

Susanne Bobzien (Oxford)

Chrysippus' Modal Logic and its Relation to Philo and Diodorus

Introduction

Chrysippus knew and discussed Philo's and Diodorus' concepts of the conditional (*συνημμένον*), and it is usually assumed that he developed his own in order to overcome their shortcomings. It is clear from the surviving testimonies that Chrysippus was acquainted with Philo's and Diodorus' modal notions as well and also that he developed his own in contrast with those of Diodorus and in some way incorporated Philo's modal notions.

The goal of this paper is to make clear the exact relations between the modal systems of the three logicians and to expound the philosophical reasons that might have led Chrysippus to modify his predecessors' modal concepts in the way he did. It should become apparent that Chrysippus in fact skilfully combines Philo's and Diodorus' modal notions, making only a minimal change to Diodorus' concept of possibility and obtaining thus a modal system which perfectly suits his Stoic philosophy.

A preliminary note: in order to understand the Hellenistic discussion of modalities, one has to ascribe to the Hellenistic logicians the idea that there is a 'right' modal system which 'fits the world' – or at least that different modal systems can have different degrees of adequacy in describing the world. It is this conception of modal logic that makes it such an important topic in Hellenistic philosophy. The 'right' modal theory will specify what is necessary and what is possible in the world and therewith to what extent the world is determined. Philo, Diodorus and Chrysippus provide different answers here.

Hellenistic propositions and Hellenistic modalities in general

I begin with some remarks about those features which are common to the modal theories of all three logicians. They all distinguish the four modalities possibility, impossibility, necessity, and non-necessity. These were primarily modalities of *ἀξιώματα*, the Hellenistic variant of propositions. The modal notions of all three logicians meet the following four basic requirements of 'normal modal logic' (cf. scheme 1):

- (1) Possibility and impossibility are contradictory to each other and so are necessity and non-necessity.
- (2) Necessity and possibility are interdefinable.
- (3) Necessity entails truth, truth entails possibility etc.

- (4) Every proposition is either necessary or impossible or contingent. (By 'contingent' I refer to propositions which are both possible and non-necessary.)¹

Scheme 1 (for any time t)

propositions		
true	false	
possible		impossible
necessary	non-necessary	
	(contingent)	

As I have mentioned above, the modalities were, for all three philosophers, primarily modalities of ἀξιώματα *i.e.* Hellenistic propositions, more precisely simple (ἀπλοῦς) or atomic Hellenistic propositions.² It is necessary to say a little about these entities in order to avoid some common pitfalls later on.

Hellenistic simple propositions describe states or events³ of quite different degrees of generality (or particularity for that matter); *e.g.* 'Dio is walking (περιπατεῖ)' and 'Dio is taking a walk tomorrow' and 'tomorrow Dio will walk in shorts through Bamberg' are all Hellenistic simple propositions, with events of different degrees of generality corresponding to them. Furthermore, Hellenistic simple propositions can be about the past, present, or future.

Typical examples of simple propositions used in Hellenistic logic are 'it is day', 'Dio is walking', 'Dio will go to Megara' (almost all examples that survived are singular propositions). For none of these propositions is there one individual event which corresponds to it. *E.g.* 'Dio is walking' corresponds to the generic or type 'event' that Dio walks and all particular walks Dio takes in his life will be covered by it. 'Dio will go to Megara' corresponds to the generic 'event' of Dio's future going to Megara and as such covers all future journeys to Megara Dio might take.

Few examples in Hellenistic logic correspond to individual events. Those which do usually describe unique events, like a person's death: because human

1 (1) – (4): cf. Boeth. *in Int.* 234.10-235.4, D.L. 7.75; Kneale & Kneale (1962) 125–6; Frede (1974a) 107ff. Bobzien (1986) 45ff; (2): the interdefinability of the Stoic modal notions has been questioned by Mignucci (1978), Vuillemin (1989); (4): for the Stoics see Boeth. *in Int.* 393.14–19. I presuppose that all three philosophers accepted for all propositions the (temporalized) Principle of Bivalence ($\forall t (T_t[p] \vee F_t[p])$) and the (temporalized) *tertium non datur* ($\forall t (p \vee \neg p)$). For the Stoics cf. Simp. *in Cat.* 406.34–407.5, Cic. *De fato* 20ff.

2 On their being modalities of propositions see D.L. 7.75 (ἀξιώματα), Boeth. *in Int.* 234.11 (*enuntiationis*), 393.14f (*enuntiationes*).

3 I use 'event' and 'state' in a non-technical sense. Furthermore, I do not claim that the Hellenistic logicians employed such a concept of event, nor that they explicitly distinguished between type events and token events.

beings have only one life, these propositions are automatically about an individual event. There are no early Hellenistic examples of 'dated propositions' as *e.g.* 'Clio walks on September 3, 1991'. Individual events seem not to have been referred to in this way.⁴

With this feature of generality or indefiniteness of Hellenistic propositions comes a different concept of truth. Truth and falsehood are not timeless or atemporal properties of propositions but belong to them 'at a time'. The proposition 'it is day' is true iff it is daytime; at night it is false (D.L. 7.65). The proposition 'it will be day' is true now if at some time later than now it will be day and the proposition 'it was day' is true now if some time earlier it was day.

As thus becomes apparent, for Hellenistic atomic propositions about past or future the truth-criteria contain – at least implicitly – a quantification over time; *i.e.* (if they do not contain time-indexicals like 'tomorrow') the criterion has the general form 'P' is true iff there is a (future/past) time at which Q'. The case of propositions which contain time indexicals is a little more complex.⁵ I will use 'T_[p]' and F_[p] in

4 But cf. Cic. *De fato* 19, *Moriatur Epicurus, cum duo et LXX annos vixerit, archonte Pytharato*, as a later example of that kind.

5 I assume that in the case of propositions with time indexicals the truth-criterion time is restricted to the time referred to by the indexical. *I.e.* 'Mia will cycle tomorrow' is true *now* iff there is a time tomorrow at which Mia cycles; 'Mia cycled last year' is true *now* iff there was a time last year at which Mia cycled. 'Theo arrives at noon' is true *now* iff Theo arrives at noon. Further, I take it that 'Mia will cycle tomorrow' will be true today and false tomorrow if she cycles tomorrow but not the day after; cf. Alex.Aphr. *Fat.* 177.7ff.

I assume that propositions about the present have the very time of their assertion as truth-criterion time. Otherwise *e.g.* the passage in Simp. *in Ph.* 1299.36ff does not make sense: it reports that 'If Dio is alive he will be alive' was seen as becoming false at the moment when Dio dies (even though this moment cannot be clearly determined, *i.e.* there is an epistemic problem). For a possibly different view of the present cf. Frede, this volume.

For propositions about the past, it seems, the general truth conditions given present no difficulties. But there are problems with propositions about the future: what happens to 'Dio will die' at the time when he dies? There are two general possibilities both of which seem unsatisfactory:

(i) The tenses are not parts of the propositions (*i.e.* of the entities which bear the truth-values) but only of the sentences which express them. The proposition expressed by the sentence "Dio will die" has to be expressed by "Dio dies / is dying" when Dio dies and by "Dio died" once Dio is dead. All three sentences are assumed to express the same proposition.

The main difficulty with this view is this: let us assume again that Dio is alive. Then the proposition 'Dio will die', which is now expressed by the sentence "Dio will die" and will later be expressed by "Dio died", is true now; but this very sentence "Dio died" *now* expresses another proposition, one which is false. One might just about swallow this, but what if we take 'Dio will be alive'? Will there be at the time of Dio's death suddenly two present propositions 'Dio is alive' and shortly after three past propositions 'Dio was alive'? What happens in the case of the proposition 'Dio will go to Athens' (assuming that Dio goes a number of times in his life)?

