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The Paradox of Counterfactual Tolerance

Counterfactuals are somewhat tolerant. Had Socrates been at least six feet tall, he need not have been *exactly* six feet tall. He might have been a little taller—he might have been six one or six two. But while he might have been a little taller, there are limits to how tall he would have been. Had he been at least six feet tall, he would not have been more than a hundred feet tall, for example. Counterfactuals are not just tolerant, then, but bounded.

This paper presents a surprising paradox: If counterfactuals are tolerant and bounded, then we can prove a flat contradiction using natural rules of inference. Something has to go then. But what?

1 Paradox

Planck lengths are incredibly small. You would quite literally need a billion trillion of them just to span the diameter of a proton. Now consider Socrates who is, we can suppose, exactly five feet tall. For all n, let s_n be the claim that Socrates is at least n Planck lengths tall. We then claim that:

Tolerance: For all n, $s_n \diamondto s_{n+1}$.
Boundedness: There are $n < m$ such that had $s_n \boxright \neg s_m$.
Heights: For all n, $s_{n+1} \boxright s_n$.

Tolerance says that had Socrates been at least six feet, he might have been at least six feet and one Planck length, and likewise for other heights. Boundedness will be true if, for example, had Socrates been at least six feet, he would not have been at least a hundred feet. Heights is just the trivial observation that had Socrates been at least six feet and one Planck length, he would thereby have been at least six feet, and likewise for other heights.

To stage our paradox, we are going to use a relatively weak counterfactual logic that David Lewis (1973) calls **V** and that we will call **B3**. This system is strictly weaker than the full system **VC** that Lewis accepts.[1]

The language of **B3** is the language of classical propositional logic, extended with a pair of two-place sentential operators $\Box\!\!\rightarrow$ and $\Diamond\!\!\rightarrow$. $A \Box\!\!\rightarrow B$ says that had it been that A, it would have been that B. $A \Diamond\!\!\rightarrow B$ says that had it been that A, it might have been that B.

We are here going to construct **B3** as a Gentzen-style sequent calculus.[2] This means that the system has both **rules** and **metarules**. Rules have the form

$$A_1, \ldots, A_n \vdash B \tag{1}$$

where $A_1, \ldots, A_n$ and B are sentences of the object language. Such rules are called **sequents**. (1) can be read as saying that if you accept all the sentences on the left of the turnstile, then you must accept the sentence on the right. Metarules have the form

$$\frac{\sigma_1, \ldots, \sigma_n}{\gamma} \tag{2}$$

where $\sigma_1, \ldots, \sigma_n$ and γ are rules rather than sentences of the object language. (2) can be read as saying that if you accept all the rules on the top, then you must accept the rule on the bottom.

B3 is constructed by extending classical logic with two basic metarules and several basic rules.[3] These are:

1. Lewis refers to **B3** as **C0** and **VC** as **C1** in his earlier (1971).

2. Lewis axiomatizes **B3** as a Hilbert-style system. For another Hilbert-style axiomatization, see my (2021).

3. By classical logic, I mean the system **LK**, which was introduced by Gentzen in his (1935). For more on LK, see Paoli (2013).

Substitution: $\dfrac{A \dashv\vdash A^*}{A \mathrel{\Box\!\!\rightarrow} B \vdash A^* \mathrel{\Box\!\!\rightarrow} B}$

Weakening: $\dfrac{B \vdash B^*}{A \mathrel{\Box\!\!\rightarrow} B \vdash A \mathrel{\Box\!\!\rightarrow} B^*}$

Identity: $\vdash A \mathrel{\Box\!\!\rightarrow} A$

Import: $A \mathrel{\Box\!\!\rightarrow} B \wedge C \vdash A \wedge B \mathrel{\Box\!\!\rightarrow} C$

Conjunction: $A \mathrel{\Box\!\!\rightarrow} B,\ A \mathrel{\Box\!\!\rightarrow} C \vdash A \mathrel{\Box\!\!\rightarrow} B \wedge C$

Disjunction: $A \mathrel{\Box\!\!\rightarrow} C,\ B \mathrel{\Box\!\!\rightarrow} C \vdash A \vee B \mathrel{\Box\!\!\rightarrow} C$

Strengthen Might: $A \mathrel{\Box\!\!\rightarrow} C, A \mathrel{\Diamond\!\!\rightarrow} B \vdash A \wedge B \mathrel{\Box\!\!\rightarrow} C$

Duality: $A \mathrel{\Box\!\!\rightarrow} B \dashv\vdash \neg(A \mathrel{\Diamond\!\!\rightarrow} \neg B)$

The new metarules are Substitution and Weakening. Substitution says that we can replace the antecedent of a counterfactual with any sentence that is logically equivalent. Weakening says that we can replace the consequent with any sentence that it logically implies.

The first two rules are Identity and Import. Identity says that every sentence counterfactually entails itself. Import says that we can import a conjunct from the consequent into the antecedent.

The next two rules are Conjunction and Disjunction, which have a kind of symmetry.[4] Conjunction says that we can conjoin the consequents of any two counterfactuals that share an antecedent. Disjunction says that we can disjoin the antecedents of any two counterfactuals that share a consequent.

The last two rules are Strengthen Might and Duality. Strengthen Might is a rule for strengthening antecedents.[5] Duality says that there is a certain equivalence between would and might counterfactuals.

As we observed earlier, Lewis accepts all the basic rules of **B3**. In addition, he also accepts:

Counterfactual Modus Ponens: $A, A \mathrel{\Box\!\!\rightarrow} B \vdash B$

Weak Centering: $A, B \vdash A \mathrel{\Diamond\!\!\rightarrow} B$

Strong Centering: $A, B \vdash A \mathrel{\Box\!\!\rightarrow} B$

4. Conjunction is sometimes called Agglomeration.

5. Given Duality, Strengthen Might is equivalent to another rule that that is sometimes called Rational Monotonicity. That rule says that $A \mathrel{\Box\!\!\rightarrow} C, \neg(A \mathrel{\Box\!\!\rightarrow} \neg B) \vdash A \wedge B \mathrel{\Box\!\!\rightarrow} C$.

Adding these rules to **B3** results in the full system **VC** that Lewis accepts. None of these rules, though, will play any role in our paradox. We will thus continue to focus on the weaker system **B3**.

Once we have these basic rules and metarules of **B3**, we can derive two further non-basic rules, which will help to simplify and structure our discussion.

Swap:	$A \wedge B \mathrel{\Box\!\!\rightarrow} C \vdash B \wedge A \mathrel{\Box\!\!\rightarrow} C$
Limited Transitivity:	$A \mathrel{\Box\!\!\rightarrow} B,\ A \wedge B \mathrel{\Box\!\!\rightarrow} C \vdash A \mathrel{\Box\!\!\rightarrow} C$

Swap follows from Substitution. It says that we can always reorder conjuncts in the antecedent of a counterfactual. Limited Transitivity follows from Identity, Disjunction, Conjunction, Weakening, and Substitution. It says that we can chain together certain counterfactuals to infer others.

