

A recipe for non-wellfounded but complete chains of explanations (and other determination relations)

December 7, 2023

Consider a series $(u_i)_{i \in I}$ whose items are each (fully) explained by their immediate successor. I can be: a) the set on the n first non-null integers $\llbracket 1, n \rrbracket$ in which case $(u_i)_{i \in I}$ constitutes a finite, non-circular chain of explanations, b) the set of non-null natural numbers \mathbb{N}^* , in which case $(u_i)_{i \in I}$ constitutes an *infinite chain of explanations*. c) I can also be the ring of integers modulo n , $\mathbb{Z}/n\mathbb{Z}$ (you can picture this as the numbers 1, 2, ..., n sequentially distributed on a circle, just like the numbers 1, 2, ..., 12 are sequentially distributed on a watch dial)¹, in which case $(u_i)_{i \in I}$ constitutes a *circular chain of explanations*.

We will say that in case (a), but not in cases (b) and (c), the chain is wellfounded. Let us say, moreover that a chain of explanation is *complete* when it leaves nothing to be explained (more on this below).

In a previous article on cosmological arguments, I have put forward a few examples of complete infinite and circular explanations, and argued that complete non-wellfounded explanations such as these might explain the present state of the world better than their well-founded theistic counterparts (Billon, 2021). Although my aim was broader, the examples I gave there implied merely causal explanations. In this article, I would like to do three things:

- Specify *some general informative conditions for complete and incomplete non-wellfounded causal explanations* that can be used to assess candidate explanations and to generate new examples of complete non-wellfounded explanations.
- Show that these conditions, which concern chains of causal explanations, easily *generalize to chains of metaphysical, grounding explanations* and even to chains involving other “determination relations” such as supervenience.

¹The ring of integers modulo n ($\mathbb{Z}/n\mathbb{Z}$) is the set of the n first integers, 1, . . . , n endowed with the addition and multiplication operations, and where (to put it rather roughly) it is assumed that for all x , $x = x + n$.

- Apply these general conditions to the recent debates against the existence of non-wellfounded chains of grounds and show, with a couple of precise examples, that the latter can be complete, and that just like in the case of causal explanations, non-wellfoundedness can in fact be an asset rather than a liability.

In the first section, I present the recent debates about non-wellfounded chains of grounds and show more broadly why the question of complete non-wellfounded chains of explanations is important. I then articulate the framework within which I will assess these questions about non-wellfounded explanations and determination relations (section 2). After that, I reconstruct an argument from Leibniz which is, I believe, the most interesting argument against complete non-wellfounded explanations (§ 3). This argument rests on a clear example of a non-wellfounded incomplete explanation. My answer to it rests on clear examples of complete well-founded explanations (§ 4). My examples involve causal (as opposed to metaphysical, grounding) explanations, but in the next sections (§ 5-6), I will put forward general formal conditions a non-wellfounded explanation must meet in order to be complete. These general criterion will then allow me to introduce examples of complete and incomplete non-wellfounded chains of *grounds* (§ 7). In the remainder of the article, I discuss a couple of objections. First, I argue that even though one might quibble about the definition of a *complete* explanation and argue that in the examples put forward our explanations still implicitly leave some things to be explained, these examples unambiguously show that *non-wellfounded explanations can do better than their wellfounded counterparts* and that there might be non-wellfounded explanations that really leave nothing at all to be explained (§ 8). I also show that complete non-wellfounded explanations are analogous and no less problematic than well-accepted explanations such as equilibrium explanations and essentialist explanations (§ 9). Finally, I discuss the possibility of infinite explanations that are as simple as (or even simpler than) finite, well-founded explanations (§ 10).

1 Completeness and non-wellfoundedness

We all wish we could have complete explanations of some things: explanations, that is, leaving nothing to be explained. Such explanations are the Grail of metaphysical inquiries (think of Leibniz (1989)'s search for the radical origin of things) but also of scientific inquiries (think of Einstein's quest for a "theory of everything" (Schilpp, 1949, p. 63), see also Hawking and Mlodinow (2010, p. 181)). In their vast majority, however, researchers believe that if such explanations exist, they must be wellfounded. This is true in the case of causal explanations, but it has recently come to the fore in the context of debates concerning metaphysical, grounding explanations.

An interesting objection, or cluster of objections, against the existence or the very possibility of non-wellfounded chains of grounds centers indeed on the idea that they would be somehow

explanatory defective because they cannot be complete. Fine (2010) has for example claimed that in cases such as (b) and (c), u_1 would not have a completely satisfactory explanation:

(...) there is still a plausible demand on ground or explanation that we are unable to evade. For given a truth that stands in need of explanation, one naturally supposes that it should have a completely satisfactory explanation, one that does not involve cycles and terminates in truths that do not stand in need of explanation (Fine 2010, 105).

Most often, this objection seems to appeal, more or less implicitly, to a version of the principle of sufficient reason (PSR), to the effect that everything must have a (full) explanation. Thus, Schaffer 2010 claims that:

There must be a ground of being. If one thing exists only in virtue of another, then there must be something from which the reality of the derivative entities ultimately derives. (Schaffer, 2010, 37)

As I understand it, the objection is that non-wellfounded chains of grounds are incomplete in that they leave something to be explained, which is bad by the PSR.

Against this explanatory deficiency objection, advocates of non-wellfounded grounding have argued that wellfounded grounding chains face the very same explanatory problem: in case (a), the last item u_n of the series seems in need of an explanation too, and this explanation is lacking (Bliss (2014), Bliss and Priest (2018, pp. 20-21)). Yet this “tu quoque” reply might be disputed by philosophers who believe that some items are by their very nature somehow self-explanatory, or at least “autonomous” in the sense that they do not call for an explanation (see Dasgupta (2016), see also Miller (1996) and other Theists on the existence and simplicity of God).

Various philosophers have recently tried to assess precisely whether as suggested by Schaffer Fine and others, non-wellfounded grounding chains need fare worse, explanatorily than wellfounded ones (see the contributions in Bliss and Priest (2018) and the very useful introduction). Some, such as Bliss and Priest, seem to assume that non-wellfounded explanations will never be complete (Bliss (2013, p. 408)), Priest (2014, p. 187), Bliss (2019), Cameron (2022, p. 130)) but reject the request for a complete explanation. Others have underlined the fact that arguments for the incomplete character of non-wellfounded explanations are often unsound or simply lacking (Oberle, 2022). Although they bring up interesting points, these discussions remain at a very abstract level and never rely on concrete examples of would-be complete explanations.

2 A framework for explanations

I will provide such concrete examples. Before that, let me make a couple of terminological points and set up the framework. I will talk, as I just have, as if grounding *were* metaphysical explanation. This might be disputed. Just like on some views causality underlies (but differs from) natural explanations, on some views, grounding only underlies metaphysical explanations. Likewise, I will often talk as if explanation were a relation (rather than, say, a sentential operator). I am not particularly keen on the views mirrored by these ways of talking, but I believe that nothing substantial depends on them here, and they will make my arguments (and my prose) much more fluid.

For simplicity, I will suppose that the relata of the explanation relation (and hence our “items”) are facts or sets of facts, where a fact is understood liberally as the referent of a true proposition. To make my prose more fluid and discuss some texts that seem committed to that view, I will sometimes speak as if the relata of explanations could be tropes or individuals. It should be clear, however, that by trope/individual x explains trope/individual y I only mean that the identity and/or existence of the former explains that of the latter. Except otherwise noted, by “A explains/grounds B”, I will always mean “A *fully* explains/grounds B”.

More substantially, I will admit that basic explanations in which one item explains another (as opposed, e.g. to complex chains of simple explanations) have a triadic structure, involving:

- a “final” item,
- an “initial” item,
- and a link accounting for the transition from the final item to the initial item, which I will consider to have the form of a law.

The final item, along with the laws, explains the initial item.²

By accounting for the transition between the final and the initial item the laws do the explanatory work. On some accounts, the link between the items plays no genuine explanatory role or does not have a lawlike structure. I will ignore them here.³ We shall see, in any case, that my understanding of lawhood is extremely minimal. The triadic framework is less orthodox in the grounding literature than in the causal explanation literature. In the former, it is associated with the works of Schaffer (2017); Litland (2017); Bader (2017); Kment (2014); Glazier (2016); Rosen (2017); Schaffer (2017).

²So yes the final item explains the initial item (and not the other way around). The terminology is a bit awkward here but it has to be so because the main focus of this paper is infinite *descending* chains of explanations.

³As emphasized by Schaffer (2017, 308), it is difficult for these accounts to understand the role explanations play in making sense of the world.

In our series $(u_i)_i$ each item u_{i+1} explains, along with the law L_i , the antecedent item u_i . Once laws are introduced, it is natural to wonder whether some laws can themselves be explained by more basic laws (as when we explain the laws of thermodynamics by those of statistical mechanics). Analogously, and this hypothesis shall prove very important in what follows, we might wonder whether laws can explain some items all by themselves, that is, without the final item — call that cases of zero-explanation or pure-law-explanation.⁴ In the literature on the “sublime question” *Why is there anything? Why this?*, many atheists have for example looked for answers that only mention laws (see Nozick (1981, ch. II), Leslie (1979)). In a couple of recent articles Kappes (2022, 2023) distinguishes a restrictive sense of “explain” (in which only the initial item) from a more inclusive sense of explain (in which laws can also be said to explain something). He also argues that the first one corresponds more closely to because-statements. I am not completely sure about this last linguistic claim, but if there really are two senses here to differentiate, then it is definitely the inclusive sense I will be using all along this article.

The triadic structure of explanation also allows us to make a distinction that we have omitted and that can prove useful in certain contexts. If we want to be very rigorous we should not identify, as we have until now, *a series of items* such as $(u_i)_{i \in I}$ in which items are explained by their successor, to *a chain of explanations*. A chain of explanations is rather a series of items *and* a series of laws accounting for the transitions between items (or equivalently from a formal point of view, a series of triplets containing a final item, a law, and an initial item). For convenience, the laws are often left implicit when they are not the target of our explanation or when we are not dealing with zero-explanations. We will follow this convention and often talk as if our series of items $(u_i)_i$ were, by itself, a chain of explanation.

With this triadic characterization of explanation, we can also define *complete* chains of explanations a bit more precisely. As I have used the term, a chain of explanation is complete when it leaves nothing to be explained (concerning the chain of explanation) except the laws. We can call “ultimate” or “supercomplete” a chain of explanations that leaves nothing at all to be explained (concerning the chain of explanation), not even the laws it relies on.

Unfortunately, these definitions are neither very informative nor very useful by themselves. It is probably hard to see, while reading them, why there should be some non-wellfounded series of explanations that are not complete (and Hume is widely held to have claimed that there could be none, see Rowe (1970)). We shall see that this is not the case, but it will take a bit of work.

⁴Authors who deny that grounding involves laws nevertheless have an analogon of our “metaphysical explanation by the laws alone”, namely what (Fine, 2012) calls zero-grounding.

3 Leibniz’s argument against complete non-wellfounded explanations

Leibniz has put forward what I take to be the most interesting argument against the completeness of non-wellfounded explanations. Although Leibniz is not exactly concerned with what we call “causal explanations” or “metaphysical explanations”, but rather with “explanations by reasons” (which seems to include causal and teleological explanations), we shall see later that his argument (and my reply to it) have purely causal and ground-theoretic analogs.

3.1 Against the completeness of infinite explanations

While exposing his version of the cosmological argument, to the effect that there must be a complete explanation of things and that these necessarily involve God, Leibniz puts forward an objection against the idea that infinite chains of explanations could be complete.

