**Practising Philosophy of Mathematics with Children**

 *what, why and how?*

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**ABSTRACT**

*This article examines the possibility of philosophizing about mathematics with children. It aims at outlining the nature of the practice of philosophy of mathematics with children in a mainly theoretical and exploratory way. First, an attempt at a definition is proposed. Second, I suggest some reasons that might motivate such a practice. My thesis is that one can identify an intrinsic as well as two extrinsic goals of philosophizing about mathematics with children. The intrinsic goal is related to a presumed inherent importance of presenting children with some philosophical questions about mathematics. The extrinsic goals consist of first the positive effects such a practice can have on mathematical learning and abilities and second the fostering of children's understanding of philosophical method of inquiry and thinking and therefore of their philosophical thinking competences. Third, some examples found in the literature of previously developed ways of practising philosophy of mathematics with children are presented. This article aims at giving a general outlining picture of the issues surrounding the practice of philosophy of mathematics with children and should therefore be read as an encouragement to further development and studies.*

# INTRODUCTION

New Philosophical Practices (or Philosophy for Children, P4C) address a wide variety of themes and issues. However, certain subjects seem to enjoy a certain prevalence when setting up activities having philosophical aims, especially with children; this seems to be the case of subjects from ethics or philosophical anthropology. On the contrary, other subjects, such as philosophy of mathematics, although they occupy a central place in the history of philosophical thought, are hardly dealt with in these new practices.

This article proposes to question the possibility of philosophizing about mathematics with children and in particular to address with children ontological questions about the existence of mathematical objects and epistemological questions about the status of mathematical knowledge. The present aim is not to develop a complete practical program for philosophizing with children about mathematics, but rather to clarify the nature and meaning of such a practice.

Confronting children with philosophical considerations about mathematics is of considerable interest, particularly because of the level of abstraction required to question the status of mathematical objects and knowledge. Moreover, philosophical reflection may be able to provide a new and interesting “flavour” to school-based mathematical knowledge and learning[[1]](#footnote-0). Philosophical reflection on mathematics might also improve the attitude children maintain regarding mathematics. Mathematics seems to be commonly considered, more than any other school subject, (in)famous for putting pupils in difficulty and for lacking any intrinsic interest; therefore, being able to modify this commonplace perspective using philosophy of mathematics would obviously present a great pedagogical interest. It then seems that there might be both an *intrinsic* and an *extrinsic* goal to develop a practice of philosophy of mathematics with children.

In this paper, I will try to answer the questions of *what is it* to philosophize about mathematics with children, *why* to do itand *how* to do it. I will first attempt to elaborate a conceptual framing in order to propose a problematizing definition of the practice of philosophy of mathematics with children[[2]](#footnote-1). Then, different reasons or motivations for conducting such a practice with children will be identified and discussed. Finally, I will briefly present several examples of practical implementations found in the literature.

# PHILOSOPHY OF MATHEMATICS WITH CHILDREN

Before looking at the ways in which it is possible to philosophize about mathematics with children and at the interests of such an activity, it is firstly necessary to clarify what is meant here by “philosophy of mathematics” and secondly what it means to philosophize about mathematics *with children.* This second clarification will be provided by identifying the necessary conditions for carrying out such a practice, both in terms of content and form.

## **What is philosophy of mathematics?**

Mathematics and philosophy have been closely related in the history of thought. In particular, philosophy has been interested in mathematics, conceived as a very special area of human knowledge, and several fundamental philosophical questions have traditionally been associated with it. In this section, I aim to sketch – obviously in a far from exhaustive way – a brief outline of some important philosophical questions about mathematics. I do not make any attempts to develop or to problematize them, since the goal of the present discussion is solely to give an overview of some philosophical problems raised by mathematics, mainly in the perspective of figuring out what questions could be addressed when philosophizing with children about mathematics.

Gandon and Smadja (2013) identify “four modalities in which the relationship between mathematics and philosophy has been established in history” (Gandon and Smadja, 2013, p. 8, *my translation*). First, philosophers have often considered mathematical knowledge as a model for philosophical knowledge, setting certain epistemological standards, and mathematical method as a model for any rational enterprise. Thus, philosophy finds in mathematics a “source of inspiration” (Gandon and Smadja, 2013, p. 8) for its norm and rigour. But sometimes it is mathematics that calls upon philosophy to justify some theoretical developments or practical applications. Thirdly, mathematical knowledge, which seems to be characterized by a certain certitude and necessity, has found itself questioned philosophically, especially in its relationship to sensory experience. The origin and reality of its apparent necessity and certainty are also objects of philosophical questioning (Gandon and Smadja, 2013, pp. 9-10). Gandon and Smadja therefore identify the presence of a genuine philosophical questioning of mathematics. As we will see below, epistemological questions are strongly linked to questions concerning the ontological status of mathematical objects (e.g., numbers, functions, geometric figures, etc.). Finally, Gandon and Smadja mention that several great philosophers were also great mathematicians, such as Descartes and Leibniz (Gandon and Smadja, 2013, pp. 10-11). One may rightly wonder whether this association is purely contingent or whether there is a more essential relationship between these two areas of human thinking and knowledge.

It is mainly the third identified modality, i.e. the one concerning the philosophical questioning of mathematics, that interests us here. Shapiro (2000) categorizes questions, problems and discussions of philosophy of mathematics under four main themes: 1) the question of mathematical knowledge and its seemingly necessary and *a priori* character, 2) the question of the status of mathematical objects (ontological questions), 3) the question of the link between mathematics and the physical world (questions related to the application of mathematics)[[3]](#footnote-2), and 4) “local” questions, i.e. those that concern a particular mathematical problem, concept or result (Shapiro, 2000, pp. 21-39). It should be noted that this categorization seems to have a primarily pedagogical or presentational purpose, since the topics in these different categories are not isolated and independent from each other, but are variously linked. It is not possible, for example, to deal with the nature of mathematical knowledge without saying something about the ontological status of mathematical objects – since 1) this status will have a consequence on the nature of mathematical truth and 2) knowledge is intrinsically linked to truth – or to discuss the application of mathematics to the physical reality without questioning the supposed *a priori* character of mathematical knowledge.

This interrelation finds itself well illustrated by a famous dilemma in philosophy of mathematics: the dilemma of mathematical truth raised by Benacerraf (1973). One question concerning the truth of mathematical assertions is whether it is independent or dependent on the human mind and its constructs. For example, is the truth of “2 + 2 = 4” independent from human mental activity or does it depend on it in any way?[[4]](#footnote-3) This question about mathematical truth is strongly related to the question of the ontological status of mathematical objects, i.e., does “mathematics deals with objective features of the world ?” (Shapiro, 2000, p. 31). Benacerraf (1973) suggests that the conceptions of mathematical truth are primarily motivated by two types of consideration. The first is to attribute to mathematical language a similar functioning to that of ordinary language, i.e. a similar syntax and semantics. According to this consideration, the assertions of a mathematician must be understood in the same way as those of a physicist for example. When a mathematician says for instance that “there is an infinity of prime numbers” and his assertion is true, it means that *there is* an infinity of prime numbers. The mathematician’s assertion should therefore not be taken as metaphorical, i.e. as not really meaning that there is an infinity of prime numbers. Thus, this semantic consideration leads to a certain ontological realism. The second consideration identified by Benacerraf (1973) is epistemological: we *have* mathematical knowledge and this mathematical knowledge is explainable. However, if one accepts ontological realism with respect to mathematical objects, it is difficult to account for the fact that we can acquire knowledge about them, since they would be abstract objects with which it seems that we cannot have direct relationships. Thus, if we take seriously mathematical language and its potential to convey truth, we cannot explain mathematical knowledge, but if we postulate an anti-realist ontology of mathematical objects in order to accommodate the epistemological requirement, then we must reject that “mathematicians mean what they say and most of what they say is true” (Shapiro, 2000, p. 32). This dilemma has been received as a challenge in the field of philosophy of mathematics and has generated a great deal of debates. Moreover, it provides a good illustration of what the concerns of philosophy of mathematics may be, and thus gives an idea of the kinds of questions that the practice of philosophy of mathematics with children proposes to address with them[[5]](#footnote-4).

