

Mood-based temporal reference

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Abstract

Last time we introduced the notion of an *illocutionary perspective*. The basic idea is that the very act of speaking up introduces several discourse referents. The speech act itself (e_0) is introduced as the *central perspective point* ($\top\varepsilon$). In addition, all the *speech spheres* (p_0) where this speech act is realized, as well as the *worlds* of each sphere ($w_0 \in p_0$) are introduced as modal topics ($\top\Omega$ and $\top\omega$).

On this view, the *speech time* is the time of the speech act in a given speech sphere (i.e. the time t s.t. $\forall w \in p_0: t = \vartheta_w e_0$). The speech time is only one of the parameters that are relevant to temporal discourse reference. Other parameters include the *consequent state* of the speech act ($\triangleright e_0$), since speaking has consequences, as well as the world-dependent *culmination point* in worlds where the consequent state culminates ($(\blacktriangleright e_0)_w$). Moreover, if the speaker ($\uparrow e_0$) reports on a prior speech event, or issues a directive, they also include the *source event* ($\hat{\leftarrow} e_0$), or the world-dependent event that realizes this *directive* in the compliance worlds ($(\hat{\leftarrow} e_0)_w$).

English has a tense-based temporal system, which relates verbal eventualities to the speech time ($\vartheta_{\top\omega} \top\varepsilon$) and/or to other top-ranked times ($\top\tau$, $\perp\tau$, and/or $\vartheta_{\top\omega} \perp\varepsilon$) (Ch. 3 and 5). Mandarin has an aspect-based temporal system, which relates verbal eventualities to the speech event itself ($\top\varepsilon$) and/or to other top-ranked eventualities ($\perp\varepsilon$, $\top\sigma$, $\perp\sigma$). We now extend the story to the mood-based temporal system of Kalaallisut, which relates verbal eventualities to illocutionary perspectives. An event e_1 is *verifiable* in w from the perspective of e_0 iff e_1 is realized in w prior to e_0 (i.e. $\vartheta_w e_1 < \vartheta_w e_0$, hereafter abbreviated to $e_1 <_w e_0$). This relation crucially depends on the entire illocutionary perspective, which determines both the relevant perspectival event (e_0) and the candidate speech worlds ($w_0 \in p_0$).

To explicate this idea, UC_{τ^+} is combined with UC_{ω^+} into a system dubbed *Update with Centering* (UC, see Appendix). The following examples illustrate verifiability-based temporal reference in Kalaallisut and its representation in UC.

Outline

1. Non-future: Verifiability from speech act
2. Future: Verifiable attitudinal states
3. Reports: Verifiability from other speech acts
4. Quantification: Verifiable habits
5. Toward a CG.UC fragment of Kalaallisut

1 NON-FUTURE: VERIFIABILITY FROM SPEECH ACT

• DISCOURSE-INITIAL DECLARATIVES

- (1) *Ole* {*aallar-pu-q* | *suli-pu-q* | *ulapik-pu-q*}.
Ole {leave-DEC_T-3SG | work-DEC_T-3SG | busy-DEC_T-3SG}
Ole {has left | is working | is busy}.

Model for (1) Discourse-initial DEC with ε -anchored $\{v^e | v^e | v^s\}$ -base

<i>Discourse ref.</i>	<i>Symbol: Description</i>	<i>Temporal-modal condition</i>	<i>Source</i>
$\top w_0 \in \top p_0$			
•	$\top e_0$: $\uparrow e_0$ speaks to $\downarrow e_0$	$\exists t \forall w \in p_0: t = \vartheta_w e_0$	$st e_0$
a. •	e_1 : <i>Ole</i> leaves $\pi_{w_0} \blacktriangleleft e_1$	$e_0 \subseteq_{w_0} \triangleright e_1, e_1 <_{w_0} e_0$	v^e_{ε} -DEC
b. ••••	e_1 : <i>Ole</i> works	$e_0 \subseteq_{w_0} \nabla e_1, \blacktriangleright \nabla e_1 <_{w_0} e_0$	v^e_{ε} -DEC
c. ———	s_1 : <i>Ole</i> is busy	$e_0 \subseteq_{w_0} s_1, \blacktriangleright s_1 <_{w_0} e_0$	v^s_{ε} -DEC

- (1') *Ole*^T [_{DEC} ...
 $\top[x | x =_i ole]; \partial[spk.to_{T\Omega} \langle \top \varepsilon, \uparrow \top \varepsilon, \downarrow \top \varepsilon \rangle]; \dots$
 a. ... leave^e _{ε}]-DEC_T-3SG
 $[e | leave_{T\omega} \langle e, \top \delta, \pi_{T\omega} \blacktriangleleft e \rangle, \top \varepsilon \subseteq_{T\omega} \triangleright e]; [\perp \varepsilon <_{T\omega} \top \varepsilon]; \top[p | p =_I \top \omega]_{T\Omega}$
 b. ... work^e _{ε}]-DEC_T-3SG
 $[e | work_{T\omega} \langle e, \top \delta \rangle, \top \varepsilon \subseteq_{T\omega} \nabla e]; [\blacktriangleright \nabla \perp \varepsilon <_{T\omega} \top \varepsilon]; \top[p | p =_I \top \omega]_{T\Omega}$
 c. ... busy^s _{ε}]-DEC_T-3SG
 $[s | busy_{T\omega} \langle s, \top \delta \rangle, \top \varepsilon \subseteq_{T\omega} s]; [\blacktriangleright \perp \sigma <_{T\omega} \top \varepsilon]; \top[p | p =_I \top \omega]_{T\Omega}$

• TOPIC-SETTING TEMPORAL MODIFIERS

- (2) *Ole* {*ippassaq | *aqagu*} {*aallar-pu-q* | *suli-pu-q* | *ulapik-pu-q*}.
Ole {yesterday | *tomorr.} {lv-DEC_T-3SG | work-DEC_T-3SG | busy-DEC_T-3SG}
Ole {left | worked | was busy} {yesterday | *tomorrow}.

- (2') *Ole*^T [_{DEC} yesterday^T ...
 $\top[x | x =_i ole]; \partial[spk.to_{T\Omega} \langle \top \varepsilon, \uparrow \top \varepsilon, \downarrow \top \varepsilon \rangle]; ((([t | t =_i day.bfr \langle \vartheta_{T\omega} \top \varepsilon \rangle];$
 $\top[s | \uparrow s =_i \top \delta, s \subseteq_{T\omega} \perp \tau]) \top); \dots$
 a. ... leave^e _{σ}]-DEC_T-3SG
 $[e | leave_{T\omega} \langle e, \top \delta, \pi_{T\omega} \blacktriangleleft e \rangle, e \subseteq_{T\omega} \top \sigma]; [\perp \varepsilon <_{T\omega} \top \varepsilon]; \top[p | p =_I \top \omega]_{T\Omega}$
 b. ... work^e _{σ}]-DEC_T-3SG
 $[e | work_{T\omega} \langle e, \top \delta \rangle, \nabla e \subseteq_i \top \sigma]; [\blacktriangleright \nabla \perp \varepsilon <_{T\omega} \top \varepsilon]; \top[p | p =_I \top \omega]_{T\Omega}$
 c. ... busy^s _{σ}]-DEC_T-3SG
 $[s | busy_{T\omega} \langle s, \top \delta \rangle, s \subseteq_i \top \sigma]; [\blacktriangleright \perp \sigma <_{T\omega} \top \varepsilon]; \top[p | p =_I \top \omega]_{T\Omega}$

• DISCOURSE ABOUT VERIFIABLE FACTS

- (3) i. *Meeqqat ullumi sukkanniupput.*
miiraq-t ullumi sukkanniut-pu-t
 kid-PL today race.e.o.-DEC_T-3PL
 The kids had a dogsled race today.
- ii. *Ole ajugaagami nuannaarpoq.*
Ole ajugaa-ga-mi nuannaar-pu-q
 Ole win-FCT_T-3SG_T happy-DEC_T-3SG
 Ole^T won, so he_T was happy.

