as upward monotonic operators, and moreover, the same application of Galois operators could be used to model other sorts of licensing relations that do not involve downward monotonicity of the licensors.
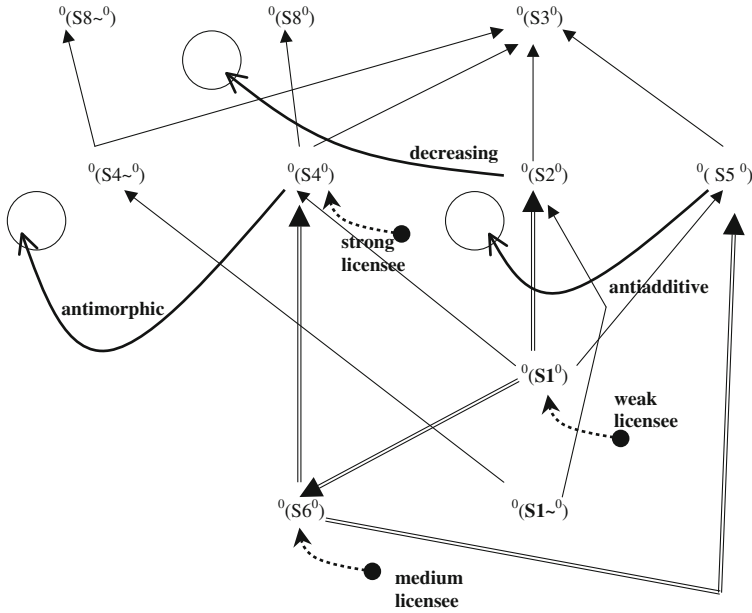
Fig. 8 Licensors and licensees with derivability relations among Incomplete categories

(33)   arg. of strong lic-or $^0(s_4^0) \longleftarrow {}^0(s_4^0)$ val. of strong lic-ee
                                              $\uparrow$
         arg. of medium lic-or $^0(s_5^0) \longleftarrow {}^0(s_6^0)$ val. of medium lice-ee
                                              $\uparrow$
         arg. of weak lic-or $^0(s_2^0) \longleftarrow {}^0(s_1^0)$ val. of weak lic-ee

Figure 8 above shows that this is indeed viable. Figure 8 exhibits double-Galois categories and their derivability relations. Recall from Sect. 5.2 that, in our case, $^0(s_a^0) \longrightarrow {}^0(s_b^0)$ iff $s_a \longrightarrow s_b$. Therefore the patterns of the derivability in Fig. 8 are familiar; they are exactly the same as the ones in Galois-free Fig. 3. Because no double-Galois category derives a Galois-free category, relations between the Galois-free ("Good-Enough") categories that are not part of the diagram cannot cause trouble. The derivability relations relevant in (33) are highlighted with double lines in Fig. 8. It is easy to see that all the relations required in (33) hold. On the other hand, $^0(s4^0)$, $^0(s2^0)$, and $^0(s5^0)$ are independent.

The diagram also contains curved arrows corresponding to the categories of linguistic expressions. The argument categories of licensees (with dotted lines) are replaced by bullets, since they are irrelevant from the present perspective and will vary with the licensees under consideration. The strong, medium and weak licensors are supposed to be antimorphic, antiadditive, and decreasing functors, respectively, which, following Sect. 5.2, point from double-Galois (Incomplete) to Galois-free (Good-Enough) categories. What their concrete value categories are is irrelevant from the general

logical perspective, and so Fig. 8 indicates them with empty circles. They are however absolutely relevant from an empirical perspective, to which we now turn; the reader is invited to fill in the circles in due course.

9.2 Is It Empirically Viable?

What we have seen demonstrates that it is possible in our calculus to assign categories to licensees and licensors in the manner envisaged in Sect. 5.2. The empirical question is whether natural language expressions can be matched up with these possibilities. In this paper we only discuss the empirical properties of licensors in detail. We simply assume that the licensees can be assigned to categories in accordance with Fig. 8.

We take the Hungarian operator poset in Fig. 4 as a point of departure. A quick glance at Fig. 4 reveals that Hungarian has suitable decreasing operators. Almost all the merely decreasing quantifiers in Hungarian are counters, assigned to $s7/s2$ in Fig. 4; this is now revised to $s7/^0(s2^0)$. The revision does not affect the word order behavior of counters, since, among the categories we use, all and only those Galois-free categories that derive $s2$ derive $^0(s2^0)$.[26]

Nem mindenki 'not_everyone' is decreasing but its word order behavior slightly differs from that of counters; its category in Fig. 4 is $s4^\sim/s3$. If *nem mindenki* is a good NPI-licensor, then its category should be revised to $s4^\sim/^0(s3^0)$. It turns out that negated universals are cross-linguistically poor licensors of even weak NPIs, compare:

(34)   *Not everyone saw anything / has ever been there.

Why this is so is something of a mystery. One possibility is to attribute the unacceptability of (34) to the intervention of *everyone* between *not* and *anything/ever*, cf. *I don't think that everyone saw anything* (Linebarger 1987). If however there is reason to analyze *not everyone* as one complex quantifier, then the intervention account becomes less obvious. Indeed, the complex quantifier analysis was motivated for Hungarian in Sect. 6.1.2. Since the judgment in (34) is replicated in Hungarian, there is no reason to assign the licensor category $s4^\sim/^0(s3^0)$ to *nem mindenki*.

Hungarian is a strict negative concord language and as such it has no antiadditive quantifier phrases comparable to English *no one*. We may however contemplate a closely related imaginary language Hungarian′ that has no strict negative concord but has an antiadditive quantifier. This imaginary item is comparable to *no one* in scope behavior: it does not scope immediately above negation but can be immediately outscoped by a decreasing counter, cf.