There are also philosophical questions about mathematics when it is conceived mainly as a *practice* and as a dynamic process of construction. Mathematics is thus seen primarily as a human activity. This perspective notably allows us to take into account a sociological reality of mathematics, that is to say to recognize that a social element is inherent to mathematical practice (Van Bendegem, 2014, p. 219). One of the main questions becomes then “does mathematics develop according to “laws” or patterns that are internal to mathematics (...) or, on the contrary, do external elements contribute and, if so, in what way(s)?” (Van Bendegem, 2014, p. 220).

## **What is philosophy of mathematics *with children*?**

What interests us here is the possibility of a genuine *practice* of philosophy of mathematics *with* children, that is of getting children *to think by themselves philosophically about mathematics*. At issue here is therefore not the teaching of philosophy of mathematics to children in a traditional sense, i.e. as a transmission of informative content from a sender-teacher to receivers-learners. It is not about teaching children elements of philosophy of mathematics, but of developing conditions in which children find themselves in a position to undertake, just like a philosopher does at his or her level, a reflection on mathematics that can be qualified as philosophical[[6]](#footnote-5). Therefore, it is the questioning and the successive discussion of a (meta-)mathematical notion[[7]](#footnote-6) that constitutes the starting point of any attempt to philosophize about mathematics with children. However, any discussion about mathematics (or about our relationship to it as human beings) is not philosophical, but some specific conditions must be met in order to ensure the “philosophicity” of the activity.

### **When is it *philosophy* of mathematics?**

Although some interactive mathematics teaching practices are reflective and collaborative[[8]](#footnote-7), they do not necessarily constitute a *philosophical* practice. In order to define the nature of the activity we are interested in here, it is useful to differentiate between several types of mathematical discourse that can take place with children.

Neubrand (2000) distinguishes four possible types of discussion and reflection about mathematics. First, it is possible to reflect on and discuss internal mathematical issues, such as the validity of a calculation or the definition of a concept, similarly to a mathematician. In the terminology of Neubrand (2000), this type of discussion is said to be at “the level of the mathematician” (Neubrand, 2000, p. 255). It is also possible to discuss issues that are internal to the practice of mathematics, such as the ways of solving a particular problem or the different techniques and methods that can be used. This is “the level of the deliberatively working mathematician” (Neubrand, 2000, p. 256). This is followed by “the level of the philosopher of mathematics”, where mathematics is considered “as a whole with critical distance” (Neubrand, 2000, p. 256). According to Neubrand (2000), this level includes the questions surrounding the application of mathematics. Considerations at this level have therefore a global scope, in that they are not limited to local concepts or states of affairs (see Ernest, 1985). They may thus concern “the nature of mathematics and the nature of meaning and truth in mathematics” (Ernest, 1985, p. 603). Ontological questions about mathematical objects are therefore situated at this level. However, philosophical questions about more local concepts, such as infinity for example, seem to belong to this level as well, particularly in that they depend largely on broader conceptions of the nature of mathematics and mathematical objects. Following the level of the philosopher, Neubrand (2000) identifies one more level in which discussions are mainly about epistemological questions, in that they concern the specificity of mathematical knowledge, both *per se* and in relation to other types of knowledge. This level, called “the level of the epistemologist”, seems to concern the philosopher of mathematics as well, since epistemological questions are inherently philosophical. If Neubrand distinguishes it from the previous level, it seems to be because he considers that the questioning that takes place at this last level is more fundamental in the sense of deeper or more absolute. By questioning the modalities of mathematical knowledge and thus its possibilities and limits, it is the very possibility and extent of the other modes of questioning that are critically examined. It thus seems that, as one rises up through the levels, one undertakes an ever deeper questioning, until one comes to question the very thinking about mathematics itself and its presuppositions. When practising philosophy of mathematics with children, it is essentially the last two levels of discussion of Neubrand’s classification that are targeted.

Hence, Neubrand distinguishes discourses on mathematics according to their content. Other authors (Daniel et al., 2005) classify discursive exchanges on mathematics according to their modes of functioning and to the socio-cognitive capacities that are mobilized by the participants. They identify five types of exchanges, ordered in a progressive perspective: 1) anecdotal exchange, 2) monological exchange, 3) non-critical dialogue, 4) semi-critical dialogue and 5) critical dialogue. The first two types of discussion are qualitatively different from the others in that they do not constitute proper instances of *dialogue*, as there is in them no consideration of the other participants’ points of view. The evolution between anecdotal and monological exchanges lies mainly in acquiring the capacity of distancing oneself from one’s personal experiences, allowing a greater focus on the question or theme under consideration. Once the exchange has taken the form of a dialogue, there is a continuous progression due to an ever greater presence of a critical dimension, until the dialogue takes the form of a “transaction among pupils, an open process in which the conclusions, when they are spoken, are open and temporary, serving as a hypothesis for future reflection” (Daniel et al., 2005, p. 340). Since these authors adopt a theoretical perspective[[9]](#footnote-8) according to which “philosophizing implicitly refers to learning to dialogue and, more specifically, to learning to engage in *critical* dialogue among peers” (Daniel, 2013, p. 65), they locate the philosophical aspect of an activity in the presence of a critical dialogical exchange among children. Thus, as Daniel (2013) points out, engaging in a truly philosophical dialogue about mathematics requires children to engage in a considerable learning process, which takes time and requests regular practice. The “philosophicity” of exchanges or reflections about mathematics is fully present only at the end of this learning process.

While it seems reasonable to consider that some minimal formal conditions (such as the presence of conceptualization, argumentation and problematization skills (Tozzi, 2012) or at least some interrogation processes and social skills) must be met for a reflection or discussion to be considered as philosophical, the nature of the conditions that are individually necessary and jointly sufficient for the “philosophicity” of a situation remains an open point for debate. In any case, if a discussion is to constitute an instance of practice ofphilosophy of *mathematics*, it is not sufficient for it to be philosophical (whatever that may imply). A condition relating to the content of the reflection or discussion must also be satisfied (as conceived by Neubrand (2000) for example).

### **When is it philosophy of *mathematics*?**

Daniel (2013) states that the material she developed with others[[10]](#footnote-9) toget children “to dialogue about and reflect on mathematics in a critical manner does not pretend to fall within the scope of the philosophy of mathematics” (Daniel, 2013, p. 60). Given her conception of what constitutes the “philosophicity” of an activity, this statement may seem surprising. Note that if the meaning of the latter is that this material (which is a philosophical novel intended for children) does not fit in and contribute to current debates in academic philosophy of mathematics, then it would only state something obvious and its relevance could be criticized. It is therefore more likely that Daniel wishes to indicate that this material does not claim to lead children to do proper philosophy of mathematics in the sense to reflect philosophically on questions recognized and debated by professional philosophers who animate this field of philosophy. Indeed, she goes on to state that “it simply aims to stimulate pupils toward an autonomous and critical comprehension of mathematical problems and concepts *included in their study programs*, as well as biases and stereotypes that are often attached to this subject” (Daniel, 2013, p. 60, *my italics*). Hence, according to the approach developed by Daniel and colleagues, philosophizing about mathematics with children involves critically questioning elements of the school curriculum and of beliefs related to mathematics primarily as a school discipline. The questions dealt with do not therefore necessarily have to be questions that interest philosophers of mathematics, but rather questions that concern mathematics as experienced in a school setting. Some questions may “fall within the scope of philosophy of mathematics” but this is not necessarily required. If therefore what makes an exchange philosophical according to Daniel and colleagues is its critical dialogical form, what makes it an instance of philosophy of *mathematics* with children is the link that its content exhibits with school mathematics.