Suppose that a book on the elements of geometry has always existed, each copy made from an earlier one, with no first copy. We can explain any given copy of the book in terms of the previous book from which it was copied; but this will never lead us to a complete explanation, no matter how far back we go in the series of books. For we can always ask: Why have there always been such books? Why were these books written? Why were they written in the way they were? (Leibniz 1989, 486)

Why does Leibniz think that this explanation is incomplete? Prima facie, one might think that his three questions can be answered easily by the proposed infinite explanation. “Why have there always been such books?” Well, for each book we can answer that it exists because of a former copy and because of a scribe who copied it. This, it seems, can provide a satisfying answer. “Why were these books written?” Well, because the scribes are instructed to make books out of other books. “Why were they written in the way they were?” Because the scribes are instructed to make faithful copies of the book they are given.

The key to understanding Leibniz’s objection, I take it, is to distinguish *each book in the series* (Leibniz’s “any copy of the book”) from *the whole series of books* (Leibniz’s “always (..) such books”, “these books”). In Leibniz’s example, each book copy is explained in terms of its successor in the series (and the law that specifies the behavior of the scribes). But the whole series of books isn’t. Indeed, it could be the case that each scribe faithfully copies the next book, as specified by the law, but that the books are all copies of the Bible rather than the Elements of Geometry. This suggests that the infinite explanation here does not explain why we have an infinite series of the Elements of Geometry rather than an infinite series of the Bible. But if this is so, it clearly leaves something unexplained, then,

namely the whole series itself.⁵

Another reason one might want to add in order to deny that the explanation is complete is that the latter can explain the content of the book copies, but not, say, whether they are made of paper or parchment, the color of the cover, or even that they exist — call that the extra-property objection. I'm not sure, however, that this second objection against the completeness of the explanation is really decisive. For it could easily be answered by enriching the laws, and specifying that the scribes make a book copy *of the same material, the same color and with the same book cover* as the book he is given, and that *they make it out of infinite stock of material and ink*, or even out of nothing. These additions would allow one to explain the material, the color of the cover and the existence of *each* book copy given the next one. It would not, however, allow one to explain why *the whole series* is made of white paper or even exists, but that is another problem. It is, in fact, the very problem we have dealt with in the preceding paragraph.

This second, extra-property objection could equally be answered by specifying the target of the explanation more precisely than Leibniz did, and claiming that the items of the series are not full books but merely the content of book copies (understood as facts of the form ‘the content of book # i is that of the Elements of Geometry’), or even, if you are really suspicious about explanations of existence⁶, by claiming that they are conditional facts concerning the content of book copies (such as ‘if book copy # i exists, its content is that of the Elements of Geometry’). Either by enriching the laws, or by impoverishing the items, we can easily dispose of the extra-property objection.

We can now conclude this discussion of Leibniz's infinite scribes case with two important conclusions. First, in order to be complete a series of explanations $(u_i)_i$ must explain not only each item of the series from its successor, but also the whole series itself from the laws alone. Conversely, of course, a chain of explanation that explains the whole series of items will be complete – what is left to explain but the laws once the whole series is explained? So,

- **Complete explanation.** A chain of explanation $(u_i)_i$ is complete if it explains not only each item from its successor but also the whole series $(u_i)_i$ from the laws alone.

⁵We thus have a simple counter-example to the so-called Hume-Edwards principle to the effect that the whole is “sufficiently explained in explaining the cause of the parts” (Hume, 1907). The name of the principles comes from Rowe (1970). See Billon (2021), p.1938, especially fn.7, see for a defense of the Hume-Edwards principle against other objections Simsek (2023, §3).

⁶A referee for this journal suggests that laws being abstract, they cannot explain the existence of concrete objects. I agree that the fact that abstract laws can regiment concrete events can seem puzzling, but I am not sure that this puzzle concerns existence specifically (how can Newton's laws “act on” this rock to make it fall on the ground?). There is, in fact, a long and influential tradition of positing laws or principles to explain existence (see Leslie (1979, 2010) (and the historical references within) on the axiarchic principle, or Nozick (1981, ch. II) on the principle of fecundity), a tradition that still has quite respectable representatives today (Parfit (2011, vol.II, pp. 623-648) is a notorious example).

This is already an interesting characterization of completeness. It shall prove quite useful.

Second, some infinite chains of explanations are not complete. This is the case of *The Infinite Scribes* series which we can reconstruct as follows:

The Infinite Scribes. Consider an infinite series of book copies. Book copy #1 is a copy of the Elements of Geometry, its content is explained by the fact that it was copied before by a scribe from an older book copy #2; this older book copy #2 is a copy of the Elements of Geometry, its content is explained by the fact that it was copied, before, by a scribe from a yet older book copy #3, and so on to infinity... where a scribe is someone who makes a faithful copy of the book he is given.

Here u_i is the fact that book copy # i (i.e. the book copy that appears at stage # i) is the Elements of Geometry. And the law L_i specifies the behavior of the scribe # i : he makes a new book with the same content as the next book (as all scribes behave in the same way, all the L_i s are actually identical).

3.2 Against the completeness of circular explanations

Interestingly, this objection to the completeness of infinite explanations generalizes to circular explanations:

The Circular Scribes. Consider two book copies. Book copy #1 is a copy of the Elements of Geometry, its content is explained by the fact that it has been copied in 1999, by a scribe, from a book copy #2. Book copy #2 is also a copy of the Elements of Geometry, and its content is explained by the fact that it was copied yesterday from book copy #1 by a scribe who then traveled through time to 1999 with book copy #2.

Here u_i and L_i are the same as in the Infinite Scribes case.

Again, this circular explanation is not complete because the specified behavior of the scribes (i.e. the laws) explains why both books have the same content, not why this content is that of the Elements of Geometry rather than, say, that of the Bible. And it fails to explain that because it leaves it open whether both books are copies of the Elements of Geometry or, say, the Bible. It does not determine that they are copies of the Elements of Geometry.

3.3 What about wellfounded explanations?

Of course, as Bliss and Priest rightly point out in another context, we might get an incomplete explanation in the wellfounded (finite, non-circular) case as well:

The Wellfounded Scribes (ordinary case). Consider a series of n book copies. Book copy #1 is a copy of the Elements of Geometry, its content is explained

by the fact that it was copied before by a scribe from an older book copy #2; this older book copy #2 is a copy of the Elements of Geometry, its content is explained by the fact that it was copied, before, by a scribe from a yet older book copy #3, and so on unto n .

Here u_i and L_i are the same as in the Infinite Scribes case.

The content of each book copy # i , where $i < n$ is fully explained in terms of its successor, but the content of book # n is left unexplained, so the explanation is incomplete. The case, in that respect, is exactly similar to that of the Infinite Scribes. Leibniz would have agreed. He believed, however, that there is a special item, namely God, that is self-explanatory because He literally explains Himself. Others have argued that although they are not really self-explanatory, some items are “autonomous” in that they do not call for an explanation (Dasgupta, 2016).

If there are autonomous or self-explanatory items we might have a complete explanation in the finite, non-circular case. Just consider *The Wellfounded Scribes (extraordinary case)*, which is exactly like the above ordinary case except that the book copy # n is very special: its content either literally explains itself or is at least autonomous.

I do not want to dispute that this toy example involving a self-explanatory book content is implausible. I want to grant, however, that there might be plausible examples of the same form as this one. My claim is that despite Leibniz’s contention *some* infinite, and more generally non-wellfounded, explanations are complete.

4 Answering Leibniz’s objection

It is not difficult to modify our Leibnizian Infinite Scribes example to get a complete series. For a trivial case, consider what happens if we replace our faithful, regular scribes, with *monomaniacal scribes*, i.e. scribes who, instead of making a faithful copy of the book they are given, always create a copy of the Elements of Geometry, whatever book copy they are given (they are so monomaniacal that they do so even if they are given no book at all).

In that case the the explanation does seem complete. The whole series indeed seems to be explained. Why? Because by explaining each transition (i.e. simply by mentioning the laws specifying the behavior of the scribes), one seems to explain why each book has the content it has. Unlike in Leibniz’s Infinite Scribes case, what explains these transitions, namely the behavior of Monomaniacal Scribes (the laws alone), does determine each item of the series. Actually, it necessitates them: necessarily, if all the scribes behave as specified, all books must be copies of the Elements of Geometry.

One might wonder whether this explanation really explains the existence (as opposed to the mere content) of the series of book copies (this is related to what we have called earlier

the “extra-property objection”). Does it really explain the whole series of facts $(u_i)_i$ (where $u_i =$ ‘book copy # i is the Elements of Geometry’) or rather the series of conditional facts $(u'_i)_i$, where $u'_i =$ ‘if book copy # i exists, it is the Elements of Geometry’? The answer is that it really explains the whole series $(u_i)_i$ (and the implied existence claims) because we have specified in the laws that Monomaniacal Scribes always create a copy of the Elements of Geometry, whatever book copy they are given *and do so even when they are given no book at all*. Had we not specified that in the laws, only the series of conditional truths would still be completely explained anyway.

The Infinite Monomaniacal Scribes is somehow trivial. There are more interesting examples of infinite chains of explanations that seem likewise complete. Consider:

The Infinite Stick Adjusters. Consider the infinite series of lengths $(l_i)_i$ of a given stick made out of a plastic lump. Given the number of molecules in the plastic lump, the length of the stick is bounded by b and B . Let l be a specific length between b and B . The length l_1 of the stick at stage #1 is explained by the fact that the stick has been adjusted before at stage #2 by a stick-adjuster from a state in which it had length l_2 , which length is explained by the fact that it has been adjusted earlier at stage #3 by another stick-adjuster from a state in which it has length l_3 , and so on to infinity... where a stick-adjuster is someone who takes a stick of length x and adjusts it so that its size becomes closer to a specific length l (where $b < l < B$):

- (i) if $l \leq x \leq B$, compresses it in order to reduce its size it by $\frac{x-l}{2}$ (so that its size becomes $x - \frac{x-l}{2}$).
- (ii) if $b \leq x \leq l$, stretches it in order to augment its size it by $\frac{l-x}{2}$ (so that its size becomes $x + \frac{l-x}{2}$).

Here L_i is the law that specifies the behavior of the stick-adjuster at stage # i , u_i is the fact ‘if the stick exists at stage # i , it has length l_i ’.

It takes little reflection to realize that necessarily if the stick-adjusters behave as specified and if the stick exists at all, the stick will always be exactly l long. This can be deduced from the laws that specify the behavior of the Stick Adjusters alone. Intuitively, stick adjusters keep adjusting the stick to make its length closer and closer to l and if you start with a finite stick, you will end up, at the limit, with a l -long stick. But as each stick in the series is bounded and has infinitely many Stick-Adjusters behind him, each stick will be l -long. More rigorously, the laws entail that $l_{i+j} = l + 2^j(l_i - l)$, so if for some i , l_i were different from l , the series $(l_{i+j})_{j \geq 1}$ would not be bounded, which is absurd by construction (unless otherwise mentioned indexes are natural numbers greater than or equal to one). This infinite explanation thus determines that the stick is always l -long — it does not leave the length of the sticks open. It accordingly seems to explain the whole series, and thus

everything there is to explain. It is arguably complete.⁷

We can come up with other, maybe simpler examples of non-trivial complete infinite explanations. Consider the Wheel-Turners.

The Infinite Wheel-Turners. Take a wheel that is divided in four identical numbered sectors (respectively 0, 1, 2, 3). The sector s_1 on which the wheel has just landed is 1 because the wheel has just been turned by a first wheel-turner at the beginning of stage #1 from a former sector s_2 , and it was in that sector because the wheel had been turned at beginning of stage #2 by a second wheel-turner from a former sector s_3 ... where a wheel-turner is someone who takes a wheel that has landed on sector x ($x \in \mathbb{Z}/4\mathbb{Z}$) and turns it so that it lands on sector $f(x) = 2x - 1$ ($f(x) \in \mathbb{Z}/4\mathbb{Z}$).