Model for (3i–ii) Topic-setting today (i), or FCT (ii), with comment by v_σ^e-DEC

		→ real time		
<u>Discourse ref.</u>	<u>Symbol: Description</u>		<u>Temporal-modal condition</u>	<u>Source</u>
$\top w_0 \in \top p'_0 \subseteq p_0$				
•	$\top e_0$: $\uparrow e_0$ speaks to $\downarrow e_0$		$\exists t \forall w \in p_0: t = \vartheta_w e_0$	$st e_0$
■■■■■■■■■■	t_1 : e_0 -day		$e_0 \subseteq_{w_0} t_1$	today
—	$\top s_1$: kids x_1 during t_1		$s_1 \subseteq_{w_0} t_1$	today
••	e_1 : kids x_1 race against $e.o.$		$\nabla e_1 \sqsubseteq_{\sigma} s_1, \blacktriangleright \nabla e_1 <_{w_0} e_0$	v_{σ}^e -DEC
~~~~~				
$w_2 \in r$				
•	$e_2$ : Ole wins $s_1$ -race		$e_2 =_{w_2} (\blacktriangleleft s_1)_{w_2}, e_2 <_{w_2} e_0$	$v^e$ -FCT
~~~~~				
$\top w_0 \in \top p''_0 \subseteq p'_0 \subseteq r$				
•	e_2 : Ole wins s_1 -race		$e_2 =_{w_0} (\blacktriangleleft s_1)_{w_0}, e_2 <_{w_0} e_0$	v^e -FCT
————	$\top s_2$: Ole fr. e_2 till $(\blacktriangleright e_2)_{w_0}$		$s_2 = (\circ e_2)_{w_0}$	FCT
—(—)	s'_2 : Ole is happy $s'_2 \sqsubseteq_{\sigma} s_2$		$\blacktriangleright s'_2 <_{w_0} e_0$	v_{σ}^s -DEC

- (3') i. kid-pl^T [_{DEC} today^T ...
 $\top[x | \mathcal{A}x \subseteq_i kid_{\top\omega} \mathbf{2}^+ \langle \mathcal{A}x \rangle]; \partial[spk.to_{\top\Omega} \langle \top\epsilon, \uparrow \top\epsilon, \downarrow \top\epsilon \rangle]; ((([t | day \langle t \rangle, \top\epsilon \sqsubseteq_{\top\omega} t]; \top[s | \uparrow s =_i \top\delta, s \sqsubseteq_{\top\omega} \perp \tau]) \top; \dots$
 ... race.e.o.^e]_σ-DEC_T-3PL
 $[e | race.e.o._{\top\omega} \langle e, \top\delta \rangle, \nabla e \sqsubseteq_i \top\sigma]; [\blacktriangleright \perp \epsilon <_{\top\omega} \top\epsilon]; \top[p | p =_I \top\omega]_{\top\Omega}]$
- ii. Ole^T [_{DEC} win^e-FCT_T-3SG_T ...
 $\top[x | x =_i ole]; \partial[spk.to_{\top\Omega} \langle \top\epsilon, \uparrow \top\epsilon, \downarrow \top\epsilon \rangle]; ((([e | win_w \langle e, \top\delta \rangle, e =_w (\blacktriangleleft \top\sigma)_w, e <_w \top\epsilon]; [\perp \epsilon =_I \mathbf{max} \{ \perp \epsilon | \perp_{\omega} \top\sigma \}]; [\top\Omega \subseteq_I \perp \omega]_{\top\sigma}]; \top[s | s =_i (\circ \perp \epsilon)_{\top\omega}]) \top; \dots$
 ... happy^s]_σ-DEC_T-3SG
 $[s | happy_{\top\omega} \langle s, \top\delta \rangle, s \sqsubseteq_i \top\sigma]); [\blacktriangleright \perp \sigma <_{\top\omega} \top\epsilon]; \top[p | p =_I \top\omega]_{\top\Omega}]$

2 FUTURE: VERIFIABLE ATTITUDINAL STATES

• REAL STATE OF INTENT & RELATED STATE OF EXPECTATION

(4) i. *Meeqqat aqagu sukkanniuniarput.*
miiraq-t aqagu sukkaniut-(niar)-pu-t*
 kid-PL tomorrow race.e.o.-intend^s_ε-DEC_T-3PL
 The kids are going to have a dogsled race tomorrow.

ii. *Ole nuannaassaaq.*
Ole nuannaar-ssa-pu-q
 Ole happy-expect(ed)^s_ε-DEC_T-3SG
 Ole^T will be happy.

Model for (4i–ii) Topic-setting tomorrow (i) with comment by v_σ-att^s

<u>Discourse ref.</u>	<u>Symbol: Description</u>	<u>Temp.-modal condition</u>	<u>Source</u>
$\top w_0 \in \top p''_0 \subseteq p'_0 \subseteq p_0$			
•	$\top e_0$: $\uparrow e_0$ speaks to $\downarrow e_0$	$\exists t \forall w \in p_0: t = \vartheta_w e_0$	${}^s e_0$
■ ■ ■ ■ ■	t_1 : e_0 -tomorrow	$t_1 = \llbracket \text{day.aft} \rrbracket (\vartheta_{w_0} e_0)$	tomorrow
—	s'_1 : kids x_1 (= $\uparrow s'_1$) intend q	$e_0 \subseteq_{w_0} s'_1, \blacktriangleright s'_1 <_{w_0} e_0$	int ^s _ε -DEC
—	s'_2 : of $p'_0, \uparrow s'_2$ expects q'	$e_0 \subseteq_{w_0} s'_2, \blacktriangleright s'_2 <_{w_0} e_0$	exp ^s _ε -DEC
~~~~~			
$w_1 \in q = \text{OPT}(p_0, \llbracket \text{int} \rrbracket (w_0, s'_1, x_1))$			
—	$\top s_1$ : kids $x_1$ during $t_1$	$s_1 \subseteq_{w_1} t_1$	tomorrow
● ●	$e_1$ : $\uparrow s'_1$ race e.o.	$\nabla e_1 \subseteq_{\sigma} s_1, \blacktriangle e_1 = (\blacktriangle s'_1)_{w_1}$	$v^e_{\sigma}$ -int ^s
~~~~~			
$w_1 \in q' = \text{OPT}(p'_0, \llbracket \text{exp} \rrbracket (w_0, s'_2, \uparrow s'_2))$			
—	s_2 : Ole is happy	$s_2 \subseteq_{\sigma} s_1, \blacktriangleright s_2 =_{w_1} (\blacktriangle s'_2)_{w_1}$	v^s_{σ} -exp ^s

(4') i. kid-PL [_{DEC} [-int tomorrow ...
 $\top [x | \mathcal{A}x \subseteq_i \text{kid}_{\top\omega}, \mathbf{2}^+ \langle \mathcal{A}x \rangle]; \partial[\text{spk.to}_{\top\Omega} \langle \top\epsilon, \uparrow \top\epsilon, \downarrow \top\epsilon \rangle]; ([w]; (([t | t =_i \text{day.aft} \langle \vartheta_{\top\omega} \top\epsilon \rangle]; \top [s | \uparrow s =_i \top\delta, s \subseteq_{\perp\omega} \perp\tau)) \top]; \dots$
 ... race.e.o._ε]-intend^s_ε]-DEC_T-3PL
 $[e | \text{race.e.o.}_{\perp\omega} \langle e, \uparrow e \rangle, \nabla e \subseteq_i \top\sigma]; [s | \blacktriangle \perp\epsilon =_i (\blacktriangle s)_{\perp\omega}]; [\text{OPT} \langle \top\Omega, \text{int}_{\top\omega} \perp\sigma \rangle \subseteq_I \perp\omega |_{\perp\sigma}]; [\uparrow \perp\sigma =_i \top\delta, \top\epsilon \subseteq_{\top\omega} \perp\sigma]; [\blacktriangleright \perp\sigma <_{\top\omega} \top\epsilon]; \top [p | p =_I \top\omega |_{\top\Omega}]$

ii. Ole^T [_{DEC} ...
 $\top [x | x =_i \text{ole}]; \partial[\text{spk.to}_{\top\Omega} \langle \top\epsilon, \uparrow \top\epsilon, \downarrow \top\epsilon \rangle]; \dots$
 ... happy^s_σ-expect(ed)^s_ε]-DEC_T-3SG
 $[s | \text{happy}_{\perp\omega} \langle s, \top\delta \rangle, s \subseteq_i \top\sigma]; [s | \blacktriangleright \perp\sigma =_{\perp\omega} (\blacktriangle s)_{\perp\omega}]; [\text{OPT} \langle \top\Omega, \text{exp}_{\top\omega} \perp\sigma \rangle \subseteq_I \perp\omega |_{\perp\sigma}]; [\top\epsilon \subseteq_{\top\omega} \perp\sigma]; [\blacktriangleright \perp\sigma <_{\top\omega} \top\epsilon]; \top [p | p =_I \top\omega |_{\top\Omega}]$

• REAL STATE OF INTENT & CONDITIONAL STATE OF EXPECTATION

(5) i. The kids are going to have a dogsled race tomorrow. (= (4i))

ii. *Ole ajugaa-gu-ni nuannaar-*(ssa)-pu-q*

Ole win-HYP_T-3SG_T happy-expect(ed)^s_ε-DEC_T-3SG

Ole^T will be happy.

