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A THEORY OF KNOWLEDGE

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To my parents,

sine qua non.

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ABSTRACT

A Theory of Knowledge

by Frode Bjordal

In this dissertation I present a new solution to the renowned Gettier problem. My solution, which in a sense represents a defense of a rather traditional epistemological approach, is based upon a distinction between primary and secondary beliefs. I argue that primary beliefs are known iff justified and true, whereas secondary beliefs are known iff they are believed on the basis of a known primary belief. Much emphasis is put upon defending this approach against potential objections, but I

also draw some epistemological and semantical consequences pertaining to such issues as the nature of epistemological justification, the Lottery Paradox, philosophical skepticism and the semantics of belief contexts.

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Introduction and overview

In 1963 Edmund Gettier[1] launched his well known and decisive criticism against a traditional analysis of knowledge according to which knowledge simply is the same as justified true belief. Gettier's criticism has generated at least four types of responses. (1) Some think that we should just give up trying to find an analysis of knowledge, (2) some people have suggested additional conditions that must hold if we know something, (3) others have suggested quite different analyses (like e.g. the causal analysis) of knowledge, and (4) others still have suggested a strengthening of the justification condition as the way out of the Gettier problem.

It is not our purpose in this essay to discuss and evaluate the different responses to the Gettier problem. I have mentioned the different kinds of responses in order to illustrate how my own response is related to the others. The analysis that I will eventually propose **might** be said to fit into the second type of response, because it introduces an extra structural requirement for knowledge when the belief of what is known is what I call a secondary belief. But such a characterization would be somewhat misleading, for the analysis that I offer is a traditional analysis in the sense that the only epistemic requirements for knowledge are justification, truth and belief. This might seem paradoxical as it stands, but will, I hope, become clearer at a later point in this exposition.

In the first chapter I argue that any further epistemic conditions which one may suggest as a necessary condition for knowledge can be built into the justification condition. I illustrate this by showing how a defeasibility condition can be built into our notion of justification by ascribing to it a certain recursive property. I then argue that any such approach is bound to fail if it fulfills certain adequacy requirements which I find it implausible to deviate from. In a second section to the first chapter I point out that there are some conflicting intuitions in the literature which justify us in not counting certain alleged Gettier-like counterexamples as genuine counter examples.

The ground work of my essay's constructive proposals is done in the second chapter, where a diagnosis of the Gettier problem is presented. I then go on to discuss and make precise a notion of <u>having a belief since some other belief is held</u>. I show that the relation is transitive, and introduce a distinction between primary and secondary beliefs.

In chapter III a more thorough treatment of the distinction between primary and secondary beliefs is given. I make use of the fact that the relation of having a belief since some other belief is held is transitive in order to reach the important conclusion that the set of primary beliefs is non-empty, and that any one secondary belief is held since some primary belief (or beliefs) is (are) held. I then go on to discuss what kind of primary beliefs there are, and i contrast these with non-inferential beliefs.

In chapter IV some set theory is used in order to motivate a recursive definition of knowledge that makes use of the distinction between primary and secondary beliefs.

Chapter V deals with six types of Gettier beliefs that have not been analyzed earlier in the essay, and I purport to show that the analysis of knowledge that I suggest also avoids these counter examples.

In chapter VI some counter examples that have been used to discard some other responses to the Gettier problem are considered. It seems that the analysis that I give is unharmed by these counter examples as well.

In chapter VII I discuss an example which may be considered to be an inductive Gettier example which is not avoided by my definition of knowledge in chapter IV. Chapter VII is divided into two sections, and the first section again divided into three subsections. In the first section I work on the basis of the assumption that there are genuine inductive Gettier examples, whereas I in the second section try to show that there are no genuine inductive Gettier examples.

In chapter VIII I apply aspects of the theory of justification and knowledge that has been developed in order to show that we with this theory can arrive at a very natural solution of the lottery paradox.

My essay includes two appendices. The first of these can be regarded as an application of the theory which I have offered, and the second explicates some semantical consequences of a disquotational principle which I have presupposed. In Appendix 1 I discuss and discard some new interpretations of the Dream Argument, and then offer my own interpretation. I end the appendix by giving an anti skeptical argument which intends to show that we indeed do know that we are awake. In Appendix 2 I defend a disquotational principle DP which I on several occasions appeal to in order to ascribe beliefs to subjects. DP depends upon some unorthodox semantical presuppositions which I go on to defend. The semantical topics discussed in Appendix 2 are of philosopical interest in their own right, but we are for the purposes of this essay mainly concerned with defending the epistemological principle DP.

I

1.0 Most analyses of knowledge which have been proposed as solutions of the Gettier problem are based on an addition of further epistemic conditions or on a strengthening of the justification condition. I will in this chapter try to argue that such solutions must fail. In the first section to follow I give a defeasibility analysis which is based on a strengthening of the justification condition, and I then go on to argue that such an analysis along with other such analyses must fail. In the second section I discuss the socalled social aspect of knowing, a category which was introduced because of an example given by Gilbert Harman, and I argue that the intuitions that underlie this and similar examples conflict with intuitions that underlie examples which have been invoked in the discussion of the defeasibility analyses of knowledge. I argue that Harman's counter example is **not** a genuine counter example to the traditional definition of knowledge.

1.1 We can think of a defeasibility analysis of knowledge as being of the following form:

S knows p iff 1) p is true
2) S believes p
3) S is justified in holding p
4) p is indefeasible

A variety of explications of "p is indefeasible" have been suggested.[2] Generally, a defeasibility condition has been introduced by means of a subjunctive conditional in such a way that p is taken to be indefeasible e.g. if, and only if, there is no further true evidence **e** such that if S were to believe **e** (or S were justified in holding **e**) then S would no longer be justified in believing p.

A notorious problem for defeasibility analyses has been to avoid the consequence that a subject S does **not** know p just because there is some misleading evidence **e**. Consider e.g. the following Tom Grabit example suggested by Paxson and

Lehrer.[3] A subject S believes that Tom Grabit took a book from the library because S believes that S saw Tom grab the book. But unbeknownst to S, Tom's demented mother Mrs. Grabit has claimed that Tom's identical twin is a notorious bookthif and was at the library at the time of the theft whereas Tom was miles away. In reality, however, Tom has no twin. The defeasibility analysis here gives the counterintuitive result that S did not know that Tom took the book as S's justification for holding that Tom took the book would, it seems, be defeated if S were to be justified in holding that Tom's identical twin was in or close to the library at the time of the theft. But why should Mrs. Grabit's false testimony matter? It seems that S really **did** know that Tom took the book. This is not the same as to say that S might not have stopped believing that Tom took the book if told by Mrs. Grabit that Tom's identical twin was in or close to the library at the time of the theft, whereas Tom wasn't.

Many epicycles have been added to the defeasibility approach in order to avoid such problems as this. Let us, however, assume that a defeasibility analysis can be given which avoids the counter examples given above with Tom

Grabit's demented mother and other such examples which are based on the existence of defective defeators, i.e. misleading counter evidence. It seems that if it were possible to give such a defeasibility analysis of knowledge by introducing a fourth condition, then it should also be possible to incorporate the defeasibility condition in the justification condition for knowledge. This could e.g. be done by saying that a person S is justified in holding p if, and only if, there is no true evidence **e** such that if S were justified in holding **e** then S would no longer be justified in holding p. The analysans in the previous sentence should, of course, be replaced by one which **does** avoid the Tom Grabit counter example and related counter examples, and also note that the previous sentence cannot amount to a definition of the notion of **justification** (as that would be circular) but can only amount to a recursive expression of a property which the notion of **justification** may have. Our point here is a simple but general one, viz. that knowledge in addition to justification, truth and belief may be transformed into a traditional analysis of knowledge by incorporating the further conditions in the justification condition. The resulting notion of **justification** may then not be our normal notion of

justification, but that would only show that it may be somewhat unnatural to make such a transformation, not that it cannot be made. And although it may be the case that the resulting notion of **justification** may be somewhat unnatural if a defeasibility condition or some other further conditions were built into it, we should on the other hand not forget but keep in mind that the further conditions for knowledge that have been proposed are in general **epistemic** conditions.

Let me in the following, in order to support my claim that a defeasibility condition can be built into the justification condition, outline a defeasibility analysis of knowledge which has the defeasibility condition built into the justification condition.

Although the analysis which I will outline in the following few pages is a defeasibility analysis of knowledge, it also differs substantially from defeasibility analyses in the following two respects: It is a recursive analysis, and it is also holistic in the sense that **all** justified information might be relevant. The defeasibility analysis which I sketch may also be claimed to supersede other defeasibility analyses as it has the advantage that it avoids the objection based on the Tom Grabit example given above.

Suppose a person S thinks himself justified in holding p. It then seems that S will be committed to the claim that any normal or superior person with any further justified information about the world apart from p who also held p would be justified in holding p; and conversely, if S were to think that any normal or superior person with any further justified information about the world would be justified in holding p, then S would thereby be committed to the claim that he himself is justified in holding p. It follows as a corollary, that when S thinks himself justified in holding p, he is committed to the claim that he himself, at any later time, on the basis of any further justified information (apart from p), would continue to be justified if he still were to hold p. As above, a converse statement also holds.

It does not seem unreasonable to think that the best explanation for why we have this is that what we are committed to **is** true when what we think **is** right, i.e. that a person **is** justified in holding p iff he at any later time, on the basis of any further justified information (apart from p), would continue to be justified if he still were to hold p.

If we let "$V_t sp$" denote the set of justified information that S has at the time t, apart from the information that p, or more formally $\{q|J_t sq\}/\{p\}$, we can specify what we have said more precisely:

$J_t sp \Leftrightarrow \Box(\forall t`)(t` > t \ \& \ (V_t sp \subset V_{t`} sp) \ \& \ B_{t`} sp \Rightarrow J_{t`} sp)$4

Here "$B_{t`} sp$" signifies that s believes that p at t`, and "$\Box$" is the symbol for necessity, such that we in this case can read the whole expression as follows: "s is justified in holding p at time t iff it is necessary that for all times t` later than t, that if (the condition) then $J_{t`} sp$."

It now seems that a kind of bifurcation can be generated from the Gettier-examples, showing that the epistemic subjects in these situations are **not** justified. Let us, in order to show this more clearly, consider Gettier's classical example of Smith and Jones. Consider the following statements:

a) Jones is the man who will get the job, and Jones has ten coins in his pocket.

b) The man who will get the job has ten coins in his pocket.

Smith has strong evidence for a), and a) entails b). Suppose Smith is aware of this entailment, and that he accepts b) because of a) at the time t. Smith is, according to Gettier, justified in believing b) at t. Now, Smith, unbeknownst to himself, gets the job, and he happens, without knowing it, to have ten coins in his pocket. In this case Smith believes a true sentence, viz. b), and is, according to Gettier, justified in holding b). But we would not want to say that Smith knows b), because b) is true for another reason than the one Smith thinks.

According to the property of justification which we have invoked, however, Smith is not justified in holding b) at time t. In some possible developments of history, Smith could at a **later** time obtain more information relevant to b), and **then** be justified in holding b), and then also know b), e.g. if he were to get the information that he gets the job **and** that he has ten coins in his pocket. But in some other developments of history he would not be justified in holding b), e.g. because he only were to gather the additional information that Jones does not get the job and that he himself got it, and not the justified information that he himself has ten coins in his pocket. In this case Smith would not be justified in believing b). It may seem that analogous arguments in similar cases can establish that Gettier-beliefs in general are not justified beliefs, but we will see below that such is not the case.

Let us next consider the argumentstrategy against traditional defeasibility analyses of knowing which was considered above in order to show that it does not raise problems for the definition of knowledge which has been developed here. Suppose s is convinced that Tom Grabit stole a book from a library because he saw him steal the book at time t. Imagine that at a later time Tom's mother says to him: "Tom has an identical twinbrother who is a notorious book thief." Call this sentence S, and the proposition it expresses B. Suppose B is false, and that Tom has no twinbrother. Tom's mother might e.g. be lying, or be suffering from dementia, or from the influence of drugs. Our intuitions are that s knows at t that Tom Grabit stole a book, and the question therefore is whether the defeasibility analysis of knowledge which we have outlined is compatible with this intuition.

It will, according to the analysis given here, be possible at a later time t`, that s has the justified information that s has at t, plus the justified information B` expressed by the sentence S' = "Tom's mother uttered the sentence S". We now claim that the information that B, as opposed to the information that B', is not justified because it is possible for s at a later time than t` to obtain more justified information than at t`, and thereby to become justified in holding ~B, since ~B is true. The claim that it is possible at a later time to become justified in holding ~B if ~B is true, amounts to saying that ~B is an **accessible truth**, where we appeal to the following definition:

$$p \text{ is accessible at } t \quad \text{iff} \quad p \Rightarrow \Diamond(\exists s)J_t sp$$

Let there furthermore be a class of sentences given by the following definition:

p is metaphysical5 at t iff $\Diamond(\exists s)J_t sp$ & $\Diamond(\exists s`)J_t s`{\sim}p$

It follows that if ~B is an accessible and non-metaphysical truth at t, which seems reasonable, then no one can be justified in holding B.

Even if B' is true and accessible, and hence someone can be justified in believing B', it does not follow that anyone is justified in holding B. B` cannot, it seems, justify B unless a set C of conditions like "Tom's mother is honest", "B is uttered in a normal context, and not as part of a play", "Tom's mother is sane", "Tom's mother is justified in holding B" etc... are themselves justified, and this set justifies the person in holding that B. The set R consisting of the negation of the members of C can be looked upon as the set of restorers of ~B. If a sentence r is a restorer of a sentence q for a person s (short: $r_s(q)$) at the time t it will be the case that $J_t s(r_s(q) \Rightarrow q)$. If we appeal to the following closure principle of justification:

ClJ $\quad (J_t sr \;\&\; J_t s(r \Rightarrow q)) \;\Rightarrow\; J_t sq$

we see that $J_t sq$ if $J_t sr_s(q)$. Substitute ~B for q. It follows from r being accessible that $\Diamond(\exists s)J_t sr_s(\sim B)$, and from ClJ and the assumption that r is a restorer of ~B for s that $\Diamond(\exists s)J_t s(\sim B)$, and from the assumption that B and ~B are non-metaphysical it follows that $\sim(\exists s`)J_t s`(B)$. But B' can only be a defeater to the extent that somebody is justified in holding B. It is at best only if somebody were to be justified in holding B, and not by being justified only in holding B', that this person would be unjustified in holding that Tom stole the book. It follows from this, that somebody who continues to believe that Tom stole the book from the library after having come to hold the justified belief B', would not be unjustified in believing this, because there are conditions (e.g. about honesty or dementia) for being justified in holding that B, which, **ex hypothesi**, are not justified in this case. But it is therefore, as can be seen from the form of $\sim J_t sp$, viz. $\Diamond(\exists t`)(t` > t \; \& \; (V_t sp \subset V_{t`} sp) \; \& \; B_{t`} sp \; \& \; \sim J_{t`} sp)$, also not the case that s is unjustified at t in holding that Tom stole the book from the library, because $V_{t`} sp$ is to consist of **justified** information. Given all of this, it would seem that we can arrive at a definition of knowledge of the traditional form as justified, true belief if we incorporate a defeasibility condition in the justification condition for knowledge.

But any such rectifications of a traditional analysis of knowledge which I have pointed out here and which rely upon an incorporation of any suggested further conditions for knowledge in the justification condition for knowledge, are, I think, bound to fail unless the resulting notion of **justification** entails truth. For suppose justification does **not** entail truth. It is then possible that S believes p and S is justified in believing p although p is false. Suppose further that S believes p or q only because S believes p, e.g. in a situation where S strongly disbelieves q but believes p or q nonetheless because S believes p. As it happens, q is true. But clearly, if S is justified in holding p then S is justified in holding p or q. The same holds for indefeasibility. If p is indefeasible then p or q is indefeasible. So even if the justification condition incorporates indefeasibility it is true that if S is justified in holding p then S is justified in holding p or q. It follows from this that all three conditions are fulfilled if justification does not entail truth, for S is in this situatiuon justified in holding p or q, and S believes p or q and p or q is true. But we do not want to say that S knows p or q. What this shows is that unless justification or one of the further epistemic conditions which have been suggested as additions to the traditional analysis of knowledge **entails** truth, the resulting analysis is bound to fail.

But it is unreasonable, I think, to hold that an epistemic condition like justification, whether or not it incorporates indefeasibility or other epistemic conditions, should entail truth. The most natural view to hold is that epistemic conditions like S being justified in holding p, or p being indefeasible for S or it being evident for S that p etc. do **not** entail that it is true that p. It would at the very least take quite a bit of non-trivial philosophical argumentation in order to show that a set of epistemic conditions short of knowledge which a proposition p fulfills for a subject S can only be fulfilled if the proposition p is true. One cannot merely stipulate that only true propositions can be justified or fulfill whatever additional conditions one wants to incorporate in the analysis which one suggests.

Given all of this, it seems to me to follow that a large family of proposed analyses of knowledge are bound to fail. We can at least draw such a conclusion if we want to hold that neither justification nor any other additional epistemic condition entails truth and that it follows from the fact that a proposition p fulfills the epistemic conditions and justification for S that also the proposition p or q fulfills those epistemic conditions and the justification condition. And it seems that these are deciderata which we should want any epistemic conditions short of knowledge to fulfill.

It should here be noted, however, that there are authors who have tried to get around the Gettier type difficulties by either holding that justification entails truth or that justification is not transmitted by deductive inferences drawn by the epistemic subject. Notably, Robert Almeder[6] has argued that one cannot be completely justified in believing a false proposition, and Irving Thalberg[7] has argued against the principle that if you are justified in believing a proposition p and you correctly deduce q from p, where p entails q, then you are also justified in believing q. If anything, these suggested ways around the Gettier problems seem desperate and utterly implausible, and that probably accounts for why these suggestions, to put it mildly, have failed to gain any influence.

1.2 There is a type of examples in the literature which are claimed to be related to the Gettier examples and to show that knowledge cannot simply be defined as justified true belief. The first one to publish this kind of example was Gilbert Harman,[8] and I cite extensively

> "Suppose that Tom enters a room in which many people are talking excitedly although he cannot understand what they are saying. He sees a copy of the morning paper on a table. the headline and main story reveal that a famous civil-rights leader has been assassinated. On reading the story he comes to believe it; it is true; and the condition that the lemmas be true has been satisfied since a reporter who witnessed the

> assassination wrote the story under his by-line. According to an empiricist analysis, Tom ought to know the assassination had occurred. It ought to be irrelevant what information other people have, since Tom has no reason to think they have information that would contradict the story in the paper.
> But this is a mistake. For, suppose that the assassination has been denied, even by eyewitnesses, the point of the denial being to avoid a racial explosion. The assassinated leader is reported in good health; the bullets are said, falsely, to have missed him and hit someone else. The denials occurred too late to prevent the original and true story from appearing in the paper that Tom has seen; but everyone else in the room has heard about the denials. None of them know what to believe. They all have information that Tom lacks. Would we judge Tom to be the only one who knows that the assassination has actually happened? Could we say that he knows this because he does not yet have the information everyone else has? I do not think so. I believe we would ordinarily judge that Tom does not know."

I do not share Harman's intuitions. My view is that this is not a genuine counter example to the standard definition of knowledge, for it seems to me to be false to say that Tom did not know that the civil rights leader was assassinated just because other people were victims of a cover up. Why should the cover up matter? I grant that Tom at a later time, if he becomes aware of the evidence which has misled other people, may himself be misled by the misleading evidence and thus no longer believe that the civil rights leader was assassinated. But that does not at all show that Tom did not know that the civil rights leader was assassinated.

Most commentators have tried to find a way around the Gettier problems while at the same time come to the result that Tom in the above example did not know that the civil rights leader was assassinated. I do not think that such a strategy is a reasonable one. For note the similarity between Harman's example and the example with the demented Mrs. Grabit. In both cases there is some misleading evidence which at a future time may stop S or Tom from believing what he did believe. It seems clear that this is a case where there are conflicting intuitions in the literature, for insofar as we want to say that S did know that Tom Grabit took the book despite the false testimony of Mrs. Grabit we should also want to say that Tom in Harman's example did know that the civil rights leader was assassinated despite the cover up stories which seduced many or most people into believing that no assassination took place. One may here retort that there is a difference between these examples because one may more plausibly be said to be justified in believing the cover up stories than in believing the demented Mrs. Grabit. And I shall be willing to grant that there is such a difference, but I do not see why that difference should make a difference. As I see it, Tom did know that the civil rights leader was assassinated.

But even if one disagrees with me and thinks that there is such a significant difference between the examples discussed above that Tom in Harman's example did not know that the civil rights leader was assassinated whereas S in Lehrer and Paxson's example did know that Tom Grabit took the book, then that is a result which one should be able to get at by some adjustments in one's theory of justification, e.g. by incorporating a defeasibility condition along the lines of our previous section. One could then hold that Tom in Harman's example was not justified in believing that the civil rights leader was assassinated, whereas S was justified in holding that Tom Grabit took the book in Lehrer and Paxson's example. Let me emphasize that I would not be in agreement with such a strategy, as I think that it would be unreasonable to say that S in Harman's example was not justified in holding that the civil rights leader was assassinated. The main point which I want to make here, however, is that Harman's example and related examples do not, as I see it, pose a threat to a standard definition of knowledge as justified true belief. We shall accordingly in this essay concentrate ourselves on more standard type Gettier examples which clearly do show that knowledge cannot simply be defined as justified true belief.

II

Let us consider one of Gettier's counter examples in order to diagnose the problem that it poses for a traditional analysis of knowledge: Smith has strong evidence which justifies him in holding that Jones owns a Ford, and he believes that Jones owns a Ford. Smith picks a city at random, Barcelona, and forms the belief that Jones owns a Ford or Brown is in Barcelona. Smith doesn't believe that Brown is in Barcelona, and has no justification for holding that. But since Smith is justified in holding that Jones owns a Ford, he is also justified in holding that Jones owns a Ford or Brown is in Barcelona. But Jones doesn't own a Ford. Smith has been mislead. Brown, however, happens by sheer luck to be in Barcelona. So Smith has a justified true belief that Jones owns a Ford or Brown is in

Barcelona. But we wouldn't want to say that Smith knew that Jones owns a ford or Brown is in Barcelona.

It seems to me that the general structure of Gettier type counter examples can be extracted by considering this, or any other Gettier type counter example. By "general structure" I mean some features that all Gettier type counter examples have in common. This is a claim that I hope to vindicate by going through several Gettier type examples in later portions of this essay. As for now, I only want to point out what I think these features are, and work from there[9].

Let us call the proposition that Jones owns a Ford p, and the proposition that Jones owns a Ford or Brown is in Barcelona q. It then seems that the Gettier type example has the following features:

1. S believes p

2. S is justified in holding p

3. p entails q

4. S believes q because S believes p

5. S is justified in holding q because of 2, 3 and 4.

6. p is not true

7. q is true

We will use the expression "because" in such a way that a sentence of the form "S believes q because S believes p" is true if and only if the sentence "S believes p" is an adequate and true answer to the question "Why does S believe q?". Or, in other words: S believes q because S believes p if and only if the fact that S believes p explains why S believes q.

