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Cubes and Hypercubes of Opposition, with Ethical Ruminations on Inviolability

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To my nephew Karl Yngve Lervåg

Abstract. We show that we in ways related to the classical Square of Opposition may define a *Cube* of Opposition for some useful statements, and we as a by-product isolate a distinct directive of being *inviolable* which deserves attention; a second central purpose is to show that we may extend our construction to isolate hypercubes of opposition of any finite cardinality when given enough independent modalities. The cube of opposition for obligations was first introduced publically in a lecture for the Square of Opposition Conference in the Vatican in May 2014.

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Let $\mathbf{O}$ be the deontical formula forming modal formula operator abbreviating *it ought to be the case that* and let $\mathbf{B}$ (which has a suppressed reference to a subject) be the agentual formula forming modal formula operator abbreviating *it is brought about (by the subject) that*. The negation of a (deontical/agentual) modal operator is itself a (deontical/agentual) modal operator, and as usual $\mathbf{P}$ abbreviate $\neg\mathbf{O}\neg$ and we let $\reflectbox{$\mathbf{B}$}\alpha$ abbreviate $\neg\mathbf{B}\neg\alpha$. Distinguish between eight *deontic-agentive* modalities formed by letting a deontical operator be followed by an agentual operator in front of a sentence α as follows: $\mathbf{OB}\alpha$, $\mathbf{OB}\neg\alpha$, $\mathbf{O}\reflectbox{$\mathbf{B}$}\alpha$, $\mathbf{O}\reflectbox{$\mathbf{B}$}\neg\alpha$, $\mathbf{PB}\alpha$, $\mathbf{PB}\neg\alpha$, $\mathbf{P}\reflectbox{$\mathbf{B}$}\alpha$, $\mathbf{P}\reflectbox{$\mathbf{B}$}\neg\alpha$.

We assume that $\mathbf{O}$ is governed by modal principles at least as strong as the modal logic D, and that $\mathbf{B}$ is governed by modal principles at least as strong as the modal logic T. This means that if something is obligatory then it is also permitted and that something is true if it is brought about. Notice moreover that consequently both $\mathbf{B}$ and $\mathbf{O}$ obey the principles of modal logic K so that $\mathbf{B}(\alpha \rightarrow \beta) \rightarrow (\mathbf{B}\alpha \rightarrow \mathbf{B}\beta)$ and $\mathbf{O}(\alpha \rightarrow \beta) \rightarrow (\mathbf{O}\alpha \rightarrow \mathbf{O}\beta)$. Further, we have necessitation for both modal operators so that $\vdash \alpha \Rightarrow \vdash \mathbf{B}\alpha$ and $\vdash \alpha \Rightarrow \vdash \mathbf{O}\alpha$.

The noted deontic-agentive modalities decorate the vertices of the cube of Fig. 1. The arrows along some of the cube's edges signify entailment, and long

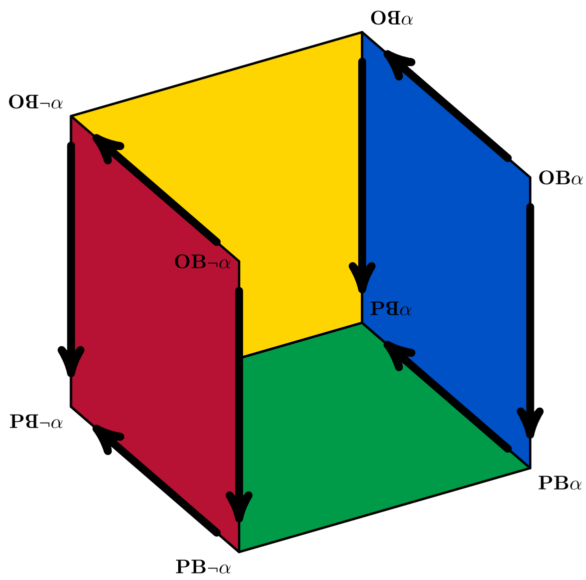

Figure 1. The cube of opposition for deontic-agentive modalities and directives

diagonals of the cube are between contradictory sentences so that we have a cube of oppositions. We say that α is *obligatory* iff **OB**α, *prohibited* iff **OB**$\neg\alpha$, *illegitimate* iff **Oꓭ**$\neg\alpha$ and *permitted* iff **PB**α. We say that α is *inviolable* iff both α and $\neg\alpha$ are illegitimate and that α is *adiaphoric* iff both α and $\neg\alpha$ are permitted.