Here u_i is the fact that the wheel if it exists, has landed on sector s_i at the end of stage # i . And L_i is the law specifying the behavior of the wheel-turner # i (again the laws are all identical).

As $f(0) = f(2) = 3$, $f(3) = 1$ and $f(1) = 1$,

- if the wheel lands on 1 it will always stay on 1 when it is turned again,
- and the wheel will always land on one (whatever the starting point) provided that it has been turned at least twice.

So for all i , $s_i = 1$. This, moreover, holds necessarily provided that the wheel-turners act as specified. This explanation accordingly seems to explain everything there is to explain. It seems complete.

The reader can check that just as the Leibnizian incomplete infinite scribes series has a circular incomplete counterpart, all these examples of complete infinite explanations have circular counterparts that are complete (just add a time-travel twist to the stories).

Finally, there is an interesting contrast to be drawn between the Stick-Stretchers case on the one hand and the Wheel-Turners and the Monomaniacal Scribes on the other. Infinity or circularity (non-wellfoundedness) indeed seems somehow more important to the completeness of the explanation in the first case than in the two others. Indeed, the reader can check that in a simple wellfounded version (u_1, u_2, \dots, u_n) of the Monomaniacal Scribes case, the laws alone suffice to explain, if not the whole series, at least its $n - 1$ first items (u_1, u_2, \dots, u_{n-1}). Roughly the same goes for the Wheel-Turner if $n \geq 4$: the laws alone will suffice to explain the $n - 3$ first items. In the Stick-Stretchers case, however, unless our

⁷The reader worried by the “extra-property objection” regarding existence can check that we can get a version of the Stick-Stretchers where u_i s are uncontroversially existence-implying facts about the length of a stick rather than more modest truths that are conditional on the existence of the stick simply by stipulating that the Stick Stretchers *create* a copy of the stick they are given and then stretch it.

series of items is infinite or circular, even the length *of the first stick* will not be determined and explained. If the series is long enough, then at stage #1 the stick will necessarily be rather close to being 1-long, its precise length will not however be determined by the laws. We can say that in the first case but not in the others, the completeness of our series of explanations is so to speak “entirely due to non-wellfoundedness”.

Cases of complete non-wellfounded explanations, and even more dramatically, cases in which the completeness is entirely due to non-wellfoundedness, show something very important, namely that far from always being a liability, infinity, and circularity can be explanatory productive and play an essential role in some explanations. We can draw an analogy here with proof theory. Despite a widespread assumption to the contrary, mathematicians do sometimes use circular or infinitely descending proofs in arithmetic. This is for example the most natural way to understand the so-called “proofs by infinite descent” (Fermat’s proof of the irrationality of $\sqrt{2}$ is a classical example, and so the classical proof of Euclid’s division lemma⁸). Now it can be shown that allowing such “non-wellfounded proofs” in Robinson Arithmetics yields classical, Peano arithmetics (Simpson, 2017).⁹ In the guise of infinity and circularity, non-wellfoundedness is proof-theoretically productive. Cases like that of the Stick-Stretchers show that what goes for proofs goes for explanations as well. Far from being an obstacle to good explanations, as suggested by the quotations of Fine and Schaffer¹⁰, infinity and circularity can do genuine explanatory work, but they will only do so in very specific cases. I would now like to find out what exactly distinguishes these cases.

5 Towards a general case: causal explanations

All the examples of complete non-wellfounded explanations we have given above imply *causal explanations*. Below I will try to get more general results. I will first abstract general conditions on the completeness of causal explanations from the examples above, and then show that the reasoning that yielded these conditions generalizes to metaphysical explanation and other “determination relations”.

We can notice, first, that in all the examples above, the i -th item seems functionally determined by the $(i + 1)$ -th item. There is, in other words, a function f_i (depending on the law L_i) that accounts for the transition from the $(i + 1)$ -th item to the i -th item.¹¹ More

⁸Euclid’s division lemma states that for two integers a and b , with $b \neq 0$, there exist unique integers q and r such that $a = bq + r$ and $0 \leq r < b$

⁹I am indebted to Léon Probst for the discovery of this very interesting result and for the realization that proofs by infinite descent can be naturally interpreted as non-wellfounded proofs.

¹⁰Schaffer and Fine are concerned with metaphysical explanations rather than causal explanations but we will see that there are ground-theoretic analogs of the Stick-Stretchers cases.

¹¹It should be emphasized that here “function” is understood in the mathematical sense where a function is just a relation R such that if xRy and xRz then $y = z$ (rather than as a causal role or as a trait selected

precisely, in all of these cases:

- There is a parameter that can take different values at different stages (the content of the book, the size of the stick, etc.)
- such that the value of this parameter at stage $\#i$ is the result of applying the function f_i to the value of this parameter at stage $\#(i + 1)$.

We can represent this functional dependence by introducing a series of functions $(X_i)_i$, where X_i associates to a possible world the value the $\#i$ -th item of the series takes in this world, and \emptyset if the $\#i$ -th item does not exist in this world. Let us also introduce the symbol “ \circ ” for the composition of functions ($f \circ g$ is the function that associates $f(g(x))$ to x). Then it seems that in all the cases we have envisioned so far,

- u_i has the form ‘ $X_i(@) = x_i$ ’ (where $@$ is the actual world and single quotes are a “fact formation device”), or in cases where the fact u_i is conditional on existence ‘Either $X_i(@) = \emptyset$ or $X_i(@) = x_i$ ’
- There is a function f_i such that $X_i = f \circ X_{i+1}$ (i.e. if the value of X_{i+1} in a world is a , then the value of X_i in this world is $b = f(a)$..

We can call f_i “the flow function” of the series.¹² In all of our examples above, $f_1 = f_2 = \dots = f_i = \dots = f$, and $L_1 = L_2 = \dots = L_i = \dots = L$ and we might say that the flow and the explanation are *uniform*. We could, however, construct explanations that are not uniform (say, by stipulating that some, but not all scribes do not copy their book faithfully, see Billon (2021, p. 1942)).

In the Infinite Regular scribe case $f = f_{RS}$ is the identity function Id over book contents, in the Monomaniacal Scribes $f = f_{MS}$ is the constant function that associates the content of the book is the Elements of Geometry to any content and even to the empty content of absent books. In the Stick-Adjusters case $f = f_{SA}$ associates the length $x - \frac{x-l}{2}$ to the length x , in the Wheel-Turner case $f = f_{WT}$ that associates the sector $(2x - 1)[4]$ to the sector x ...

In all the examples we have considered there is also a natural metric associated with the values of our X_i s. We can thus define a notion of distance and a notion of convergence on these values. I will argue that in cases such as these, the following conditions are both necessary for the non-wellfounded chain of explanation to be complete:

(CN1) f has a unique fixed point e (i.e. there is a unique value x of f such that $x = f(x)$ and $x = e$).

(CN2) for all x , the series $(f^i(x))_i$ converges toward the same item e .

by evolution for its causal role).

¹²I borrow the term from dynamical system theory, which should make sense by the end of the paper (§9).

I will also argue that, conversely, the two following conditions are jointly sufficient¹³:

(CS1) f is contractive: there is $k < 1$ such that for all x, y , $|f(x) - f(y)| \leq k * |x - y|$

(CS2) f is bounded.

Intuitively, (CS1) means that f shrinks the space.

These conditions fit most examples above: (CN1) and (CN2) are only satisfied by the Wheel-Turners the Monomaniacal Scribes and the Stick Adjusters: they are not satisfied in the Circular Scribes and Infinite Scribes cases. (CS1-CS2) are satisfied in the Stick Adjusters example but not in the Wheel Turners example (at least when $\mathbb{Z}/4\mathbb{Z}$ is fitted with the canonical metric, i.e. the distance between two points being the absolute value of their difference), which shows that (CS1-CS2) are not necessary.

By reflecting on an unbounded variant of the Stick-Adjusters case, the reader can also check that (CS1) is insufficient by itself (i.e. without (CS2)) and that (CN1-CN2) are jointly insufficient. In that unbounded variant, *for any arbitrary length l_1* , we can construct a series of sticks such that stick $\#i$ is l_i -long and has been adjusted from stick $\#(i + 1)$ by one of our Stick-Adjusters. Just take sticks such that $l_i = l + 2^{i-1}(l_1 - l)$. Accordingly, the fact that each item of a series is the length of a stick that has been adjusted by our next Stick-Adjuster *does not determine the length l_1 of the first stick*, and it does not, a fortiori, determine the whole series of lengths. But if it does not determine it, it seems that it won't explain it either.

The argument for the necessary character of (CN1) and (CN2), and for the joint sufficient character of (CS1-CS2), involves two parts. It has a philosophical component first, connecting the notions of explanation and completeness to that of functional dependence, and translating the claim that a non-wellfounded causal explanation is complete in mathematical terms. It also includes a mathematical component, demonstrating that the translated claim holds when (CS1-CS2) are satisfied and only holds when (CN1-CN2) are satisfied. The mathematical part of the argument is non-trivial, but it is philosophically uninteresting, so I will place it in the appendix. Now, I will slowly unfold the philosophical part of the argument, pausing at some interesting concepts that need to be introduced along the way.

5.1 Insensitivity to prior items

To say that a chain of explanations $(u_i)_{i \in I}$ is complete, as we have seen, is to say that it (fully) explains the whole series $(u_i)_{i \in I}$. In the non-wellfounded case, this means that by explaining the transitions from u_{i+1} to u_i , we fully explain the whole series $(u_i)_{i \in I}$. This,

¹³For these conditions to hold we need to suppose that our metric space is “complete” (in the sense that every Cauchy sequence (intuitively, every sequence whose items can become arbitrarily close to each other) has a limit), which is unproblematic in all the examples we consider.

in turn, seems equivalent to saying that what explains the transitions from u_{i+1} to u_i , (i.e. the laws $(L_i)_i$) fully zero-explains the whole series $(u_i)_{i \in I}$.

This means that in all cases of non-wellfounded complete explanations, the laws $(L_i)_i$ alone will suffice to explain the first item u_1 . Accordingly, the history $(u_2, u_3 \dots)$ of the first item will be explanatorily irrelevant. Complete non-wellfounded causal explanations will display a form of “historical irrelevance” or “insensitivity to prior items”.

5.2 The explanation-determination condition

In order to show that the Leibizian infinite explanation is incomplete, we have argued that it does not *determine* the whole series. In order to show that the Infinite Monomaniacal Scribes, the Infinite Stick-Adjusters and the Infinite Wheel-Turners are complete we have argued that these chains of explanations do determine all the items of the series.

We have relied on the following explanation-determination conditions, to the effect that the Leibnizian Infinite Scribes Series fails to be complete because and *only because* it fails to determine all their terms:

- (ED1) In order to be complete, chains of explanation such as the Leibnizian Infinite Scribes Series need to determine all their terms.
- (ED2) If a similar chain of explanations did determine all its terms it would be complete.

I tackle (ED1) and (ED2) in turn.

(ED1) stems from the fact that (full) explanation is a *determination relation*, so that a (full) explanans (a final item) must, along with a law, *determine* its explanandum (an initial item). This is true for determinist explanations. One might worry this does not hold for non-determinist explanations, as found, e.g. in quantum mechanics. However, non-determinist explanations are arguably explanations in which the probability distribution of a variable (if not its effective value) *is* determined — this is what happens in quantum mechanics. So (ED1) is still arguably true in the non-determinist case provided that we consider the explananda to be probability distributions.

