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Pendulums, Pedagogy, and Matter: Lessons from the Editing of Newton's *Principia* ⋆

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Abstract. Teaching Newtonian physics involves the replacement of students' ideas about physical situations with precise concepts appropriate for mathematical applications. This paper focuses on the concepts of 'matter' and 'mass'. We suggest that students, like some pre-Newtonian scientists we examine, use these terms in a way that conflicts with their Newtonian meaning. Specifically, 'matter' and 'mass' indicate to them the sorts of things that are tangible, bulky, and take up space. In Newtonian mechanics, however, the terms are defined by Newton's Second Law: 'mass' is simply a measure of the acceleration generated by an impressed force. We examine the relationship between these conceptions as it was discussed by Newton and his editor, Roger Cotes, when analyzing a series of pendulum experiments. We suggest that these experiments, as well as more sophisticated computer simulations, can be used in the classroom to sufficiently differentiate the colloquial and precise meaning of these terms.

1. Introduction

Teaching Newtonian physics involves, perhaps first and foremost, the replacement of students' uncritically held ideas about physical situations with a set of precise concepts appropriate for mathematical applications. The theoretical concept of 'force', for example, is known to be different than the one naturally invoked by students (Brown 1989) and novel methods for correcting this disparity have been offered (Gauld 1998; 1999). The idea of 'matter' and its theoretically precise kin 'mass', however, have been discussed less. Empirical studies regarding these either follow the work of Piaget and Inhelder by examining the relationship between 'density', 'volume', 'mass' and 'weight' in everyday reasoning (Smith et al. 1997; Baker & Susanne 2001), focus on teaching the distinction between 'weight' and 'mass' (Galili & Kaplan 1996), or test students' understanding of atomic theory and its implications regarding inter-atomic vacua (Novick & Nussbaum 1981). None examine the relationship between 'volume', 'matter', and 'mass' in a specifically Newtonian context. Although not an empirical study, this paper will suggest that

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the untutored use of these terms embodies assumptions that are foreign to their precise Newtonian meaning. Specifically, we suggest that the pre-theoretic use of the terms implies that ‘matter’ and ‘mass’ are the sorts of things that are tangible, bulky, and take up some definite volume. In Newtonian mechanics, however, the terms are given operational meaning only by their use in Newton’s Second Law: a body’s ‘mass’ or ‘quantity of matter’ is simply a measure of the body’s acceleration under an impressed force. Although there is no need to think of a massive body as spatially extended, we suggest that ‘massive’ is often taken to indicate that a body not only has a certain reaction to impressed forces, but also a determinate volume.

Grasping the distinction between ‘mass’ as a measure of the reaction to impressed forces and ‘mass’ as a representation of overall size is crucial to understanding Newtonian mechanics at both advanced and introductory levels. At an advanced level, a clear view of the distinction can be used to motivate the concepts of field theory and to teach about the implications of Newtonian mechanics for atomic physics. At an introductory level, the distinction can be used to counter students’ basic misconceptions about the relationship between the size of an object and its ability to exert force. Special care should be taken at this level, since the confusion between the two senses of ‘mass’ is embodied in our most basic physics teaching tools. In many collision diagrams, for example, the mass of an object is represented by the size of a ball. The collision of a massive body with a less massive body is represented as the meeting of a big ball with a small one (Resnik et al. 1992, pp. 210–217; Young & Freedman 1996, p. 242). This implicitly suggests that mass is related to spatial extension, and moreover, that bigger objects are necessarily more massive. Paradoxically, these diagrams are used most often in a student’s education exactly when it is most crucial that she divorce herself from the assumptions embodied in them.

In order to demonstrate how this feature of Newtonian mechanics can be better handled in the classroom, we will examine a short exchange between Isaac Newton and Roger Cotes, the editor of the *Principia*’s second edition, concerning pendulum motion. As several studies have suggested, there exists a correlation between contemporary students’ opinions about mechanics and those of pre-Newtonian scientists (Wandersee 1985; Squeira & Leite 1991), a fact which we will take for granted. Section 2 examines the historical basis for the distinction between the two senses of ‘mass’, particularly how it evolved from two different conceptions of the nature of ‘matter’. Section 3 turns to the pendulum experiments and the exchange between Newton and Cotes. The pendulum experiments demonstrate that at a fixed distance gravity depends only on the mass of an object, yet Newton attempts to draw from this further consequences regarding the composition of bodies and the nature of matter. What makes the exchange particularly interesting is that Newton himself plays the role of the pre-Newtonian scientist, and it is his editor who realizes the true implications of Newtonian mechanics. In Section 4, we draw morals from this exchange. Overall, this paper can be taken as yet another example of how the wrangling over the meaning of concepts that occurred during their discovery can be recast and utilized in contemporary pedagogy.