Model for (5i–ii) Topic-setting tomorrow (i), or HYP (ii), with comment by v_σ-att^s

Discourse ref. Symbol: Description Temp.-modal condition Source

^Tw₀ ∈ ^Tp''₀ ⊆ p'₀ ⊆ p₀

•	^T e ₀ : ↑e ₀ speaks to ↓e ₀	∃t∀w ∈ p ₀ : t = ∂ _w e ₀	^s t e ₀
■■■■■	t ₁ : e ₀ -tomorrow	t ₁ = [[<i>day.aft</i>]](∂ _{w₀} e ₀)	tomorrow
———	s' ₁ : kids x ₁ (= ↑s' ₁) intend q	e ₀ ⊆ _{w₀} s' ₁ , ▲s' ₁ < _{w₀} e ₀	int ^s _ε -DEC
—————	s'' ₂ : of r ₂ , ↑s'' ₂ expects q'	e ₀ ⊆ _{w₀} s'' ₂ , ▲s'' ₂ < _{w₀} e ₀	exp ^s _ε -DEC

w₁ ∈ q = OPT(p₀, [[*int*]](w₀, s'₁, x₁))

———	^T s ₁ : kids x ₁ during t ₁	s ₁ ⊆ _{w₁} t ₁	tomorrow
●●	e ₁ : ↑s' ₁ race e.o.	∇e ₁ ⊆ _σ s ₁ , ▲e ₁ = (▲s' ₁) _{w₁}	v ^e _σ -int ^s

w₁ ∈ ^Tr₂ ⊆ p'₀ ∩ q

•	e ₂ : Ole wins s ₁ -race	e ₂ = _{w₁} (▲s ₁) _{w₁} , e ₁ < _{w₁} e ₂	v ^e _σ -HYP
———	^T s ₂ : O. from e ₂ till (▲▷e ₂) _{w₁}	s ₂ = (◊e ₂) _{w₁}	HYP

w₁ ∈ q' = OPT(r₂, [[*exp*]](w₀, s''₂, ↑s''₂))

———	s' ₂ : Ole is happy	s' ₂ ⊆ _σ s ₂ , ▲s' ₂ = _{w₁} (▲s'' ₂) _{w₁}	v ^s _σ -exp ^s
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(5') i. kid-PL [DEC [-int tomorrow ...

^T[x|^Ax ⊆_i kid_{Tω} 2⁺<^Ax>]; ∂[*spk.to*_{TΩ}<Tε, ↑Tε, ↓Tε>]; ([w]; ([[t| t=i
day.aft](∂_{Tω} Tε)]; ^T[s| ↑s =_i Tδ, s ⊆_{⊥ω} ⊥τ])^T; ...

... race.e.o._σ^e]-intend^s_ε]-DEC_T-3PL

[e| *race.e.o.*_{⊥ω}<e, ↑e>, ∇e ⊆_i Tσ]; [s| ▲⊥ε =_i (▲s)_{⊥ω}]; [OPT<TΩ, int_{Tω} ⊥σ>
⊆_I ⊥ω||_{⊥σ}]; [↑⊥σ =_i Tδ, Tε ⊆_{Tω} ⊥σ]; [▲⊥σ <_{Tω} Tε]; ^T[p| p =_I Tω||_{TΩ}]

ii. Ole^T [DEC win^e-HYP_T-3SG_T ...

^T[x| x =_i ole]; ∂[*spk.to*_{TΩ}<Tε, ↑Tε, ↓Tε>]; ([[e| win_{⊥ω}<e, Tδ>, e =_{⊥ω}
(▲Tσ)_{⊥ω} ⊥ε <_{⊥ω} e]; [⊥ε =_I max{⊥ε||_{⊥ω} Tσ}]; ^T[s| s =_i (◊⊥ε)_{⊥ω}];

[⊥ω ∈_I Tω||_{TΩ}]; ^T[p| p =_I ⊥ω||_{Tσ}])^T; ...

... happy^s_σ-expect(ed)^s_ε]-DEC_T-3SG

[s| *happy*_{⊥ω}<s, Tδ>, s ⊆_i Tσ]; [s| ▲⊥σ =_{⊥ω} (▲s)_{⊥ω}]; [OPT<TΩ, exp_{Tω} ⊥σ>
⊆_I ⊥ω||_{⊥σ}]; [Tε ⊆_{Tω} ⊥σ]); [▲⊥σ <_{Tω} Tε]; ^T[p| p =_I Tω||_{TΩ}]

3 REPORTS: VERIFIABILITY FROM OTHER SPEECH ACTS

- (6) *Ole* {a. *aallar-nirar-pu-q* | b. *suli-nirar-pu-q* | c. *ulapik-nirar-pu-q*}
Ole { leave-say-DEC_T-3SG | work-say-DEC_T-3SG | busy-say-DEC_T-3SG }
Ole^T said: “I {a. have left | b. am working | c. am busy}.”

Model for (6) Topic-setting Ole with comment by {a. v^e | b. v^e | c. v^s }-say^e

<i>Discourse ref.</i>	<i>Symbol: Description</i>	<i>Temporal-modal cond.</i>	<i>Source</i>
$\top w_0 \in \top p'_0 \subseteq p_0$			
•	$\top e_0$: $\uparrow e_0$ speaks to $\downarrow e_0$	$\exists t \forall w \in p_0: t = \vartheta_w e_0$	$s^t e_0$
•	e'_1 : <i>Ole</i> (= $\uparrow e'_1$) says q_1	$e'_1 <_{w_0} e_0$	-say ^e -DEC

$w_1 \in q_1$

a. •	e_1 : $\uparrow e'_1$ leaves $\pi_{w_1} \blacktriangleleft e_1$		v^e -
•	e'_1 : $\uparrow e'_1$ speaks to $\downarrow e'_1$	$e_1 <_{w_1} e'_1 \subseteq_{w_1} \blacktriangleright e_1$	-say ^e
b. •••	e_1 : $\uparrow e'_1$ is working		v^e -
•	e'_1 : $\uparrow e'_1$ speaks to $\downarrow e'_1$	$e_1 <_{w_1} e'_1 \subseteq_{w_1} \blacktriangledown e_1$	-say ^e
c. —	s_1 : $\uparrow e'_1$ is busy		v^s -
•	e'_1 : $\uparrow e'_1$ speaks to $\downarrow e'_1$	$e_1 <_{w_1} e'_1 \subseteq_{w_1} s_1$	-say ^e

- (6') *Ole* [_{DEC} [-say ...
 $\top[x | x =_i ole]$; $\partial[spk.to_{\top\Omega} \langle \top\varepsilon, \uparrow\top\varepsilon, \downarrow\top\varepsilon \rangle]$; $[w]^\perp$; ...

- a. ...leave^e]-say^e]-DEC_T-3SG
 $([e | leave_{\perp\omega} \langle e, \uparrow e, \pi_{\perp\omega} \blacktriangleleft e \rangle]$; $[e p | \perp\varepsilon <_{\perp\omega} e \subseteq_{\perp\omega} \blacktriangleright \perp\varepsilon, spk.to_p \langle e, \uparrow\perp\varepsilon, \downarrow e \rangle]$;
 $[\perp\Omega \subseteq_I \perp\omega | \perp\varepsilon]$; $[say_{\top\omega} \langle \perp\varepsilon, \top\delta, \perp\Omega \rangle]$); $[\perp\varepsilon <_{\top\omega} \top\varepsilon]$; $\top[p | p =_I \top\omega | \top\Omega]$
- b. ...work^e]-say^e]-DEC_T-3SG
 $[e | work_{\perp\omega} \langle e, \uparrow e \rangle]$; $[e p | \blacktriangledown \perp\varepsilon <_{\perp\omega} e \subseteq_{\perp\omega} \blacktriangledown \perp\varepsilon, spk.to_p \langle e, \uparrow\perp\varepsilon, \downarrow e \rangle]$;
 $[\perp\Omega \subseteq_I \perp\omega | \perp\varepsilon]$; $[say_{\top\omega} \langle \perp\varepsilon, \top\delta, \perp\Omega \rangle]$); $[\perp\varepsilon <_{\top\omega} \top\varepsilon]$; $\top[p | p =_I \top\omega | \top\Omega]$
- c. ...busy^s]-say^e]-DEC_T-3SG
 $[s | busy_{\perp\omega} \langle s, \uparrow s \rangle]$; $[e p | \blacktriangleleft \perp\sigma <_{\perp\omega} e \subseteq_{\perp\omega} \perp\sigma, spk.to_p \langle e, \uparrow\perp\sigma, \downarrow e \rangle]$;
 $[\perp\Omega \subseteq_I \perp\omega | \perp\varepsilon]$; $[say_{\top\omega} \langle \perp\varepsilon, \top\delta, \perp\Omega \rangle]$); $[\perp\varepsilon <_{\top\omega} \top\varepsilon]$; $\top[p | p =_I \top\omega | \top\Omega]$

- (7) *Ole* {*ippassaq* | **aqagu*} {*aallar-* | *suli-* | *ulapik-*}-*nirar-pu-q*.
Ole {yesterday | *tomorrow} {leave- | work- | busy-}-say-DEC_T-3SG
 A. *Ole* said: “I {left | worked | ’ve been busy} {yesterday | *tomorrow}”.
 B. *Ole* said {yesterday | *tomorrow}: “I {left | am working | am busy}”.