Let us now move on to (ED2). It captures the idea that the only reason why the Leibnizian Infinite Scribe series is not complete is that it fails with regard to (ED1). Importantly, (ED2) does not imply that determination suffices for a full explanation: there are classical counterexamples to this claim, involving asymmetry, overdetermination, or “pre-emption”, see Billon (2021, § 6). It only implies (and in fact it means) that if a series of explanations $(u_i)_i$ is such that the laws determine the full series, then the explanation is complete. And this claim is arguably true because when we talk about explanations of a series of items $(u_i)_i$ by laws, obstacles to the entailment from determination to explanation such as asymmetry,

pre-emption, and overdetermination are not a real threat. The question of asymmetry does not even make sense in this context (the laws are not an explanandum here). As for the question of pre-emption and over-determination, they might make sense in cases where the laws are not uniform. Yet, if the laws determined the whole series but did not explain it because of pre-emption or overdetermination, a proper subset of the laws would arguably explain the whole series and we would still have a complete explanation of the series.

Now (ED1) and (ED2) entail that in our examples, the series we consider is complete iff (B) follows from (A):

- (A) for all i , u_{i+1} (along with L_i) fully explains u_i
- (B) The laws $(L_i)_i$ alone determine the whole series $(u_i)_{i \in I}$

Now it is arguable that if there is something in (A) that can entail a determination condition such as (B) it is only the following determination condition that is entailed by (A):

- (A*) for all i , u_{i+1} (along with L_i) determines u_i

If that is so (and I will admit that it is), (B) follows from (A) iff it follows (B) from (A*). That is, iff

- **Completeness Condition (first version).** That for all i , each item u_{i+1} (along with L_i) determines its antecedent u_i entails that the series of laws $(L_i)_i$ determines the series of items $(u_i)_{i \in I}$

5.3 The functional account of determination

One might wonder how we should analyze the sense of “determine” in the claim that explanation entails determination and in our first completeness condition. I must say it is very tempting to analyze it in terms of necessity (this is a temptation to which I have informally yielded a couple of times above, using modal considerations to assess determination claims). We might want to claim for example that an initial item determines a final item only if it necessitates it. This corresponds to what we might call “the strong functional account of determination”. If U_i is a function that associates with a possible world the value the i -th item of the series takes in this world (U_i associates u_i to our world: $U_i(@) = u_i$), this account of determination says that

- **The strong functional account of determination.** The $(i+1)$ -th item determines its antecedent (the i -th item) if there is a function g_i (depending on L_i) such that one of the following equivalent conditions is satisfied:
 - (i) $U_i = g_i \circ U_{i+1}$
 - (ii) Necessarily, if the $(i+1)$ -th item U_{i+1} is a in some world then the i -th item U_i is $b = f(a)$ in that world.

Even though I believe that determination can indeed be understood as necessitation and that it is useful to think of it that way in what follows, it is not totally uncontroversial to do so, and it is not, strictly speaking, required. We can provide a broader account of determination below: the *weak functional account of determination*. It relies on a weakening of the conditional (ii) so that it becomes (a-b):

- **The weak functional account of determination.** The $(i + 1)$ -th item determines its antecedent (the i -th item) if there is a function g_i (depending on L_i) such that one of the following equivalent conditions is satisfied:
 - (a) $g_i(u_{i+1}) = u_i$, and in close possible worlds where $U_{i+1} = u_{i+1}$ the value of U_i is still $g_i(u_{i+1}) = u_i$.
 - (b) had the (value of the) $(i + 1)$ -th item been slightly different because of a local miracle (say equal to u'_{i+1}) then the (value of the) i -th item would have been $u'_i = g_i(u'_{i+1})$.

The weak functional account of determination construes it not as necessitation but, merely, as a counterfactually supporting functional relation. Notice that (a) and (b) are equivalent to claiming that that U_i and $g_i \circ U_{i+1}$ only coincide in a certain subset Ω of all possible worlds (a subset that contains the actual world and very close worlds), i.e. that $U_i|_{\Omega} = g_i \circ U_{i+1}|_{\Omega}$.

Why think that explanation must entail determination in this sense? Well, as far as causal explanations are concerned, good scientific explanations all seem underwritten by equations that yield, at least locally, a form of functional determination of this sort. Connectedly, the fact that causal explanations always yield a functional determination in this sense is entailed by the structural equation account of the “structural equations framework” of causation and causal explanation (Menzies and Beebe, 2020, §V), which precisely stems from scientific practice (Schaffer, 2016). It is equally entailed by the more general “functional conception” of explanatory laws (Schaffer, 2017).¹⁴ More deeply, the claim that causal explanation requires such a “functional determination” stems from the fact that a cause must determine its effect and that an explanation follows a law. (a) and (b) are arguably the minimal conditions capturing these two facts.¹⁵

¹⁴The functional characterization of determination bears a strong resemblance to the structural equations framework of causation and the more general “functional conception” of explanatory laws (Schaffer, 2017). There are important differences, though. First, my account is an account of determination, which I take to be a necessary condition of explanation, not an account of explanation itself. Moreover, as I understand them, both the structural equation framework and the functional conception of laws aim at accounting for the fact that explained item *really depends* counterfactually on x (i.e. is sensitive to x), and so they require (at least) that the function f be non-constant. My “functional account of determination”, on the other hand, is neutral regarding real counterfactual dependence, and thus less demanding. It only aims at accounting for the fact that the explaining item determines the item it explains.

¹⁵One might wonder why (b) and not just (a) is required to capture the idea that a cause determine its effect. (a) concerns the tokens u_{i+1} and u_i that happen to be the $i + 1$ -th item and the i -th item in our world and require that the latter token be modally fixed by the former. (b) concerns the types represented

The reader who would not be convinced that either the condition (ED1-ED2) and the functional account of determination universally hold should still grant that it holds rather generally (and in particular it holds in all the examples we have put forward until now and in those we will consider in what follows). This should be sufficient to maintain his interest in the conclusions of this paper.

5.4 A mathematical formulation of the completeness condition

The distinction between the weak and the strong version of the determination condition is important philosophically but not very important formally. In what follows, I will, for the sake of simplicity, suppose that our function $(U_i)_i$ are only defined on Ω and accordingly omit the restriction and consider that U_{i+1} determines U_i iff $U_i = g_i \circ U_{i+1}$.

Now interestingly, when, like in all of our examples, the items are facts u_i of the form ‘ $X_i(@) = x_i$ ’, or of the form ‘Either $X_i(@) = \emptyset$ or $X_i = x_i$ ’ it can be checked that the determination condition on $(u_i)_i$ (there is g_i such that $U_i = g_i \circ U_{i+1}$) is equivalent the corresponding determination condition on $(x_i)_i$:

- There is f_i such that $X_i = f_i \circ X_{i+1}$.
- where f_i is exactly what we have called before the “flow function”.

Using these conventions (with capital letters for functions other than f_i and f , and with the associated minuscules for the items which are their values), we can recapitulate:

- The item $\#(i + 1)$ (along with L_i) determine the item $\#i$

means that

- there is a function f_i (that depends of L_i) such that $X_i = f_i \circ X_{i+1}$.

(In our examples, the explanations are uniform so neither L_i nor f_i really depends on (i.e. is sensitive to) the index i .)

Similarly, to say that

- the laws $(L_i)_i$ determine each u_i in $(u_i)_i \in I$ *all by themselves*

means that

- there is a series $(E_i)_i$ of *constant functions* (i.e. functions whose output is insensitive to the input and so “depend on nothing”) such that for all i , $X_i = E_i$.

by the functions U_{i+1} and U_i and require that the latter be determined by the former. Now suppose that the conditions (a) held but not the condition (b): imagine that had U_{i+1} been u'_{i+1} ($\neq u_i$) then then U_i would have been indeterminate (say that it could equally have been many different token items and that there is no fact of the matter regarding which it would have been). In such a case, one might still claim that the token u_{i+1} causally explains the item u_i but it would be hard to maintain that this causal explanation follows laws.

Our chain of explanation is hence complete iff

- **Completeness condition (second version).** (For all i , $X_i = f_i \circ X_{i+1}$) implies that (for all i , X_i is constant).

As shown in the appendix A, this second version of the Completeness Condition is all we need to get the mathematical running and show that (CS1-CS2) are jointly sufficient for completeness while (CN1) and (CN2) are both necessary.

6 From causal explanation to metaphysical explanation and other determination relations

We have isolated general formal conditions (CN1-CN2) and (CS1-CS2) on the completeness of chains of causal explanations. They could be used to generate other examples of complete and incomplete such chains and to check whether current cosmology supports the idea that our universe might actually be explained by a complete non-wellfounded chain of causes.

Do the conditions (CN1-CN2) and (CS1-CS2) generalize to chains of metaphysical explanations? The answer is that they do. Why? Because in our reasoning, the fact that we were dealing with causal explanations, as opposed to some other relations, only intervened in our argument to the effect that causal explanations satisfy the Explanation-Determination conditions (and also, though only verbally, in our choice of dubbing the “insensitivity to prior items” in §5.1, “historical irrelevance”). Yet if, as we have supposed metaphysical explanations follow laws, they should functionally determine their explanandum as well: if the $(i + 1)$ -th item fully grounds its antecedent, and if it does that according to a law, it should determine it in the required sense of a counterfactual-supporting functional dependence specified by (a-b) (see §5.3, fn.17 and also Schaffer (2017) who develops a couple of arguments to that effect). In other words, a metaphysical explanation must be a determination condition at least in the weak functional sense isolated above. (ED1) should accordingly be satisfied. The same goes for (ED2) because the obstacles to the entailment from determination to explanation are arguably the same in the causal and in the metaphysical case, and they lose their grip when we consider the completeness question for chains of explanations (the problem of asymmetry does not arise in this context, and even if the laws determine without explaining the full series because of overdetermination or pre-emption, we would still have to say that a subset of the laws explains the whole series and that the explanation is complete).

More broadly, let us call an R -chain $(u_i)_i$ a chain of R -related facts of the form ‘ $X_i(@) = x'_i$, where R is a relation

- (I) whose logical form is $(x, L)Ry$ where x is an item or nothing (\emptyset), L a law that can be kept implicit, and y an item or a series of items.

- (II) which is a determination relation in the sense that $u_{i+1}Ru_i$ entails that there is a flow function such that $X_i = f_i \circ X_{i+1}$.

Suppose that the R -chain is uniform in that the laws and the flow functions are always the same (for all i , $f_i = f$). Suppose, also that we can define a metric on our items¹⁶.

Say, finally, that the R -chain $(u_i)_i$ is complete just in case

$$(\emptyset, L)R(u_i)_i$$

and say that it is quasi-complete if the laws alone L determine the whole series. We can easily show that the conditions (CS1-CS2) and (CN1-CN2) are respectively sufficient and necessary conditions for the quasi-completeness of the R -chain. We can easily show that (CN1-CN2) are also necessary for completeness. Conversely, (CS1-CS2) will be sufficient for completeness when completeness is entailed by quasi-completeness.

As mentioned earlier, some philosophers believe that we should distinguish grounding from metaphysical explanation. Even if we have assumed that they were identical, these philosophers can still take R =grounding and get for grounding the exact same conclusions that we got for metaphysical explanations. Finally, this result might also apply to other determination relations, and in particular to R =supervenience (for supervenience the questions of asymmetry preemption and overdetermination do not arise¹⁷ so the analog of (ED2) should trivially hold).

7 Non-wellfounded chains of grounds

Now that we have sufficient and necessary conditions for the completeness of non-wellfounded chains of ground, we could try to use them to put forward “concrete” examples of complete and incomplete chains of grounds. As the conditions are formally similar to those that obtain in the case of causal explanations, we could also just try to adapt the examples we have already put forward. After all, if, as we have supposed grounding is metaphysical explanation, the only relevant difference between a case in which an item x causally explains another y and a case in which x grounds y is that the laws regimenting the transition are natural, causal laws in the first case and metaphysical, grounding laws in the second. Maybe simply specifying that Scribes, Stick-Adjusters and Wheel-Turners are gods moved by metaphysical laws could do the trick?