2. Two senses of 'Matter' and 'Mass'

Newton's *Principia* appeared in 1687, when the 'Mechanical Philosophy' dominated the scientific and philosophical scene in Europe. According to this philosophy, all physical phenomena were to be explained by the motion and configuration of bits of matter, where 'matter' was taken to mean 'a substance extended, divisible, and impenetrable' (Boyle 1666 [1979], p. 18). There was no well-formed notion of 'force' in this program, and a rejection of the idea that bits of matter could affect one another at a distance.[1] All physical interaction was thought to be reducible to direct contact of homogeneous, though differently sized, material particles. The Mechanical Philosophy was in large part a critical reaction to the ontological excesses of Scholastic philosophers, who postulated a multiplicity of 'substantial forms' to explain the observable characteristics of matter. For example, the inherence in a body of the substantial form 'red' was thought to explain why that body was red; the form 'heaviness' was thought to explain why it was heavy, etc. The Mechanists, on the other hand, sought to minimize explanatory principles and account for the observable characteristics of matter through only a few of its basic properties (e.g., size and impenetrability) which were thought to be truly essential to it – without them, matter would cease to exist *as* matter.[2] This explanatory strategy suggested that an explicit identification of matter *as such* with its essential properties was philosophically valuable. After all, if all the observable characteristics of matter could be explained by its essential properties, and if these properties were definitional of what matter is in itself, it would behove a philosopher to invoke them directly instead of invoking the less fundamental notion of 'matter'.

A radical version of this identification was endorsed by Rene Descartes, who claimed that matter was nothing other than spatial extension. He based this on the idea that spatial extension is the only property of matter which we cannot imagine eliminated from our experience and is thus matter's only truly essential property:

> *There is no real difference between space and corporeal substance.*
> It is easy for us to recognize that the extension constituting the nature of a body is exactly the same as that constituting the nature of a space ... Suppose we attend to the idea we have of some body, for example a stone, and leave out everything we know to be non-essential to the nature of body: we will first of all exclude hardness, since if the stone is melted or pulverized it will lose its hardness without thereby ceasing to be a body; next we will exclude colour, since we have often seen stones so transparent as to lack colour ... (Descartes 1644 [1985], p. 227)

After similarly discussing heaviness, heat, and cold, Descartes concludes:

> After all this, we will see that nothing remains in the idea of the stone except that it is something extended in length, breadth and depth. (Descartes 1644 [1985], p. 227, original emphasis)

The very idea of matter is thus reduced to the idea of spatial extension.

However absurd this identification may seem to the modern reader, it allowed Descartes to give a quantitative measure of matter: the quantity of matter in a body was simply the amount of space which it filled. The question of how to measure quantities of matter has a venerable history, dating back to various attempts by St. Thomas Aquinas and his followers in the 13th century to explain the preservation of the observable characteristics of the Eucharist through transubstantiation.[3] In the 17th century, however, the problem needed to be answered for the sake of getting any mathematical physics off the ground. Although his physics was an overall failure, Descartes understood the need to mathematize matter and stated unequivocally that what is important about matter is simply its quantity. He thus wrote shortly before the passage quoted above:

> *The distinction between quantity or number and the thing that has quantity or number is merely a conceptual distinction*. (Descartes, 1644 [1985], p. 226, original emphasis)

In other words, the quantity of matter in a body is not on equal footing with other properties of that body; it is the *only one* we need to understand in order to understand the body at all.

Without delving into further details, we note that analogous positions were held by a variety of mechanical philosophers. Galileo, for example, conceived of matter primarily in terms of its spatial limits. In this passage from *The Assayer*, he wrote that he cannot even entertain the notion of matter that is not contained in some volume:

> Now I say that whenever I conceive any material or corporeal substance, I immediately feel the need to think of it as bounded, and as having this or that shape; as being large or small in relation to other things, and in some specific place at any given time . . . (Galilei 1623 [1957], p. 274).