We should also note that our analysis is intended to be synchronistic. The analysis offered is of the knowledge of a subject at a given time t, so the because-relation which we invoke should not be taken in a temporal sense.

Feature 5 might raise the following question. Are 2, 3 and 4 generally sufficient for S to be justified in holding q? I think they are. But one might think of something like the following as providing a counter example. Suppose that the Peano axioms, unbeknownst to S, entail Fermat's Theorem. S believes, or claims to believe Fermat's Theorem because S believes the Peano axioms for arithmetic, and is justified in holding the Peano axioms. The features or conditions 2, 3 and 4 above are thus fulfilled, but would we in such a case as this say that S was justified in holding Fermat's Theorem? No. Not at all. Nor would we, however, say that S believes Fermat's Theorem simply because S believes the Peano axioms, as S crazily claims. And this is so because the fact that S believes the Peano axioms does **not** explain why S believes Fermat's theorem. It must in some sense be evident that p supports q in order for it to be the case that the fact that S believes p explains why S believes q. For S to believe q because S believes p, we shall accordingly require that the fact that p supports q has to be recognized by anyone that understands p and q. Note that the evidence requirement that we have introduced is not an **additional** requirement for what it takes to say that someone believes something because she believes something else. We still have that S belives q because S believes p if and only if the fact that S believes p explains why S believes q. The evidence requirement that we are suggesting is built into the notion of **explanation** that we used in the definiendum in the definition of "S believes q because S believes p". With this evidence requirement for explanations, which is thereby also a requirement for what it takes to believe something because of something else that is believed, I think that 5 is quite plausible. So if S is justified in holding p, and p entails q and S believes q because S believes p then S is justified in holding q.

It should at this point be noted that "because" in the formulation "S believes q because S believes p" is not a transitive relation. That is to say, that even if S believes q because S believes p, and S believes r because S believes q, it need not be the case that S believes r because S believes p. For the evidence requirement we have been invoking may not hold for the entailment from p to r. This may be easier to see if the explanatory chain is somewhat long. Suppose S were fortunate enough to find a proof of Fermat's theorem (FT) in Peano arithmetics, and started his proof with the Peano axioms (PA). It is by no means evident to anyone who understands PA and FT that PA supports FT, so it cannot be the case that S believes FT because S believes PA. But we were imagining that S had found a proof of FT based upon PA. So S must have gone through a series of steps from PA=s(1) through s(2), s(3).... to FT=s(n) in his proof. For each i, $1 \le i \le n$, it must be the case that S believes s(i+1) because S believes s(i), as we do require mathematical proofs to be transparent to those who understand what is going on. But this shows that believing something because you believe something else is not a transitive relation, as S does not believe FT simply because S believes PA. There is, however, an important sense in which S's belief in FT is based upon his belief in PA, and we will in the following try to explicate the sense in which this can be said.

It seems that we can divide the set of beliefs that a person has into those beliefs which (1) she has because of some other belief or beliefs that she has and which (2) are entailed by the latter, and those that do not stand in both of these relations to other beliefs. We shall call the former type of belief **secondary beliefs**, and the latter type we will call **primary beliefs**. (A more refined definition willl be given later.) The reason why we are interested in secondary beliefs should be clear from the list of the seven features of Gettier type beliefs given above. If we consider features 3, 4, 6 and 7, we can see that Gettier type of beliefs do stand in these two relations to other beliefs. In addition, Gettier beliefs are such that they do not entail the belief or beliefs because of which they are held. Features 6 and 7 make sure that the Gettier belief does not entail the belief because of which it is held, and feature 3 and 4 only restate the two relations that I have been invoking in drawing the distinction between primary and secondary beliefs. Since 4 entails 1, we can see that the two relations invoked capture features 1, 3, and 4 of the features that I have claimed are common for Gettier type beliefs.

On the basis of these considerations, our strategy will roughly be to hold that primary beliefs are known iff they are justified and true beliefs and that secondary beliefs are known iff there is a primary belief because of which the secondary belief is believed and this primary belief is also a known, i.e. a justified and true, primary belief. In our example above, we can then see that Smith does not know that Jones owns a Ford or Brown is in Barcelona. For Smith believes that Jones owns a Ford or Brown is in Barcelona because Smith believes that Jones owns a Ford. Smith does not, however, know that Jones owns a Ford, since it is not true that Jones owns a Ford. If Smith's belief that Jones owns a Ford is a primary belief, and there are no other beliefs than Smith's belief that Jones owns a Ford because of which Smith believes that Jones owns a Ford or Brown is in Barcelona, our strategy can easily be seen to work as the primary belief because of which Smith believes that Jones owns a Ford or Brown is in Barcelona is not known. Suppose Smith's belief to the effect that Jones owns a Ford is a secondary belief. There is then at least one primary belief p which entails that Jones owns a Ford and is such that Smith believes that Jones owns a Ford because Smith believes p. But p cannot possibly be a justified and true primary belief. For if it were true that p and p entails that Jones owns a Ford it would also be true that Jones owns a Ford. But it is not true that Jones owns a Ford. So there can be no justified and true primary belief because of which Smith believes that Jones owns a Ford which also entails that Jones owns a Ford. This shows that our strategy gives the desired result that Smith does not know that Jones owns a Ford or Brown is in Barcelona.

One might at this point wonder why I have not included a third condition of the following form in my definition of secondary beliefs: "A belief because of which a secondary belief is believed is not entailed by the secondary belief." After all, Gettier beliefs seem to have this property that they do not entail the beliefs because of which they are held (features 6 and 7 make that clear), and it is with Gettier type of beliefs that we are concerned.

But the suggested requirement would, or at least could, give rise to a problem of the following kind. Suppose S believes q because S believes p, where p is a primary belief which is false and justified and true, whereas q is a secondary belief fulfilling the suggested requirement (i.e. q does not entail p). Suppose further that S believes r because S believes q, where q is logically equivalent with r. (Since circular explanations are no good, it cannot at the same time be the case that S believes q because S believes r.) Suppose further that there are no other beliefs because of which S believes r. If we were to adhere to the requirement that a secondary belief cannot entail a belief because of which it is held, we would have to conclude that r is a primary belief. But then r would, if we were to follow the strategy that we are considering, be known iff r is a justified and true belief. But we have assumed that q, the belief because of which S believes r, is a Gettier type of belief. So q is not known iff q is a justified and true belief. But r is logically equivalent with q. So r cannot be known iff r is a justified true belief. So r should not be counted as a primary belief, but would have to be counted as such a belief if we were to adopt the suggested requirement.

Furthermore, if we made use of such a non-entailment requirement we would not be able to think e.g. of a mathematical theorem q which is shown in a proof by S to be equivalent to a theorem p as something q which S believes because S believes p. This should suffice as an explanation for why we do not include the suggested non-entailment requirement in our definition of secondary beliefs.

In order to arrive at a working definition of knowledge, however, we need to define a relation that holds between the beliefs of a subject and which is also a transitive relation. We need, in other words, to find a way of stating that a belief q of a subject S is a belief that S has either because it is the case that S believes q because S believes p where p is a primary belief, or because it is the case that there are beliefs p and p' such that S believes q because S believes p' and S believes p' because S believes p and p is a primary belief, and so on for longer explanatory chains of beliefs. We also need to keep the requirement that p entails q. This we do by defining the ancestral of the relation "S believes q because S believes p and p entails q". We need to sharpen, or slightly revise, our distinction between primary and secondary beliefs. In order to do that, we have to invoke a series of definitions.

First we define:

1 $B_S pq =_{df}$ S believes q because S believes p and p entails q.

We then define the notion of B_S-heredity:

2 α is B_S-hereditary$=_{df}(p)(q)((p\in\alpha \ \& \ B_S pq) \supset q\in\alpha)$

Informally, this says that a class of propositions α is BS-hereditary iff for any two propositions p and q where p is in α and S believes q because S believes p and p entails q then also q is in α. The empty set is trivially BS-hereditary, as are all sets that contain none of the beliefs that S has. Also, a set can be BS-hereditary and contain some or all of the beliefs that S has, and it may in addition contain propositions that S does not believe. BS-heredity is therefore by itself not sufficient to arrive at anything of substance. We therefore define:

3 q is secondary to p for S $=_{df} (\alpha)((\alpha$ is B_S-hereditary $\& \ (r)(B_S pr \supset r\in\alpha)) \supset q\in\alpha)$

This definition requires some explanation. First note, that another way of expressing that a set α is BS-hereditary is to say that the set α is closed under the relation BS, i.e. if p is in α and BSpq then also q is in α. Definition 3 thus says that a belief q is secondary to p for S just in case q is in all sets α which are closed under the BS relation and also contain all beliefs r which stand in the BS relation to p. What do we achieve by this definition? Suppose a belief q of S is such that S believes q because S believes p_1, and S believes p_1 because S believes p_2, ... and S believes p_{n-1} because S believes pn, where n≥1. This is what we informally mean by saying that q is a belief of S secondary to S's belief p_n. By our definition, q is secondary to p_n just in case q is in all sets closed under the BS-relation which also contain all beliefs r that stand in the BS-relation to p_n. Let β be any such set of statements, and suppose q is secondary to p_n for S. If, in fact, S believes p_n (as we have assumed), then, since β contains all beliefs that stand in the BS-relation to p_n, also p_{n-1} is in β. But β is closed under the BS-relation, and since $BSp_{n-1}p_{n-2}$, ..., BSp_1q, clearly also q is in β. We have thus shown that the definiendum holds if q is in fact a belief of S which is secondary to a belief p of S in our informal sense.

Let us, before we show that the truth of the definiendum in 3 guarantees that a belief q is secondary, show that S must believe p in order for it to be the case that some belief q is secondary to p for S. Suppose S does not believe p. In that case the second conjunct of the antecedent in the definiendum in 3, i.e. the sentence $(r)(B_S pr \supset r \in \alpha)$, is true for all sets α, since it for all r is false that BSpr. So it follows from definition 3 that if q were secondary to p for s and S did not believe p then q would be in all BS-hereditary sets α. But the empty set is, as we pointed out, a BS-hereditary set, so a statement q cannot be in all BS-hereditary sets. So if q is secondary to p for S then S believes p. In other words, the definiendum in 3 is always false if it is not the case that S believes p. This shows that our definition gives the desired result that if q is secondary to p for S then S believes p.

Let us now show that the truth of the definiendum guarantees that a belief q is secondary to p for S in the informal sense provided there are beliefs r that stand in the BS-relation to p, i.e. such that BSpr is true. Suppose the definiendum is true with this proviso. We then know that S believes p and that q is in all BS-hereditary sets which also contain the beliefs r that stand in the BS-relation to p. In other words, all BS-hereditary sets which contain the beliefs r such that BSpr also contain q. But then also the BS-hereditary set β which only contains the beliefs r such that BS-pr and in addition those beliefs that must be in β because of β's BS-heredity must contain q. Clearly β is a subset of all other sets that fulfill the condition of the antecedent, so if q is in β then q must be in all sets fulfilling the antecedent of the definiendum. It therefore suffices to assume that q is in the described set β. But if q is in β, then either q is one of the r's such that BSpr, or there is one r such that BSpr and BSrq, or there is one r and one s such that BSpr, BSrs, and BSsq, and so on. So if there are beliefs r such that BSrp then the truth of the definiendum guarantees that q is secondary to p for S. Also note that this shows that if q is secondary to p for S then S believes q, so that we in combination with our result in the last paragraph have: If q is secondary to p for S then S believes p and S believes q.

It only remains to consider the situation when there are no beliefs r such that BSpr. In that case q cannot be secondary to p, so we want the definiendum to be false. But if there are no beliefs r such that BSpr then BSpr is always false so the second conjunct of the definiendum is true for all sets α. But the statement q cannot be in all BS-hereditary sets since the empty set is BS-hereditary. So the definiendum is false if there are no beliefs r such that BSpr.

We will now give some definitions of vocabulary:

4 q is a secondary belief for S $=_{df}$ $(\exists p)$(q is secondary to p for S)

Next:

5 p is a primary belief for S $=_{df}$ S believes p & p is not a secondary belief for S

Furthermore:

6 S believes q since S believes p $=_{df}$ q is secondary to p for S

7 S believes q on the basis of p $=_{df}$ p is a primary belief for S and S believes q since S believes p.

8 p induces q for S $=_{df}$ S believes q on the basis of p.

We will finally define the following notation:

9 $B_S(q, p_1, \ldots, p_n)$ $=_{df}$ "p1, ..., pn" is a complete list of beliefs that induce q for S.

It follows from these definitions that "$p_1, \ldots, p_n$" in $B_S(q, p_1, \ldots, p_n)$ is a list of all and only those beliefs that are primary beliefs for S that are such that S believes q since S believes $p_1, \ldots$ etc. (If q is a primary belief, i.e. if n=0, we simply write $B_S(q)$.)

Although such lists as the ones referred to in 9 exist, it would of course be difficult, not to say impossible, for us to list any of them, and different individuals will not generally have the same list corresponding to the same secondary beliefs. Our notation will, however, be useful in stating the definition of knowledge that we give in chapter IV.

Let us finally make the note, that we will assume that if $B(q,p_1,\ldots,p_n)$, $n>0$, then the beliefs $p_1,\ldots, p_n$ are severally sufficient for S to believe q. We moreover assume, that if S were to have the same beliefs that she has except the beliefs p_i, $0<i\leq n$, then she would also not believe q. This assumption follows from what we have said about the nature of the because-relation which we are

invoking.

III

I think that we with the definitions in the previous chapter have done much of the groundwork for arriving at an adequate definition of knowledge. This is something I hope to show in the next chapter.

In this chapter I want to show that the since-relation which we have invoked in "S believes q since S believes p" is a transitive, anti symmetric and anti reflexive relation, and that the set of primary beliefs for that reason is nonempty. I then go on to say more about what kind of beliefs that are primary, and to show that primary beliefs should not be confused with basic beliefs in the foundationalist sense.

That the since-relation is anti symmetric and anti reflexive follows from the fact that we do not accept circular explanations as genuine explanations, and the since-relation is defined in terms of the because-relation, which in its turn is defined in terms of the notion of an explanation. As the fact that S believes q cannot explain why S believes q, it cannot be the case that S believes q because S believes q. Could it be the case that there were a chain of the form: S believes q because S believes p_1 and .... and S believes p_n because S believes q? We would then have to say that the fact that S believes q explains why S believes p_n and .... the fact that S believes p_1 explains why S believes q. But we do not want to accept such a circular chain of propositions which are supposed to explain each other. Explanations should not be circular. It therefore follows that the since-relation is anti reflexive and anti symmetric.

It can also be proved that the since-relation is a transitive relation. In fact, the ancestral of any relation can be shown to be transitive, and the since-relation is, as we know, the ancestral of the relation BSpq (i.e. S believes q because S believes p and p entails q). But let us give an informal proof of this. Suppose S believes q since S believes p and S believes r since S believes q. In this case it is clear that there is a finite number n of beliefs t, t', t''.... such that S believes r because S believes t, S believes t because S believes t', S believes t' because S believes t'' etc. , where each t followed by i "'"'s, $0<i\leq n-1$, entails the t which is followed by i-1 "'"'s. One of these t's followed by a finite number of "'"'s would be q, and the one with n-1 "'"'s would be p. As entailment is transitive, it is then clear that S believes r since S believes p. So the since-relation is clearly a transitive relation.

We can, as pointed out in the previous chapter, define a subset of the beliefs that S has which are believed since some other beliefs that S has (and consequently entailed by those latter beliefs) are believed (secondary beliefs), and those beliefs that do not stand in that relation to other beliefs (primary beliefs). We now want to show that the set of primary beliefs must be non-empty.

We will let "Bp" stand for "S believes p", and "B" stand for "the set of all beliefs that S has". The corresponding abbreviations for justification ("Jp" and "J") and knowledge ("Kp" and "K") will be used later. We have defined B^S (secondary beliefs) as the set of propositions q such that there is at least one proposition p such that q is believed since p is believed.

More formally we could express this as

$$B^S = \{q:(\exists p)(Bq \text{ since } Bp)\}$$

We call the set B^S the set of secondary beliefs. The definition above could also be stated by saying that a belief q is secondary if and only if q is believed since some other belief p_1 is believed and........and q is believed since some other belief p_n is believed, where n is larger than or equal to one. We call the set $B^P = B/B^S$ (B minus B^S), i.e. all beliefs that are not secondary beliefs, the set of primary beliefs.

The set B^P cannot be empty. We could show this by giving examples of primary beliefs. That is something which we will do later in this chapter. Let us at this point be more systematical.

Suppose B^P is empty. It would then be true for each of my beliefs that I have that belief since I have some other belief(s) (where the latter entail(s) the former). But the relation that we are invoking is transitive, anti-symmetric (i.e. if I believe q since I believe p then it is not the case that I believe p since I believe q) and anti-reflexive (i.e. I do never believe p since I believe p). This has as a consequence that we can not have a cluster of beliefs such that I believe one since I believe any of the other. The since-relation forms a unidirectional tree, or sometimes a chain. So if all of my beliefs stand in the since relation to some other belief that I have, it must be the case that I have infinitely many beliefs.

But can it be the case that I have infinitely many beliefs? At least I cannot have infinitely many beliefs that can be verbalized in the sense of our disquotation principle DP (see Appendix 2), and it is with beliefs that can be verbalized that the Gettier problem is concerned since otherwise we would be at pains of stating the problem.

But why can I not have infinitely many verbalizeable beliefs? Because there must be a finite limit of the number of characters in a sentence for me to be able to understand that sentence, and there is only a finite number of characters that I can understand, so there is only a finite number of sentences that I can understand. But I have to understand what I believe, so I can only have a finite number of verbalizeable beliefs.

Some people may, however, hold that we have an infinite number of beliefs in some de re contexts. One might e.g. want to say that we believe of each natural number that it is nonnegative, and not only that we believe that all natural numbers are nonnegative. If so, it would not, I think, present a problem for my arqument to the effect that the set B^P is non-empty. For these alleged **de re** beliefs could not form an infinite branch or chain of the form: S believes A(1) since S believes (A2), and S believes (A2) since ... etc. We would rather have an infinite branching of the tree, such that all the de re beliefs of particular natural numbers are beliefs I have since I have the de dicto belief that all natural numbers are nonnegative.

Since we have only a finite number of beliefs, and since the set B^P is not empty for that reason, we can arrive at the following useful result by appealing to König's Lemma which says that a tree with an infinitely long branch has an infinite number of nodes. It then follows by modus tollens that we cannot have an infinite series of the form "S believes p_1 since S believes p_2 and S believes p_2 since..........". This means that any such series, whether a chain or a branch of a tree, must terminate in a belief that is in B^P. As the since relation is transitive we can then see that any secondary belief is induced by at least one primary belief.

And note that this is at it should be. For we have been assuming that the fact that S believes q because S believes p iff the fact that S believes p explains the fact that S believes q. So if a proposition q such that S believes q were preceeded by an infinite chain of beliefs primary to q, we would be faced with a situation where we would have an infinitely long explanation of the fact that S believes q. But explanations must come to an end, or at least so we assume.

On the basis of these considerations we can therefore conclude that the set B^P cannot be empty. And this gives us the result which we wanted to establish, viz. that there **are** primary beliefs.

What kind of beliefs, then, are primary beliefs? We know that a primary belief is a belief which is such that it is not believed because of some other belief which entails it. Given our definition of the because-relation, it then follows that a belief p is a primary belief for S iff there is no belief p' which S has that is such that the fact that S believes p' explains why S believes p, and p' entails p. There are therefore two different kinds of primary beliefs that a subject S can have, viz. those beliefs that S has which are such that there are no beliefs which explain why S has them, and those that are such that although there are beliefs that S has which explain why S has them, none of these explanatory beliefs are entailing them. I shall here focus upon the first type of cases, i.e. on the kind of primary beliefs where it can be said that there are no beliefs p' which S has that are such that the fact that S believes p' explains why S believes p, as we shall spend a fair amount of time on the second kind of primary beliefs in a later portion of this essay.

One may think that the kind of primary beliefs that we are focusing upon can be identified with non-inferential beliefs, as it seems plausible to assume that a belief q is inferred from a belief p by S iff the fact that S believes p explains, either directly or through an explanatory chain, why S believes q. There is, however, an objection to this assumption which goes along the following lines. What if S jumps to a conclusion q on the basis of p in a situation where it is not at all the case that it is recognized by anyone who understands p and q that p supports q? Given the evidence requirement that we introduced in chapter II, we would then have a situation where it is not the case that the fact that S believes p explains why S believes q. But S inferred q from p. It therefore seems that there are cases where S believes q and there is no belief p which is such that the fact that S believes p explains why S believes q, but q is, it seems, an inferential belief nonetheless.

What should we say about this? We should first note that such cases are not likely to be really important ones, and, more importantly, that it will not be a problem for our account that there are such cases. It will only go to show that there can be inferential beliefs which are primary because no beliefs explain why they are held. One may think that one can make a good case for holding that there can be no such situations as the one which was just suggested. For, given our disquotational principle DP (see Appendix 2), it seems that it must be the case that S should be able to account for her own inference from p to q in such a way that it is obvious to anyone that understands p and q that p supports q. S should be able to account for the inference in the somewhat indirect sense that S should, so we may assume, be disposed to assent to sentences which express propositions which are intermediaries of p and q and thus make the inference from p to q more transparent. Only, this does not seem to work if S has committed a rather stupid fallacy, for it is then **not** obvious to anyone who understands p and q plus the extra reasons which could be given by S that p supports q. This goes to show, then, that the set of primary beliefs which are not held because of some other belief is not identical with the set of non-inferential beliefs. Rather, the latter set is a proper subset of the former set, i.e. all non-inferential beliefs are primary beliefs which are not held because of some other belief, but not all primary beliefs which are not held because of some other belief are non-inferential belief. This is a consequence of the fact that we have built in an evidence requirement into our notion of **explanation.**

But although the set of primary beliefs which we are now considering cannot be identified with the set of non-inferential beliefs, it seems to be safe to assume that the members of that set for the most part are non-inferential beliefs. In particular, this is likely to be true if the epistemic subject we are considering is a rational subject. We shall accordingly in the following focus upon the primary beliefs which are non-inferential beliefs, and we shall in later portions of this essay even pretend that non-inferential beliefs can be identified with the beliefs that are such that no other beliefs explain why they are held.

Our question then becomes: What kind of beliefs are non-inferential beliefs? We do not intend to make any attempts to deviate from the tradition by trying to categorize any beliefs as non-inferential which are not commonly thought of as being non-inferential, but we shall suggest the following linguistic criterion for identifying beliefs as non-inferential beliefs.

LC   S's belief in proposition p is non-inferential if S would give an appropriate answer to the question "Why do you, S, believe p?" by asserting a sentence s which expresses the proposition p.

