Directly depicting granular ontologies

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Abstract. We propose an ontological theory that is powerful enough to describe both complex spatio-temporal processes and the enduring entities that participate in such processes. For this purpose we distinguish between ontologies and meta-ontology.

Ontologies are based on very simple directly depicting languages and fall into two major categories: ontologies of type SPAN and ontologies of type SNAP. These represent two complementary perspectives on reality and result in distinct though compatible systems of categories. In a SNAP (snapshot) ontology we have the enduring entities in a given domain as they exist to be inventoried at some given moment of time. In a SPAN ontology we have perduring entities such as processes and their parts and aggregates.

We argue that both kinds of ontology are required, together with the meta-ontology which joins them together. On the level of meta-ontology we are able to impose constraints on ontologies of a sort which can support efficient processing of large amounts of data.

1 Introduction

Ontologies have been recognized to be of importance in almost all parts of computer science and engineering. They are critical at least for the Semantic Web [FvHH+00], for data exchange among information systems [Gua98], and for communication between software agents [MIG00]. An important aspect of the use of ontologies in such contexts is that they need to be represented by means of formalisms that guarantee certain nice computability properties. Usually description logics are used as representation formalisms, since they are held to provide the optimal compromise between expressive power and efficiency of the underlying reasoning [BHS03].

However, given the vast amount of information on the web one might argue that even more efficient reasoning techniques are required, i.e., that even more sacrifices at the level of expressive power should be made, for example by representing ontologies as simple diagrams or lists of things that exist or as simple finite trees. That such simplified representations can be useful is shown by the wide-spread use of maps in geography or of simple category trees in philosophy and biology. More recent representation of similar kind are simple relational databases.

A map is a specific, simplified and therefore highly efficient representation of the ontology of a certain part of geographic space. It is an ontology because it is an inventory of things that exist in a certain part of the world and of some of the properties and relations between them. From a logical perspective a map can be considered as a vast conjunction of sentences in a language that gives us the facilities to express propositions. Such a language is *directly depicting* [Deg78,SM82,Smi92]. This means that all the terms in such a language correspond to entities and the sentences depict, in effect, arrangements of such entities.

Obviously, directly depicting languages are, due to their limited expressiveness, not suitable for the expression of complex statements about ontological structures. We argue here that for this purpose we need to distinguish

- ontologies, which are formulated in conformity with the principles of a directly depicting language, and
- *meta*- ontology, which draws on greater expressive resources and expresses properties of and relationships between (directly depicting) ontologies.

We hereby draw on ideas first set out in [Gre03].

Ontologies themselves, because they use only directly depicting languages and are very simple and efficient. Meta ontologies are more complex. They provide contexts for ontologies and establish relationships between them. Consider, again, the case of a map. Here meta-level information is attached in the form of scale, legend, and method of projection. If you use the map then you pay attention to these matters (which might be rather complex) only perishably. You are focussed rather on what is depicted on the map itself, on estimating distances, making decisions as to which path to take, and so on.

All phenomena in reality are in one or other respect dynamic in nature. They are processes or are subject to change. Moreover, most phenomena in reality fall into one or the other of two disjoint classes: endurants, on the one hand, and perdurants, on the other. Endurants are entities which exist in full in every instant at which they exist at all. Perdurants unfold themselves over time in successive temporal parts or phases. This dichotomy needs to be reflected in our treatment of ontology. In this paper we distinguish two different kinds of ontologies:

- SNAP – for snapshot – ontologies which represent enduring entities as they exist at a certain moment of time;
- SPAN ontologies which represent perduring entities (such as processes) from what we can think of as a a god's-eye perspective.

In both cases we have ontologies employing very simple directly depicting languages.

Obviously, there are relationships between processes and the enduring entities which participate in them. Consider the process of *John kissing Mary*. The lips of John and Mary move towards each other and eventually meet before becoming once more separated. This process depends on the enduring entities John and Mary. To establish and characterize these kinds of relationship and to give a formal characterization is the job of meta ontology.

It is the purpose of this paper to formally define what we mean by ontologies and to show how they are related to each other when viewed on the meta-level.

2 Formal ontology of endurants and perdurants

In this section we provide that part of the meta-ontology which deals with spatial and spatio-temporal particulars. Here we focus only on specific aspects. More comprehensive approaches can be found in [ANC$^+$] and [Grept].

2.1 Mereology

Basic tool for formalizing meta ontology is Mereology [Sim87]. We assume a first order logic with identity. We use $x \leq y$ in order to signify that x is a part of y. In terms of parthood we can define the relations proper part $<$, overlap O, and underlap U, where x is a proper part of y iff (if and only if) it is a part of y but not identical to y ($D_<$). x and y overlap iff they share a part (D_O) and they underlap iff there exists a z which contains both as parts (D_U).

The part-of relation is governed by the axioms of reflexivity (M1), antisymmetry (M2), transitivity (M3), and the strong supplementation principle (M4) which says that if x is not a part of y then there exists a z which is part of x and which does not overlap y. Here and in all that follows we omit leading universal quantifiers.

