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Infinite Regress Arguments and Infinite Regresses

This paper explains what an infinite regress argument is. Part 1 contains some examples of infinite regress arguments. Part 2 presents a schema for all such arguments and defines an infinite regress argument as one that approximates to the schema. Part 3 tests the schema on the examples. Part 4 contrasts my account of infinite regress arguments with that given by Passmore and shows that Passmore's theory succumbs to objections. Part 5 distinguishes an infinite regress argument from an infinite regress and defines an infinite regress. Part 6 explains the dyslogistic force of "infinite regress".

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1 Examples

Infinite regress arguments have been used throughout the history of western philosophy. Here are some examples:

Plato's Third Man

In the *Parmenides* Plato presents the following argument, which purports to show that the theory of forms leads to an infinite regress:

> *Parmenides:* I fancy the consideration which leads you to imagine the existence of these various unitary forms is to this effect: when you have judged a number of things to be large, you presumably pronounce, in a review of them all, that they present one and the same pattern, and this is why you regard the large as one thing.
> *Socrates:* Precisely so.
> *Parmenides:* But what of *the* large and other large things? When you pass them all mentally in review in the same fashion, must this not again give rise to the appearance of a single large something, in virtue of which they all appear large?
> *Socrates:* Presumably.

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> *Parmenides:* Consequently a second form of magnitude will present itself, distinct alike from just magnitude, and from the things which participate of magnitude. On a fuller view of all these cases, we shall discover yet a further form, in virtue of which they will all be large; thus you see, every one of your forms will no longer be one, but an indefinite plurality.[1]

Aristotle on things desired for themselves

In a parenthesis in the *Nicomachean Ethics*, Aristotle states an infinite regress argument for the thesis that some things are desired for themselves:

> Now, if there exists an end in the realm of action which we desire for its own sake, an end which determines all our other desires; if, in other words, we do not make all our choices for the sake of something else — for in this way the process will go on infinitely so that our desire would be futile and pointless — then obviously this end will be the good, that is, the highest good.[2]

Sextus Empiricus on epistemic justification

Sextus Empiricus defends scepticism by arguing that justification involves an infinite regress. The quotation is from a passage in which Sextus is discussing the different theories of the material elements:

> If we shall prefer any one standpoint, or view, to the rest, we shall be preferring it either absolutely and without proof or with proof. Now without proof we shall not yield assent; and if it is to be with proof, the proof must be true. But a true proof can only be given when approved by a true criterion, and a criterion is shown to be true by means of an approved proof. If, then, in order to show the truth of the proof which prefers any one view, its criterion must be proved, and to prove the criterion in turn its proof must be pre-established, the argument is found to be the circular one which will not allow the reasoning to go forward, since the proof keeps always requiring a proved criterion, and the criterion an approved proof. And should any one propose to approve the criterion by a criterion and to prove the proof by a proof, he will be driven to a regress *ad infinitum*. Accordingly, if we are unable to assent either to all the views held

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[1] Plato (1973): *The Collected Dialogues*, Princeton: Princeton University Press. The Parmenides contains two arguments to this effect. Both are known as the Third Man argument. The first, quoted here, is at 132a1-b2 and the second at 132d1-133a6.

[2] Aristotle (1962): *Nicomachean Ethics*, Indianapolis: Bobb-Merrill Educational Publishing, I.2.

> about the elements or to any one of them, it is proper to suspend judgment about them.[3]

Aquinas' First Way

Aquinas present his First Way as follows:

> It is certain as a matter of sense-observation that some things in this world are in motion. Now whatever is in motion, is moved by something else ... Moreover, this something else, if it too is in motion, must itself be moved by something else, and that in turn by yet another thing. But this cannot go on for ever ... And so we must reach a first mover which is not moved by anything: and this all men think of as God.[4]

Bradley on relations

The remaining examples are from modern philosophy. The fifth is one of Bradley's arguments that relations and their terms are incompatible.[5] In the quoted passage the concept of a relation is associated not with that of a term but with the concept of a quality, the reason being that Bradley expounds his view of relations in the course of a discussion of the phenomenalist analysis of things as qualities in relation. The talk of qualities is not essential to the argument.

> Let us abstain from making the relation an attribute of the related, and let us make it more or less independent. "There is a relation C, in which A and B stand; and it appears with both of them." But here again we have made no progress. The relation C has been admitted different from A and B, and no longer is predicated of them. Something, however, seems to be said of this relation C, and said, again, of A and B. And this something is not to be the ascription of one to the other. If so, it would appear to be another relation, D, in

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[3] Sextus Empiricus (1976): *Outlines of Pyrrhonism*, London: William Heinemann Ltd., III, 34-36. Sextus' writings contain many versions of the problem.

[4] Aquinas, St. Thomas (1894): *Summa Theologiae*, Rome: Leonine, Ia.2.3. The translation is from Kenny, Anthony (1972): *The Five Ways: St. Thomas Aquinas' Proofs of God's Existence*, London: Routledge & Kegan Paul, 6-7. The argument is developed at length in Aquinas, St. Thomas (1894): *Summa Contra Gentiles*, Rome: Leonine, 1.13.

[5] Bradley, Francis H. (1930): *Appearance and Reality*, Oxford: Clarendon Press. Bradley approaches the thesis of incompatibility both from the side of terms and from the side of relations. In each case he uses an infinite regress argument. The argument from the side of terms is at 26-27. There are two arguments from the side of relations, one at 17-18, the other at 27-28. The quoted passage is from 17-18.

> which C, on one side, and, on the other side, A and B, stand. But such a makeshift leads at once to the infinite process. The new relation D can be predicated in no way of C, or of A and B; and hence we must have recourse to a fresh relation, E, which comes between D and whatever we had before. But this must lead to another, F; and so on, indefinitely. Thus the problem is not solved by taking relations as independently real. For, if so, the qualities and their relations fall entirely apart, and then we have said nothing. Or we have to make a new relation between the old relation and the terms; which, when it is made, does not help us. It either itself demands a new relation, and so on without end, or it leaves us where we were, entangled in difficulties.

Ryle on "the intellectualist legend"

Ryle argues as follows against the view he calls the intellectualist legend:

> The crucial objection to the intellectualist legend is this. The consideration of propositions is itself an operation the execution of which can be more or less intelligent, less or more stupid. But if, for any operation to be intelligently executed, a prior theoretical operation had first to be performed and performed intelligently, it would be a logical impossibility for anyone ever to break into the circle.[6]

Wittgenstein on criterial knowledge of referents

Wittgenstein gives a characteristically elliptic argument against the view that knowledge of referents is criterial:

> How is he to know what colour he is to pick out when he hears "red"? — Quite simple: he is to take the colour whose image occurs to him when he hears the word. — But how is he to know which colour it is "whose image occurs to him"? Is a further criterion needed for that?[7]

Kelsen on legal validity

One of the functions of the *Grundnorm* in Kelsen's thought is to be the source of the validity of all the norms in a legal system, and one reason for giving the *Grundnorm* this function is the belief that otherwise there would be an infinite regress of validation:

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[6] Ryle, Gilbert (1969): *The Concept of Mind*, London: Hutchinson, 30.

[7] Wittgenstein, Ludwig (1972): *Philosophical Investigations*, Oxford: Basil Blackwell, I.239.