The practice of philosophy with children can also find itself related to mathematics in a solely contextual manner. This is the case when discussions having a philosophical form take place in the context of mathematical activities (such as in mathematics teaching at school). Calvert et al. (2017), in a paper aptly named *Philosophizing with Children in Science and Mathematical Classes,* address such cases. What is important to them is then that discussions of a philosophical nature take place within mathematics lessons, but these discussions can belong to all levels identified by Neubrand (2000), as presented above. It is also possible that the mathematical situation is only the starting point for a philosophical discussion about questions belonging to another field of philosophy, such as ethics or political philosophy (English, 2013, cited in Calvert et al., 2017, p. 197). It therefore seems inadequate to consider all these discussions as practice of philosophy of mathematics with children, since a simple contextual concomitance between mathematics and philosophy is not sufficient for it to be so recognized[[11]](#footnote-10). The content of the philosophical discussion must be kept in a sufficiently close relationship with mathematics[[12]](#footnote-11). Moreover, a philosophical discussion with such content is not necessarily a discussion of philosophy of mathematics, for example if the philosophical dialogue is used for problem solving only, i.e. is entirely centered on technical mathematical practice. In addition to being significantly concerned with mathematics, the content of a (critical) discussion intending to be a practice of philosophy of mathematics with children must (be able to) have a certain philosophical dimension *itself*.

Assuming a philosophical mode of discussion and a content concerning mathematics (given that the context is mathematical lessons at school), Kennedy (2007) distinguishes three possible types of discussion, i.e. she identifies three ways of using a philosophical methodological approach to discuss mathematics. The philosophical methodology in question is that of a research community, as developed by Lipman (e.g. Lipman, 2003) and also adopted by Daniel and her collaborators[[13]](#footnote-12). According to Kennedy (2007), this philosophical approach can be used for: 1) “do and talk mathematics”, 2) “talk about mathematics” or 3) “talking about doing mathematics” (Kennedy, 2007, p. 296). Kennedy (2007) explains that it is mainly when a philosophical device is used in the second mode, i.e. to talk *about mathematics,* that a genuine philosophical dimension can emerge in a reflexive or discursive mathematical activity. This mode is characterized by a “collaborative inquiry into mathematical concepts (...), and the posing of philosophical questions that concern mathematics as a system (...) and their relations to human experience” (Kennedy, 2007, p. 296). In terms of content, this type of discussion or of reflection mode seem to correspond to the last two levels of Neubrand (2000) – the level of the philosopher and that of the epistemologist[[14]](#footnote-13). Kennedy (2007) is also explicit about the inclusion of considerations related to “local” concepts in this philosophical mode of reasoning.

By thus respecting both formal and thematic constraints, this mode of reflection or discussion seems to me to adequately constitute what can be considered as a practice of philosophy of mathematics with children (“philosophizing about mathematics with children”). This is not the case of the first and third modes described by Kennedy (2007). The first (“doing and talking in mathematics”) focuses on questions of mathematical problem-solving and thus corresponds in terms of content to the first two levels of Neubrand (2000). The third is intended to be a meta-reflective analysis of the way in which discussions in the first two modes take place. It means that the discussion carried out at that level will not have a (meta-)mathematical content, but a theme related to group dynamics and to the particularities of the experiences of a particular research community.

At the end of this reflection on the nature of the practice of philosophy of mathematics with children, it appears that a double constraint is imposed on any activity that intends to be an instance of this practice: a formal or structural constraint that requires that the activity be qualified as a practice of *philosophy* with children, but also a thematic constraint that concerns the content of the reflections or discussions that are undertaken. To use a terminology proposed by Daniel et al. (1999), the reflection or discussion must be about philosophical-mathematical concepts (“whose essence is philosophical and content is mathematical” (Daniel et al., 1999, p. 427)) or meta-mathematical ideas (i.e. ideas of common sense that concern mathematics).

# WHY PHILOSOPHIZING ABOUT MATHEMATICS WITH CHILDREN?

According to Aristotle, the purpose of a thing (the reason for which that thing was established) has a certain priority in the (scientific) explanation of that thing[[15]](#footnote-14). Without committing oneself to the whole Aristotelian philosophy, it seems relevant to note that the reason for which a certain practice of philosophy of mathematics with children is established may have important consequences on the way it is practised and even on the way its essence or nature is to be conceived. For example, if one considers that the primary purpose of philosophizing with children about mathematics is to promote school learning of mathematics, then it seems relevant to focus that practice (and conceived it as so focused) on school mathematics, and perhaps to weaken the thematic requirements proposed by Kennedy (2007).

While a purely extrinsic reason related to improving purely mathematical abilities may be the first reason that comes to mind when asking why philosophizing about mathematics with children, other types of motivations may also be identified. It is these different motivations underlying the practice of philosophy of mathematics with children that I now wish to explore.

Jankvist and Iversen (2013) propose a binary classification of the reasons to use philosophy of mathematics in mathematics education, based on their extrinsic (“philosophy as a tool”) or intrinsic (“philosophy of mathematics as a goal”) nature. Jankvist and Iversen consider the reasons for philosophizing about mathematics with children from the perspective of mathematical (school-based) education, and consequently identify only one type of extrinsic goal: that of promoting children’s understanding of mathematics. Based on their classification, I will propose an additional type of extrinsic goal: that of broadening children's philosophical experience and fostering their philosophical abilities. We can label it accordingly as “philosophy as a goal and philosophy of mathematics as a tool”.

## Extrinsic reason 1: for mathematics (“philosophy as a goal”)

According to a study conducted by the British association SAPERE[[16]](#footnote-15), the practice of philosophy for children appears to have a favourable effect on mathematical skills in children aged between 7 and 11 years (Gorard et al., 2015). There is therefore some evaluation of the *effects* of practising philosophy with children on mathematical learning. But it is a general practice of philosophy with children that is here evaluated, and not the particular practice of philosophy of *mathematics*. However, one of the goals of this particular practice is to have a similar effect, i.e. to promote mathematics learning. This positive effect does not necessarily and only have to concern mathematical skills assessed by tests of mathematical competence (standardized evaluative tests), but might also include any improvement in children's understanding of mathematics as well as in their attitude towards mathematics (notably in terms of motivation, confidence or creativity)[[17]](#footnote-16). Philosophizing about mathematics with children is thus first presented as a tool for teaching and learning mathematics, in that it “may assist students in their sense-making of (...) *in-issues* of mathematics”(Jankvist et al., 2014, pp. 206-207).