Note that we are not suggesting that a belief in a proposition p is non-inferential **only if** a sentence which expresses p is an appropriate answer to the question "Why do you believe p?", as there may, for all I know, be other types of propositions which are plausibly categorized as non-inferential beliefs. We are only suggesting that a large family of non-inferential beliefs can be identified by means of criterion LC.

Given LC, it follows that many, or maybe most, non-inferential beliefs are beliefs which are intimately related to our sources of knowledge, e.g. to perception or to memory, or to our faculty of reasoning in the case of beliefs which we think of as *a priori* beliefs.

If e.g. S believes that S sees that there is a green tree, it would be appropriate for S, if queried, to answer the question "Why do you, S, believe that you see that there is a green tree?" with the sentence "I see that there is a green tree". Similar remarks can be made with respect to other predicates which take agents and propositions as argument values and are intimately related to our sources of knowledge, e.g. "remember that", "hear that", "being told by _that" and "read that". Some of these, such as "see that" and "remember that" must be understood in a veridical sense, whereas e.g. "read that" may not always be veridical. One cannot see that there is a green tree if there is no green tree, but one may read that Malta is an ugly island without it being the case that Malta is an ugly island.

Note that we in the case of "see that" also would want to include the nonperceptual notion of "see" which is used e.g. in sentences like "She could finally see that the sum of 18 and 5 equals 23". The use of "see" in the sentence mentioned is only metaphorically related to the visual faculty, and does instead signify that the child by means of her faculty of reasoning came to realize that 18 and 5 is 23. By incorporating this use of "see", we see that our analysis is able to accomodate *a priori* knowledge, at least if we, as seems reasonable, by *a priori* knowledge mean knowledge which can possibly be arrived at solely by means of our faculty of reasoning.

Let us at this point emphasize that the set of primary beliefs is not a set of some kind of foundational beliefs or basic beliefs. For it follows from our discussion that primary beliefs, unlike foundational or basic beliefs, can be utterly unjustified and/or false. A person may e.g. have the primary belief that he can trust the missionary who told him that there is one God, Allah, and that Mohammed is his prophet. Many of us would think that the belief is both false and unjustified. And no one would think that all such primary beliefs, if we e.g. consider similar primary beliefs that pertain to other world views, could be true and justified.

But although primary beliefs need not be justified, I think that primary beliefs which are non-inferential are at least *prima facie* justified. If e.g. a person believes that she sees a green tree, then she is, I think, as a rule of thumb, justified in believing that she sees a green tree. The person is, however, in such a case, not justified in believing that she sees a green tree simply because she believes that she sees one, or because such a belief, given our LC, would count as a non-inferential belief. That this must be so, should be pretty obvious, for a person may e.g. believe that she sees a green tree while under the voluntary influence of a hallucinatoric drug, or she may have been informed by a reliable physician that her colorvision is unreliable, and so on. In more normal cases, however, we would want to say that a person who believes that she sees a green tree is justified in believing that she sees a green tree.

One might also want to hold, possibly somewhat more controversially, that a person S is **prima facie** justified in believing that she has been informed by T that it is the case that p if S believes that she has been informed by T that it is the case that p. If such were the case, it would not mean that it would be extremely rarely the case that one is not justified in believing what one is being told although one does believe it. What one would want to say, rather, would be, I take it, that it takes something out of the ordinary for us not to be justified in believing that we are being informed (in the veridical sense) if we believe that we are being informed, whereas we in ordinary cases do not need any specific **reason** in the sense that we have to believe certain specific propositions in order to trust the people that we communicate with. This would, it seems, provide some evidence for holding that a belief to the effect that one is being informed by someone should count as a non-inferential belief. But even if such a view is tenable, note that if we e.g. are justified in holding that the person we are dealing with is a notorious liar, or a politician, this would seem to undermine our justification in believing that we have been informed by that person. Similarly, if we have good reasons not to believe what we are being told because of other justified beliefs that we may have, this may undermine our justification in believing that we have been correctly informed about anything.

Note, however, that one in the case with "informed that" cannot appeal to LC in order to show that a belief that S has to the effect that she was informed by T that it is the case that p should count as a non-inferential belief. For one may plausibly argue that it would be appropriate for S to answer a why-question by saying that she can trust T and T told her that p, and that it would **not** be appropriate for her to answer such a question by saying that she was informed by T that it is the case that p. But it does, however, seem pretty clear that S's belief to the effect that T told her that p **would** count as a non-inferential belief according to our criterion LC. The more controversial aspect of the claim above to the effect that our beliefs about being informed by someone are prima facie justified can then, if the analysis suggested here is appropriate, be reduced to the question as to whether our beliefs to the effect that we can trust people are *prima facie* justified.

But to believe that you can trust someone is something you do because you believe **in** that person, and there seems to be an important difference between *believing that* and *believing in*. We here seem to have a distinction which is similar to the distinction between *knowing that* and *knowing how*. That S knows that p signifies that S stands in a specific relation to the true proposition p, so that we may say that S has a propositional attitude when S knows that p. But when we e.g. say that S knows how to drive a car, no such propositional attitude is to be found which can serve as an analysans. Simlarly, I take it, when S believes that p that signifies that S has a propositional attitude, viz. a belief, towards the proposition p, whereas when a person S believes **in** a person T, we have to do with a non-propositional attitude. To believe in a person is to trust that person. The relevance of this is that it seems unnatural to ask for any beliefs which are primary to S's belief to the effect that S can trust T. Yes, we may say that S believes that she can trust T because S believes in or trusts T, but to believe in T is not the same as to have a propositional belief. All of this, then, goes to show that we can treat S's belief to the effect that she can trust T as a primary belief. It still makes sense, though, it seems, to ask whether one is justified in believing in a person. But it seems that one in normal situations are *prima facie* justified in believing in the people with which one communicates, and that it takes something out of the ordinary for one to not be justified in believing in them. Our argument for this would here be the same as the one presented above, viz. that we do not seem to need any **reasons** in the sense that we need to believe certain propositions in order to trust someone. If we at all can be said to have any **reasons** for believing in people in normal situations, then these would, it seems, be based on factors such as context and body language, and such reasons would not be such that we could verbalize them. But also note, that we are not committed to the view that we do not need any propositional reasons in order to trust someone. If we are right in holding that a belief to the effect that one can trust some person T is a primary belief which relies upon the fact that one trusts or believes in T, then it does not matter to our analysis whether the justification that one has for trusting T relies upon further propositional beliefs or not.

We have seen that one can make a very good case for holding that a belief to the effect that one has been informed by someone should not be counted as a non-inferential belief. We shall, however, in parts of this essay for the sake of simplicity pretend that they are in fact non-inferential. This should, I think, create no problem, as we have seen how we can find an inducing belief for such beliefs. For we have argued that S believes that she has been informed by T that it is the case that p because S believes that she can trust T and that she was told that p is the case by T, and this latter belief which S has is a primary belief which induces the former.

All of this, however, raises the question as to what it means to say that someone is justified in believing something. I shall have more to say about that problem in other parts of this essay, but let it here be remarked that I intend to be as neutral as possible with respect to the question as to what theory of justification which is the most appropriate one. The solution which I propose of the Gettier problem does not depend upon any particular theory of justification.

IV

We will in this chapter explicate an important assumption which underlies the traditional definition of knowledge as justified true belief by means of some elementary set theoretic reasoning, and we then go on to give an argument by analogy in order to show that the underlying assumption must be given up. The chapter then concludes with a suggested revision of the traditional analysis of the concept of knowledge which relies upon our distinction between primary and secondary beliefs.

Let "T" denote some very large set of true propositions that includes all true propositions that S ever believes. We will introduce the following notation: $p \in C(A)$ (read p is a member of the consequences of A, or shorter: p is a consequence of A) iff A entails p. The traditional definition of knowledge that is presupposed in the Gettier type examples can then, if we incorporate our distinction between primary and secondary beliefs, be stated as follows:

I $\quad p \in K$ iff $p \in C(J \cap B^P) \cap C(T) \cap B$

(p is known iff p is a consequence of a justified primary belief and a consequence of something true and p is believed)

But, since $B = B^P \cup B^S$, this is equivalent to

II $p \in K$ iff $p \in C(J \cap B^P) \cap C(T) \cap (B^P \cup B^S)$

(p is known iff p is a consequence of a justified primary belief and a consequence of something true and either p is a primary belief or p is believed since a primary belief is believed (i.e. p is a secondary belief))

This gives us

III $p \in K$ iff $p \in C(J \cap B^P) \cap C(T) \cap B^S$

(p is known if p is a consequence of a justified primary belief and a consequence of something true and p is a secondary belief)

We can now see why the Gettier example succeeds. Let p be "Jones owns a Ford" and q be "Brown is in Barcelona", and let p' be the primary belief such that Smith believes p since Smith believes p'. p' could e.g. be "Jones has truthfully and correctly told me (i.e. informed me) that he owns a Ford', or p' could simply be p (e.g. if Smith's evidence for p does not entail p? See below.). pvq is in K because p' is in $J \cap B^P$ (and hence pvq is in $C(J \cap B^P)$) and q is in T (and hence pvq is in C(T)) and pvq is in B^S.

There is some evidence for holding that the traditional definition of knowledge is not captured by I, but that it should, given our set theoretical apparatus, rather be rendered as follows:

IV $p \in K$ iff $p \in C(J) \cap C(T) \cap B$

Nicholas Rescher10 has e.g. given the following argument against the traditional definition of knowledge as justified true belief. Suppose S is justified in holding p and p is true, but p does not at all believe p, and suppose further that S believes q but that S is not justified in holding q and q is not true. Rescher assumes that the following two closure principles for belief and justification are true, viz. that if a subject is justified in believing a proposition p then that subject is justified in believing the proposition that p or q, and that if a subject believes a proposition q then that subject also believes the proposition that p or q. But it then follows that S, who believes the unjustified and false proposition q and who is justified in holdinq p, where p is true but not believed by S, has a justified true belief in the proposition that p or q. But we would as before not in such a situation want to say that S knows that p or q.

Rescher's argument, we should note, suffers from two defects. His argument is not supported by a genuine example, and it seems furthermore to be quite implausible to assume the closure principle for beliefs which he makes use of. It is, it seems, not generallly true that beliefs are closed under logical consequences. If our analysis in Appendix 2 is correct, it is a necessary condition that a proposition be grasped by the subject in order for it to be believed by the subject. So it cannot be true in general that if a subject believes q then she believes p or q. To see this, suppose e.g. that the subject is a child who believes that snow is white. We do not then want to say that she e.g. believes that snow is white or the continuum hypothesis is true, for she doesn't even grasp the proposition that snow is white or the continuum hypothesis is true.

It may, however, it seems, on the basis of our disquotational principle DP be possible to generate a genuine counterexample to the traditional definition of knowledge as it is rendered by IV without assuming any closure principles for belief statements. Suppose Andrea, who is the mother of John, is the victim of wishful thinking. She has overwhelming evidence for believing that John is into drugs and that he does not attend school. But she believes that John is drug free and attending school nonetheless. So we can say of Andrea that she is justified in holding that John is into drugs and that she does not believe that John is into drugs. On a given day when John is supposed to be at school, Andrea would be disposed to assent to the sentence "John is at school", so, given DP, she believes that John is at school. As it turns out, John isn't at school, rather, he is using drugs. But clearly Andrea would also be disposed to assent to the sentence "John is at school or John is using drugs", even though she would **not** assent to the sentence "John is using drugs". So Andrea believes that John is at school or John is using drugs, and her belief is justified and true. But we would not want to say that Andrea knows that John is at school or John is using drugs. If one thinks that this example is somewhat less than entirely convincing, one may want to replace "John is at school" with some other sentence which expresses an utterly unjustified and false belief which Andrea may have.

Whether I or IV should be taken to be the most appropriate rendering of the traditional definition of knowledge may be a matter of controversy. The question does, however, not have any bearings on the considerations which are to follow, and I shall therefore be assuming that I gives an adequate rendering of the traditional definition. The conclusions reached would be unaffected if we were to assume that IV gives the adequate rendering of the traditional definition instead of assuming that I gives an appropriate rendering of it.

But I differs from

V $\quad p \in K$ iff $p \in C(J \cap B^P \cap T) \cap B$

(p is known iff p is a consequence of a justified true primary belief and p is believed)

As a consequence, II and III differ from the corresponding statements that we get by using V instead of I:

VI $p \in K$ iff $p \in C(J \cap B^P \cap T) \cap (B^P \cup B^S)$

(p is known iff p is a consequence of a justified true primary belief and either p is a primary belief or p is believed since a primary belief is believed (i.e. p is a secondary belief))

VII $p \in K$ if $p \in C(J \cap B^P \cap T) \cap B^S$

(p is known if p is a consequence of a justified true primary belief and p is a secondary belief)

To see that V differs from I it suffices to show that

$C(J \cap B^P) \cap C(T)$ is not logically equivalent to $C(J \cap B^P \cap T)$ (because of the tautology $p \supset ((p \& q \equiv p \& r) \equiv (q \equiv r))$, and because something known has to be believed). To show this it is enough to show that there are sets A and B such that $C(A \cap B)$ is different from $C(A) \cap C(B)$.

Consider the sets $A=\{p\}$ and $B=\{q\}$, where pvq is different from any logical truth. We then have $p \vee q \in C(A)$ and $p \vee q \in C(B)$, since both p and q entails $p \vee q$. So $p \vee q \in C(A) \cap C(B)$. But $A \cap B$ is the empty set, and the empty set only entails logical truths. As $p \vee q$ was supposed to be different from any logical truths, we have that $p\vee q\notin C(A\cap B)$. It follows that $C(A\cap B)$ is not generally equivalent to $C(A)\cap C(B)$.

At this point I want to appeal to an analogy in order to justify that we should use V and not I in order to capture the notion of knowledge. Consider the statement "A bachelor is an unmarried man". We can from this form the true statement that the set of bachelors (Ba) is identical to the intersection of the set of unmarried people (U) and the set of men (M), i.e. $Ba = U\cap M$. Suppose we now want to talk about the set of all mothers of bachelors Mo(Ba). Is this set identical to $Mo(U)\cap Mo(M)$ or to $Mo(U\cap M)$? It is identical to the latter but not to the former. For someone, say Tina, could be the mother of an unmarried woman and the mother of a married man but not the mother of an unmarried man, so $Tina\in Mo(U)\cap Mo(M)$ and $Tina\notin Mo(Ba)$. Because we have introduced the distinction between primary beliefs and secondary beliefs which enabled us to use the C- operator in the analysis, we can interpret the Gettier problem as giving us a structurally similar argument for considering V and not I as the most proper explication of knowledge.

Note that it is only in this more constructive phase of our investigation that we have had a real need for the incorporation of our distinction between primary and secondary beliefs. If we did not have these constructive concerns, we could have formulated I without making any appeals to the distinction between primary and secondary beliefs, and our argument by analogy would even in such a case provide strong evidence for doubting that such a formulation could be an adequate explication of the concept of knowledge.

We now have to be a bit careful though. V as it stands is not entirely adequate as an explication of the concept of knowledge. Consider the following possibility. p and q are both primary beliefs that entail r, but r is believed (and so $r \in B^S$) only since q is believed and not since p is believed. The epistemic subject does not make the connection between p and r. p is justified and true, so $r \in C(J \cap B^P \cap T)$ for that reason. But q, the only belief such that r is believed since it is believed, is not both justified and true. It seems that we are now getting a Gettier type of situation again: We do not want to say that the epistemic subject knows r, but r satisfies V.

The way to avoid this problem is to make explicit reference to the primary beliefs that are such that the particular secondary belief is believed since the primary beliefs are believed. We cannot use the C-operator, but rather an operator that explicitly mentions the primary beliefs in B^P that are such that the subject believes the relevant secondary belief since he believes the mentioned primary beliefs. But we introduced such an operator at the end of the second chapter of this essay, and this can now be put to use. I think we on the basis of such a strategy can provide an explicit definition of knowledge that captures both the insight expressed by formulation V, and the moral that can be drawn from the counter example that was just given. The definition that I will suggest is as follows:

If p is a primary belief then p is known iff p is a justified true belief.

If p is a secondary belief then p is known iff p is a justified true belief and at least one of the primary beliefs on the basis of which p is believed is known.

This recursive definition can, given the notational convention introduced at the end of the first chapter, be expressed more succinctly as follows:

$$q \in K \text{ iff } q \in J \;\&\; B(q, p_1, \;.\;\;.\;\;.\;\; , p_n) \;\&\; q \in T \;\&\; (Kp_1 \vee \ldots \vee Kp_n),$$

If n=O, i.e. if $q \in B^P$, we simply get

$$q \in K \text{ iff } q \in J \;\&\; B(q) \;\&\; q \in T$$

Informally, the definition of knowledge presented here simply says that a secondary belief is known if and only if at least one of the inducing primary beliefs, i.e. the primary beliefs that are such that the secondary belief is believed since they are believed, is known. A primary belief is known if an only if it is a justified true belief.

Since what is known is also justified and true, as is anything which is believed because of something which is known, we get the following result:

$$B(q, p_1, \ . \ . \ . \ , p_n) \supset (q \in K \equiv (Kp_1 \vee \ldots \vee Kp_n))$$

Informally: If q is believed on the basis of $p_1, \ldots, p_n$, then q is known iff p_1 is known or ..... or p_n is known.

V

We will in this chapter deal with some Gettier type examples in order to vindicate the analysis given above.

There seems to me to be seven different types of Gettier counter examples in the literature. We have in the previous chapter already shown that the analysis which we are proposing is capable of dealing with one type of Gettier examples, viz. the one that arises when a disjunctive statement is believed since one of the disjuncts is believed. We will therefore in this chapter only discuss the six remaining types of Gettier examples. One type of Gettier situation occurs when an existential generalization of a statement is believed since a particular instance of the statement is believed.

A second type occurs when a particular instance of a statement is believed since a (restricted) universal quantification of the statement is believed.

A third type of Gettier situation occurs when a statement is believed since it is believed that what the statement says is or was perceptually experienced (e.g. when someone says that he/she believes that the mail man came since he/she believes that he/she saw the mail man come).

The fourth type that I will consider is exemplified by Gettier's first counter example, as it does not fit into either of the six other categories.

A fifth type of counter example is illustrated by the so called pyromaniac example.

I finally consider a counter example given by Richard Feldman.

We will in going through these examples for the most part assume that there is only one primary belief such that the secondary Gettier belief is believed since that primary belief is believed. We could naturally operate with several primary beliefs in all examples, but no such beliefs would, I think, be different from the ones that I have specified in any relevant or important respects, and one of them would have to be known in each example. I do therefore not think that the outcome of my discussion of these examples would be any different if I all the time assumed that there were several primary beliefs for the secondary Gettier belief.

But it is at this point, I think, that the reader should look for a possible retort. If it can be shown that there is a primary belief that I know and is such that I believe the Gettier belief since I believe that primary belief, my analysis fails. I think that there are good reasons, though, to think that there are no such beliefs.

First type. This type can be illustrated by Keith Lehrer's following example[11]:

"A pupil in S's office, Mr. Nogot, has given S evidence e that justifies S in believing `Mr. Nogot, who is in the office, owns a Ford,' from which S deduces p: `Someone in the office owns a Ford.' But, unsuspected by S, Mr. Nogot has been shamming and p is true because another person in the office, Mr. Havit, owns a Ford."

In this case S's primary belief would presumptively be expressed by "Mr. Nogot, who is in the office, has (truthfully and correctly) informed me that he owns a Ford." But the primary belief is not known, as it would have to be for S to know that p according to our definition.

Second type. Suppose Mr. Smith has deceived Ms. Jones and told her that the grapes on the plate have an excellent taste. The grapes are in fact clever made fake decoration grapes, but someone has put one similar looking real grape onto the plate. Ms. Jones decides to taste one of them. Ms. Jones believes that she is about to taste a grape, and she has justification for this belief. She reaches out, and by chance she grabs the single grape of the heap. Did she know that she was about to taste a grape? It seems that she did not.

In this example[12] Ms. Jones believes that the particular fruit she is about to pick is a grape since she believes that all the things on the plate are grapes, and she might be said to believe that all the things on the plate are grapes since she has the primary beliefs that she sees that the things on the plate are grapes and that Mr. Smith has truthfully and correctly told her that they are all good tasting grapes. But neither of these two primary beliefs are known. So Ms. Jones' belief that she is about to taste a grape does not count as knowledge according to our definition of knowledge.

Third type. Mr. John Duosmith[13] is estranged from his wife and in financial troubles. A man's body is found in a hotel room, shot in his head with Duosmith's revolver in the hand and a "suicide note" signed by Duosmith. Mrs. Duosmith identifies the handwriting and the corpse as her husband's, as they are totally similar. What happened, though, is that Mr. John Duosmith received a secret visit from his identical twin Jim, of whom Mr. Duosmith never told his wife because Jim was a notorious criminal and an embarrassment to the family. Jim was going to beg John for help to escape some former accomplices who were seeking retaliation for something he had done. John, however, saw his opportunity to avoid his financial troubles and to escape his wife and begin a new life. So John killed Jim, wrote a "suicide note" and made it look like it was he, John, who were lying dead on the bed. But after John left the hotel he was mistakenly identified as Jim by Jim's pursuers, and killed by them.

In this case John Duosmith's wife was justified in believing that her husband was dead since she had the primary belief that she saw the dead body of her husband in the bed of the hotel room. But she did not know that she saw her dead husband in the bed, as she saw Jim and not John. According to our analysis she did therefore not know that her husband was dead.

Fourth type. This type is illustrated by Gettier's first counter example. Smith has evidence for thinking that Jones is the one who will get the job and that Jones has ten coins in his pocket (p). Smith therefore believes that the one who will get the job has ten coins in his pocket (q). But Jones does not get the job. Smith gets the job, and Smith, unbeknownst to himself, happens to have ten coins in his pocket. So q is justified, true and believed, but not known.

We can see that Smith believes q since Smith believes p. Let us refer to the primary belief that is such that Smith believes p since he believes that primary belief as p'. p' might e.g. be expressed by "The director has truthfully and correctly informed me that Jones will get the job and I have seen that Jones has ten coins in his pocket." But p' is not known. As it is only on the basis of p' that Smith believes q, it follows that Smith does not know q according to our analysis.

Fifth type. This type is illustrated by the following example14, often considered to be a "causal" counter example to a standard definition of knowledge:

"Striking a match, S infers that it will light directly from S's knowledge that it is a dry match of a brand ("Sure-Fire" matches) that has often and always lit for S when dry and struck. However, unsuspected by S this one cannot be lit by friction because of impurities and is going to light only because of a burst of rare -radiation."

One could be strongly tempted to discard this as a genuine counter example if one has some sceptical inclinations. But the nonsceptical reader may observe, that even if it is granted that S is justified in holding that the next match will light, he does not know that it will according to our definition, because his primary belief that the match is like previously struck Sure-Fire matches in all respects relevant to ignition[15] is not known. More simply: It seems that S believes that the match will light on the basis of S's belief to the effect that his striking of the match will cause the match to light. The latter primary belief is not known as it is not the case that S's striking of the match causes the match to light.