   *No one didn't laugh.
   Few men saw no one.

These properties are guaranteed by assigning it to the category $s2/s5$, to be revised as $s2/^0(s5^0)$ because it is an NPI-licensor.

---

[26] Although both 'more than six men' and 'few men' are counting quantifiers, their categories are now distinguished: 'more than six men' is $s7/s2$ but 'few men' is $s7/^0(s2^0)$. Thus while their word order behavior is otherwise the same, only the latter is a licensor.

Hungarian has even two anti-morphic operators: lo-neg and hi-neg. Are they both strong licensors? If not, which of the two is? It turns out that the choice of lo-neg is simply incompatible with our most basic assumptions. If it were a strong licensor, its category would be $s1^\sim/^0(s1^0)$. But $^0(s1^0)$ is the bottom element of our small set of categories, and indeed the "center" of the whole (infinite) set of categories defined by our calculus. If the value category of strong licensees (or medium licensees, for that matter) derived $^0(s1^0)$, then it would derive the argument categories of all licensors. That move would wipe out all the strong/weak distinctions we are trying to accommodate. Therefore lo-neg is not in the game. Fortunately, we can resort to the hi-neg version of *nem*, previously assigned to category $s4^\sim/s4$. This is now revised as $s4^\sim/^0(s4^0)$. $^0(s4^0)$ is the right value category for strong licensees, because it does not derive the argument categories of either weak or medium licensors, and of course it derives itself in the capacity of being the argument category of the strong licensors.

The assumption that hi-neg is a licensor but lo-neg is not is empirically less strange than it may initially sound. In (35), where the finite verb is preceded by just one negation and by no focus or counter, this negation could be an instance of either lo-neg or hi-neg. The category of the inflected verb, $s1$ derives the argument categories of both $s1$ and $s4$.

(35) **Nem** hiszem, hogy **valaki is** hallotta volna a hírt.
     not think.1sg that someone even heard aux the news
     'I don't think that anyone heard the news'

Since (35) contains the NPI *valaki is* 'someone even', we will simply take its *nem* to be hi-neg. The one case where hi-neg and lo-neg are distinguishable is where a focus or counter precedes the negation. Our analysis makes the prediction that (36), which cannot but involve lo-neg, is unacceptable. As linguists often say, the judgment is subtle, but the example is certainly less natural than (35):

(36) [??]**ÉN nem** hiszem, hogy **valaki is** hallotta volna a hírt.
     I not think.1sg that someone even heard aux the news
     'It is me who doesn't think that anyone heard the news'

Hi-neg licenses an NPI only in the absence of an intervening focus, cf. (37). Since Linebarger (1987) NPI-licensing has been known to be sensitive to operator intervention. Whatever technique is employed to capture this, it will rule out (37):

(37) *__Nem én__ hiszem, hogy **valaki is** hallotta volna a hírt.
     not I think.1sg that someone even heard aux the news
     'It is not me who thinks that anyone heard the news'

All in all, it is not unreasonable to assume that in this licensing domain hi-neg, but not lo-neg, represents the strong, antimorphic licensor and, given the fact that lo-neg is at the bottom of our category set, this is indeed the only option.

To summarize, the following licensors fit the recipe and work for the reincarnation of Hungarian dubbed Hungarian′:

    Strong NPI-licensor: $s4^\sim/^0(s4^0)$ – example: hi-neg *nem*
    Medium NPI-licensor: $s2/^0(s5^0)$ – example: imaginary 'no one'
    Weak NPI-licensor: $s7/^0(s2^0)$ – example: any decreasing counter

Notice that in this theory not only NPI-licensing is a licensing relation. Any structure that must be immediately outscoped by a particular kind of operator is a "licensee"—one whose value category is an Incomplete category, wherefore its superstructure can only derive $s11$, the category of Complete sequences if it is brought back to the "Good-Enough plane". Does our theory predict that no licensee might call for, or allow for, lo-neg as a licensor in Hungarian (or in Hungarian$'$)? It does not. Recall from Sect. 5.2 that infinitely many distinct "Incomplete planes" can be formed by adding new pairs of Galois-operators. The iteration of different pairs, i.e. $^0(\cdot^0)$ followed by $(^0\cdot)^0$ and conversely $(^0\cdot)^0$ followed by $^0(\cdot^0)$ produces inequalities, $(^0A)^0 \longrightarrow {}^0(((^0A)^0)^0)$ but $^0(((^0A)^0)^0) \not\longrightarrow (^0A)^0$ and similarly for the other combination.

(38)

Good-Enough		**Incomplete**		**Incomplete**	
s_b	$\longrightarrow$	$^0(s_b^0)$	$\longrightarrow$	$(^0(^0(s_b^0)))^0$	$\longrightarrow \dots$
$\uparrow$		$\uparrow$		$\uparrow$	
s_a	$\longrightarrow$	$^0(s_a^0)$	$\longrightarrow$	$(^0(^0(s_a^0)))^0$	$\longrightarrow \dots$

Suppose that some licensee has the value category $(^0(^0(s_1^0)))^0$, and that is the argument category of lo-neg. Since $(^0(^0(s_1^0)))^0$ does not derive $^0(s_1^0)$, this addition does not interfere with (38). Essentially, each kind of licensing relation may be associated with a different "Incomplete plane".

9.3 Semantics in the Syntax? Is Licensing Truly Monotonic?

In (31) it was observed that licensors exhibit a semantic inclusion relation: anti-morphic $\subseteq$ anti-additive $\subseteq$ decreasing. Perhaps the most appealing way to accommodate the monotonicity of licensing would be to import the corresponding derivability relations between argument and value categories into the syntax.