> The quest for the validity of a norm is not — like the quest for the cause of an effect — a *regressus ad infinitum*; it is terminated by a highest norm which is the last reason of validity within the normative system.[8]

2 Definition of an Infinite Regress Argument

Although infinite regress arguments are common in philosophy, few philosophers have thought to say what an infinite regress argument is.[9] I propose the following definition. Let "x_1" and "x_2" be unrestricted variables, "A" a schematic predicate and "R" a schematic expression for a binary relation. Then an infinite regress argument is one that approximates to the following seven-step schema:

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[8] Kelsen, Hans (1945): *General Theory of Law and State*, Cambridge, Mass.: Harvard University Press, 111.

[9] Sanford has proposed a schema for infinite regress arguments (Sanford, David H. (1975): "Infinity and Vagueness," The Philosophical Review 84: 520-35) but he does not explicitly connect his schema with the concept of an infinite regress. (The connection is made in MacKay, Alfred F. (1980): *Arrow's Theorem: the Paradox of Social Choice*, New Haven: Yale University Press, 115-17.) The main differences between Sanford's schema and mine are, first, that Sanford uses a premiss of asymmetry in place of my premiss of irreflexivity, second, that he uses only a schematic relational expression while I also use a schematic predicate and, third, that he does not use the concept of a sequence.

Further observations on the nature of infinite regress arguments and infinite regresses can be found in Beth, Evert W. (1959): *The Foundations of Mathematics*, Amsterdam: North-Holland, 9-12; Carruthers, Peter (1981): "Frege's Regress," Proceedings of the Aristotelian Society 81: 17-32; Gardner, Martin (1965): "The Infinite Regress in Philosophy, Literature and Mathematical Proof," Scientific American 212: 128-32; Gardner, Martin (1971): "The Orders of Infinity, the Topological Nature of Dimension and 'Supertasks'," Scientific American 224: 106-9; Henry, D.P. (1972): *Medieval Logic and Metaphysics*, London: Hutchinson University Library, 56-67; Hughes, Patrick, and Brecht, George (1984): *Vicious Circles and Infinity*, Harmondsworth: Penguin Books; Lakatos, Imre (1984): *Proofs and Refutations*, Cambridge: Camrbidge University Press, 53; Passmore, John (1961): *Philosophical Reasoning*, London: Duckworth, ch. 2; Russell, Bertrand (1979): *The Principles of Mathematics*, London: George Allen & Unwin, 348-49; Sellars, Wilfrid (1955): "Vlastos and 'The Third Man'," The Philosophical Review 64: 406-7; Vlastos, Gregory (1955): "Addenda to the Third Man Argument: a Reply to Professor Sellars," The Philosophical Review 64: 443; and Yalden-Thomson, D.C. (1964): "Remarks about Philosophical Refutations," The Monist 48: 505-12.

i) $(\forall x_1)[Ax_1 \rightarrow (\exists x_2)(Ax_2 \,\&\, x_1Rx_2)]$.

This is a schematic premiss of *generation*: informally, (i) says that, if a thing has the property A, it bears the relation R to a thing that has A.

ii) $(\exists x_1)(Ax_1)$.

This is a schematic premiss of *existence*: informally, it says that something has A.

iii) *R/R is irreflexive.

This is a schematic premiss of *linearity*. (iii) says that R has an irreflexive proper ancestral. Roughly, the proper ancestral of R is the relation that x_1 bears to x_2 where x_1 bears R to x_2 or bears R to something that bears R to x_2 or … . To say that a relation is irreflexive is to say that nothing bears the relation to itself.

iv) $(\exists s)[\mathrm{Inf}(R(s)) \,\&\, (\forall)(i \in D(s) \rightarrow As_i \,\&\, As_{i+1} \,\&\, s_iRs_{i+1})]$,

where "s" and "i" are variables ranging respectively over sequences and the positive integers. Informally, (iv) says that there is a sequence with infinite range, each of whose elements has A and bears R to its successor. Note that the claim is that the sequence has infinite range, not just that it is infinite; for an infinite sequence may consist of an infinite iteration of a finite number of elements, and infinite regress arguments do not concern repetitive sequences of that kind. (iv) is derived in three stages: first, an inductive procedure is specified for generating from (i)-(iii) a sequence that satisfies the second conjunct of (iv); second, the range of the sequence is proved to be infinite; third, (iv) is inferred with the rule for introducing the existential quantifier. The deduction is sketched below.

v) ~(iv).

This is a schematic premiss of *finitude*: (v) is the negation of (iv).

vi) (iv) & ~(iv).

This contradiction follows from (iv) and (v) by the rule for introducing conjunction.

vii) ~(i) / ~(ii) / ~(iii).

The conclusion, deduced by *reductio ad absurdum*, is the negation of one of the schematic premisses (i)-(iii). *Reductio* also licenses an inference to the negation of (v), but that conclusion would not be drawn, as it would make the last three steps of the argument redundant: ~(v) is equivalent to (iv). Usually an infinite regress argument will conclude either to the negation of its premiss of generation or to that of its premiss of existence.[10] The premiss of linearity is normally taken for granted and seldom made explicit.

The premisses of an infinite regress argument need not be used in a formal proof: usually they may be used informally to make a paradox.[11] A paradox can be defined as a set of propositions each of which is plausible but not all of which can be true. Normally each premiss of an infinite regress argument has some plausibility, but not all the premisses can be true, as those of generation, existence and linearity entail the negation of the premiss of finitude. When the four premisses are presented as a paradox, the task is to resolve it by rejecting one or more of the four as false. This approach has two advantages over the use of the premisses in an infinite regress argument. First, it allows the premiss of finitude to be rejected; as already explained, that premiss cannot be rejected without redundancy in an infinite regress argument. Second, it allows more than one premiss to be rejected, whereas an infinite regress argument has room for the rejection of only one.

An assignment of truth-values to the premisses will partly determine the range of admissible theories about the relevant property and relation. Staying at the schematic level: if (i) (generation) is false, a component of a form of foundationalism in respect of A and R is true. It is true, that is, that something is basic either in the strong sense that it has A without bearing R to anything, or in the weak sense that it has A without bearing R to anything that has A. An A-less

[10] *Pace* Sanford, who claims that premisses answering to his schematic existence-proposition (similar to my (ii)) are usually beyond question: op. cit. (note 9 above), p. 520.

[11] I took this approach in Black, Oliver (1996): "A Paradox of Legal Validity," in Koller, Peter, and Puhl, Klaus, eds., *Current Issues in Political Philosophy*, Kirchberg am Wechsel: The Austrian Ludwig Wittgenstein Society, 34-39.

x_2 might itself be called basic where some x_1 with A bears R to x_2. These claims constitute only parts of foundationalist theories; a full theory will also state that, and how, non-basic things with A come to have A through a relation to basic things. It is likely to characterise the derivation recursively. Note that the *truth* of (i) is compatible with claims made by other forms of foundationalism, for example the claim that something with A is basic in the sense that it bears R to itself. Note also that, if (i) is construed as a material conditional, (ii) (existence) is true if (i) is false. If (ii) is false, a form of nihilism in respect of A is true: nothing has A.