This extrinsic goal seems to be the most attributed goal to the practice of philosophy of mathematics with children in the existing literature on the latter. This is mainly explained by the fact that most authors are firstly interested in mathematical education (in school setting) and are only interested in the practice of philosophy in a second stage or indirectly, i.e. as a didactic means that can improve mathematical capacities. In this framework, the practice of philosophy of mathematics with children is presented as adequate to achieve this goal, in view of its potential effects on cognitive, affective and social aspects related to mathematical learning. Certain cognitive and affective aspects concerned by the practice of philosophy of mathematics with children will now be developed[[18]](#footnote-17).

### Cognitive aspects

According to Roy and Schubnel (2017), “two modes of thinking would be indispensable intellectual tools for solving mathematical problems (…): critical thinking and creative thinking” (Roy and Schubnel, 2017, p. 20, *my translation*). Learning mathematics would therefore make it possible to develop these two thinking abilities that “contribute to the formation of enlightened citizens” (Roy and Schubnel, 2017, p. 20, *my translation*). Mathematics can thus be “instrumentalized” for the development of transversal skills. However, the influence seems to be reciprocal, in that improving these thinking abilities results in better mathematical skills (Roy and Schubnel, 2017, p. 26). This is suggested by Kroesbergen and Schoevers (2017) regarding creativity, who suggest a positive relationship between creativity and mathematical performance (Kroesbergen and Schoevers, 2017)[[19]](#footnote-18).

These thinking abilities are general but it is also possible to identify a critical or creative *mathematical* thinking*,* i.e. a *specific* creativity or critical attitude which consists of a creative or critical reasoning applied to mathematical content[[20]](#footnote-19). It is probable that the ability to apply these general transversal thinking abilities to mathematical content is *critical* to improving mathematical performance. But achieving this specific application is not obvious and easy. It is perhaps in this respect that the practice of philosophy of mathematics can be of considerable help. By philosophizing about mathematics, children learn to reason critically and creatively *about mathematics*. If the practice of philosophy in general develops critical and creative reasoning[[21]](#footnote-20), children may then still experience difficulties in applying such reasoning to a mathematical content[[22]](#footnote-21). Interestingly, this *transitional* difficulty may eventually be reduced if the content of the philosophical discussion is mathematical.

*Creativity*

Slade (in De la Garza et al., 2001, pp. 92-97) argues that philosophizing about mathematics with children fosters the development of personal mathematical creativity understood as the ability to develop new ideas and alternatives and to use one’s “imagination about the conceptual spaces of mathematics” (Slade, in De la Garza et al., 2001, p. 94) to explore its nature and limits. According to her, when applied to mathematics, philosophical methods based on the questioning of ideas and of their presuppositions and on the development of conceptual alternatives would enable children to acquire creative mathematical thinking.

Roy and Schubnel (2017) suggest that creative thinking is often associated with divergent thinking “which has four essential components: fluency, flexibility, originality, and elaboration” (Roy & Schubnel, 2017, p. 25, *my translation*). Referring to the production of (new) ideas, fluidity indicates their number (i.e. facility of production), flexibility signals their diversity, originality is linked to their new or unusual character, and elaboration concerns the strength and depth of their explanation. Creative mathematical thinking can thus be characterized by the presence of these four properties in the production of mathematical ideas, methods, solutions and problems. According to Roy and Schubnel (2017), their presence is dependent (at least in part) on certain attitudes through which the subject adopts a posture enabling creative thinking to develop. These attitudes include openness, curiosity, imagination, and daring to take intellectual risks. By bringing children to philosophize about mathematics, a new perspective on the subject might appear to them, notably through the awareness that mathematics can be questioned and that they *themselves* can be the actors of this questioning. This new perspective and awareness might in turn encourage the adoption of these four attitudes towards mathematics.

With respect to curiosity, Kennedy (2012, pp. 85-86) suggests that the practice of philosophy of mathematics with children has the potential to generate a curiosity for mathematics in children[[23]](#footnote-22). Indeed, it seems to me that, for example, the discovery of ontological questions about the status of mathematical objects can awaken in children a powerful sense of astonishment. This feeling can then provoke an “epistemological curiosity” (Kennedy, 2012, p. 86), i.e. a curiosity that pushes children to question their relationship to knowledge, and thus to adopt a critical attitude. But this curiosity needs to be stimulated and nurtured in order to develop into a critical attitude of inquiry (Kennedy, 2012, p. 86).

*Critical Thinking*

The cognitive movement occurring when children begin to *question mathematics* – or perhaps even sooner when they realize that mathematics *is problematic* (in the sense of “questionable”) – seems to be at the root of the facilitating effects that a practice of philosophy of mathematics with children could induce. This movement is made possible by the reflective and critical attitude that is encouraged by any philosophical reflection or discussion. As mentioned above, philosophical reflection or discussion is mainly characterized by this very critical dimension (Daniel, 2013). According to Kennedy (2016), “By philosophical inquiry I understand (...) the process of arriving at *critical judgments* regarding philosophical questions or issues that have become a focusing point of a given group dialogue” (Kennedy, 2016, p. 2). Philosophy thus seems to consider critical thinking as its goal as well as its method[[24]](#footnote-23).

Based on a reflexive and evaluative approach, critical thinking interrogates every presupposition, including those on which it is *itself* based, in that it is (also) self-critical. Aiming at getting rid of anything that might in one way or another constrain reasoning, it tries, so to speak, to “wipe away” presuppositions in order to create a maximally free conceptual space in which only what has been subjected to previous scrutiny is present. In this maximized space, free from any unexamined constraints, thinking can then genuinely operate by itself and can construct its conceptual world according to the examined elements that it has itself put at its own disposal and to its capacity to arrange them (and here creativity seems to intervene).

Critical mathematical thinking includes the ability to distance oneself from mathematical problems in order to analyze the data, to identify potential problems, to situate them within the conceptual field of mathematics, to devise ways to solve them and evaluate their relevance. Having such an attitude regarding a mathematical problem undoubtedly seems to favour the development of good mathematical performance, especially that of *autonomy* in mathematics.

According to Roy and Schubnel (2017), the adoption of critical thinking[[25]](#footnote-24) is dependent on certain attitudes (as in the case of creativity), including “a concern for clearly stating the problem in a situation, examining different perspectives, a tendency to apply critical thinking skills, and an expression of mind openness” (Roy and Schubnel, 2017, p. 23, *my translation*). Learning to adopt these attitudes towards mathematics through the practice of philosophy of mathematics can facilitate the adoption of these attitudes in the context of mathematical problem solving (which may, as has been suggested, have beneficial consequences on mathematical performance). Here we find again the idea that the practice of philosophy of mathematics acts as a bridge between transversal abilities, i.e. between abilities that can be developed through the practice of philosophy in general and their application to mathematical content (which is crucial for the improvement of mathematical skills).

*Construction of meaning*

It is often reported that children struggle to find meaning in learning mathematics (Daniel, 1999; 2013) and that this can have a demotivating effect on them and thus can impact their performance. Philosophy of mathematics might appear as a way to help children to make sense of mathematics and its learning.

According to Pallascio (2000), the meaning of ideas “lies in their practical applications” (Pallascio, 2000, p. 128, *my translation*). From this perspective, making sense of mathematics (and thus of mathematics learning) means knowing what the practical applications of mathematics (and of what one learns in school mathematics) are. However, it seems a bit too reductive to consider that the practical dimension of something exhausts its meaning. Indeed, it is often possible to recognize a non-practical but intrinsic meaning of something.