($D_<$) $x < y \equiv x \leq y \wedge \neg(x = y)$
(D_O) $O\ xy \equiv (\exists z)(z \leq x \wedge z \leq y)$
(D_U) $O\ xy \equiv (\exists z)(x \leq z \wedge y \leq z)$

(M1) $x \leq x$
(M2) $x \leq y \wedge y \leq x \rightarrow x = y$
(M3) $x \leq y \wedge y \leq z \rightarrow P\ xz$
(M4) $\neg x \leq y \rightarrow (\exists z)(z \leq x \wedge \neg O\ yz)$

(M1-M3) make the part-of relation a partial ordering. The role of M4 is twofold: (i) it ensures that every entity has at least two non-overlapping proper parts (T1); (ii) it ensures that entities which overlap the same entities are identical (T2). This is the so called extensional principle.

$$T1 \qquad x < y \rightarrow (\exists z)(z < y \wedge \neg O\ xz)$$
$$T2 \qquad x = y \leftrightarrow (z)(O\ zx \leftrightarrow O\ zy)$$

The theory formed by M1-M4 is somewhat non-standard in the sense that there is no axiom that ensures that for any collection of entities there is an entity that is the sum formed by these entities. We omit this axiom since we need to deal with categories of entities which do not form joint sums. For an extended version of the present paper which uses a form of layered mereology [Don03] see [BS03a].

2.2 Entities and regions

We distinguish two disjoint categories of things: entities and regions. Both have a mereological structure. This means that in both domains we have a part-of relation satisfying M1-M4. At the formal level we now distinguish variables $x, y, z, c, g, d, x_1, x_2, \ldots$ for entities and variables $r, s, r_1, r_2, \ldots$ for regions. Which part-of relation is referred to will be obvious from the type of variables.

The relationship between entities and regions is characterized by the relation of location. $L\,xr$ holds if and only if the entity x is exactly located in region r. For example, you are exactly located, at any given moment of your life, in a certain region of space. Your life, on the other hand, is exactly located in a certain region of space-time. We state that if an entity is exactly located in some region then it is exactly located in a single region (L1) and we demand that if the entity x is a part of the entity y and x is exactly located in r and y is exactly located in s and r and s underlap then r is a part of s (L2). This ensures that parthood is monotonic with respect to location. (The need for the underlap in the antecedent of L2 will become clear in the next subsection.) For an extended discussion of the notion of location and its axiomatization see [CV95].

(L1) $(L\,xr \wedge L\,xs) \rightarrow x = y$
(L2) $x \leq y \wedge L\,xr \wedge L\,ys \wedge U\,rs \rightarrow r \leq s$

D_{CE} $CE\,x \equiv (\exists r) L\,xr$
D_{AE} $AE\,x \equiv \neg CE\,x$
(T3) $\neg(AE\,x \wedge CE\,x)$

We now introduce definitions to capture the distinction between abstract and concrete entities: A concrete entity (D_{CE}) is an entity that is located at some region. Examples of concrete entities are you, your dog, a table, your life, World War II. An abstract entity (D_{AE}) is an entity that is not located at some region . Examples are numbers, propositions, ontologies. It follows that no entity can be abstract as well as concrete (T3).

2.3 Space, time, and spacetime

We now introduce the primitive constants $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n$ which are interpreted as three dimensional space at different time indexes, and the constant $\mathcal{ST}$ which is interpreted as four dimensional spacetime. We demand that spaces with different time indexes do not underlap (ST1) and that non of the spaces underlaps spacetime (ST2). It follows immediately from the definitions of overlap and underlap that spaces with different time indexes do not have parts in common – they do not overlap (T4). They form disjoint *layers* in spacetime in the sense of [Don03]. It also follows that spacetime does not overlap any of the the time-indexed spaces (T5). Again spacetime and time-indexed spaces form different layers. It goes beyond the scope of the current paper to go into the details of this layered structure. We refer the reader to [BS03a].

We define a spatial region, $SR\,r$, as a region which is part of some time-indexed space (DSR) and a spatio-temporal region $STR\,s$, as a part of space (DSTR). It immediately follows that time-indexed spaces are spatial regions (T6) and that spacetime is a spatio-temporal region (T7). We then can prove that no region can be a spatial and a spatio-temporal region (T8), that every part of a spatial region is a spatial region (T9), and that every part of a spatio-temporal region is a spatio-temporal regions (T10).

(ST1) $U\,\mathcal{S}_i\mathcal{S}_j \rightarrow \mathcal{S}_i = \mathcal{S}_j$
(ST2) $\bigvee_{1\leq i\leq n} \neg U\,\mathcal{ST}\mathcal{S}_i$
(T4) $\neg(\mathcal{S}_i = \mathcal{S}_j) \rightarrow \neg O\,\mathcal{S}_i\mathcal{S}_j$
(T5) $\bigvee_{1\leq i\leq n} \neg O\,\mathcal{ST}\mathcal{S}_i$
(DSR) $SR\,r \equiv \bigvee_{1\leq i\leq n} r \leq \mathcal{S}_i$
(DSTR) $STR\,r \equiv r \leq \mathcal{ST}$

(T6) $\bigwedge_{1\leq i\leq n} SR\,\mathcal{S}_i$
(T7) $STR\,\mathcal{ST}$
(T8) $\neg(STR\,r \wedge SR\,r)$
(T9) $(SR\,r \wedge s \leq r) \rightarrow SR\,s$
(T10) $(STR\,r \wedge s \leq r) \rightarrow STR\,s$

T8 might be slightly controversial since it implies that a spatial slice of four dimensional space is a spatio-temporal region in the sense of (DSTR) rather then a spatial region in the sense of (DSR). This, however, is a consequence of the layered structure in which regions can *coincide* without sharing parts. T8 tells us that being a spatio-temporal region is not a matter of dimensions. Again, for details see [BS03a].