Would that be possible? We have already seen some empirical reasons why it would not be. First and foremost, the semantic approach would force us to treat hi-neg and lo-neg alike. Or, if they can be semantically distinguished, lo-neg might end up as "the" anti-morphic operator. But lo-neg is at the bottom of both the semantic inclusion hierarchy and the syntactic category hierarchy. Therefore, as was observed above, if the value category of strong licensees derived the argument category of lo-neg, it would inescapably derive the argument categories of medium and weak licensors as well, and all the licensing distinctions would be lost.

Secondly, notice that for the sake of the argument we considered a Hungarian$'$ which is not a strict negative concord language (and thus has anti-additive generalized quantifiers) and whose NPIs are exactly like NPIs in English or Dutch. These two related properties of Hungarian$'$ do not hold of plain Hungarian. Progovac (1994) observed that the distribution of English *anything* is covered by two complementary items in Serbo-Croatian: *ništa* in the context of clause-mate negation and *išta* elsewhere. *Ništa* is a strict negative concord item in our sense, and *išta* a NPI. But the licensing of *išta* is non-monotonic: while it is licensed by clause-mate 'few men', it is not licensed by clause-mate 'not'. The same holds for Hungarian: *senki* is the equivalent of *ništa* and *valaki is* of *išta*. (The latter would have the value category $^0(s2^0)$, not $^0(s1^0)$ when its licensor is clausemate.) If derivability relations corresponding to semantic inclusion

were part of the syntax, licensing should always be monotonic, which Serbo-Croatian and Hungarian show is not the case.[27]

The conclusion is that semantic inclusion does not amount to syntactic inclusion. It is not true that a semantically stronger operator can do everything in syntax that a semantically weaker one can. It may have restrictions of its own that the weaker one lacks. Therefore, simply importing semantic inclusion relations into the syntax is not possible.

The natural explanation of the mismatch between semantic properties and word order behavior is that each expression has many semantic properties, whereas our syntax builds all word order properties into the syntactic category of the expression. (This is indeed the basic idea of categorial grammar. If I know your category, I know how you behave.) But then we cannot expect one particular semantic property to correspond to a syntactic category. Our syntax differs from the Minimalist syntax employed in Stabler (1997), for example, where each lexical item is a bundle of syntactic features, including [determiner], [decreasing], [singular], etc. Stabler's idea is to couple that syntax with a Natural Logic, whose inference schemata are anchored to some of the features, e.g. [decreasing]. Stabler's framework would probably lend itself more easily to studying whether a thorough-going match between genuine semantic properties and syntactic behavior can be found.

10 Summary

This paper has argued that using a partially ordered set of categories offers a unified theory for solving the problem of complement selection in the presence of optional categories and accommodating licensing relations. The partial ordering on the set of categories could always be stipulated. If instead the category labels are logical formulae, then the ordering is given by the derivability relation of the logic. This fact has at least two important advantages. One is that the logic will predict what categories or feature structures can combine; this is how Johnson (1991), Johnson and Bayer (1995), and Blackburn and Spaan (1993) use their logics. Another is that the logic will allow one to create systematic relationships between certain, smaller or larger, sets of categories. This use of the logic is more novel, to our knowledge.

(i) The categories $s1$, $s2$, and $s3$ are members of the same basic poset and are ordered as $s1 \longrightarrow s2 \longrightarrow s3$. This models the situation where expressions of category $s1$ or $s2$ can satisfy a higher head that selects for a complement of category $s3$, i.e. where $s2$ and $s3$ are optional.

(ii) Such a poset, or parts of such a poset, can be multiplied by the use of different modes. The categories $s1$ and $s1^{\sim}$ belong to two distinct modes and are not ordered with respect to each other. However, just as $s1 \longrightarrow s4 \longrightarrow s3$, $s1^{\sim} \longrightarrow s4^{\sim} \longrightarrow s3$. Therefore there are two minimally distinct ways to derive $s3$. This models the situation where $s1$ and $s1^{\sim}$ differ from each other in one respect and some other categories are sensitive to the distinction; in all other respects

---

[27] Empirically even the English data are more complicated, see De Decker et al. (2005). In this paper we investigated the idealization that forms the basis of the consensus in the literature.

however $s1$ and $s1^\sim$ as well as those other categories behave identically. We used two modes to capture some quantifiers' constraints with respect to negation in their immediate scope.

(iii) The basic poset (together with its distinct modes) is fully replicated by arbitrarily many other posets unidirectionally derived from it: $s1' \longrightarrow s2' \longrightarrow s3', s1'' \longrightarrow s2'' \longrightarrow s3''$, etc. Fully replicated means that the exact same derivability relations obtain in each copy: $s1 \longrightarrow s2$ iff $s1' \longrightarrow s2'$, and unidirectionality means that $s1 \longrightarrow s1' \longrightarrow s1''$ but never the other way around. Given this unidirectionality, an expression whose category belongs to one of the "primed" copies can only be part of a Good-Enough sentence (whose category is in the basic poset) if a wider scoping operator maps it back to the basic poset. This models the situation where an otherwise well-formed expression is Incomplete in that it requires licensing by a particular wider scoping operator; each "primed" copy corresponds to one kind of licensing need. We used such an Incomplete copy to assign categories to expressions containing an unlicensed NPI.

The fact that the different modes and copies have identical internal derivability relations ensures that "other things" are always kept equal.