With our premises and counterfactual logic in place, we can now prove[6] a flat contradiction as follows:

1.	$s_j \mathrel{\Box\!\!\rightarrow} \neg s_k$	Boundedness
2.	$s_j \mathrel{\Diamond\!\!\rightarrow} s_{j+1}$	Tolerance
3.	$s_j \wedge s_{j+1} \mathrel{\Box\!\!\rightarrow} \neg s_k$	1, 2, Strengthen Might
4.	$s_{j+1} \mathrel{\Box\!\!\rightarrow} s_j$	Heights
5.	$s_{j+1} \wedge s_j \mathrel{\Box\!\!\rightarrow} \neg s_k$	3, Swap
6.	$s_{j+1} \mathrel{\Box\!\!\rightarrow} \neg s_k$	4, 5, Limited Transitivity

This reasoning is paradoxical because it can be iterated. After $k - j - 1$ applications we have:

$$s_{k-1} \mathrel{\Box\!\!\rightarrow} \neg s_k \tag{3}$$

But Tolerance tells us that

$$s_{k-1} \mathrel{\Diamond\!\!\rightarrow} s_k \tag{4}$$

which gives us a flat contradiction by Duality. So something has to go: We must either reject one of the premises or reject one of the rules or metarules. Call this the **paradox**

6. **B3** is a sequent calculus. But sequent proofs can be hard to follow, so we will generally use informal natural deduction proofs instead. These get the basic idea across, but can also be easily converted into the official form.

of counterfactual tolerance, or just the **tolerance paradox** for short.

2 Premises

The tolerance paradox has two major premises, which are Tolerance and Boundedness. Should the paradox be resolved by denying one of them?

Boundedness says that some height has a counterfactual bound, where m is a **counterfactual bound** of n when $s_n \mathrel{\Box\!\!\rightarrow} \neg s_m$. Denying Boundedness thus means accepting that had Socrates been taller, he might have been any height whatsoever. But any height *whatsoever*? Surely not. Even if Socrates had been taller, the Spartans would not have thrown their weapons into the sea. But if Socrates might have been any height *whatsoever*, this is simply false. Had Socrates been taller, he might have been a thousand feet. But had Socrates been a thousand feet, the Spartans *would* have thrown their weapons into the sea. Who wants to oppose the Great Giant of Athens? This illustrates a general problem: If we deny that counterfactuals are bounded, so many would counterfactuals will turn out false that we might as well give up counterfactual reasoning altogether.

Two more observations about Boundedness. First, Boundedness says that some height has a counterfactual bound. It does not say that *all* heights have a counterfactual bound. Sufficiently outlandish heights might still be unbounded. Second, boundedness does not require there to be a height with a *least* counterfactual bound. If it did, there might be some concern that vagueness will result in indeterminacy. But since Boundedness only requires there to be a height with some counterfactual bound, these concerns can be sidestepped.

If we cannot deny Boundedness, perhaps we should deny Tolerance? Denying Tolerance means accepting the existence of singularities, where a **singularity** is any height n for which $\neg(s_n \mathrel{\Diamond\!\!\rightarrow} s_{n+1})$.

As it turns out, there are certain reasons to accept singularities. First, suppose that you accept Strong Centering. Suppose also that Socrates is exactly n Planck lengths tall, which is to say that:

$$s_n \wedge \neg s_{n+1} \tag{5}$$

But then by Strong Centering and Duality:

$$\neg(s_n \mathrel{\Diamond\!\!\rightarrow} s_{n+1}) \tag{6}$$

The actual height of Socrates thus turns out to be a singularity, given Strong Centering.

You might also accept singularities because you accept certain bridge principles connecting necessity, possibility, and counterfactuals. For example, consider the following rule, with the diamond and box expressing metaphysical possibility and necessity.

Modal Constraint: $\Diamond A, \Box B \vdash A \mathrel{\Box\!\!\rightarrow} B$

Suppose that A is metaphysically possible and B is metaphysically necessary. Given Modal Constraint, it then follows that had it been that A, it would have been that B.

Now suppose that there is a necessary limit to the height of Socrates. That is, suppose that there is some n such that $\Box(\neg s_n)$. Given the Law of Excluded middle, there is then some k such that:

$$\Diamond(s_k) \wedge \Box(\neg s_{k+1}) \tag{7}$$

This k is what you might call the **least possible upper bound** on the height of Socrates. Modal Constraint and Duality then tells us that:

$$\neg(s_k \mathrel{\Diamond\!\!\rightarrow} s_{k+1}) \tag{8}$$

Thus, the least possible upper bound on the height of Socrates turns out to be a singularity.

There are various ways that we could deal with such difficulties. We could deny Strong Centering. We could deny Modal Constraint. Fortunately, there is no need to do either. Tolerance says that there are no singularities whatsoever. But this claim is stronger than needed. All we need is the weaker claim that there are certain *intervals* within which there are no singularities.

Say that an interval $[j, k]$ is bounded when $s_j \mathrel{\Box\!\!\rightarrow} \neg s_k$ and tolerant when $s_n \mathrel{\Diamond\!\!\rightarrow} s_{n+1}$ for every n such that $j \leq n < k$. An interval is **paradoxical** when it is both tolerant and bounded.

All we need to generate the paradox is the existence of some paradoxical interval. Moreover, that there are such intervals would seem to be obvious. For example, suppose that Socrates is in fact five feet tall. The interval between six feet and ten feet would then seem to be paradoxical. After all, had Socrates been at least six feet, he would not have been at least ten feet. And for any height between six feet and ten feet, had Socrates been at least that tall, he might have been at least a Planck length taller. Strong Centering is no longer a problem, since five feet is strictly less then six feet. If there is a least possible upper bound on the height of Socrates, it is presumably greater than ten feet. Thus, Modal Constraint is also no longer a problem. So the interval between six feet and ten feet would seem to be paradoxical.

What this shows is that to defuse the paradox by denying Tolerance, we need do more than just accept the existence of singularities. We in fact need to deny that there are, or even could be, any paradoxical intervals. But why would that be? Why *must* every bounded interval include a singularity?

One response would be to simply assert that the rules and metarules used in the paradox are correct. Thus, given Heights, logic *itself* guarantees that every bounded interval has a singularity. And what better explanation could you want?

Maybe. But the plausibility of this response depends on the plausibility of the rules and metarules, which in turn depends on whether there are simple and elegant views on which they fail. In the second half of this paper, I will defend a view on which one of the rules fails. Thus, I do not find this line of response convincing.