Sixth type. Richard Feldman has presented an example[16] that he takes to be a Gettier type counter example where the knowledge claim does not rest on a false or unjustified belief. Let me cite extensively:

"Suppose Mr. Nogot tells Smith that he owns a Ford and even shows him a certificate to that effect. Suppose, further, that up till now Nogot has always been reliable and honest in his dealings with Smith. Let us call the conjunction of all this evidence (m). Smith is thus justified in believing that Mr. Nogot who is in his office owns a Ford (r) and, consequently, is justified in holding that someone in his office owns a Ford (h).

As it turns out, though, m and h are true but r is false. So, the Gettier example runs, Smith has a justified true belief in h, but he clearly does not know h.

What is supposed to justify h in this example is r. But since r is false, the example runs afoul of the disputed principle (The disputed principle is that false evidence can justify a belief, FB). Since r is false, it justifies nothing. Hence, if the principle is false, the counter-example fails.

We can alter the example slightly, however, so that what justifies h for Smith is true and he knows that it is. suppose he deduces from m its existential generalization:

(n) There is someone in the office who told Smith that he owns a Ford and even showed him a certificate to that effect, and who up till now has always been reliable and honest in his dealings with Smith.

n, we should note, is true and Smith knows that it is, since he has correctly deduced it from m, which he knows to be true. On the basis of n Smith believes h - someone in the office owns a Ford. Just as the Nogot evidence, m, justified r = Nogot owns a Ford - in the original example, n justifies h in this example. Thus Smith has a justified true belief in h, knows his evidence to be true, but still does not know h."

I think Feldman is successful in rebutting the response to the Gettier problem that simply consists in holding that false evidence cannot justify a belief. I think that we are in fact sometimes justified by false evidence. It frequently happens that we have justified false beliefs.

In particular, I do not deny that Smith has a justified true belief in h.

But Feldman's example is, I think, not successful in rebutting the analysis that I have presented. Note that it is not the case that n entails h, for it does not follow from the fact that you are told that h is the case that it is the case that h. Feldmann's example is therefore not one that disconfirms our analysis in the sense of finding beliefs P and Q that is such that S believes Q since S believes P and S knows P and it is implausible to say that S knows Q. For since n does not entail h it is not the case that Smith believes h since he believes n.

But, given that Smith believes h, it must also be the case that Smith believes

(n') Someone in the office _informed_ Smith that he owns a Ford.

In fact, given Feldmann's scenario, Smith believes h since he believes n', i.e. if we, for the sake of simplicity, assume that n' is a primary belief.17 But Smith does not know n', for n' is false.

Feldmann's example is thus not succesful in rebutting our analysis, which is not the same as to say that our analysis has been proven to be true. As is the case for most philosophical theories, it may be impossible to prove the theory which we are suggesting. If the theory is false it can be proved to be false by finding a falsifying example. If the theory is true, it is still most likely that it is impossible to **prove** that it is true. But we may gain confidence in the theory from the fact that it holds in many and all instances in which the theory has been tested.

VI

We will in this chapter consider objections that have been raised against four other types of responses to the Gettier problem.

One response to the Gettier problem which we already discussed in the first chapter consists in adding a fourth condition stating that the justified true belief must be indefeasible, wher "indefeasibility" means something like

"The subject's justification must be such that no further addition to his evidence would undermine his justification." A lot of epicycles have later been added to this kind of approach, as it soon became clear that such a condition, as it stands, is too strong. The following example should make this clear[18]:

"S believes that his acquaintance, Tom Grabit, stole a book from the library since S saw Tom do it. But, unsuspected by S, Tom has an identical twin brother who was in the library near the time of the theft."

The Tom Grabit example shows that the defeasibility analysis is too strong. How is the analysis that we have presented affected by the Tom Grabit counter example? It is not affected at all. It is simply not relevant that Tom Grabit's twin was in the library near the time of the theft. S knows that Tom Grabit stole the book, because S's primary belief (viz. that S saw Tom take the book) is justified and true, and hence known. S might of course loose his belief if he is later informed about the twin's presence.

Another type of response consists in adding a fourth condition that prohibits false propositions to constitute the evidence for or the justification for the proposition. There is the following counter example[19],:

"S is told by Mr. Nogot and by Mr. Havit who are in his office that they own Fords. S infers that someone in his office owns a Ford. But Mr. Nogot is shamming, whereas Mr. Havit tells the truth."

In this case we would want to say that S did know that someone in the office owned a Ford, but the proposal to eliminate false evidence does not get this result. The analysis is therefore too strong. Our analysis, however, is not harmed by this example, because the primary belief that Mr. Havit has informed S (truthfully and correctly) that he owns a Ford is known. So S knows that someone in the office owns a Ford.

A third type of response may be exemplified by the so called causal analyses of knowledge. There are many different variants of these. One, due to Max Steiner[20], consists in adding the following condition: "The sentence "p" must be used in a causal explanation of S's believing that "p" is true." The following example due to Alvin Goldman[21] is considered to be a counter example:

"The Careless Typesetter. On a newspaper known to be generally reliable, a typesetter carelessly misprints details of a story which S misreads because of eyestrain in such a fashion as to believe the true story."

The example is not a counter example to our analysis of knowledge, however, for S believes the true story only since he has the primary belief that the newspaper presents the true story. But that primary belief is not known, so S does not know the true story.

We will finally consider a counter example to an analysis of knowledge due to Ernest Sosa[22]. The counter example is due to Gilbert Harman[23]:

"The unobtainable unopened letter. S knows p: `Norman is in Italy,' thanks to being told upon phoning Norman's office that he is spending the summer in Rome. In addition, Norman tried to deceive S by having a friend in

San Fransisco mail a letter from Norman to S claiming that he is spending the summer in San Fransisco. The letter will continue to lie unopened in a building to which the postman misdelivered it on its way from San Fransisco to S."

But the unopened letter does not matter any more than the presence of Tom Grabit's twin in the library for our analysis of knowledge. S believes that Norman is in Italy since he has the primary belief that he has been correctly informed that Norman is in Italy, and that primary belief is justified and true. So, according to our analysis, S knows that Norman is in Italy.

There are many more responses to the Gettier problem. And it would probably exceed the proper limits of any essay to discuss them all. As far as I can see, all of the responses that have been suggested have their problems. I do not at this point see any problems with my own analysis. But, as the art of giving counter example is pretty sophisticated, it would be somewhat premature to conclude that a final solution of the Gettier problem has been presented. We will in the next chapter study a different type of counter example which seems to threaten the analysis which I have suggested as a solution of the Gettier problem.

VII

7.0.

Our analysis of knowledge is amongst other things supposed to avoid the Gettier type examples, and we have, if I am right, in the previous two chapters seen that it does indeed avoid many of them, but a problem of the following kind may arise with the account which has been given up to now.

Suppose Smith believes that Jones owns a Ford simply because Smith believes that he has often and always seen Jones drive a Ford. In this case the belief because of which Smith believes that Jones owns a Ford does not entail that Jones owns a Ford. So, given this description, Smith's belief that Jones owns a Ford would have to be counted as a primary belief according to our analysis, and it would seem that his belief is justified. If so, Smith knows that Jones owns a Ford if Jones owns a Ford. Suppose Jones in fact does own a Ford, but not the one that Smith has seen him drive which is one that he has leased, or it is a company car. We now seem to get a Gettier situation which our analysis is not able to account for, for we would not want to say that Smith knows that Jones owns a Ford, but according to our analysis it seems that he does.

In general, how do we account for Gettier type situations where the evidence does not entail the Gettier belief, in which case our analysis would classify the Gettier belief as a primary belief and hence as knowledge if justified and true? We will call this an inductive Gettier situation.

I will divide my answer to this question into two sections. In the first section I will work on the basis of the assumption that there indeed are such Gettier beliefs which are not entailed by a belief because of which they are held. In the second section I will suggest that there are no such Gettier beliefs. If my argument in the second section is correct, we avoid some potential difficulties associated with the account given in the first section. In particular, we will then have a theory which arguably has less problems while remaining uncommitted to foundationalism and skepticism. On the other hand it seems that the approach of the second section will be committed to a **moderate** scepticism with respect to at least some unobserved matters of fact, but that is a form of skepticism which I think that we should be willing to embrace. If the conclusion of my argument in the second section should be false, i.e. if there indeed are inductive Gettier cases, the approach that I have given in the first section will remain as an alternative, but, maybe less plausible account. Let me, however, remark that the difference between the two following accounts should not be exaggerated. They both seem to me to be viable and somewhat similar solutions to the inductive Gettier problem. Maybe the most important reason why I opt for the second account is that it is less complicated and also more congenial to traditional accounts of inductive arguments.

In a second subsection of the first section I give an outline of a strategy which can be used in order to adopt the approach of the first subsection of that section while at the same time avoiding any commitments to humean skepticism with respect to induction. I am not claiming that the strategy that I outline is altogether satisfactory, but then again, nor are, to my knowledge, any other accounts of inductive reasoning.

In a third subsection of the first section I show how one on the basis of the discussion which precedes may arrive at an alternative formulation of our analysis which **may** be thought of as a certain simplification of the analysis which we have proposed earlier, but I then argue that we should not adopt any such alternative formulations because the connection between a secondary belief and its inducing primary beliefs then gets lost.

Although I do not, at least not fully, endorse the approach given in the first section to follow, I do think that some important observations on the nature of justification are made in the discussion there. Some of these observations should, I think, be taken seriously independently of the inductive Gettier problem, and it is for that reason that I devote the next chapter to an analysis of the so called lottery paradox.

7.1.1.

It is not, it would seem, reasonable to hold that a belief q, which S believes because S believes p, is justified if S's belief in p is justified, unless p in fact entails q. We are here assuming that S does not have some other belief p' which is justified and entails q and is such that S believes q because S believes p'. To see that it may be claimed to be unreasonable to think that something less than an entailing inference can transmit justification, suppose e.g. that Tim is justified in holding that John comes from the USA, but does not at all know from where in the USA. Tim knows the USA very well, and reasons as follows: "Most people from USA do not come from Bakersfield, California, so John most likely comes from USA'=USA minus Bakersfield. Most people from USA' do not come from Barstow, California, so John most likely comes from USA''=USA' minus Barstow..... and so on. So John most likely comes from 212 84th Street in New York City."

Tim is, however, clearly no more justified in believing that John comes from Manhattan than he is in believing that he comes from anywhere else in the USA, including Bakersfield. But if one thinks that something less than entailment from p to q is sometimes sufficient to preserve justification, then why isn't Tim justified in holding that John comes from 212 84th Street? Let us call the problem which we have pointed out for *the paradox of justification*.

One might reply that the paradox of justification depends upon a sorites type argument, and that Tim stops being justified somewhere down the line in his chain of reasoning. And it is true enough that the paradox of justification is based upon a sorites type argument, but I cannot see that one can evade the paradox just by so labeling the argument upon which the paradox is based. If one holds that Tim is justified in believing that John comes from, say, USA'''', but that Tim is not justified further on in his chain of reasoning, then it seems clear that one's notion of justification is a somewhat vague one that admits of different degrees, one of which would presumptively be considered minimal. Let us call this weak justification. As weak justification is vague, we will not attempt to make this notion precise.

While what I have here called weak justification may not be an unreasonable or useless notion, it conflicts with the more strict notion of justification which may seem appropriate as a condition for knowledge, and which we, following Ayer, can define as having the right to be certain. Applied to our examples above, it is then clear that while Tim may be justified in believing that John comes from the USA, it is not the case that he is justified in believing that John comes from USA', for he has no right to be certain that John comes from USA' since he doesn't have the right to be certain that John does not come from Bakersfield, California. Likewise, while Smith may be justified in believing that he has often and always seen Jones drive a Ford even in the strong sense of having the right to be certain, this does not, it seems, give him the the right to be certain that Jones owns a Ford. It seems that the paradox of justification shows that a belief q, which is inferential in the sense that it is believed because a belief p is believed, must be entailed by a strongly justified belief p' which is such that q is believed because p' is believed in order for q to be strongly justified.

Perhaps the following example would bring out the difference between the notions of strong and weak justification that we have discerned. Suppose Smith has bought one Lotto ticket. According to the weak notion of justification, it would be appropriate to say that Smith is justified in holding that he is not going to win. But Smith does not have the right to be certain that he is not going to win, so he is not what we can call **strongly justified** in holding that he is not going to win.

It seems to be beyond any doubt, that someone that subscribes to weak justification and holds that we are sometimes justified in holding q when we believe q because we believe p where p does not entail q, need not, and in fact cannot, hold that **all** non-deductive inferences preserve justification. For there are cases when it is obvious that the non deductive inference does not preserve justification. A hasty generalization is one such case. The problem for an adherent of weak justification is therefore to give an account of "justifies" that decides when non-deductive inferences preserve justification, and when they do not preserve justification. The paradox of justification shows that this may be a difficult problem indeed, and that such an account will most likely have difficulties with preserving our intuition that if p justifies q and q justifies r then p justifies r. It will have such difficulties because, as I have tried to show with my example, epistemic closure principles for nondeductive inferences like the one expressed by the sentence "a is an F and the vast majority of F's are G's, therefore a is a G" must be false. I may be completely justified or even know that a is an F and also that the vast majority of F's are G's without being justified or knowing that a is a G. This last statement is obviously true for knowledge, as it may be the case that it is false that a is a G. But it is, as my argument above has shown, also false for justification if we by "justified" mean "having the right to be certain".

Maybe an adherent to weak justification could make a good case for allowing one or a few applications of an epistemic closure principle as the one I have indicated. But if so, he would have to give up the principle that if p justifies q and q justifies r then p justifies r, since one cannot accept an indefinite number of applications of such an epistemic closure principle. This is a problem which is avoided if one adheres to what I have called strong justification.

It seems, though, that there may be a way around the paradox of justification without giving up one's acceptance of what I have called weak justification. We will here try to give a rough sketch of such an approach to justification which shows that one **may** adopt a weaker notion of justification than the one which says that one must have the right to be certain in order to be justified, and at the same time avoid the paradox of justification and also the related lottery paradox which we will discuss in the next chapter. I am not suggesting that the approach which I outline should be adopted.

Think of probabilities as in some sense being determined by the subjective expectations of a person. The expression $P(q)=1$ may e.g. **roughly** signify that S expects that q with complete certainty. Let there be some number t which is smaller than 1 but close enough to 1 so that the following relationship between justification and probability may be said to hold according to the adherent of weak justification:

$$J(q) \supset P(q) \geq t$$

Let us now define the following relationship between propositions:

$$(q \Rightarrow r) \equiv P(r \backslash q) \geq (1 + t - P(q))$$

(Or in words: $(q \Rightarrow r)$ if, and only if, the probability of r given q is larger than or equal to $1 + t$ minus the probability of q.)

Note that $q \Rightarrow r$ is not defined if $P(q)<t$. Suppose it is the case that $q \Rightarrow r$. Since the probability of r, i.e. $P(r)$, is given by $P(q)$ multiplied by $P(r \backslash q)$, we see that

$$P(r) = (t + t^2 - t \cdot P(q)) \geq (t + t^2 - t^2) = t$$

So if $q \Rightarrow r$ then $P(r) \geq t$.

It seems that we now have a relationship between propositions which on the one hand is weaker than entailment but which on the other hand is strong enough for the following relation to hold:

$$(Jq \mathbin{\&} (q \Rightarrow r) \mathbin{\&} q>r) \supset Jr$$

(Here "q>r" abbreviates "S believes r because S believes q".) Suppose $P(q)=1$. By the definition of $(q \Rightarrow r)$, the probability of r given that q is the case must then be larger than or equal to t, i.e. $P(r \backslash q) \geq t$. Suppose $P(q)=t$. The definition of $q \Rightarrow r$ now gives $P(r \backslash q)=1$. So in the case when $P(q)=t$ the relationship $q \Rightarrow r$ will hold if, and only if, the probability that r obtains given that q obtains equals 1.

One **might** want to consider the possibility that the relation $q \Rightarrow r$ could serve instead of entailment in our analysis of knowledge, since "=>" is transitive. In order to decide whether this is possible one would, I think, need to do some research on the metaphysics of probability and on the relationship between probability and justification. It seems, though, that one may at least use such an approach as the one which I have outlined here in order to avoid the paradox of justification and the lottery paradox. I do, however, not think that such an approach would be successful, and there are three reasons for that. It seems that the numerical value of the parameter t which we have used in stating this approach could only be settled by a rather arbitrary stipulation. As soon as one were to stipulate a numerical value N(t) of t, it seems that one could reasonably ask for a justification for why one should use the number N(t) rather than e.g. the number N(t) minus 0.002. Secondly, an approach like the one I have outlined would have to rely upon a certain metaphysics of probability which may be befuddled with methodological problems. How can we possibly measure the probability of a proposition if the probability of a statement is to be thought of in terms of the subjects expectations? Thirdly, although there may be ways around the former problem, e.g. along the lines suggested by Ramsey, the connection between subjective probability and justification which is presupposed is at the very least questionable. For it seems that we are sometimes in epistemic situations where we are justified in holding a proposition without believing that proposition, and the subjective notion of probability which was presupposed in the approach outlined above would essentially be based on a reduction of high probability measures to high degrees of belief.

It seems, then, that the sketched alternative strategy, which basically relied upon a technique that made it possible to avoid the paradox of justification while adhering to a weak notion of justification, runs into insuperable difficulties. Given these problems with the alternative approach, our policy will be to use the word "justification" in the strong sense. We will, in other words, treat the word "justified" as being synonymous with "having the right to be certain", because it is, it seems, in this strong sense that we need to be justified in order to know. At least there is this sense of being justified, and it is worthwile to explore the scope and limits of the notion of knowledge which it gives rise to.

A further argument for using this strong sense of justification is that it is consistent with a restriction as to when inferential beliefs are justified which may help us to avoid the inductive Gettier problem. Let us, in order to show this, again make use of the distinction between inferential and noninferential beliefs. Let us, for the sake of simplicity assume, contrary to what we have shown to be the case earlier24, that a belief is inferential iff it is believed because some other belief is believed, and a belief is noninferential iff it is not inferential. Recall that a belief q is believed by S because a belief p is believed by S iff "S believes p" is a true answer to the question "Why does S believe q?". We are thus using "because" as before, but do not now have the requirement that an inferential belief q is believed because of a belief p which also entails q.

Given this distinction, we see readily that any secondary belief is inferential. But it seems clear that also a primary belief may be inferential. This latter possibility is the one that gives rise to the inductive Gettier problem. We will in the following suggest a way to overcome the inductive Gettier problem by relying upon a restriction as to when inferential beliefs are justified.

Let "p>q" abbreviate "S believes q because S believes p". A principle which it seems can be invoked in order to avoid the inductive Gettier problem can then be stated as follows:

JP $\quad (\exists p)(p>q) \supset (Jq \equiv (\exists p)(p \text{ entails } q \;\&\; p>q \;\&\; Jp))$

In other words: An inferential belief is justified iff it is entailed by a justified belief because of which it is believed. It follows by iteration that this amounts to the same as holding that an inferential belief is justified iff it is a secondary belief which is induced by a justified primary belief.

One should note that the principle JP has as a consequence that a subject S is not justified in holding q if S believes q because S believes some proposition p1 and ...... S believes q because S believes pn, where none of p1,..., pn entails q but a conjunction of some of p1,..., pn does entail q, even though S may be completely justified in believing the conjuncts of that conjunction. This seems to be an implausible result. In order to avoid this result, we need to avoid a situation in which no single one of the propositions pi as specified above entails q but a conjunction of some of them does. We will therefore assume that the following holds: If p1 and p2 are logically distinct, i.e. if neither p1 is a logical consequence of p2 nor p2 is a logical consequence of p1, and S believes q because S believes p1 and S believes q because S believes p2 then S also believes q because S believes p1 and p2. In symbols:

CP If $(p1,p2)>q$ and $\sim\vdash p1\supset p2$ and $\sim\vdash p2\supset p1$ then $(p1, p2, p1\&p2)>q$.

CP may be taken as an additional stipulation concerning the because-relation which we have been using all along.

I now want to discuss the relationship between the principle JP and the analysis of knowledge as presented earlier. For it may on reflection seem that the principle JP supersedes the analysis of knowledge that has been presented, and that JP can provide us with a rather simple solution to the Gettier problem. May it, so one may ask, not simply suffice to require that the evidence for a belief must entail the belief in order for it to count as knowledge? But such a strategy will not work, for the evidence for the belief, i.e. its inducing belief, may be false. We have already stressed that primary beliefs can be false. S may e.g. believe that Tegucigalpa is the capital of Guatemala since S believes that the geography teacher informed S that Tegucigalpa is the capital of Guatemala. But the geography teacher **mis**informed S. The latter belief is a false, although justified primary belief. But this fact will allow Gettier problems to remain since S e.g. may form a disjunctive belief to the effect that Tegucigalpa is the capital of Guatemala or Brown is in Barcelona, and yet though S knows nothing about Brown's whereabouts, Brown happens to be in Barcelona.

But may it not suffice, in order to deal with the Gettier examples, to require, as in JP, that the evidence for a belief must entail the belief in order for the belief to be **justified**? Can we in such a manner preserve the traditional definition of knowledge as justified true belief? The answer is no, and for the same reason as above. Consider the example we just gave, where S has a justified false belief to the effect that Tegucigalpa is the capital of Guatemala. Let us call the statement that Tegucigalpa is the capital of Guatemala p. Let q be any statement such that S disbelieves q or S does not believe q and q is true, and suppose S reasons and accepts the statement that either p or q. Let us call this disjunction r. It is clear that r is a justified true belief, given that we, as I think we should, count S's belief in p as a justified belief. But we do not want to say that S knows r. This shows that JP cannot replace our analysis of knowledge, but can only supplement it in order to avoid the inductive Gettier examples.

There is, I think, more to be said for principle JP than the fact that it makes it possible to get around the difficulties we encountered with the inductive Gettier example.[25] I think that as a matter of fact most of our inferential beliefs are such that they are entailed by some of the beliefs because of which they are held. These are examples of questions that we have to consider in order to test this intuition:

A Suppose you are inside a house and look out. You see the branches of the trees moving in a certain way and infer that the wind is blowing. Is it the case that

a) You believe that the wind is blowing because you believe that you see the branches of the trees move like they do?

Or:

b) You believe that the wind is blowing because you believe that you see the branches of the trees move like they do **and** if the wind weren't blowing then you wouldn't see the branches of the trees move like they do?

B Suppose you come to believe that Smith owns a Ford after having often and always seen Smith drive a Ford. Is it the case that

a) You believe that Smith owns a Ford because you believe that you have often and always seen Smith drive a Ford?

Or:

b) You believe that Smith owns a Ford because you believe that you have often and always seen Smith drive a Ford and if Smith didn't own a Ford then you wouldn't often and always have seen him drive a Ford?

It seems that b) is the most reasonable answer to give to these questions.