Precisely four modalities corresponding to the vertices of a face of the cube of Fig. 1 are true in any given situation, and exactly the four faces of the cube that do not contain the vertices decorated by **OB**α and **OB**$\neg\alpha$ have vertices that correspond to a possible combination of the modalities. We say that such possible combinations form *deontic-agentive directives* for an agent with respect to a given proposition α, and we assign these directives designations and color codes as follows relating to Fig. 1: (blue) α is mandatory, (red) α is forbidden, (green) α is adiaphoric and (yellow) α is inviolable.

The author does not know that the deontic-agentive directive of being inviolable has been scrutinized or discussed in the literature. Nevertheless, in certain cases where the agent is obligated to not interfere some true propositions clearly entail the inviolability directive. To see this, let e.g. α be the state of affairs that *she is innocent* and let the subject be her father and suppose that the female in question is adult. Elaborations upon such examples make it clear that there are genuine cases of inviolability which are not subsumable under the directives of being mandatory, forbidden or adiaphoric. There seem to be many cases of genuine inviolability on account of the obligation we have of respecting the autonomy of others. Observe that objects taken to be holy are taken to be inviolable in the sense that they enter many propositions which are taken to be inviolable to many.

There are 28 lines connecting two vertices of the deontic cube, and they all express logical relations between the deontic modalities that decorate the vertices. Four of the lines are long diagonals, and these are between opposing vertices decorated by contradictory statements. Eight lines coincide with those that are decorated with an arrow in Fig. 1, and the lines signify entailment; as entailment is transitive also the line from the vertix decorated by $\mathbf{OB}\alpha$ to $\mathbf{P\reflectbox{B}}\alpha$ and the line from the vertix decorated by $\mathbf{OB}\neg\alpha$ to $\mathbf{P\reflectbox{B}}\neg\alpha$ correspond with an entailment from the first modality unto the second. The horizontal line connecting $\mathbf{O\reflectbox{B}}\alpha$ and $\mathbf{O\reflectbox{B}}\neg\alpha$ and the horizontal line connecting $\mathbf{PB}\alpha$ and $\mathbf{PB}\neg\alpha$ as well as the short diagonal connecting $\mathbf{O\reflectbox{B}}\alpha$ and $\mathbf{PB}\alpha$ and the short diagonal connecting $\mathbf{O\reflectbox{B}}\neg\alpha$ and $\mathbf{PB}\neg\alpha$ are each between *logically independent* sentences in the sense that either one of the pairs of sentences or both of them or none of them may be true. The line connecting $\mathbf{OB}\alpha$ and $\mathbf{OB}\neg\alpha$ is between conventionally contrary sentences, and the line between $\mathbf{P\reflectbox{B}}\alpha$ and $\mathbf{P\reflectbox{B}}\neg\alpha$ is between conventionally subcontrary sentences. This leaves eight short diagonals unaccounted for: The two short diagonals so far unaccounted for from the vertix decorated with $\mathbf{OB}\alpha$ and the two short diagonals so far unaccounted for from the vertix decorated with $\mathbf{OB}\neg\alpha$ are between *neoterically* contrary sentences that cannot both be true. The two short diagonals so far unaccounted for from the vertix decorated with $\mathbf{P\reflectbox{B}}\alpha$ and the two short diagonals so far unaccounted for from the vertix decorated with $\mathbf{P\reflectbox{B}}\neg\alpha$ are between *neoterically* subcontrary sentences that cannot both be false. One might want to express finer distinctions between neoteric and conventional contrariety and subcontrariety, but in our analysis here the deontic cube exhibits five pairs of contrary sentences and five pairs of subcontrary sentences.