Model for (7_A) Topic-setting yesterday with comment by v_σ-say^e

<u>Discourse ref.</u>	<u>Symbol: Description</u>	<u>Temporal-modal cond.</u>	<u>Source</u>
$\top w_0 \in \top p'_0 \subseteq p_0$			
●	$\top e_0$: $\uparrow e_0$ speaks to $\downarrow e_0$	$\exists t \forall w \in p_0: t = \vartheta_w e_0$	$st e_0$
■■■■■	t_1 : e ₀ -yesterday	$t_1 = \llbracket \text{day.bfr} \rrbracket (\vartheta_{w_0} e_0)$	yesterday
●	e'_1 : Ole (= $\uparrow e'_1$) says q_1	$e'_1 <_{w_0} e_0$	-say ^e -DEC
~~~~~			
$w_1 \in q_1$			
—	$\top s_1$ : Ole during $t_1$	$s_1 \subseteq_{w_1} t_1$	yesterday
●	$e_1$ : $\uparrow e'_1$ leaves $\pi_{w_1} \blacktriangleleft e_1$	$e_1 \subseteq_{w_1} s_1$	$v^e_{\sigma}$ -
●	$e'_1$ : $\uparrow e'_1$ speaks to $\downarrow e'_1$	$e_1 <_{w_1} e'_1 \subseteq_{w_1} \blacktriangleright e_1$	-say ^e

(7_A') Ole [_{DEC} [-say yesterday...]

$\top [x | x =_i \text{ole}]$ ;  $\partial [\text{spk.to}_{\top \Omega} \langle \top \varepsilon, \uparrow \top \varepsilon, \downarrow \top \varepsilon \rangle]$ ;  $[w]^\perp$ ;  $(([t | t =_i \text{day.bfr} \langle \vartheta_{\top \omega} \top \varepsilon \rangle])$ ;  
 $\top [s | \uparrow s =_i \top \delta, s \subseteq_{\perp \omega} \perp \tau])^\top$ ; ...

... leave^e_σ]-say^e_ε]-DEC_τ-3SG

$[e | lv_{\perp \omega} \langle e, \uparrow e, \pi_{\perp \omega} \blacktriangleleft e \rangle]$ ;  $[e p | \perp \varepsilon <_{\perp \omega} e \subseteq_{\perp \omega} \blacktriangleright \perp \varepsilon, \text{spk.to}_p \langle e, \uparrow \perp \varepsilon, \downarrow e \rangle]$ ;  
 $[\perp \Omega \subseteq_I \perp \omega | \perp \varepsilon]$ ;  $[\text{say}_{\top \omega} \langle \perp \varepsilon, \top \delta, \perp \Omega \rangle]$ ;  $[\perp \varepsilon <_{\top \omega} \top \varepsilon]$ ;  $\top [p | p =_I \top \omega | \top \Omega]$

*Model for (7_B) Topic-setting yesterday with comment by v_σ-say^e*

<u>Discourse ref.</u>	<u>Symbol: Description</u>	<u>Temporal-modal condition</u>	<u>Source</u>
$\top w_0 \in \top p'_0 \subseteq p_0$			
●	$\top e_0$ : $\uparrow e_0$ speaks to $\downarrow e_0$	$\exists t \forall w \in p_0: t = \vartheta_w e_0$	$st e_0$
■■■■■	$t_1$ : e ₀ -yesterday	$t_1 = \llbracket \text{day.bfr} \rrbracket (\vartheta_{w_0} e_0)$	yesterday
—	$\top s_1$ : Ole during $t_1$	$s_1 \subseteq_{w_0} t_1$	yesterday
●	$e'_1$ : Ole (= $\uparrow e'_1$ ) says $q_1$	$e'_1 \subseteq_{w_0} s_1, e'_1 <_{w_0} e_0$	-say ^e _σ -DEC
~~~~~			
$w_1 \in q_1$			
●	e_1 : $\uparrow e'_1$ leaves $\pi_{w_1} \blacktriangleleft e_1$		v^e -
●	e'_1 : $\uparrow e'_1$ speaks to $\downarrow e'_1$	$e_1 <_{w_1} e'_1 \subseteq_{w_1} \blacktriangleright e_1$	-say ^e

(7_B') Ole [_{DEC} yesterday [-say ...]

$\top [x | x =_i \text{ole}]$; $\partial [\text{spk.to}_{\top \Omega} \langle \top \varepsilon, \uparrow \top \varepsilon, \downarrow \top \varepsilon \rangle]$; $(([t | t =_i \text{day.bfr} \langle \vartheta_{\top \omega} \top \varepsilon \rangle])$;
 $\top [s | \uparrow s =_i \top \delta, s \subseteq_{\top \omega} \perp \tau])^\top$; $([w]^\perp$; ...

... [leave^e]-say^e_σ]-DEC_τ-3SG

$[e | lv_{\perp \omega} \langle e, \uparrow e, \pi_{\perp \omega} \blacktriangleleft e \rangle]$; $[e p | \perp \varepsilon <_{\perp \omega} e \subseteq_{\perp \omega} \blacktriangleright \perp \varepsilon, \text{spk.to}_p \langle e, \uparrow \perp \varepsilon, \downarrow e \rangle]$; $[\perp \Omega \subseteq_I \perp \omega | \perp \varepsilon]$;
 $[\text{say}_{\top \omega} \langle \perp \varepsilon, \top \delta, \perp \Omega \rangle, \perp \varepsilon \subseteq_{\top \omega} \top \sigma]$; $[\perp \varepsilon <_{\top \omega} \top \varepsilon]$; $\top [p | p =_I \top \omega | \top \Omega]$

- (8) i. *Aanip meeqqat aqagu sukkanniunniarnerarpai.*
Aani-p miiraq-t aqagu sukkanniut-niar-nirar-pa-i.
 Ann-ERG kid-PL tomorrow race.e.o-intend-say-DEC_{T⊥}-3SG.3PL
 Ann^T said the kids[⊥] were going to have a dogsled race tomorrow.
- ii. *Olegooq ajugaaguni nuannaassaaq.*
Ole=guuq ajugaa-gu-ni nuannaar-ssa-pu-q.
 Ole=RPT win-HYP_T-3SG_T happy-expect(ed)-DEC_T-3SG
 If Ole^T wins, [she] said, he_T'll be happy.
- (8') i. Ann-ERG^T kid-PL[⊥] [_{DEC} [-say [-int tomorrow ...
^T[x| x =_i ann]; [x| ^Ax ⊆_i kid_{Tω} 2⁺<^Ax>]; ∂[spk.to_{TΩ}<Tε, ↑Tε, ↓Tε>];
 (([w]; (([t| t =_i day.aft<∂_{Tω}Tε>); ^T[s| ↑s =_i ⊥δ, s ⊆_{⊥ω} ⊥τ))^T; ...
 ... race.e.o.^e]_σ]-intend^s]-say^e]-DEC_T-3PL
 [e| race.e.o._{⊥ω}<e, ↑e>, ∇e ⊆_i Tσ]); [s| [▲]⊥ε =_i ([▲]s)_{⊥ω}]; [w| OPT<TΩ, int_w ⊥σ>
 ⊆_I ⊥ω||_{⊥σ}]; [e| p| ↑⊥σ =_i ⊥δ, [▲]⊥σ <_{⊥ω} e ⊆_{⊥ω} ⊥σ, spk.to_p<e, ↑e, ↓e>]; [⊥Ω
 ⊆_I ⊥ω||_{⊥ε, ⊥δ}]; [say_{Tω}<⊥ε, Tδ, ⊥Ω>]; [⊥ε <_{Tω} Tε]; ^T[p| p =_I Tω||_{TΩ}]
- ii. Ole^T=RPT_⊥^T [_{DEC} ...
^T[x| x =_i ole]; ∂[spk.to_{TΩ}<⊥ε, ↑Tε, ↓Tε>]; ^T[p| spk.to_p<[<]Tε, ↑[<]Tε, ↓[<]Tε>, <[<]Tε =_i ⊥ε, p ⊆_i ⊥Ω>]; ^T[w| w ∈_i TΩ]^T; (∂[spk.to_{TΩ}<[<]Tε, ↑[<]Tε, ↓[<]Tε>];
 ... win^e-HYP_T-3SG_T ...
 (([e| win_{⊥ω}<e, Tδ>, e =_{⊥ω} ([▲]Tσ)_{⊥ω} Tε <_{⊥ω} e]; [⊥ε =_I max{⊥ε||_{⊥ω} Tσ}&];
 [t| t =_{⊥ω} ([○]⊥ε)_{⊥ω}]; [⊥ω ∈_I Tω||_{TΩ}]; ^T[p| p =_I ⊥ω||_{⊥τ}])^T; ...
 ... happy^s_τ-expect(ed)^s]-DEC_T-3SG]_{RPT}
 ([s| hpp_{⊥ω}<e, Tδ>, [▲]⊥σ ⊆_{⊥ω} ⊥τ]; [s| [▲]⊥σ =_{⊥ω} ([▲]s)_{⊥ω}]; [OPT<TΩ, exp_{Tω} ⊥σ>
 ⊆_I ⊥ω||_{⊥σ}]; [Tε ⊆_{Tω} ⊥σ]); [[▲]⊥σ <_{Tω} [<]Tε]; ^T[p| p =_I Tω||_{TΩ})]

4 QUANTIFICATION: VERIFIABLE HABITS

Figure 1 Temporal-modal relations to habits

$$\blacktriangleright A_{at} <_w B := \exists u_a (u \in A \wedge u <_w B)$$

$$B \sqsubseteq_w A_{at} := \exists u_a \exists u'_a (u \in A \wedge u' \in A \wedge u <_w B <_w u')$$

- (9) *Ole ullumikkut {aallartarpoq | sulisarpoq | ulapittarpoq}*
Ole ullumi-kut {aallar-(tar)-pu-q | suli-*(tar)-pu-q | ulapik-*(tar)-pu-q}.*
 Ole today-VIA {lv-*(habit)-DEC_T-3SG | work-*(hab.)-... | busy-*(hab.)-...}
 Ole {goes away | works | is busy} these days.