2.4 Endurants and Perdurants

We say that x is a *spatial* part of y if and only if x is a part of y and both are located at underlapping spatial regions r and s (DSP). The underlap of r and s here ensures that x and y are located at spatial regions which belong to spaces with identical time indexes. For example, your arm is a spatial part of you, Montana is a spatial part of the United States. These examples show that in our common use of language the fact that the parameters of the spatial-part-of relation are parts of the same time-indexed space is taken for granted.

Similarly we say that x is a *temporal* part of y if and only if x is a part of y and both are located at spatio-temporal regions (DTP)[1]. Your youth is a temporal part of your life, a soccer game has the first and the second half as parts.

(DSP) $SP\,xy \equiv x \leq y \wedge (\exists r)(\exists s)(SR\,r \wedge SR\,s \wedge U\,rs \wedge L\,xr \wedge L\,ys)$
(DTP) $TP\,xy \equiv x \leq y \wedge (\exists r)(\exists s)(STR\,r \wedge STR\,s \wedge L\,xr \wedge L\,ys)$
(T11) $\neg(SP\,xy \wedge TP\,xz)$

We then can prove that nothing can be a spatial part as well as a spatio-temporal part of some whole (T11).

We now define an endurant x as an entity which is a spatial part of itself (DEnd). Prototypical endurants are substances like you and me, your computer, planet Earth, etc. A perdurant is an entity x is a temporal part of itself (DPerd). Prototypical perdurants are processes like the process of your life, the flow of air in and out your lungs, the process of global warming, etc.

(DEnd) $End\,x \equiv SP\,xx$
(DPerd) $Perd\,x \equiv TP\,xx$

It follows trivially that no entity can be an endurant and a perdurant. Consider yourself and your life. You are an enduring entity and you located at a certain region within some time-indexed space at every moment in time. Your life, on the other hand is located at a region of spacetime.

There exist complex relations between endurants and perdurants: The process of your life *depends* on you – the endurant. On the other hand you *participate* in the process of your life. See [SG,PSng] for an extended discussion.

[1] Temporal parts are parts located in spacetime. From this perspective it would be more appropriate to call them spatio-temporal parts. In order to be consistent with the literature we call them temporal parts.

3 Ontologies

We now consider ontologies as subjects of study from a meta-ontological perspective. We write $\mathcal{O}\ \omega$ in order to signify that ω is an ontology. As already mentioned, specific representations of ontologies include maps, figures, lists of names, category trees, partonomies, etc. In this section however we abstract from specific forms of representation.

3.1 Ontologies as abstract entities

An ontology is an abstract entity (O1) and so are all its parts (O2). Therefore ontologies are disjoint from the domain of concrete entities. We introduce the notion of a constituent of an ontology, signified by the binary predicate Co. We also introduce the notion of ontological projection $\Pi\ xy$ in order to signify that the constituent x projects (ontologically) onto the entity y. Hereby the relation of ontological projection between a constituent x and target y is similar to the relation between the name 'Mount Everest' and the corresponding mountain. A constituent of an ontology ω is a part of ω which projects upon or refers to something (DCo) that is not itself a constituent of this ontology (O3). We require, finally, that no ontology is empty (O4).

(O1) $\mathcal{O}\ \omega \rightarrow AE\ \omega$

(O2) $(\mathcal{O}\ \omega \wedge x \leq \omega) \rightarrow AE\ x$

(DCo) $Co\ x\omega \equiv (\mathcal{O}\ \omega \wedge x \leq \omega \wedge (\exists y)\Pi\ xy)$

(O3) $(Co\ x\omega \wedge \Pi\ xy) \rightarrow \neg Co\ y\omega$

(O4) $\mathcal{O}\ \omega \rightarrow (\exists x) Co\ x\omega$

If an ontology is represented as a map, then the constituents of the ontology are represented as features on the map. Consider the map in the left part of Figure 1. Constituents of this ontology are abstract entities which project onto China, Australia, North America, South America, the Pacific Ocean, the relevant body of water, and the trade winds represented by the arrow.

Fig. 1. The El Niño phenomenon: Due to the weakening of the trade winds (blue arrows) the warm waters of the western Pacific (red regions) migrate eastward to the South American coast. (From the University of Illinois WW2010 Project.)

We now continue to consider ontologies as collections of abstract entities, their constituents, that have a particular projective relationship to external entities. We postpone the question about the exact nature of the projective relationship and the question about the structure of ontologies until Section 4. For the moment it will be sufficient to assume that such a projective relationship exists and that the constituents of ontologies are structured in an appropriate manner.

An ontology ω acknowledges an entity x if and only if there is some constituent of the ontology which projects onto x (DAckn).

$$\text{(DOAckn)} \qquad \mathit{Ackn}\, \omega x \equiv (\exists y)(Co\, y\omega \wedge \Pi\; yx)$$

Consider again the map in the left part of Figure 1. The ontology represented by this map acknowledges: China, Australia, North America, South America, the Pacific ocean, a certain body of water, and the trade winds. In the case of maps the ontology does not merely acknowledge the existence of these objects – it also represents their location.