(ii) An alternative view is this: the tenses *are* part of the propositions (*i.e.* of the bearers of truth-values). 'Dio will die' and 'Dio will be alive' become false at the very moment he dies and remain false thereafter. There is some evidence that some Stoics at some point took this view (Alex.Aphr. *Fat.* 10; Alex.Aphr. *in Apr.* 403). But as far as Diodorean modalities are concerned, this view gives some strange results: 'Dio will die' for instance would not be Diodorean

order to symbolize ‘p’ is true-at-the-time-t’ and ‘p’ is false-at-the-time-t’, with ‘p’ standing for Hellenistic propositions about past, present or future as introduced above.

It will prove useful for what follows to distinguish between the time of assertion of a proposition, (which for reasons of convenience I will henceforth call *now* – although this *now*, of course, runs over all ‘real’ points of time, as it were) and the time the truth-criterion is concerned with. In the case of propositions about the present this latter time is the same as *now*, for propositions about the future it is the future relative to *now* and for propositions about the past it is the past relative to *now*. I will use the index n for ‘now’ (e.g. $T_n[p]$ for ‘p’ is true-*now*’).

As a consequence of this temporalized concept of truth, many Hellenistic propositions change their truth-value over time; ‘it is day’ for instance changes its truth-value twice daily. These propositions were called ‘changing propositions’ (μεταπίπτοντα).⁶

The existence of future and past propositions together with the time-related concept of truth has one disconcerting consequence: it enables Hellenistic logicians to talk about future (and past) events in two different ways: on the one hand they can speak of the future truth of propositions about the – relative – present (they can say that ‘Pia goes to Athens’ will be true). On the other, of the present truth of propositions about the – relative – future (they can also say that ‘Pia will go to Athens’ is true). This double covering of the future is responsible for a good deal of logical muddle, especially in the realm of modalities.

Stoic modalities seem to be properties of propositions (rather than operators), just as truth and falsehood; and, if one follows Boeth. *in Int.* 234 and Epict. *Diss.* 2.19.1–5 this could be true of Diodorus’ and Philo’s modalities as well.⁷ The modalities are temporalized in the same way as the truth-values (possible-at-t etc.). Thus I will use $M_n[p]$ for ‘(the proposition) ‘p’ is possible-*now*’ and $L_n[p]$ for ‘p’ is necessary-*now*’, with ‘p’ for Hellenistic propositions about past, present or future, as introduced above.⁸

necessary. (It could of course be the case that Diodorus’ and the Stoics’ concept of a proposition differ in this respect.)

I do not see a satisfactory way out of these difficulties. But it is important to be aware that the Diodorean or Chrysippean modalities of a proposition about the future might depend on the view one adopts about their truth conditions. Generally, I assume that this problem was never properly thought through in early Hellenism. I suspect that the truth-conditions of propositions about the future were often seen as determined by the context or the specific concepts used. ‘Dio will die’ might have been used ‘elliptically’ for ‘as long as Dio is alive there will be a future time at which he will die’; ‘Thea will come and visit’ might have been understood as ‘... while she is in Athens this year’ or ‘... today’, depending on the respective context, and the exact truth conditions would differ each time.

6 Cf. D.L. 7.76. (cf. Frede (1974a) 44ff; Bobzien (1986) 21ff); actually, most examples of Hellenistic atomic propositions are μεταπίπτοντα, for Diodorus and Philo as well as for the Stoics (cf. S.E. M. 8 and P. 2; in D.L. 7 the ‘Diocles fragment’; Plu. *De Stoic. rep.* 1055F).

7 Limitations of space do not permit me to discuss this question here.

8 The positioning of the time index in $M_n[p]$ etc. (‘p’-no-time-index is possible-*now*’ etc.) is not accidental:

I will now introduce the modal notions of the three philosophers separately. In the case of Philo and Diodorus I will mainly concentrate on those features of their modalities that are germane to the comparison with Chrysippus' modalities. For reasons of time and simplicity, I will primarily focus on their concepts of possibility – in line with the fact that the Hellenistic logicians obviously were primarily interested in this modal concept. Due to the regularities mentioned above, the remaining three modal concepts can always be easily derived.

Philo

Philo's modal definitions are the least well reported and their exact meaning cannot be reconstructed with certainty. Only Boethius gives all four Philonian modal definitions (*in Int.* 234.10–22). The other three sources, all Aristotle commentators as well (Alex. Aphr. *in Apr.* 183f; Phlp. *in Apr.* 169; Simp. *in Cat.* 195f), confine themselves to Philo's notion of possibility, contrasting it with others; and it is not always clear what is part of the definition and what is part of the contrast.⁹ I will rely primarily on Boethius. According to his report, a proposition is Philonian possible, iff it is capable of *truth* according to the *proposition's* own nature or as far as the proposition itself is concerned; otherwise it is impossible.¹⁰ Thus, it seems, what is required for Philonian possibility is some sort of intrinsic consistency of the proposition. The propositions '(this) piece of wood burns' (Simp. *in Cat.* 196.1), 'Diocles is alive', 'it is night' would all be consistent in this sense.

The evidence is too sparse and heterogeneous to allow one to give a clear account of the type of consistency Philo had in mind. As it is also not essential for what follows, I leave the concept of consistency uninterpreted.

Consistency seems to be a common and reasonable criterion for possibility; still, due to the temporalized concept of truth, it works a little differently for Hellenistic propositions than for atemporal propositions.

In the case of atemporal truth a proposition will either have the required internal capability of (atemporal) truth or not – as in the cases of 'at noon 3 September 1991

M_p (it is possible-*now* that p-at-t)

M_{p_n} (it is possible-no-time-index that p-*now*)

M_{p_t} (it is possible-no-time-index that p-at-t)

or the reading of ' $M_n[p]$ ' as 'p is possibly true-*now*' all lead to nonsense in the case of Diodorean and Chrysippean modalities, as should become clear later. The reading of ' $M_n[p]$ ' as 'it is possible-*now* that 'p' is true-atemporally' does not square with the Hellenistic concept of truth.

- 9 I doubt for example that the expressions 'μόνη', 'ψιλήν', 'ψιλός' (Alex. Aphr. *in Apr.* 184; Simp. *in Cat.* 195.33, 196.16), *si nil extra prohibeat* (Boeth. *in Int.* 234.13f) were part of Philo's original account of possibility.
- 10 *Philo enim dicit possibile esse, quod natura propria enuntiationis suscipiat veritatem ... Eodem autem modo idem ipse Philo necessarium esse definit, quod cum verum sit, quantum in se est, numquam possit susceptivum esse mendacii. Non necessarium autem idem ipse determinat, quod quantum in se est possit suscipere falsitatem. Impossibile vero, quod secundum propriam naturam numquam possit suscipere veritatem.* (Boeth. *in Int.* 234.10–21)

Dio is walking through Bamberg' or 'triangles are square'. Referring to the uninterpreted consistency or capability of truth as 'it is ok that 'p' is true' and abbreviating 'OK T[p]', one might formalize:

$$M[p] =_{df} \text{OK } T[p]$$

(A proposition is possible iff it is ok that it is true.)