Suppose our logic has the formulas $\mathbf{B\reflectbox{B}}\alpha \to \mathbf{B}\alpha$ and $\mathbf{B}\alpha \to \alpha$ as axiomatical theses. It is a fact that the semantics of our bi-modal logic then contains the condition on the accessibility relation that accounts for $\mathbf{B}$ that if a state u sees a state v then u sees a state w which sees v and only sees v, and also the condition that it is reflexive; from these we know that the bi-modal logic only holds in frames where the agentual accessibility relation on these states are reflexive and autistic so that states see themselves and only see themselves. This means that the semantics of our logic verifies the trivial system with the characteristic formula $\alpha \leftrightarrow \mathbf{B}\alpha$. Indeed, we can prove syntactically that if $\mathbf{B\reflectbox{B}}\alpha \to \mathbf{B}\alpha$ had been an axiom then also $\alpha \to \mathbf{B}\alpha$ would hold as a matter of logic: We have by assumption $\mathbf{B}\neg\mathbf{B}\alpha \to \neg\mathbf{B}\alpha$, i.e. $\mathbf{\reflectbox{B}B}\alpha \vee \mathbf{\reflectbox{B}}\neg\alpha$, as axiomatic. This entails $\mathbf{\reflectbox{B}}(\alpha \to \mathbf{B}\alpha)$ by elementary principles so that $\mathbf{B\reflectbox{B}}(\alpha \to \mathbf{B}\alpha)$ follows by necessitation. But as $\mathbf{B\reflectbox{B}}(\alpha \to \mathbf{B}\alpha) \to \mathbf{B}(\alpha \to \mathbf{B}\alpha)$ is supposed to hold axiomatically, it follows that $\mathbf{B}(\alpha \to \mathbf{B}\alpha)$ and consequently that $\alpha \to \mathbf{B}\alpha$.

As the derivation of $\alpha \to \mathbf{B}\alpha$ in the previous paragraph depends upon the assumption that the logic for $\mathbf{B}$ is normal, i.e. that $\vdash \alpha \Rightarrow \vdash \mathbf{B}\alpha$, one may explore the possibility of including the schema $\mathbf{B}\neg\mathbf{B}\alpha \to \mathbf{B}\neg\alpha$ while discarding the normalcy assumption. However, here we assume that $\mathbf{B}\neg\mathbf{B}\alpha$ may be true while $\mathbf{B}\neg\alpha$ is false as this in situations where it is called for helps us to draw some rather fine and useful distinctions which we do not relate here.

It may of course be that someone brings about that something ought to be the case, and so $\mathbf{BO}\alpha$ is well-formed and an *agentive-deontic* modality which potentially may be of great conceptual interest in languages as here that combine modalities. We have here focused upon one particular succession of the deontic and agentual modalities in order to describe the resulting 3-cube of opposition, and this should not be taken as advice to restrict the language.

Suppose we have n independent modalities each in isolation extending the modal logic D, and assume that each modality $\boxed{n}$ is weaker than the trivial modal logic so that $\nvdash \alpha \rightarrow \boxed{n}\alpha$; we say that such modalities as these are *appropriate*. We assume that there are directives expressed by a string of n such modalities in a particular succession such as with the *deontic-agentive* modalities above for $n = 2$, and we have an associated $n+1$-dimensional cube such as above for $n = 2$ or for some other natural number n greater than 0; the 2-dimensional cube is the square. Find next an $n + 1$st modality independent of the n preceding modalities used in the $n+1$-dimensional cube presupposed, and such that the independent $n + 1$st modality $\blacksquare$ found also is appropriate. Next prolong the modality of each vertex of the $n + 1$-dimensional hypercube by preceding it with $\blacksquare$. Next build a hypercube with one more dimension by using the next dimension for an entailment from the $\blacksquare$-decorated modalities of the vertices of the $n + 1$-dimensional hypercube to descending vertices where the same original modalities are decorated with $\blacklozenge$ instead. The directives of hypercubes of increasing dimensionality will increase as in a geometric progression and the $n + 1$-cube for $n > 0$ will have 2^n oppositional hyperdiagonals. The author hopes that cubes and hypercubes of opposition will be studied insightfully and more thoroughly by others in the future.

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