3.2 SNAP and SPAN ontologies

A SNAP ontology is such that its constituents project onto entities which are located at parts of some time-indexed space (DSnap). Consequently, every SNAP ontology has a unique temporal index. A SPAN ontology is such that its constituents project onto things which are located at parts of spacetime (DSpan). It follows that no ontology is both of type SNAP and of type SPAN (T13).

$$\text{(DSnap)} \quad SNAP\; \omega\mathcal{S}_i \equiv \mathcal{O}\; \omega \wedge (\forall x)(\forall y)((Co\; x\omega \wedge \Pi\; xy) \rightarrow (\exists s)(s \leq \mathcal{S}_i \wedge L\; ys))$$
$$\text{(DSpan)} \quad SPAN\; \omega \equiv \mathcal{O}\; \omega \wedge (\forall x)(\forall y)((Co\; x\omega \wedge \Pi\; xy) \rightarrow (\exists s)(s \leq \mathcal{ST} \wedge L\; ys))$$
$$\text{(T13)} \quad \neg(\text{SNAP}\; \omega\mathcal{S}_i \wedge \text{SPAN}\; \omega)$$

Consider Figure 1. Each map represents a SNAP ontology with a specific time index. A SPAN ontology corresponding to this sequence of snapshots acknowledges processes like the weakening of the trade winds and the migration of the warm waters of the western Pacific eastward.

A SNAP entity is an entity which is acknowledged by some SNAP ontology (DSnapEnt) and a SPAN entity is an entity which is acknowledged by a SPAN ontology (DSpanEnt). We then can prove that SNAP entities are endurants (T15) and that SPAN entities are perdurants(T16). It also follows that SNAP entities and SPAN entities form disjoint domains (T14).

$$\text{(DSnapEnt)} \quad SnapEnt\; x \equiv (\exists\omega)((\textstyle\bigvee_{1 \leq i \leq n} SNAP\; \omega\mathcal{S}_i) \wedge \mathit{Ackn}\; \omega x)$$
$$\text{(DSpanEnt)} \quad SpanEnt\; x \equiv (\exists\omega)(SPAN\; \omega \wedge \mathit{Ackn}\; \omega x)$$
$$\text{(T14)} \quad \neg(SnapEnt\; x \wedge SpanEnt\; x)$$
$$\text{(T15)} \quad SnapEnt\; x \rightarrow End\; x$$
$$\text{(T16)} \quad SpanEnt\; x \rightarrow Perd\; x$$

Relationships between endurants and perdurants are mirrored by cross-ontological relationships between SNAP and SPAN entities. For example: (a) the process of weakening (a SPAN entity) depends of certain air masses moving in a certain uniform way in a certain direction. Those air masses are SNAP entities which themselves depend on a certain process of movement: (b) the process of migration (a SPAN entity) depends on a body of water (a SNAP entity) with certain qualities such as temperature (also a SNAP entity).

It is important to see that these cross-ontological relations belong to the meta-level and are thus outside the scope of ontologies themselves. This means that their complexity does not add to the complexity of the ontologies.

4 Directly depicting ontologies and their hierarchical structure

In the previous section we characterized constituents of an ontology as abstract entities which project onto something that is not a constituent of this ontology. In this section we concentrate on the structures formed by constituents of ontologies together with their projective relation to the entities in their target domains. We will show that ontologies form granular partitions in the sense of [BS03b].

The theory of granular partitions has two main components:

- Theory A governs the way constituents of ontologies (cells) are organized into nesting structures (the nested boxes in Figure 2).
- Theory B governs the way these cell-structures project onto reality indicated by the arrows connecting cells to portions of reality (the nested ellipses).

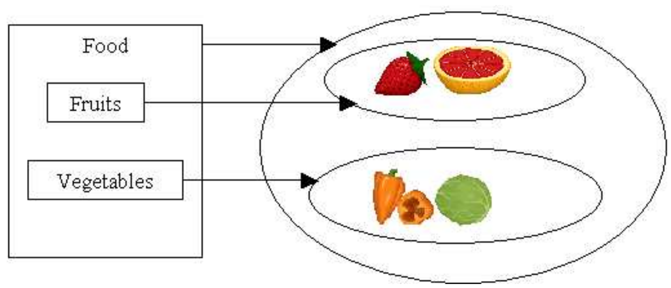

Fig. 2. Relationships between cells and entities (left)

We now focus on ontologies of individuals rather than on ontologies of classes of things. As a first example we will use a SNAP ontology whose constituents target parts of the body of some human being named Tom. Tom's body is subdivided into head, torso, and limbs, which are subdivided further into: arms, legs, and so on. The representation of this ontology as a tree is given in the left part of Figure 3 (an alternative way of representation is to use Venn-diagram like nested ovoids as in Figure 2). As a second example for the representation of a SNAP ontology we use a map of the subdivision of the United States into states, a part of which is shown in the right part of Figure 3.

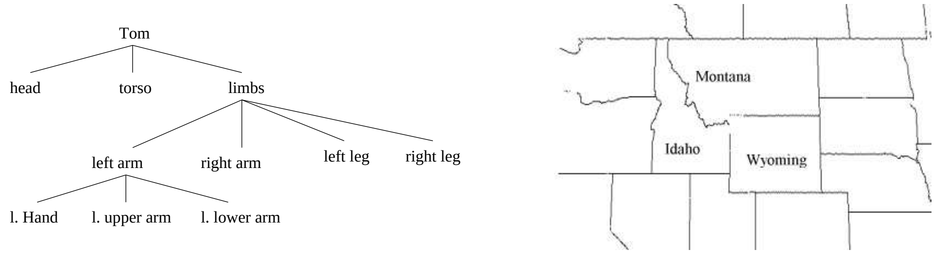

Fig. 3. Hierarchical subdivision of the human body (left) and a map of parts of the United States (right).