Astolfi (2008) points out that “the same movement that apparently creates nonsense in fact makes it possible to expand the space of available meaning” (Astolfi, 2008, p. 34, *my translation*). In the case of mathematics, if moving away from their practical applications even more than what propose the situations in which mathematics is learned at school by proposing a philosophical reflection on mathematics – notably through epistemological and ontological questions – may, at first sight, seem undesirable because it increases children's sense of loss of meaning, it may on the contrary be at the origin of a *new* construction of meaning. The space, i.e. the considerations, in which to search for meaning in mathematics increases through philosophical reflection. It is no longer limited only to considerations related to the practical applications of mathematics and to what mathematics is used for, although these practical considerations can (and surely must) obviously be included in the construction of the overall meaning of mathematics. Becoming aware of the possibilities for philosophical (especially ontological and epistemological) reflection on mathematics creates a *positive pull away* from what is known and familiar, as it allows one to move away from a purely pragmatic perspective and thus to develop a new approach to mathematics. To use Astolfi's (2008) expression, it seems to me that philosophical questioning about mathematics allows children to develop an “extraordinary view” (Astolfi, 2008, p. 19) on mathematics, which goes beyond that of sufficient knowledge for daily life (or even practice in general). And this new perspective might be a source of meaning.

Philosophical reflection on mathematics allows conceptual deepening. More specifically, it makes it possible to grasp a mathematical notion not only as encountered in school mathematics exercises, but also to perceive its genesis, its “raison d'être”, its consequences, its limits as well as the questions it generates. This might bring some depth to mathematical knowledge, instead of being satisfied with a superficial understanding limited to the ways in which a notion is used to solve certain exercises. This depth may foster the solidity of mathematical knowledge, leading to autonomy and flexibility in mathematics. It is a source of greater “intellectual freedom” (Pallascio, 2000, p. 127).

### Emotional aspect: mathematical anxiety

Mathematical anxiety is defined as the fear or apprehension of mathematics or more precisely as a negative emotional reaction in situations involving the solving of a mathematical problem (Young, Wu & Menon, 2012). Several studies have shown a significant relationship between mathematical anxiety and mathematical performance (see Ramirez et al., 2016, p. 84 for references). Mathematical anxiety has a negative impact on performance because it induces avoidance of mathematical situations but also because it disrupts the cognitive resources needed to solve mathematical problems, i.e. working memory (Ramirez et al., 2016, p. 84). These effects are already observed in young children. In the study by Ramirez et al. (2016), 26% of the children tested (with an average age of about 7 years) experience moderate to high anxiety (Ramirez et al., 2016, p. 89). Mathematical anxiety is thus a major impediment to the proper development of mathematical skills that must be taken into account in school instruction.

Ramirez et al. (2016), after showing that anxiety impacts mathematical performance by influencing which problem-solving strategies are used, note that “enabling students to more effectively deploy the strategies that predict success in mathematics will require not only teaching students math content but also providing them with ways in which to alleviate the anxiety they experience when engaging in mathematical thinking” (Ramirez et al., 2016, p. 97). Lafortune et al. (2003) hypothesize that philosophizing about mathematics with children could be a way to reduce this anxiety. In the study following this hypothesis, they observe that the effect of the philosophical approach to mathematics is mainly stabilizing - rather than reducing - on the development of anxiety. Knowing that the level of anxiety tends to increase with the time spent at school, this result is rather positive. In addition, the level of satisfaction (pleasure) and investment when performing mathematical tasks seems to increase following philosophical practice (Lafortune et al. 2003). However, in order to determine with greater validity the real effects of practising philosophy of mathematics on mathematical anxiety, further empirical studies are needed[[26]](#footnote-25).

However, it is possible to be specified why it seems *a priori* that philosophizing about mathematics can help to reduce mathematical anxiety. Firstly, if philosophical discussions directly confront this theme of math anxiety, for example by asking questions such as “why can mathematics be scary?” or “is mathematics a desirable or undesirable activity and why?”, it may be that anxious children gain a better understanding of (the origin of) their attitude and may then, through some process of demystification, regain a more realistic attitude towards mathematics. Deconstructing certain stereotypes about success in mathematics, particularly in relation to gender criteria, also seems to be beneficial in this respect – knowing that Lafortune et al. (2003) observe that girls have significantly higher anxiety levels than boys.

Moreover, it seems to me once again that the simple fact of realizing that mathematics can be questioned and that there is, behind the somewhat algorithmic practice sometimes experimented at school, a whole “universe” of philosophical questions concerning mathematics can prove beneficial in reducing anxiety. Anxious children might feel that they have no control over mathematics, no way of thinking about it, no way of handling it, and therefore that they are not able to do mathematics *themselves*. This feeling might partly explain why they get stuck in thinking they do not have an answer they should have. Philosophy of mathematics may enable these children to experience mathematics as something that can be grasped, as something that can be questioned in the light of their personal experiences and thoughts, and to experience themselves as valid actors, thinkers and interlocutors as regards mathematics. Ideally, these children will perhaps be able, after a certain amount of time necessary for internalization[[27]](#footnote-26), to transpose this feeling of agentivity to their mathematical practice and thus reducing their emotional “blockage”. In general, philosophical discussion about mathematics can enable some children to have a (first) positive experience in a mathematical context, notably by personally acknowledging their ability to express themselves about mathematics and the relevance of their opinion. In other words, some children may regain some confidence during philosophical discussions and maintain it (at least in part) in future mathematical activities. Lafortune et al. (2003) identify two affective aspects relating to decreasing math anxiety: confidence in one’s mathematical abilities and the sense of control. As I suggested, philosophy of mathematics with children could perhaps help to improve both of them.

## Intrinsic reason: for philosophy of mathematics (“philosophy of mathematics as a goal”)

Jankvist and Iversen suggest that the practice of philosophy of mathematics with children can also be motivated by an intrinsic goal, i.e. one that is not external to the practice and to what is being discussed during the practice. Philosophizing about mathematics with children thus presents itself as a goal in itself and is justified by the fact that philosophy of mathematics (in a broad sense) is a field that deserves attention for its own sake. Note that Jankvist and Iversen speak of “philosophy as a goal” and not of “philosophy *of mathematics* as a goal”. However, it seems to me that the second expression fits their description better. Also, it makes it possible to insist on the intrinsic character of this goal, whereas if it were philosophy in general that was aimed at, the goal would (at least in part) also be extrinsic. This extrinsic philosophical goal will be developed below.

Philosophizing about mathematics can be a source of pleasure and given the particularity of the object of discussion or reflection, this pleasure may have a particular and proper flavour. If this is the case, allowing children to experience it may well be a goal in itself. The level of abstraction traditionally associated with the objects of mathematics gives this science a special status within human knowledge and thus within human experience. Questioning its nature can thus provoke a sense of self-esteem, recognizing oneself as a being capable of thinking about and questioning such abstract things. Moreover, the interrogation process seems to be associated with a certain control (or even with a certain superiority with respect to what is being interrogated – though possibly only temporary) on the part of the questioner. Thus, this “control feeling” applied to mathematics can be pleasant.

Philosophy in general makes it possible to get away from everyday considerations, often linked to practical concerns. This distancing can generate a certain feeling of freedom. It allows one to “escape from emergencies” (Astolfi, 2008, p. 31, *my translation*) of daily life and from its dominant pragmatic dimension. Philosophizing *about mathematics* allows one to distance oneself even further from everyday considerations and thus to make new conceptual experiences, which can generate a certain (intellectual) pleasure. Certainly, the reality and nature of this supposedly pleasurable cognitive phenomenology in philosophical thinking about mathematics needs to be clarified and validated by further examination. But if it proves to be present, then it would speak in favour of the fact that philosophizing about mathematics is a goal (and a good) in itself. Astolfi (2008) speaks of an “erotic dimension of knowledge” (Astolfi, 2008, p. 37), which children often discover, according to him, long after they have finished school. Perhaps philosophical reflection on mathematics might help children to feel this dimension earlier.