If (iii) (linearity) is false, there are R-circles; that is, there is either (a) a finite sequence of things such that each bears R to its successor, if any, and the last bears R to the first or (b) — the limiting case where a circle contracts into a point — a thing that bears R to itself. If that thing also has A, it is basic in the sense noted above which is compatible with (i). If (v) (finitude) is false, there are infinite sequences of the kind whose existence it denies.

Sketch of a proof of (iv) from (i)-(iii)

(ii) $(\exists x_1)(Ax_1)$. Denote one of the relevant values of "x_1" by "X_1" and let X_1 be the first element S_1 of the sequence S. If $(\exists x_2)(Ax_2 \;\&\; S_1Rx_2)$, denote one of the relevant values of "x_2" by "X_2" and let X_2 be S_2. By (i), $X_2 = S_2$. Carry on in the same way: for S_i, if $(\exists x_j)(Ax_j \;\&\; S_iRx_j)$, let one of those values of "x_j" be S_{i+1}. (This specification of S presupposes the principle of dependent choice, which I take to be true. For, granted the recursion rule, the principle is a consequence of the axiom of choice, which I assume to be true.)

Suppose $R(S)$ is finite. Then there is a finite list of its members, $X_1, \ldots, X_n$, and a finite least place in S by which each of $X_1, \ldots, X_n$ has occurred at least once. That is:

$$(\exists k \geq 1)\{k \in D(S) \;\&\; (\forall x_i \in R(S))(x_i = S_1 \vee \ldots x_i = S_k) \;\&\;$$
$$(\forall m)[1 \leq m < k \rightarrow (\exists x_i \in R(S))\sim(x_i = S_1 \vee \ldots x_i = S_m)]\}.$$

Denote the relevant value of "k" by "K". Suppose S_K is the last element of S. Then, by the specification of the procedure for generating S, $\sim(\exists x_j)(Ax_j \;\&\; S_KRx_j)$. But, by (i), $(\exists x_j)(Ax_j \;\&\; S_KRx_j)$. So $(\exists x_j)(Ax_j \;\&\; S_KRx_j) \;\&\; \sim(\exists x_j)(Ax_j \;\&\; S_KRx_j)$. Hence, by RAA, S_K is not the last element of S. Therefore there is a K+1th element.

$S_{K+1} \in R(S)$. Since each of $X_1, \ldots, X_n$ has occurred at least once in S by the Kth place, $(\exists m)(1 \leq m \leq K \ \& \ S_m = S_{K+1})$. Denote one of the relevant values of "m" by "M". By the specification of the procedure for generating S, S_M*R/R S_{K+1}. Therefore S_{K+1}*R/R S_{K+1}. Hence *R/R is not irreflexive. (iii) *R/R is irreflexive. Hence *R/R both is and is not irreflexive. Therefore, by RAA, there is no K+1th element of S. Therefore there both is and is not a K+1th element. Therefore, by RAA, *R*(S) is infinite. Application of EI yields (iv).

Formerly I used two premisses in place of (iii), namely that (viii) R is irreflexive and (ix) R is transitive, to derive (iv).[12] The present formulation is more frugal because, first, (iii) is true if (viii) and (ix) are true but, second, it is not the case that (viii) and (ix) are true if (iii) is true.[13] The first of these two claims is established by deducing a contradiction from the hypothesis that (viii) and (ix) are true but (iii) false. If (iii) is false, something (X) bears *R/R to itself. By (ix), XRX. By (viii), ~(XRX). Therefore XRX & ~(XRX). The second claim is true because some non-transitive relations — eg, ... is the father of ... — have an irreflexive proper ancestral: there are no circles of fatherhood.

3 Application to the Examples

Infinite regress arguments diverge from the schema to various degrees. As the examples in part 1 make clear, such arguments are seldom stated so formally or explicitly, and even a rational reconstruction may differ in containing modal operators, in using conditional proof and *modus tollendo tollens* instead of &-introduction and *reductio ad absurdum*, or in other respects. The vague word "approximates" in the definition of an infinite regress argument is intended to leave room for these variations.

I shall now reformulate the arguments from part 1 in a way that makes clear their correspondence to the schema. The aim is just to display the structure of the arguments, not to reproduce all their details or to assess their cogency.

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[12] See Black, Oliver (1987): "Induction and Experience: an Alleged Infinite Regress," Fundamenta Scientiae 7: 391-405.

[13] I owe this point to Dorothy Edgington.

Plato's Third Man
Parmenides' reasoning in the quoted passage can be represented as follows:

i_1) For any x_1, if x_1 is large, there is an x_2 such that x_2 is a form, x_2 is large and x_1 participates in x_2 (an application of the theory of forms).
ii_1) There are large things.
iii_1) Participation has an irreflexive proper ancestral.
iv_1) There is a sequence, with infinite range, of forms each of which is large and participates in its successor.
v_1) There is no such sequence.
vi_1) There both is and is not such a sequence.
vii_1) Not-(i_1); so the theory of forms is false.

(i_1) captures Parmenides' first speech and the first sentence of his second: when he talks of "*the* large and other large things" he clearly implies that *the* large — the form of the large — is itself large. (i_1), using only two variables, does not reproduce the reference to "a number of things" and "a review of them all", but that is inessential to the argument; the reference could be made explicit by rewriting (i_1) as a proposition about a set of things $x_1, \ldots, x_n$ and a form x_{n+1}. (ii_1) is presupposed by Parmenides' question "But what of *the* large and other large things?" (iii_1) involves a concept that was not clearly articulated in Plato's day, but this premiss is needed for the derivation of the remaining steps; it is thus implicit in the argument. (iv_1) is a more precise version of the final speech. The last three steps are taken by Plato as too obvious to be worth stating.

Aristotle on things desired for themselves
The argument in the passage from Aristotle is condensed, but can be explicated as follows:[14]

i_2) For any x_1, if N desires x_1, there is an x_2 such that N desires x_2 and N believes that x_1 is a means to x_2.
ii_2) N desires something.

[14] For a discussion of the substantive issues raised by this argument, see Black, Oliver (1994): "Ends, Desires, and Rationality," International Philosophical Quarterly 34: 75-88.

iii$_2$) The relation (N believes that … is a means to …) has an irreflexive proper ancestral.
iv$_2$) There is a sequence, with infinite range, of things such that N desires each of them and believes that each is a means to its successor.[15]
v$_2$) There is no such sequence.
vi$_2$) There both is and is not such a sequence.
vii$_2$) Not-(i$_2$); so some things are desired for themselves.

Sextus Empiricus on epistemic justification
The passage from Sextus is obscure, and various arguments can be extracted from it. One concerns justified belief:[16]

i$_3$) If a belief is justified, it is supported by a justified belief.
ii$_3$) There are justified beliefs.
iii$_3$) … is supported by … has an irreflexive proper ancestral.
iv$_3$) There is a sequence, with infinite range, of justified beliefs each of which is supported by its successor.
v$_3$) There is no such sequence.
vi$_3$) There both is and is not such a sequence.
vii$_3$) Not-(ii$_3$); that is, there are no justified beliefs.

Aquinas' First Way
Aquinas' argument becomes:

i$_4$) If a thing is in motion, it is moved by something that is in motion.
ii$_4$) Something is in motion.
iii$_4$) …is moved by … has an irreflexive proper ancestral.
iv$_4$) There is a sequence, with infinite range, of things in motion each of which is moved by its successor.
v$_4$) There is no such sequence.