We now focus on two aspects of such ontologies: (a) the way their constituents form *hierarchical structures* like the ones we know from partonomies and category trees; and (b) the way their constituents *project* onto their target domains (i.e. onto the collection of entities targeted by the constituents of an ontology).

4.1 Hierarchical structure

At the abstract level we enforce the tree structure by defining a specific partial order among constituents of an ontology. Again, this corresponds to theory A in the framework of granular partitions.

We introduce a subcell relation $\sqsubseteq$ which holds among constituents of a single ontology (OA0). Consider the left part of Figure 3. Here the constituents are the nodes of the tree with their respective labels; the subcell relation holds wherever an edge connects such nodes.

Using the primitive subcell relation $\sqsubseteq$ we define the relations proper subcell (D$_{\subset}$), immediate proper subcell (D$_{\overline{\subseteq}}$), cell-overlap (D$_{OC}$) and predicates for the root cell (D$_{root}$) and for atoms (D$_{At}$):

D$_{\sqsubseteq}$	$c \sqsubset g \equiv c \sqsubseteq g \wedge \neg(c = g)$
D$_{\overline{\subseteq}}$	$c \overline{\subseteq} g \equiv c \sqsubset g \wedge \neg(\exists d)(c \sqsubset d \wedge d \sqsubset g)$
D$_{OC}$	$OC\ cg \equiv (\exists d)(d \sqsubseteq c \wedge d \sqsubseteq g)$
D$_{root}$	$\text{root}(c) \equiv (\forall g)(g \sqsubseteq c)$
D$_{At}$	$At\ c \equiv \neg(\exists g)(g \sqsubset c)$

The proper subcell relation $c \sqsubset g$ holds if c is a subcell of g but c and g are distinct entities. The constituent c is an immediate subcell of g if and only if c is a proper subcell of g and there is no proper subcell between them. $OC\ cg$ is the relation of overlap between constituents. The predicates *root* and *At* hold if the entity to which they are applied is the root of a tree structure or an atom, i.e., a constituent without proper subcells.

The subcell relation $\sqsubseteq$ is governed by the following axioms:

$$
\begin{array}{ll}
\text{(OA0)} & x \sqsubseteq y \rightarrow (Co\; x\omega \wedge Co\; y\omega) \\
\text{(OA1)} & c \sqsubseteq c \\
\text{(OA2)} & (c_1 \sqsubseteq c_2 \wedge c_2 \sqsubseteq c_1) \rightarrow c_1 = c_2 \\
\text{(OA3)} & (c_1 \sqsubseteq c_2 \wedge c_2 \sqsubseteq c_3) \rightarrow c_1 \sqsubseteq c_3 \\
\text{(OA4)} & (\exists c)\text{root}(c) \\
\text{(OA5)} & OC\; c_1 c_2 \rightarrow (c_1 \sqsubseteq c_2 \vee c_2 \sqsubseteq c_1) \\
\text{(OA6)} & c \subset g \rightarrow (\exists d)(d \subset g \wedge \neg OC\; cd) \qquad\qquad WSP \\
\text{(OA7)} & (\exists g)(At\; g \wedge g \sqsubseteq c) \\
\text{(OA8)} & \neg At\; d \rightarrow (\exists c_1, \ldots, c_n)((\bigwedge_{1 \leq i \leq n} c_i \subset d) \wedge \\
 & \qquad (g) g \subset d \rightarrow \bigvee_{1 \leq i \leq n} g = c_i))
\end{array}
$$

Here OA0 restricts the scope of $\sqsubseteq$ to constituents of a certain ontology ω. OA1-3 ensure that $\sqsubseteq$ is a partial ordering. In OA4 we demand that there is a root cell which has all constituents of a certain ontology as subcells. Using OA2 we can then prove that there exists exactly one root in every ontology. OA5 rules out the possibility of partial overlap of cells. From this it follows that there cannot occur any cycles and the resulting structure is a tree. OA6 rules out cases where a constituent has only a single proper subcell. OS6 is known in the literature as the weak supplementation principle and mirrors T1 on the mereological level. OA7 ensures that every cell has at least one atom as subcell. Finally OA8 is an axiom schema which ensures that every constituent is either an atom or has finitely many subcells.

Using OA1, OA5, and OA6 we then can prove that the strong supplementation principle (SSP) holds (T16). From SSP then immediately follows the extensionallity of overlap (T17).

$$
\begin{array}{ll}
\text{(T16)} & \neg(c \sqsubseteq g) \rightarrow (\exists d)(d \sqsubseteq c \wedge \neg OC\; dg) \\
\text{(T17)} & c = g \leftrightarrow (w)(OC\; wc \leftrightarrow OC\; wg)
\end{array}
$$

It follows that the organizational structure of ontologies is simpler than the underlying mereology of entities and regions – it is a finite tree structure – but it mirrors important structural properties such as partial ordering, weak and strong supplementation, and extensionallity.

Consider the right part of Figure 2. Here the hierarchy is rather flat. We have one root constituent – projecting onto the United States – and one constituent for every state. But still – it satisfies (OA1–8).

4.2 Projection onto reality

The projective relationship between constituents of an ontology and the entities in its target domain is complex. In the context of this paper we focus on ontologies with particularly well-defined projection relations. For a more general approach and an extensive discussion of the axioms below see [BS03b].