On the other hand, in the case of temporalized truth, one must ask: internally capable of being true-*at-what-time(s)*? And the answer can only be: capable of being true-*at-some-time*. Take as proposition of the Hellenistic type 'Dio dies of rabies'. At what time is that proposition meant to be capable of being true? It is Philonian possible if it is capable of being true-*at-some-time*. It would presumably not be capable of being true-*at-all-times*, because Dio could not die of rabies at all times, in fact, he could not even die of rabies twice. In order to make the time relatedness of truth in the Philonian criterion visible, I reformulate Philo's possibility criterion as: a proposition is possible *now*¹¹ iff it is capable of being true-*at-some-time* (or: iff it is ok that it is true-*at-some-time*). Formalized Philo's definitions then become

$$\begin{array}{llll} (P/M) & M_n[p] & =_{df} & \text{OK } \exists t T_t[p] \\ (P/\neg M) & \neg M_n[p] & =_{df} & \neg \text{OK } \exists t T_t[p] \\ (P/L) & L_n[p] & =_{df} & \neg \text{OK } \exists t F_t[p] \\ (P/\neg L) & \neg L_n[p] & =_{df} & \text{OK } \exists t F_t[p] \end{array}$$

Without having to fiddle with the negations and existential quantifiers I think it is obvious that the following theorems hold: If it is not the case that 'p' is capable of being true-*at-some-time*, then 'p' is always false

$$(P1) \quad \neg \text{OK } \exists t T_t[p] \rightarrow \forall t F_t[p]$$

If it is not the case that '-p' is capable of being true-*at-some-time*, then 'p' is always true

$$(P2) \quad \neg \text{OK } \exists t F_t[p] \rightarrow \forall t T_t[p]$$

i.e. for Philo, if a proposition is necessary it is true at all times and, if impossible false at all times. A Philonian contingent proposition, on the other hand, can be consistent and still always false¹².

$$(P3) \quad \neg((\forall t F_t[p]) \rightarrow \neg M_n[p]).$$

Three of our four sources stress this point. *E.g.* '(this) piece of wood burns' is said to be Philonian possible even if the wood will never burn. This theorem (P3) was essential for the determinism debate.

11 It cannot be made out from the sources whether or not Philo's modalities were temporalized, as those of Diodorus and Chrysippus; hence, whether to write $M_n[p]$ or just $M[p]$ etc. Such time relatedness must however be assumed for the Philonian accounts when they form part of Chrysippus' modal definitions (see below; cf. also note 13).

12 Correspondingly, there can be Philonian non-necessary propositions which are never false

$$(P4) \quad \neg((\forall t T_t[p]) \rightarrow L_n[p]).$$

Despite the temporalized concept of truth, it should be possible to ascertain the Philonian modality of a proposition by simply contemplating the proposition itself, as the definitions do not contain references to any specific time or circumstances. And we are told that it is Philonian possible for some straw to burn even at times when it rests at the bottom of the ocean (Alex.Aphr. *in Apr.* 184); hence the particular circumstances in which the proposition is uttered seem not to matter for its modality. This suggests that a proposition, though it might change its truth-value, will not change its Philonian modalities.¹³

Diodorus

As in the case of Philo, for Diodorus the full set of modal definitions is only reported by Boethius (*in Int.* 234.22-6). Yet, the definition of possibility is confirmed in some other sources (Alex.Aphr. *in Apr.* 183f.; Phlp. *in Apr.* 169; Simp. *in Cat.* 195; Boeth. *in Int.* 412), and we have further valuable information about Diodorus' modal theory in Epictetus, Cicero, and Plutarch (Epict. *Diss.* 2.19.1-5; Cic. *De fato* 12, 13, 17 and *Fam.* 9.4; Plu. *De Stoic. rep.* 1055E-F).

For Diodorus, a proposition is possible iff it either is true or will be true¹⁴. This definition is perfectly illustrated by the example given in Alexander:

According to him <*i.e.* Diodorus> 'that I am in Corinth' is possible if either I am in Corinth or, at any rate, I will be <in Corinth>. If I am not <*i.e.* ever in Corinth> it is not possible either. (*Apr.* 183.34ff).¹⁵

Making explicit the time-relatedness we can hence say that a proposition is Diodorean possible *now* if it is true at least once from *now* on. If it is false at least once from now on it is non-necessary. Continuous truth from now on makes it necessary, continuous falsehood impossible.¹⁶

$$\begin{array}{lll} (D/M) & M_n[p] & =_{df} \exists t (t \geq n \wedge T_t[p]) \\ (D/\neg M) & \neg M_n[p] & =_{df} \forall t (t \geq n \rightarrow F_t[p]) \end{array}$$

13 Whether the circumstances are never in any way relevant for the Philonian modalities and hence whether changes of modality could never occur depends in the end on what "theory of reference" Philo subscribed to. Thus we might wonder whether 'Socrates will die' is still Philonian possible once Socrates is dead and 'Dio is teething' when Dio just celebrated his 87th birthday. Should any such propositions turn out not to be Philonian possible, the account would have to be changed to something like:

$$(P/M) \quad M_n[p] =_{df} \text{OK}_n \exists t T_t[p]$$

14 Diodorus *possibile esse determinat, quod aut est aut erit; impossibile, quod cum falsum sit non erit verum; necessarium, quod cum verum sit non erit falsum; non necessarium, quod aut iam est aut erit falsum.* (Boeth. *in Int.* 234.22-26)

15 τὸ γὰρ ἐμὲ ἐν Κορίνθῳ γενέσθαι δυνατόν κατ' αὐτόν, εἰ εἴην ἐν Κορίνθῳ, ἢ εἰ πάντως μέλλοιμι ἔσεσθαι· εἰ δὲ μὴ γενοίμην, οὐδὲ δυνατόν ἔστιν. Simplicius understands it in the same way: for Diodorus it is possible that something is known if it either is or will be known (*in Cat.* 196.4-5).

16 Note that for *p* in each case Hellenistic 'atomic' propositions about the present, the past and the future can be substituted. But cf. note 36 below for propositions with time-indexicals.

$$\begin{aligned} \text{(D/L)} \quad L_n [p] &=_{df} \forall t (t \geq n \rightarrow T_t [p]) \\ \text{(D}/\neg\text{L)} \quad \neg L_n [p] &=_{df} \exists t (t \geq n \wedge F_t [p]) \end{aligned}$$

The Diodorean modality of a proposition at a time hence depends on the range of truth-values which that proposition has from that time onwards. In order to make this clearer I will take a little detour.

I will introduce a set of modal notions which I call 'Proto-Diodorean'. A proposition is Proto-Diodorean possible at a time iff it is true at that time:

$$\text{(PD/M)} \quad M_n [p] =_{df} T_n [p]$$

There is a certain similarity between this and the Megarian concept of possibility which is reported in Aristotle's *Metaphysics*: something (an event or activity) is possible only at those times when it is actual (*Metaph.* 1046b29ff).

$$\text{(M/M)} \quad M p_n =_{df} p_n$$

I can ride a bike only while I am pedalling along; as soon as I get off, cycling becomes – temporarily – impossible for me.

On the level of events, this is a concept of possibility which denies the existence of unactualized dispositions or potentialities. As such it seems to be an ontological concept of possibility to which no ontological concept of necessity correlates.

On the level of (time-related) propositions, the time the possibility-criterion is concerned with coincides with the time of the truth-criterion of the respective proposition. When I *now* make the (meta-)statement 'It is day' is Proto-Diodorean possible', I offer information about its being day *at this very time*: at this moment 'day-being' has a certain property, which makes the proposition possible – the property is its being the case. Unlike the case of the Philonian modalities, propositions can hence change their Proto-Diodorean modalities easily. In fact, they do so whenever they change their truth-value.

If one wants to construct a whole set of modal concepts, based on Proto-Diodorean possibility, one obtains:

$$\begin{aligned} \text{(PD/M)} \quad M_n [p] &=_{df} T_n [p] \\ \text{(PD}/\neg\text{M)} \quad \neg M_n [p] &=_{df} F_n [p] \\ \text{(PD/L)} \quad L_n [p] &=_{df} T_n [p] \\ \text{(PD}/\neg\text{L)} \quad \neg L_n [p] &=_{df} F_n [p] \end{aligned}$$

This turns out to be a time-related variety of those modal systems which collapse into a non-modal calculus. At no time can it contain a contingent proposition, for contingency requires that a proposition is both possible and non-necessary, which for Proto-Diodorus would mean that it is true and false at the same time

$$(T_n [p] \wedge F_n [p])$$

and this is logically precluded by the – temporalized – Principle of Bivalence

$$\forall t (T_t [p] \vee F_t [p]).$$

One consequence of this is that Proto-Diodorus is deterministic: As contingency is ruled out for logical reasons, all propositions that correspond to actual, individual events become necessary.