3 Duality

Duality says that would and might counterfactuals are duals. This rule is controversial, and so you might wonder whether the paradox could be resolved by denying it. After all, suppose that like Stalnaker (1968), you accept Counterfactual Excluded Middle:

CEM: $\vdash (A \mathrel{\Box\!\!\rightarrow} B) \vee (A \mathrel{\Box\!\!\rightarrow} \neg B)$

In that case, given Duality, it follows that

$$\vdash \neg(A \mathrel{\Diamond\!\!\rightarrow} B) \vee \neg(A \mathrel{\Diamond\!\!\rightarrow} \neg B) \tag{9}$$

But this is not correct. If a fair coin were flipped, it might have landed heads and, if a fair coin were flipped, it might not have landed heads. Thus, those who accept CEM have good reason to deny Duality.

While there may be good reasons to deny Duality, doing so will not solve the paradox, and this for two reasons. First, even without Duality, we could use similar reasoning to conclude that:

$$s_k \mathrel{\Box\!\!\rightarrow} \neg s_k \tag{10}$$

This strikes me as obviously false. Thinking of k Planck lengths as a hundred feet, this says that had Socrates been at least a hundred feet, he would *not* have been at least a hundred feet. Even worse, given Identity and Conjunction, this entails:

$$s_k \mathrel{\Box\!\!\rightarrow} \bot \tag{11}$$

But surely not. Had Socrates been at least a hundred feet, the world would have been very different. Maybe the laws of physics would have been different. Maybe the Great Giant of Athens would have changed the course of geopolitical history. Still, however things would have been, there would not have been true contradictions, nor would there have been all the things that *follow* from true constrictions. There would not have been square circles, married bachelors, even primes greater than two, and so on.

The other reason that denying Duality will not help is that we can distinguish between strong and weak counterfactual operators. In English, we express the weak would operator $\mathrel{\Box\!\!\rightarrow}$ and the weak might operator $\mathrel{\Diamond\!\!\rightarrow}$. It is then an open question whether these operators are duals. Suppose, though, that they are not. In that case, we can introduce strong operators that are by definition duals of the weak operators.

$$A \mathrel{\Box\!\!\Rightarrow} B \dashv\vdash \neg(A \mathrel{\Diamond\!\!\rightarrow} \neg B) \tag{12}$$

$$A \mathrel{\Diamond\!\!\Rightarrow} B \dashv\vdash \neg(A \mathrel{\Box\!\!\rightarrow} \neg B) \tag{13}$$

The strong would operator $A \mathrel{\Box\!\!\Rightarrow} B$ can be read as saying that it is false that had it been that A, it might not have been that B. The strong might operator $A \mathrel{\Diamond\!\!\Rightarrow} B$ can be read as saying that it is false that had it been that A, it would not have been that B.

Once we have our strong operators, we can reformulate the paradox by simply replacing the weak would operator with the strong would operator. Since the strong would operator and weak might operators are duals by definition, Duality is no longer a problem. The reformulate premises are just as compelling. Boundedness, for example, becomes the claim that there are $j < k$ such that it is false that had it been that j, it might have been that k. And this would seem to be correct: It is false that had Socrates been at least six feet, he might have been at least ten lightyears. The reformulated rules and metarules are also equally compelling. Denying Duality, then, will not solve the paradox.

4 Limited Transitivity

Limited Transitivity is compelling. For example, suppose that had it been sunny, it would have been warm. Suppose also that had it been *sunny and warm*, Alice would have gone for a run. From this, it would seem to follow that had it been sunny, Alice would have gone for run. This is the sort of reasoning licensed by Limited Transitivity, and would seem to be correct.

Besides being compelling in its own right, Limited Transitivity also follows from other compelling rules and metarules. These include Identity, Conjunction, Disjunction, Weakening, and Substitution. In my own case, I accept all of these rules and metarules, and so also accept Limited Transitivity.

That said, there are views on which Limited Transitivity fails.[7] These are generally views, though, on which Substitution holds. But so long as we have Substitution, we can reason to paradox, even without Limited Transitivity.

To see why, note that we have said nothing about the logical structure of the sentences s_n. For all we have said, they could be atomic sentences. Suppose, though, that we let them be infinite disjunctions

$$s_n = p_n \vee p_{n+1} \vee p_{n+2} \vee \ldots \tag{14}$$

with each p_i being an atomic sentence saying that Socrates is exactly i Planck lengths. In that case, the premises are just as compelling, but we can reason directly from line three to line six using only Substitution.

Thus, we cannot block the inference from line three to line six by simply denying Limited Transitivity. We would have to deny Substitution as well.[8] But the cost of denying *both* Limited Transitivity and Substitution is simply too great, or so it seems to me. So we have reason to want a different solution.

5 Strengthen Might

This leaves us with few remaining options for resolving the paradox. We could try to deny Heights. This would mean accepting that had Socrates been at least $n + 1$ Planck lengths, he might not have been n Planck lengths. But how could that be? Moreover, even if we were to deny Heights, we could reason from line three to line six directly using Substitution, as we pointed out in the last section. Thus, denying Heights would not even solve the basic problem.

Another option would be to deny Swap. All that Swap lets us do, though, is

7. See for example Bacon (forthcoming) and Leitgeb (2012a, 2012b).

8. For a view on which Substitution fails, but Limited Transitivity holds, see Fine (2012).

rearrange the order of conjuncts in the antecedent of a counterfactual. But how could *rearranging conjuncts* fail to preserve truth?

The only remaining option is to deny Strengthen Might. This, I think, is the right solution. The second half of this paper will develop a positive view on which Strengthen Might fails. For now, we will just say a bit about what the rule says and why you might accept it.

In classical logic, the antecedent of a material conditional can always be strengthened. From the fact that $A \supset C$, it follows that $A \wedge B \supset C$ for any B whatsoever. As Lewis (1973) observes, though, the same is not true for counterfactuals. That is, the following rule is not correct:

Strengthening: $A \mathrel{\Box\!\!\rightarrow} C \vdash A \wedge B \mathrel{\Box\!\!\rightarrow} C$

Suppose that had Alice gone to the party, she would have had a good time. From this, it does not follow that had Alice gone to the party *and learned that an asteroid was about to destroy the earth*, she would have had a good time. In fact, she most certainly would not have.

The question then arises: If we cannot in general strengthen antecedents, then under what conditions can we strengthen them? Lewis's preferred system **B3** gives two answers. The first takes the form of a derived rule called Strengthen Would, which follows from Conjunction and Import.

Strengthen Would: $A \mathrel{\Box\!\!\rightarrow} C, A \mathrel{\Box\!\!\rightarrow} B \vdash A \wedge B \mathrel{\Box\!\!\rightarrow} C$

When $A \mathrel{\Box\!\!\rightarrow} B$, we will say that A **counterfactually entails** B. What Strengthen would tells us, then, is that we can strengthen an antecedent with any sentence that it counterfactually entails.