Intuitively, we can compare an ontology, ω, with a rig of spotlights projecting down onto an orchestra during the performance of a symphony. Each constituent of ω corresponds to some spotlight in the rig. Some constituents (spotlights) will project upon single players, others onto whole sections of the orchestra (string, wind, percussion, and so forth). One constituent (spotlight) will project upon the orchestra as a whole. Note that the spotlights do not hereby create the objects which they cast into relief. When once the rig has been set, and the members of the orchestra have taken their places, then it will be an entirely objective matter which objects (individuals and groups of individuals) are located in which illuminated cells.

Consider the left part of Figure 3. Here the projection is given by the obvious interpretation of the labels as depicting body parts. Consider the right part of Figure 2. Here the projection is such that the constituent labeled 'Montana' projects onto the state of Montana, and so on.

From axiom (O3) we know that every constituent of an ontology projects onto something that is not a constituent of this ontology. We then demand that the projection relation Π is a mapping (OB1) which is one-one (OB2), i.e., every constituent projects onto one entity in the target domain and each entity in the target domain is targeted by at most one constituent. (OB1) rules out that an ontology has a single constituent projecting at the same time onto the Republic of China and the People's Republic of China as if they were one single object. (OB2) rules out ontologies with distinct constituents projecting on the same entity (for example one projecting onto the morning star another projecting on the evening star). One can easily verify that (OB1) and (OB2) are satisfied in the case of map representations of ontologies.

Corresponding to Π there is a converse relation $\overline{\Pi}$ holding between entities and constituents (OB3). [BS03b] call this relation location. Here we just call it the converse of the relation of projection in order to keep it separate from the notion of spatial location (L1-2). Based on OB1-3 we can define the functional counterparts of π and $\overline{\pi}$ (D_π and $\mathrm{D}_{\overline{\pi}}$). (OB4) makes π a total function and (OB5) ensures that π preserves the ordering structure.

D_π	$x = \pi\ c \equiv \Pi\ cx$	(OB2)	$(\Pi\ xy \wedge \Pi\ zy) \rightarrow x = z$
$\mathrm{D}_{\overline{\pi}}$	$c = \overline{\pi}\ x \equiv \overline{\Pi}\ xc$	(OB4)	$(\exists x)(\Pi\ cx)$
(OB1)	$(\Pi\ xy \wedge \Pi\ xz) \rightarrow y = z$	(OB5)	$c_1 \subseteq c_2 \leftrightarrow (\pi c_1) \leq (\pi c_2)$

Consider Figure 4 and let the z_i range over constituents and the o_i range over targeted entities. Configuration (a) satisfies (OB5). However configuration (c) is ruled out by the left-to-right direction of (OB5) and configuration (b) is ruled out by the right-to-left direction of (OB5).

From D_π, $D_{\overline{\pi}}$, and (OB1-4) then immediately follows that π and $\overline{\pi}$ behave like inverse functions wherever $\overline{\pi}$ is defined (T18 and T19). Using OB3, OB5, D_π, $\mathrm{D}_{\overline{\pi}}$, and T18 we then can prove (T20) which tells us that π indeed is an order homomorphism. We then prove (T21) and (T22) using T16, OB5, and T20. This tells us that both π and $\overline{\pi}$ preserve the tree structure.

$T18$	$c = \overline{\pi}(\pi\ c)$
$T19$	$(\exists c)(\overline{\Pi} xc) \rightarrow x = \pi(\overline{\pi}\ x)$
$T20$	$((\exists c)(\Pi\ cx_1) \wedge (\exists c)(\Pi\ cx_2)) \rightarrow (x_1 \leq x_2 \leftrightarrow \overline{\pi}\ x_1 \subset \overline{\pi}\ x_2)$
$T21$	$\neg c_1 \subset c_2 \rightarrow (\exists z)(z \leq (\pi\ c_1) \wedge \neg O\ z(\pi\ c_2))$
$T22$	$((\exists c)(\Pi\ cx_1) \wedge (\exists c)(\Pi\ cx_2)) \rightarrow$ $\{\neg x_1 \leq x_2 \rightarrow (\exists d)(d \subset (\overline{\pi}\ x_1) \wedge \neg O\ d(\overline{\pi}\ x_2))\}$

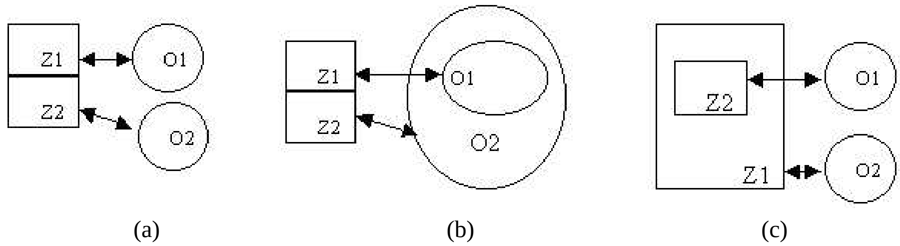

Fig. 4. Ontologies preserve mereological structure: (OB5) rules out configuration (b) and (c).

It follows that if the left part of Figure 3 represents an ontology then we can trust that the partonomic structure of the human body is indeed the way it is depicted in the tree.