At any rate, such a modal system is not very useful. It might also go counter to Proto-Diodorus' initial intentions. For it is one thing to 'reduce' possibility to truth or actuality, but quite another to equate actuality with necessity. Moreover, that a proposition happily oscillates from being impossible to being necessary and back might go against the intuitive ideas of a more sensitive modal logician. For instance the modal theorem for tensed propositions that what is necessary is always true does certainly not hold for Proto-Diodorus.

If Proto-Diodorus is serious about the criterion of present actuality, he cannot easily integrate modalities of future and past propositions. However, he might talk about future and past modalities in the following way: 'Pia is walking' was possible yesterday afternoon (between 3.10 and 4.20).' *etc.*¹⁷

Now, why did I call these modal notions 'Proto-Diodorean'?

Let us imagine that Diodorus was at some time in his modal research confronted with the unpleasant consequences of the concept of Proto-Diodorean possibility. And let us assume that for some reason he wanted a modal system which kept the spirit of the Proto-Diodorean *possibility* without going counter to the intuitive concept of necessity and without the embarrassing – or boring – collapse into a non-modal system. So how could he improve on Proto-Diodorus? One way is this: one can imagine that he chose to apply a procedure analogous to that with which he seems to have developed his concept of a sound conditional from Philo's.

– A conditional is Philonian-true *now* iff *now* it is not the case that its antecedent is true and its consequent false.

– A conditional is Diodorean-true *now* iff it is Philonian true at all times.¹⁸

In the Philonian criterion for a true conditional the time of assertion and the criterion-time are the same (*now* in both cases). In the Diodorean criterion the Philonian criterion is adopted, but has been quantified over time. The conditional 'If I walk, I move' is true *now* for Diodorus because at no time is the antecedent true and the consequent false.

Similarly, in the case of modality, a proposition is Proto-Diodorean necessary *now* iff it is true *now*. A proposition is Diodorean necessary *now* iff it is Proto-Diodorean necessary at all times from now on. Accordingly, a proposition is Proto-

17 If one applies the Proto-Diodorean modal concepts on future and past propositions as well one obtains a system which I call Pseudo-Diodorus and which is occasionally taken to be Diodorus'. In this system 'Dio will walk' is possible *now* iff 'Dio will walk' is true *now*, i.e. iff there is a future time at which Dio walks, and *then* it is necessary as well. 'Dio will walk' is impossible *now* iff it is false *now*, i.e. iff there is no future time at which Dio walks, and then it is non-necessary as well. Accordingly for propositions about the past. This is not Diodorus' modal system, because it does not square with Diodorus' modal accounts (cf. Boeth. *in Int.* 234.22–26, and note 14).

18 cf. S.E. P. 2.110ff. It is in fact a little more complicated; for one interpretation see Denyer (1981) 39f.

Diodorean possible *now* iff it is true *now*. A proposition is Diodorean possible *now* iff it is Proto-Diodorean possible at some time from *now* on. Again, a criterion is adopted in which originally criterion-time and time of assertion have been equivalent, but that criterion is then quantified over time.

One might of course ask: Why does the definition refer only to 'present and future' or 'the time from *now* on'? Why is not possible *now* what is at some time, past or present or future, the case (and necessary *now* what is at all times, past, and present, and future, the case)?

I cannot offer a fully satisfactory answer to this. But it might be worth noting that there is at least one common use of modal expressions which is exclusively concerned with present and future, and that this use is relevant for the discussion of the determinacy and indeterminacy of events. Often, if we want to know whether something is possible or impossible we want to know something about its future (plus present) chances of being the case. If there was a past occurrence of an event which made a proposition true, but for the future this is definitely ruled out, we do not want to call such a proposition or event possible any longer. Take for instance the proposition 'Dio is walking' and assume that Dio died last Sunday. Then – in this sense of possible – we would no longer want to say that 'Dio is walking' is possible or that it is possible for Dio to walk, even if Dio had taken plenty of walks over the last 87 years. (What we would do instead is move back our reference point in time and say 'Dio is walking' was possible' or 'It was possible for Dio to walk' – and in the case of Diodorus' modalities that works as well.) That this use of modal expressions is relevant for the question of determinism becomes clear when one substitutes 'it is in X's power' for 'it is possible for X'. The restriction of the criterion-time for modalities to present and future is hence backed up by one usage of modal expressions which is of relevance in Hellenistic philosophy.

At all events, Diodorus' modalities constitute in various ways an improvement on those of Proto-Diodorus:

- The strange Proto-Diodorean oscillation from impossible to necessary and *vice versa* cannot occur any longer. Propositions can change their Diodorean modalities over time but only from contingency to necessity or impossibility – and this is borne out by common language use, as the above example made clear.¹⁹
- Moreover, the concept of necessity is now more satisfactory: in some sense at least the theorem that a necessary proposition is always true is preserved; *viz.* if 'always' is understood as 'always from *now* on'.

19 The reason for this restriction on possible changes is that if a proposition is always true (false) from a certain time *now* onwards, then at all times *t* after *now* the proposition will always be true (false) from *t* onwards. This is easily exemplified with the true 'Dio walked' or the false 'Dio will walk'.

A consequence of this is that for Diodorus all past truths are necessary, which is the first theorem of the 'κυριεύων λόγος'; however, it does not follow from this that for Diodorus all past falsehoods are impossible (cf. *now* the presumably false 'I have finished reading this article').

Furthermore, all false future propositions become impossible; what happens to true future propositions depends on what one takes their truth-conditions to be, cf. note 5.

- And finally, Diodorus managed to avoid the collapse into a non-modal system.²⁰ The class of contingent propositions is not empty. For a proposition is Diodorean contingent iff it is Diodorean possible and non-necessary, *i.e.*

$$(D/C) \quad C_n [p] \quad =_{df} \quad (\exists t (t \geq n \wedge T_t[p])) \wedge (\exists t (t \geq n \wedge F_t[p]))$$

a proposition is contingent *now* iff from *now* onwards it will be true at some time and false at some time. That is, exactly those propositions which change truth-value in the future are Diodorean contingent. Examples are 'it is day', which is true (or false) now and will be false (or true) later, and 'I am walking'.

So, obviously, in Diodorus' modal logic contingency is not precluded. But is his modal theory because of this also indeterministic? Not at all. It is fully in the spirit of the Proto-Diodorean modalities. In Proto-Diodorus, contingency was logically precluded because it would lead to

$$\exists t (T_t[p] \wedge F_t[p])$$

In Diodorus we do have contingency; however there are certain cases of contingency which are – again – logically impossible: there can be no contingent propositions which fulfil the criteria for possibility and non-necessity *at the same time*, for this would mean

$$\exists t (t \geq n \wedge (T_t[p] \wedge F_t[p])).^{21}$$

And, assuming that Diodorus accepts the temporalized Principle of Bivalence, this cannot be – again, for logical reasons.

And this means that individual events, which as such are connected with a specific (though possibly unspecified) point of time (and propositions which describe them), can still be only either Diodorean non-necessary (and impossible) or Diodorean possible (and necessary) but never both – just as in the case of Proto-Diodorean modalities. For instance, the proposition 'Dio is walking' could be Diodorean contingent. There could be a present-or-future time at which Dio walks and another present-or-future time at which he does not walk. But if we think, say, of the individual event of Dio's walking at noon today, this does not work. The proposition 'Dio is walking at noon, 3 September 1991' cannot be Diodorean contingent, as this would mean that Dio would both have to walk and not to walk simultaneously. And so in all cases of individual events. And as determinism and indeterminism are concerned with the predetermination of individual events, Diodorus' modal system, although it allows for contingent propositions, is – in fact – not one iota less 'deterministic' than the Proto-Diodorean system.²²

20 The reason for this is the expansion of the modal-criterion time and the subsequent differentiation of necessity and possibility and of impossibility and non-necessity due to universal or existential quantification (respectively) over the criterion time.