¹⁶A metric, more precisely, that makes the space of items metric complete (in the mathematical sense of the term, see fn.13).

¹⁷This is precisely the reason why supervenience, which has long been used to capture something like metaphysical explanation has largely been replaced by ground in this role.

7.1 The Infinite Simulation and the Infinite Truth-Teller

More convincingly, we could rely on the idea that the world contains various layers of reality that are grounded on each other but might closely resemble each other. This is an idea we can find in some interpretations of Plato (where forms resemble concrete reality which resemble representations thereof...), but that is also popular among digitalists who believe that we might live in a simulation that is being run in an “upper” world that it itself being simulated in an “upper” world, etc. (Chalmers, 2022). Some even suppose that this could go on indefinitely (Steinhart, 2014). Of course when A is a simulation of B there is normally a causal story to tell: A has for example been programmed by someone to simulate B. Yet B is realized and grounded on A. Likewise, Plato famously provides (in the *Timaeus*) causal stories to explain the relationship between the Forms and the concrete objects we interact with. Yet these relations seem to involve grounds.

Now, we can obtain a ground-theoretic version of the Infinite Scribes that way if we imagine that our layer of reality is likewise grounded on another layer which is itself grounded on another layer... and that this series is infinite. In the example below, I adopt Chalmers (2022)’s theory of simulation according to which a simulation of X is a digital object having the same causal structure as X so that a simulation of a simulation of X is still a simulation of X.

The Infinite Simulation. Layer #1 of reality contains just a digital object d_1 which has the same causal structure t as that of a small tree and simulates the latter. This simulation is realized (and grounded) in layer #2 on another object d_2 (which is part of a computer of that layer). d_2 is realized and grounded in layer #3 on another object d_3 (which is part of a computer at that layer)... Let x_i be the causal structure of d_i . Here u_i is the fact that the causal structure of the object at layer # i is t (with the same notations as above, $u_i = 'X_i(@) = t'$). The laws specify that each layer contains a simulated object realized in the next layer.

Here the chain of ground is incomplete. Indeed the fact that each object is a simulation of the next does not explain why our series is a simulation of a tree rather than one of (say) a bacteria. The reader can check that the flow function is the identity function over causal structures and has every causal structure as a fixed point. We have a simple example of incomplete non-wellfounded chain of grounds.

The following Infinite Truth-Teller, which relies on truth-making rather than simulation/realization is similar to the Infinite Simulation and to the Infinite Scribes case (the flow function is the identity over semantic values). It is an incomplete infinite chain of grounds as well.

The infinite Truth-Teller. Let $(v_i)_i$ be a series of sentences, such that $v_i = \text{“}v_{i+1} \text{ is true”}$. Let $(x_i)_i$ be the series of the truth-values of the v_i s. Let u_i be the fact

that the semantic value x_i of v_i is 1 ($u_i = 'X_i(@) = 1'$). u_1 is grounded on u_2 which is grounded on u_3 , etc.

I find it harder to find an intuitively plausible ground-theoretic analog of our Strick-Stretcher example using iterated simulations or infinite chains of sentences whose truths are grounded on each other.¹⁸ Below, I argue that we can come up with interesting cases of *complete* and *incomplete* chains of grounds if we focus on the way facts about certain objects are grounded on facts about smaller objects (think about the way chemical facts are grounded in microphysical facts).

7.2 Rep-tiles and fractals

Chemists often use tilings by dominoes as models of the composition of solids. Facts about a solid modeled after a region of space can be considered as being grounded on facts about the arrangement of molecules (modeled after the dominoes) tiling that region of space. More broadly we can consider a world whose inhabitants are geometrical figures grounded on tilings thereof.

Below, I consider two such worlds. The first one involves rep-tiles. The second involves fractals. I did not find these by accident. Indeed, (CN1-CN2) imply that if u_1 is grounded on an infinite and complete chain of grounds, then u_1 can be obtained, at the limit, by the recursive iteration of the flow function f . This provides a nice recipe for candidates complete chains of grounds.

The first world is a rep-tile world. Rep-tiles are “self-replicating figures”: figures whose copies can be assembled to produce a bigger figure with the exact same shape — figures that can, equivalently, be dissected into smaller copies of the same shape (see Gardner (2001, pp. 46-58), and figure 1 for an illustration).¹⁹ The second involves a fractal, i.e. a geometrical object whose structure is identical at every scale (we sometimes say that such an object is “self-similar”).

A Rep-tiles World.

Let us start with rep-tiles, then. We can divide rep-tiles according to the number of copies of themselves needed to make a bigger version of themselves. Here, we will focus on rep-4 tiles, that is, on figures that can compose bigger versions of themselves composed of *four* copies of themselves. Every triangle and every parallelogram is a rep-4 tile (they are not the

¹⁸It might be possible to construct a truth-making analog of the Stick-Stretchers using a supervaluationist semantics. It might as well be possible to construct a “simulationist” analog of the Stick-Stretchers by specifying that the degree of relativity decreases geometrically with iterated simulations and by considering facts such as $u_i =$ ‘the degree of reality at level i is zero’. I have not, however, been able to find simple and intuitively convincing examples of such analogs.

¹⁹A more complex, and probably more realistic example involves a generalization of rep-tiles called “self-tiling tile sets” or “setisets” for short (Sallows, 2014).

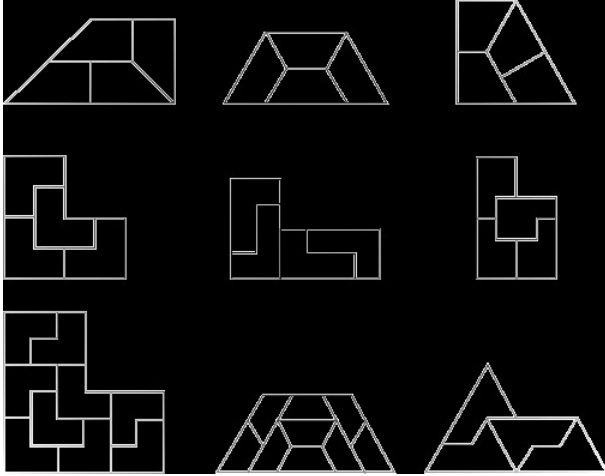


Figure 1: Examples of rep-tiles : the first two lines display rep-4 tiles, the last line displays two rep-9 tiles and one rep-4 tile.

only rep-4 tiles, see figure 1, but we will focus on these rep-4 tiles to make things simpler). For every triangle and every parallelogram O , there is a unique rep-4 tile $f_r(O)$ made of O and three other copies of O and such that O is on the bottom left corner of this rep-4 tile.

Conversely, for every triangle and every parallelogram O there is a unique tiling (or “dissection”) of O in four identical parts of the same shape as O , but with sides that are half the size of O ’s side. We can represent a tiling of O as a set of tiles (understood as compact regions, i.e. bounded set of points that is topologically closed) whose union is O and whose intersection is reduced to the border of neighboring tiles.²⁰ We can label these tiles of O “ O_a ”, “ O_b ”, “ O_c ” and “ O_d ”, using the left-to-right and up-to-down order.

Now each of these tiles likewise admits a unique tiling in four similar parts. O_a is tiled by $O_{aa}, O_{ab}, O_{ac}, O_{ad}$; O_b is tiled by $O_{ba}, O_{bb}, O_{bc}, O_{bd}$, etc. We can call “ i -iterated rep-4 tiling” a tiling obtained by i iterations this operation (see figure 2).

The inverse of f_r is the function f_d which is such that $f_d(O) = O_a$.

Now imagine a world that can only contain triangular or rectangular rep-4 tiles and in which each rep-4 tile is, at the next level, composed of its dissection in 4 tiles, whose tiles are in turn composed of their own dissections, etc. Consider the following series of rep-4 tiles indexed by levels. At the level 1, the figure is an equilateral triangle $x_1 = abc$. At the level 2, it is the smaller figure x_2 at the left bottom corner of x_1 such that x_1 is composed of

²⁰We stipulate that figures are all compact and hence topologically closed, so the intersection of two bordering figures is non-empty and we do not get a genuine partition of the original figure O . We could slightly modify the case to get a genuine partition but that would make things uselessly more complex.

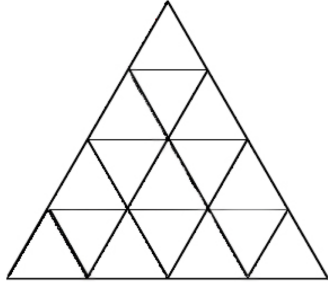


Figure 2: A triangle to which the dissection operation has been applied twice, and that is accordingly tiled in 16 similar triangles.

three copies of x_2 ($x_2 = f_d(x_1)$). At the level 3, it is the smaller figure x_3 at the left bottom corner of x_2 such that x_2 is composed of three copies of x_3 ($x_3 = f_d(x_2)$), etc. If we assume that facts about parts are explanatorily prior to facts about the whole they compose, the fact that the figure at level 1 is x_1 is grounded on the fact that level 2 is $x_2 = f_d(x_1)$ ²¹. This is grounded on the fact that at level two, the figure at the bottom left corner is the yet smaller equilateral triangle $x_3 = f_d(x_2)$... The flow function here is f_r , the inverse function of f_d .

The Infinite Rep-4 Tiles World. Consider a world that contains a rep-4 tile at level 1 and at each other level a basic rep-4 tile on the bottom left corner of which the rep-4 tile at the preceding level is composed. At level #1 the figure is an equilateral triangle $x_1 = abc$, which (fact) is grounded in the fact that at level #2 the figure is its tile x_2 (where $x_2 = f_d(x_1)$ and $x_1 = f_r(x_2)$), which (fact) is grounded on the fact that at level #3, the figure is x_2 's tile x_3 (with $x_3 = f_d(x_2)$ and $x_2 = f_r(x_3)$), etc. We have an infinite chain of grounds. Here u_i is the fact that the figure at level i is x_i : $u_i = 'X_i(@) = x_i'$, the flow function is f_r , and $X_i = f_r \circ X_{i+1}$. The metaphysical laws specify that the world contains only rep-4 tiles and that each rep-4 tile at level # i is composed of its tiling at level # $(i + 1)$.

Now in this case, the fact that each item of this series of grounds is grounded on its successor according to the laws leaves it open whether they are all triangles or (say) squares (compare with the Infinite Scribes series). It also leaves completely open the size of the first item or

²¹Priority monists such as Schaffer (2010) believe that, on the contrary, facts about parts are grounded on facts concerning the wholes they compose. The reader can check that the example can be modified to suit priority monism: consider a world that contain iterated tilings (rather than iterated dissections) and replace f_d with f_r .

its very existence. So the series is not complete. In fact, it can be checked that the flow function f_r has no fixed point at all (it maps a figure to one of its proper parts), so the case does not satisfy (CN1).

A Fractal World.

We can now move on to the fractal case. A dilation of factor x and center O is a function that regularly dilates the space of a factor x around O . Such a dilation will, for example, transform a circle of center O and radius 1 meter into a circle of center O and radius x meters (if $x < 1$ the dilation will actually shrink the space).