The grammar outlined in this paper was formulated using a version of the Lambek calculus. However, the ideas are independent both of the Lambek calculus and of our particular additions. The same ideas pertaining to the role of partial ordering can be implemented in theories that do not use these particular techniques, as was briefly demonstrated.

11 Part III: More Logic

11.1 Sound and Complete Systems

Below we present the extensions of the Lambek system (NL) by means of residuated unary operators ($\mathsf{NL}(\Diamond)$), and Galois operators ($\mathsf{NL}(\Diamond,\cdot^{\mathbf{0}})$), we have been using through the paper.

11.1.1 Proof Theory

Definition 11.1.1 ($\mathsf{NL}, \mathsf{NL}(\Diamond), \mathsf{NL}(\Diamond,\cdot^{\mathbf{0}})$: *Axiomatic System*) The system NL is defined by the axioms below. Given $A, B, C \in \mathsf{FORM}$

$$
\begin{aligned}
&\text{[REFL]} \quad A \longrightarrow A,\\
&\text{[TRANS]} \quad \text{If } A \longrightarrow B \text{ and } B \longrightarrow C, \text{ then } A \longrightarrow C,\\
&\text{[RES}_2\text{]} \quad A \longrightarrow C/B \text{ iff } A \bullet B \longrightarrow C \text{ iff } B \longrightarrow A\backslash C.
\end{aligned}
$$

$\mathsf{NL}(\Diamond)$ is obtained by adding the following Axiom:

$$
\text{[RES}_1\text{]} \qquad \Diamond A \longrightarrow B \ \text{ iff } \ A \longrightarrow \Box B
$$

Furthermore, $\mathsf{NL}(\Diamond)$ can be extended with [GC].

$$[\text{GC}] \quad A \longrightarrow {}^{\mathbf{0}}B \text{ if and only if } B \longrightarrow A^{\mathbf{0}}.$$

Alternatively, one adds [A1], [A2] and the rules [R1], [R2].

$$[\text{A1}] \ A \longrightarrow {}^{\mathbf{0}}(A^{\mathbf{0}}).$$
$$[\text{A2}] \ A \longrightarrow ({}^{\mathbf{0}}A)^{\mathbf{0}}.$$
$$[\text{R1}] \ \text{From } A \longrightarrow B \text{ infer } B^{\mathbf{0}} \longrightarrow A^{\mathbf{0}}.$$
$$[\text{R2}] \ \text{From } A \longrightarrow B \text{ infer } {}^{\mathbf{0}}B \longrightarrow {}^{\mathbf{0}}A.$$

It is easy to show that [GC] is a derived rule in this setting. A similar alternative presentation could have been given while introducing $\mathsf{NL}(\Diamond)$. There as well, we could obtain an axiomatic system based on the composition of residuated type forming operators and on their monotonicity properties.

Gentzen Sequent Calculus presentations of these systems have been given in (Kurtonina and Moortgat 1995) and (Areces et al. 2003), respectively, where they have been proved to be equivalent to the axiomatic presentation above and decidable.

11.1.2 Model Theory

The Lambek calculi and their extensions with unary operators are modal logics. Standard models for modal logics are Kripke models, or relational structures.

In (Kurtonina 1995) $\mathsf{NL}(\Diamond)$ has been proved to be sound and complete with respect to the Kripke semantics for all frames and valuations.

Definition 11.1.2 (*Kripke Models*) A model for $\mathsf{NL}(\Diamond)$ is a tuple $\mathcal{M} = (W, R_{\bullet}^3, R_{\Diamond}^2, V)$ where W is a non-empty set, $R_{\bullet}^3 \subseteq W^3$, $R_{\Diamond}^2 \subseteq W^2$, and V is a valuation $V : \mathsf{ATOM} \to \mathcal{P}(W)$. The $R_{\bullet}^3$ relation governs the residuated triple $(\backslash, \bullet, /)$, the $R_{\Diamond}^2$ relation governs the residuated pair $(\Diamond, \Box)$. Given a model $\mathcal{M} = (W, R, V)$ and $x, y \in W$, the *satisfiability relation* is inductivly defined as follows

$$\mathcal{M}, x \Vdash A \text{ iff } x \in V(A) \text{ where } A \in \mathsf{ATOM}.$$
$$\mathcal{M}, x \Vdash \Diamond A \text{ iff } \exists y[R_{\Diamond}xy \ \& \ \mathcal{M}, y \Vdash A].$$
$$\mathcal{M}, y \Vdash \Box A \text{ iff } \forall x[R_{\Diamond}xy \to \mathcal{M}, x \Vdash A].$$
$$\mathcal{M}, x \Vdash A \bullet B \text{ iff } \exists y \exists z[R_{\bullet}xyz \ \& \ \mathcal{M}, y \Vdash A \ \& \ \mathcal{M}, z \Vdash B].$$
$$\mathcal{M}, y \Vdash C/B \text{ iff } \forall x \forall z[(R_{\bullet}xyz \ \& \ \mathcal{M}, z \Vdash B) \to \mathcal{M}, x \Vdash C].$$
$$\mathcal{M}, z \Vdash A \backslash C \text{ iff } \forall x \forall y[(R_{\bullet}xyz \ \& \ \mathcal{M}, y \Vdash A) \to \mathcal{M}, x \Vdash C].$$

The Kripke style semantics of $\mathsf{NL}(\Diamond)$ has been extended to $\mathsf{NL}(\Diamond, \cdot^{\mathbf{0}})$ in (Areces et al. 2003). A model for $\mathsf{NL}(\Diamond, \cdot^{\mathbf{0}})$ is a tuple $\mathcal{M} = (W, R_{\bullet}^3, R_{\Diamond}^2, R_{\mathbf{0}}^2, V)$, where W, V and the accessibility relations $R_{\bullet}^3$ and $R_{\Diamond}^2$ are as before. The new binary relation $R_{\mathbf{0}}^2$ governs the Galois connected pair $({}^{\mathbf{0}}\cdot, \cdot^{\mathbf{0}})$ as defined below.