It also seems to be a fact, and I think in a certain sense a necessary fact, about us, that we believe a whole range of conditional propositions like: If the water is heated then it will boil, not freeze. I will burn if I touch the fire. If I don't eat and drink I'll die. If (I see that) the branches of the trees are moving in certain ways then the wind blows. Such conditional beliefs play a role in making inferential beliefs into beliefs that are entailed by some of the beliefs because of which they are held. At least so one may claim. But not only may they play such a role. They also seem to play a more important and irreplaceable role in our biological survival. It seems that we couldn't possibly **not** have a whole range of such action guiding conditional beliefs as the one's mentioned, and survive. Why would we take the stairs or the elevator instead of jumping out the window from a tall building to leave it if we didn't believe that the latter strategy would be a fatal one?

But this raises the question as to what the nature of these conditionals is. Are they simply material conditionals? No. It would seem not. They must be some kinds of subjunctive conditionals or causal statements. We do not simply believe that the water will boil if it is heated in the material and truthfunctional sense that either the water is not heated or it will boil. We believe that the fact that the water is heated will in some sense **cause** the water to boil. Similarly, we believe that the fact that we touch the fire will cause us to burn, and so on for similar cases.

But do such causal if-then-statements ensure that the consequent is entailed by the truth of the statement plus the truth of the antecedent? The nature of causal statements is a controversial issue in metaphysics, and we do not want to get sidetracked by getting involved in that discussion. For our purposes it should suffice to think of causal statements in terms of subjunctive conditionals, but it then seems that a bifurcation must occur. We may, it seems, in some situations understand the causal proposition that the state of affairs p causes the state of affairs q as being identical with the proposition expressed by "If q weren't the case then p wouldn't have been the case".26 Let us call a causal statement which is natural to interpret in such a way an **antecedent-causal statement.** An example of an antecedent-causal statement would be the statement that the man died because he jumped from the Eifell tower. In some other situations it seems to be more appropriate to understand the proposition that the state of affairs p causes the state of affairs q as being identical with the proposition expressed by "If p weren't the case then q wouldn't have been the case".[27] We will call such causal statements **consequent-causal statements.** An example of a consequent-causal statement would be the statement that the fetus was conceived because the lovers had intercourse. Given such an approach as this we see that the truth of an antecedent-causal statement together with the truth of the antecedent entail the consequent. For q certainly follows by modus tollens if it is both the case that p and that if q weren't the case then p wouldn't have been the case. Similarly, we see that the truth of a consequent-causal statement together with the truth of the consequent of such a statement entail the antecedent.

One may take all of these considerations to support the thesis that many, maybe most inferential beliefs are beliefs that are entailed by some of the beliefs because of which they are held. In fact, it would even seem to lend support to the thesis that inferential beliefs in general **are** entailed by some of the beliefs because of which they are held, and that we could lay down the following principle:

BP $(\exists p)(p>q) \supset (\exists p)(p\vdash q \wedge p>q)$

Or informally: If there is a belief because of which S believes q then there is a belief because of which S believes q and which also entails q.

But I do not think that BP is true. There seem to be strong reasons to think that there are cases where I believe something q but also realize that my evidence for the belief q is far from conclusive. I may e.g. believe that the wine will taste good (p) simply because I believe that the wine comes from France. The latter does not entail the former. At best it makes it somewhat probable. Or I may believe that red will come up next on the roulette table (q) because I believe that black has come up fifteen times in a row, without believing something that **entails** that red will come up next. I may in a sense just choose to believe that red will come up next because I believe that black has come up so many times. In the same way, I may choose to believe that red will not come up fifteen times in a row. Principle BP would require there to be beliefs I have which entail p and q respectively, and that seems to be an unreasonable requirement, as p and q both have the status of being more or less educated guesses.

The reason why I bring principle BP into the discussion is that I want to refute the following somewhat seducing considerations that one could marshall in **favour** of principle BP:

The following two axioms for belief statements seem reasonable:

B1 $((Bp \vee B{\sim}p) \,\&\, Bq) \supset B(p \supset q)$

B2 $(Bp \,\&\, Bq) \supset B(p \,\&\, q)$

If these axioms are true, and I in fact do believe that red will come out next because I believe that black has come out fifteen times in a row, it seems to follow that I by that very fact also believe that black has come out fifteen times in a row **and** if black has come out fifteen times in a row then red will come out next. It then **seems** natural to say that I believe that red will come out next because I believe that black has come out fifteen times in a row and if black has come out fifteen times in a row then red will come out next. Likewise, it seems natural, given our example above, to say that I believe that the wine will taste good because I believe that the wine is French and if the wine is French then the wine will taste good.

One mistake in the above argument is that it presupposes that the truth functional conditional can always be used in an adequate rendering of if-then-statements. But it is far from plausible to hold that a belief in a material conditional which a subject has as a result of B1 can be understood as a belief in an if-then-statement. The reasons why this is not plausible are well known. If we e.g. let p be the proposition that 2+2=4 and q the proposition that the earth is round then we get as a result that anyone who believes both of these also believes the proposition (2+2=4 ⊃ the earth is round). So far, so good.

But we do not ordinarily say that anyone believes that if 2+2=4 then the earth is round, so we shouldn't interpret a subject's belief in the proposition (2+2=4 ⊃ the earth is round) as a belief in the proposition that if 2+2=4 then the earth is round. It is for this reason not reasonable to make such a use of principle B1 as is presupposed in the argument above.

A second mistake in the above argument is that it seems to reverse the order of things. Yes, B1 and B2 seem to be plausible principles, but rather than showing that a subject who believes q because he or she believes p thereby also believes q because he or she believes p & (p ⊃ q), it seems to show that someone who believes e.g. p and q thereby believes (p ⊃ q) because she believes p and q. And the subject need not believe both of p and q in order to believe (p ⊃ q) for that reason. For (p ⊃ q) is equivalent to (~p ∨ q), so a subject may therefore believe (p ⊃ q) simply because she believes q, or simply because she believes ~p.

There may, on the other hand, be another way in which someone in fact believes that a certain wine is good because she believes that the wine is French and if the wine is French then the wine is good. But then the conditional statement believed by the subject would presumptively be an instance of a law-like proposition, or it could be a belief in a causal conditional of the kind that we have discussed above. A person who believes such a conditional might e.g. be under the false impression that all wines that are French are good. But a belief in such a law-like proposition or causal conditional cannot be inferred from B1 plus the fact that someone believes that a wine is good because he or she believes that it is French. Nor can we infer that the person has such a belief in the general goodness of French wines by means of any other plausible epistemological principles from the mere fact that he or she believes that the wine is good because he or she believes that it is French.

All of this supports my claim that BP is false. But BP is neither needed for our analysis, nor, as I have tried to show, desirable in its own right. As we have pointed out, there seem to be cases where we have beliefs which are inferential and not entailed by any of the beliefs because of which they are held. They would thus count as primary beliefs according to our theory. By principle JP, these would all be beliefs that are not justified. This is a consequence which must be argued for.

Suppose it is the case that I believe that the wine is good because I believe it is French, but I have no further beliefs which explain why I believe that the wine is good. I do e.g. not believe that all French wines are good or that if it weren't good then it wouldn't be French. Maybe I believe that most French wines are good, but I am not enough of a wine expert to give any further arguments for my belief about this particular wine. Do I have the right to be certain that the wine is good given that I have the right to be certain that the wine is French and that most French wines are good? It seems not. I may be said to be justified to a certain degree if we speak in terms of weak justification, but I do not have the right to be certain.

Note that I would lack justification for the belief that a particular French wine were good, in the strong sense of having the right to be certain, even if I were a great wine expert and I were justified in holding that the vast, vast majority of French wines were good. This would be a consequence of JP, and it can be argued for in the same way as we did above when we pointed out the paradox of justification. For we can imagine a partition of French wines into a variety of categories, and we can suppose that I were a wine expert. Suppose I were justified in holding that the wine is good because I were justified in holding that the wine is French and that the vast, vast majority of French wines are good. Take another category A, which is such that I were justified in holding that the vast, vast majority of good French wines are A. A could e.g. be the proposition that the wine does not come from a certain chateau in Bordeaux. It would then seem that I would be justified in holding that the wine is good and A. We next in turn take other categories B, C, ... and so on with which I am familiar, where each of these are so related to the previous categories that I were justified in holding that the vast, vast majority of those that fall within the previous categories also fall within it, and I would end up being justified in holding that the wine is good and A and B and C and......and Z. But only a few good French Wines are A and B and C and .... and Z, so I am not justified in holding that the wine is good and A and B and C and .... and Z.

Not only would I in the case with the French wine lack the **right** to be certain that the wine is good. I would not even **be** certain that the wine is good. At least not if I, given my actual lack of expertise, were epistemologically reasonable. I would rather believe to a certain degree that the wine is good, without being quite sure. But it seems adviseable to not only follow Ayer in requiring that we should have the **right** to be certain when we know, but also to follow him and e.g. Ramsey in holding that we should **be** certain about what we know. It does e.g. not seem correct to say that I **know** that the lecture starts at 14.00 if I have to check my schedule to verify it. That we should thus adopt Ayer's and Ramsey's position also seems to be supported by the fact that it is seldom if ever the case that one has the right to be certain if one is not certain. If I have to check the schedule in order to verify that the lecture starts at 14.00, i.e. if I thus display a lack of certainty with respect to when the lecture starts, it would be odd indeed if I could be said to have the **right** to be certain that it starts at 14.00.

These comments which I have here made with regard to the requirements that beliefs be certain are not essential to the present account. The requirement is e.g. not needed to reach the conclusion that I do not have the **right** to be certain that the wine is good in the example above.

But does a restriction to beliefs that are certain restore the validity of principle BP? This would seem to be a question of psychology. Could it be the case that a person were certain that q because he or she were certain that p, without it being the case that the person were certain about anything which entails q and is not entailed by q? I do not see why not. But maybe one could make a case for adhering to principle BP if one restricts oneself to beliefs that are certain. However that may be, we shall not follow such a course of action. We shall, in other words, not assume principle BP, although one possibly could make a case for that principle if one restricts oneself to beliefs that are certain. One important reason for following such a policy is that we thereby avoid making the restriction to beliefs that are certain an essential part of the analysis which is being offered.

Let us, before we proceed, sum up some of the main points so far of the discussion in this section. I have given arguments for a principle (JP) which says that an inferential belief q is justified only if q is believed because of some belief p which is justified and which also entails q. The arguments I have given are sorites type arguments which show that justification is not preserved by inductive inferences. I have also considered but discarded a principle (BP) which says that all inferential beliefs in fact are secondary beliefs.

Let us now apply this principle (JP) to the examples that we have considered in order to see what kind of consequences it gives rise to.

S looks out the window and sees the branches of the trees move, and S forms the belief that the wind blows. Since in this case S believes that the wind blows because S believes that S sees the branches move like they do, we have, according to JP, that S is justified in holding that the wind blows only if there is some belief p such that p entails that the wind blows and S is justified in holding that p. We have already pointed out one plausible candidate for such a belief, viz. the belief in the conjunction that S sees the branches move like they do and if the wind weren't blowing then S wouldn't see the branches move like they do. One may claim, then, that it is because S is justified in holding this latter belief that S is justified in holding that the wind blows. This raises a skeptical question, which we will address in the last subsection of this section: In virtue of what is S justified in holding that if the wind weren't blowing then S wouldn't see the branches move like they do?

Suppose I believe that the wine is good because it is French. According to our principle JP, I am only justified in holding that the wine is good if there is some further belief p such that p entails that the wine is good and I believe that the wine is good because I believe p. Maybe I do have such a belief primary to my belief that the wine is good. But if so, it would, given my lack of expertise in wines, certainly not be a justified belief.

Suppose someone believes that red will come out next because black has come out fifteen time in a row on the roulette table. Maybe that person does have a belief primary to his or hers belief that red will come out next.

But, given the nature of the game, such a belief would certainly **not** be justified.

We finally consider the example which started us on the discussions of this section. Smith believes that Jones owns a Ford because Smith believes that he has often and many times seen Jones drive a Ford. According to our principle JP, Smith is justified in holding that Jones owns a Ford only if Smith's belief to the effect that Jones owns a Ford is secondary to a justified belief. We have suggested one plausible candidate for a justified belief primary to Smith's belief in Jones' Ford ownership, viz. the belief that Smith has often and many times seen Jones drive a Ford and if Jones didn't own a Ford then Smith wouldn't often and many times have seen Jones drive a Ford. I have no problems with accepting that Smith is justified in holding the latter belief. But Smith does not in this case **know** that Jones owns a Ford according to our analysis because it is not true that if Jones didn't own a Ford then Smith wouldn't often and always have seen Jones drive a Ford.

7.1.2

We will in the following subsection suggest an alternative formulation which may be thought of as a certain simplification of our original analysis which can be adopted if one accepts principle JP. Let us again take a look at the principle JP:

JP $(\exists p)(p>q) \supset (Jq \equiv (\exists p)(p \text{ entails } q \ \& \ p>q \ \& \ Jp))$

After having seen the potential usefulness of principle JP for justification, a natural question to ask is whether one can get an adequate analysis of knowledge by in addition to principle JP also adopting an analogue principle for knowledge:

KP $(\exists p)(p>q) \supset (Kq \equiv (\exists p)(p \text{ entails } q \ \& \ p>q \ \& \ Kp))$

And KP does, it seems, facilitate a certain simplification of the theory which we have been propounding. Given principles KP, JP, and CP we now only need a principle NP which says that a belief which is non-inferential is known iff it is a justified true belief, in addition to a principle NC to exclude circular explanations, in order to arrive at an analysis of knowledge which one might think supersedes the analysis which we have suggested and which at the same time is an analysis that is founded upon the same insights as those upon which the earlier analysis was founded. At least so it may seem.

A more formal formulation of principle NP looks like this:

NP $\sim(\exists p)((p)>q) \supset Kq \equiv Jq \& Bq \& q$

In order to formulate the principle NC which excludes circular explanations, one must make use of the ancestral of the relation denoted by "because" in S believes q because S believes q. We use the symbol ">>" to denote this ancestral relation:

NC $p>>q \supset \sim(q>>p)$

The motivation for adopting principle NC is based on the fact that "S believes p" in "S believes q because S believes p" is intended to be an explanation of the fact that S believes q. And we do not normally accept circular explanations. NC is thus only an explication of what we mean, and have meant all along, by "because".

If one accepts principle JP, one can thus in a certain sense simplify the analysis of knowledge which we have been proposing by adhering to the principles JP, CP, NC, KP and NP. We reproduce these here for the reader's convenience:

JP $(\exists p)(p>q) \supset (Jq \equiv (\exists p)(p \text{ entails } q \ \& \ p>q \ \& \ Jp)$

CP If $(p1,p2)>q$ and $\sim\vdash p1\supset p2$ and $\sim\vdash p2\supset p1$ then $(p1, p2, p1\&p2)>q$.

NC $p>>q \supset \sim(q>>p)$

KP $(\exists p)(p>q) \supset (Kq \equiv (\exists p)(p \text{ entails } q \ \& \ p>q \ \& \ Kp)$

NP $\sim(\exists p)((p)>q) \supset Kq\equiv Jq\&Bq\&q$

This, then, is one way in which one can deal with the Gettier examples, and at the same time avoid the inductive Gettier cases. As I, for reasons which will become clear in the next section, do not think that JP is an entirely plausible principle (or at least I do not think that we have sufficient evidence for adopting JP), I shall leave it as an excercise to figure out how these six principles are interrelated. Also note that there is not much, if anything to be gained by such a "simplification" as the one suggested by the above reformulation even in one finds that JP is a plausible principle. On the contrary, it seems that we would lose some insights. It would e.g. no longer be possible to introduce the expressions "p induces q for S" and "S believes q on the basis of p" if we were to adopt such a revision, for the relationship between a secondary belief and a primary belief which induces the former would get lost if we were to adopt such a revised formulation of our theory. It is furthermore open to question whether the above reformulation really is a simplification of our official analysis.

Before we move on to the the next section, we will in the following subsection show how the approach which we have outlined in this section may possibly be worked out in such a way as to avoid humean skepticism with respect to induction.

7.1.3.

Consider again S's belief to the effect that if S sees the trees move like they do then the wind blows. The humean skeptic may claim that S is not justified in holding that if S sees the trees move like they do then the wind blows, although he may grant that S is justified in holding that S sees the trees move like they do. "Why does S believe that if S sees the trees move like they do then the wind blows?" the humean skeptic may ask. And the skeptic would probably in his own answer to the question claim that S must ultimately appeal to some kind of uniformity principle which is not justified. The skeptic may claim that S cannot rule out that the trees move by themselves without being caused to do so by the wind, since S, by assumption, is inside and cannot feel that the wind blows. One cannot in response to this just claim that the trees **could** not be moving by themselves. It is at least logically possible that they should be, and if they actually were then S would not know that the wind blows, although S would, one may claim, have a justified belief to the effect that the wind blows. But it is, I presume, this latter claim that the skeptic would disagree with.

One may, when faced with such a skeptical challenge, try to trace an inducing primary belief for S's belief in the consequent causal statement that if S sees the trees move like they do then the wind blows, or alternatively stated, the conditional belief that if the wind weren't blowing then S wouldn't see the trees move like they do, in order to see if in fact S is not justified in holding it after all.

Let p abbreviate "the wind blows" and let q stand for "S sees the trees move like they do". Why, then, does S believe $p \rightarrow q$, where "$\rightarrow$" signifies that p causes q or that if p weren't the case then q wouldn't have been the case? His belief in $p \rightarrow q$ seems to be one that he is so to speak conditioned to have on the basis of past experience. We could maybe end our questioning here, and claim that beliefs that we are thus conditioned to believe are justified. That may be a possible way out, but I think that we should consider this to be an excessively defensive strategy.

But one may want to suggest that S believes $p \rightarrow q$ since S believes that * and that $(* \supset (p \rightarrow q))$, where "*" abbreviates "S has experienced that p (e.g. felt that p) on many and all similar occasions with evidence like the one expressed by q". But is S justified in holding $* \& (* \supset (p \to q))$? The humean could again grant that S is justified in holding the first conjunct, but deny that S is justified in holding the second. For why may not the trees on this occasion just be moving by themselves?

But why, then, does S believe that $(* \& (* \supset (p \to q)))$? The humean would say that S believes that because S believes in some uniformity principle. Why not? Let us for the sake of argument accept this.

Let us assume that S believes that U=there is some regularity in nature and that $C = (U \supset (* \supset (p \to q)))$. One may then claim that S is justified in holding $H = U \& * \& p \& C$ and that S believes p since S believes H, so S knows q if H is true and a primary belief for S. The humean, I assumed, already granted that S is justified in believing * and p, so let us take it to be established that S knows that * and p. It thus remains to consider U and C.

Note that U is a very weak statement. No matter what course the events in the world take, there will be **some** regularity in the world. And S is clearly justified in believing so. But why does S believe that there is some regularity in the world? One may hold that S's belief in U is induced by S's belief that EU=S has experienced that U. EU seems to be a truly primary belief according to our criterion LC. For if you ask S why S believes EU then it would be appropriate of S to answer that he or she believes EU because it is the case that EU. And EU is, it seems, both justified and true. So S knows that EU. H, then, cannot be a primary belief for S, but if we substitute EU for U in C and H and get H' we seem to get at a primary belief which induces q for S. If this is right, then S knows q if S knows H'. If H' is primary, then S knows H' iff S has a justified true belief that H'. We have already argued that S has a justified true belief in EU, in * and in p, and all of these seem to be primary beliefs for S according to our ceriterion LC, for S would for all these beliefs invoke the content of the belief as the reason why he holds the belief.

It remains to consider whether C', i.e. C with U replaced by EU, is also a justified true primary belief for S in order to decide whether H' is in fact a primary belief.

And it seems that C' is a primary belief, but in this case it would not help to appeal to our criterion LC. Only, it would not, it seems, be reasonable for S to base the belief in C' on some other belief. That is to say that we should not expect S to have any further beliefs which explain why S belives C'. C' is, so to speak, a fundamental empirical hypothesis for S which is justified by its own content and not by other beliefs. Note that this is not the same as to say that C' is analytically true. We have said that C' may be **justified** by its own content, and not that C' is **true** because of its meaning.

One may claim, then, that S has a justified true belief in H' and that H' is a primary belief. If so, then S knows that H'. But S's belief that q is induced by S's belief that H'. So according to this analysis S knows that the wind blows by looking out the window and observing how the branches of the trees are moving.

This, then, is one approach that one may suggest that we should use in order to respond to humean skepticism while adhering to principle (JP). The response raises questions that need to be dealt with. Let me first give the following argument to show that the strategy is not necessarily inferior to a more standard approach which would accept the use of inductive arguments. I take it that such an approach would grant that S is justified in holding EU and * and q, but would proceed directly from S's justification in holding q to S's justification in holding p. I will also assume that S is justified in holding p only if S is justified in holding that $p \rightarrow q$. But then the following holds:

$$J(EU \,\&\, * \,\&\, q) \supset (Jp \equiv J(EU \supset (* \supset (p \rightarrow q))))$$

We have here assumed the principles that justification is closed under justified implication and justified consequent causal statements:

JI $\quad Jp \,\&\, J(p \supset q) \supset Jq$

JA $\quad Jq \,\&\, J(p \rightarrow q) \supset Jp$

To see that the above principle holds, given our assumptions, suppose first that S is justified in holding (EU & * & q) and p. But we assumed that S is justified in holding p only if S is justified in holding $(p \rightarrow q)$, so it follows that S is justified in holding $(p \rightarrow q)$. It then follows that S is justified in holding $(EU \supset (* \supset (p \rightarrow q)))$.

Suppose next that S is justified in holding (EU & * & q) and $(EU \supset (* \supset (p \rightarrow q)))$. It follows that S is justified in holding EU and $(EU \supset (* \supset (p \rightarrow q)))$. By JI it follows that S is justified in holding $(* \supset (p \rightarrow q))$. But S is justified in holding *, so by JI S is justified in holding $(p \rightarrow q)$. Since S is justified in holding q, it follows by JA that S is justified in holding p.

S is, in other words, justified in holding p iff S is justified in holding C', provided that S is justified in holding EU and * and q. But we have assumed that S is justified in holding EU and * and q. So S is, given these assumptions, justified in holding p iff S is justified in holding C'.

This shows that the present analysis does not diverge too much from a standard approach which would accept the use of inductive arguments. The approach is even consistent with such a standard approach. The difference between this and a standard approach is that it instead of accepting that the use of inductive arguments preserves the justification value of the premises suggests that we have primary beliefs which, if justified, confer justification upon the beliefs that they induce. The difference between this and a standard approach is therefore similar to the difference between using e.g. mathematical induction as an inference rule and producing the same results by having axioms or an axiom schema of induction.