Model for (9) Topic-setting today-VIA with comment by $\{v^e | v^e | v^s\}_\sigma$ -habit-

<u>Discourse ref.</u>	<u>Symbol: Description</u>	<u>Temporal-modal condition</u>	<u>Source</u>
$\top w_0 \in \top p'_0 \subseteq \top p_0$			
●	$\top e_0$: $\uparrow e_0$ speaks to $\downarrow e_0$	$\exists t \forall w \in p_0: t = \vartheta_w e_0$	$st e_0$
... ■■■ ...	t_n : day within $\circ e_0$	$t_n \subseteq_{w_0} \circ e_0$	today-VIA
—	$\top s_n$: Ole during day t_n	$s_n =_{w_0} t_n$	VIA
	$\top S_1 = \{s_1, s_2, \dots\}$	$e_0 \subseteq_{w_0} S_1$	VIA
	$\forall s_n \in S_1: (a) (b) (c)$		VIA...-habit
a. ... ● ...	e_n : Ole leaves $\pi_{w_0} \blacktriangleleft e_n$	$e_n \subseteq_{w_0} s_n$	v^e_σ
	$E_1 = \{e_1, e_2, \dots\}$	$\blacktriangleright E_1 <_{w_0} e_0$	v^e_σ -habit-DEC
b. ... ●● ...	e_n : Ole works	$\nabla e_n \subseteq_\sigma s_n$	v^e_σ
	$E_1 = \{e_1, e_2, \dots\}$	$\blacktriangleright E_1 <_{w_0} e_0$	v^e_σ -habit-DEC
c. ... — ...	s'_n : Ole is busy	$s'_n \subseteq_\sigma s_n$	v^s_σ
	$S'_1 = \{s'_1, s'_2, \dots\}$	$\blacktriangleright S'_1 <_{w_0} e_0$	v^s_σ -habit-DEC

(9') Ole $[_{DEC}$ today-VIA...

$\top [x | x =_i ole]; \partial [spk.to_{\top\Omega} \langle \top\epsilon, \uparrow \top\epsilon, \downarrow \top\epsilon \rangle]; [t | day \langle t \rangle, t \subseteq_{\top\omega} \circ \top\epsilon];$
 $(\top [s | \uparrow s =_i \top\delta, s =_{\top\omega} \perp \tau])^\top; (\top [S | S =_I \top\sigma]_{\top\omega}); [\top\epsilon \subseteq_{\top\omega} \top\sigma]^\top; \dots$

a. ... leave $^e_\sigma$ -habit]-DEC $_{\top}$ -3SG

$([e | lv_{\top\omega} \langle e, \top\delta, \pi_{\top\omega} \blacktriangleleft e \rangle, e \subseteq_{\top\omega} \top\sigma]; [\perp\epsilon =_I \mathbf{max} \{ \perp\epsilon |_{\top\omega, \top\sigma} \}]; [\top\sigma t =_I \top\sigma]_{\top\omega});$
 $[E | E =_I \perp\epsilon]_{\top\omega}); [\blacktriangleright \perp\epsilon t <_{\top\omega} \top\epsilon]; \top [p | p =_I \top\omega]_{\top\Omega}$

b. ... work $^e_\sigma$ -habit]-DEC $_{\top}$ -3SG

$([e | work_{\top\omega} \langle e, \top\delta \rangle, \nabla e \subseteq_i \top\sigma]; [\perp\epsilon =_I \mathbf{max} \{ \perp\epsilon |_{\top\omega, \top\sigma} \}]; [\top\sigma t =_I \top\sigma]_{\top\omega});$
 $[E | E =_I \perp\epsilon]_{\top\omega}); [\blacktriangleright \perp\epsilon t <_{\top\omega} \top\epsilon]; \top [p | p =_I \top\omega]_{\top\Omega}$

c. ... busy $^s_\sigma$ -habit]-DEC $_{\top}$ -3SG

$([s | busy_{\top\omega} \langle s, \top\delta \rangle, s \subseteq_i \top\sigma]; [\perp\sigma =_I \mathbf{max} \{ \perp\sigma |_{\top\omega, \top\sigma} \}]; [\top\sigma t =_I \top\sigma]_{\top\omega});$
 $[S | S =_I \perp\sigma]_{\top\omega}); [\blacktriangleright \perp\sigma t <_{\top\omega} \top\epsilon]; \top [p | p =_I \top\omega]_{\top\Omega}$

(10) i. *Meeqqat sapaatikkut sukkanniuttarput.*

Miiraq-t sapaat-kut sukkanniut-tar-pu-t.

kid-PL Sunday-VIA race.e.o-habit-DEC $_{\top}$ -3PL

The kids $^\top$ have dogsled races on Sundays.

ii. *Ole (unammigaangami) (amerlanertigut) ajugaasarpoq.*

Ole (unammik-gaanga-mi) (amirlaniq-kut) ajugaa-tar-pu-q.

Ole (compete-HAB $_{\top}$ -3SG $_{\top}$) (most-VIA) win-habit-DEC $_{\top}$ -3SG

(When he $_{\top}$ competes,) Ole (usually) wins.

- (10') i. **kid-PL** [_{DEC} **Sunday-VIA** ...
 $\top[x | \mathcal{A}x \subseteq_i \text{kid}_{\top\omega} \mathbf{2}^+ \langle \mathcal{A}x \rangle]; \partial[\text{spk.to}_{\top\Omega} \langle \top\varepsilon, \uparrow\top\varepsilon, \downarrow\top\varepsilon \rangle]; (([t \text{ sunday} \langle t \rangle, t \subseteq_{\top\omega} \circlearrowleft \top\varepsilon]; \top[s | \uparrow s =_i \top\delta, s =_{\top\omega} \perp \tau]; \top[S | S =_I \top\sigma ||_{\top\omega, \top\delta}]; [\top\varepsilon \subseteq_{\top\omega} \top\sigma]) \top;$
 ... **race.e.o.**^e**-habit**]-DEC_T-3PL
 $([e | \text{race.e.o} \langle e, \top\delta \rangle, \nabla e \subseteq_i \top\sigma]; [\perp\varepsilon =_I \mathbf{max} \{ \perp\varepsilon ||_{\top\omega, \top\sigma} \}]; [\top\sigma =_I \top\sigma ||_{\top\omega, \top\delta}]; [E | E =_I \perp\varepsilon ||_{\top\omega, \top\delta}]); [\blacktriangleright \perp\varepsilon t <_{\top\omega} \top\varepsilon]; \top[p | p =_I \top\omega ||_{\top\Omega}]$
- ii. A_n : topic-setting update(s), B: comment
- A₁. **Ole** [_{DEC} ...
 $\top[x | x =_i \text{ole}] \top; (\partial[\text{spk.to}_{\top\Omega} \langle \top\varepsilon, \uparrow\top\varepsilon, \downarrow\top\varepsilon \rangle]; ...$
- A₂. **Ole** [_{DEC} **most-VIA** ...
 $\top[x | x =_i \text{ole}] \top; (\partial[\text{spk.to}_{\top\Omega} \langle \top\varepsilon, \uparrow\top\varepsilon, \downarrow\top\varepsilon \rangle]; ((\top[S | \mathbf{most} \langle \top\sigma, S \rangle]; [\top\varepsilon \subseteq_{\top\omega} \top\sigma]) \top; ...$
- A₃. **Ole** [_{DEC} **compete**^e**-HAB**_T-3SG_T...
 $\top[x | x =_i \text{ole}] \top; (\partial[\text{spk.to}_{\top\Omega} \langle \top\varepsilon, \uparrow\top\varepsilon, \downarrow\top\varepsilon \rangle]; (([e w | \text{compete}_w \langle e, \top\delta \rangle, \nabla e \subseteq_i \top\sigma]; [\perp\varepsilon =_I \mathbf{max} \{ \perp\varepsilon ||_{\perp\omega, \top\sigma} \}]; [\top\Omega \subseteq_I \perp\omega ||]; \top[s | s =_i (\circlearrowleft \perp\varepsilon)_{\top\omega}]; \top[S | S =_I \top\sigma ||_{\top\omega}]; [\top\varepsilon \subseteq_{\top\omega} \top\sigma]) \top; ...$
- A₄. **Ole** [_{DEC} **compete**^e**-HAB**_T-3SG_T **most-VIA** ...
 $\top[x | x =_i \text{ole}] \top; (\partial[\text{spk.to}_{\top\Omega} \langle \top\varepsilon, \uparrow\top\varepsilon, \downarrow\top\varepsilon \rangle]; (([e w | \text{compete}_w \langle e, \top\delta \rangle, \nabla e \subseteq_i \top\sigma]; [\perp\varepsilon =_I \mathbf{max} \{ \perp\varepsilon ||_{\perp\omega, \top\sigma} \}]; [\top\Omega \subseteq_I \perp\omega ||]; \top[s | s =_i (\circlearrowleft \perp\varepsilon)_{\top\omega}]; [S | S =_I \top\sigma ||_{\top\omega}]; [\top\varepsilon \subseteq_{\top\omega} \perp\sigma]) \perp; ((\top[S | \mathbf{most} \langle \perp\sigma, S \rangle]; [\top\varepsilon \subseteq_{\top\omega} \top\sigma]) \top; ...$
- B. ... **win**^e**-habit**]-DEC_T-3SG
 $([e | \text{win} \langle e, \top\delta \rangle, e =_{\top\omega} (\blacktriangleleft \top\sigma)_{\top\omega}]; [\perp\varepsilon =_I \mathbf{max} \{ \perp\varepsilon ||_{\top\omega, \top\sigma} \}]; [\top\sigma =_I \top\sigma ||_{\top\omega, \top\delta}]; [E | E =_I \perp\varepsilon ||_{\top\omega, \top\delta}]); [\blacktriangleright \perp\varepsilon t <_{\top\omega} \top\varepsilon]; \top[p | p =_I \top\omega ||_{\top\Omega}]$

5 TOWARD A CG.UC FRAGMENT OF KALAALLISUT

K1 (Kalaallisut categories)

- s and $\text{pn}_\delta, \text{pn}_\tau, \text{pn}_\omega,$ are Kalaallisut categories
- If X and Y are Kalaallisut categories, then so are (X/Y) and $(X \setminus Y)$.