The second answer takes the form of the basic rule Strengthen Might. Say that A is **counterfactually consistent** with B when $A \mathrel{\Diamond\!\!\rightarrow} B$. In that case, what Strengthen Might tells us is that we can can strengthen the antecedent of a would counterfactual with any sentence with which it is counterfactually consistent.

Strengthen Would and Strengthen Might together represent a simple and elegant theory of antecedent strengthening. The concern is that if we reject Strengthen Might, we will be left with a theory that is not only less simple and elegant, but materially inadequate. I think that this challenge can be met, and so we will return to this issue in §12.

6 Not the Sorites Paradox

At this point, you might be convinced that we have a paradox, but concerned that we do not have a new paradox. Why is this not just the sorites paradox in disguise?

To build a sorites paradox, we need a scale and something like a vague predicate. We might observe, for example, that one grain of sand is not a heap, but that a thousand grains of sand is a heap. We then deny that there is a sharp cutoff between then number of grains that do not form a heap and the number of grains that do. But in that case, we can prove a contradiction using classical logic.

Now in the case of counterfactuals, we certainly *can* build a sorites paradox. We can use heights as the scale and the counterfactual operators in place of a vague predicate. The main observation, though, is that the resulting sorites paradox is not the tolerance paradox. The arguments have different premises. They rely on different inference rules.

To construct a counterfactual sorites paradox, we can start by accepting Boundedness. We then add two further premises:

Non-Triviality: For all n, $s_n \diamondto s_n$.

No Sharp Cutoffs: For all n and m, $\neg((s_n \diamondto s_m) \wedge \neg(s_n \diamondto s_{m+1}))$.

Non-Triviality says that had Socrates been at least six feet, he might have been at least six feet, and likewise for other heights. No Sharp Cutoffs says that there is No Sharp Cutoffs to how tall Socrates might have been. We can then use these premises to prove a contradiction as follows:

1.	$s_j \boxright \neg s_k$	Boundedness
2.	$s_j \diamondto s_j$	Non-Triviality
3.	$\neg((s_j \diamondto s_j) \wedge \neg(s_j \diamondto s_{j+1}))$	No Sharp Cutoffs
4.	$\neg(s_j \diamondto s_j) \vee \neg\neg(s_j \diamondto s_{j+1})$	3, DeMorgans
5.	$s_j \diamondto s_{j+1}$	2,4 DS, EXP, DNE

The rules used on line five are Disjunctive Syllogism, Explosion, and Double Negation Elimination. This reasoning is paradoxical because, after repeated application, we get $s_j \diamondto s_k$, which is equivalent to $\neg(s_j \boxright \neg s_k)$ by Duality. But this contradicts line one, so we have a paradox.

We can now observe several important differences between the two paradoxes. The first is that while the sorites paradox requires No Sharp Cutoffs, the tolerance paradox does not.[9] The two paradoxes thus require different premises. They also require different inference rules. The tolerance paradox requires substantial rules of counterfactual logic like Strengthen Might, Limited Transitivity, and Substitution. The sorites paradox does not.[10] Going the other direction, the sorites paradox uses classical rules like DeMorgans, Disjunctive Syllogism, Double Negation Elimination, and Explosion. These rules, though, play no role in the tolerance paradox.

Maybe you are willing to grant that the two paradoxes are not literally the same. Still, you might think, there is a more general sense in which they depend on similar reasoning, and so should be solved in similar ways.

For example, suppose you are a supervaluationist, and so think that vagueness is the result of semantic indecision. Our semantic practice requires there to be a sharp cutoff somewhere in the sorites series, but we have not yet decided where to draw it. You thus accept that determinately, there is a sharp cutoff, but deny that there is a *determinate* sharp cutoff. The sorites paradox then results from our tendency to conflate these two claims. From the fact that there is no determinate cutoff, we tend to conclude that determinately, there is no cutoff. Once these claims are distinguished, though, the sorites paradox can be resolved, since we can reject No Sharp Cutoffs as determinately false.

You might then say something similar about singularities. Suppose that we are considering a particular bounded interval. You then say that our semantic practice requires us to place a singularity somewhere, but we have not yet settled on where to place it. You thus accept that determinately, there is a singularity in the interval, but deny that there is a determinate singularity in the interval. The tolerance paradox then arises because we tend to conflate these two claims. From the fact that there are no determinate singularities, we tend to conclude that determinately, there are no singularities. Once these claims are distinguished, though, the tolerance paradox can be resolved, since we can reject Tolerance as determinately false.

I agree that a supervaluationist solution to the tolerance paradox should be on the table. This kind of solution, though, will face all the challenges faced by supervalua-

9. We know this because we can build models in which the premises of the tolerance paradox are all true, but No Sharp Cutoffs is false. For one such model, see §11.

10. As formulated above, the sorites paradox does appeal to Duality, but this could be eliminated by taking the dual of Boundedness as a premise.

tionist theories of vagueness, of which there are many.[11]

Moreover, even if you accept a supervaluationist theory of vagueness, there is not the same motivation for accepting a supervaluationist theory of tolerance. The best reason to be a supervaluationist about vagueness is that this lets us preserve classical logic. But the correctness of classical logic is not at stake in the tolerance paradox. What is at stake is the correctness of certain rules of counterfactual logic. These do not have the same standing, and so rejecting them does not come at the same cost. There is thus weaker motivation for adopting a supervaluationist theory of tolerance.

7 Towards a Solution

A common view is that counterfactuals can be correctly modeled in terms of the relative closeness of possible worlds. In particular, with the limit assumption in place, you might think that:

Closest: $A \mathrel{\Box\!\!\rightarrow} B$ is true iff all of the closest A worlds are B worlds.
$A \mathrel{\Diamond\!\!\rightarrow} B$ is true iff some of the closest A worlds are B worlds.

Here, an A world is just a world at which A is true. A world is among the closest A worlds when it is an A world and there are no other A worlds that are strictly closer.

What can be shown is that if Closest is true, then **B3** is sound and complete.[12] But in that case, the rules and metarules used in the paradox are sound, and so we must reject one of the premises.

Those of us who accept all of the premises, then, are committed to rejecting Closest. But if Closest is rejected, what should be put in its place?

For my own part, I agree with Lewis that counterfactuals are determined by the closeness of worlds. What I deny is that they are determined by which worlds are *closest*. Instead, I think that they are determined by which worlds are *nearly* closest. Assuming that we have the limit assumption in place, the view is that:

11. See chapter five of Williamson (1996), for example.
12. Lewis (1971, 1973).

Nearly Closest: $A \mathrel{\Box\!\!\rightarrow} B$ is true iff all of the nearly closest A worlds are B worlds.
$A \mathrel{\Diamond\!\!\rightarrow} B$ is true iff some of the nearly closest A worlds are B worlds.