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[15] Objections may be raised to (i$_2$) and (iv$_2$) on the grounds that they quantify into opaque contexts and possibly quantify over the non- (or not-yet-) existent.

[16] I have discussed a version of this argument in Black, Oliver (1988): "Infinite Regresses of Justification," International Philosophical Quarterly 28: 421-37, and Black, Oliver (1988): "Justification and Context," in Weingartner, Paul, and Schurz, Gerhard, eds., *Logic, Philosophy of Science and Epistemology*, Vienna: Hölder-Pichler-Tempsky.

vi_4) There both is and is not such a sequence.
vii_4) Not-(i_4); that is, there is an unmoved mover.

(i_4) is implied by the second and third sentences of the quoted passage. The second says that if a thing is in motion it is moved by something; and Aquinas clearly assumes that the mover is itself in motion, for otherwise the third sentence is a *non-sequitur*. (ii_4) embodies the first sentence, except for the reference to sense-observation. (iii_4) and (iv_4) are implicit. (v_4) corresponds to the claim that "this cannot go on for ever", (vi_4) is taken for granted and (vii_4) corresponds to the final sentence of the passage.

Bradley on relations
Bradley's argument is less easy than the foregoing to bring into line with the schema. The object of the argument is to establish that relations and their terms are incompatible. How Bradley conceives the incompatibility is obscure, but it may be assumed that he believes at least this:

1) $\sim(\exists x_1, x_2)(x_1 R x_2)$,

where "R", as before, is schematic. (1) says in effect that no two things are related, and I shall take it to express the desired conclusion. It may be suggested that Bradley is aiming for the stronger conclusion that it is impossible for two things to be related, or for a conclusion that embraces relations with more than two places, but these refinements will be ignored in the interest of simplicity.

In the quoted passage C holds between A and B; D holds between C on the one hand and A and B on the other; E holds between D on the one hand and C, A and B on the other; and so on. Let "R_H" means "holds between the members of". Then Bradley's thought may be expressed by saying that $CR_H\{A, B\}$ implies $DR_H\{C, \{A, B\}\}$, $ER_H\{D, \{C, \{A, B\}\}\}$ and so forth, the member of R_H's range increasing in this way each time. It seems then that Bradley is arguing that from any proposition that two things are related it follows that:

2) $(\exists s)(\exists x_1, x_2)[\mathrm{Inf}(R(s))\ \&\ s_1 R_H\{x_1, x_2\}\ \&\ s_2 R_H\{s_1, \{x_1, x_2\}\}\ \&$
$(\forall i)(i \in D(s)\ \&\ i > 2 \rightarrow s_i R_H\{s_{i-1}, \{\ldots s_1, \{x_1, x_2\} \ldots\}\})]$,

where each s_i is a relation. To say that (2) follows from any such proposition is equivalent to saying that (2) follows from:

3) $(\exists x_1, x_2)(x_1 R x_2)$.

The question then is this: how can Bradley move from (3) to (2)?

The quoted passage begins by assuming that, if two things stand in a relation, the relation satisfies a certain condition. The condition is expressed by saying that the relation is "more or less independent" of the terms and again that it is "independently real". Other locutions, apparently articulating the same idea, occur in the neighbourhood of the passage: a relation is said to "be something" to the terms and described as "standing alongside of its terms" and as "a solid thing".[17] Such phrases are baffling, but perhaps the condition can be captured by a proposition quantifying over relations:

4) $(\forall x_1, x_2)[x_1 R x_2 \rightarrow (\exists r)(r R_H \{x_1, x_2\})]$.[18]

(3) and (4) allow a sequence S of relations to be generated as follows. Assume (3). Denote one of the relevant values of "x_i" by "X_i" and of "x_2" by "X_2". By (4), $(\exists r)(r R_H \{X_1, X_2\})$. Let one of the relevant values of "r" be S_1. But "R_H" is itself a relational expression and may therefore be substituted for "R" in the schema (4). So, by (4) again, $(\exists r)(r R_H \{S_1, \{X_1, X_2\}\})$. Let one of the relevant values of "r" be S_2. Applying the same procedure, $(\exists r)(r R_H \{S_2, \{S_1, \{X_1, X_2\}\}\})$. Let one of the relevant values of "r" be S_3. Carry on in the same way: for $S_i>1$, if $(\exists r)(r R_H \{s_i, \{\ldots s_1, \{X_1, X_2\} \ldots\}\})$, let one of the relevant values of "r" be S_{i+1}. Suppose it is proved that the range of S is infinite. Then:

5) $\mathrm{Inf}(R(S))\ \&\ S_1 R_H \{X_1, X_2\}\ \&\ S_2 R_H \{S_1, \{X_1, X_2\}\}\ \&$
$(\forall i)(i \in D(S)\ \&\ i > 2 \rightarrow S_i R_H \{S_{i-1}, \{\ldots S_1, \{X_1, X_2\} \ldots\}\})$.

[17] Op. cit. (note 5 above), 18, 27 and 28.

[18] The condition which (4) is intended to express is stated as the conclusion of Bradley's argument (ibid., 16-17) that relations are neither identical to nor attributes of their terms. The part of the argument to the effect that they are not attributes is repeated in different form at 27.

Quantification yields (2).

The move to (2) has not yet been justified, for a proof is still needed that S's range is infinite. A natural thought, in the light of the schema and the previous examples, is that a proof can be given by assuming that $*R_H/R_H$ is irreflexive. But that assumption cannot be applied in the same way as the corresponding assumptions in the other arguments. There the relation whose proper ancestral is asserted to be irreflexive holds between adjacent elements of the sequence concerned; R_H however links each element not directly to one next to it but to a class that contains the immediately preceding element if any. Hence a proof by way of the assumption that $*R_H/R_H$ is irreflexive will not conform to the schema. Nevertheless another relation R_S, in which each element of S does stand to its successor, may be defined in terms of R_H, and the assumption that $*R_S/R_S$ is irreflexive used to prove that S's range is infinite. With that assumption the proof conforms to the schema. The definition of R_S is:

6) $R^N{}_1 R_S R^N{}_2 = \text{df } (\exists s)(\exists x)[s_1 R_H x \ \&\ s_2 R_H\{s_1, x\} \ \&\ (\forall i)(i \in D(s) \ \&\ i > 2 \rightarrow s_i R_H\{s_{i-1}, \{ \ldots s_1, x \ldots \}\}) \ \&\ (\exists j)(j \in D(s) \ \&\ j+1 \in D(s) \ \&\ R^N{}_1 = s_j \ \&\ R^N{}_2 = s_{j+1})]$,

where "$R^N{}_1$" and "$R^N{}_2$" are schematic names of relations.

Bradley's argument can now be completed as follows: (2); but ~(2); hence (2) & ~(2); hence ~(3); that is, (1). Thus reconstructed, the argument corresponds fairly closely to the schema. The schema's (i) is answered by (4), though (4) diverges from (i) to the extent that it would be procrustean to designate any of (4)'s component expressions as twins for (i)'s "Ax_1", "Ax_2" and "x_1Rx_2". (ii) in the schema is answered by (3), and (iii) by the assumption that $*R_S/R_S$ is irreflexive. (iv) is answered by (2), (v) by the negation of (2), (vi) by the conjunction of (2) and its negation, and (vii) by (1), the negation of (3).

