The Construction of Logical Space, by Augustín Rayo. Oxford: Oxford University Press, 2013. Pp. xix+220. H/b £35.00.

In *The Construction of Logical Space*, Augustín Rayo develops a form of deflationary realism about mathematical objects, centered on the idea that the existence of pure mathematical objects requires nothing from the world. As he puts it, "For the number of dinosaurs to be Zero just is for there to be no dinosaurs."(p.3) He suggests that accepting this kind of deflationary realism will allow us to avoid access worries concerning mathematical objects. He also proposes a non-foundationalist approach to metaphysical possibility, which assigns a central role to 'just-is' statements like the one above.

Rayo introduces his key 'just-is' operator (which he treats as a primitive) by using examples like, "For Susan to be a sibling just is for her to share a parent with someone else"(p.3) and "For the glass to be filled with water just is for it to be filled with H_2O." A claim of the form 'for ϕ just is for ψ' expresses the idea that there's "nothing else God would have to do"(p.3) to make it the case that ϕ, beyond what she'd have to do to make it the case that ψ (and vice versa). Such a claim is true whenever ϕ and ψ both "describe the same feature of reality."(p.3) Thus, unlike the notion of grounding, Rayo's 'just-is' operator is symmetric.

Rayo also develops some big-picture ideas concerning philosophy of language, metaontology, and how we ought to decide which 'just-is' statements to accept, and uses these ideas to support his proposals about mathematics and metaphysical possibility. In a familiar neo-Carnapian vein, he rejects the idea that there is a single metaphysically preferred way of carving up 'features of reality' which the logical structure of true sentences must reflect. He further maintains (as clarified in Agustín Rayo. Reply to critics.

Inquiry, 57(4):498–534, 2014) that one can, in principle, successfully stipulate any assignment of truth conditions – in the sense of something like sets of possible worlds – which respects logical entailment, to the sentences of a first-order language. Additionally, all expressions which syntactically behave like terms and figure in true existence statements in some such language will refer. This leaves a question of what to say about the references of terms in languages which (appear to) talk in terms of more objects than our own language. However, Rayo does not explicitly address this question.

Regarding the epistemology of 'just-is' statements, Rayo suggests that we should decide which 'just-is' statements to accept on the basis of a cost-benefit analysis, which combines consideration of "the way the world is" with consideration of our theoretical aims. For example, when deciding whether to adopt 'just-is' statements connecting the experience of red with the property of being in certain brain-states, we should consider whether distinguishing between pain and brain state claims, "would...lead to fruitful theorizing [for example, whether this distinction would] help us understand [the] cognitive accomplishment involved when [Jackson's Mary] is first exposed to a red tomato."(p. 63) But we should also consider questions about our aims, such as whether *we find* the theoretical fruitfulness of distinguishing between pain and brain-states sufficient "to justify the need to explain" things like "why Mary experiences the sensation of seeing red, rather than an inverse sensation."(p.63) Notably, Rayo offers no explicit argument against more traditional approaches on which there's an objective fact about how heavily one epistemically ought to weight such costs.

Next, Rayo connects 'just-is' facts to metaphysical possibility and intelligible demands for explanation. He proposes that a first-order sentence ϕ is logically consistent with the set of true 'just-is' statements if and only if it is metaphysically possible that ϕ. He suggests that the resulting picture of the space of metaphysical possibilities avoids certain unappealing foundationalist assumptions of Lewis' picture of the space of metaphysical possibilities. Similarly, he maintains that one can intelligibly ask for a certain kind of explanation of why ϕ if and only if ϕ is not a logical consequence of true 'just-is' statements (equivalently, iff ϕ is not metaphysically necessary). The latter restriction can seem somewhat counterintuitive. For, imagine someone who knows by testimony that no one has actually squared the circle but not does not realize that this fact is a metaphysical necessity rather than a historical accident. It would seem that such a person *could* intelligibly say (using Rayo's preferred formulation of the relevant demand for explanation), "I can see exactly what it would take to satisfy the truth conditions of 'no one has squared the circle,' but I wish to better understand why the world is such as to satisfy them."

Finally, Rayo turns to philosophy of mathematics. He argues that his subjective cost-benefit approach to when we should accept 'just-is' statements supports adopting a trivialist approach to mathematical objects. He then attempts to "give a precise statement of trivialism to someone who ... [already] understand[s] mathematics, by saying exactly what truth-conditions a trivialist would associate with each arithmetical sentence – and doing so in such a way that the resulting assignment of truth-conditions can be recognized as delivering trivialism regardless of whether one happens to be a trivialist."(pg 85) He does this by providing something like disquotational truth conditions for a language including some mathematical vocabulary – with the important twist that Rayo's principles specify which sentences are supposed to be true at a given possible world while only appealing to non-mathematical facts about *that world*. For example, Rayo's principles say that "a car exists" is true at world w just if a car exists at w while "a prime number exists" is true at w just if a prime number exists — not at

w, but simpliciter. However, one might like more explanation of how these principles are supposed to 'deliver trivialism.' As Rayo recognizes, the truth conditions which his principles assign to mathematical sentences would be accepted by nearly all realists about mathematical objects (trivialists and non-trivialists alike). So how exactly is asserting these principles supposed to help state trivialism?