The result of replacing Closest with Nearly Closest is that Strengthen Might fails. But while that may be, the other rules and metarules that we would like to keep are still valid. This, I claim, resolves the paradox.

This is a rough sketch of the positive proposal. The rest of this paper will be spent filling in the details.

8 Burgess Models

There are different approaches to modeling counterfactuals in terms of relative closeness. Lewis (1971, 1973) uses systems of spheres. Stalnaker (1968) uses selection functions. Here, we are going to use Burgess models, which are most general.[13]

Definition 8.1: A **frame** $\mathcal{F} = \langle W, L, \preceq \rangle$ consists of a non-empty domain of worlds W, a function L assigning every world x a **local domain** $L_x \subseteq W$ of worlds, and a function $\preceq$ assigning every world x an **accessibility relation** $\preceq_x \subseteq L_x \times L_x$.

Each world in a model has a local domain, which includes those worlds that are **counterfactually possible** relative to that world. For every world, there is then an accessibility relation on this local domain. When $b \preceq_x a$, we say that b is **accessible** from a relative to x. We can then use $b \prec_x a$ as shorthand for $b \preceq_x a$ and $a \not\preceq_x b$ and read this as saying that b is **strictly accessible** from a relative to x.

Definition 8.2: A **Burgess model** $\mathcal{M} = \langle \mathcal{F}, V \rangle$ consists of a frame $\mathcal{F}$ and a valuation function V assigning every atomic sentence p of $\mathcal{L}$ a denotation $V(p) \subseteq W$. A sentence A is **true at a world** x in a model $\mathcal{M}$ when $\mathcal{M}, x \vDash A$, with this relation

13. Burgess models were introduced in his (1981). For another interesting application of Burgess models, see Field (2016).

defined recursively:

$\mathcal{M}, x \vDash p$	iff	$x \in V(p)$
$\mathcal{M}, x \vDash \neg A$	iff	$x \nvDash A$
$\mathcal{M}, x \vDash A \vee B$	iff	$x \vDash A$ or $\vDash B$
$\mathcal{M}, x \vDash A \wedge B$	iff	$x \vDash A$ and $\vDash B$
$\mathcal{M}, x \vDash A \mathrel{\Box\!\!\rightarrow} B$	iff	for every $a \vDash A$ such that $a \in L_x$, there is a $b \vDash A$ such that $b \preceq_x a$ and, for all c such that $c \preceq_x b$, if $c \vDash A$ then $c \vDash B$
$\mathcal{M}, x \vDash A \mathrel{\Diamond\!\!\rightarrow} B$	iff	there is an $a \vDash A$ such that $a \in L_x$ and, for all $b \vDash A$ such that $b \preceq_x a$, there is a c such that $c \preceq_x b$ with $c \vDash A$ and $c \vDash B$

Once we have truth at a world in a model, soundness and completeness in a class of models is defined in the usual way. When a system is sound and complete in a class of models, we will say that the class of models **generates** that system.

The class of all models generates a system that is too weak to be a realistic logic for counterfactuals. After all, there are Burgess models in which Identity fails. But surely, if there are any logical truths involving counterfactuals, one of them is that had it been that A, it would have been that A.

To generate more realistic systems, then, we need to use smaller classes of models. These classes are generally specified by placing constraints on accessibility relations. Some common constraints are listed below.

Property	**Definition**
Reflexive	$a \preceq a$
Connected	$(a \preceq b) \vee (b \preceq a)$
Transitive	$(c \preceq b) \wedge (b \preceq a) \supset (c \preceq a)$
Preorder	reflexive, transitive
Total Preorder	connected, transitive

The question now is, which of these constraints should we impose if we want to generate the correct logic of counterfactuals?

9 Lewis

Lewis thinks that we should require accessibility relations to be total preorders. More concretely, you might think of this as the view that we should require accessibility relations to be *as close* relations.

What are *as close* relations? Imagine that you are standing in the middle of a grassy field. You are surrounded by several brightly colored balls at various distances. These balls stand in *as close* relations, where b is as close as a when the distance of b is no greater than the distance of a. Such *as close* relations are both connected and transitive, and so form total preorders. Lewis, then, can be thought of as endorsing the view that a world b is accessible from another world a relative to x if and only if b is at least as close as a to x.[14]

Such relations are both connected and transitive. They are connected because, for any two balls, either a is at least as close as b or b is at least as close as a. They are transitive because, for any three balls, if c is at least as close as b and b is at least as close as a, then c is at least as close as a. Since *as close* relations are both connected and transitive, they are also total preorders.

The problem is that if we require accessibility relations to be total preorders, the resulting class of models validates **B3**, and so validates all of the rules and metarules used in the paradox. Thus, if we accept the premises, we cannot require accessibility relations to be total preorders, and so cannot require them to be both connected and transitive. At least one of those properties has to go.

10 Pollock

John Pollock (1975, 1976a, 1976b) thinks that we should require accessibility relations to be preorders. Unlike Lewis, then, he denies that we should require them to be connected, and so denies that we should require them to be *total* preorders. In terms of distance relations, you might think of this as the view that counterfactual accessibility relations are *determinately as close* relations.

Filling in the details a bit, you might think that the closeness of possible worlds is

14. When we say that b is as close as a to x, we sometimes mean that it is *exactly* as close, rather than at least as close. For our purposes, when we say that b is as close as a to x, we always mean that it is *at least* as close.

determined by our modal practice. Our modal practice, though, is somewhat indeterminate. Thus, while possible worlds stand in *as close* relations on all precisifications, our practice is compatible with many precisifications. A world b is then **determinately as close** as a to x when b is as close as a to x on all precisifications.

Such *determinately as close* relations are reflexive and transitive, but need not be connected. Thus, while *determinately as close* relations are always preorders, they need not be total preorder. You might think of Pollock, then, as endorsing the view that a world b is accessible from a relative to x if and only if b is determinately as close as a to x.

If we require accessibility relations to be preorders, the resulting class of models generates a system that we will call **B1**.[15] That system can be axiomatized using the same basic rules and metarules that we used to axiomatize **B3**. The only difference is that Strengthen Might is simply deleted from the list of basic rules. Thus, since Strengthen Might is not a correct rule in **B1**, Pollock would seem to have given us a natural solution to the paradox.

The problem for Pollock is that while his proposal invalidates Strengthen Might, it also invalidates other rules we would like to keep. For example:

Distribution: $A \vee B \mathrel{\Box\!\!\rightarrow} C \vdash (A \mathrel{\Box\!\!\rightarrow} C) \vee (B \mathrel{\Box\!\!\rightarrow} C)$

Distribution says that we can always distribute a would counterfactual over a disjunction in the antecedent. So for instance, suppose that:

Had it either rained or snowed, Ophelia would have been pleased. (15)

Given Distribution, it then follows that at least one of the following is true.