Ryle on "the intellectualist legend"

Ryle's argument can be represented thus:

i_5) If an agent performs an action intelligently, he has already performed an intellectual action intelligently (the intellectualist legend).

ii_5) Agents sometimes perform actions intelligently.

iii$_5$) Temporal succession has an irreflexive proper ancestral.

iv$_5$) There is a sequence, with infinite range, of intelligent actions all of which are performed by the same agent and each of which is performed after its successor in the sequence.

v$_5$) There is no such sequence.

vi$_5$) There both is and is not such a sequence.

vii$_5$) Not-(i$_5$); that is, the intellectualist legend is false.

There are some exegetic difficulties here. Ryle presents the thesis that "the consideration of propositions [a paradigmatically intellectual action] is itself an operation the execution of which can be more or less intelligent" as an *objection* to the intellectualist legend. This suggests that it is not part of the legend that, as (i$_5$) asserts, the intellectual action preceding an intelligent action is itself intelligent. But Ryle's thought seems to be that a proponent of the legend, once he acknowledges the objection, will amplify his claim to include this condition. (i$_5$) therefore represents the intellectualist legend as modified to encompass the objection. (ii$_5$), although not stated explicitly, is central to the argument; Ryle's point after all is that the intellectualist legend is false because incompatible with the obvious truth that people perform intelligent actions. (iii$_5$) is taken for granted. The remaining steps of the reconstruction constitute the simplest path to the conclusion, though the obscurity of Ryle's language makes it uncertain whether they reproduce what he has in mind. His talk of a "logical impossibility" (the word "logical" is surely out of place here) suggests that a fuller representation of his reasoning would contain a modal operator, and the reference to a "circle" suggests that he may be thinking of something different from nonrepeating sequences of the kind specified by (iv$_5$). On the other hand, Ryle attaches little importance to the word "circle"; in the next sentence he talks instead of "this regress".

Wittgenstein on criterial knowledge of referents

The passage from Wittgenstein can be expanded to:

i$_6$) For any x that refers to something, if a person knows what x refers to, he knows the referent of a criterion for the application of x.

ii$_6$) Someone knows the referent of something.

iii$_6$) The criterial relation has an irreflexive proper ancestral.

iv_6) There is a sequence, with infinite range, of referrers such that, first, the same person knows the referents of all and, second, each is a criterion for applying its predecessor if any.
v_6) There is no such sequence.
vi_6) There both is and is not such a sequence.
vii_6) Not-(i_6); so not all knowledge of referents is criterial.

Kelsen on legal validity
Kelsen's argument for *Grundnormen* can be represented as follows:[19]

i_7) A norm is valid only if validated by a valid norm.
ii_7) There are valid norms.
iii_7) Validation, on the class of norms, has an irreflexive proper ancestral.
iv_7) There is a sequence, with infinite range, of valid norms each of which is validated by its successor.
v_7) There is no such sequence.
vi_7) There both is and is not such a sequence.
vii_7) Not-(i_7); so there is a *Grundnorm*.

4 Passmore's Account of Infinite Regress Arguments

My account of infinite regress arguments differs from that given by Passmore in chapter 2 of *Philosophical Reasoning*. This is clear from the following passage:

> Philosophical regresses ... demonstrate only that a supposed way of explaining something or "making it intelligible" in fact fails to explain, not because the explanation is self-contradictory, but only because it is, in the crucial respect, of the same form as what it explains.[20]

Passmore is concerned only with *philosophical* infinite regress arguments, but this limitation may be disregarded, for the objections I shall raise apply whether or not

[19] I have discussed a version of this argument in "A Paradox of Legal Validity" (note 11 above) and in Black, Oliver (1996): "Legal Validity and the Infinite Regress," Law and Philosophy 15: 339-368.
[20] Op. cit. (note 9 above), 33.

they are restricted to arguments in philosophy. The difference between my view and Passmore's may be put like this. According to me, infinite regress arguments conclude to the negation of a proposition. Expressed semantically: they prove that a proposition is false. According to Passmore they prove not that a proposition is false, but that an explanation is inadequate. This distinction can of course be obscured by saying that in Passmore's eyes too they demonstrate the falsity of a proposition, the proposition that a certain explanation is adequate.

It will be useful to make a terminological ruling here. "X explains Y" may or may not imply that X fulfils the aims of explanation, and "explanation" itself may or may not connote adequacy in respect of those aims. Passmore's talk of a "supposed" way of explaining something and, in the passage to be quoted next, of an "alleged" explanation suggests that he is employing "explain" and its paronyms with the connotation of adequacy: in this usage a so-called explanation that fails to meet the requirements is no explanation. My claim, that Passmore views infinite regress arguments as proving that an explanation is inadequate, uses "explanation" without that connotation, for otherwise it would unkindly be attributing to Passmore the outlandish view that an infinite regress argument contains a contradiction not as a lemma, as I have maintained, but as its conclusion. For simplicity the following discussion will use "explain" and its paronyms without the connotation of adequacy.

Two further features of Passmore's account must be noted. The first is that the explanatory failure which he thinks is proved by the arguments consists in the relevant explanans' being "of the same form" as its explanandum. He adds a qualification: it is "in the crucial respect" of the same form as what it explains. The second is that he seems to regard the generation of a sequence with infinite range as in each case inessential to the desired conclusion. The point is not that it is one of a variety of methods each of which will achieve the result, but the stronger thesis that the generation of a sequence is no more than a rhetorical embellishment to emphasise a conclusion reached by other means:

> It is the first step in the regress that counts, for we at once, in taking it, draw attention to the fact that the alleged explanation or justification has failed to advance matters; that if there was any difficulty in the original situation, it breaks out in exactly the same form in the alleged explanation.[21]

[21] Ibid., 31.

A propos of Wittgenstein's argument against the criterial theory of referential knowledge, he says:

> So we could never use the criterion. To point this out *underlines* the unsatisfactory character of the original explanation and makes it perfectly clear, too, that we cannot evade the difficulty by introducing a third criterion into the story.[22]

It underlines, but apparently is unnecessary to establish, the inadequacy of the explanation.

Passmore's account needs to be amplified in two ways before it can be assessed. First, it must be determined what he takes the explanandum and explanans to be in an infinite regress argument. Second, it needs to be decided what he means by saying that the explanans has the same form, at least in "the crucial respect", as the explanandum.

Judging by his discussion of the argument from the *Parmenides*,[23] it seems that the explanandum and explanans he has in mind may be identified by reference to the first step of the schema:

i) $(\forall x_1)[Ax_1 \rightarrow (\exists x_2)(Ax_2 \ \&\ x_1Rx_2)]$.