K2 (Kalaallisut category-to-type rule)

- $\mathbf{tp}(s) = []$, $\mathbf{tp}(\text{pn}_a) = sa$
- $\mathbf{tp}(X/Y) = \mathbf{tp}(X \setminus Y) = (\mathbf{tp}(Y) \mathbf{tp}(X))$

ABBREVIATIONS (categories and types)

$$\begin{array}{llll} \underline{s} := s \backslash \text{pn}_\omega & s_a := s \backslash \text{pn}_a \text{pn} := \text{pn}_\delta & \text{D} := s \delta & \text{W} := s \omega \\ \text{iv} := \underline{s} \backslash \text{pn}_\delta & \text{cn}_a := (s_a \backslash \text{pn}_\omega) \backslash \text{pn}_\tau & \text{cn} := \text{cn}_\delta & \text{I} := s \tau \quad [] := (st)st \end{array}$$

Figure 2 Algebraic correlates of eventualities

$$\begin{array}{ll} \circ e := (\nabla \blacktriangleleft e \sqcup \blacktriangleright e) & (\circ u)_w := \sigma(\varepsilon u \sqcup (\blacktriangleleft \sigma u)_w) \\ \sigma u_a := u \quad \text{if } a = \sigma & \varepsilon u_a := \blacktriangleright u \quad \text{if } a = \sigma \\ \in \{ \blacktriangleright u, \nabla u \} \quad \text{if } a = \varepsilon & \in \{ u, \blacktriangleleft u, \blacktriangleright \nabla u \} \quad \text{if } a = \varepsilon \end{array}$$

Kal. \vdash CG category: UC translation

$$\text{win}^e \vdash \text{iv}: \lambda x \lambda w ([e]; [\text{win}_w \langle \perp \varepsilon, x \rangle, \perp \varepsilon =_{(w)} (\blacktriangleleft \top \sigma)_w])$$

$$\text{work}^e \vdash \text{iv}: \lambda x \lambda w ([e]; [\text{work}_w \langle \perp \varepsilon, x \rangle])$$

$$\text{busy}^s \vdash \text{iv}: \lambda x \lambda w ([s]; [\text{busy}_w \langle \perp \sigma, x \rangle])$$

$$\text{-int}^s \vdash \text{iv} \backslash \text{iv}: \lambda P \lambda x \lambda w (P \uparrow \perp a \perp \omega \perp; [s | \varepsilon \perp a =_i (\blacktriangleleft s)_{\perp \omega}]; [\text{OPT} \langle \top \Omega, \text{int}_w \perp \sigma \rangle \subseteq_I \perp \omega |_{\perp \sigma}]; [\uparrow \perp \sigma =_i x])$$

$$\text{-say}^e \vdash \text{iv} \backslash \text{iv}: \lambda P \lambda x \lambda w (P \uparrow \perp a \perp \omega \perp; [e p | \varepsilon \perp a <_{\perp \omega} e \sqsubseteq_{\perp \omega} \sigma \perp a, \text{spk.to}_p \langle e, \uparrow \perp a, \downarrow e \rangle]; [\perp \Omega \subseteq_I \perp \omega |_{\perp \varepsilon}]; [\text{spk.to}_w \langle \perp \varepsilon, x, \downarrow \perp \varepsilon \rangle])$$

$$\text{-habit} \vdash \text{iv} \backslash \text{iv}: \lambda P \lambda x \lambda w (P x w \perp; [\perp a =_I \mathbf{max} \{ \perp a |_{w, \top \sigma} \}]; [\top \sigma \tau =_I \top \sigma |_{\perp w}]; [A | A =_I \perp a |_{\perp w}]; [\top \varepsilon \sqsubseteq_w \perp a \tau])$$

$$\perp(\cdot) \vdash \underline{s} \backslash \underline{s}: \lambda V_{[w]} \lambda w_w ([w] \perp; V w)$$

$$(\cdot)_\varepsilon \vdash \underline{s} \backslash \underline{s}: \lambda V_{[w]} \lambda w_w (V w \perp; [\top \varepsilon \sqsubseteq_w \sigma \perp a]) \quad a \in \{ \varepsilon, \sigma \}$$

$$(\cdot)_\sigma \vdash \underline{s} \backslash \underline{s}: \lambda V_{[w]} \lambda w_w (V w \perp; [(\sigma) \perp a \sqsubseteq_{(w)} \top \sigma])$$

$$(\cdot)_\tau \vdash \underline{s} \backslash \underline{s}: \lambda V_{[w]} \lambda w_w (V w \perp; [(\varepsilon) \perp a \sqsubseteq_w \perp \tau])$$

$$\text{-FCT} \vdash (\underline{s} / \underline{s}) \backslash (s \backslash \text{pn}_\omega): \lambda V'_{[w]} \lambda V_{[w]} \lambda w_w ((V' \perp \omega \perp; [\varepsilon \perp a <_{\perp \omega} ? \varepsilon]); [\perp a =_I \mathbf{max} \{ \perp a |_{\perp \omega, \dots} \}]; [\top \Omega \subseteq_I \perp \omega |_{\dots}]; [\top [s | s =_i (\circ \perp a)_{\top \omega}] \top]; V w)$$

$$\text{-DEC} \vdash s \backslash (s \backslash \text{pn}_\omega): \lambda V_{[w]} (\partial [\text{spk.to}_{\top \Omega} \langle \top \varepsilon, \uparrow \top \varepsilon, \downarrow \top \varepsilon \rangle]; (V \top \omega \perp; [\varepsilon \perp a <_{\top \omega} \top \varepsilon]); \top [p | p =_I \top \omega |_{\top \Omega}])$$

$$\text{-OPT} \vdash s \backslash (s \backslash \text{pn}_\omega): \lambda V_{[w]} (\partial [\text{spk.to}_{\top \Omega} \langle \top \varepsilon, \uparrow \top \varepsilon, \downarrow \top \varepsilon \rangle]; (V \perp \omega \perp; [\varepsilon \perp a =_{\perp \omega} (\blacktriangleleft \top \varepsilon)_{\perp \omega}]); [p | p =_I \perp \omega |_{\top \delta}]; [\text{OPT} \langle \top \Omega, \text{des}_{\top \omega} \circ \top \varepsilon \rangle \subseteq_i \perp \Omega]; [\top \Omega =_I \top \omega |_{\top \Omega}])$$

APPENDIX:
UPDATE WITH CENTERING (UC, Bittner 2012: Ch. 7)

D1 The set of UC *types* is the smallest set Θ such that: (i) $t, \delta, \varepsilon, \sigma, \tau, \omega, s \in \Theta$, and (ii) $(ab) \in \Theta$ if $a, b \in \Theta$. The subset $\text{DR}(\Theta) = \{\delta, \varepsilon, \sigma, \tau, \omega, \delta t, \varepsilon t, \sigma t, \tau t, \omega t\}$ is the set of *discourse referent types*.