Similarly, Robert Boyle, referring to the properties of the atomic constituents of matter, wrote:

> [We] must admit three essential properties of each entire or undivided, though insensible, part of matter: namely *magnitude* (. . . which . . . we in English oftentimes call the *size* of a body), *shape*, and either *motion* or *rest*. (Boyle 1666 [1979], p. 20, original emphasis)

These examples should make clear that in the mid-17th century the commonplace scientific opinion was that one of matter's definitional properties was its spatial extension.[4] What we call a *geometrical* conception of matter was thus one of the core commitments of the mechanical philosophy. Although the scientists above did not use the term 'mass' to describe the geometrical measure of a quantity of matter before Newton popularized the term, what is important for our purposes is that insofar as they were concerned with quantifying matter, they invoked a geometrical measure. As will become clearer later, when the term 'mass' came to stand for 'quantity of matter' (after the *Principia*'s publication), the question still remained which quantification strategy was appropriate.

Newton himself held the geometrical view of matter throughout his long scientific career.[5] For example, in *De Gravitatione*, a treatise that may have been written around 1684 as part of the composition sequence of the *Principia*, Newton described his conception of matter in terms of matter's creation by God. Bodies, he wrote, were nothing other than:

determined quantities of extension which omnipresent God endows with certain [additional] conditions . . . :
(1) that they [the quantities of extension] be mobile; and therefore I did not say that they are numerical parts of space which are absolutely immobile, but only definite quantities which may be transferred from space to space;
(2) that two of this kind cannot coincide anywhere; that is, that they may be impenetrable . . . ;
(3) that they can excite various perceptions of the senses and the fancy in created minds. (Hall & Hall 1962, p. 140, original emphasis)

What is important about this passage is that it begins the discussion of matter with the assumption that bodies are "quantities of extension". Newton did not doubt that the physics of material bodies – for which he is setting the scene by discussing the impenetrability and motion of bodies – must be directly concerned with volumes. A later passage in the same treatise confirms the importance of the geometrical conception. Newton offered the following definition of momentum ("motion", as he called it):

[M]otion is either more intense or more remiss, as the space traversed in the same time is greater or less, for which reason a body is usually said to move more swiftly or more slowly . . . motion is more or less in extension as the body moved is greater or less, or as it is acting in a larger or smaller body. And the absolute quantity of motion is composed of both the velocity and the magnitude of the moving body. (Hall & Hall 1962, p. 149)

Newton argued that momentum is proportional to the product of the velocity and the size ("larger or smaller") of the body. For modern readers, this is a curious statement, since momentum should be equal to the product of velocity and mass, not velocity and size! It seems that Newton believed the two to be equivalent, at least as far as the atomic particles of matter were concerned.

During the same period, however, Newton was establishing a new conception of matter that was radically different from the one outlined above. This innovative *dynamical* conception of matter related the mass of a body not to its size, but to its ability to resist impressed forces; i.e., to its inertia.[6] Since this conception is familiar to all those who have some facility in Newtonian mechanics, we only need to mention that Newton's second law, put anachronistically as $F = ma$, is the ultimate embodiment of this view. According to this law, given a force a body's mass can be calculated based on the observable acceleration of the body in response to that force. Since the second law is one of the foundational principles of Newtonian mechanics, it is clear that the *dynamical* sense of matter underlies Newtonian mechanics. An appeal to the *geometrical* sense of matter, on the other hand, is completely out of place: geometric properties of the fundamental particles are irrelevant to Newton's mechanics.

It should be noted, however, that when coming to explicitly *define* mass, Newton did not state its relation to impressed forces. Rather, Newton defined mass as "a measure of the matter that arises from its density and volume jointly" (Newton, 1726 [1999], Defn. 1, p. 403). This more geometrical definition is seldom used in the *Principia* and is virtually forgotten in later treatments of Newtonian mechanics. Starting with Euler's *Mechanica* and ending in modern textbooks, the concept of mass has been traditionally introduced exclusively as a measure of a body's resistance to impressed forces.[7]

The history of physics in the 18th and 19th centuries shows that this *dynamical* conception was one of Newton's greatest contributions to physical theory. In the 18th century, Roger Boscovich suggested that matter should be conceived as a set of point-sized particles, each considered only as the locus of forces, while in the 19th century, the work of Faraday and Maxwell developed the idea of point sources into the field concept that still dominates physics today. Overall, the shape and size of the fundamental constituents of matter turned out to be of no importance for doing theoretical physics and the *geometrical* conception was left by the way-side. Nevertheless, the intuitive appeal of this conception continues to exert its power. The opinions of 17th century scientists demonstrate that even at a sophisticated level it is possible to believe that the concept of 'matter' appropriate for doing physics implies that matter is to be understood through its spatial properties. Yet as far as the theoretical structure of Newtonian mechanics is concerned, this is an unwarranted belief. We now turn to the exchange between Newton and Cotes that illustrates this theme.