Rayo claims that adopting trivialism lets us avoid access worries for mathematical realists. He notes that, on the trivialist view, mathematical truths require nothing of the world, so that "Once one gets clear enough about the ... truth conditions [of a statement in pure mathematics] one has done all that needs to be done to determine that the sentence is true"(p.98). However, it's unclear how this fact is supposed to help deal with the kind of modern formulations of the access problem Rayo invokes. For, these modern formulations ask for an explanation of how mathematicians reliably accept true rather than false axioms (Øystein Linnebo. Epistemological challenges to mathematical platonism. *Philosophical Studies*, 129(3):545–574, 2006)(Hartry Field. *Science Without Numbers.* Princeton University Press, 1980), and merely recognizing that true mathematical sentences 'require nothing from the world' does nothing to provide such an explanation.

Later on, Rayo does say something relevant to these modern access worries, by defending the claim that all conservative stipulations introducing new kinds of mathematical objects will succeed. However, he does not explicitly discuss how this story is to be extended to account for human accuracy about mathematics in general. For example, we don't *seem* to have introduced the numbers by any kind of act of (explicit) stipulation. Moreover, since a theory recognizing infinitely many objects can't be a conservative extension of a theory recognizing only finitely many objects, it's unclear how this story could explain the introduction of infinitely many mathematical objects if one starts off speaking a language which recognizes only finitely many objects.

Additionally, Rayo provides no satisfying answer to the remaining problem of explaining how creatures like us are able to recognize the conservativeness, or even consistency, of some powerful mathematical postulates such as the ZFC axioms. He suggests that we might learn to recognize which axiom systems are coherent by (among other things) gaining experience working with them. However, we should note that Rayo appeals to languages which go beyond first-order logic (as needed to precisely describe the natural numbers), and that in such systems coherence requires more than the inability to derive a contradiction. He also notes, very abstractly, that our ability to recognize consistent axiom systems might sometimes involve having a feeling for, "what a model for the axiom system in question would be like"(p. 84-85). This tame assertion is surely plausible, but it does nothing to dispel intuitive access worries about how creatures like us could have gotten in a position to recognize the possibility of models for very ontologically demanding mathematical theories, especially those which (like those characterizing the sets) plausibly require the existence of too many objects to have a physical model.

Lastly, Rayo considers how to make sense of cognitive accomplishment in mathematics. Insofar as pure mathematical claims don't rule out any possible worlds, how are we to think about the difference between mathematical knowledge and ignorance? Rayo answers this question by suggesting we associate a single person's mind with many different fragments, each corresponding to a different task. Each of these fragments has its own belief state, in the sense of a (partial) assignment of probabilities to sets of possible worlds. He explains our apparent lack of logical omniscience by saying that the fragment of our minds associated with the task of answering a question about a complex logical tautology can know the trivial proposition, but fail to recognize the contingent fact that we are currently speaking a language in which the relevant string of sounds expresses a question about this proposition . In situations where it would normally be said that a person learns additional truths of mathematics, the knowledge possessed by certain fragments of their mind (such as the one relevant to answering, “Does every number have a successor?”) becomes available to other fragments (such as those relevant to answering, “What is the square root of 81?” or buying tiles to cover a yard). This explains why learning mathematics is helpful, and why it constitutes a cognitive accomplishment.

Rayo does not say why he thinks it is necessary (or even desirable) to explain our (apparent) ability to accept structured propositions, in terms of mental fragments which accept unstructured Lewis-Stalknaker style propositions. This question is especially pressing because Rayo doesn’t doubt the existence of structured propositions (as most philosophers who attempt such a reduction do). Additionally, standard structured-proposition models of belief include an attractive account of which behaviors accepting a mathematical claim is likely to influence, namely those which are connected to the claim by short arguments (from the speaker’s other beliefs). For example, learning the formula for the area of a circle is more likely to influence carpet purchases for a circular room than a polygonal room. It is far from clear that Rayo’s framework can duplicate or replace this kind of explanation. Yet allowing some such systematic story about the connection between mathematical talk and action seems absolutely essential if Rayo’s theoretical constructs are intended to help explain or predict actual human behavior.

As we have seen, Rayo argues for putting ‘just-is’ claims at the center of a unified and wide-reaching philosophical project. Given these sweeping ambitions, it is unsurprising that certain details can seem under-argued or under-explained. Many aspects of Rayo's philosophy of mathematics are familiar. For example, Thomasson's story about mathematical objects in (Amie L. Thomasson. *Ordinary Objects*. Oxford University Press, 2007) shares all the features which Rayo cites as distinguishing his story about mathematical objects from neo-Fregean proposals (though Thomasson does not motivate her position by appeal to anything like Rayo's epistemology of 'just-is' statements). Nonetheless, Rayo's overall picture is alluring and brings plenty of interesting new material to the table, such as his development of Stalnaker's work on making room for mathematical knowledge in within (something like) the Lewis-Stalnaker picture of propositions(Robert Stalnaker. The problem of logical omniscience, i. *Synthese*, 89(3):425–440, 1991) and his appealingly-detailed account of how facts about metaphysical possibility systematically reflect facts about logical possibility and true 'just-is' claims.

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