Had it rained, Ophelia would have been pleased. (16)

Had it snowed, Ophelia would have been pleased. (17)

And this would seem to be correct. Pollock, though, is committed to the bizarre view that this reasoning is invalid.[16]

Distribution, it seems to me, is obviously valid. It also plays an important role our ordinary counterfactual practice. By way of illustration: Suppose that you and I have

15. See Burgess (1981).

16. See my (2021) for a countermodel to Distribution.

a disagreement. I accept (15), but you do not. You thus set out to change my mind. How might you do that? One natural strategy would be for you to try and convince me to reject (16) and (17). For if I reject both of those claims, I cannot rationally go on believing (15). That is, if you convince me to *reject* the claim that had it rained, Ophelia would have been pleased and *reject* the claim that had it snowed, Ophelia would have been pleased, then I cannot go on accepting that had it either rained or snowed, Ophelia would have been pleased. But if Distribution fails, this strategy is not available. For even if you convince me to reject both (16) and (17), I can happily go on accepting (15), since there is no logical inconsistency.

It may be worth mentioning that Distribution bears a passing resemblance to a more controversial principle called Simplification.

Simplification: $A \vee B \boxright C \vdash (A \boxright C) \wedge (B \boxright C)$

The difference is that where Simplification has a conjunction in the consequent, Distribution only has a disjunction. Thus, if you accept (15), Simplification requires you to accept that (16) and (17) are *both* true. Distribution only requires you to accept that *at least one* is true.[17]

11 Near Closeness

This brings us to my positive proposal for resolving the tolerance paradox. Rather than requiring accessibility relations to be total preorders, as Lewis suggests, or requiring them to be preorders, as Pollock suggests, I think we should require them to be semiorders. More concretely, you might think of this as the view that accessibility relations are *nearly as close* relations, rather than *as close* relations or *determinately as close* relations.

What are *nearly as close* relations? Imagine again that you are standing in a grassy field and are surrounded my several brightly colored balls. These balls stand in *as close* relations. They also stand in *nearly as close* relations. A ball b is **nearly as close** as a to x when b is no more than t farther away than a from x. The value of t is what we will

17. As further illustration of the difference between the two, we might note that while Distribution is licensed by Lewis's **B3**, Simplification is not.

call the **tolerance margin**.

So for example, suppose there is a blue ball that is three feet away and a red ball that is four feet away. Suppose also that the tolerance margin is one foot. In that case, the red ball is *nearly as close* as the blue ball, even though it is not *as close* as the blue ball.

In ordinary contexts, when we say that something is nearly as close, this often implies that it is not as close. For example, if you tell a friend that the coffee shop is nearly as close as the bakery, this will often imply that the coffee shop is not as close as the bakery. Whether this implication is semantic or pragmatic is an interesting question. For our purposes, we are going to stipulate that there is no such implication. If b is nearly as close as a, it could be that b is as close as a. It could even be that b is strictly closer.

Once we have *nearly as close* relations, we can define other useful relations. We will say that b is **much closer** than a when b is nearly as close as a and a is not nearly as close as b. The distance of b and the distance of a is **roughly equal** when b is nearly as close as a and a is nearly as close as b.

So for example, suppose again that we have a one foot tolerance margin. In that case, one ball is much closer than another when it is more than one foot closer. The distance of two balls is roughly equal when neither is more than one foot closer.

Like *as close* relations, *nearly as close* relations are connected. For suppose that we have a tolerance margin of one foot, and consider any two balls a and b. Either a is no more than one foot farther away than b or b is no more than one foot father away a. But then, given our one foot tolerance margin, either a is nearly as close as b or b is nearly as close as a.

Unlike *as close* relations, *nearly as close* relations are not transitive. After all, suppose that a is three feet away, b is four feet away, and c is five feet away. In that case, using a one foot tolerance margin, c is nearly as close as b and b is nearly as close as a, but c is not nearly as close as a.

While *near closeness* relations may not be transitive, they still satisfy a weaker property that we will call semitransitivity.[18]

Semitransitive:
$$\begin{aligned}&(d \prec c) \wedge (c \prec b) \wedge (b \preceq a) \supset (d \prec a)\\&(d \prec c) \wedge (c \preceq b) \wedge (b \prec a) \supset (d \prec a)\\&(d \preceq c) \wedge (c \prec b) \wedge (b \prec a) \supset (d \prec a)\end{aligned}$$

18. Candeal, Induráin, and Zudaire (2002) call this property generalized pseudotransitivity.

Semitransitivity is the conjunction of three conditions. The first says: Suppose that d is much closer than c, that c is much closer than b, and that b is nearly as close as a. In that case, it follows that d is much closer than a. The other two conditions are then similar, except that the pattern of *much closer* and *nearly as close* relations in the antecedent is permuted. This condition clearly hold for *nearly as close* relations.[19]

When a relation is connected and semitransitive, it forms a **semiorder**.[20] What the above discussion shows, then, is that *nearly as close* relations are semiorders. However, they need not be total preorders or even preorders.

As we saw earlier, Lewis thinks that we should require accessibility relations to be total preorders. Pollock thinks that we should require them to be preorders. What I think is that instead, we should require them to be semiorders. Since *nearly as close* relations are semiorders, you might think of this as the view that accessibility relations are *nearly as close* relations. The result is that when the limit assumption is in place, Closest is replaced with Nearly Closest.

If we require accessibility relations to be semiorders, the resulting class of models generates a system that we will call **B2**.[21] This system is the result of adding two basic rules to **B1**. The first is Distribution, which was described earlier. The second is Diamond, which will be describe shortly. **B2** is strictly intermediate in strength between Pollock's **B1** and Lewis's **B3**.

The key observation is that Strengthen Might is not a correct inference of **B2**. Thus, if we only require accessibility relations to be semiorders, we have a solution to the tolerance paradox.

That we do in fact have a solution can be shown by building a model in which the premises are true at every world. To that end, let t be any natural number. We then

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19. For the first, suppose that d is much closer than c and c is much closer than b. Using a tolerance margin of one foot, this means that d is more than two feet closer than b. Now suppose that b is nearly as close as a. This means that at most, a is exactly one foot closer than b. But in that case, it follows that d is more than one foot closer than a. So d is much closer than a. Showing that the other conditions hold is similar.

20. Semiorders are usually credited to Luce (1956), who introduced them to model apparent cases of intransitive preferences. However, they were in fact introduced almost forty years earlier by Wiener (1914), a mathematician who studied under Bertrand Russell. See Fishburn and Monjardet (1992) for more on the early history of semiorders.