D2.1 A UC *frame* is a set $\mathcal{F} = \{\mathcal{D}_a \mid a \in \Theta\}$ such that:

- i. $\mathcal{D}_t = \{0, 1\}$, $\mathcal{D}_\delta, \mathcal{D}_\varepsilon, \mathcal{D}_\sigma, \mathcal{D}_\tau$ and \mathcal{D}_ω are non-empty pairwise disjoint sets
 $\mathcal{D}_\tau = \{t \mid t \text{ is a non-empty convex set of integers}\}$
- ii. $\mathcal{D}_s = \cup_{n \geq 0, m \geq 0} \{\langle \langle d_1, \dots, d_n \rangle, \langle d'_1, \dots, d'_m \rangle \rangle : d_i, d'_j \in \mathcal{D}_{dr}\}$,
where $\mathcal{D}_{dr} = \cup \{\mathcal{D}_a : a \in \text{DR}(\Theta)\}$
- iii. $\mathcal{D}_{ab} = \{f \mid \emptyset \subset \text{Dom } f \subseteq \mathcal{D}_a \ \& \ \text{Ran } f \subseteq \mathcal{D}_b\}$

D2.2 An \mathcal{F} -*mereology* is a structure $\mathbf{M} = \langle \{\mathcal{D}_{\delta^+}, \mathcal{D}_{\delta^-}\}, \sqsubseteq, \overset{\mathcal{A}}{\cdot}, \nabla, \blacktriangle \rangle$ such that:

- i. $\{\mathcal{D}_{\delta^+}, \mathcal{D}_{\delta^-}\}$ is a partition of \mathcal{D}_δ (into *objects* \mathcal{D}_{δ^+} and *masses* \mathcal{D}_{δ^-}).
- ii. $\forall a \in \{\delta_+, \delta_-, \varepsilon, \sigma, \tau\}$: $\langle \mathcal{D}_a, \sqsubseteq_a \rangle$ is a join-semilattice
 $t_1 \sqsubseteq_\tau t_2 \quad :\Leftrightarrow \quad t_1 \subseteq t_2$ (temporal inclusion)
- iii. $\forall a \in \{\delta_+, \varepsilon, \tau\}$:
 $y \in \overset{\mathcal{A}}{\cdot} x \quad \Leftrightarrow \quad y \sqsubseteq_a x \ \& \ \forall z : z \sqsubseteq_a y \rightarrow z = y$
 $y \in \overset{\mathcal{A}}{\cdot} \mathcal{D}_a \quad \Leftrightarrow \quad \exists x \in \mathcal{D}_a : y \in \overset{\mathcal{A}}{\cdot} x$
- iv. $\forall a \in \{\delta_+, \varepsilon\}$:
 $x = \blacktriangle y \ \& \ y \in \mathcal{D}_a \Rightarrow y \notin \overset{\mathcal{A}}{\cdot} \mathcal{D}_a \ \& \ x \in \overset{\mathcal{A}}{\cdot} \mathcal{D}_a$
 $z = \nabla y \ \& \ y \in \mathcal{D}_a \Rightarrow y \notin \overset{\mathcal{A}}{\cdot} \mathcal{D}_a \ \& \ z \in \mathcal{D}_{\pm(a)}$, where $\pm(\delta_+) = \delta_-$ & $\pm(\varepsilon) = \sigma$

D2.3 An \mathbf{M} -*network* is a structure $\mathbf{N} = \langle \mathcal{D}_v, \vartheta, \pi, \prec_\tau, \triangleright, \blacktriangleleft, \blacktriangleright, \blacktriangleup, \blacktriangledown, \uparrow, \downarrow, \prec, \succ \rangle$ s.t.:

- i. $\mathcal{D}_v = \mathcal{D}_\varepsilon \cup \mathcal{D}_\sigma$ (eventualities)
 $\pi : \mathcal{D}_v \rightarrow [\mathcal{D}_\omega \rightarrow \mathcal{D}_{\delta^-}]$ (place)
 $\vartheta : \mathcal{D}_v \rightarrow [\mathcal{D}_\omega \rightarrow \mathcal{D}_\tau]$ (run time)
 $t_1 \prec_\tau t_2 \quad :\Leftrightarrow \quad \forall n \in t_1 \forall m \in t_2 : n < m$ (temporal precedence)
 $t_1 \prec_\tau t_2 \quad :\Leftrightarrow \quad t_1 \prec_\tau t_2 \ \& \ \neg \exists t : t_1 \prec_\tau t \prec_\tau t_2$ (immediate precedence)
- ii. $e' = \blacktriangle e \ \& \ \vartheta_w e = t \Rightarrow \vartheta_w e' = t$ (atomic-equivalent)
 $s = \nabla e \ \& \ \vartheta_w e = t \Rightarrow \vartheta_w s = t$ (state-equivalent)
 $e' = \blacktriangleleft e \ \& \ \vartheta_w e = t \Rightarrow e \in \overset{\mathcal{A}}{\cdot} \mathcal{D}_\varepsilon \ \& \ e' \in \mathcal{D}_\varepsilon \ \& \ \vartheta_w e' \prec_\tau t$ (prep.-process)
 $s = \triangleright e \ \& \ \vartheta_w e = t \Rightarrow e \in \overset{\mathcal{A}}{\cdot} \mathcal{D}_\varepsilon \ \& \ s \in \mathcal{D}_\sigma \ \& \ t \prec_\tau \vartheta_w s$ (consequent state)
 $e = \blacktriangleright s \ \& \ \vartheta_w s = t \Rightarrow e \in \overset{\mathcal{A}}{\cdot} \mathcal{D}_\varepsilon \ \& \ s \in \mathcal{D}_\sigma \ \& \ \vartheta_w e \sqcup_\tau \vartheta_w \triangleright e = t$ (start pt.)
 $e = (\blacktriangleleft s)_w \ \& \ \vartheta_w e = t \Rightarrow e \in \overset{\mathcal{A}}{\cdot} \mathcal{D}_\varepsilon \ \& \ s \in \mathcal{D}_\sigma \ \& \ \vartheta_w s \prec_\tau t$ (culmination pt.)
- iii. $x = \uparrow u \Rightarrow u \in \mathcal{D}_v \ \& \ x \in \mathcal{D}_\delta$ (central individual)
 $y = \downarrow u \Rightarrow u \in \mathcal{D}_v \ \& \ y \in \mathcal{D}_\delta \ \& \ \exists x : x = \uparrow u$ (background individual)
 $x = \uparrow u \Leftrightarrow x = \uparrow f(u)$ $f \in \{\nabla, \blacktriangle, \triangleright, \blacktriangleleft, \blacktriangleright, \blacktriangleup\}$
 $u_1 \sqsubseteq_a u_2 \ \& \ f(u_2) \in \mathcal{D}_b \Rightarrow f(u_1) \sqsubseteq_b f(u_2)$ $f \in \{\vartheta, \pi, \uparrow, \downarrow, \nabla\}$
- iv. $e' = \prec e \ \& \ \vartheta_w e = t \Rightarrow \uparrow e' \neq \uparrow e \ \& \ \vartheta_w e' \prec_\tau t$ (speaker source)
 $e' = (\triangleright e)_w \ \& \ \vartheta_w e' = t \Rightarrow \uparrow e' = \downarrow e \ \& \ t = \vartheta_w (\blacktriangle \triangleright e)_w$ (speaker directive)

D3 A UC *model* is a tuple $\mathcal{M} = \langle \mathcal{F}, \mathbf{M}, \mathbf{N}, \llbracket \cdot \rrbracket \rangle$ s.t. \mathcal{F} is a UC-frame, \mathbf{M} is an \mathcal{F} -mereology, \mathbf{N} is an \mathbf{M} -network, $\llbracket \cdot \rrbracket$ maps any $A \in Con_a$ to $\llbracket A \rrbracket \in \mathcal{D}_a$, and:

- i. $\langle e, \uparrow e, \downarrow e \rangle \in \llbracket spk.to \rrbracket(w) \Rightarrow \exists e': e' = (\blacktriangleleft e)_w$
 $e' = \sphericalangle e \ \& \ \partial_w e = t \quad \Rightarrow \langle e, \uparrow e, \downarrow e \rangle \in \llbracket spk.to \rrbracket(w)$
 $e' = (\widehat{e})_w \quad \Rightarrow \langle e, \uparrow e, \downarrow e \rangle \in \llbracket spk.to \rrbracket(w)$
- ii. $\forall A \in Con_{\omega\delta\dots t}, w \in \mathcal{D}_\omega, u \in \mathcal{D}_\nu, d, d' \in \mathcal{D}_\delta$:
 $\langle u, d, \dots \rangle \in \llbracket A \rrbracket(w) \Leftrightarrow \langle u, \uparrow u, \dots \rangle \in \llbracket A \rrbracket(w) \ \& \ \uparrow u = d$
 $\langle u, d, d', \dots \rangle \in \llbracket A \rrbracket(w) \Leftrightarrow \langle u, d, \downarrow u, \dots \rangle \in \llbracket A \rrbracket(w) \ \& \ \downarrow u = d'$
- iii. $\forall a \in DR(\Theta), i \in \mathcal{D}_s$:
 $\llbracket \top a \rrbracket(i) \doteq ((\textcircled{1}i)_a)_1 \quad \llbracket \top' a \rrbracket(i) \doteq ((\textcircled{1}i)_a)_2 \quad \llbracket \top^\Rightarrow a \rrbracket(i) \doteq \{((\textcircled{1}i)_a)_n : n \geq 1\}$
 $\llbracket \perp a \rrbracket(i) \doteq ((\textcircled{2}i)_a)_1 \quad \llbracket \perp' a \rrbracket(i) \doteq ((\textcircled{2}i)_a)_2 \quad \llbracket \perp^\Rightarrow a \rrbracket(i) \doteq \{((\textcircled{2}i)_a)_n : n \geq 1\}$

D4 (UC syntax)