5 Meta-level relations between ontologies

In the previous sections we have seen that at the meta level we have a powerful language at our disposal in order to describe ontologies and the ways in which they relate to their targets in reality. This allowed us to formalize constraints on the structure of ontologies which support their representations using very simple and directly depicting languages. This means that all the terms (constituents) in such a language correspond (project onto) to entities, and that the sentences depict arrangements of such entities. Arrangement hereby is characterized in terms of conjunctions of statements involving the subcell relation.

The discussion in the previous section (in particular the axioms OA0-8, OB1-5 and the theorems T16-22) has shown that at the meta-level ontologies can be represented or modeled as (finite) tree-structure preserving mappings. In the remainder of this section we will use this fact and talk about ontologies in terms of the mathematical language of (finite) tree-structure preserving functions.

Above we have also seen that we can distinguish ontologies of type SNAP and SPAN, depending on whether the targeted entities are endurants which exist at a certain instant of time or perdurants. At the meta-level now the question about relations between distinct ontologies and their targeted entities arises. At a very coarse level we can distinguish relations that hold between ontologies of the same kind and relations between ontologies of different kind, i.e., relations of signature SNAP $\times$ SNAP, SPAN $\times$ SPAN, SNAP $\times$ SPAN, and SPAN $\times$ SNAP. At a finer level we can relations between SNAP ontologies with identical and with different time indexes, i.e., relations of signature $\text{SNAP}_{\mathcal{S}_i} \times \text{SNAP}_{\mathcal{S}_i}$ and $\text{SNAP}_{\mathcal{S}_i} \times \text{SNAP}_{\mathcal{S}_j}$. An analysis of relations between ontologies will be related to relations between entities acknowledged by those entities. For an analysis of the latter kinds of relations see [SG,PSng,PSng].

In the remainder of this paper we focus on a specific class of relations of signature $\text{SNAP}_{\mathcal{S}_i} \times \text{SNAP}_{\mathcal{S}_i}$ and SPAN $\times$ SPAN. The relations in focus are based the selective and granular character of ontologies. Our examples will focus on relations between SNAP ontologies but can be extended easily to relations between SPAN ontologies.

5.1 Refinement relations

Consider the SNAP ontologies ω_1 (left) and ω_2 (right) in Figure 5. Both project onto the individual Fred in the middle and one can see that the corresponding ontologies stand in a kind of refinement relation to each other. We will use the symbol $\preceq$ to refer to this relation and write $\omega_1 \preceq \omega_2$ to express the fact that the ontology ω_1 is a refined by the ontology ω_2.

Fig. 5. Refinement relations between the ontologies ω_1 (left) and ω_2 (right).

We give a formal account of the relation $\preceq$ as follows. First of all we represent ontologies on the meta-level as mappings of signature $\pi : \Omega \to \Delta$. Here π is the projection mapping introduced in (D$_\pi$). Ω is the collection of constituents of the ontology ω considered as a set and Δ is the set of entities targeted by the elements of Ω. The set Ω is structured by the relation $\sqsubseteq$ for which the axioms OA1-8 hold. The set Δ is structured by the relation $\leq$ for which the axioms $M1 - 4$ hold.

We then say that ω_1 is refined by ω_2, $\omega_1 \preceq \omega_2$, if and only if there exists a one-one and into mapping $f : \Omega_1 \to \Omega_2$ such that

(a) f is *order-preserving*, i.e., $z_i \subseteq z_j$ iff $f(z_i) \subseteq f(z_j)$,
(b) f is *target-preserving*, i.e., $\pi_1(z) = \pi_2(f(z))$.

The existence of the mapping f with its particular properties ensures that we can map constituents in ω_1 to constituents in ω_2 in such a way that: (a) If two constituents in $z_i, z_j \in \Omega_1$ are subcells of each other then so are their counterparts in $f(z_1), f(z_2) \in \Omega_2$ and vice versa; (b) The target $\pi_1(z)$ of the constituent $z \in \Omega_1$ is identical to the target $\pi_2(f(z))$ of its counterpart $f(z) \in \Omega_2$. In other words we demand that there exists an order and target-preserving mapping f such that the left diagram in Figure 6 commutes. From (a) and (b) it follows that $\Delta_1 \subseteq \Delta_2$.

We can show that the relation refined-by, $\preceq$, is reflexive (ref) and transitive (tr):

(ref) We have $\omega \preceq \omega$ since there exists an identity map, defined by $z = id(z)$, such that id is order-, and target-preserving.

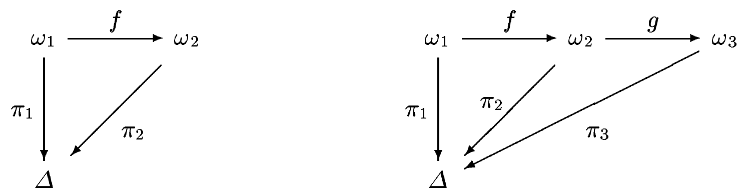

Fig. 6. Commutative diagram illustration for: (Left) the definition of $\preceq$; and (Right) the proof of transitivity of $\preceq$.

(tr) For transitivity we have to show that if $f : \Omega_1 \rightarrow \Omega_2$ and $g : \Omega_2 \rightarrow \Omega_3$ are order- , and target-preserving then so is their composition $g \circ s : \Omega_1 \rightarrow \Omega_3$. That this is the case can be seen in the right diagram in Figure 6.