In what, then, does the advantage of this approach consist? If the approach, which depends upon the adoption of principle (JP), helps us to solve the inductive Gettier problem then that would, it seems, be a sufficient advantage. Another advantage that the approach seems to have is that it avoids the paradox of justification which we discussed at the beginning of the preceding section. In developing that paradox we assumed that Tim knows the USA very well, so Tim should reason as follows: "I am justified in holding that: John comes from USA. The probability that John does not come from Bakersfield, California, given that John comes from USA, equals P1 which is very close to 1. So the probability that John comes from USA'=USA minus

Bakersfield equals P1. The probability that John does not come from Barstow california given that John comes from USA' equals P2 which is very close to 1. So the probability that John comes from USA''=USA' minus Barstow equals P1 times P2. And so on.... So the probability that John comes from 212 84th Street on Manhattan equals P1 times P2 times...........times Pn, where each of Pi, $1 \le i \le n$, are close to 1, but their product would be close to zero."

Instead of becoming justified in holding that John comes from a certain place in Manhattan, which would be absurd, the analysis gives Tim justification for holding that the probability that John comes from this spot is very low indeed.28 This clearly shows that our present analysis results in a more proper attitude towards the paradox of justification as opposed to the analysis which was based upon the weak notion of justification, for it is obvious that Tim is no more justified in holding that John comes from a certain place on Manhattan than he is in holding that John comes from Bakersfield, California.

The analysis outlined here does not, I think, refute humean skepticism. What I have shown, though, is that the analysis, in addition to having certain advantages over a standard approach, does not presuppose or lead to humean skepticism. The analysis is therefore in that respect not inferior to a standard approach.

7.2.

The account given in the previous section of this chapter suffers, I think, from at least one important defect. This defect is that principle JP does not follow from, but is only consistent with, the fact that justification is not generally preserved by non-deductive inferences. The paradox of justification **does** show that justification is not generally preserved by certain enumerative types of inductive arguments, but it does **not** show that justification is **never** preserved by non-deductive arguments, or that we, as principle JP has it, always need justified entailing evidence for a proposition q for which we have evidence, in the sense that we believe q because we believe something else, if q is to be counted as justified. And note in this connection that it would take an **inductive** argument to arrive at the conclusion that no inductive argument preserves justification. But if so, the conclusion that inductive arguments never preserve justification is not itself justified.

One may also think that a second and third defect in the account given in the previous section is that principle JP will commit us to foundationalism and a certain type of skepticism. For, in order for us to be justified in holding some inferential belief q, there must, if JP is true, be at least one justified primary belief which we do not believe because of any other beliefs which we have and is such that q is believed because of it. And this seems in some cases to be a somewhat doubtful assumption. Am I not justified in believing that a fire would burn me if I put my hand in it? It seems that I am, and it also seems clear that the belief is inferential. But do I believe that a fire would burn me if I put my hand in it since I believe some proposition p which entails that a fire would burn me if I put my hand in it? If JP is true there must be such a belief in a proposition p which is a justified primary belief if my inferential belief in the proposition that a fire would burn me if I put my hand in it is justified. But this requirement seems both to presuppose a strong foundationalism and to lead to skepticism. Maybe the scepticism and foundationalism which JP gives rise to are true theories, but why think that the inductive Gettier problem which launched us on the discussion which gave rise to JP provides sufficient evidence for such theories? Such a question does not seem unreasonable given that the paradox of justification, as I have been suggesting in the previous paragraph, does not provide sufficient evidence for principle JP and for the scepticism and foundationalism which one may claim that it gives rise to.

It is, however, I think, not obviously the case that the account in the previous section **would** commit us to foundationalism or skepticism any more than our resolve to think of justification as having the right to be certain. And we have seen that we do need to think of justification as having the right to be certain in order to avoid the paradox of justification. Our theory will, it seems, be a foundationalist one at least in the very modest sense that it holds that there are non-inferential beliefs which are justified, but that is not the same as to say that the way such non-inferential beliefs cohere with other beliefs of the subject does not play any role in justifying the belief in question. Recall that we do not hold that all non-inferential beliefs are justified, rather we hold that **some** types of non-inferential beliefs are **prima facie** justified. But this leaves open the possibility that these beliefs are justified because of some kind of coherence condition which they fulfill. And so it is that our analysis may be claimed to be uncommitted to foundationalism, and this would be so even if we were to adopt the account given in the previous section.

Let us next consider the accusation that the account of the previous section is committed to certain types of skepticism. I think there is more to be said for such an accusation than there is to be said for the accusation that the approach will be commited to full fledged foundationalism. Basically, the principle JP will compell us to hold that we in **all** cases where a skeptic questions our knowledge of a proposition q, where q is an inferential belief, need to know a proposition p which entails q and is such that we believe q because we believe p. But this only seems to give the skeptic more ways of attacking our claim to know the proposition q.

It seems that it because of all this would be better if we could abandon principle JP while at the same time avoid the inductive Gettier problem. We could then adopt the quite plausible standard view that **some** but not **all** inductive arguments preserve justification. I will in the following argue that it is possible to adopt such an alternative strategy.

I think the fundamental mistake of the discussion in the previous section was in its assessment of the Gettier example which we called an inductive Gettier example. Let us therefore consider the example again. Smith believes that Jones owns a Ford simply because Smith believes that he has often and always seen Jones drive a Ford. Jones does in fact own a Ford, but not the one that Smith has seen him drive. In the example it was further assumed that Smith does not believe that Jones owns a Ford because of some belief that Smith has which also entails that Jones owns a Ford. But is this a true assumption? I do not think it is. Is it not the case that Smith believes of the car which he has often and always seen Jones drive that it is a Ford which belongs to Jones?29 It is because of our disquotational principle DP that we can say that Smith believes of that car that it is owned by Jones, for Smith would assent to the sentence "Jones owns the car which you (Smith) have seen Jones drive, and it is a Ford", and the definite description in this sentence would in this case be used referentially in order to pick out the car that Smith has seen Jones drive.

But the discussion in the previous paragraph only refutes one particular candidate for an inductive Gettier example. Could there, so one may ask, not be other more plausible candidates? If we consider the different types of Gettier beliefs that we studied in chapter V, one will, assuming that my classification of the different types of "standard" Gettier beliefs is adequate, on reflection realize that it is only in the case when an existential generalization of a statement of the form $(\exists x)(Fx)$ is inferred from some particular evidence that an inductive Gettier problem could arise. Suppose, then, that some evidence E inductively justifies S in holding that $(\exists x)(Fx)$, and that it is true that $(\exists x)(Fx)$ and that S believes that $(\exists x)(Fx)$, and assume further that S's belief in $(\exists x)(Fx)$ is an inductive Gettier belief. But what would make S's belief in $(\exists x)(Fx)$ a Gettier belief if not a fact to the effect that it is an object **b** which is F and not the object **a** which S thought was an F? I cannot see that there could be any other possibilities here. But if so, it is more than reasonable to hold that S believes $(\exists x)(Fx)$ because S believes F**a**, and F**a** does, of course, entail $(\exists x)(Fx)$. If I am right, this shows that there are no inductive Gettier examples.

My suggestion, then, or I should say hypothesis, is that all Gettier type beliefs are in fact entailed by some belief because of which they are believed. My hypothesis is, in other words, that Gettier beliefs are always secondary beliefs, and I have tried to provide ample evidence for this hypothesis. I may, however, be proven wrong. Someone may one day come up with an example which falsifies my hypothesis. But as far as I can see, the hypothesis holds true for all Gettier type examples which have been suggested in the literature. If I see far enough, the hypothesis is at least confirmed by a significant amount of examples, and by some reasonably good arguments as well.

If I am right in holding that all Gettier type beliefs are in fact also secondary beliefs, we can revert back to the analysis which was offered in chapter III, and if I am wrong, the analysis offered in the previous section would remain as an, admittedly less plausible, alternative which could be adopted. There may be other alternatives.

VIII

In the previous chapter we discussed a sorites type of problem faced by weak justification. A further, although related disadvantage of weak justification as compared to our approach to justification can be seen in the former's failure to avoid the lottery paradox[30]. In the lottery paradox the following is assumed: (A) One is justified in believing that which is very likely, and (B) If one is justified in holding p and justified in holding q then one is justified in holding p and q. Suppose there is a lottery, justifiably recognized to be fair by everyone, with as many tickets as it takes (e.g. 1000 tickets) for an adherent of weak justification to claim that one is justified in holding that e.g. ticket 1 will not win. One is thus justified in holding that ticket 1 will not win. But it is at least as unlikely that ticket 2 will win as it is that ticket 1 will win, given that the lottery is fair, so one is justified in holding that ticket 2 will not win. By assumption B one is then justified in holding that neither ticket 1 nor ticket 2 will win. By reasoning in this fashion, we may then eventually conclude that one is justified in holding that no ticket will win. But the lottery was recognized to be fair by everyone, so one is also justified in holding that some ticket will win. By assumption (B) one is therefore justified in holding that no ticket will win and some ticket will win. As it is not reasonable to hold that one can be justified in holding something which can be recognized to be inconsistent, the lottery paradox shows that a theory of justification cannot both fulfill condition (A) and (B).

The theory of justification which we are defending does not fulfill condition (A). For it does not on our account follow from the fact that one is justified in holding that a certain item is one of the lottery tickets and that if something is one of the lottery tickets then it is very likely that it will not win that one is justified in holding that the ticket will not win. It does, however, follow that one is justified in holding that it is very likely that the ticket will not win, and that it in that sense is rational to accept that the ticket will not win. But note that it does not follow from the fact that one is justified in holding that it is very likely that ticket 1 will not win and justified in holding that it is very likely that ticket 2 will not win that one is justified in holding that it is very likely that ticket 1 will not win and ticket 2 will not win. This inference is blocked because of the rules of probability theory which governs the use of the operator "it is very likely that _ ", and not by giving up assumption (B). It may be objected that it is still very likely that neither ticket 1 nor ticket 2 will lose, given that there are, say, 1000 tickets in the lottery. But this is only because "very likely" is a vague notion. As soon as this notion is made precise, it will be seen that it does not follow from the fact that event x is very likely and that event y is very likely that it is very likely that both x and y will occur.

It **does**, however, on our account follow from the fact that one is justified in holding that it is very likely that ticket 1 will not win and justified in holding that it is very likely that ticket 2 will not win that one is justified in holding that it is very likely that ticket 1 will not win and very likely that ticket 2 will not win.

Given all of this, we can see that the lottery paradox does not arise for our notion of justification because assumption (A) of the lottery paradox is not satisfied. Nor does the lottery paradox arise for the weaker notion of "rational acceptance" if we by "it is rational of S to accept p" mean that "S is justified in holding that it is very likely that p", for assumption (B) of the lottery paradox is, as can be seen from or discussion above, not satisfied by the compound operator "S is justified in holding that it is very likely that _".

As weak justification **does** accept condition (A), it must reject condition (B) on justification in order to avoid the lottery paradox. This is a very high price to pay, especially if one is interested in a notion of justification which is to serve in an analysis of knowledge. For the principle that if someone knows p and knows q then s/he knows p and q has strong intuitive appeal. In fact, I think it is true. But if condition (B) for justification is given up then this principle can only be maintained by holding that the rules governing our use of "justified" are sensitive as to whether what is justified does also fulfill the remaining necessary conditions for knowledge. If it does then (B) holds for justification, if it doesn't then (B) may not hold. But this seems to be a somewhat arbritrary decision.

More importantly, it seems that principle (B) for justification plays an important role in our use of arguments by **reductio ad absurdum.**[31] For if it can be shown that a contradiction can be derived from a set of assumptions then that gives me a reason to discard at least one of the assumptions since I am justified in holding that a contradiction cannot obtain. But if I have a reason to discard at least one of the assumptions it must follow that there is at least one assumption which is not justified, since I cannot be justified in holding something which I have a reason to discard. This reasoning does not hold, however, if it is possible, as it is if principle (B) for justification is rejected, to be justified in holding each of the assumptions in isolation and not to be justified in holding the conjunction of the assumptions. But the reasoning in the argument above is sound, so this seems to prove that principle (B) for justification is true.

Given these considerations, the lottery paradox virtually proves that condition (A) cannot be fulfilled for justification in the lottery situation. This is, I think, a fact which lends very strong support to our theory of justification, i.e. to the view that we by "justified" should mean the same as "having the right to be certain".

Appendix 1:

The argument presented in Descartes' first meditation has been one of the most debated and influential arguments in the history of philosophy. But although the Dream Argument has been commented upon by such a vast number of authors throughout the centuries, some philosophers[32] have recently claimed to shed some important new light upon the argument by using some rather elementary principles and alleged theorems of epistemic logic. One author, viz. David Gordon, even comes close to say that the Dream Argument does not stand if we subscribe to an adequate theory of knowledge. These claims deserve our attention if we are either interested in how to interpret Descartes or struggling with the challenge of skepticism.

Steiner claims that the following would be a reasonable paraphrase of the Dream Argument: "If I am in fact dreaming, then I do not know that I am sitting down. I do not know that I am not dreaming. So I do not know that I am sitting down." We can symbolize this, and let D abbreviate "I am in fact dreaming", let S abbreviate "I am sitting down" or some other sentence that holds true of the subject that does the Cartesian meditation. If we thus abbreviate Steiner's paraphrase of the Dream Argument we get:

P1 $D \supset \sim K(S)$

P2 $\sim K(\sim D)$

c $\sim K(\sim S)$

But this argument is not valid. Some other premises or rules of inference are needed in order to support the skeptical conclusion. Seeing this, Steiner proposes a rather lengthy argument as the adequate Dream Argument. It is most likely not thought of by Steiner as the argument that Descartes actually had in mind. But he would probably argue that his is the most adequate representation of the Dream Argument, that it is Cartesian in spirit, and that it might be thought of as a proper explication of what

Descartes actually had in mind. Steiner's argument runs as follows:

1 $K(S) \supset \sim D$ Premise

(If I know that I'm sitting down then I'm not dreaming.)

2 $K(K(S) \supset \sim D)$ 1, Necessitation

(I know that if I know that I'm sitting down then I'm not dreaming.)

3 $KK(S) \supset K(\sim D)$ 2, Distribution of K over "$\supset$".

(If I know that I know that I'm sitting down then I know that I'm not dreaming.)

4 $K(S) \supset KK(S)$ The KK Principle

(If I know that I'm sitting down then I know that I know that I'm sitting down.)

5 $K(S) \supset K(\sim D)$ 3,4

(If I know that I'm sitting down then I know that I'm not dreaming.)

6 $\sim K(\sim D) \supset \sim K(S)$ 5, Contraposition

(If I don't know that I'm not dreaming then I don't know that I'm sitting down.)

7 $\sim K(\sim D)$ Premise

(I don't know that I'm not dreaming.)

C $\sim K(S)$ 6,7

(I don't know that I'm sitting down.)

This argument appeals to three principles of epistemic logic, viz. Necessitation for the K-operator, Distribution of K over "⊃" and Hintikka's KK-principle. Each of these principles might be challenged. In particular, as Steiner points out, Hintikka's KK- Principle is implausible for reasons that are independent of the skeptical Dream Argument. Steiner cites as an example a student who is credited with knowledge of a test answer that she has little confidence in. We would not say of such a student that she knows that she knows the answer, but the student would be justified in complaining if points were deducted for that reason. (We must assume that Steiner did not have an oral exam in mind when he gave this example.)

There is also, one should note, strong textual evidence for holding that Descartes would have ohjected to the KK-principle, for Descartes in his Sixth Replies, writes:

> "It is true that no one can be certain that he is thinking or that he exists unless he knows what thought is and what existence is. But this does not require reflective knowledge, or the kind of knowledge that is acquired by means of demonstrations; still less does it require knowledge of reflective knowledge, i.e. knowing that we know, and knowing that we know that we know and so on ad infinitum. This kind of knowledge cannot possibly be obtained' about anything. It is quite sufficient that we should know it by that internal awareness that always precedes reflective knowledge."33

It is clear from this passage, that Descartes would have had to object to the KK-principle, for the KK-principle implies that we have what Descartes calls reflective knowledge, and knowledge of reflective knowledge, and so on ad infinitum. But Descartes states that we cannot possibly have such reflective knowledge about anything.

This leaves us with a situation where it seems that the Dream Argument does not stand. Steiner argues, however, that we can replace the KK-principle with the following rationality principle:

*   If one is committed to ~K(P) ['P' for any sentence], then it is irrational to assert P.

We can arrive at the conclusion ~KK(S) without the KK-principle from premise 1,2,3 & 7. We are thus committed to ~KK(S). So if * is correct, it is irrational to assert K(S). This is a conclusion that might satisfy the skeptic. If we furthermore accept the rationality principle

**   If it is irrational to assert K(P), then it is irrational to assert P.

then we arrive at the conclusion that it is irrational to assert S (e.g. that I am sitting down). A skeptical conclusion indeed.

But is Steiner's interpretation of the Dream Argument correct? It seems, at least on the face of it, that Descartes' argument has a significantly simpler structure than the argument presented by Steiner. Might there not be interpretations of Descartes that retain this simplicity while having the Dream Argument stand? This is exactly what Schlesinger thinks is the case.

Schlesinger introduces the symbol 'JB*sp' to mean the same as 'objectively speaking, it is rational to accept p on the basis of information in s's possession'[34]. He then establishes, or, as we will later see, tries to establish, the following equivalent theorems:

(d1) [(p => q) & Ksp] => JB*sq

(If both if p then q and s knows p then it is objectively speaking rational to accept q on the basis of information in s's possession.)

(d2) [(p => q) & -JB*sq] => -Ksp

(If both if p then q and it is objectively speaking not rational to accept q on the basis of information in s's possession then s does not know p.)

It should be noted that we cannot replace 'JB*' with 'K', because we do not know all consequences of what we know.

If we now substitute 'KS' ('I know I'm sitting') for 'p' and '~D' ('I'm not dreaming) for 'q' in (d2) we get:

(d2*) [(KS => ~D) & ~JB*s~D] => ~KKS

Schlesinger interprets Descartes as holding ~JB*~D. By using (d2*) and an appeal to Steiners principle * or **, Schlesinger is able to derive the skeptical conclusion of the Dream Argument.

But Schlesinger's proposal suffers from a fundamental unclarity: What does the first '=>' in (d2*) mean. If the '=>' is the sign for material implication, the principle is obviously wrong. It would (by (d1)) be rational, objectively speaking; to accept all true sentences q if we know one sentence p. Suppose '=>' means 'strictly implies'. But this alternative does not help a bit. All necessary statements are strictly implied by any sentence. Suppose e.g. that Fermat's Theorem (FT) is true. It is then necessarily true. Suppose furthermore that '=>' abbreviates 'strictly implies'. We then, according to (d1), have:

$[(\text{Grass is green} \Rightarrow FT) \ \& \ Ks\text{Grass is green})] \supset JB^*s(FT)$

But this result is not at all reasonable. On the contrary, it is a bizarre result, for even preliterate children know that grass is green, and it is not, objectively speaking, rational to accept FT on the basis of information that these children possess.

But if '=>' does not mean material or strict implication, then what **does** it mean? This should have been made clear by the author. As it stands, Schlesinger's analysis is quite unsatisfactory. But let us try to see whether his analysis can be amended in any plausible ways.

Suppose Schlesinger restated his analysis, and e.g. gave us (d1') as an amendment of (d1):

(d1') $[(\vdash p \supset q) \ \& \ Ksp] \Rightarrow JB^*sq$

Let us suppose that "$\vdash$" means "Is provable in the logical system A". But this does not work. Suppose e.g. that q (e.g. FT or some non-trivial mathematical or logical theorem) is provable in A. We then again have $\vdash p \supset q$ for any sentence p. So we would also in this case be committed to say that it is, objectively speaking, rational to accept q (e.g. FT) on the basis of information possessed by preliterate children. Schlesinger might in reply try to suggest (d1'') instead of (d1') as an alternative to (d1):

(d1'') $[(\vdash p \supset q) \ \& \ \text{Not-}\vdash q \ \& \ Ksp] \Rightarrow JB^*sq$

But such a suggestion would, of course, put severe restrictions on the logical system A. Suppose e.g. that q is a non-trivial tautology of the sentential calculus. We would then not want to say that it is rational to accept q on the basis of information possessed by preliterate children. So we would have to say that q is not a theorem of A, i.e. that q is not provable in A. So the logical system A would, at least in some respects, have to be significantly weaker than the sentential calculus.

As Schlesinger, in his text, is unaware of the aporia which we have pointed out, we are not offered any clues by him as to how to avoid the problem. Suppose, though, for the sake of argument, that the problem that I have pointed out could be solved. This would shift our attention from how to read

'=>' to the operator 'JB*'. There is an exegetical as well as a systematical difficulty with the way this operator has been introduced by Schlesinger. It is at least open to question whether Descartes really held that it was not rational to accept 'I am not dreaming' on the basis of the information he possessed. Descartes did hold that he could not be certain that he was not dreaming. But that is not the same as to say that he would have been irrational were he to think that he was not dreaming.

The systematical difficulty with the 'JB*' operator remains even if we disregard the exegetical problem. The systematical difficulty connects with the problem we discussed above concerning how to read '=>'. Suppose $p \supset q$ is provable in the system A and that q is, although A is a weak logical system, a relatively nontrivial sentence whose truth depends upon the truth of p and upon properties of A. Suppose furthermore that s knows p. It is then, according to Schlesinger, objectively speaking rational to accept q on the basis of the information possessed by s. But it may not be. This all depends upon what the system A is like. If A contains some reasonably strong rules of inference and axioms, we have a problem reminiscent of the one pointed out above. If A does not contain some reasonably strong rules of inference and axioms, then Schlesinger would face the difficulty of convincing us that the sentence (KS ⊃ -D) is provable in the system A. But it seems that the system A cannot be all to weak in order for it to be able to prove (KS ⊃ -D). One way out would be to take (KP ⊃ -D) [for any sentence 'P', except those that express knowledge that we may have even if it should be true that we were dreaming] as an axiom of the system A. I think, though, that the best strategy for Schlesinger would be to redefine his operator by means of a reference to the system A. He could e.g. introduce JB*' as follows:

$$JB^{*\prime}sq =_{Df} \vdash(\text{Information possessed by } s \supset q)$$

(q is provable in the system A on the basis of the information possessed by s.)

We then get

(d1''') $[\vdash(p \supset q)\ \&\ \text{Not-}\vdash q\ \&\ Ksp] \Rightarrow JB^{*\prime}sq$

and finally

(d2*''') [⊢(KS ⊃ ~D) &Not-⊢~D & ~JB*'~D] => -KKS

(If it can be proved in A that if I know that I'm sitting down then I'm not dreaming, and it cannot be proved in A that I'm not dreaming, and it cannot be proved in A from the information that I possess that I'm not dreaming, then I don't know that I know that I'm sitting down.)

(d1''') and (d2*''') are, as opposed to (d1) and (d2) as given by Schlesinger, not **obviously** unreasonable to adopt as epistemic principles. Let us, as seems reasonable, suppose that (d2*''') expresses what Schlesinger wanted to express with (d2*). We then seem to have an interpretation of the Dream Argument that is arguably simpler in structure than the interpretation or reconstruction offered by Steiner. But although one might hold that the interpretation offered by Schlesinger, or rather our revision of Schlesinger, has the virtue of being simpler and more reasonable as an interpretation of Descartes than Steiner's reconstruction of the Dream Argument, one can at least not claim that it is a simple and natural interpretation.