$$\mathcal{M}, x \Vdash A^{\mathbf{0}} \text{ iff } \forall y(Rxy \to \mathcal{M}, y \not\Vdash A).$$
$$\mathcal{M}, x \Vdash {}^{\mathbf{0}}A \text{ iff } \forall y(Ryx \to \mathcal{M}, y \not\Vdash A).$$

See (Areces et al. 2003) NL($\Diamond, \cdot^{\mathbf{0}}$) for the proof of soundness and completeness. Recall that using the Lambek Calculi as a grammar means looking at the worlds of the Kripke models as sets of linguistics structures. Hence, for instance, the collection of such structures associated with $\Diamond A$ is the set of all y's s.t. $M, y \Vdash \Diamond A$.

12 Properties of Unary Residuated Operators

The proofs of the properties of residuated unary operators are given below. All the derivability relations among two formulae decorated with $\Diamond$ and $\Box$ are due to these properties. The arrows in the Fig. 3 are the results of different orders of application of these properties.

(a) Unit: $\Diamond\Box A \longrightarrow A$

    i. $\Box A \longrightarrow \Box A$ [Axiom]
    ii. $\Diamond\Box A \longrightarrow A$ [Residuation]

(b) Co-unit: $A \longrightarrow \Box\Diamond A$

    i. $\Diamond A \longrightarrow \Diamond A$ [Axiom]
    ii. $A \longrightarrow \Box\Diamond A$ [Residuation]

(c) Monotonicity of $\Box$

    i. $\Diamond\Box A \longrightarrow A$ [by Unit]
    ii. $A \longrightarrow B$ [*Hypothesis*]
    iii. $\Diamond\Box A \longrightarrow B$ [from i. and ii. by trans]
    iv. $\Box A \longrightarrow \Box B$ [from iii. by Residuation]

(d) similarly for the Monotonicity of $\Diamond$

Not all iterations of unary operators patterns produce formulae that are not interderivable with simpler ones. In particular, the iteration of $\Box\Diamond$ is unproductive, i.e. we obtain interderivability of formulae: $\Box\Diamond\Box\Diamond A \longleftrightarrow \Box\Diamond A$; similarly for $\Diamond\Box$, $\Diamond\Box A \longleftrightarrow \Diamond\Box\Diamond\Box A$.

On the other hand, if we compose $\Box\Diamond$ with the other pair $\Diamond\Box$ we obtain inequalities, viz. $\Diamond\Box A \longrightarrow \Box\Diamond\Diamond\Box A$ and similarly for $\Diamond\Box A \longrightarrow \Diamond\Box\Box\Diamond A$. But neither $\Box\Diamond\Diamond\Box A$ nor $\Diamond\Box\Box\Diamond A$ derive $\Diamond\Box A$, though they derive $\Box\Diamond A$. Other productive patterns of unary operators are what we call *center embeddings*. In particular, if we plug the pair $\Diamond\Box$ into the middle of $\Diamond\Box$ we obtain an inequality among the formulae, namely $\Diamond\Diamond\Box\Box A \longrightarrow \Diamond\Box A$ but not the other way around. Similarly, if we plug $\Box\Diamond$ into the middle of $\Box\Diamond$ we obtain a new formula, viz. $\Box\Diamond A \longrightarrow \Box\Box\Diamond\Diamond A$. In the following, we highlight the embedded pairs by underlining them.

Unproductive Iteration: Unproductive iterations are due to the fact that both $\Diamond\Box\cdot$ and $\Box\Diamond\cdot$ are closure operators.

(a) $\Box\Diamond A \longleftrightarrow \Box\Diamond\Box\Diamond A$

    i. $\Diamond A \longrightarrow \Diamond A$ [Axiom]
    ii. $\Diamond\Box\Diamond A \longrightarrow \Diamond A$ [from i. by Unit]
    iii. $\Box\Diamond\Box\Diamond A \longrightarrow \Box\Diamond A$ [from ii. by Mon. of $\Box$]

i'. $\Box\Diamond A \longrightarrow \Box\Diamond\Box\Diamond A$ [by Co-unit]

(b) $\Diamond\Box A \longleftrightarrow \Diamond\Box\Diamond\Box A$

    i. $\Box A \longrightarrow \Box A$ [Axiom]

    ii. $\Box A \longrightarrow \Box\Diamond\Box A$ [from i. by Co-unit]
    iii. $\Diamond\Box A \longrightarrow \Diamond\Box\Diamond\Box A$ [from ii. by Mon. of $\Diamond$]
    i'. $\Diamond\Box A \longleftarrow \Diamond\Box\Diamond\Box A$ [by Unit]

Productive Iterations: Productive iterations are obtained in two ways, (I) by combining different pairs of unary operators, namely $\Diamond\Box$ with $\Box\Diamond$ and conversely, and (II) by center embeddings.[28] The derivations are spelled out below.

