Chaos Theory.
A Quick Immersion

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Robert C. Bishop

CHAOS THEORY

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Advanced Praise

"Books on chaos theory tend to fall one of two ways. The first are those that cannot resist the hype. The exotic sound of "chaos," like "artificial intelligence," invites authors to lean into its mysteries. The second group of books is overly technical and beyond the reach of most readers. Physicists and mathematicians especially find it hard not to include all sorts of arcane detail. This book avoids both pitfalls. Robert Bishop deftly takes the reader through each of the key topics raised by chaos, including its historical roots, and shows why so many disciplines take note. Mathematicians provide fractal geometry as a tool. Computer scientists reveal how chaos puts limits on numerical simulations. Physical scientists have discovered that chaos is ubiquitous in nature. And philosophers explore its implications for a range of longstanding questions. With one foot in physics and another in the philosophy of science, Bishop illustrates each using minimal mathematics with important technical terms explained throughout. There is no better introduction to the topic available and no scholar better suited to the task."

Jeffrey Koperski, Professor of Philosophy, Saginaw Valley State University

"Sensitivity to initial conditions and path dependence, nonlinearities, strange attractors, and fractals…Bishop explains – in enviably clear language – the surprising properties and emergence of chaotic dynamics in mathematical models and real-world systems, and the differences between the two. The last chapter, on the limits chaos places on knowledge and predictability, provides insights on how individuals as well as scientists can work within those limits while at the same time embracing the remarkable wisdom they offer. Highly recommended."

Alicia Juarrero, author of Context Changes Everything: How Constraints Create Coherence (MIT Press)

"It won't come as a surprise to anyone who has been following Professor Bishop's work over the years on this topic that this book represents the best primer on chaos theory and nonlinear dynamics in existence today."

Michael Silberstein, Professor of Philosophy, Elizabethtown College

"Chaos Theory: A Quick Immersion is a clear and engaging introduction to chaos theory. Assuming no prior knowledge, and using helpful analogies and examples from everyday life, it familiarizes readers with key concepts and findings of this fascinating field. The discussion is readily accessible to those new to the topic, yet without skirting over important nuances. In addition to surveying conceptual foundations, Chaos Theory nicely illustrates how ideas and tools from the mathematical study of chaos have been applied in science – in weather forecasting, ecology, physiology, physics, and more. Along the way, it calls attention to oft-overlooked challenges involved in relating mathematical and computer models to the physical world and emphasizes the importance of recognizing and navigating limits when seeking knowledge. This delightful little book will be useful to a wide range of readers interested in understanding what chaos theory is and how its insights can make a difference in science."

Wendy Parker, Professor of Philosophy, Virginia Polytechnic University

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Acknowledgments

I have benefited greatly from conversations over the years with Harald Atmanspacher, Fred Kronz, Tim Palmer, and Lenny Smith on the topics of chaos and nonlinear dynamics. Although formally distinct from my Stanford Encyclopedia of Philosophy article on chaos, it is hard to pull apart the influence of past writings on such a fascinating topic when revisiting it with fresh eyes.

Introduction

Chaos is a word with many meanings in our everyday world. We often feel we experience chaos when our day is jumbled and disorganized, when things keep happening that throw us off balance. We may feel that achieving our goals for the day were blocked by the unforeseen events that threw off our plans. Then there is the Marvel universe enemy organization, Chaos, which funds armies and countries to lead the world down a pathway to a particular future. Here, disorganization is deliberately sown to achieve a plan. Or we often use the word chaos as synonymous with randomness, as lawlessness, a complete lack of order or pattern.

By contrast, there is chaos as mathematicians and scientists talk about: small changes producing surprisingly large effects that appear unpredictable yet exhibit an exquisite kind of order. The **butterfly effect** illustrates this extreme sensitivity to tiny changes. The term originates from the idea that the flapping of a butterfly's wings in Brazil could cause a tornado in Texas three weeks later.

You might be thinking, "So what? We've always known that small changes can have big effects. What's special about this mathematical sense of chaos?" For instance, on July 7th, 2005, because of a delay of just a few seconds, a friend and her family missed boarding one of the London Underground trains attacked by a terrorist bombing. Like the organization Chaos, such terrorist attacks are intended to sow chaos and fear by violently disrupting ordinary life.