Note that both Steiner and Schlesinger depend upon the rationality principles * and ** in order to arrive at the skeptical conclusion of the Dream Argument. But are these so called rationality principles plausible principles?

Gordon points out that * and ** fail for the same reason as the one Steiner gives as evidence for holding that Hintikka's KK-principle fails. Consider ** and the example given by Steiner in order to criticize that KK-principle. Although it is irrational for the student who lacks confidence in the answer he gives to the test to say that he knows the answer, it is not irrational for him to give the answer. Consider *. Suppose e.g. that my sister tells me that she is getting married, and that my girlfriend and I are invited. I am committed to say (if pressed) that I do not know that my sister is getting married, because she might, for all I know, change her mind and decide not to marry, or something else might happen. But is it irrational of me to assert "My sister is getting married" as I tell my girlfriend that we are invited? It seems that it isn't, so principle * must be false.

Gordon concludes his note by saying: "It seems to me that Professor Steiner has failed to show that modifications of Hintikka's principle can be produced which are consistent with the Causal Theory while allowing a version of the Cartesian argument to stand." Gordon here invokes the Causal Theory because he seems to think that the example with the student that invalidates the KK-principle and * and ** must be derived from such a particular theory of knowledge. Although also Steiner makes such a connection, I do not think that such a claim is justified. It seems to me that the counter example with the student, and other such counter examples, are genuine and independent of any particular theory of knowledge and justification.

Gordon seems to be entirely right in his criticism of the rationality principles * and **. But if this is so, it follows that if either Steiner's reconstruction or our revision of Schlesinger's interpretation of the Dream Argument is a correct interpretation of the Dream Argument, and if the criticism of the KK-principle and the rationality principles * and ** holds, then the Dream Argument does not stand. But I think there are compelling reasons to hold that none of these interpretations of the

Dream Argument are adequate, as simpler and more natural interpretations are available.

In an unpublished paper **Epistemic Logic and The Dream Argument** which was written before the publication of Schlesinger's book, Anthony Brueckner responded to Gordon's claim, and argued that neither the KK-principle nor * nor ** are needed in deriving the skeptical conclusion of the Dream Argument.

Brueckner considers the conceptual truths:

1 $D \supset \sim(A \,\&\, S)$

(If I'm dreaming then I'm not awake and sitting down.)

and

2 $\sim(A \,\&\, S) \supset \sim K(A \,\&\, S)$

(If I'm not awake and sitting down then I don't know that I'm awake and sitting down.)

We then get

3 $D \supset \sim K(A \& S)$

(If I'm dreaming then I don't know that I'm awake and sitting down.)

by a hypothetical syllogism from 1 and 2. If we contrapose 1 we get

4 $(A \& S) \supset \sim D$

(If I'm awake and sitting down then I'm not dreaming.)

By the rule of necessitation, we get:

5 $K((A \& S) \supset \sim D)$

(I know that if I'm awake and sitting down then I'm not dreaming.)

By distributivity of K over $\supset$ we get

6 $K(A \& S) \supset K\sim D$

(If I know that I'm awake and sitting

down then I know that I'm not dreaming.)

Since we assume

7 $\sim K(\sim D)$

(I don't know that I'm not dreaming)

we get

8 $\sim K\sim(A \ \& \ S)$

('I don't know that I'm awake and sitting down')

from 6 & 7 by modus tollens.

Brueckner thus gets what he takes to be an adequate formulation of the skeptical conclusion of the Dream Argument by only appealing to the necessitation rule of epistemic logic and to the distributivity of K over $\supset$.

But one of Schlesinger's motives for giving an interpretation that differs from Steiner's is that he questions the necessitation rule for knowledge, and I think that Schlesinger is essentially correct when he questions the necessity rule. Note that even Hintikka[35], who in an important sense may be claimed to have started the whole enterprise of epistemic logic, admits that the necessitation rule for knowledge as used in an epistemic logic has as a consequence that the epistemic subject becomes omniscient. All theorems of the epistemic logic become known to the epistemic subject if the necessitation rule for knowledge is adopted, so in this sense an epistemic logic with the necessitation rule for knowledge is a very idealized theory indeed. Clearly, no **real** epistemic subject knows all the theorems of any plausible epistemic logic. It is for this reason more than just merely advisable to not rely upon any appeals to a necessitation rule for knowledge when interpreting the Dream Argument. And if such a simplification is not possible, it seems to me that one on the basis of the current literature may plausibly claim that there are no available interpretations of the Dream Argument which do not rely upon a use of quite questionable and implausible epistemic principles.

But there seems to be interpretations of the Dream Argument which are even simpler than the one offered by Brueckner, and which do not appeal to the necessitation rule for knowledge. Consider the sentence

A $\qquad \sim K(\sim D) \supset \sim K(A)$

('If I don't know that I'm not dreaming then I don't know that I'm awake')

A seems to be as much a conceptual truth as 1 and 2, although Brueckner uses some effort to derive A in a part of his paper.

We also have

B $\qquad \sim K(A) \supset \sim K(A \,\&\, S)$

(If I don't know that I'm awake then I don't know that I'm awake and sitting down.)

So if we assume

C $\qquad \sim K(\sim D)$

('I don't know that I'm not dreaming')

we arrive at the desired conclusion

D $\sim K(A \& S)$

('I don't know that I'm awake and sitting down')

from A, B and C by means of a repeated use of modus ponens. This interpretation of the Dream Argument does not appeal to any principles that are special for an epistemic logic. We can call this *the uncomplicated interpretation* of the Dream Argument.

It might be objected to Brueckner's and the uncomplicated interpretation that D is not the conclusion of the Dream Argument. The dream alternative allows us to have knowledge of conceptual truths like the proposition that 2 + 2 = 4, whereas Brueckner's interpretation or mine both seem not to allow this. But such an objection would be wrong, although it is true that the arguments do not allow for knowledge of the form 'I know that I'm awake and two and two is four'. But it does allow us to have 'I know that two and two is four'. There is a difference here between the proposition that I'm sitting down and the proposition that two and two is four, which is of the following kind. If I claim to know that I'm sitting down then "what I would like to claim to know is that I am sitting ....while awake" (Brueckner, p. 3). But there is no such claim of being awake involved if I claim to know that two and two is four.

We can because of this phenomenon provide an even simpler interpretation of the Dream Argument which we call *the simple interpretation* if we add:

E $\quad \sim K(A \ \& \ S) \supset \sim K(S)$

(If I don't know that I'm awake and sitting down then I don't know that I'm sitting down.)

Principle E does not hold for any sentence S. It does e.g. not hold for the sentence '2 + 2 = 4'. But it seems to hold for a large family of sentences that express empirical propositions, and those are the ones that are challenged by the Dream Argument. In particular, E holds for the sentence 'I'm sitting down'. From A and B and E, by a repeated hypothetical syllogism, we get

F $\sim K(\sim D)$

$\supset \sim K(S)$

The uncomplicated interpretation depends only upon the truth of A, whereas the simple interpretation depends upon the truth of A and the truth of E for the relevant set of sentences that express empirical propositions which are damaged by the Dream Argument. If both A and E are true, and I think that there are compelling reasons for holding that they are, then the skeptical conclusion of the Dream Argument can be seen to simply follow from C and F by modus ponens. If only A is true, then the skeptical conclusion will again follow by modus ponens, but the conclusion will in this case be of the form ~K(A & S), i.e. that I don't know that I'm awake and sitting down. This, as Brueckner points out, seems to be an adequate formulation of the skeptical conclusion.

It follows from my discussion, that the only inference rule that is needed in order to arrive at the skepcical conclusion of the Dream Argument, i.e. if we accept either the simple or the uncomplicated interpretation, is modus ponens.

At this point I want to bring in the analysis of knowledge which I have suggested in order to show what kind of bearing it has upon the problem of Cartesian skepticism. I take it to be obvious, given our linguistic criterion LC, that my belief to the effect that I see that there is a computer in front of me is a primary belief. Let us abbreviate this belief as S(C). Likewise, my belief to the effect that I see that if I see that there is a computer in front of me then I am not dreaming is a primary belief. Let us abbreviate this second belief as $\Sigma(S(C) \supset {\sim}D)$. Note that we two sentences ago used the word "see" in two different ways, and we symbolize the first way we used it by using "Σ" and the second way by using "S" when abbreviating the belief.[36] Both of the two ways we used the word "see", however, are veridical uses of the word, hence if it is the case that $\Sigma(S(C) \supset {\sim}D)$ then it is the case that $(S(C) \supset {\sim}D)$, and if it is the case that S(C) then it is the case that C.

Consider now my belief to the effect that I am not dreaming. What I want to suggest is that I believe that I am not dreaming because I believe that I see that there is a computer in front of me and I see that if I see that there is a computer in front of me then I am not dreaming. If we abbreviate, we get

$$B(\sim D) \text{ because } B(S(C) \ \& \ \Sigma(S(C) \supset \sim D))$$

If I am right in identifying the belief S(C) and the belief $\Sigma(S(C) \supset \sim D)$ as primary beliefs, it follows, given our analysis of knowledge, that I know S(C) and $\Sigma(S(C) \supset \sim D)$ if the beliefs are both justified and true. But the skeptic would not want to deny that I do see that there is a computer in front of me, nor would he want to deny that I see that I see that if I see that there is a computer in front of me then I am not dreaming. And I think that it would be unreasonable of the skeptic to say that I am not justified in believing that I see that there is a computer in front of me or that I am not justified in believing that I see that if I see that there is computer in front of me then I am not dreaming. If I am justified in holding anything, then it seems that I would at least be justified in having these two beliefs. If this is right, it follows that I do know that I see that there is a computer in front of me and that I know that I see that if I see that there is a computer in front of me then I am not dreaming. But it then follows, given our analysis, that I know that I am not dreaming.

In order to critizise my approach to the problem, the skeptic will have to critizise one or a combination of the following assumptions which I have made, viz. (1) that the analysis of knowledge which has been presented is adequate, or (2) that the beliefs $S(C)$ and $\Sigma(S(C) \supset \sim D)$ are primary, or (3) that the beliefs are true or (4) that the beliefs are justified, or (5) that I believe that I am not dreaming **because** I have the primary belief $(S(C) \,\&\, \Sigma(S(C) \supset \sim D))$. Needless to say, but I think that each of the assumptions (1) through (5) are quite reasonable, and in fact true.

Appendix 2:

We will in the following explore some consequences of the fact that we cannot take only occurrent or conscious beliefs into account in our analysis. For also tacit or non-occurrent beliefs play a role in our belief systems. It would e.g. be correct of us to ascribe the belief that

2 + 326 = 328 to most people we know, although few, if any, of them do actually think consciously that 2 + 326 = 328 at the time at which we ascribe it as a belief which they hold at that time.

In order to include tacit beliefs in our analysis, we need to think of beliefs in dispositional terms. We will accordingly think of a belief in a proposition p as a disposition to assent to a sentence or internal representation which expresses the proposition p. It then follows that a person S believes p, where p is a proposition, if, but not only if (see below), S is disposed to assent to **some** sentence s which expresses the proposition p. We will refer to this as our Disquotation Principle, abbreviated as DP. For reference:

DP S believes proposition p if S is disposed to assent to some sentence s which expresses p.

We are naturally assuming that S understands the sentence s to which S assents, and we must also assume that S assents to the sentence s as meaning p and not as some code for something else. Note that a person S, given this criterion, may believe a proposition p (e.g. that the Morning Star is identical with the Evening Star) and at the same time not assent to, or even assent to the negation of, one sentence s which expresses p (e.g. the sentence "The Morning Star is identical with the Evening Star") because S is assenting to some other sentence s' (e.g. the sentence "The Morning Star is identical with the Morning Star") which expresses p, provided that the proposition p in question is expressed by both sentences. One might of course dispute that the proviso given in the previous sentence is fulfilled in the case where s="The Morning Star is identical with the Evening Star" and s'="The Morning Star is identical with the Morning Star". One may hold that s and s', so defined, are sentences which express different propositions. Such is Frege's way of dealing with the situation. While it is not our main concern in this essay to deal with Frege's Puzzle or with the problem of how to deal with socalled non-extensional contexts, it turns out that these are problems which we cannot fully ignore. For it is our goal to provide an account of knowledge, and since believing a proposition is a necessary condition for knowing it, it follows that a reasonably full account of knowledge must come to terms with what it means to say that someone believes a proposition. (For the same reason, a reasonably full acoount of knowledge must also com to terms with the concepts of **truth** and **justification.** In this essay, however, we shall only tacitly assume some kind of minimalist theory of truth, and we shall try to be as neutral as possible with respect to different theories of **justification.**) But we are suggesting that DP gives a sufficient condition for what it takes for someone to believe something, and DP has consequences for how to deal with Frege's Puzzle. And the consequences are such that they need to be defended.

We will assume it as uncontroversial that different sentences may express the same proposition. An important consequence of DP and the fact that a person S may assent to one sentence s which expresses a proposition p and at the same time assent to the negation of a sentence s' which also expresses the proposition p is then that S may believe both the proposition p and its negation. S may even believe the contradictory proposition p and not-p. Insofar as one thinks that this is an unacceptable result, one must also think that at least one of the following three principles is unacceptable, viz. DP; the principle that different sentences may express the same proposition; the principle that propositions are the objects of our beliefs. I take it, like I have already stated, as being uncontroversial that different sentences may express the same proposition, and I will also assume that propositions are the objects of our beliefs. In order to defend principle DP, which plays an important role in our analysis, we therefore have to defend the possibility of there being situations where a subject believes a proposition and its negation, since DP, together with the truth of the two other principles mentioned, virtually entails that there are such situations.

Nathan Salmon has, I think, in his book *Frege's Puzzle*[37], made a strong case for holding that the principle of substitutivity of coreferential terms holds in belief contexts, and that there are such cases as we have pointed out where a person believes a proposition p and its negation because the person is disposed to assent to one sentence s which expresses p and at the same time to assent to a sentence s' which expresses the negation of p. Salmon also considers a situation where S is disposed to assent to one sentence s which expresses the proposition p and at the same time expressly witholding judgment with respect to the **same sentence** s, and his analysis of the situation gives, I think, a plausible account of what is going on. The reader is referred to his discussion. In the following I will give a brief sketch of Salmon's analysis. I then make some refinements of his analysis which make it somewhat more transparent why many, in fact most, people have had the intuition that we e.g. cannot infer that S believes that Tully is an author from the fact that Cicero is Tully and S believes that Cicero is an author. I also want to suggest that my refinements make it possible to provide a new solution to the problem as to when we can quantify into belief contexts, and to account for Donnellan's distinction between a referential and an attributive use of definite description. Before suggesting these refinements I give some examples which should make it clear, I hope, that some analysis along the lines suggested by Salmon must be the appropriate kind of analysis. The examples should also provide ample evidence for my principle DP. Principle DP is, as the reader should note, stronger than the disquotational principle suggested by Kripke.

Salmon suggests the following analysis of the proposition that S believes p, where p is a proposition:

BS $(B_S p) = (\exists x)$(S grasps p with x & BEL(S,p,x))

Salmon makes notes of three ways in which a negation sign may alter BS. We first of all have the situation where it is not at all the case that S believes p:

~BS $\sim(B_S p) = \sim(\exists x)$(S grasps p with x & BEL(S,p,x))

We also have the situation where S believes not-p:

BS~ $(B_S \sim p) = (\exists x)$(S grasps ~p with x & BEL(S,~p,x))

There is also, as Salmon points out, a sense in which a person may withhold judgment with respect to a proposition p which does not entail that the person withholds judgment with respect to p in the sense of ~BS:

$\underline{\text{BS}}$ $(\sim B_S p) = (\exists x)$(S grasps p with x & ~BEL(S,p,x))

In this latter case we shall, following Salmon, say that the subject S **withholds belief** from p. My use of parentheses in order to distinguish between the different cases should be self explanatory.

But note that there is in addition a fourth place which a negation sign can occupy, for also <u>BS</u> can be negated. In that case we get

<u>~BS</u> $\sim(\sim B_S p) = (\forall x)(S \text{ grasps } p \text{ with } x \supset BEL(S,p,x))$

Note that in one kind of circumstance, both $\sim(\sim B_S p)$ and $\sim(\sim B_S \sim p)$ can be true. If they are, it signifies that the subject does not even grasp the proposition p. This is one way in which a person may be said to fail to judge whether p which differs from what we ordinarily think of as a suspension of judgment. For when we say that a person suspends judgment as to whether p is the case we usually think of a situation where the subject grasps the proposition p but withholds belief from p and also withholds belief from ~p. We may e.g. say of a child that it **fails to judge** whether the continuum hypothesis is true simply because the continuum hypothesis is beyond the grasp of, or at least not in fact grasped by, the child, whereas we would say of some mathematicians that they **suspend judgment** as to whether the continuum hypothesis is true without thereby implying that they don't grasp the continuum hypothesis. In the first case we have a situation where $\sim(\sim B_S p)$ and $\sim(\sim B_S \sim p)$ are both true if S denotes the child we are talking about and p denotes the continuum hypothesis. In the second case, i.e. if we take S to denote one of the undecided mathematicians and p to denote the continuum hypothesis, neither $\sim(\sim B_S p)$ nor $\sim(\sim B_S \sim p)$ is true, but both $(\sim B_S p)$ and $(\sim B_S \sim p)$ would be true.

It is clearly, I think, the case that $B_S p$ implies $(\sim B_S \sim p)$. For if there is an x such that the subject grasps the proposition p with that x and BEL(S,p,x) holds, i.e. S believes p relative to x, then there must surely be an x such that S grasps the proposition ~p with that x and it is not the case that BEL(S,~p,x). This is not so because it must be impossible to believe a contradiction. In fact, I don't think that **is** impossible. Rather, I think that the fact that $B_S p$ implies $(\sim B_S \sim p)$ reflects some kind of psychological law.

One should not think that it is an objection to my claim in the previous paragraph to point out that we may have difficulties with grasping a proposition which is expressed by a sentence which begins with a long series of "not"'s, such as "It is not the case that not not not not not not not not not snow is white". One may plausibly argue against such a point as Ramsey does by pointing out that it is an accidental feature of our language, whether we talk about formal languages or natural ones, that negation is expressed by letting a sentence be preceded by a negation operator. We could e.g. imagine that the negation of a sentence was expressed by reversing it, so that the negation of the sentence "Snow is white" would be "Etiwh si nows". But such is, fortunately enough, not the case. It is, however, more than plausible to hold that the proposition expressed by a sentence s preceded by two negation operators is identical with the proposition expressed by the original sentence s. But it then follows, given our disqutational principle DP, that the proposition expressed by the sentence "It is not the case that not not

not not not not not not not snow is white" is grasped if one assents to the sentence "Snow is white", since one, according to DP, believes the proposition that snow is white if one assents to the sentence "Snow is white", and because one must grasp what one believes.

One might here want to raise something like the following as an objection to what I have just argued. The fact remains, it would seem, that there **are** some ways of formulating negations, viz. the ordinary ones, which can trick the subject in such a way that he or she becomes confused and does assent to a negation of a sentence to which the subject has earlier assented. But I do not disagree with that. What I have stated as a principle above is that if a subject does believe a proposition in the ordinary sense then the subject also withholds belief from the negation of that proposition in the ordinary sense. If we spell out the principle that $B_S p$ implies $(\sim B_S \sim p)$ in terms of the definitions which I have given above, we see that it means the same as to say that the fact that $(\exists x)$(S grasps p with x & BEL(S,p,x)) implies the fact that $(\exists x)$(S grasps ~p with x & ~BEL(S,~p,x)). But this principle does not in any way contradict the possibility which was envisaged in the argument which we have now just considered. For it may still be the case that $(\exists x)$(S grasps ~p with x & BEL(S,~p,x)).

We can on the basis of the observations that we have made above distinguish between seven distinct epistemic attitudes which a subject S may have vis à vis a proposition p. We have (1) the case where the proposition p is not even grasped by S and (2) the case where S grasps p but, as Nathan Salmon would say, **actively suspends judgment** as to whether p is the case. We also have (3) the situation where S believes p and (4) the one where S believes ~p, and nothing funny is going on. In addition we have (5) the Kripke situations where it is both the case that S believes p and that S believes ~p. And we finally have two types of Salmon situations, viz. in the first place when we both have (6) that S believes p ($B_S p$) and S withholds belief from p when S grasps the proposition in a different manner ($\sim B_S p$) and in the second place when (7) the same holds for S relative to ~p. Let me make a table which shows how I think that these seven distinct epistemic attitudes differ in their truth value ascriptions to the four expressions $B_S p$, $B_S \sim p$, $(\sim B_S p)$ and $(\sim B_S \sim p)$:

	$B_S p$	$B_S \sim p$	$(\sim B_S p)$	$(\sim B_S \sim p)$
(1)	False	False	False	False
(2)	False	False	True	True
(3)	True	False	False	True
(4)	False	True	True	False
(5)	True	True	True	True
(6)	True	False	True	True
(7)	False	True	True	True

It is a pleasant exercise to verify that the following two axioms for belief statements allow for exactly the seven different epistemic attitudes which I have discerned. By this I mean that the seven distinct epistemic attitudes are the only possible ones given that the following two axioms are true:

A(1) $(B_S p) \supset (\sim B_S \sim p)$

A(2) $(B_S p) \vee (B_S {\sim}p) \vee (({\sim}B_S {\sim}p) \equiv ({\sim}B_S p))$

A(1) may here be understood as saying that if there is a way in which S believes p then there is also a way in which S disbelieves ~p. A(2) can be understood as saying that if there neither is a way in which S believes p nor a way in which S believes ~p, then it is either the case that S actively suspends judgment as to whether p is the case or it is the case that S fails to judge whether p because S does not even grasp the proposition p. The "or" in the previous sentence must be understood in its exclusive sense.

Of the seven distinct epistemic attitudes which I have discerned, a more orthodox approach would only be able to include three, or, at the most, four. It is, I think, a quite serious defect of orthodox approaches that they have to conflate (1) and (2). For, intuitively, there **is** a difference between a subject who is actively suspending judgment with respect to a proposition that he or she grasps and one who fails to judge whether the proposition is true because he or she doesn't grasp the proposition. But since there is such a difference, the difference should have to be captured in order for an account to be counted as an adequate account of belief. One should also, I think, not try to camouflage the peculiarities in the situations described by Kripke and Salmon. The situations described **are** peculiar and different from ordinary situations. The difference between Kripke situations and Salmon situations on the one hand and ordinary situations on the other hand should therefore be captured by an adequate theory. But orthodox theories do not capture this difference, so they are not adequate.