Roughly, the explanandum is any proposition of the form "AX_1", which results from substitution in the antecedent of (i)'s conditional, and the explanans is the corresponding proposition of the form "$(\exists x_2)(Ax_2 \ \&\ X_1Rx_2)$", which results from the same substitution in the consequent. In the argument from the *Parmenides*, for example, the explanandum is any proposition of the form "X_1 is large" and the explanans is not the specified application of the theory of forms but the corresponding instance of the consequent of its conditional. That is, the explanans is not:

i_1) For any x_1, if x_1 is large, there is an x_2 such that x_2 is a form, x_2 is large and x_1 participates in x_2,

[22] Ibid., 31. Passmore's emphasis.
[23] Ibid., 19-25.

but a proposition of the form "There is an x_2 such that x_2 is a form, x_2 is large and X_1 participates in x_2". Similarly in Ryle's argument the explanandum is a proposition of the form "N performs X_1 intelligently" and the explanans is not the intellectualist legend (i_5) but a proposition of the form "There is an intellectual action x_2 that N has already peformed intelligently".

It is now clear what Passmore means by saying that the explanans is of the same form in the crucial respect as the explanandum. The point is that the same property A is ascribed to both X_1 and the relevant values of "x_2". That is, the object is to explain how something has a certain property, and the explanation is that it is by virtue of its bearing a certain relation to something that has the same property.

Three objections to Passmore's account

Passmore's account is subject to three objections from which mine is free. The first is that a coincidence of form between explanandum and explanans need not make an explanation inadequate. The second is that the thesis of superfluity — the view that the generation of a sequence with infinite range is merely a rhetorical embellishment — is implausible, especially in the light of a claim Passmore makes about infinite regress arguments in philosophy. The third is that infinite regress arguments are not usually concerned with explanation.

First objection. Passmore says that, if the explanans has the same form as the explanandum, the explanation "has failed to advance matters". It is not clear what he takes the matters concerned to be and in what sense he thinks they ought to be advanced. Perhaps there is little to be said in general terms about the purpose of explanation and the criteria whereby good explanations are distinguished from bad. The objection needs no general theory of explanation, however, for it can be mounted from paradigm cases. Consider a standard genetic explanation of something's having an inherited characteristic: so-and-so has blue eyes because both his parents have blue eyes and … . This is satisfactory, but explanans and explanandum are of the same form in the proposed sense.

Second objection. It is implausible to maintain that in an infinite regress argument the generation of a sequence with infinite range is merely a rhetorical embellishment designed to emphasise an explanatory failure due to formal identity between explanans and explanandum. Generally, in the presentation of an argument, the importance of an idea is roughly reflected by the degree to which it is explicitly stated; this rule breaks down only when the proponent has misconceived his own argument or when he has some sophisticated intention that requires obliquity. It is therefore an objection to Passmore's thesis of superfluity that, whereas to my knowledge no infinite regress argument asserts that an explanation fails through formal identity, most such arguments fairly explicitly generate a sequence with infinite range.

Even if the thesis were not exegetically implausible, there would remain the *ad hominem* consideration that Passmore regards infinite regress arguments as embodying one of "the major forms of philosophical reasoning".[24] If the thesis is true, he is claiming that one of the major forms of philosophical reasoning is a type of argument part of which consists of merely rhetorical embellishment. That is absurd. Passmore might reply that arguments like the ones considered contain as a part a direct argument — one not involving a sequence — to display the coincidence of form between explanans and explanandum,[25] and that it is only the type exemplified by these sub-arguments that he intends to dignify as a major form of philosophical reasoning. But, if the form does not comprise the generation of a sequence with infinite range, it is inappropriate for Passmore to label it "the infinite regress".[26]

Third objection. The contention that infinite regress arguments are not usually concerned with explanation embraces three more precise claims. The first is that standardly the proponent of the proposition answering to (i) in the schema does not intend the relevant substitution-instances of its consequent to explain the corresponding instances of its antecedent. The second is that usually an infinite regress argument does not prove that the instances of that proposition's consequent fail to explain the corresponding instances of the antecedent. The third is that usually the proponent of an infinite regress argument does not intend it to prove this.

As regards the first point, it is unlikely for example that a champion of the intellectualist legend will hold that propositions of the form "There is an intellectual action x_2 that N has already performed intelligently" explain propositions of the form "N performs X_1 intelligently". All his theory implies, and all he need say, is that the performance of the earlier action is a necessary condition for that

[24] Ibid., 19.

[25] At ibid., 20-21, he presents such an argument which, he says, is "readily suggested" by the argument of the *Parmenides*.

[26] Ibid., 19.

of the later. There is no reason why he should think that the statement of the necessary condition gives an explanation. Passmore might maintain that in most cases the proponent of the proposition answering to (i) will have the concept of explanation at the back of his mind. But that is unprovable.

As regards the second point, if an infinite regress argument standardly proves that the instances of the proposition's consequent fail to explain the corresponding instances of the antecedent, it does so, according to Passmore, by showing that the former have the same form as the latter. In that case we should expect to find infinite regress arguments typically embodying the following syllogistic pattern: any explanation is inadequate in which the explanans has the same form as the explanandum; in these explanations the explanans and explanandum have the same form; therefore these explanations fail. There is no trace of this pattern of reasoning in any infinite regress argument known to me. As mentioned in the second objection, no such argument asserts that an explanation fails by virtue of formal identity.

The third point hangs on the first two. If for example the proponent of the intellectualist legend is not offering an explanation, to interpret Ryle as intending to prove an explanatory failure is to accuse him of *ignoratio elenchi*. Moreover, if Ryle's argument does not establish a failure of explanation, the interpretation charges him with another form of incompetence. The interpretation is therefore forbidden by charity.

These objections refute Passmore's account of infinite regress arguments.

5 Infinite Regresses

Some philosophers talk of infinite regresses as if infinite regresses were themselves arguments. Passmore, as already noted, says that philosophical regresses demonstrate a failure of explanation and that the infinite regress is one of the major forms of philosophical reasoning.[27] This should be regarded as a loose way of speaking. If an infinite regress were an argument, it would presumably be an infinite regress argument. In that case the "argument" in the phrase "infinite regress argument" would be redundant. However, it is natural to construe "infinite

[27] Ibid., 19, 33. Other examples of this usage are found in Nerlich, Graham (1960): "Regress Arguments in Plato," Mind 69: 89, and in Vlastos, op. cit. (note 9 above), 443.

regress argument" not as pleonastic but as synonymous with "argument *to* an infinite regress". But this clearly does not signify an argument to an argument. An infinite regress is best defined, not as an argument, but as something *referred to* in an infinite regress argument.

The obvious referent for the term "infinite regress" is the sequence generated in an infinite regress argument in the course of moving from (i)-(iii) to (iv). This suggests that a definition may be achieved by identifying those properties that are attributed to the relevant sequences in all the sample arguments. "Properties" and "attributed" are intended loosely: the point is merely that the definition is to be extracted from the things the arguments say, and the things more or less strongly implied by what they say, about the sequences.[28] There are two assumptions here. The first is that the members of the class of properties in question are not jointly possessed by something that is plainly not an infinite regress. The second is that a claim made by all the arguments about the sequences they generate is a component of, ie is implied by definition by, the claim that such a sequence is an infinite regress.

It will be useful to have a list of the sequences:

The argument from the *Parmenides* generates a sequence, with infinite range, of forms each of which is large and participates in its successor.

Aristotle generates a sequence, with infinite range, of things such that someone desires each of them and believes that each is a means to its successor.

Sextus generates a sequence, with infinite range, of justified beliefs each of which is supported by its successor.