21 In other words, to any Diodorean contingent proposition there corresponds a function of Proto-Diodorean modalities over time which takes only the two modal-value combinations necessity-plus-possibility and impossibility-plus-non-necessity.

22 *pace* Denyer (1981) 52–3.

Amongst others, Chrysippus recognized this and thought that Diodorus' concept of possibility was a threat to human freedom (Cic. *De fato* 13f; cf. Plu. *De Stoic. rep.* 1055E–F; Boeth. *in Int.* 235).²³ This answers the question: why did Chrysippus not take over Diodorus' concept of possibility?

But we have to be careful here. Individual events, marked out as such by a date which is specified non-indexically, and the dated propositions corresponding to them, were not part of the ancient 'standard ontology' and 'terminology' (cf. the above section on Hellenistic propositions). So even if Chrysippus and other philosophers 'sensed' the deterministic consequences of Diodorus' modal logic – how could they have expressed this? How could *they* have argued that Diodorean possibility threatens freedom?

They could *not* argue that for Diodorus everything that happens is necessary, invoking that for him every true proposition is necessary and every false one impossible. For, Diodorus' system allows for contingency, as we have seen: 'It is day' will always be Diodorean contingent and 'Dio is walking' will be contingent as long as Dio will have another walk. So they could not (straightforwardly) accuse Diodorus of endorsing universal determinism or the necessity of all that happens. As a matter of fact, none of our testimonies claims that universal determinism is a consequence of Diodorus' modal logic.

For Diodorus, it is not the case that *all* false propositions are impossible; it is only those propositions which correspond to type events tokens of which will never happen (or never happen again) which are Diodorean impossible. These form a proper subclass of false propositions. Hence only the possibility of type events tokens of which never happen (again) is threatened directly by Diodorus. I will call them 'counterfactual events'. To make the threat of determinism clear within Hellenistic ontology, Diodorus' opponents therefore had to come up with examples of (propositions which describe) events tokens of which would never (again) happen. And that is exactly what Chrysippus did (Plu. *De Stoic. rep.* 1055E–F; Cic. *De fato* 13f).²⁴

One type of event which will satisfy this requirement is what I will call 'unique events', *i.e.* events tokens of which, if they happen, can do so only once. The notorious example 'Fabius – or whoever – will die on land' (or 'Fabius dies on land' – the tense does not matter here) is such a case. As human beings die only once, if this proposition is false, it is Diodorean impossible. Examples which are not (intrinsicly) unique in this way, require the extra assumption 'and it neither does nor ever will happen'.²⁵ Take 'Thea goes to Bamberg' and assume that, as a matter of fact, she never goes there. Then, even if she is just deliberating or tries to decide whether or not to go, it is not Diodorean possible for her to go. It is Diodorean impossible for her to go. And realizing this, Thea might get slightly upset should she

23 A modal logic is a threat to freedom iff it renders necessary or impossible those propositions which correspond to states of affairs that are considered to be up to human beings.

24 This also holds true of all other sources which criticize the Diodorean modalities.

25 This kind of exposition was employed in the case of the example 'I am in Corinth', see note 15 above.

be a
that I
kind;
V
was
unive
day,
what
moda
liber
'
rejec
refut
notic
conc

One
did
poss
ann
nistr
with
tion
cati

eith
abo
eas
wal
am
eve
Sin
mi
soa

be a 'libertarian'.²⁶ Our testimonies all back up their (explicit or implicit) criticism that Diodorus' modalities lead to determinism with arguments and examples of this kind; they focus only on the impossibility of that which never happens.

We can hence conclude – fully in accord with the surviving passages – that what was seen as a threat to freedom was not that Diodorus' modal notions implied universal determinism, *i.e.* that 'everything that happens is necessary' – for that it is day, for instance, is not Diodorean necessary. Rather what was feared was that whatever in fact never happens is impossible or, in other words, that Diodorus' modalities rule out counterfactual events. But this is surely enough to worry a libertarian or a soft determinist such as Chrysippus.

So Chrysippus – wanting to retain 'counterfactual possibility' – clearly had to reject Diodorus' notion of possibility. And he did this, as everyone knows, by refuting the 'κυριεύων λόγος', the argument with which Diodorus established his notion of possibility as the (only) right one. Still, the question remains: what concept of possibility should Chrysippus adopt instead?

Chrysippus and Philo

One might ask, why – if Chrysippus thought Diodorus' modal system inadequate – did he not just take over Philo's modal system? As we saw, for Philo there are possible propositions which are never true. Hence Philo's modal notions do not annul 'counterfactual possibilities'. They seem unfit to suggest any kind of determinism at all. However, on the other hand, Philo's modal notions are also compatible with any kind of 'event-determinism', as they do not seem to supply any information about individual events (*qua* individual events). All they do is allow a classification of (individual events *qua* belonging to) certain type events.

So we can surmise why Chrysippus was not content with Philo's modal notions either. He might have thought that the 'true' modal system must say something about individual events and their occurrence, not only about type events. We could easily imagine Chrysippus pondering about the absurdity that for Philo 'Dio is walking' is possible even if Dio is chained to the wall or has both his legs amputated, or – to take a Philonian example from our sources – that chaff can burn even while it lies deep down at the bottom of the sea (Alex. Aphr. *APr.* 184.6–10; cf. *Simp. in Cat.* 196.1–2) (one might call this the 'Philonian Paradox'). Chrysippus might have decided that for Dio then it is *not* possible to walk and that that chaff, soaked as it is, *cannot* burn and that a modal system has to account for these cases.

26 If, on the other hand, one assumes that Thea will go to Bamberg, in, say, 10 years, then 'Thea goes to Bamberg' is in fact now Diodorean possible. Hence *in terms of Diodorean modalities* in this case there is no reason for Thea to worry about her general freedom to go to Bamberg.

Chrysippus²⁷

So, for Chrysippus, Diodorus' modal system does not generate enough contingent propositions and Philo's generates too many. What, then, should he do? Being a logician of his calibre there is only one answer: he has to introduce his own concept of possibility – finally the right one. And this is what he did.

For Chrysippus, a proposition is possible *now* iff it satisfies the following conjunction of requirements: first, that it is capable of being true; secondly, that it is not hindered by external factors from being true. For the account of impossibility Chrysippus accordingly ends up with a disjunction: A proposition is impossible iff either it is not capable of being true or – being capable of being true – it is hindered by external factors from being true.

In the same way, the account of non-necessity is a conjunction, that of necessity a disjunction.²⁸ Hence every definition consists of two parts, connected with either 'and' or 'or'.²⁹

The *first parts* of his accounts look very similar to Philo's; in Boethius they are formulated in the very same words (cf. above and Boeth. *in Int.* 234–5).³⁰ As Chrysippus must have known the Philonian definitions I assume that these parts actually are intended to reproduce Philo's definitions.

The meaning of the *second parts* is less clear. In order to understand them fully two questions have to be answered: The first question is this: what are the 'external things that hinder propositions from having a certain truth-value'?

It should be safe to assume that the external things hindering a proposition from being true (false) are precisely those external things which hinder the corresponding state of affairs from obtaining (not obtaining). Furthermore, one might suppose that 'external' ('ἐκτός') is contrasted with something 'internal'. The best I can come up with is that this is something 'internal to the proposition'; *i.e.* what hinders truth internally is some kind of inconsistency of the proposition; as *e.g.* the inconsistency

27 Testimonies on the modal theory of Chrysippus and the Stoics in general are: D.L. 7.75; Boeth. *in Int.* II 234–5, 393; Plu. *De Stoic. rep.* 1055D–F; Cic. *De fato* 12–15; Epict. *Diss.* 2.19.1–5; Alex.Aphr. *Fat.* 10; *Quaest.* 1.4.1; *in Apr.* 177–8; Simp. *in Cat.* 195. As for [Plu.] *Fat.* 570F and Alex.Aphr. *in Apr.* 184, I doubt that they give a Stoic account.