However, it seems that Jankvist and Iversen (2014) have something else in mind when they suggest an intrinsic goal to the practice of philosophy of mathematics with children. They seem to accept that at least some questions of philosophy of mathematics are important and interesting enough to be addressed for their own sake, without other extrinsic goals. They mention questions relating to the foundations of mathematics[[28]](#footnote-27)  or to the epistemology and ontology of mathematics and questions of the role and application of mathematics. This last category of questions seems to me to illustrate the fact that certain philosophical questions about mathematics are important for our very understanding of the worldand therefore need to find a place in educational programs[[29]](#footnote-28). Since nowadays the world is characterized by an increasing use of mathematics for scientific and technological purposes and thus by a certain omnipresence of mathematics in our daily lives, understanding and questioning the links between mathematics and the physical world (and consequently physical sciences) seems to be necessary to become a lucid and independent individual. Any ethical reflection on mathematics and its actual and potential uses presupposes, moreover, some knowledge and thinking about the application of mathematics.

## Extrinsic reason 2: for philosophy (“mathematics as a tool and philosophy as a goal”)

Philosophizing about mathematics with children also seems to me to be a way of improving their *philosophical* skills. This third reason for philosophizing about mathematics with children is therefore an extrinsic reason, but one which, instead of taking as its goal the improvement of mathematical skills, aims to lead children to a better understanding of the nature of the philosophical enterprise and to a better mastery of its tools of investigation. In this sense, while Kennedy (2012) states that philosophizing about mathematics with children “represents a potential expansion of student's mathematical experiences” (Kennedy, 2012, p. 81), I suggest that the same is true of their *philosophical* experience. Moreover, it seems likely that practising philosophy of mathematics with children will improve the quality of philosophical exchanges on other philosophical subjects, such as ethical issues, that can take place between them.

The philosophical experience offered by philosophy of mathematics seems to be very specific, especially for individuals who are beginning to discover philosophical reflection and who are therefore starting the process of learning to philosophize. As mentioned above, to philosophize has to be learned and this learning is characterized by the acquisition of various skills, among which is the (complex[[30]](#footnote-29)) ability to distance oneself from one’s personal experience and one’s representation of the world in order to conceive (reasonable) alternative models or possibilities and to evaluate them in terms of their presuppositions and consequences. This complex capacity is critical to the philosophical act and posture. A repeated practice of philosophical discussions and activities allows children to develop it progressively. However, my suggestion is that the practice of philosophy of mathematics could have an accelerating effect on the acquisition of this capacity.

First, philosophizing about mathematics requires an important distancing from one's sensible and daily experiences. The need for this distancing quickly becomes obvious to anyone who tries to philosophize about mathematics, in that appealing to one's personal experience often seems irrelevant in this case. This appearing irrelevance of unexamined personal experience may have the effect of “blocking” the natural temptation to appeal to it. If for example the theme of a philosophical discussion with children is “friendship” and its aim is to reflect on the nature of friendship, then it is very tempting for children to share their personal experiences of friendship. Children who are not yet fully familiar with the type of acceptable answers to a philosophical question about the nature of something consider these reports of experience to be adequate answers. The discussion then risks not going beyond the anecdotal level, which does not allow to get close to an attempt of definition. If on the other hand, the question is “what is a number?”, children will find it more difficult to identify personal experiences that they feel relevant to propose, which may lead them to mobilize other skills and other types of data to develop an answer.

Hence, the use of daily life experiences often seems to come naturally when developing responses. It is an easy and heuristic way to respond. In order not to do so, it requires conscious control, which necessitates the ability to reflect on oneself and on the present situation. Having this ability comes only after a consequent learning process. However, if the question or theme is such that it does not trigger this natural tendency to report one’s personal experience because it presents itself from the outset as being too far removed from it, as seems to be the case with many philosophical questions about mathematics, the need to resort to other ways of elaborating an answer will be more quickly detected.

Similarly, the unfamiliarity we have with philosophically questioning mathematics seems to create a “dead time” or a time for reflection between the moment the question is asked and the moment we feel like giving an answer. Likewise, unless we are explicitly aware that we are in a philosophical context and have a knowledge of the aims and expectations of such a context, to the question “what is a friend?” we naturally feel like answering it *immediately,* without feeling the need to take time for reflection, because we feel that we “know” or that we have an idea of an answer or at least that we have something to say about it. But if the question is “what is a number?” it is likely that we need to take time to reflect on this. We hardly feel like answering right away, even without an explicit awareness of the specificities of philosophical discussion or reflection, because in this case we have the feeling that we do *not* know. But whatever the question, this moment of reflection is often necessary for the elaboration of a quality answer, which is not purely intuitive but is rather the result of critical and creative thinking.

The general hypothesis I want to put forward here is that through the specific practice of philosophy of mathematics, children have the opportunity to grasp the nature of philosophical reflection more directly than through the discussion of other themes that are closer to their daily experiences. If they then succeed in using this learning of what it means to philosophize when they are engaged in philosophical reflections or discussions about other types of subjects (like “friendship”), the quality of these latter philosophical activities would then be improved.

It seems to me that ontological and epistemological questions about mathematics are perhaps those that have the greatest impact on the enrichment of philosophical experience. Specifically, ontological questions about the nature of the existence of mathematical objects, including questions about the existence of abstract objects, appear to constitute unusual reflective experiences, which thus have the potential to change the perspective maintained on mathematics but also on philosophy and on the possibilities of philosophical imagination. As far as epistemological questions are concerned, they seem to me to be paradigmatic of a critical questioning of knowledge and generally of the need for creative philosophical thinking in order to build explanatory models[[31]](#footnote-30).

Philosophical questioning about mathematics thus seems to me to be able to help children to enter fully into thought and into philosophical reflection, as they might feel freed from the “weight of the sensible” which can sometimes be detrimental to the understanding of the nature of the philosophical movement. Philosophizing about mathematics can thus be an opportunity for a genuine experimentation in thought and for philosophical imagination, which can then be applied to themes where this freedom is more difficult to feel in the first place, because they are more constrained by the sensible world and the human experience of it.

Before concluding this section on the motivations for philosophizing about mathematics with children, it should be pointed out that the distinction made above between three goals attributable to this practice does not claim to refer to a real and exhaustive distinction. It is only useful “to provide analytical clarity and insight as to why philosophy of mathematics is - or could be - used” (Jankvist and Iversen, 2014, p. 207). For example, the issue of constructing meaning in mathematics is as close to an intrinsic purpose as it is to an extrinsic purpose (that of learning maths). Nevertheless, as already mentioned, the specificity of the practice may vary depending on the primary goal of its establishment.

# HOW TO PHILOSOPHIZE ABOUT MATHEMATICS WITH CHILDREN?

The last part of this paper is a bit more practice-oriented. It intends to present some examples of practical implementation of the philosophy of mathematics with the children found in the literature.