Aquinas generates a sequence, with infinite range, of things in motion each of which is moved by its successor.

Bradley generates a sequence, with infinite range, of relations each of which stands in R_S to its successor.

Ryle generates a sequence, with infinite range, of intelligent actions all of which are performed by the same agent and each of which is performed after its successor in the sequence.

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[28] Likewise I am using a broad concept of reference, in maintaining that an infinite regress is referred to in an infinite regress argument. Given that, as I shall argue, infinite regresses do not exist, I am allowing that reference can be made to the non-existent.

Wittgenstein generates a sequence, with infinite range, of referrers such that, first, the same person knows the referents of all and, second, each is a criterion for applying its predecessor if any.

Kelsen generates a sequence, with infinite range, of valid norms each of which is validated by its successor.

In all the arguments the existence of the relevant sequence is denied. This suggests that infinite regresses have it in common that they do not exist. It might be objected that this cannot be incorporated in a definition, on the grounds, first, that the project of defining an infinite regress has been described as involving the identification of the properties attributed to the sequences and, second, that nonexistence is not a property. But rephrasing will avoid this problem, and in any case it has been conceded that "properties" here carries no theoretical weight. Combining the present suggestion with the obvious point that in the arguments the sequences are attributed infinite range:

D1) An infinite regress is a nonexistent sequence with infinite range.

(D1) does not connect the nonexistence of the sequence with its having infinite range. It is reasonable to hold that it is by virtue of having infinite range that the specimen sequences do not exist. There are for example *finite* sequences of intelligent actions all of which are performed by the same agent and each of which is performed after its successor in the sequence. Granted that this thought is implicit in each argument's denial that its sequence exists, the definition may be strengthened to:

D2) An infinite regress is a sequence that by virtue of having infinite range does not exist.

"By virtue of" needs analysis. The following gloss will suffice here: a sequence is nonexistent by virtue of having infinite range if and only if the proposition that it has infinite range is an essential part of some adequate explanation of its nonexistence. This of course is no clearer than the concepts of an adequate explanation and of an essential part of an explanation. The definition now expands to:

D3) An infinite regress is a sequence S such that:
   a) S has infinite range;
   b) S does not exist; and
   c) The proposition that S has infinite range is an essential part of some adequate explanation of S's nonexistence.

(D3) fails to encompass the fact that each of the sample arguments represents the elements of the sequence it generates as things of the same sort connected to their successors by the same relation: in the Third Man they are all forms linked by participation, in the First Way they are all things in motion linked by the relation ...is moved by ..., and so on. This feature is captured by adding a clause to (D3) so that the definition becomes:

D4) An infinite regress is a sequence S such that:
   a) S has infinite range;
   b) S does not exist;
   c) The proposition that S has infinite range is an essential part of some adequate explanation of S's nonexistence; and
   d) There is a property A and a relation R such that, for every i in the domain of S, S_i has A and stands in R to S_{i+1}.

This I propose as the best definition.

Further clauses?

Temporal priority. It might be suggested that the elements of an infinite regress have more in common not merely with others of the same sequence, but with those of other infinite regresses, and that these further common properties should be incorporated in the definition. For example it might be supposed that infinite regresses involve the relation of temporal priority. Among the examples there is an explicit reference to it only in Ryle's argument: each intelligent action is preceded by the next element in the sequence. But the claim might be made that the other arguments, although they do not explicitly characterise the relevant sequences in terms of temporal priority, nevertheless imply propositions that affirm temporal relations between their elements. For example Wittgenstein's might be thought to imply that, before the subject knows the referent of the first element, he must know that of the second, and so on. Similarly Bradley's argument can be construed in temporal terms. Consider the following passage from Wollheim's commentary:

> I have assumed that, in asserting that for a relation to relate its terms, it must be related to them, the expression "For a relation to relate its terms", meant something like "Before a relation can relate its terms" or "In order for a relation to relate its terms". In other words, I have taken the expression as stating a prior condition or prerequisite of relationship.[29]

There are three objections to the addition of a clause concerning time to the definition of an infinite regress. First, it is doubtful that propositions affirming relations of temporal priority among the elements of the sequences in question can indeed be extracted from the arguments of Wittgenstein and Bradley. In this passage Wollheim also gives a reading in terms of means and ends: "in order for a relation to relate its terms". The means-end relation is not temporal. Second, even if such propositions are implied by these two arguments, it is more doubtful that they are implied by all the others. Third, supposing that all the sample arguments do imply a temporal order between the elements of the sequences, the implication in most of them is too weak — that is, falls too far short of an entailment from anything explicitly said — to be incorporated in the definition.

A definitional connection. Another suggestion is that for any infinite regress there is a property possessed by all elements of the regress and such that the proposition, regarding any element, that it possesses the property implies by definition the corresponding proposition regarding the element's successor. Russell has a definitional connection between the elements in mind when he says that, in what he calls an infinite regress of the objectionable kind:

> Two or more propositions join to constitute the meaning of some proposition; of these constituents, there is one at least whose meaning is similarly compounded; and so on *ad infinitum*. This form of regress commonly results from circular definitions.[30]

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[29] Wollheim, Richard (1969): *F.H. Bradley*, Harmondsworth: Penguin Books, 114. Vlastos also hints at a temporal ordering in his account of an infinite regress: op. cit. (note 9 above).

[30] Op. cit. (note 9 above), 348.

Similarly Passmore, when discussing Ryle's argument, maintains that a regress cannot be generated from the intellectualist legend if that theory is taken merely as an empirical hypothesis. It must rather, Passmore claims, be read as a "constitution-explanation":

> Against the mere assertion that: "Every intelligent action is in fact preceded by intelligent thinking", the infinite regress argument certainly does not apply, any more than there is a regress involved in asserting that "every happy marriage is preceded by a happy engagement" ... But the thesis Ryle is considering is not such a straightforward psychological assertion; it is in fact what we might call a "constitution-explanation" and these are subject to philosophical criticism. The thesis is that the intelligence of an action is somehow *constituted* by the fact that it is preceded by an intelligent mental operation: just as in the *Parmenides* case a thing's property is supposed to be constituted by its relation to a form.[31]

These remarks seem to imply that in the regresses generated from the intellectualist legend and the theory of forms the elements are connected by definition in the manner stated.

There are four objections here. First, the concept of a constitution-explanation is unclear. In particular, the distinction between empirical hypotheses and constitution-explanations, if it can be sustained at all, may not be sharp: given that constitution-explanations assert definitional truths, the distinction is no sharper than that between the synthetic and the analytic in general. The remaining objections correspond to those raised to the proposal about temporal priority. The second — a consequence of the first — is that it is doubtful that either the intellectualist legend or the theory of forms is a constitution-explanation; the suggestion has already been rejected that infinite regress arguments are standardly concerned with explanation at all. The third — again a consequence of the obscurity of "constitution-explanation" — is that, even if the intellectualist legend and the theory of forms are constitution-explanations, it is doubtful that this is the case with the propositions answering to the schema's (i) in the other sample arguments. The fourth is that, even if all the propositions answering to (i) in the examples are constitution-explanations, this is not sufficiently explicit to motivate the addition to (D4) of a clause mentioning the constitutive relation. Hence, if this relation and the definitional relation are identical, (D4) should not be elaborated

[31] Op. cit. (note 9 above), 26. Passmore's emphasis.

to mention the definitional relation. If the two relations are distinct, a clause about the definitional relation should be added only if some other reason is found to suppose that the relation holds between every pair of adjacent elements in the specimen sequences.