3. Pendulum Motion and Conceptual Confusion

The conflation of the two conceptions of matter discussed above was first noted by Newton's young editor, Roger Cotes. Cotes' prodigious mathematical talent earned him an appointment as the first Cambridge Plumian Professor of Astronomy and Experimental Philosophy at the tender age of 26. In 1709 he was chosen as the editor of the *Principia*'s second edition, and his correspondence with Newton over the ensuing four years reveals that Cotes was one of the *Principia*'s most astute readers. Here we will focus only on an exchange between Newton and Cotes in the winter of 1712. Cotes noticed that Newton himself combined the two conceptions of 'matter' in drawing out the implications of ingenious pendulum experiments reported in the *Principia*, and attempted to dispel the confusion in exchanges with Newton.

Newton's predecessors thought that gravity (or 'heaviness') could depend upon a wide variety of a body's properties. Advocates of the geometrical conception usually explained gravity in terms of a fluid aether pressing upon bodies, and in this context it was natural to suppose that gravity might depend on a body's volume or shape (Descartes 1644 [1985], p. 268). Others thought that gravity resembles magnetism, which clearly affects some bodies differently than others. Newton devised a pendulum experiment to rule out the dependence of gravity on anything but mass (Newton 1726 [1999], Book III, Prop. 6). As Newton notes, this experiment is a more controlled version of Galileo's famous experiment comparing the free fall of bodies with different densities. Newton constructed two pendulums of equal length side by side, filled the empty pendulum bobs with equal weights of a variety of materials, and set them in motion. He had earlier derived the following relationship between the period of oscillation, the weight of a pendulum, and its mass, $m \propto w \cdot p^2$ (Book II, Prop. 24). In addition, the period of a simple pendulum depends only its length and the accelerative force due to gravity acting on the bob. Since the pendulums had the same length, any variation in the periods would reflect a difference in the accelerative force gravity imparts to different materials. Newton found that the pendulums exhibited *nearly identical periods* for all the materials he tested, and thereby confirmed that the mass and weight of an object are always directly proportional to one another, $m \propto w$, far more accurately than the Galilean experiment. In modern parlance, this proportionality shows that the 'inertial mass' m in $F = ma$ (which measures the response of the bob to the force acting on it) and the 'gravitational mass' m in $W = m\mathbf{g}$ (which measures the magnitude of the force acting on the bob) are one and the same.

This novel feature sets the gravitational force apart from other forces encountered in physics. In the corollaries to Prop. 6, Newton draws attention to this feature via a comparison to magnetism: unlike gravity, the force of magnetism has strikingly different effects on various materials. In anachronistic terms, gravitational 'charge' in the force law for gravitation is exactly equal to the inertial mass, unlike other force laws (such as Coulumb's law) which include a separate charge that determines the magnitude of the force. For forces with a separate charge one can imagine experimentally varying the ratio of the electric charge to the inertial mass to distinguish inertial effects from those due to the force. However, the exact equality of inertial and gravitational mass prevents this in the case of gravitation. Newton's notion of 'mass' is thus uniquely distinguished from other properties of matter. A little over two centuries later, Einstein would take this surprising fact to be one of the guiding principles – he called it the 'equivalence principle' – in his successful search for a new theory of gravitation.[8]

Newton thought that the pendulum experiments established more than just the proportionality between mass and weight. In the first edition, he goes on to argue in a corollary (Book III, Prop 6, Cor. 2) that matter must be filled with empty spaces![9] Newton's argument is based on two additional premises. The first is that pendulum motion would be impossible if the density of the pendulum bob was equal to the density of the medium in which it was moving. This is just a consequence of the fact that bodies cannot rise and descend unless their specific gravity is different from that of the ambient medium. Newton's second premise is that matter is composed of atoms of fixed density. If atoms have a fixed density yet there is a variation in the densities of gross bodies, the variation must be due to the presence of a different number of atoms in equal volumes of gross bodies. This can happen only if different amounts of empty spaces separate the atoms in each of these bodies.