28 δυνατὸν μὲν [ἐστὶν ἀξίωμα] τὸ ἐπιδεκτικὸν τοῦ ἀληθῆς εἶναι τῶν ἐκτὸς μὴ ἐναντιουμένων πρὸς τὸ ἀληθῆς εἶναι. ἀδύνατον δὲ τὸ μὴ ἐστὶν ἐπιδεκτικὸν τοῦ ἀληθῆς εἶναι [ἢ ἐπιδεκτικὸν μὲν ἐστὶ, τὰ δ' ἐκτὸς αὐτῶ ἐναντιοῦται πρὸς τὸ ἀληθῆς εἶναι]. ἀναγκαῖον δὲ ἐστὶν ὅπερ ἀληθῆς οὐκ ἔστιν ἐπιδεκτικὸν τοῦ ψεύδους εἶναι ἢ ἐπιδεκτικὸν μὲν ἐστὶ, τὰ δ' ἐκτὸς αὐτῶ ἐναντιοῦται πρὸς τὸ ψεύδους εἶναι. οὐκ ἀναγκαῖον δὲ ἐστὶν ὃ καὶ [εἰ] ἀληθῆς ἐστὶν καὶ ψεύδος οἶον τε εἶναι τῶν ἐκτὸς μηδὲν ἐναντιουμένων [πρὸς τὸ ψεύδους εἶναι]. (D.L. 7.75)

In the reconstruction of the sources which report Chrysippus' modal definitions I follow Frede (1974a) 107–114; cf. Bobzien (1986) 40–49. I will not argue here for the fact that these are Chrysippus' accounts.

29 This idea is not new: Diodorus already used it in his modal definitions. There, however, the accounts of possibility and non-necessity were disjunctions.

30 He leaves out the phrases *natura propria enuntiationis* and *quantum in se est*. Is this merely an ellipsis? Or does it show that Philo's specific concept of consistency has been given up by Chrysippus?

of square circles hinders 'this circle is square' from being true. 'Εκτός' would then serve to contrast the second parts of the definition with the first Philonian-type parts.

Some examples for external hindrances would help us further. But unfortunately no such examples were handed down. The only helpful passage is Alex. Aphr. *in Apr.* 185, from which we can at least surmise that the relevant external factor or circumstance for the proposition 'this chaff burns' is the ocean surrounding the chaff (or that the ocean surrounds the chaff). Accordingly, I shall assume that the external hindrances would include such prosaic cases as the following: Leo's being tied to a chair, having his legs paralyzed, being severely ill, might all hinder Leo from walking and thus be relevant to the modality of the proposition 'Leo is walking'.³¹

The second question is when – relative to the time at which the proposition is asserted – does the truth/falsehood have to be hindered/not hindered? Here the introduction of another set of proto-modal notions might be helpful. I shall call these 'Proto-Chrysippean'.³²

A proposition is *Proto-Chrysippean* possible at a certain time iff it is Philonian possible and is not hindered by external things from being true at that time. 'Dio is walking' is then possible at all and exactly those times when Dio is not tied to his chair, does not have his legs paralyzed, has not been killed *etc.* At some of these times the proposition will be true (when he actually walks) at others false (when he is in bed or cycles through town *etc.*). At all those times when Dio *is* tied to his chair *etc.*, the same proposition is impossible.³³

For a more formal description I shall introduce a 'hindrance-operator'³⁴:

$H T_t[p]$: External factors hinder that the proposition p is true-at- t . (External factors hinder p from being true-at- t .)

The four Proto-Chrysippean modalities then become, formally :

(PC/M)	$M_n[p]$	= _{df}	$OK \exists t T_t[p]$	$\wedge \neg H T_n[p]$
(PC/ \neg M)	$\neg M_n[p]$	= _{df}	$\neg OK \exists t T_t[p]$	$\vee H T_n[p]$
(PC/L)	$L_n[p]$	= _{df}	$\neg OK \exists t F_t[p]$	$\vee H F_n[p]$
(PC/ \neg L)	$\neg L_n[p]$	= _{df}	$OK \exists t F_t[p]$	$\wedge \neg H F_n[p]$

Proto-Diodorus and Proto-Chrysippus are similar in one respect: in both cases the time at which the proposition has the modality and the time at which the

31 It is not easy to conceive of external hindrances to falsehood; but perhaps the belt in Leo's green trousers keeps the proposition 'Leo is wearing his green trousers' from being false.

32 I do not claim that any Hellenistic philosopher ever put forward these modal accounts.

33 For propositions about the future and the past: 'Pia will walk' is Proto-Chrysippean possible *now* if Pia will not be hindered from walking at some future time from *now*, and 'Pia walked' if she was not hindered from walking at some past time from *now*.

34 I do not add a time index to the operator H itself (resulting in something like $H_t T_t[p]$) since our sources say nothing about when the hindrances have to be present. In the 'normal case' one would expect them to be present (absent) when truth or falsehood are (are not) hindered. But I do not want to rule out the possibility that *preceding* circumstances might influence later truths (for instance Dio's having died yesterday might hinder his walking today), since this could be relevant for Stoic fate doctrine.

modality-criterion has to be satisfied are the same. But whereas for possibility Proto-Diodorus required truth, Proto-Chrysippus requires only the absence of hindrances to truth. For example, 'Mia is walking' is Proto-Diodorean possible now iff Mia is now walking. For Proto-Chrysippus it is enough that nobody and nothing external keeps Mia from walking now. In general, whereas Proto-Diodorus requires truth or falsehood, Proto-Chrysippus requires the presence or absence of hindrances to truth or falsehood.

A consequence of this difference is that whereas Proto-Diodorus 'collapses', Proto-Chrysippus does not. Contingency is no longer logically precluded. A proposition is Proto-Chrysippean contingent at those times at which neither truth nor falsehood is prevented:

$$(PC/C) C_n [p] =_{df} ((OK \exists t T_n [p]) \wedge (OK \exists t F_n [p])) \wedge ((\neg H T_n [p]) \wedge (\neg H F_n [p]))$$

Is, then, Proto-Chrysippus Chrysippus? Some people have thought so. But I think this is very unlikely, for the following two reasons:

- First, the theorem that what is necessary is always true would (again) clearly not hold, although it seems that it was held valid by the Stoics.³⁵ For instance, for Proto-Chrysippus 'Dio is sitting' is necessary now if something now keeps him from standing up. Five minutes later the same proposition can happily be non-necessary and false and ten minutes later necessary again.
- Secondly, we have good evidence that for Chrysippus the modal-criterion time was not restricted to the respective present (*now*). For Chrysippus 'Dio is dead' is possible *now* if it can be true at some time ($\mu\omicron\tau\acute{\epsilon}$) and 'this one (*i.e.* Dio) is dead' would not be impossible if, although being false *now*, it could sometime later ($\upsilon\sigma\tau\epsilon\rho\acute{o}\nu$ $\mu\omicron\tau\epsilon$) be true (Alex.Aphr. *in Apr.* 177f., cf. below). And this clearly does not square with Proto-Chrysippus.

Hence I conclude that Proto-Chrysippus is not Chrysippus.

But if the Proto-Chrysippean modalities differ from the Chrysippean ones, what are the Chrysippean? We saw above that the time-relatedness of Diodorus' modal accounts (to present and future) can be understood as making explicit the time-relatedness of one common use of modal expressions. When I say something is (now) possible, I might want to say that it 'could be' at some present-or-future time. This might also have been the philosophical idea underlying Chrysippus' formulation of his modal definitions.