I conclude that (D4) is adequate as it stands.

6 The Dyslogistic Force of "Infinite Regress"

Many authors use the expression "infinite regress" in a way which implies that an infinite regress as such is a bad thing.[32] Others distinguish vicious from benign infinite regresses. Russell contrasts what he calls objectionable and unobjectionable infinite regresses,[33] and Yalden-Thomson writes:

> Should a concept or statement or explanation entail what is called an "infinite regress", this in itself does not necessarily result in the concept or statement being false, meaningless or self-defeating ... While some "infinite regresses" are logically vicious, others are benign; a third category — one does not know quite what to say — are bizarre, absurd or depressing, while a fourth type of regress results in mere nonsense."[34]

Gardner even claims that every infinite series is an infinite regress.[35] Passmore uses a variety of expressions: at one point he contrasts "vicious" and "harmless" infinite regresses, a dichotomy he immediately glosses as that between an "infinite regress" and an "infinite series" and later as "the difference between infinite process and infinite regress".[36] He seems to have in mind a distinction

[32] See for example Popper, Karl R. (1977): *The Logic of Scientific Discovery*, London: Hutchinson, 29.

[33] Op. cit. (note 9 above), 348-49.

[34] Op. cit. (note 9 above), 505. The assumption that an infinite regress need not be a bad thing is also made in Carruthers, op. cit. (note 9 above), esp. 19, 23 and 26, and in the published answers to a problem set in Analysis: see Radford, Colin (1983): "Report on Analysis 'Problem' No. 19", Analysis 43: 113-15. Narskii thinks that an infinite regress can be "a stimulus to dialectical investigation": Narskii, I. S. (1964): "The Concept of Formal Analysis and Dialectics," Soviet Studies in Philosophy 2: 53.

[35] "The Infinite Regress in Philosophy, Literature and Mathematical Proof" (note 9 above), 130.

[36] Op. cit. (note 9 above), 28, 30.

similar to Russell's; but, whereas Russell clearly represents it as a distinction *within* the class of infinite regresses, Passmore first expresses it thus but then as a distinction *between* the class of infinite regresses and a complementary class of series or processes.

I take "infinite regress" to have a dyslogistic force, and must therefore explain how the force arises. The materials for an explanation are supplied by the definition (D4) and the schema of an infinite regress argument. The core of the explanation is this. Infinite regresses do not exist. So any proposition P that implies that an infinite regress exists is false. Likewise any state of belief with P as object is false. It is bad for a belief to be false. The dyslogistic force of "infinite regress" arises by association with the badness of false beliefs taking as object a proposition that implies that an infinite regress exists.

A brief remark is in order about the thesis that it is bad for a belief to be false. The thesis might be developed in either or both of two ways. First, it might be said that truth is a good internal to beliefs, as it is a function of beliefs to be true, just as pumping blood is a good internal to hearts because that is the function of hearts.[37] Second, it might be said that the negative value of false beliefs is instrumental and pragmatic: a belief's being false hinders the success of purposive actions based on it, and it is good, other things equal, for a purposive action to be successful.[38] Clearly, each version of the thesis would need considerable refinement.

The explanation of the dyslogistic force of "infinite regress" is insufficient as it stands, for if it were sufficient every term would have a dyslogistic force. Consider the following reasoning, parallel to the argument just stated: dogs are mammals; so any proposition P implying that a dog is not a mammal is false; similarly any belief with P as object is false; it is bad for beliefs to be false; hence

[37] A sophisticated version of this approach is found in Millikan, Ruth G. (1993): *White Queen Psychology and Other Essays for Alice*, Cambridge, Mass.: MIT Press. See in particular ch. 3.

[38] This approach is found in Chisholm, Roderick M. (1964): "Theory of Knowledge," in Chisholm et al., eds., *Philosophy*, Englewood Cliffs: Prentice Hall, 328-30; Dewey, John (1929): *Experience and Nature*, New York: W. W. Norton, 323-24; and Lewis, Clarence I. (1971): *An Analysis of Knowledge and Valuation*, La Salle: Open Court, 258. Compare Edgley, Roy (1969): *Reason in Theory and Practice*, London: Hutchinson University Library, ch. 3, and Levi, Isaac (1967): *Gambling with Truth*, New York: Alfred A. Knopf, ch. 1.

"dog" has a dyslogistic force by association with the badness of false beliefs taking as object a proposition implying that a dog is not a mammal.

The first step in amplifying the explanation to avoid this consequence is to reason as follows. We are standardly concerned with an infinite regress only when it is being used in an infinite regress argument: we are interested in it for its role in the argument. (This perhaps explains why some philosophers are led to refer to infinite regresses as arguments.) The infinite regress is generated from the premisses of the argument in order to refute one of them. Thus, in the standard context, we are interested in an infinite regress for its bearing on the falsity of a proposition which, in conjunction with the other premisses of the relevant argument, implies that it exists. We are not typically interested in dogs for their bearing on the falsity of propositions implying that dogs are not mammals. The connection between infinite regresses and falsity, then, is important with respect to our standard concerns in a way in which that between dogs and falsity is not. This is why "infinite regress", but not "dog", derives a dyslogistic force from the connection with falsity.

There is still a lacuna in the explanation. The core account explains the dyslogistic force of "infinite regress" in terms of the falsity of beliefs, but in the amplification of the account no mention is made of beliefs: it is said rather that we are interested in infinite regresses for their bearing on the falsity of *propositions*. A connection with the falsity of propositions cannot be used to account for the force; for, although it is bad for a belief to be false, it is at best untrue and at worst senseless to say that it is bad for a proposition to be false.

The lacuna can be filled as follows. An infinite regress argument is not usually a mere mental exercise: it seeks to refute the premiss in question because someone believes the premiss. (Compare the point in part 2, that the premisses of such an argument can usually be employed to form a paradox.) That is, the final aim of the argument is to prove that this belief is false. Thus, when an infinite regress is generated in an infinite regress argument, the final purpose of its generation is to prove that a belief — namely a belief in one of the premisses from which it is generated — is false. Given, then, that we are usually concerned with an infinite regress for its role in an infinite regress argument, we are usually interested in its bearing on the falsity of a belief. The dyslogistic force is thus explained in terms of the connection with the falsity not just of a proposition but of a belief.

This reply is slightly overstated, for sometimes an infinite regress argument attacks not a proposition which is definitely believed but one for which there exists no more than an inclination to belief. For example, earlier in the *Parmenides* Socrates confesses a doubt that "trivial and undignified objects" such as hair, mud and dirt all participate in separate forms;[39] this shows a lack of conviction about the theory of forms. Again, the criterial theory of referential knowledge (i_6) is perhaps a view which many people — particularly, Wittgenstein would say, when they are gripped by a certain misleading picture — are inclined to hold, rather than one with committed champions. The explanation of the dyslogistic force of "infinite regress" is easily refined to accommodate this point.

[39] Op. cit. (note 1 above), 130c-d.