21. See my (2021) for soundness, completeness, and decidability results.

let:

$$\begin{array}{lcl} W & = & \mathbb{N} \\ L_x & = & \mathbb{N} \\ b \preceq_x a & = & \{\langle b, a\rangle \mid (|b-x| - |a-x|) \leq t\} \\ V(s_n) & = & \{x \mid x \geq n\} \end{array}$$

In this model, the domain of worlds is the set of natural numbers and each world has the set of natural numbers as its local domain. You might think of each n as a world in which Socrates is exactly n Planck lengths tall. We then assign accessibility relations using t as the tolerance margin. Finally, we choose a valuation function that assigns each atomic sentence s_n to the set of worlds at which Socrates is exactly n Planck lengths or greater.

Several facts about this model can be easily confirmed. First, Tolerance, Boundedness, and Heights are all true at every world. Second, every accessibility relation is a semiorder. Third, Strengthen Might fails in this model. Thus, Strengthen Might is invalid in the class of all Burgess models whose accessibility relations all form semiorders. Fourth and finally, No Sharp Cutoffs is false at every world. Thus, we can consistently accept all the premises of the tolerance paradox while rejecting the main premise driving the sorites paradox.

We said earlier that the basic rules of **B2** include a rule called Diamond.[22] That rule may be unfamiliar, but can be derived in **B3**, and so is accepted by Lewis, among others.

Diamond: $(A \vee B \mathrel{\Box\!\!\rightarrow} \neg A) \wedge (B \vee C \mathrel{\Box\!\!\rightarrow} \neg B) \vdash (A \vee D \mathrel{\Box\!\!\rightarrow} \neg A) \vee (D \vee C \mathrel{\Box\!\!\rightarrow} \neg D)$

What is this rule telling us? Suppose that Alice had been given the choice between one dollar, two dollars, three dollars, and one beer. Suppose also that:

Had Alice chosen either one dollar or two dollars, she would not have chosen one dollar. (18)

Had Alice chosen either two dollars or three dollars, she would not have chosen two dollars. (19)

22. I am calling this rule Diamond because, when thinking in terms of Burgess models, it expresses a property that I call diamond directedness in my (2021).

What Diamond tells us is that in that case, at least one of the following is true.

Had Alice chosen either one dollar or one beer, she would not have chosen one dollar. (20)

Had Alice chosen either one beer or three dollars, she would not have chosen one beer. (21)

The reasoning here is complex, and I grant that its correctness is not completely obvious. Nevertheless, I think that Diamond is in fact a correct rule of inference.[23]

12 Strengthen Easy

Suppose that had Alice flipped a coin, Margaux would have laughed. Suppose also that had Alice flipped a coin, it might have landed heads. From this, it would then seem to follow that had Alice flipped a coin *that landed heads*, Margaux would have laughed.

This reasoning would seem to be in perfectly good order and is an instance of Strengthen Might. The question is, if the general rule is not correct, then why does reasoning like this often seem to be correct? This section aims to answer that question.

Our modal practice includes not only counterfactuals, but also what you might call **comparative modals**. These come in a variety of flavors but, for our purposes, we will focus on claims of the form:

It could have as easily been that A as that B. (22)

So for example, you might think that it could have as easily been that a fair coin was flipped and landed heads as that a fair coin was flipped and landed tails. Or you might think it could have as easily been that there was life on Mars as that there was life on Venus.

To regiment such claims, we will add a two-place operator $\trianglelefteq$ to our language. We then read $A \trianglelefteq B$ as saying that it could have as easily been that A as that B.

Once we have both comparative modals and counterfactuals on the table, there is the question of how they are related. Lewis, for example, accepts the following

23. For those skeptical of Diamond, there is another system **B1.1** that results from adding Distribution, but not Diamond, to **B1**. That system is generated by the class of all interval orders. See my (2021).

equivalence.[24]

$$A \trianglelefteq B \dashv\vdash (A \vee B \mathrel{\Diamond\!\!\rightarrow} A) \vee (B \mathrel{\Box\!\!\rightarrow} \bot) \tag{23}$$

Given this, Strengthen Might is equivalent to another rule that we are going to call Strengthen Easy.[25]

Strengthen Easy: $A \mathrel{\Box\!\!\rightarrow} C,\ (A \wedge B) \trianglelefteq (A \wedge \neg B) \vdash A \wedge B \mathrel{\Box\!\!\rightarrow} C$

Suppose that had it been that A, it would have been that C. Suppose also that it could have as easily been that A and B as that A and $\neg B$. In that case, Strengthen Easy tells us that had it been that A and B, it would have been that C.

If we follow Lewis and accept (23), there is no hope of accepting Strengthen Easy while rejecting Strengthen Might. The two rules are simply equivalent. But while that may be, I reject the right-to-left direction of (23). I thus accept Strengthen Easy while rejecting Strengthen Might.

To show that the right-to-left direction of (23) fails, suppose that Socrates is h Planck lengths tall and consider any $n > h$. Tolerance tells us that:

$$s_n \mathrel{\Diamond\!\!\rightarrow} s_{n+1} \tag{24}$$

Which in turn entails:

$$s_{n+1} \vee s_n \mathrel{\Diamond\!\!\rightarrow} s_{n+1} \tag{25}$$

The right-to-left direction of (23) then gives:

$$s_{n+1} \trianglelefteq s_n \tag{26}$$

But this is false. Socrates could have *more* easily been at least n Planck lengths than at least $n + 1$ Planck lengths. It would not have been *much* easier, but it would have been easier.

Now return to the case from the beginning of this section. We started by accepting

24. See (Lewis 2001) pp. 52-6 and pp. 118-42. For further discussion of comparative modals and the role they play in our modal practice, see Kment (2014).

25. More precisely, the rules are equivalent in any system that includes(23) and extends **B1**. The proof is straightforward and so left to the reader.

two claims.

$$a \Box\!\!\rightarrow m \tag{27}$$

$$a \Diamond\!\!\rightarrow h \tag{28}$$

Had Alice flipped the coin, Margaux would have laughed. Moreover, had Alice flipped the coin, it might have landed heads. From this, it would seem to follow that:

$$a \wedge h \Box\!\!\rightarrow m \tag{29}$$

Had Alice flipped a coin that landed heads, Margaux would have laughed. This reasoning seems to be correct. But why would that be if Strengthen Might is not itself a correct rule of inference? The answer, I think, is that when we accept (28), we also accept:

$$(a \wedge h) \trianglelefteq (a \wedge \neg h) \tag{30}$$

That is, we accept that it could have as easily been that Alice flipped a coin that landed heads as that Alice flipped a coin that did not land heads. Using standard metarules, Strengthen Easy entails:

$$A \Box\!\!\rightarrow C,\ A \Diamond\!\!\rightarrow B,\ (A \wedge B) \trianglelefteq (A \wedge \neg B) \vdash A \wedge B \Box\!\!\rightarrow C \tag{31}$$

What this rule tells us is that so long as we have (30) as a background assumption, we *can* correctly infer (29) from (27) and (28). Since we often have such background assumptions in place, this explains why many instances of Strengthen Might would seem to be correct.