Let abc be a filled equilateral triangle with 1 meter sides, and let f_{sa} be the dilation of center a and factor $1/2$, f_{sb} be the dilation of center b and factor $1/2$ and f_{sc} be the dilation of center c and factor $1/2$. Consider the function $f_s = f_{sa} \cup f_{sb} \cup f_{sc}$. It is, so to speak, a “shrinking and duplication” function that associates to a figure O (understood as the shape of a compact set of points) three shrunk copies of it disposed at the extremities of the equilateral triangle. Now consider the figure s obtained at the limit by applying f_s iteratively to the filled equilateral triangle abc . This figure is called the Sierpinski gasket (or the Sierpinski sieve or the Sierpinski triangle) of corners a , b , and c .²²

This way of generating the Sierpinski gasket might suggest a causal process (imagine someone repetitively shrinking triangles and assembling them...). Pace constructivists, however, we do not need to construe this way of generating the Sierpinski gasket (or indeed others) as really requiring some kind of diachronic construction. Moreover, even if constructivists were right to claim that the only good definition of s involves a causal or quasi-causal construction process, this causal construction story would be compatible with the following grounding claims concerning the output of this process. Indeed, by construction the first figure $x_1 = s$ is composed of three shrunk copies (scale $1/2$) of a second figure x_2 (take the one on the left bottom corner) such that $x_1 = f_s(x_2)$, x_2 is likewise composed of three shrunk copies (scale $1/2$ again) of a third figure x_3 such that $x_2 = f_s(x_3)$, etc. If we assume, again, that facts about parts are explanatorily prior to facts concerning the whole they form, then the fact that the figure at level i in the series is x_i is grounded on the fact that the figure at level $i + 1$ is x_{i+1} where $x_i = f_s(x_{i+1})$.

The Sierpinski Gasket World. Consider a world that contains an infinity of levels. At level 1, there is a figure x_1 which is composed of three shrunk copies (scale $1/2$) of the figure x_2 at level 2 ($x_1 = f_s(x_2)$), the figure at level 2 is itself composed of three shrunk copies (scale $1/2$) of the figure at level 3 ($x_2 = f_s(x_3)$)... The figure at level #1, x_1 is the Sierpinski gasket s , which (fact) is grounded on the fact that the figure at level #2, x_2 , is s as well, which is grounded, on the fact that the figure at level #3, x_3 is s as well, etc.

²²This way of generating the Sierpinski sieve is called “Iterated functions system”, see e.g. Falconer (2004, ch. IX).

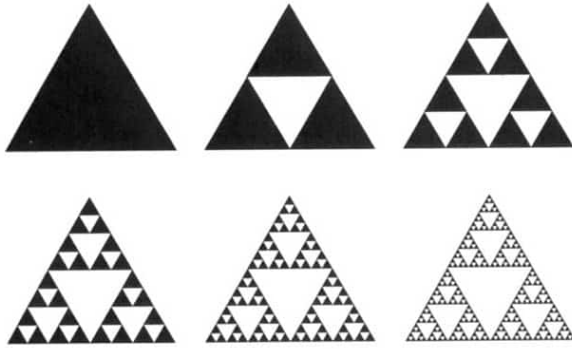


Figure 3: Sierpinski gasket obtained by iteration of f_s on a filled triangle.

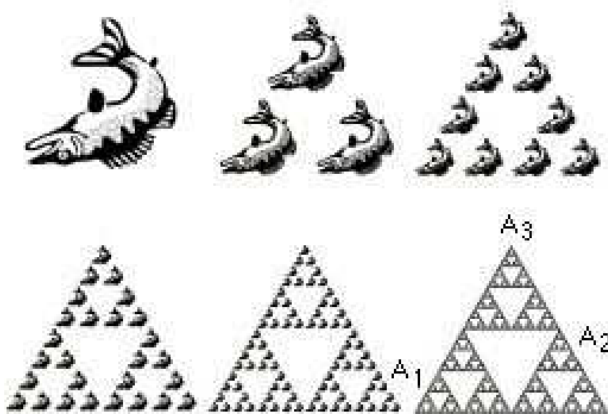


Figure 4: Sierpinski sieve obtained by iteration of a f_s on a fish... (from Barnsley et al. (2003))

Here u_i is the fact that figure x_i at level $\#i$ is $s : u_i = 'X_i(@) = x_i'$. The flow function is f_s and $X_i = f_s \circ X_{i+1}$. The metaphysical laws state that there exists at least a figure (a compact set of points) and regiment the way (reflected by the flow function f_s) each figure is composed of three shrunk copies of the figure at the next level.

Here the laws alone determine both the shape of the figures in our series ($x_1 = x_2 = x_3 = \dots = x_n = \dots$ is the Sierpinsky gasket s) and their existence. It thus determines the whole series u_i .

Indeed it can be shown that f_s is contractive²³ and we can delimit our world so that it is

²³The canonical, euclidian metric of the plane is not defined for figures (compact sets of points) but only

bounded (we can specify that f_s is only defined on a bounded portion of space including the triangle abc). This means that flow function f_s satisfies (CS1) (contractive character) and (CS2) (boundedness).

The fact that our series is complete is also connected to a very peculiar property of f_s : whatever figure x we start with, the iteration of f_s on x will always yield the same figure. x can be a triangle (figure 3) a filled square, or even a fish (figure 4), the iteration of f_s on x will always yield the Sierpinski gasket s at the limit. The reader can check that this peculiar property is in fact equivalent to the satisfaction of (CN1) and (CN2).

Using CN1, CN2 and (CS1-CS2), we can construct other examples of complete and incomplete non-wellfounded explanations. These might help us understand better what the difference between them amounts to. In the appendix B, I put forward simpler (if less graphic), unidimensional versions of the above rep-tiles and fractal world : the Zeno world, and the Cantor Set World.

7.3 What about circular chains of ground?

Our examples of complete non-wellfounded chains of grounds involve infinite chains. Could we modify them, as we have modified the Stick-Stretchers example, to put forward an example of a circular chain? Formally, this is not particularly problematic. The problem is rather to make metaphysical sense of the formal model – we have no simple ground-theoretic analog of time-travel to make sense of circular metaphysical explanations.

Nolan (2018) does try to make sense of something like a circular version of our Sierpinsky Gasket by describing a world in which “what appears to be our entire universe is just a sub-atomic particle in a larger universe, which is but a sub-atomic particle in a yet larger “universe”, and so on” but where if you “go up through enough stages (...) you will arrive back at one of our sub-atomic particles”. Even though, I find the scenario conceivable myself, I must say that most people I have met – and a couple of referees for this journal – don’t.

There are in any case simpler, and less controversial (if less graphic) ways to construct complete circular chains of grounds. Consider:

The No-Yes-Yes sentences.

- (1) “(2) is not true”
- (2) “(1) is true and (2) is true”

The semantic value of (1) is grounded on that of (2) which is grounded on that of (1) and (2). So we have a circular (non-uniform) chain of grounds. Here u_1

for points, so in order to rigorously show that f_s is contractive we need to introduce a distance on compact sets. This is typically done using Hausdorff distance (Falconer, 2004).

is the fact that the semantic value of sentence (1) is true and u_2 is the fact that the semantic value of sentence (2) is false (see below).

Classical logic and the naïve T-schema²⁴ show that (2) must be false, and that (1) must be true. Indeed, if (2) is true, by one conditional T-out of the naïve T-schema, (1) is true, which means by T-out again that (2) is not true. This implies that (2) must be untrue, and, by the other conditional T-in of the naïve T-schema, that (1) must be true. So the fact that (and the way) the semantic values of (1) and (2) are grounded on each other determine their semantic values.²⁵ We have an example of a *complete but circular chain of grounds*.

Note that we can put that reasoning in functional terms to match the other cases of grounding chains presented in this article. The semantic value [1] of (1) is such that $[1] = 1 - [2]$, and $[2] = \min([1], [2])$ so $[2] = \min(1 - [2], [2])$. The function that associates to a semantic value x the semantic value $\min(1 - x, x)$ has only one fixed point, however, which is 0. This implies that (2) is false and (1) true. The fact that all orbits of the function $\min(1 - x, x)$ (all series of that result from the iteration of that function) converge to 0 moreover implies that the second version of the completeness condition is satisfied.²⁶

8 Supercomplete explanations and the extra-property objection again

In order to deny the significance of complete non-wellfounded chains of grounds, one might try to downplay the contrast between my examples of complete and incomplete chains of explanation cases by claiming that their comparison is not totally fair.

Consider for instance the contrast between my rep4-tiles case and my Sierpinski fractal case. For one thing, we could add “degrees of freedom” to the Sierpinski gasket case so

²⁴The naïve T-schema says that “p” is true entails p (T-out) and that p entails that “p” is true (T-in), where “p” is replaced by an arbitrary sentence. This naïve T-schema is notorious for giving rise to semantic paradoxes when conjoined with classical logic and the existence of certain sentences such as the liar-sentence ‘this sentence is false’. One way to solve such paradoxes, once popular, consists in brutally restricting the naïve T-schema to prevent self-referential truth-talk. Since the work of Kripke (1975), it is widely held that such an approach is too costly.

²⁵In other words, we are in a case in which condition (B) follows from (A).

²⁶This example is a non-paradoxical and non-hypodoxical variant of the truth-teller hypodox and the no-no paradox (cf. Billon (2019)). The reader can check that this example can also be modified very simply to yield an *infinite* (and partly circular) complete chain of grounds:

- (1) “(2) is not true”
- (2) “(1) is true and (3) is true”
- (3) “(4) is not true”
- (4) “(3) is true and (5) is true”
- (.) ...

that the chain of grounds becomes incomplete. Suppose, for example, that s is red but that our world allows for the possibility of blue and green figures. The color of s , unlike its shape, would not be determined by the infinite chain of grounds. The fact that s_1 was obtained by the iteration of f_s would indeed leave its color totally open. The latter would not be determined and it would not be explained. Accordingly, the chain of ground would be incomplete.

Conversely, we could modify the rep-4 tile case so as to determine certain features that were not determined by our description of the case. For example, we might specify that our world contains only equilateral triangles and accordingly, that f_d only ranges over such triangles. The fact that our initial figure is an equilateral triangle (but, it should be emphasized, not the size of this triangle) would thus be determined and explained by our infinite chain of grounds. Similarly, we could specify that there exists at least one figure in the world, as we did in the fractal case: the fact that there exists a figure (though not its shape) would then be determined and explained by our series.

The upshot is that in both examples, *the description of the case presupposes what can vary and needs to be grounded and what is fixed by the (more or less implicit) laws regimenting our example* (where these laws are understood broadly enough to include “structural features” of our cases, such as a specification of the possible entities it involves). But these presuppositions can be called into question, and what they presuppose (certain laws) might itself call for an explanation.

This is a fair point. In answer, we might concede that the morals of the fractal and rep-tiles examples is somehow modest. Indeed, it is only that:

- *some features* (here, for example, the shape)
- can be explained by *certain infinite or circular chains of grounds but not by others*,
- and that these features would not be explained by the corresponding finite, wellfounded chains of ground (unless they start with an element that is self-grounded or, maybe, autonomous).