(I)   i. $A \longrightarrow A$
      ii. $\Diamond\Box A \longrightarrow \Diamond\Box A$ [by Mon. of $\Box$ and Mon. of $\Diamond$]
      iii. $\Diamond\Box A \longrightarrow \Box\Diamond\Diamond\Box A$ [by Co-Unit]

      i. $A \longrightarrow A$
      ii. $A \longrightarrow \Box\Diamond A$ [Co-Unit]
      iii. $\Diamond\Box A \longrightarrow \Diamond\Box\Box\Diamond A$ [Mon. of $\Diamond$ and Mon. of $\Box$]

(II)   a) $\Diamond\underline{\Diamond\Box}\Box A \longrightarrow \Diamond\Box A$

      i. $\Box A \longrightarrow \Box A$ [Axiom]
      ii. $\Diamond\Box\Box A \longrightarrow \Box A$ [from i. by Unit]
      iii. $\Diamond\Diamond\Box\Box A \longrightarrow \Diamond\Box A$ [from ii. by Mon. of $\Diamond$]
      whereas, $\Diamond\Box A \not\longrightarrow \Diamond\Diamond\Box\Box A$

    b) $\Box\Diamond A \longrightarrow \Box\underline{\Box\Diamond}\Diamond A$

      i. $\Diamond A \longrightarrow \Diamond A$ [Axiom]

      ii. $\Diamond A \longrightarrow \Box\Diamond\Diamond A$ [from i. by Co-unit]
      iii. $\Box\Diamond A \longrightarrow \Box\Box\Diamond\Diamond A$ [from ii. by Mon. of $\Box$]
      whereas, $\Box\Box\Diamond\Diamond A \not\longrightarrow \Box\Diamond A$

13 Properties of the Galois Operators

Similar observations hold for the Galois operators. In this case as well, we have to consider the two different pairs, namely, $(^0\cdot)^0$ and $^0(\cdot^0)$. For the sake of transparency we notate the second pair as $^\bullet(A^\bullet)$.

As in the case of the residuated operators, iteration yields an equality, $(^0A)^0 \longleftrightarrow (^0((^0A)^0))^0$ and $^\bullet(A^\bullet) \longleftrightarrow {}^\bullet((^\bullet(A^\bullet))^\bullet)$. On the other hand, the composition of different pairs produces inequalities, namely $(^0A)^0 \longrightarrow (^0(^\bullet(A^\bullet)))^0$, $(^0A)^0 \longrightarrow {}^\bullet(((^0A)^0)^\bullet)$, and the same holds for the formula $^\bullet(A^\bullet)$.

---

[28] These patterns have been pointed out to us by Eytan Zweig during the visit of the first author to the NYU Linguistics Department.

Furthermore, in this case as well productive patterns are obtained by means of center embeddings. We can embed $^0(\cdot^0)$ within another pair of the same sort $^0(\cdot^0)$ obtaining $^0(^0((A^0)^0)) \longrightarrow^0 (A^0)$ and similarly for the other pair.

(a) Co-unit': $A \longrightarrow (^0A)^0$

    i. $^0A \longrightarrow {}^0A$ [Axiom]
    ii. $A \longrightarrow (^0A)^0$ [Galois]

(b) similarly for the other pair

(c) Monotonicity of $\cdot^0$

    i. $A \longrightarrow (^0A)^0$ [Co-unit']
    ii. $B \longrightarrow A$ [Axiom]
    iii. $B \longrightarrow (^0A)^0$ [from i. and ii. by trans.]
    iv. $^0A \longrightarrow^0 B$ [from iii. by Galois def.]

(d) similarly for $^0\cdot$

Unproductive Iterations:

(a) $(^0A)^0 \longleftrightarrow (^0((^0A)^0))^0$

    i. $^0A \longrightarrow^0 A$ [Axiom]
    ii. $^0A \longrightarrow^0 ((^0A)^0)$ [from i. by Co-Unit']
    iii. $(^0((^0A)^0))^0 \longrightarrow (^0A)^0$ [from ii. by Mon. of $\cdot^0$]
    i'. $(^0A)^0 \longrightarrow (^0((^0A)^0))^0$ [by Co-unit']

(b) similarly for the other pair.

Productive Iterations:

    (I) $(^0A)^0 \longrightarrow^\bullet (((^0A)^0)^\bullet)$ simply by Co-unit. Whereas, $^\bullet(((^0A)^0)^\bullet) \not\longrightarrow (^0A)^0$

    (II)   (a)   i. $A^0 \longrightarrow A^0$ [Axiom]
                      ii. $A^0 \longrightarrow^0 ((A^0)^0)$ [from i. by Co-Unit']
                      iii. $^0(^0((A^0)^0)) \longrightarrow^0 (A^0)$ [from ii. by Mon. of $^0\cdot$]

        (b)   similarly with the other pair.

For the application in this paper it is important to pay particular attention to the following difference between Galois-connected and residuated operators: while the pair of residuated operators $\Diamond\Box$ can disappear from a formula by means of the Unit $\Diamond\Box A \longrightarrow A$, there is not such possibility for the pairs of Galois, there is neither $(^0A)^0 \longrightarrow A$ nor $^0(A^0) \longrightarrow A$. The failure of both these derivability relations is easily checked: in the definition of Galois operators both $^0\cdot$ and $\cdot^0$ are on the right side of the $\longrightarrow$ and hence they cannot be brought on the left as it happens for the $\Diamond$ (see the derivation of the Unit above 12.)

This fact is relevant for us, since we use pairs of Galois operators to mark Incomplete expressions, and of course, they should not have the power of becoming Good-Enough by themselves, but rather only when a proper operators (a licensor) take scope over them.

14 Isomorphic Copies

We want to show that the copy of any poset will be isomorphic to the original one. In other words, that (a) given s_i and s_j of a poset s.t. $s_i \longrightarrow s_j$, their copies are also in the derivability relation in the copied poset, i.e. $(^0s_i)^0 \longrightarrow (^0s_j)^0$ (similarly for the other pair of Galois); and (b) given s_i and s_j s.t. $s_i \not\longrightarrow s_j$, their copies are not in a derivability relation either, i.e. $(^0s_i)^0 \not\longrightarrow (^0s_j)^0$.