Let us now try to apply Salmon's analysis to the following example. Suppose S at some point met Paderewski while he was performing, so S believes that Paderewski is an accomplished musician. S later gets to know about Paderewski as a statesman without realizing that he is the same as the man who played the piano, and S is not, in this situation, disposed to assent to the sentence "Paderewski is an accomplished musician" if he takes "Paderewski" to refer to the statesman. In fact, he is disposed to assent to the negation of that sentence if he takes "Paderewski" to refer to the statesman. So S is in the peculiar situation that he is disposed to assent to the sentence

"Paderewski is an accomplished musician" and at the same time to assent to the negation of that sentence. Salmon would here say that S takes the single sentence "Paderewski is an accomplished musician" to be two different sentences.[38] This bars him from thinking of the third relatum of the BEL-predicate in the definitions above as some function of subjects, times and sentences in the general case.[39] For in the Paderewski case both the subject and the time and the sentence is the same, although the subject erroneously takes what is one sentence to be two different sentences. I think that there are additional reasons for not thinking of the third relatum as such a function, but I am now anticipating some of what I am going to say below. The main thing to note at this point is that Salmon leaves us with no general account of the third relatum of the BEL-predicate in the definitions above. I will in the following try to rectify this by providing at least some rough outlines of such an account.

Let us, however, first take a look at how a Fregean theory would fare when faced with the Paderewski example. What would a Fregean say? He would probably say that the single name "Paderewski" is associated by S with two different senses. But the Fregean cannot, I think, deny that S believes that Paderewski is a musician, nor can he deny that S believes that Paderewski is not a musician. To see this, consider two other people T and U. T was with S at the concert where Paderewski performed, and T and S expressly agreed and assented to the sentence "Paderewski is an accomplished musician". And T died the next day. U was another friend of S, and U only knew about Paderewski as a statesman, and S and U expressly agreed and assented to the sentence "Paderewski is not an accomplished musician". But clearly, T believed that Paderewski was an accomplished musician and U believed that Paderewski was not an accomplished musician. If not, noone can ever have believed that Paderewski was an accomplished musician or that he wasn't. So the Fregean must concede both that T believed that Paderewski was an accomplished musician and that U believed that Paderewski was not an accomplished musician. But if so, he must also concede that S believed that Paderewski was an accomplished musician and that S believed that Paderewski was not an accomplished musician. At least the Fregean must concede this insofar as he is willing to say that S and T at one time, and S and U at another time, believed the same thing. But we have assumed that S and T were in agreement, as were S and U at a later time. So the Fregean should be willing to say, in whatever way he can, that S and T at one time, and S and U at another time, believed the same thing about Paderewski. And it should then follow that S believed that Paderewski was an accomplished musician and that S believed that Paderewski was not an accomplished musician.

All of this goes to show that we **should** say about S in our example that S believes that Paderewski is an accomplished musician and that S believes that Padereswki is not an accomplished musician. And the Fregean can, insofar as he wants to offer an analysis of propositions expressed by sentences like "T believes that Paderewski is an accomplished musician", not avoid saying that S in our example has two beliefs which contradict each other if we, as I think is reasonable, say that two contingent propositions are contradictories iff it is necessarily the case that any fact which makes one of them true (false) will make the other false (true). In particular, the Fregean does then not avoid the fact that S believes propositions which contradict each other by maintaining that a sense_1 of "Paderewski" in the true proposition expressed by "S believes that Paderewski is an accomplished musician" is different from a sense_2 of "Paderewski" in the true proposition expressed by "S believes that Paderewski is not an accomplished musician". For sense_1 and sense_2 of "Paderewski" would in this case determine the same referent, viz. Paderewski, so, necessarily, the fact that Paderewski was an accomplished musician makes the one proposition which is believed by S true, and the other false, and if it were a fact that Paderewski was not an accomplished musician, then that fact would make one of the propositions believed by S true and the other false. In short, the two propositions that are believed by S contradict each other, and this remains so even if one thinks of propositions along the lines suggested by Frege.[40]

Note that although the above argument makes an extremely strong case for saying that S in the situation described does have contradictory beliefs, the argument does **not** establish that different coreferential terms are substitutable **salva veritate** in belief contexts. A Fregean inspired philosopher may concede that there are such odd situations as the one described where the subject has contradictory beliefs, without thereby giving up his fundamental idea that **different** coreferential names like e.g. "Cicero" and "Tully" are not substitutable **salva veritate** in belief contexts. This may sound like an odd position, but it clearly is a possible one. I therefore do not think that the above argument suffices to show that different coreferential names are substitutable **salva veritate** in belief contexts, and I shall for that reason present arguments which I think make a very strong case for holding that such is indeed the case.

It is natural, but I think in a sense unfortunate, that disussions in the field of philosophy of language has primarily, if not exclusively, centered around lingual creatures and their language and access to reality. This is, I think, somewhat unfortunate because we should not forget that we also use our language to make reports about nonlingual creatures. In particular, we use our language to make reports about the inner states of animals and young children. We may e.g. say of an animal that it is in pain. And it is obvious, or at least it should be, that we can sometimes truly ascribe belief states to animals. Even Quine, who is a sententialist, would concede that much.

Let me try to bring out the relevance of all this by providing the following example. Imagine that we were in a position to observe the behaviour of one of Cicero's dogs, assuming he had some. Since we don't know the names of any of his dogs, we might as well call this one "Fido". We observe Fido throughout some time and make notes of his habits. In particular we make note of the fact that Fido salivates when Cicero is preparing food for him. In these situations we may obviously say that Fido believes that it is going to be fed by Cicero. But insofar as we may say that, we may also say that Fido believes that it is going to be fed by Tully. So we clearly have that coreferential names are substitutable **salva veritate** in the belief contexts of nonlingual creatures, and in particular in the case of Fido. And note that this is not something which holds because Fido knows that the names "Cicero" and "Tully" are coreferential or because Fido knows a **Fregean** identity proposition to the effect that Cicero is Tully. The dog may have no beliefs whatsoever concerning the names "Cicero" and "Tully", and it would therefore make no sense to talk about the dog's Fregean senses of "Tully" and "Cicero". But it does make sense to say that Fido believes that Cicero will feed it, as well as to say that Fido believes that Tully will feed it. If I am right, it follows, and at this point maybe uncontroversially, that it must be wrong to assume that the fact that a name lacks a sense for an individual should have as a consequence that the name cannot figure within the scope of the belief operator of a sentence which ascribes a belief to the individual.

I take the above argument to provide very strong evidence for the view that coreferential names **are** substitutable in belief contexts. There is at least one type of objection that a Fregean may raise. One may say that the dog **doesn't** believe that it will be fed by Cicero. This would either be extremely anthropocentric and implausible to say, or it would have to rely upon the distinction between **de re** and **de dicto** beliefs. Maybe a Fregean would claim that a nonlingual being only has objectual beliefs. It would therefore strictly speaking be false to say, **de dicto**, that Fido believes that Cicero will feed it. Fido, so one may claim, only has the **de re** belief of Cicero to the effect that he will feed it.

There are two reasons why I would not be impressed with such a reply. Firstly, I am convinced that **de re** beliefs are reducible to **de dicto** beliefs. This is something for which I will argue below. Secondly, an even stronger case for the view that coreferential names are substitutable can, I think, be made on the basis of beliefs which we ascribe to human beings. I will first try to make this stronger case, and then try to show how **de re** beliefs are reducible to **de dicto** beliefs and that they are so reducible even in the case of nonlingual beings.

I do not know much about Cicero's personal history, but let us suppose that he was bald and that he had a son whom we call "Antonio". When Antonio was less than one year's old, Cicero was already bald. Cicero, so we assume, was a compassionate father who spent much time with Antonio, and Antonio loved to stroke Cicero's bald head. And Antonio had not yet learned to speak and was not yet familiar with the use of proper names. In particular then, he did not know that "Cicero" or "Tully" are names of Cicero. But it would be quite implausible to say, I think, that Antonio did not believe that Cicero was bald. Clearly, Antonio must have had a variety of beliefs, and I do not see any reasons why we should not say that Antonio believed that Cicero was bald. Granted, Antonio may not have had a very sophisticated concept of baldness, but then again, most people don't. And the concept of baldness is not essential to my example. One may, however, if one thinks there is a problem here, e.g. want to replace "bald" with "kind", or whatever. In the following I shall pretend that there is no problem, but if one disagrees, I suggest that one replace "bald" with some word which one finds less problematic.

Let me be granted, then, at least for the sake of argument, that Antonio believed that Cicero was bald. But clearly, if such was the case, Antonio also believed that Tully was bald. And, as was the case in our example with Fido, the substitution which we are allowed to make within this belief context does **not** rely upon any fact such as that Antonio knows that "Cicero" and "Tully" are coreferential names, or that Antonio knows a **Fregean** proposition to the effect that Cicero is Tully. For Antonio was not at this stage even aware of the names "Cicero" and "Tully".

So far our example does not differ much from our previous example with Fido and Cicero. But let us consider a new stage in Antonio's development. Antonio had now learned how to speak, and he was quite familiar with the uses of the name "Cicero" in order to refer to his father. But Cicero had kept it a secret that he had used "Tully" as a pseudonym, and Antonio was not yet let in on this secret.

It was agreed above that Antonio, before he learned latin, did believe that Cicero was bald and that he believed that Tully was bald. Do we have any good reasons to hold that Antonio now did no longer believe that Tully was bald but did believe that Cicero was bald? I shall think not. Insofar as we admit that Antonio did believe that Tully was bald before Antonio learned a language, we must also admit that Antonio now that he did speak Latin still believed that Tully was bald. We have no good reasons at all to think that Antonio ever stopped believing that Tully was bald. On the contrary, it would, it seems, be patently absurd to propound the view that Antonio lost any knowledge that he had concerning Tully just because he learned how to speak Latin and to use the name "Cicero".41

But Antonio, after he learned latin, was in a situation which is not in any relevant respects different from that of a modern day schoolgirl who has learned about Cicero and believes that Cicero is an author but never learned that "Tully" refers to Cicero. If we accept the substitution of "Tully" and "Cicero" in one of these belief contexts, we should also accept the substitution in the other belief context. All of this, then, goes to show that coreferential names, contrary to what the tradition has held, indeed **are** substitutable in belief contexts.

We will now, partly in order to make the above result more palatable to the skeptical reader, discuss the nature of the third relatum in Salmon's BEL-predicate. Clearly, since it is the case that also non-lingual beings have beliefs, it cannot be the case that the third relatum is some sort of function from times, sentences and subjects. Neither the dog Fido nor the young Antonio grasps any proposition by means of sentences.

How, then, can we provide a general account of the third relatum? Let us think about what is going on in the simple case when Antonio grasps the proposition that Cicero is bald, where we think of that proposition as being identical with the ordered triple[42] $\langle \lambda\Phi\lambda u.\Phi u$, baldness, Cicero$\rangle$, and let us assume that the time at which this occurs is held constant. I shall suggest that what is going on is that Antonio somehow manages to pick out Cicero by means of a referential device α relative to one of Antonio's mental states m. Let us abbreviate this as **R**(Antonio,α,m,Cicero). α may be some mental picture or element of an internal language, whereas m is maybe most useful to think of as a dispositional state of Antonio's mind. Similarly, I shall suggest that Antonio is somehow able to pick out the property **baldness** by means of some kind of referential or expressional[43] device δ relative to a mental state m', or short **R**(Antonio,δ,m',baldness).

It is reasonable to assume that there is a very intimate connection between the second and the third argument of the **R**-predicate which I have introduced, i.e. between the mental disposition and the referential or expressional device, and I think the following principle which we stipulate captures this intimacy:

$$(\forall S)(\forall\alpha)(\forall\beta)(\forall m)(\alpha\neq\beta\supset((\exists x)\mathbf{R}(S,\alpha,m,x)\supset \sim(\exists x)\mathbf{R}(S,\beta,m,x)))$$

This principle, which we may call our Principle of Intimacy, or PI, simply suggests that each one of a subjects mental states or dispositions can be paired at most with one referential device such that the **R**-predicate holds of the subject, the referential device, the mental state and some object or property which is being picked out by the subject with the referential device relative to that mental state. The referential devices which a subject has can, on the other hand, be paired with more than one of his or her mental states or dispositions, where the **R**-predicate holds of the subject, the referential device, the mental state and the referent. This was e.g. the case in the example above where the subject S picked out Paderewski with "Padwerewski" relative to two different mental states. The mental states of a subject, then, can, given PI, be thought of as being more fine grained than the refererential devices which are at the subjects disposal.

By means of our technical vocabulary, we can now state what we take to be the analysis of the proposition that Antonio believes that Cicero is bald. We have:

Antonio believes that Cicero is bald $=_{Df}$

$$(\exists\alpha)(\exists\beta)(\exists m)(\exists m')[\mathbf{R}(\text{Antonio},\alpha,m,\text{Cicero})\ \&\ \mathbf{R}(\text{Antonio},\delta,m',\text{baldness})\ \&\ \text{BEL}(\text{Antonio},\langle\lambda\Phi\lambda u.\Phi u, \text{baldness}, \text{Cicero}\rangle,\langle m,m'\rangle)]$$

Note that, given our story about Antonio, the first of the two following existential instantiations of the definiendum above is true whereas the second is false:

I1 $(\exists\beta)(\exists m)(\exists m')[\mathbf{R}(\text{Antonio},\text{"Cicero"},m,\text{Cicero})\ \&\ \mathbf{R}(\text{Antonio},\delta,m',\text{baldness})\ \&\ \text{BEL}(\text{Antonio},\langle\lambda\Phi\lambda u.\Phi u, \text{baldness}, \text{Cicero}\rangle,\langle m,m'\rangle)]$

I2 $(\exists\beta)(\exists m)(\exists m')[\mathbf{R}(\text{Antonio},\text{"Tullius"},m,\text{Cicero})\ \&\ \mathbf{R}(\text{Antonio},\delta,m',\text{baldness})\ \&\ \text{BEL}(\text{Antonio},\langle\lambda\Phi\lambda u.\Phi u, \text{baldness}, \text{Cicero}\rangle,\langle m,m'\rangle)]$

The fact that I1 is true and I2 is false might lead some to think that something like I1 would be the proper analysis of the proposition that Antonio believes that Cicero is bald. But we should resist such suggestions. There are at least two reasons for that. Firstly, such a metalinguistic approach would fail because of Alonzo Church's translation argument[44]. Secondly, we would, if such a view were to be adopted, not be able to ascribe beliefs to nonlingual beings like Fido and the young Antonio. But I regard it as being extremely implausible to say that animals and young children don't have beliefs.

The truth of I1 and falsity of I2 does, however, I think, to a certain extent make it understandable that people have had the intuition that coreferential names are not substitutable in belief contexts. But we have seen that there is very strong evidence for holding that this traditional view is false.

Because of PI, there should be no danger in letting the third argument of the BEL-predicate consist of the ordered pair <m,m'>, for the mental states are, as we have seen, more fine grained than the referential devices available to a person. In the case of more complex propositions, the third argument will be an n-tuple of mental states, where n>2. It is useful and appropriate to think of the third argument of the BEL-predicate as signifying **how** the subject believes the proposition which is the second argument of the BEL-predicate. In some discussions about the explanatory force which a person's beliefs may have, e.g.

in explaining the person's behaviour, one should, ideally speaking, not only consider **what** a person believes but also **how** the person believes what he or she believes. Needless to say, but it will in general be quite difficult, and perhaps even impossible, to describe how a person believes a proposition. This would, however, it seems, be a problem for the philosophy of mind, and it is a problem with which we shall not be concerned in this essay.

We can on the basis of our present analysis also draw Donnellan's distinction between the attributive and the referential use of definite descriptions. If e.g. Smith while in mental state m uses the definite description "The man who murdered Jones" referentially in the sentence "The man who murdered Jones is mad", then the definite description functions in much the same way as a name, viz. as a second argument of the **R**-predicate. We assume that Smith believes the proposition which he expresses by using the sentence "The man who murdered Jones is mad", where the definite description is used referentially. Let us also assume that Smith refers to Anderson, who is the innocent man who is charged with the murder of Jones, with his referential use of the definite description. We then have:

RBB

$$(\exists m,m')[\mathbf{R}(\text{Smith},\text{"The man who murdered Jones"},m,\text{Anderson}) \wedge \mathbf{R}(\text{Smith},\text{"mad"},m,\text{madness}) \wedge \text{BEL}(\text{Smith},\langle\lambda\Phi\lambda u.\Phi u, \text{madness}, \text{Anderson}\rangle,\langle m,m'\rangle)]$$

Note that the belief which RBB ascribes to Smith may be true even if it is not the case that Anderson, the man who Smith refers to, murdered Jones. For Smith may say something true about the innocent man who is being prosecuted for the murder of Jones.

Note that RBB is not a very natural analysis of the proposition that Smith believes that the murderer of Jones is mad, rather, it more naturally analyses the proposition that Smith believes of Anderson that he is mad. This is so because **we**, knowing, by virtue of our construction of the example, that Anderson didn't commit the murder, would be unwilling to use "The murderer of Jones" referentially in order to pick out Anderson.

Suppose the definite description "The murderer of Jones" is being used attributively by Smith as Smith uses the sentence "The murderer of Jones is mad", i.e. as a statement about the man, whoever he may be, who murdered Jones. The proposition that Smith believes that the murderer of Jones is mad is then to be analysed as follows, where we let "S" be short for "Smith" and "M" be short for "is the murderer of", "I" be short for "is mad", "j" be short for "Jones" and we use "**T**" to denote the definite description function:

ABB

$$(\exists m,m',m'',m''')[\mathbf{R}(S,\text{"the"},m,\mathbf{T}) \wedge \mathbf{R}(S,\text{"murderer of"},m',\lambda xy(Mxy)) \wedge (\mathbf{R}(S,\text{"Jones"},m'',\text{Jones}) \wedge \mathbf{R}(\text{Smith},\text{"mad"},m''',\lambda x(Ix)) \wedge \text{BEL}(S,\langle\lambda\Phi\lambda X\lambda\Psi\lambda u.\Phi(X(\lambda x\Psi xu)),I,\mathbf{T},M,j\rangle,\langle m,m',m'',m'''\rangle)]$$

Note that the man who murdered Jones is not an argument of the **R**-predicate in ABB, nor is he, assuming the murderer was a man, an element in the proposition which is believed, i.e. in the second argument of the BEL-predicate. This is a very important point in connection with the problems we have had with making sense of the relational sense of believing something. The fact that the man who murdered Jones is not an argument of the **R**-predicate in ABB may be a consequence of the fact that the man who murdered Jones, i.e. the value of the **T**-function as applied to the property of being a murderer of Jones, may be outside the subject's area of acquaintaince (Cfr. Russel). The main thing to note is that we cannot on the basis of ABB existentially generalize on an individual in such a way that we get the result that there is someone who Smith believes is mad.

The same cannot, or so I claim, be the case in RBB. For the **R**-predicate cannot hold unless the subject succeeds in referring to an object with the second argument of the **R**-predicate, and that presupposes that the object in some nontrivial sense is within the subjects area of acquaintaince. My claim, then, is that a **de dicto** belief is a **de re** belief just in case we can existentially generalize as we can in RBB. A person S must be in a position to refer to an individual I in order for S to have a **de re** belief about the individual I. And in the case of definite descriptions my claim is that it is only the referential use of definite descriptions which succeeds in referring to the descriptum in such a way that a singular proposition is expressed, and consequently in such a way that the person who uses that description may be said to have a **de re** belief about the descriptum. It is for this reason that we cannot infer that there is someone whom Ralph believes to be a spy from the fact that Ralph believes that the shortest spy is a spy. For it would, presumably, only be when the description "The shortest spy" were to be used attributively that Ralph would assent to the sentence "The shortest spy is a spy".

Note that our present criterion for identifying **de re** beliefs would also classify the beliefs of nonlingual beings as **de re** if, and only if, we can existentially generalize as in RBB. In the case of nonlingual beings the second arguments of the BEL-predicate will of course be nonlinguistic referential devices, such as mental pictures or elements of an internal mental language. It may therefore be, and probably is, the case that Fido believes **of** Cicero that Cicero will feed it, but only if it is also the case that Fido believes **that** Cicero will feed it. If I am right, one cannot argue against my use of the examples above by saying that nonlingual beings only have **de re** beliefs, for what I have argued is that the set of **de re** beliefs which a subject has is a subset of the set of **de dicto** beliefs which that subject has. If this is right, it may still be the case that nonlingual beings only have **de re** beliefs, but not in such a way that it e.g. in our example with Fido would be false to say that Fido believes, de dicto, that Cicero will feed it. The **de re** beliefs are also **de dicto** beliefs, and we should not be seduced by the etymologi of "de dicto" into believing that nonlingual creatures can have no **de dicto** beliefs.

Note that a metaphysical consequence, or maybe rather a metaphysical presupposition, of our discussion has been that thinking is prior to language in the sense that thinking can occur without any use of language, whereas no use of language can occur without thinking. For to believe something is to think, so if nonlingual beings may have beliefs, it follows that nonlingual beings may think. So it follows that thinking can occur without any use of language. That no use of language can occur without thinking should be pretty obvious, and I am here naturally assuming that computers don't use a language.45 It seems, however, that it is less obvious to many people that thinking can occur without the use of language. I am puzzled by such a reluctance to admit what ought to be obvious. For how else can we understand the ontogenesis and fylogenesis of language? The view that no thinking can occur without the use of language leaves the origin of language a complete mystery, and would, it seems, ultimately have to appeal to some kind of divine intervention.

We should at this point assess the epistemological relevance of our discussion in this chapter. If our analysis is rigth, it clearly follows that the converse of our principle DP is false. For we have argued that Fido believed that Cicero was going to feed it, and that the young Antonio believed that Cicero is bald. But clearly, neither Fido nor the young Antonio were disposed to assent to any sentence s which expressed the proposition of their belief, although both Antonio and Fido must have had some internal representation which made it possible for them to grasp the propositions which they believed.

But since Fido was in a position where he believed that Cicero was going to feed it without being in a position where he would assent to a sentence s which expressed the proposition that Cicero was going to feed it, there should be no problem in assuming that principle DP must be true.

For since it is the case that a nonlingual being can believe a proposition p without being in a position to assent to any sentence s which expresses p, it must also be the case that a lingual being is in a position where he or she believes a proposition p if it is the case that he or she assents to **any** sentence s which expresses the proposition p. And we have seen that the somewhat controversial consequences which principle DP has are indeed defensible ones.

Principle DP is important to our analysis of knowledge because, if true, it refutes the idea that a fluent monolingual English speaking subject believes that P if, and only if, he or she assents to "P". Here P stands for any sentence, so that we e.g. would have: S believes that snow is white if, and only if, S assents to "Snow is white". Given DP, such a quotational/disquotational principle must be false, even in the case of a monolingual fluent English speaking subject.

The importance of all this is that we, given principle DP, will find that it should on many occassions be less controversial to ascribe beliefs to subjects. And we may even in many situation be in a position to ascribe a belief in a proposition p to a subject S even though S fails to assent to a sentence s which expresses p, or even though S may assent to the negation of s, as long as it seems to be the case that S would assent to some other sentence s' which also expresses the proposition p. This fact will be useful throughout this essay, as the analysis, and in particular the technique I use to get around the Gettier type difficulties, to a large extent depends upon us being able to ascribe beliefs to the epistemic subject.