5.2 Equivalence

We now continue by defining an equivalence relation on ontologies ω_1 and ω_2 as follows:

$$\text{D}_\sim \qquad \omega_1 \sim \omega_2 \equiv \omega_1 \preceq \omega_2 \ \wedge \ \omega_2 \preceq \omega_1 .$$

In other words, two ontologies are equivalent if and only if they stand in the refinement relation $\preceq$ to each other, i.e., there exist order and target preserving mappings f and f' with $f'(f(c)) = c$ between them.

The relation $\sim$ is an equivalence relation, i.e., reflexive, symmetric, and transitive. The reflexivity of $\sim$ follows immediately from the reflexivity of $\preceq$. The symmetry of $\sim$ follows from the commutativity of the conjunction in its definition. To see the transitivity of $\sim$ assume $\omega_1 \sim \omega_2$ and $\omega_2 \sim \omega_3$. Therefore we have $\omega_1 \preceq \omega_2$ and $\omega_2 \preceq \omega_1$ and similarly $\omega_2 \preceq \omega_3$ and $\omega_3 \preceq \omega_2$. From the transitivity of $\preceq$ it follows that we have $\omega_1 \preceq \omega_3$ and $\omega_3 \preceq \omega_1$ and hence $\omega_1 \sim \omega_3$. The corresponding set of equivalence classes is defined as

$$\text{D}_{[\omega]} \qquad [\omega] \equiv \{\omega_1 \mid \omega_1 \sim \omega\}$$

Consider Figure 7. Our intuition tells us that all three figures are distinct representations of the same ontology of type SNAP. All three are pairwise refinements of each other, i.e., equivalent in the sense of $\text{D}_\sim$.

Fig. 7. Distinct representations of the same ontology.

$D_\sim$ and $D_{[\omega]}$ help us to distinguish between an ontology and its (many) representations. As stated in (O1) an ontology is an abstract entity and is therefore not located in space and time (D_{AE}). Representations of ontologies like maps or lists are concrete entities and are therefore located in space and time (D_{CE}). However the underlying ontology and all representations are equivalent in the sense of $D_\sim$. Consequently, every ontology has a corresponding equivalence classes of representations. Examples of such equivalence classes are: The set of all current maps of the federal states of the United States – each representing the ontology of the US at a given point in time; and the set of periodic tables of the elements given in the different textbooks of chemistry – each representing the ontology of chemical elements at a given point in time.

5.3 The refinement ordering of ontologies

Given that ontologies are characterized by the equivalence classes of their representations then we can define a partial ordering, the refinement ordering between ontologies, in terms of these equivalence classes.

Consider equivalence classes $[\omega_1]$ and $[\omega_2]$. The relation $\preceq$ now induces a partial ordering $\ll$ as follows:

$$D_\ll \qquad [\omega_1] \ll [\omega_2] \equiv \omega_1 \preceq \omega_2$$

To show that $\ll$ is well defined, suppose $\omega_1 \preceq \omega_2$ and $x \in [\omega_1]$ and $y \in [\omega_2]$. Then $x \preceq \omega_1$, $\omega_1 \preceq \omega_2$, and $\omega_2 \preceq y$ and by transitivity $x \preceq y$.

The relation $\ll$ is a partial ordering. The reflexivity and transitivity of $\ll$ immediately follow from the reflexivity and transitivity of $\preceq$. It remains to show that $\ll$ is antisymmetric, i.e., that if $[\omega_1] \ll [\omega_2]$ and $[\omega_2] \ll [\omega_1]$ then $[\omega_1] = [\omega_2]$. Assume $[\omega_1] \ll [\omega_2]$ and $[\omega_2] \ll [\omega_1]$ holds. Assume $x \in [\omega_1]$. Then $x \preceq \omega_1$ and $\omega_1 \preceq \omega_2$ and by transitivity $x \preceq \omega_2$ and similarly $\omega_2 \preceq \omega_1$ and $\omega_1 \preceq x$ and by transitivity $\omega_2 \preceq x$. Therefore $\omega_2 \preceq x$ and $x \preceq \omega_2$ that is $x \in [\omega_2]$. The other direction from $x \in [\omega_2]$ to $x \in [\omega_1]$ is similar and omitted here. Together this yields $[\omega_1] = [\omega_2]$.

It follows that the relation $\ll$ is the refinement relation between ontologies. It abstracts from particular representations. $D_\ll$ then tells us that in order to determine whether or not one ontology is a refinement of another it is sufficent to look at one of their representations.

6 Conclusions

The theory outlined above contains the resources to describe both complex spatio-temporal processes and the enduring entities which participate therein. We argued that to deal with such phenomena we need a plurality of ontologies together with a meta-ontological framework to deal with the relations between them. Ontologies are based on directly depicting languages which are very simple and are therefore quite efficient from a computational point of view. From the constraints imposed on the structure of ontologies it follows that computation within ontologies can be reduced to computation in finite tree structures and there exists a wide variety of efficient algorithms for performing operations on such structures [BHS03]. Meta ontology is more complex and requires more expressive power. It provides the context for ontologies and for establishing relations between them.

We distinguished two major categories of ontologies: ontologies of type SPAN and ontologies of type SNAP. These ontologies represent orthogonal inventories of reality – one (SNAP) acknowledging enduring entities, and the other (SPAN) acknowledging perduring entities. We showed that the distinction between perduring and enduring entities itself needs to be established on the meta-level since it is outside the scope of ontologies themselves.

We also showed that the constraints on ontologies allow us to talk in a very efficient way on the meta-level about relations between ontologies. Thus we were able to use very simple mathematics in order to formalize refinement relations between ontologies of the same kinds.

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