Recall that s_i are formulae built from the atomic formula s by means of pair of residuated operators only. Furthermore, sentential categories within the same Incomplete poset are decorated by (i) the same number of Galois pairs; (ii) the exact same patterns of Galois pairs. This means that none of the two situations below will occur.

(i) Assume we would allow two sentential categories of the same poset to be decorated by a different number of Galois pairs. Then, the isomorphism would not be preserved.
Take s_i and s_j to be $(^0p)^0$ and p, respectively, then $s_i \not\longrightarrow s_j$, but $(^0s_i)^0 \longrightarrow (^0s_j)^0$, i.e. $(^0((^0p)^0))^0 \longrightarrow (^0p)^0$, as it is spelled out below.

$$\frac{\dfrac{\dfrac{p \longrightarrow p}{^0p \longrightarrow^0 p}}{^0p \longrightarrow^0 ((^0p)^0)}}{(^0((^0p)^0))^0 \longrightarrow (^0p)^0}$$

This is due to the fact that by applying the Galois pair to s_i we have formed a closure, i.e., $(^0((^0p)^0)^0 \longrightarrow p$. Since the number of Galois pairs decorating sentential categories within the same Incomplete poset is the same, no such s_i and s_j exists within the same Incomplete copied poset.

(ii) Assume that in a poset there are sentential categories decorated by different pairs of Galois. Take s_i and s_j to be $(^0p)^0$ and $^0(p^0)$, respectively, then $s_i \not\longrightarrow s_j$, but $(^0s_i)^0 \longrightarrow {}^0(s_j^0)$, i.e. $(^0((^0p)^0))^0 \longrightarrow^0 ((^0(p^0))^0)$. By applying the Galois pair to s_i and s_j we have formed a closure, i.e., $(^0((^0p)^0))^0 \longrightarrow p$, and $^0((^0(p^0))^0) \longrightarrow p$. Hence, $(^0((^0p)^0))^0 \longrightarrow^0 ((^0(p^0))^0)$, simply reduces to $p \longrightarrow p$. Since sentential categories within the same Incomplete poset are decorated by the exact same Galois pair patterns, no such s_i and s_j exist within the same Incomplete copy.
Notice that the example in (ii) can be used to observe that it is not true that if $(^0(A))^0 \longrightarrow (^0(B))^0$ then $A \longrightarrow B$. Take A and B to be $(^0p)^0$ and p, respectively, $(^0p)^0 \not\longrightarrow p$, but $(^0(^0p)^0)^0 \longrightarrow (^0p)^0$ as shown above.

We can now, look at the proof of (a) and (b),

(a) The first part is trivial, it simply follows by the monotonicity of the Galois pairs.

1. Let s_i and s_j to be of the Good-Enough poset, i.e. they are decorated only by residuated unary operators. Starting from $(^0s_i)^0 \longrightarrow (^0s_j)^0$ we can apply the monotonicity rules of the two Galois and arrive to $s_i \longrightarrow s_j$. Hence, since there exists a derivation, $\mathcal{D}$, of $s_i \longrightarrow s_j$, it holds that $(^0s_i)^0 \longrightarrow (^0s_j)^0$.

Take for instance, $s_i = \Diamond\Box p$ and $s_j = p$:

$$
\frac{\dfrac{\begin{array}{c}\mathcal{D}\\ \vdots\end{array}}{\dfrac{\Diamond\Box p \longrightarrow p}{\dfrac{{}^0 p \longrightarrow ({}^0(\Diamond\Box p))}{({}^0(\Diamond\Box p))^0 \longrightarrow ({}^0 p)^0}}}{}
$$

since there exists a derivation, $\mathcal{D}$, of $\Diamond\Box p \longrightarrow p$, it holds that $({}^0\Diamond\Box p)^0 \longrightarrow ({}^0 p)^0$.

Similarly, for the other pair of Galois.

2. Let $P(s_i)$ and $P(s_j)$ be two sentential categories of an Incomplete poset, where P stands for the pattern of pairs of Galois operators decorating the atomic formula. By induction hypothesis (a) holds for $P(s_i)$ and $P(s_j)$ when the number of pair is n. The derivation would consists of applications of Galois rules so to arrive to the atomic formula decorated only by residuated pairs as in the basic case.

3. Let $P(s_i)$ and $P(s_j)$ be two sentential categories of an Incomplete poset, and let the number of pairs in P be $n + 1$. (a) holds by I.H., since the only thing we can do is to first eliminate the out most pair of Galois operators and get back to the I.H. case.

(b) The second part is guaranteed by the fact that iteration of Galois never form closure operators, and that sentential categories within the same poset are decorated by the same order of pairs of Galois operators.

1. Let s_i, s_j be two sentential categories in the Good-Enough poset, such that $s_i \not\longrightarrow s_j$, we want to show that $({}^0 s_i)^0 \not\longrightarrow ({}^0 s_j)^0$. Again, this is guaranteed by the fact that s_i and s_j are decorated only by pairs of residuated unary operators. Since there is no way for Galois and residuated unary operators to interact with each other, the only rules that can be applied are the ones eliminating the Galois operators till we reach atomic formulas decorated only by residuated operators. Hence, the proof goes as before. We look at the following derivation by means of example. Take $s_i = p$ and $s_j = \Diamond\Box\Box\Diamond p$,

$$
\frac{\dfrac{\begin{array}{c}\mathcal{D}\\ \vdots\end{array}}{\dfrac{p \longrightarrow \Diamond\Box\Box\Diamond p}{\dfrac{{}^0(\Diamond\Box\Box\Diamond\Diamond p) \longrightarrow {}^0 p}{({}^0 p)^0 \longrightarrow ({}^0(\Diamond\Box\Box\Diamond\Diamond p))^0}}}{}
$$

since there is no such derivation, $\mathcal{D}$, we conclude that $({}^0 p)^0 \not\longrightarrow ({}^0(\Diamond\Box\Box\Diamond\Diamond p))^0$.