Understanding what is distinctive about the phenomenon and properties of chaos as mathematicians and scientists characterize and study it is what this book is about. The mathematical sense of chaos—often called **chaotic dynamics**—looks random at first glance but represents a surprising kind of order. This book will help you understand this surprising kind of order and how sensitivity to the smallest of changes has revolutionized how scientists think about the behavior of our world.

Randomness

Before we begin that journey, it is important to clear up one confusion about randomness or random behavior. In everyday talk, our tendency is to use the word random to mean a lawless or completely unordered behavior. Scientists never use the term random to mean this for an important reason: There are no examples of lawless disorder in any of the physical phenomena we study.

Confusion arises because systems behaving randomly appear to lack any order when we're watching their behavior. Scientists call this **apparent randomness** when a system looks random to us but has an underlying deterministic order to it. Think of a roulette wheel. The outcome of each spin with the ball landing in a particular numbered slot looks like there is no order. Yet suppose we were able to know the speed of the wheel's spinning, the initial velocity of the ball as it enters the wheel, the friction slowing the wheel down, the friction the ball experiences as it rolls around the wheel and eventually bounces into a slot, among several other factors. Given these factors, the final slot the ball settles in is fully determined. We might not be able to calculate this due to the many factors involved and the limits on our knowledge, but there is an underlying order to the system determining where the ball will land. It appears random to us because we cannot track all the factors involved. Nevertheless, the ball's behavior is fully determined in an ordered way.

There is a second form of randomness scientists study known as **irreducible randomness**. When the full set of physical conditions determine the probability for outcomes, but not the specific outcome in a system at a particular time, it is irreducibly random. Nonetheless, the irreducible randomness of these outcomes still conforms to fixed probabilities. These probabilities are constrained by statistical laws rather than deterministic laws. This means irreducible randomness is a different form of order than the deterministic order we experience with mechanical systems such as engines and computers. It definitely is not lawless chaos.

An example of irreducible randomness is radioactive decay. All the relevant factors in a sample of a radioactive element, such as uranium, will not determine when any specific nucleus in the sample will undergo a decay event. Nevertheless, the sample will behave as described by a statistical law constraining how many nuclei will decay on average during a given time interval. Scientists make use of such irreducible randomness all the time in medical treatments for cancer and in nuclear power plants.

Over the course of this book, you will see how these two forms of scientific randomness intersect with chaotic dynamics. Moreover, you will see that perhaps the most important lesson of chaos is that of limits, the limits of what can be forecast and the wisdom of working within these limits.

With this brief introduction, our journey begins.

Chapter 1

Sensitive Dependence

I began approaching chaotic dynamics with the idea of very small changes now producing very large effects in the future. The illustration of butterfly wing flaps in Brazil causing a tornado in Texas three weeks later illustrates **amplification**—a small change rapidly growing into a large effect. A butterfly wing flap disturbs a small number of air molecules. For this tiny disturbance to amplify into a tornado thousands of miles away suggests an exquisite sensitivity to such minute disturbances. Scientists call this **sensitive dependence**, a property of a system such that the smallest of changes now can rapidly produce very large effects in the system's future behavior.

But how sensitive are we talking about? Consider a car at a stop sign. When it is your turn to go, if you press the accelerator a little bit, the car inches forward slowly. Press the accelerator a little more and the car increases speed a little more. The small changes in the accelerator do not lead to rapid increases in the car's speed. This is not the sensitive dependence characteristic of chaos.

Consider another example. Suppose a perfectly symmetric cone is precisely balanced on its tip with only the force of gravity acting on it. Absent any other influences, the cone would maintain this unstable equilibrium forever. In the actual world, the perfect balance is unstable because the smallest nudge from an air molecule colliding with the cone will cause it to tip over. However, the cone could tip over in any direction due to the slight differences in various perturbations arising from suffering different collisions with different molecules.

This example illustrates that variations in the slightest of causes produce dramatically different effects. If we plotted the tipping over of the unstable cone, we would see that from a small ball of potential starting conditions representing the variations in how the air molecules strike the cone (apparent randomness), several different directions for the cone's falling issue forth from this small ball of uncertainty. Dramatic, indeed, but not what scientists typically mean by the sensitive dependence of chaotic dynamics.