- i. $Con_a \cup Var_a \subseteq Term_a$
- ii. $(A_a = B_a) \in Term_t$, if $A_a, B_a \in Term_a$
 $(A_\tau < B_\tau) \in Term_t$, if $A_\tau, B_\tau \in Term_\tau$
 $(A_s \lesssim B_s) \in Term_t$, if $A_s, B_s \in Term_s$
 $(A_a \sqsubseteq B_a) \in Term_t$, if $A_a, B_a \in Term_a \ \& \ a \in DR(\Theta)$
- iii. $\mathbf{n}(A), \mathbf{n}^+(A) \in Term_t$, if $A \in Term_{at}$ and $\mathbf{n} \in \{1, 2, \dots\}$
 $\mathbf{most}(A, B) \in Term_t$, if $A, B \in Term_{at}$
- iv. $\neg\varphi, (\varphi \wedge \psi) \in Term_t$, if $\varphi, \psi \in Term_t$
- v. $\exists u_a \varphi \in Term_t$, if $u_a \in Var_a$ and $\varphi \in Term_t$
- vi. $\lambda u_a(B) \in Term_{ab}$, if $u_a \in Var_a$ and $B \in Term_b$
- vii. $BA \in Term_b$, if $B \in Term_{ab}$ and $A \in Term_a$
- viii. $\partial_W A \in Term_\tau \ \& \ \pi_W A \in Term_\delta$, if $W \in Term_\omega \ \& \ A \in Term_\varepsilon \cup Term_\sigma$
 $\uparrow A, \downarrow A \in Term_\delta$, if $A \in Term_\varepsilon \cup Term_\sigma$
 $\sphericalangle A \in Term_\varepsilon \ \& \ \triangleright A \in Term_{\omega\varepsilon}$, if $A \in Term_\varepsilon$
 $\blacktriangleleft A \in Term_\varepsilon \ \& \ \blacktriangleright A \in Term_\sigma$, if $A \in Term_\varepsilon$
 $\blacktriangleright A \in Term_\varepsilon \ \& \ \blacktriangleleft A \in Term_{\omega\varepsilon}$, if $A \in Term_\sigma$
 $\blacktriangle A_a \in Term_a$, if $a \in \{\delta, \varepsilon\} \ \& \ A \in Term_a$
 $\nabla A \in Term_{f(a)}$, if $a \in \{\delta, \varepsilon\}, A \in Term_a, f(\delta) = \delta, f(\varepsilon) = \sigma$
 $\overset{A}{\blacktriangleright} A_a \in Term_{at}$, if $a \in \{\delta, \varepsilon, \tau\} \ \& \ A_a \in Term_a$
- ix. $(A^\top \bullet B), (A^\perp \bullet B) \in Term_s$, if $A \in Term_\varepsilon$ and $B \in Term_s$
- x. $(A^\top; B), (A^\perp; B) \in Term_{(st)st}$, if $A, B \in Term_{(st)st}$

D5 (UC semantics)

- i. $\llbracket A \rrbracket^g = \llbracket A \rrbracket$, if $A \in Con_a$
 $\llbracket A \rrbracket^g = g(A)$, if $A \in Var_a$
- ii. $\llbracket (A_a = B_a) \rrbracket^g = 1$, if $\llbracket A_a \rrbracket^g = \llbracket B_a \rrbracket^g$; else, 0
 $\llbracket (A_\tau < B_\tau) \rrbracket^g = 1$, if $\llbracket A_\tau \rrbracket^g <_\tau \llbracket B_\tau \rrbracket^g$; else, 0
 $\llbracket (A_s \lesssim B_s) \rrbracket^g = 1$, if $\llbracket A_s \rrbracket^g \lesssim_s \llbracket B_s \rrbracket^g$; else, 0
 $\llbracket (A_a \sqsubseteq B_a) \rrbracket^g = 1$, if $\llbracket A_a \rrbracket^g \sqsubseteq_a \llbracket B_a \rrbracket^g$; else, 0

- iii. $\llbracket \mathbf{n}(A) \rrbracket^g = 1$, if $\llbracket A \rrbracket^g = n$; else, 0
 $\llbracket \mathbf{n}^+(A) \rrbracket^g = 1$, if $\llbracket A \rrbracket^g \geq n$; else, 0
 $\llbracket \mathbf{most}(A, B) \rrbracket^g = 1$, if $\llbracket A \rrbracket^g \cap \llbracket B \rrbracket^g \geq \llbracket A \rrbracket^g \setminus \llbracket B \rrbracket^g$; else, 0
- iv. $\llbracket \neg \varphi \rrbracket^g = 1$, if $\llbracket \varphi \rrbracket^g = 0$; else, 0
 $\llbracket (\varphi \wedge \psi) \rrbracket^g = 1$, if $\llbracket \varphi \rrbracket^g = 1$ and $\llbracket \psi \rrbracket^g = 1$; else, 0
- v. $\llbracket \exists u_a \varphi \rrbracket^g = 1$, if $\{d \in \mathcal{D}_a \mid \llbracket \varphi \rrbracket^{g[u/d]} = 1\} \neq \emptyset$; else, 0
- vi. $\llbracket \lambda u_a (B) \rrbracket^g(d) \doteq \llbracket B \rrbracket^{g[u/d]}$, if $d \in \mathcal{D}_a$
- vii. $\llbracket BA \rrbracket^g \doteq \llbracket B \rrbracket^g(\llbracket A \rrbracket^g)$
- viii. $\llbracket f_w A \rrbracket^g \doteq f(\llbracket W \rrbracket^g)(\llbracket A \rrbracket^g)$, if $f \in \{\vartheta, \pi\}$
 $\llbracket fA \rrbracket^g \doteq f(\llbracket A \rrbracket^g)$, if $f \in \{\uparrow, \downarrow, <, >, \blacktriangleleft, \blacktriangleright, \blacktriangle, \blacktriangledown, \mathcal{A}\}$
- ix. $\llbracket (A \top \bullet B) \rrbracket^g \doteq \langle (\llbracket A \rrbracket^g \cdot \textcircled{1}\llbracket B \rrbracket^g), \textcircled{2}\llbracket B \rrbracket^g \rangle$
 $\llbracket (A \perp \bullet B) \rrbracket^g \doteq \langle \textcircled{1}\llbracket B \rrbracket^g, (\llbracket A \rrbracket^g \cdot \textcircled{2}\llbracket B \rrbracket^g) \rangle$
- x. $\mathbf{c}\llbracket (A \top; B) \rrbracket^g = \{k \in \mathbf{c}\llbracket A \rrbracket^g \llbracket B \rrbracket^g \mid \exists i \in \mathbf{c} \exists j \in \mathbf{c}\llbracket A \rrbracket^g \exists a \in \text{DR}(\Theta): (\textcircled{1}j)_1 \in \mathcal{D}_a$
 $\& \textcircled{1}i < \textcircled{1}j \& (\textcircled{1}i)_a = (\textcircled{1}k)_a \& \llbracket B \rrbracket^g \neq \llbracket B[\top a / \perp a] \rrbracket^g\}$
 $\mathbf{c}\llbracket (A \perp; B) \rrbracket^g = \{k \in \mathbf{c}\llbracket A \rrbracket^g \llbracket B \rrbracket^g \mid \exists i \in \mathbf{c} \exists j \in \mathbf{c}\llbracket A \rrbracket^g \exists a \in \text{DR}(\Theta): (\textcircled{2}j)_1 \in \mathcal{D}_a$
 $\& \textcircled{2}i < \textcircled{2}j \& (\textcircled{2}j)_a = (\textcircled{2}k)_a \& \llbracket B \rrbracket^g \neq \llbracket B[\perp a / \top a] \rrbracket^g\}$

D6 For $(st)st$ -term K , model \mathcal{M} , assignment g , info-state \mathbf{c} , and $i \in \mathbf{c}$:

- i. $\mathcal{I}_{\mathcal{M},g}(i, \mathbf{c}) = \llbracket \lambda w (\exists k (Ik \wedge \top \Omega i = \top \Omega k \wedge w = \top \omega k)) \rrbracket^{g[i/i][\mathbf{c}/\mathbf{c}]}$
 ii. K is *at-issue* iff $\exists \mathcal{M} \exists \mathbf{c} \exists i \in \mathbf{c} \exists j: i \prec_s j \& \forall g: \mathcal{I}_{\mathcal{M},g}(j, \mathbf{c}\llbracket K \rrbracket^g) \subset \mathcal{I}_{\mathcal{M},g}(i, \mathbf{c})$

D7 (truth, falsity) For $(st)st$ -term K , model \mathcal{M} , info-state \mathbf{c} , and world w :

- i. $\mathcal{M}, \mathbf{c}, w \models K$ iff K is at-issue & $\exists i \in \mathbf{c} \forall g: w \in \mathcal{I}_{\mathcal{M},g}(i, \mathbf{c}\llbracket K \rrbracket^g)$
 ii. $\mathcal{M}, \mathbf{c}, w \not\models K$ iff K is at-issue & $\neg \exists i \in \mathbf{c} \forall g: w \in \mathcal{I}_{\mathcal{M},g}(i, \mathbf{c}\llbracket K \rrbracket^g)$

D8 (*startup info-state*)

$${}^{st}\mathbf{e} = \{\langle \langle w, p, e \rangle, \langle \rangle \rangle \mid \exists t: w \in p \subseteq \{v \mid \langle e, \uparrow e, \downarrow e \rangle \in \llbracket spk.to \rrbracket(v) \& \vartheta_v e = t\}\}$$

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