In other words, we can my conclusion would only be that *infinity or circularity (non-wellfoundedness) can do some explanatory work*. The fact that a figure s^* results from 12 iterations of f_s does not determine the shape of this figure or of its successors (it only determines that s^* will loosely resemble a Sierpinski gasket). The fact that it results from an infinity of iterations does determine its shape. Far from being an obstacle to good explanations, as suggested by the quotations of Fine and Schaffer, infinity and circularity can do genuine explanatory work, but they will only do so in cases in cases where the completeness condition is satisfied.²⁷

²⁷Morganti (2015) distinguishes between the transmission model and the emergence model of being and argues that the prejudice against infinite chains of grounds stems from a neglect of the emergence model:

In any case, it should be emphasized that granting, as I have just done, that the right conclusion is just that non-wellfoundedness can do some explanatory work, and that non-wellfounded explanations might only yield complete explanations in cases where the laws themselves are in need of explanation, is not as concessive as it might seem. For one thing, many people believe that (what we consider as) the causal or even metaphysical laws of the world call for some explanations: fundamental physics tries to explain and unify the acknowledged laws, some metaphysicians ask for grounds of grounds (Litland, 2017). For another, once it is granted that infinity or circularity can do some explanatory work, one could start wondering whether the features that are not determined and explained by the chain of explanations itself, and that we “hold fixed” by putting them in the laws L implicit in the description of the case (e.g. that shapes have no colors in the fractal case) could themselves be explained completely by another non-wellfounded chain of explanations. Who knows, the law L might be explained by some further law \mathcal{L}_1 which might be explained by \mathcal{L}_2 , etc. And this chain of explanations might be complete. For sure, this chain of explanations will presuppose what we might call meta-laws ML , but they might likewise be explained by a complete, infinite chain of explanations... And once it is understood that infinite chains of explanations can be complete, and can, more broadly, be explanatorily productive, infinite regresses should not scare us anymore — not even if we are looking for a complete explanation (where nothing but the laws call for explanation) or an ultimate explanation (where not even the laws call for explanation).²⁸

9 Equilibrium explanations and essentialist explanations

I have argued that non-wellfounded chains of explanations are somehow on a par with non-wellfounded chains of explanations with regard to completeness: the first can be complete

The “transmission model” of being, whereby the being of an entity at a given level of reality L_n is fully obtained, in a yes/no, all-or-nothing fashion, from the entity or entities at the immediately prior level L_{n-1} .

(...) According to the emergence model of being, then, the metaphysical structure of priority and dependence gives rise to a dynamics analogous to that underlying the convergent [Zeno/geometric] series $1/2 + 1/4 + 1/8...$ which converges towards 1 as n approaches infinity (and never becomes higher than 1) (560-2).

However, if I understand him correctly, Morganti fails to draw the the relevant distinction between cases where infinity does and where infinity does not do any explanatory work. Indeed, some examples of infinite chains of grounds seem to fit perfectly the emergence model but are incomplete nonetheless: this is the case, notably, of the Rep-4 Tiles World and of the Zeno World.

²⁸Let us call “weirdly” explained a fact that is explained in a case where $(u_i)_i$ is non-wellfounded but that would not be explained if $(u_i)_i$ were wellfounded. As we have just seen, the fact that the first figure of the series s is a Sierpinski gasket inscribed in abc is weirdly explained in the fractal case, not the fact that there exists a figure. An interesting question is whether some facts cannot, because of their very nature, be weirdly explained by any kind of chain of explanations. If there were such facts, they could only be explained by being put explicitly in the laws. Many philosophers exposed to the arguments in this article have suggested that the fact that there exists something could not be weirdly explained.

if the flow function meets certain conditions (such as being bounded and contractive), and the second can be complete if they start with a self-explained item (in the sense of an item x such that x or a proper part of x fully explains x) or an autonomous item (in the sense that it does not call for an explanation). One might even think that they have a decisive advantage. Indeed, self-grounded or autonomous items, maybe in part because they have been often invoked by Theists, are sometimes considered spooky or supernatural, but complete non-wellfounded explanations seem to be just as kosher as the bounded and contractive flow functions underlying them. In this section, I would like to show that complete non-wellfounded explanations are indeed unproblematic by answering an important objection against that claim and showing that we already appeal to them ordinarily.

We have seen in section 5.1 that all complete non-wellfounded explanations $(u_i)_i$ will display a form of “historical irrelevance” or “insensitivity to prior items”: u_1 will not depend on the successors that explain it. (ED1) moreover entails that if our non-wellfounded chain of explanation is complete, the series $(u_i)_i$ is determined by the series of laws $(L_i)_i$. Assuming, as we have, that determination is functional, this, in turn, entails that if our chain of explanation is uniform (and for all i , $L_i = L_1$), then all x_i will all be equal to x_1 and all u_i will be of the form ‘ $x_i = x_1$ ’. We can check that this is what happened in all our examples of complete non-wellfounded chains of explanations (except for the No-Yes-Yes sentence which is not uniform).

Now it might be wondered if such explanations, in which everything is determined by the laws alone rather than by prior items, are acceptable. I believe they are totally OK. One reason is that we do in fact commonly use such explanations. Below I consider two rather common types of explanations that display “insensitivity to prior items” and in which the explanandum is determined by the laws alone: strict equilibrium explanations and essentialist explanations.

Consider, in the case of causal explanations, the so-called equilibrium explanations. The statistician Ronald Fisher explained why the sex ratio of males and females is approximately one by the fact that any deviation from this ratio would be progressively canceled by natural selection. This is a classical equilibrium explanation and it displays, like our complete infinite and circular explanations, a form of Historical Irrelevance: to the extent that this explanation is correct, the sex ratio should always have been approximately one, and one can deduce that it is approximately one today without inquiring about its former values. Consider, to take another example, a lead ball in a closed bowl submitted to the law of gravity. One can explain why, after some time, the ball rests at the bottom of the bowl by the fact that it is the only equilibrium of the system.

More formally, an equilibrium explanation is an explanation of the state of a dynamical system i.e. of a system whose state is described by a point Y in a geometrical space that depends functionally on a variable X , usually temporal: $Y = f(X)$. An equilibrium

explanation explains the present state of the system by the fact that this state is an equilibrium of the system, and that the present state is the result of the iteration of f on a given initial state x . The series $(f^i(x))_i$ is called the orbit of x . The equilibria of a dynamical system are determined by the explanatory laws. They are fixed points of f .

In many cases, equilibrium explanations are partial or elliptical. Sometimes, for example, we just state that the system is in state e because it is an equilibrium, but there are multiple equilibria of the system in which the system could end up being as well, or there is only one equilibrium e , but not all orbits $(f^i(x))$ converge toward e , or else all orbits converge towards e but some converge so slowly that the system could fail to be even close to the equilibrium even after a huge amount of time. We can call “strict” an equilibrium explanation in which the system has only one equilibrium e and all orbits $(f^i(x))$ converge toward e , and “supers-strict”, one in which f is bounded and contractive. A super-strict equilibrium explanation, is intuitively, a strict equilibrium explanation whose orbits converge very quickly (geometrically). It seems that a strict equilibrium explanation in which the prior states of the system are infinitely many is a full explanation of why the system is in the equilibrium state e . Moreover, such a strict equilibrium explanation is an explanation in which the prior states of the system are irrelevant: it is an explanation by the laws alone. In fact, the reader can check that the function f regimenting the dynamical system, here, is exactly what we have called the flow function before, and that, formally speaking, all non-wellfounded complete chains of explanations can be understood as such strict equilibrium explanations with an infinite number of prior states.²⁹ Even though classical equilibrium explanations are causal rather than metaphysical, we might thus consider complete infinite chains of grounds as a ground-theoretic version of equilibrium explanations.

Take essentialist explanations, now, such as the classical theist explanation that explains the existence of God by the fact that existence is part of His essence, or, lower on Earth, this essentialist explanation put forward by Kappes (2022, p. 444): the fact that either the sun is shining or it is not shining is explained by the essence of (classical) disjunction and negation. (Note that we do not need to commit to the precise essence of classical disjunction and negation to make the claim that they explain such a fact.)³⁰ Essentialist explanations are, or at least can be, perfectly kosher. It also seems that they can be understood, at least sometimes, as explanations by the laws alone: in the classical logic example, we might say that the explanation relies on some laws of logic that are part of what define disjunction and negation, i.e. are essential to them.

²⁹There is a close connection between equilibrium explanations and optimality explanations, i.e. explanations, often found in biology or in certain interpretations of physics (cf. the Maupertuis-Leibniz interpretation of classical mechanics and (geometrical) optics), that explain the state of a system by the fact that it is in some sense optimal. This comes from the fact, exploited by optimization algorithms, that the optima of a (regular enough) function are the fixed point of a certain flow function, and in the good cases, the unique fixed point of a certain flow function towards which all orbits converge.

³⁰Kappes (2022) calls explanations by the laws alone “empty-base explanations” and provides many other interesting examples of such explanations.

We can conclude that there is at least one rather ordinary and unproblematic explanations by the laws alone, and that there might even be two: equilibrium explanations and essentialist explanations.³¹

10 How simple are non-wellfounded complete explanations?

In the introduction, I mentioned the fact that foes of non-wellfounded chain of ground sometimes argue that they are explanatorily defective because they are incomplete. I have argued that they are wrong to suppose that non-wellfounded explanations need be incomplete.

There is, however, another, a weaker version of the “explanatorily defective objection” against infinite chains of grounds. Instead of the principle of sufficient reason or one of its cognates, the latter invokes theory-choice considerations such as unity or simplicity and concludes that even though they are strictly speaking possible, infinite chains of grounds simply do not occur in the actual world. Thus, says Cameron (2008):

It would be better to be able to give a common metaphysical explanation for every dependent entity [every item in the chain that is grounded on another one]. We can do that only if every dependent entity has its ultimate onto-logical basis in some collection of independent entities; so this provides reason to believe the intuition against infinite descent in metaphysical explanation (Cameron, 2008, 12).

Interestingly, the examples we have used to answer the stronger PSR-based objection against non-wellfounded chains of ground allow us to dismiss this objection against infinite chains of ground. For in all our examples of complete explanations, we have a simple explanation “for all dependent entities”: it involves a simple structural feature of the chain of ground, namely the fact that its flow function f satisfies the second version of the completeness condition (see §5.4), and in all but the Wheel-Turners case, the even simpler fact that f is bounded and contractive.

It might not be trivial to compare two explanations for their theoretical virtues (a point rightly emphasized Bliss and Priest (2018) in response to Cameron), but I think Cameron’s point rests on the following comparison. Consider an ascending chain of grounds that starts with a foundational element v_1 which explains v_2 , which explains v_3 ... Such a wellfounded chain provides a simple explanation because, even if we have an infinity of items, the

³¹An interesting question, which I will not have the time to address here is whether some essentialist explanations can be analyzed as (ground-theoretic analogues of) equilibrium explanations. I raise this question because the connection between equilibrium explanations and optimality explanations noted in fn.29 suggests a fascinating (if speculative) possibility, namely that teleological essentialist explanations found in certain broadly Aristotelian or Leibnizian metaphysics might be underwritten by complete non-wellfounded chains of explanations.

infinite chain can so to speak be factorized: v_1 explains all the following items. By contrast, suggests Cameron, a descending infinite chain such as the one we have considered, where u_1 is grounded on u_2 which is grounded on u_3 , etc., cannot be factorized because there is no Ur-item on which all the others are grounded. So such a descending infinite explanation, concludes Cameron, must necessarily be complex. What is wrong with Cameron's argument is that he supposes that the only way to factorize or simplify an infinite (descending or ascending) chain of explanation involves a foundational item. This is wrong: in all the complete infinite cases we have considered the descending infinite chains can be so to speak factorized if we invoke the fact that the laws (and in the uniform cases the law) suffice to explain all the items.³²

11 Conclusion

Most philosophers assume that non-wellfounded explanations are either impossible, non-existent, or at least incomplete or complex. Friends of non-wellfounded explanations usually accept that they cannot be complete, but argue that this should not be counted against them. I have argued that non-wellfounded chains of explanations, be they causal or metaphysical, can be complete and simple, and indeed perfectly satisfying and *not* defective. The examples I have provided in support of that claim also show, I hope, that such explanations are also perfectly possible. Those who want a complete explanation of the world need not restrict their attention to foundationalist explanations starting with a self-explanatory or

autonomous item. They can – in fact, they should – consider non-wellfounded explanations very seriously.