Hence, since $s_i \not\longrightarrow s_j$, then $s_i^1 \not\longrightarrow s_j^1$.

2. Similarly, it follows that this holds also for further copies of the Incomplete poset.

15 Proof of Non-Derivability

Here we show that in $\mathsf{NL}(\Diamond,\cdot^{\mathbf{0}})$, for any s_i, s_j in Fig. 3, such that there is no derivability arrow from s_i to s_j, then $s_i \nrightarrow s_j$. We first look at the Good-Enough poset, then to the Incomplete ones.

The easiest way to see that the derivation of a theorem fails is to use a Gentzen Sequent Calculus since it's decidable, viz. we only have a finite number of options at each step in the derivation—in the bottom up reading.

Let us first look at the poset of Good-Enough sentences. As the reader could check, given any s_i, s_j in Fig. 3, such that there is no derivability arrow from s_i to s_j, a derivation of $s_i \longrightarrow s_j$ will arrive to one of the following dead-ends:

1. $\Box \ldots p \longrightarrow \Diamond \ldots p$
2. $\Box \ldots p \longrightarrow p$
3. $p \longrightarrow \Diamond \ldots p$

When arrived at this stage there are no rules to apply, the derivation fails. We illustrate these schema by means of the examples below. Let's take (I) $s_8 \longrightarrow s_3$ and (II) $s_3 \longrightarrow s_4$. Recall, s_8, s_3, s_4 stand for $\Box\Box\Diamond\Diamond p, \Box\Diamond p, \Box\Diamond\Diamond\Box p$, respectively.

15.1. Example (I)   The first step can only be ($\Box R$), after applying this rule there are two options ($\Box L$) in (Ia) or ($\Diamond R$) in (Ib) as it is spelled out below.

(Ia)
$$\cfrac{\cfrac{\cfrac{\text{FAIL}}{\Box\Diamond\Diamond p \longrightarrow \Diamond p}\ \text{No rules!}}{\langle\Box\Box\Diamond\Diamond p\rangle \longrightarrow \Diamond p}\ (\Box L)}{\Box\Box\Diamond\Diamond p \longrightarrow \Box\Diamond p}\ (\Box R)$$

(Iab)
$$\cfrac{\cfrac{\cfrac{\text{FAIL}}{\Box\Box\Diamond\Diamond p \longrightarrow p}\ \text{No rules!}}{\langle\Box\Box\Diamond\Diamond p\rangle \longrightarrow \Diamond p}\ (\Diamond R)}{\Box\Box\Diamond\Diamond p \longrightarrow \Box\Diamond p}\ (\Box R)$$

(Ia) and (Ib) are examples of failure due to the dead-end 1. and 2. above, respectively.

15.2. Example (II)   Here as well there are only two cases:

(IIa)
$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\text{FAIL}}{p \longrightarrow \Diamond\Box p}\ \text{No rules!}}{\langle p\rangle \longrightarrow \Diamond\Diamond\Box p}\ (\Diamond R)}{\Diamond p \longrightarrow \Diamond\Diamond\Box p}\ (\Diamond L)}{\langle\Box\Diamond p\rangle \longrightarrow \Diamond\Diamond\Box p}\ (\Box L)}{\Box\Diamond p \longrightarrow \Box\Diamond\Diamond\Box p}\ (\Box R)$$

(IIb)
$$\cfrac{\cfrac{\cfrac{\text{FAIL}}{\Box\Diamond p \longrightarrow \Diamond\Box p}\ \text{No rules!}}{\langle\Box\Diamond p\rangle \longrightarrow \Diamond\Diamond\Box p}\ (\Diamond R)}{\Box\Diamond p \longrightarrow \Box\Diamond\Diamond\Box p}\ (\Box R)$$

(IIa) and (IIb) are examples of failure due to the dead-ends 3. and 1. above, respectively.

Since we have shown that the Incomplete posets are isomorphic to the Good-Enough one, it follows that all the sentential categories that are not marked to be in a derivability relation in Fig. 3 are indeed not derivable from each other.

15.3. Model theoretical proofs   Here we build two counter-examples to the two derivabilities seen above, namely (I) $s_8 \rightarrow s_3$ and (II) $s_3 \rightarrow s_4$.

(I) We build a model in which s_8 is true and s_3 is false. Take $\mathcal{M}$ to be a model in which: Ryx and such that $\mathcal{M}, x \nVdash p$. Then $\mathcal{M}, x \Vdash \Box\Box\Diamond\Diamond p$ trivially holds, but $\mathcal{M}, x \nVdash \Box\Diamond p$ since it would require $\mathcal{M}, y \Vdash \Diamond p$ to be true and this cannot be since p is false in x and there are no other possible worlds in the model related to y.

(II) We build a model in which s_3 is true and s_5 is false. Take a model in which: Ryx and $\mathcal{M}, x \Vdash p$. Then by definition $\mathcal{M}, x \Vdash \Box\Diamond p$, but $\mathcal{M}, x \Vdash \Box\Diamond\Diamond\Box p$ cannot be true since it would require $\mathcal{M}, y \Vdash \Diamond\Diamond\Box p$ to hold, and hence x to be related to a world where p is true.

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