For instance imagine that someone asks 'Is it possible for Dio to die of tuberculosis?' and the reply is: 'No, it is not possible; he was vaccinated only three weeks ago'. The answer implies that it is impossible for Dio to die of TB since from now on the vaccination will prevent him from dying of TB. Last year, though, it might have been possible, because he had not yet been vaccinated. Or take 'Thea goes to Bamberg'; at the moment Thea might lack the money for a ticket or be locked up somewhere. Still, we might say that 'Thea goes to Bamberg' is possible since she might get the money or escape from her prison at some future time. On the

35 Cf. Alex.Aphr. *Fat* 177.8-9 and *in Apr.* 180.13f.

other
escap
F
from
hinde
C
thoug
- ']
n
- ']
y
The q
walk
- ']
is
h
Simil
even
- ']
h
- ']
fi
Henc
T
howe
177f.
($\mu\omicron\tau\acute{\epsilon}$
it cou
it is i
V
not h
the p
some
seco
T
expo
36 T
p
t
n
E
e
t

other hand, should we know that she will never earn any money or be unable ever to escape we would no longer say that 'Thea goes to Bamberg' is possible.

Following this thought, Chrysippean impossible might be what is hindered from being true (at least) from *now* on, and Chrysippean possible what is not hindered from being true-at-some-time-from-*now*-on.

One obtains the same outcome via a different route if one pursues the following thought:

- 'Dio is taking a walk (περιπατεῖ) today' is presumably possible *now* if sometime (from *now* on) today he is or will be free to take a walk.
- 'Dio goes to Verona this year' is possible *now* if sometime (from *now* on) this year he is not or will not be prevented from going to Verona.

The question is: when do hindrances have to be absent in order for 'Dio is taking a walk (περιπατεῖ)' to be possible *now*? I think the answer would naturally be:

- 'Dio is taking a walk' is possible *now* if there is a time from *now* on at which he is not prevented from walking; *i.e.* if either *now* or at some later time he is not hindered from walking.

Similarly for the presence of hindrances in the case of impossibility (here it seems even more obvious):

- 'Dio is taking a walk today' is *now* impossible if today at all times Dio is hindered from walking.
- 'Dio is taking a walk' is *now* impossible if from *now* on always Dio is hindered from taking a walk. (*I.e.* *now* and later Dio is prevented from walking.)

Hence, this way, we reached the same results.

The best argument in favour of this interpretation of Chrysippus' modalities is however, that we have testimonial evidence for it: according to Alex. Aphr. *in Apr.* 177f., for Chrysippus 'Dio is dead' is possible (*now*) if it can be true at some time (ποτέ) and 'this one is dead' would not be impossible if, although being false *now*, it could be asserted sometime later (ὑστερόν ποτε), after this one's death – for then, it is implied, it would be true.

We only have to read 'can be true' as a short version for 'is capable of truth and not hindered from being true'. And this seems legitimate since the point at issue in the passage is not the modal accounts but the fact that 'Dio is dead' can be true sometime later, whereas 'this one is dead' cannot and hence only the first but not the second is Chrysippean possible.

This interpretation of Chrysippus' modal notions leads to the following formal exposition:³⁶

36 Thus the criterion times for truth-values and modalities differ. In the case of truth-value, a proposition about the present 'p' and the proposition 'p, now' are equivalent ('Dio talks', 'Dio talks now'). In the case of modalities a proposition about the present 'p' and the proposition 'p, now or later' are interchangeable. ('Dio talks' is possible iff 'Dio talks now or later' is possible). Both exposition and formalization are, of course, again oversimplifications which do not hold *e.g.* in the case of propositions with time indexicals. And in the discussion of ancient modal theories, propositions with time indexicals are important – think of the sea-battle tomorrow. For

(C/M)	$M_n[p]$	=	df	$OK \exists t T_i[p] \wedge \exists t (t \geq n \wedge \neg H T_i[p])$
(C/ \neg M)	$\neg M_n[p]$	=	df	$\neg OK \exists t T_i[p] \vee \forall t (t \geq n \rightarrow H T_i[p])$
(C/L)	$L_n[p]$	=	df	$\neg OK \exists t F_i[p] \vee \forall t (t \geq n \rightarrow H F_i[p])$
(C/ \neg L)	$\neg L_n[p]$	=	df	$OK \exists t F_i[p] \wedge \exists t (t \geq n \wedge \neg H F_i[p])$

(A proposition is Chrysippean possible *now* iff it is internally capable of being true-at-some-time and nothing external hinders it from being true-at-some-time-from-*now-on*, etc.)

Contingency would accordingly be:

$$(C/C) \quad C_n[p] =_{df} ((OK \exists t T_i[p]) \wedge (OK \exists t F_i[p])) \wedge ((\exists t (t \geq n \wedge \neg H F_i[p])) \wedge (\exists t (t \geq n \wedge \neg H T_i[p])))$$

i.e. a proposition is Chrysippean contingent iff it is Philonian contingent and from now on there is both a time when it is not hindered from being true and a time when it is not hindered from being false. These times do not have to (but could, in principle) be the same.³⁷ In the case of the proposition 'it is day', they might never be the same. In the case of 'I am talking' there are many times at which there are no hindrances either way – namely whenever both nothing external keeps me from speaking and nothing external forces me to speak.

Chrysippus and Diodorus

If this interpretation of the Chrysippean modal notions is correct, it becomes very likely that Chrysippus developed his notions by way of slightly modifying Diodorus' concept of possibility.

those propositions I suggest the following interpretation in which the modal criteria for 'standard' propositions are slightly modified in accordance with ordinary language use: For modalities of propositions without time indexicals the relevant period is present-plus-future. If a time-indexical ('tomorrow', 'at noon', 'this year') is added to such a proposition without any date, the criterion time of the modalities is restricted from present-plus-future to the (relative) time the time-indexical refers to. This holds equally for Diodorus and Chrysippus. Take 'Dio is taking a walk ($\pi\epsilon\rho\iota\pi\alpha\tau\epsilon\tau$)'. The relevant time period is *now-plus-later*. Next take 'Dio is taking a walk today'. The relevant time period is then restricted to *now-until-before-tomorrow*. If there is a point in time during this period at which Dio is not hindered from taking a walk, the proposition is Chrysippean possible. If there is a point in time during this period when Dio does take a walk, the proposition is Diodorean possible. Or take 'There will be a sea-battle tomorrow'. The relevant time period is *tomorrow*. If there is a time after *today* and before the day after *tomorrow* at which a sea-battle takes place, the proposition is Diodorean possible. If there is such a time at which a sea-battle is not hindered from taking place, the proposition is Chrysippean possible. Finally, in the case of 'Dio walks now' the relevant time is *now*; *i.e.* here we have, as it were, Proto-Diodorus and Proto-Chrysippus as special cases of Diodorus and Chrysippus.

Hence, in the case of propositions with time indexicals, the difference between truth-criterion time and modality-criterion time vanishes. In the case of truth, the indexical transfers and/or extends the relevant time period. In the case of modality the indexical transfers and/or restricts the relevant time period.

37 Note that the account of Chrysippean contingency is not

$$((OK \exists t T_i[p]) \wedge (OK \exists t F_i[p])) \wedge \exists t (t \geq n \wedge (\neg H T_i[p] \wedge \neg H F_i[p]))$$

Diodorus' requirement for the possibility of 'Dio is walking' is that Dio walks now or later. Chrysippus' requirement is that nothing hinders Dio from walking now or later. Where for Diodorus it matters whether Dio walks at some time from now on (possibility) or never from now on (impossibility), for Chrysippus it matters whether Dio is not tied to his chair at some time from now on (possibility) or whether he is tied to his chair from now to his death (alas!) (impossibility).