References

- Bader, R. M. (2017). The grounding argument against non-reductive moral realism. *Oxford Studies in Metaethics*, 12.
- Barnsley, M., Hutchinson, J. E., and Stenflo, Ö. (2003). V-variable fractals and superfractals. *arXiv preprint math/0312314*.
- Billon, A. (2019). Paradoxical hypodoxes. *Synthese*, 196:5205–5229.

³²In his latest book, Cameron (2022) grants that there *are* non-wellfounded chains of ontological dependence or grounds but he argues that non-wellfounded chains of ontological dependence cannot be explanatory and that non-wellfounded chains of grounds are not normally explanatory (ch.3). He relies, to that effect, on the quite unorthodox claim that grounding is not tied to metaphysical explanation. According to him, metaphysical explanation is indeed tied to understanding in a way that grounding is not. I do not have the room to discuss his view and his arguments here in any detail. I just want to mention that all my examples of infinite chains of grounds seem perfectly explanatory to me, and that they do seem to provide a better understanding of the grounded items.

- Billon, A. (2021). Are infinite explanations self-explanatory? *Erkenntnis*, 88(5):1935–1954.
- Bliss, R. (2014). Viciousness and circles of ground. *Metaphilosophy*, 45(2):245–256.
- Bliss, R. (2019). What work the fundamental? *Erkenntnis*, 84(2):359–379.
- Bliss, R. and Priest, G. (2018). *Reality and its Structure: Essays in Fundamentality*. Oxford, UK: Oxford University Press.
- Bliss, R. L. (2013). Viciousness and the structure of reality. *Philosophical Studies*, 166(2):399–418.
- Cameron, R. P. (2008). Turtles all the way down: Regress, priority and fundamentality. *Philosophical Quarterly*, 58(230):1–14.
- Cameron, R. P. (2022). *Chains of Being: Infinite Regress, Circularity, and Metaphysical Explanation*. Oxford University Press.
- Chalmers, D. J. (2022). *Reality+: Virtual worlds and the problems of philosophy*. Penguin UK.
- Dasgupta, S. (2016). Metaphysical rationalism. *Nous*, 50(2):379–418.
- Falconer, K. (2004). *Fractal geometry: mathematical foundations and applications*. John Wiley & Sons.
- Fine, K. (2010). Some puzzles of ground. *Notre Dame Journal of Formal Logic*, 51(1):97–118.
- Fine, K. (2012). Guide to ground. In Correia, F. and Schnieder, B., editors, *Metaphysical Grounding*, pages 37–80. Cambridge University Press.
- Gardner, M. (2001). *The colossal book of mathematics: classic puzzles, paradoxes, and problems: number theory, algebra, geometry, probability, topology, game theory, infinity, and other topics of recreational mathematics*. WW Norton & Company.
- Glazier, M. (2016). Laws and the completeness of the fundamental. In Jago, M., editor, *Reality Making*, pages 11–37. Oxford, UK: Oxford University Press.
- Hawking, S. and Mlodinow, L. (2010). *The Grand Design*. Bantam Books.
- Hume, D. (1907). *Dialogues concerning natural religion*. William Blackwood.
- Kappes, Y. (2022). Self-explanation and empty-base explanation. *Journal of the American Philosophical Association*, 8(3):436–453.
- Kappes, Y. (2023). Grounding ground and the (in-)escapable ill-foundedness of the inclusive "explains". *Dialectica (this issue)*, x(x):xxx–xxx.

- Kment, B. (2014). *Modality and Explanatory Reasoning*. Oxford University Press.
- Kripke, S. (1975). Outline of a theory of truth. *Journal of philosophy*, 72(19):690–716.
- Leibniz, G. W. (1989). On the radical origination of things. In *Philosophical Papers and Letters*, pages 486–491. Springer.
- Leslie, J. (1979). *Value and existence*. Roman and Littlefield, Totawa, NJ.
- Leslie, J. (2010). *Infinite Minds*. Lexington Books, New York.
- Litland, J. (2017). Grounding grounding. *Oxford Studies in Metaphysics*, 10.
- Menzies, P. and Beebe, H. (2020). Counterfactual Theories of Causation. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Winter 2020 edition.
- Miller, B. (1996). *A most unlikely God: a philosophical enquiry into the nature of God*. University of Notre Dame Press Notre Dame.
- Morganti, M. (2015). Dependence, justification and explanation: Must reality be well-founded? *Erkenntnis*, 80(3):555–572.
- Nolan, D. (2018). Cosmic loops. In Bliss, R. and Priest, G., editors, *Reality and Its Structure*, pages 91–106. Oxford, UK: Oxford University Press.
- Nozick, R. (1981). *Philosophical Explanations, Cambridge*. Harvard University Press.
- Oberle, T. (2022). No work for fundamental facts. *Philosophical Quarterly*, 73(4):983–1003.
- Parfit, D. (2011). *On what matters*. Oxford Univ Press, New York.
- Priest, G. (2014). *One: Being an Investigation into the Unity of Reality and of its Parts, including the Singular Object which is Nothingness*. OUP Oxford.
- Rosen, G. (2017). Ground by law. *Philosophical Issues*, 27(1):279–301.
- Rowe, W. L. (1970). Two criticisms of the cosmological argument. *The Monist*, 54(3):441–459.
- Sallows, L. (2014). More on self-tiling tile sets. *Mathematics Magazine*, 87(2):100–112.
- Schaffer, J. (2010). Monism: The priority of the whole. *Philosophical Review*, 119(1):31–76.
- Schaffer, J. (2016). Grounding in the image of causation. *Philosophical Studies*, 173(1):49–100.
- Schaffer, J. (2017). Laws for metaphysical explanation. *Philosophical Issues*, 27(1):302–321.

Schilpp, P. A. (1949). *The Library of Living Philosophers, Volume 7. Albert Einstein: Philosopher-Scientist*. Open Court.

Simpson, A. (2017). Cyclic arithmetic is equivalent to peano arithmetic. In *International Conference on Foundations of Software Science and Computation Structures*, pages 283–300. Springer.

Simsek, M. (2023). Determination relations and metaphysical explanations. *Dialectica (this issue)*, x(x):xxx–xxx.

Steinhart, E. (2014). *Your digital afterlives: Computational theories of life after death*. Springer.

A From the second completeness condition to (CS1-CS2) and (CN1-CN2)

Let us use the symbol “Im” for the image of a function. Let us also call:

$$\Pi_i = \text{Im } f_i \cap \text{Im}(f_i \circ f_{i+1}) \cap \text{Im}(f_i \circ f_{i+1} \circ f_{i+2}) \cap \dots \cap (f_i \circ f_{i+1} \circ f_{i+2} \circ \dots \circ f_{i+j}) \cap \dots$$

Our completeness condition will be satisfied iff for all i , Π_i is reduced to a singleton whose member is the only value X_i can take.

It can be checked that if the flow is uniform ($L_i = L$ and $f_i = f$ do not depend on i),

$$\Pi_i = \text{Im } f \cap \text{Im } f^2 \cap \text{Im } f^3 \cap \dots \cap \text{Im } f^j \dots = \Pi$$

does not depend on i and the completeness condition is simply that Π is a singleton.

- **Completeness condition (uniform case, third version).** There is e such that

$$\Pi = \text{Im } f \cap \text{Im } f^2 \cap \text{Im } f^3 \cap \dots \cap \text{Im } f^j \dots = \{e\}$$

As for all i , $\text{Im } f^{1+i} \subset \text{Im } f$, this condition can be simplified

- **Completeness condition (uniform case, fourth version).** There is e such that $\text{Im } f^i$ converges towards $\{e\}$.

We can now show that if the completeness condition is satisfied (CN1) will be satisfied too. If $x \in \Pi$ and the completeness condition (take the uniform case, first version) is satisfied, there is y such that $x = f(y)$. But as $(x =)f(y) \in \Pi$ entails $y \in \Pi$ too, this means that if Π is a singleton and $x \in \Pi$ then x is a fixed point of f : $x = f(x)$. Conversely, if e is a fixed point of f , by the completeness condition (take the first version again) it belongs to Π . So if Π is a singleton f has a unique fixed point e and $\Pi = \{e\}$.

Similarly, it can be checked that if (CN2) failed $\text{Im } f^i$ could not converge toward $\{e\}$ and so the completeness condition (take the uniform case, second version) would not be satisfied.

Conversely if f is bounded by m and contractive, it can be shown that $|f^n(x) - f^n(y)| \leq k^{n-1} * 2m$, which implies that f has a unique fixed point e and that all “orbits” $(f^i(x))_i$ converge towards e and, more importantly, that $\Pi = \{e\}$ (this is a variant of the Banach-Picard fixed point theorem). So (CS1-CS2) are jointly sufficient for completeness.

B The Zeno world and the Cantor set world

Here is a unidimensional version of the Rep-4 Tiles World: the Zeno world. Let f_h be the dichotomic function that associates to an interval $[a, b]$ its first tile $[a, \frac{b-a}{2}]$. Let us call f_z its inverse. $x_1 = [a, b]$ is composed of two copies of $x_2 = f_h([a, b]) (= [a, \frac{b-a}{2}])$. Which is composed of two copied of $x_3 = f_h(x_2)$...

The Zeno World. The fact that the interval at level i (two copies of which the interval at the preceding level is composed) is x_i is grounded on the fact that the interval at the level $i + 1$ is x_{i+1} , where $x_{i+1} = f_h(x_i)$, that is $x_i = f_z(x_{i+1})$. Here u_i is the fact that interval at level $\#i$ is $x_i : u_i = 'X_i(@) = x_i'$, the flow function is f_z and $X_i = f_z \circ X_{i+1}$.

Here again, the first fact is grounded on the second which is likewise grounded on the third, etc., but that does not determine the first fact. It leaves completely open what the first interval is: it could very well be $[0, 2]$ or $[0, 17]$... More deeply, the flow function f_z has no fixed point at all, so it does not even satisfy (CN1).

We can likewise put forward a simpler (albeit less graphic) one-dimensional version of the Sierpinski Gasket. This one is known as the standard Cantor Set. Let f_{ca} and f_{cb} be functions on compact set of real numbers which associate to a set the image of this set by $g_{ca}(x) = \frac{x}{3}$ and $g_{cb} = \frac{2}{3} + \frac{x}{3}$ respectively. The Cantor set can be obtained by iteratively applying the shrinking (factor 1/3) and duplicating function $f_c = f_{ca} \cup f_{cb}$ to a any compact set of real numbers. To fix the ideas, f_c associates $[0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$ to $[0, 1]$ and associates $[0, \frac{1}{9}] \cup [\frac{2}{9}, \frac{1}{3}] \cup [\frac{2}{3}, \frac{7}{9}] \cup [\frac{8}{9}, 1]$ to $[0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$, etc.

The Cantor Set World. At level 1, there is a set x_1 which is composed of two shrunk copies (scale 1/3) of the set x_2 at level 2 (in the sense that $x_1 = f_c(x_2)$), the figure at level 2 is itself composed of of two shrunk copies (scale 1/3) of the figure at level 3 (in the sense that $x_2 = f_c(x_3)$)... The figure at level $\#1$, x_1 is the Cantor set s , which (fact) is grounded on the fact that the set at level $\#2$ is x_2 , which is grounded, on the fact that the figure at level $\#3$ is x_3 , etc.

Here u_i is the fact that figure at level $\#i$ is $x_i : u_i = 'X_i(@) = x_i'$, the flow function is f_c and $X_i = f_c \circ X_{i+1}$. The metaphysical laws state that there exists

at least a compact set and regiment the way (reflected by the flow function f_c) each set is composed of two shrunk copies of the set at the next level.

Here again (CS1) (contractive character) is satisfied and we can define the domain of f_c so that (CS2) (boundedness) is satisfied as well. The infinite chain of grounds is complete.