## P4CM and *The Mathematical Adventures of Mathilde and David*

In the late 1990s, inspired by the development of philosophy for children (P4C) by Lipman, a Quebec research team (M.-F. Daniel, L. Lafortune, R. Pallascio and P. Sikes) developed an approach to philosophy for children adapted to mathematics (P4CM[[32]](#footnote-31)). Lipman suggested starting philosophical reflections with children by reading a philosophical novel specifically created for this purpose in order to stimulate reflection but also to offer a model of philosophical dialogue for children, since the characters in the novel engage themselves in *communities of philosophical research*. Moreover, children could identify themselves in these reflective dialogues. In continuity with this practice, M.-F. Daniel and colleagues developed a philosophical novel for children aged between 9 to 13, entitled *The Mathematical Adventures of David and Mathilde* – originally written in French under the title *Les Aventures mathématiques de David et Mathilde* (Daniel et al., 1996, (2)) – as well as a teacher manual aiming at facilitating philosophical discussions following the reading of the novel (Daniel et al., 1996, (1))*.*

According to the authors, “the pedagogical principles of the *Philosophy for Children Applied to Mathematics* program should enable young people to develop a better relationship with mathematics and sciences” (Daniel et al., 1996, (1), p. 9, *my translation*). Thus, an important part of the proposed reflections in the novel deal with issues related to attitudes towards mathematics (e.g. affective or doxastic attitudes). Daniel (in De la Garza et al., 2001) explains that “the novel we wrote depicts pupils’ daily life experiences in relation to philosophico-mathematical concepts and problems. The narrative is based on young people’s personal concerns (...)” (De la Garza et al., 2001, pp. 98-99). But the companion guide also includes exercises in logic and mathematics. In addition, in keeping with what was mentioned above regarding Daniel’s understanding of the nature of philosophy of mathematics with children, a section is devoted to discussing and questioning mathematical concepts themselves. In this way, children have the opportunity to “construct and experiment their own mathematical theories, principles and problems, in order to feel the same fascination, excitement and pride the first mathematicians felt when they elaborated their mathematical laws” (De la Garza et al., 2001, p. 99).

Initially, this material was conceptualized for use in mathematics classrooms for one hour per week (De la Garza et al., 2001, p. 99). However, nothing prevents its use from being adapted to different situations. Moreover, it should be noted that the authors recommend a classic Lipmanian methodology for the structure and conduct of the activity: some students reading aloud a chapter of the novel, question gathering, individual reflection preceding the discussion, and finally philosophical discussion in a research community.

To give a more figurative overview of the content of this material, let us briefly describe as an example one chapter of it. In the eighth chapter of the philosophical novel[[33]](#footnote-32), the characters begin a discussion on the theme of infinity. The context is that of a party among friends during which a group of children starts playing chess. A child, named Rosalie, wonders if the possibilities of moving in a chess game are infinite. The children consequently wonder about the nature of infinity and the possibility of thinking about it, and then they look for examples of infinite things or sets. The case of the number of grains of sand present on earth is considered. Subsequently, they make a distinction between an infinite and an indefinite number. The example of a straight line is proposed and then the example of numbers. On reading this short passage, we can see that the theme is only slightly problematized or developed. In accordance with the authors' wishes, it is up to the children to propose philosophical questions following the reading; the text is only a stimulator. In the companion guide (Daniel et al., 1996 (1)), there is some advice for dealing with those questions (Daniel et al., 1996 (1), pp. 141-142). After a short introductory paragraph on the concepts involved, a discussion outline is provided in the form of a list of questions that allows the facilitator to initiate a reflection and discussion. These questions help to situate the notion of infinity among other notions, such as immensity, eternity, the indefinite, divisibility (and indivisibility), imagination (including the link between conceptualization and possibility or reality) and numbers. Afterwards, three exercises are proposed, including a conceptualization exercise concerning infinite division and two more mathematical exercises (extending a sequence of fractions and wondering when it ends, introducing thus the notion of limit, and comparing two infinite sets of numbers, integers and peer numbers). Finally, a strictly mathematical activity related to chess game is suggested. This mathematical activity is moreover embellished with questions permitting to launch a philosophical discussion about it.

## Nadia Kennedy and *Mathematical Research Communities*

Nadia Kennedy, like M.-F. Daniel and her collaborators, adopts a Lipmanian approach to philosophy for children. She develops the idea of a “discipline-based inquiry practice” (Kennedy, 2012, p. 82), that is a community of mathematical inquiry (CMI[[34]](#footnote-33)) that “embodies most of the essential characteristics of a community of philosophical inquiry as conceived by P4C, but introduces some further field-specific differences” (Kennedy, 2007, pp. 293-294). The main difference between the approach favoured by Kennedy and the one developed by Daniel and her colleagues presented above seems to be that Kennedy focuses on cases where the philosophical discussion emerges of its own or directly follows a mathematical teaching situation. As Kennedy (2007) notes, a philosophical discussion about mathematics can be initiated in two ways: either it is planned and prepared in advance (in which case specific material, such as a philosophical novel, can be used), or it emerges from a strictly mathematical discussion. In the latter case, “the facilitator has a choice whether to embrace the emerging philosophical impulse and allow the discussion to unfold, or to forestall it by adhering strictly to the mathematical inquiry” (Kennedy, 2007, p. 305).

Kennedy (2007, 2012) reports the example of a philosophical discussion about a philosophical theme that emerged as a result of a mathematical teaching situation. It is interesting to note that the mathematical problem that gave rise to this philosophical discussion, i.e. the comparison of two infinite sets, corresponds to an exercise recommended by Daniel et al. (1996 (1)) in the accompanying guide to follow up the philosophical discussion on infinity. What therefore in one case constitutes the “end” of the philosophical activity is found in another case as the stimulator of this activity. It would be illegitimate to see any contradiction in this, as it might be the very expression of a dynamic process of reasoning that moves back and forth between mathematical considerations and philosophical reflections, leading to mutual enrichment (although it is “quite difficult to draw a line between philosophy and mathematics proper” (Kennedy, 2007, p. 305)).

More recently, Kennedy (2016) has also developed a practice of philosophy of mathematics with children based on upstream preparation and on the use of specific materials. She also proposes exercises that can precede philosophical discussions (apparently fostering individual reflection), notably on the theme of the application of mathematics (Kennedy, 2016)[[35]](#footnote-34).

This comparison of examples of practice of philosophy of mathematics with children has made it possible to distinguish between two ways of conceiving its implementation: either as a spontaneous practice following a mathematical questioning (but which then seems to presuppose a particular mode of teaching mathematics), or as a practice for which a specific time is granted and planned in advance, with a certain organization of the session beforehand. It is interesting to consider the advantages and disadvantages of these two possibilities. Among other things, it seems that the spontaneous[[36]](#footnote-35) emergence ofphilosophical questioning has the advantage of generating a more direct integration of reasoning and thus perhaps of facilitating the construction of the meaning of mathematics and its learning. On the other hand, it likely limits the scope of philosophical reflection to a particular problem, and perhaps prevents a more general view of mathematics and of the possibilities of questioning it, as well as of the many questions that surround a concept and thus of the interactions between different concepts and domains. Ideally, these two types of philosophical discussions would be present, complementing each other.

This question raises another important question for the practice of philosophy of mathematics with children, which has however not been directly addressed in this paper, that of knowing whether this practice is essentially linked to the context of mathematics teaching, and thus to the school context and more specifically to the classroom environment, or whether it can be extended to other settings in which philosophy workshops for children are regularly practices (libraries, extra-curricular activities, festivals, etc.). *A priori* no reason seems to me to speak against the extension of this practice. It even seems to be beneficial in the sense that extending considerations on mathematics in extra-curricular contexts can help children not to limit their understanding of mathematics to the field of school, but to broaden it to that of human experience as a whole. However, the question of the aims of this practice resurfaces and depending on these a practice that is more or less rooted in the mathematics classroom setting will be advocated. Considerations regarding the need for regular practice of philosophy of mathematics may also tip the balance in favour of its inclusion in mathematics education[[37]](#footnote-36).