Cotes challenged this argument. Although Cotes' objection may appear to be a minor quibble related to an unimportant corollary, it actually focuses clearly on the theme of this paper – the notion of 'matter' and its measure via 'mass' that is appropriate in Newtonian mechanics. Cotes asked Newton if it was not possible to imagine two identically sized atoms which nonetheless possess different masses. If this is possible, Cotes held, then Newton's argument fails to show that gross bodies must possess internal vacua. It would be possible for two identically sized pendulum bobs to have different masses by virtue of being made from two types of atomic matter (each with a different density), rather than from the same type of atomic matter differently distributed in space. One could even image a matter that is so dense that gross bodies made of it contain no vacua whatsoever. What is important about this observation, however, is not so much what it says about the existence of vacua, but what it says about the Newtonian concept of 'mass'.

Cotes claims that it is inappropriate, in the strict context of Newtonian mechanics, to treat bodies as if they were spatially extended. The *only* concept of 'mass' that is relevant for Newtonian mechanics is the one given by the second law. In Cotes' own words:

> Let us suppose two globes A & B of equal magnitudes to be perfectly fill'd with matter without any interstices of void Space; I would ask the question whether it be impossible that God should give different *vires inertia* [i.e., forces of inertia] to these Globes. I think it cannot be said that they must necessarily have the same or an equal *Vis Inertia*. Now You do all along ... estimate the quantity of matter [i.e., mass] by the *Vis Inertia* ... Tis possible then, that ye equal spaces possess'd by ye Globes A & B may be both perfectly fill'd with matter, so no void interstices remain, & yet that the quantity of matter in each space shall not be the same. Therefore when You define or assume the quantity of Matter to be proportionable to its *Vis Inertia*, You must not at the same time define or assume it to be proportionable to ye space which it may perfectly fill ... (Turnbull 1977, Volume V, Doc. 893, p. 228)

In other words, the *dynamical* conception of mass essential to the formulation of Newtonian mechanics is in no way tied to the geometrical conception advocated by Newton and the other mechanical philosophers. When Newton assumed that similarly sized atomic particles must necessarily possess the same mass – i.e., that their volume was a measure of their quantity of matter – , he was stepping beyond the limits of his own mechanics. As Cotes noted, it is possible for similarly sized atoms to possess different masses because 'mass' is simply a measure of a body's inertia – it need not be at the same time a measure of a body's volume. Although the experiments fix the relationship between inertial and gravitational mass, they have no bearing on the relationship between mass and volume of the fundamental particles.

The remainder of the exchange between Newton and Cotes is fascinating, yet beyond the scope of this paper (Biener & Smeenk 2001 for further discussion). It shows that although Newton ultimately acquiesed and formulated the corollaries as conditionals, he did not fully understand Cotes' complaint. Instead, Newton kept insisting that the proportionality between mass and volume, just as the proportionality between inertial and gravitational mass, was proved by the pendulum experiments. As we've discussed, however, only the latter was strictly proved by these experiments. Newton's belief in the former seems to have been due to a subtle confusion between the conception of matter he invented and the one generally accepted at his time. In other words, he failed to sufficiently differentiate in his proofs when 'mass' and 'quantity of matter' were used to indicate inertia and when they were used to indicate volume. The unfounded conclusions drawn from his pendulum experiments were simply artifacts of this confusion.

4. Morals for Teachers

If Newton's misconceptions about his own theory are any indication, physics teachers would do well to actively counter students' insufficient discrimination between the geometrical and dynamical senses of 'matter' and their quantification via the concept of 'mass'. This involves both an explicit confrontation with students' confused views and a more subtle use of existing teaching tools, so that they will not exacerbate existing confusions.

Newton's pendulum experiment can be easily modified for use in the classroom. Although some students might be familiar with the idea that the weight of bodies depends only on their mass, reproducing the experiment will provide a vivid demonstration of their knowledge. The easiest way to reproduce the experiment would be to construct two pendulums side by side, with small boxes for pendulum bobs, and ask students to fill the boxes with a variety of materials. Alternatively, one could use a single pendulum and have students measure the period for different materials. In either case, students should be asked to discuss the patterns they have observed and what they imply regarding the nature of the gravitational force. In the theoretical treatment of the experiments, it is crucial to distinguish inertial and gravitational 'mass' as mentioned above rather than simply assuming their equality. Students will discover that the two are very precisely equal, but it is important for them to recognize that m plays two distinct roles.