13 Burgess Models Again

If we are going to use Strengthen Easy to explain the apparent correctness of Strengthen Might, then we need a way to simultaneously model counterfactual operators and comparative modal operators. Moreover, we need to make sure that our models do in fact validate Strengthen Easy.

Fortunately, we can do this using the Burgess models we already have. The first observation is that every counterfactual accessibility relation $\preceq$ determines what you might call a **comparative modal accessibility** relation $\preceq^*$. This is done with:

$$b \preceq^* a \text{ iff } \forall x(((a \preceq x) \supset (b \preceq x)) \wedge ((x \preceq b) \supset (x \preceq a))) \tag{32}$$

Thus, we can think of every Burgess model as assigning not only a counterfactual accessibility relation to every world, but also a comparative modal accessibility relation to every world. These comparative modal accessibility relations can then be used to give the semantics for our comparative modal operator.

$$\mathcal{M}, x \vDash A \trianglelefteq B \quad \text{iff} \quad \text{for every } b \in L_x \text{ such that } b \vDash B\text{, there is an } a \vDash A \text{ such that } a \preceq^*_x b$$

Strengthen Easy is then valid in the class of all Burgess models. The proof is straightforward, and so left to the reader.

Putting the idea somewhat more intuitively, suppose that we have assigned a *nearly as close* relation to every possible world. We can then use these relations to give the semantics for counterfactuals in the way already described. Something else we can do, though, is use each *nearly as close* relation to determine a unique *as close* relation. Once we have these additional *as close* relations, we can use them to give truth conditions for comparative modal claims. In particular, it could have as easily been that A as that B if and only if for every B world, there is an A world that is at least as close. The result is that Strengthen Easy is valid.

14 Three Coins

Boylan and Schultheis (forthcoming) have recently suggested a counterexample to Strengthen Might. Since I think of the tolerance paradox as also being a kind of counterexample to Strengthen Might, I want to close by saying a bit about how our counterexamples differ.

The Boylan and Schultheis counterexample goes like this. Suppose that Alice, Billy, and Carol flip three fair coins.[26] The flips are simultaneous and causally independent. Alice flips heads, Billy flips tails, and Carol flips heads. Now consider the following claims:

Had Alice and Billy flipped the same, Carol would have flipped heads. (33)

Had Alice and Billy flipped the same, all three might have flipped the same. (34)

26. Boylan and Schultheis use a more complicated example involving dice and cash payouts, but the basic structure is the same.

Had Alice and Billy and Carol all flipped the same, all three would have flipped heads. (35)

Boylan and Schultheis claim that (33) and (34) are true, but (35) is false. If so, then we have a counterexample to Strengthen Might. Call this the **coin counterexample**.

The coin counterexample succeeds only if there are plausible uniform standards on which (33) and (34) are true, but (35) is false. The problem is that I can find no such standards.

Say that a coin at a possible world is **incongruent** if it lands differently than it does at the actual world. Now suppose that we use strict standards on which, had either n or m coins been incongruent, n coins would have been incongruent, when $n < m$. (33) and (34) are then true. The problem is that (35) is also true.

Alternatively, suppose that we use more tolerant standards. These could be standards on which, had either n or m coins been incongruent, then n coins might have been incongruent, so long as $n - 1 \leq m$. (34) is then true and (35) if false. The problem is that (33) is false.

The counterexample arguably goes through if we accept Simplification and understand x and y flipping the same as the disjunction of either x and y both flipping heads or x and y both flipping tails. After all, (33) and (34) would seem to be true. (35) is then false since, by Simplification, it entails that had all three flipped tails, all three would have flipped heads, which is clearly false.

The problem is that while these are uniform standards on which the coin counterexample goes through, they are not *plausible* standards, since Simplification is not a plausible rule of inference.

Suppose that Boundedness is true because, had Socrates been at least six feet, he would not have been at least ten feet. From this, it follows that had Socrates been either six feet or ten feet, he would not have been ten feet. But then by Simplification, it follows that had Socrates been ten feet, he would not have been ten feet. This is false. So Simplification is invalid.

Boylan and Schultheis and I agree that Strengthen Might is invalid. Why, then, is it so hard to find plausible uniform standards on which the coin counterexample goes through? The answer, I think, is Strengthen Easy. After all, given that Alice actually flips heads and Billy actually flips tails, it would seem that:

It could have as easily been that Alice and Billy both flipped heads as that Alice and Billy both flipped tails. (36)

Since Carol actually flips heads and all three flips are independent, it should then follow that:

> It could have as easily been that Alice and Billy and Carol all flipped the same as that Alice and Billy flipped the same and Carol flipped differently. (37)

But now given Strengthen Easy, (33) and (37) entail (35). Thus, plausible uniform standards on which (33) is true and (35) is false would have to be standards on which Strengthen Easy fails. This means that if Strengthen Easy is a correct rule of inference, as I maintain, that would explain why there are no such standards.[27]

Summing up, there are at least two important differences between the coin counterexample and the tolerance paradox. First, where the coin counterexample may or may not depend on Simplification, Simplification is incompatible with the tolerance paradox, since it is incompatible with Boundedness. Second, where the coin counterexample would seem to require Strengthen Easy to fail, the tolerance paradox does not. This is a nice feature because Strengthen Easy gives us an especially simple and straightforward explanation of why instances of Strengthen Might often seem to be correct, even if the general rule itself is not.

15 Conclusion

The first half of this paper presented the paradox of counterfactual tolerance. We defended the premises and considered the possibility of rejecting each of the various inference rules. The rule that should be rejected, I think, is Strengthen Might.

The second half of this paper presented a positive proposal for resolving the paradox. The positive proposal is to give up the common idea that counterfactuals should be understood in terms of *as close* relations. Rather, they should be understood in terms of *nearly as close* relations. Thinking in terms of Burgess models, this corresponds to

27. Boylan and Schultheis might respond by saying that Strengthen Easy does in fact fail. For my own part, I find this implausible, since I can still find no plausible uniform standards on which the counterexample goes through. I am also unsure how to square this response with their positive proposal, which is to follow Pollock and require accessibility relations to be preorders. In that case, Strengthen Easy is valid, given our proposed semantics for comparative modals. Thus, if Boylan and Schultheis deny Strengthen Easy, they will need to give a different semantics for comparative modals.

the idea that we should require counterfactual accessibility relations to be semiorders rather than total preorders or even preorders. Finally, we suggested an explanation for why instances of Strengthen Might are often compelling, even though the principle itself is invalid. In particular, in many contexts, there are background assumptions in place that allow the relevant reasoning to go through using a different rule called Strengthen Easy.

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