In general, the only significant formal difference between Diodorean and Chrysippean modal accounts is that where Diodorus requires that the proposition be either true or false, for Chrysippus the absence or presence of hindrances to its truth or falsehood is required. That is, we have the very same difference we observed in the comparison of Proto-Diodorus and Proto-Chrysippus.

With this slight modification Chrysippus overcame the unwelcome deterministic consequences of Diodorus' modalities. The possibility of counterfactual events is guaranteed. In Chrysippus' modal system there can be propositions which are never (or never again) true and still possible – as long as there is some future time at which they are not hindered from being true. Take the proposition 'Thea will go to Bamberg' and assume that she never will (*i.e.* that Thea will go to Bamberg is a counterfactual event). According to Diodorus, this proposition is impossible, as it is and will be false. For Chrysippus, it is possible and hence contingent, if we assume such things as the fact that Thea has access to the basic means necessary for such a journey *etc.* For even though she never actually sets off to Bamberg, there are times at which she is not externally hindered from doing so.

The crucial case of contingency which was logically precluded in Diodorus, namely that the conditions for non-necessity and possibility are met *at the same time* is not ruled out in Chrysippus. Nothing is logically wrong with the assumption that at the same time a proposition is prevented neither from being true nor from being false.

Contingency and Freedom

Still, this does not justify the conclusion that Chrysippus ends up with a concept of contingency according to which everything that is contingent is 'undetermined' or 'up to us', as some people have assumed.

In fact, this assumption goes counter to Chrysippus' own examples, above all the proposition 'it is night' which according to Chrysippus himself is possible and false (at daytime) (Alex.Aphr. *in Apr.* 177.25ff.), hence contingent. And we would probably agree that there are times at which nothing external hinders it from being night and times at which nothing external hinders it from being day. However, it is not ever up to us whether or not it is night.³⁸ Generally, at least in all cases in which

38 The following examples for Chrysippean contingent propositions also do not (necessarily) express that something is in the power of a human being: 'Diocles is alive' (D.L. 7.75), 'Dio is dead' (Alex.Aphr. *in Apr.* 177), 'He (or that one) dies (or will die) on land' (Plu. *De Stoic. rep.* 1055F), 'He will die at sea' (Boeth. *in Int.* 235), the same with 'Fabius' as subject in Cic. *De fato* 12 and 14.

the condition for contingency is satisfied, but not for possibility and non-necessity at the same time, we will have contingent propositions to which no events 'up-to-us' correspond. This is because events up-to-us are individual events and as such are attached to a specific time, and *at this time* the criteria both for non-necessity and for possibility have to be met. Chrysippus' contingency hence provides a necessary condition for things being up to us, but not a sufficient one.

Equally, there is no reason for assuming that Chrysippus' modal concepts guarantee freedom by way of rendering all propositions about the future contingent.³⁹ This seems to be a typical case of throwing out the baby with the bath water: Chrysippus might want to be able to say something about some necessary future events – which would then be ruled out *a priori*.

Apart from such obvious and less interesting cases as 'Christmas day will be on a Sunday next year' as future necessity and 'Socrates will be alive' as future impossibility, there are also cases like: 'Dio will be dead (*i.e.* soon)', taking it that Dio has been wounded in the heart. In such a case death was then seen as inevitable in Hellenistic times, probably also by the Stoics (cf. S.E. M. 8.254f), and if Chrysippus' modal notions did not cover such cases they would not be very expedient.⁴⁰

In my interpretation, they do account for such cases: at all times up to his death the wound in Dio's heart prevents the proposition 'Dio will be dead (*i.e.* soon)' from being false by keeping Dio from being alive much longer. (After that, Dio being dead, that will keep it from being false.) So Chrysippus' modal system is in fact more accurate and powerful than has often been assumed.

Chrysippus and Philo again

One question is still open: why did Chrysippus add the Philonian notions as a first criterion in his definitions? This is something that Diodorus did not do. The answer is not difficult to find: there are propositions which are hindered neither from being false nor from being true by any external circumstances and which Chrysippus still could not have wanted to be contingent.

Take the proposition 'This circle is square'. There are future times at which it is not externally hindered from being true and others at which it is not hindered from being false. Still, this does not give the circle any real chance of ever being square. There is something intrinsically wrong with the proposition which throws it out of the game before the hindrance-criterion can be applied. Chrysippus could not want propositions of this type to be contingent. The addition of the Philonian criterion guarantees that they come out as impossible.

Note that there was no such need for this (Philonian) clause in the case of Diodorus. Being false at all times, the propositions of the kind at issue passed his requirement for impossibility anyway.⁴¹ Or, the other way round, a proposition's

39 This is assumed for instance by Reesor (1978), Rist (1969).

40 Similar cases for impossibility would be 'Thea will die of tuberculosis' if Thea has received a TB injection; or 'Dio will walk' if Dio is tied to his chair until his death (alas!).

41 (P1 from above) $\neg \text{OK} \exists t T_t[p] \rightarrow \forall t F_t[p]$
 $\forall t F_t[p] \rightarrow \forall t (t \geq n \rightarrow F_t[p])$

actually being true at some time from *now* seems to entail that, as far as its nature is concerned, the proposition is capable of being true at some time. Chrysippus' slight modification of the Diodorean possibility hence forced him to add some further criterion – and the Philonian one was just right.

Final Comparison between Philo, Diodorus, Chrysippus

It should have become sufficiently clear by now how close the links are between Chrysippus and the two dialecticians. With the help of a general scheme I will summarize the results and indicate where Chrysippus ends up – *i.e.* 'somewhere in the middle' between Philo and Diodorus.

SCHEME 2 (for any time *t*)

1	2	3	4	4'	3'	2'	1'
always true	always true ≥ <i>t</i>		sometimes true, sometimes false ≥ <i>t</i>		always false ≥ <i>t</i>		always false
true 'by nature'	$HF_t [p]$ $\forall t \geq t$	simply true $\forall t \geq t$	'metapiptonta' ≥ <i>t</i>		simply false $\forall t \geq t$	$HT_t [p]$ $\forall t \geq t$	false 'by nature'
true				false			
P-necessary		P-contingent				P-impossible	
C-necessary			C-contingent			C-impossible	
D-necessary			D-contingent		D-impossible		
1	2	3	4	4'	3'	2'	1'

Columns 1 and 1':

Propositions which cannot be false or cannot be true (*i.e.* are true or false because of their nature) meet the criteria for necessity and impossibility of all three logicians.

Columns 2 and 2':

Consistent propositions which are externally hindered from being false from *now* on are Philonian non-necessary, but necessary for Chrysippus and Diodorus (and accordingly for false propositions).

Columns 3 and 3':

Propositions which are always true from *now* on but not always externally

hindered from being false are necessary only for Diodorus (and accordingly for false propositions).

Columns 4 and 4':

Propositions which change their truth-value at some future time are granted contingency by all three logicians.

Thus, Chrysippus chose the 'happy medium'. By substituting 'not hindered from being true' for 'true' in the possibility account (and filling the unwanted gaps which thereby occurred by adding Philo's criterion), he achieved exactly what he needed for his philosophical system. Necessary propositions are always true (*i.e.* always from *now* on); true propositions about past events are necessary; and there are counterfactual possibilities. Moreover, his concept of possibility comes fairly close to the 'ordinary language use' of the word – at any rate closer than those of both Philo and Diodorus – and this might be a sign of his coming closer to the 'true' notion of possibility than either of them.^{42 43}

42 As in the case of the conditional, this improvement (which escapes paradoxes and comes closer to ordinary language use) is made at the expense of formal manageability of the concept. Chrysippus' system no longer shares the advantages both Philo's and Diodorus' modal logics have for easy formalization.

43 I am grateful to many participants of the Bamberg Symposium for their helpful criticism of this paper. In particular I would like to thank Mario Mignucci and Hermann Weidemann for their detailed and valuable comments, and Jonathan Barnes, who commented on an earlier draft of the paper.