On a more introductory level, a series of experiments can be run with pendulum bobs of different sizes but equal lengths. A ball should be placed at the lowest point in the pendulum trajectory, so that a collision occurs when the pendulum is moving directly in the horizontal direction. As students can see, changing the size of the pendulum bob without changing the mass will not affect the motion of the struck ball. Changing the mass, however, will affect this motion. From this, the teacher can stress the point that almost all of a body's properties – save its *dynamical* mass – are irrelevant for Newtonian mechanics. In this experiment, the overall length of the pendulum should be large in relation to the size of the pendulum bobs, so that the difference in the length of the pendulum trajectory will be negligibly changed by a change in the bob size.

Several computer simulations can also be used to press home the idea that a body's geometrical properties are irrelevant to the dynamical notion of mass. For example, a computer simulation in which the volume of a pendulum bob may be varied by the student *while* the pendulum is oscillating illustrates that no change in the motion of the pendulum occurs. From this students should conclude that the volume of the bob is of no consequence. A similar simulation involving collisions can make an even more powerful teaching tool. In this case, both the volume and *mass* of colliding balls may be changed by the student during the course of collision. Students can see that when volumes are changed, no change in the motion results. However, a change in mass affects the motion. We believe that such a demonstration will be most useful if it is performed as a proper part of teaching Newton's Third Law and divorcing students' from the view that a larger body necessarily contains a larger 'internal force' (Gauld 1999). The idea of momentum and its relation to mass can then be developed more forcefully.

On a more subtle level, a better use of diagrams can also discourage the confusion in the two senses of 'mass'. As noted earlier, collision diagrams ordinarily use the size of an object as a representation of its mass. This enforces the notion that mass and volume are somehow related and even suggest that they are related linearly. As a corrective, collision diagrams should represent the mass of an object by internal shading, not size. Different masses should no longer be represented as differently sized balls, but as differently filled ones. It is important to use fillings which have a straightforward numerical interpretation (such as a set of dots with a certain number per unit area) and not color gradients, whose mathematical interpretation is less clear.

Before concluding, we must sound a single cautionary note: there are plenty of applications of Newtonian mechanics where size *does* matter. For example, in calculating angular momentum, fluid resistance, or coefficients of friction the size of the bodies involved determines the magnitude of the forces. The point of this paper is not that 'size doesn't matter', but rather that distinguishing between 'mass' as a dynamical measure and 'mass' as geometrical measure is crucial for understanding the foundations of Newtonian mechanics. The historical precedent suggests that students are likely to confuse these senses and thus teachers should combat this confusion at all pedagogical education.

5. Conclusion

The concept of 'mass' (i.e., a 'quantity of matter') in Newtonian mechanics is different than the one used colloquially. The difference lies in the fact that the Newtonian concept is derived from other precise concepts – force and acceleration – while the colloquial concept is derived from our everyday interaction with more or less massive material bodies. In learning Newtonian mechanics, students need to divorce themselves from the colloquial concept and understand that a Newtonian 'mass' (i.e., a 'quantity of matter') is a different notion than the one they are accustomed to. The difference between these two notions was historically noted in connection with pendulum experiments and we believe that such experiments can still be used today to clarify their conceptual interrelations.

Notes

[1] See Westfall (1971) for the evolution of the force concept in the 17th century.

[2] See Dijksterhius (1960), especially §III.G.

[3] See Jammer 1961, chap. 4) on the medieval notion of 'quantitas materiae' and its theological origins.

[4] See Hall (1963) for an extended analysis of similar matter theories.

[5] For more on the impact of this view on Newton's physical theories, in connection with his views on the explanation of gravitation by aetherial mechanisms, see Biener and Smeenk (2001).

[6] Newton was not the only one to recognize that the mechanical philosophy's conception of matter as bare, inert geometric extension was insufficient for physics. Newton's contemporary (and rival) Leibniz also prominently criticized the mechanical philosophy (see, for example, Leibniz 1989, pp. 245–256).

[7] See Jammer (1961, Chap. 8) for an analysis of 'mass' in Newton's successors. An example of a modern definition of 'mass' via Law II can be found in Resnick and Halliday (1960, §5.4.

[8] See Einstein (1916) for Einstein's brief statement of the principle, and Norton (1985) for a detailed discussion of the principle and its role in Einstein's discovery of general relativity.

[9] Newton revised these corollaries in light of his discussion with Cotes, and the second corollary from the first edition was revised and expanded into corollaries three and four in later editions.

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