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1. This notion of “flavour of knowledge” comes from Astolfi (2008). [↑](#footnote-ref-0)
2. I will be asking what are the conditions that any discursive activity about mathematics must meet in order to be qualified as a practice of *philosophy of mathematics with children*. The exploration of this question will be dialectically conducted and will take into consideration different and sometimes complementary proposals, without definitively deciding on the correct answer: this is why it is called a “problematizing” definition. [↑](#footnote-ref-1)
3. On this topic, see Wigner (1960). Questions concerning the applicability and application of mathematics, especially to physics, will not be developed in this paper. However, they seem to be of particular interest when discussing about mathematics with children. [↑](#footnote-ref-2)
4. The question “if there were no human beings on earth, would “2 + 2 = 4” still be true?” can be an interesting question for a philosophical discussion with children. [↑](#footnote-ref-3)
5. It is in this sole aim that this dilemma has been presented here. Of course, a legitimate philosophical analysis of it would require much more considerations, but it is not the purpose to furnish them here. [↑](#footnote-ref-4)
6. As notes Prediger (2007) « This shift to the activity itself can well be expressed by creating the verb *philosophize* » (Prediger, 2007, p. 46). [↑](#footnote-ref-5)
7. These may be concepts, notions, biases, processes, or beliefs inherent to mathematics and to its practice (Daniel et al., 2005, p. 336). [↑](#footnote-ref-6)
8. When, for example, students are asked to imagine different strategies for solving a problem and then compare them in an argumentative discursive exchange. [↑](#footnote-ref-7)
9. That is, a Lipmanian perspective. Lipman (see e.g. Lipman, 2003, chap. 4) develops an approach to philosophy for children centered on what he calls “research communities”, which must meet certain conditions requiring significant learning during which participants in a developing community develop "higher-order thinking skills and cooperative behaviours" (Daniel et al., 1999, p. 428). [↑](#footnote-ref-8)
10. The nature of this material will be developed below, in the third part of this paper. [↑](#footnote-ref-9)
11. Which, by the way, Calvert et al. (2017) recognize perfectly. [↑](#footnote-ref-10)
12. Notwithstanding, it is reasonable to consider that the degree of closeness does not need to be clearly determined, but allows for some flexibility. [↑](#footnote-ref-11)
13. Kennedy uses the notion of a *mathematical research community*, which is largely inspired by Lipman's notion of a research community but is adapted to mathematics (see, for example, Kennedy, 2009). [↑](#footnote-ref-12)
14. See especially Kennedy (2007, p. 298) for the correspondence with the level of the epistemologist. [↑](#footnote-ref-13)
15. See, e.g.*,* *Physics,* II, chaps. 7 and 8. [↑](#footnote-ref-14)
16. *Society for the Advancement of Philosophical Enquiry and Reflection in Education (SAPERE)*  [↑](#footnote-ref-15)
17. What is anyway considered to have a positive influence on learning and thus on mathematical skills. [↑](#footnote-ref-16)
18. However, this selection of aspects is not exhaustive. For example, no social aspect (related to stereotypes or to mathematical learning climate) will be addressed. [↑](#footnote-ref-17)
19. In this study (Kroesbergen and Schoevers, 2017), it is especially noted that creativity is an important predictor of mathematical *excellence*. [↑](#footnote-ref-18)
20. In their study, Kroesbergen and Schoevers (2017) claim to measure both a general creativity and a specific mathematical creativity. [↑](#footnote-ref-19)
21. According to De la Garza (2001), « philosophy is a discipline that takes into consideration alternative ways of acting, thinking or creating. To discover these alternatives, philosophers examine presuppositions, question what is taken for granted and speculate imaginatively, taking into account broader frames of reference. That is why *philosophy brings to education critical and creative thinking* » (De la Garza, 2001, p. 89, *my italics*). [↑](#footnote-ref-20)
22. This difficulty may be due to the fact that children are not used to reflecting at the level of abstraction required by mathematics, as they make little experience of it in their daily lives. But socio-affective explanations also seem to be relevant: widespread belief that mathematics is not accessible to personal/individual thought or action or that it is unmanageable because it is fixed and immutable, anxiety that prevents complex and reflective thinking from taking place when faced with mathematics, etc. [↑](#footnote-ref-21)
23. Because it allows « a more integral connection and communication between past and current experiences in mathematics by empowering students to draw more extensively on previous knowledge, to think in more complex terms about the world, and to begin to overcome those yawning epistemological gaps between mathematical practices and the world of everyday reasoning and perception. » (Kennedy, 2012, p. 86). [↑](#footnote-ref-22)
24. Perhaps one should distinguish between critical thinking, critical mind, critical attitude and critical sense. According to Roy and Schnubel (2017, p. 24), critical thinking can be defined as “a set of attitudes that cause an individual to tend to be critical” (Roy and Schnubel, 2017, p. 24, *my translation*). [↑](#footnote-ref-23)
25. Or rather a critical posture, i.e. a disposition to think critically (an habit or *hexis*). [↑](#footnote-ref-24)
26. Every study should also recognize that there are several ways of understanding the nature of this practice (as described in the first part of this work) and that variations in these understandings can have significant effects on the results of such surveys. [↑](#footnote-ref-25)
27. On this process of internalization, of Vigotskian inspiration, see Kennedy (e.g. 2007, pp. 294-295; 2012, pp. 84-85). [↑](#footnote-ref-26)
28. It should be noted that these issues are highly complex and it is reasonable to ask whether it is possible to treat them with children (or even with adults without specific mathematical knowledge). Moreover, Jankvist and Iversen seem to be mainly interested in young adults rather than children, since the example they give in presenting this intrinsic goal concerns young people at the end of high school or even in their first year of university study in mathematics! [↑](#footnote-ref-27)
29. The traditional epistemological and ontological questions of the philosophy of mathematics also seem to me to be important to this understanding, but perhaps the link is less obvious than in the case of the application of mathematics. [↑](#footnote-ref-28)
30. In the sense of non-fundamental, i.e. which can be broken down into several simpler, more fundamental abilities. [↑](#footnote-ref-29)
31. For example, Balaguer (1995) develops a theory that, according to him, makes it possible to ensure and explain mathematical knowledge while accepting a realistic conception of mathematical objects that seems creative and philosophically interesting (called “full-blooded platonism”). [↑](#footnote-ref-30)
32. *Philosophy for Children adapted to Mathematics* (P4CM). [↑](#footnote-ref-31)
33. More specifically, pp. 67-69. [↑](#footnote-ref-32)
34. *Community of Mathematical Inquiry.* [↑](#footnote-ref-33)
35. Here is an example of such an exercise : « Choose one thing from the list below and try to answer the two questions: 1) Can these things be described mathematically? 2) If so, how would one go about it? A barn; A dance; A soccer game; A card game; A game of chess; A human life; A conversation; A rabbit warren; The economy; An election campaign; A thunderstorm; A dream; Personal “coolness”; Happiness; Anxiety. » (Kennedy, 2016). [↑](#footnote-ref-34)
36. The adjective “spontaneous” may not be so appropriate, because, at least at the beginning, philosophical discussion does not come of itself, but requires the intervention of the teacher, who knows how to seize the right moment to launch a philosophical discussion. [↑](#footnote-ref-35)
37. This paper grew out of my thesis, written for the “University Certificate” in Philosophy of Children at the University of Nantes (France) in 2019. [↑](#footnote-ref-36)