

# History and Philosophy of Logic



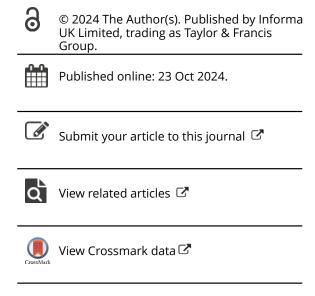
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# Frege, Hirzel, and Stoic logic

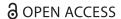
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# Frege, Hirzel, and Stoic logic

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#### **ABSTRACT**

This paper is a discussion of Gabriel, Hülser and Schlotter's pioneering 2009 article on a possible causal relation between Stoic logic and Frege. It provides detailed argument for why Rudolf Hirzel should not be taken as the qualified middleman in philosophical discussion with whom Frege learned what he 'borrowed' without acknowledgement from Stoic logic. Additionally, this paper offers some very modest findings about some aspects of Frege's and Hirzel's lives and work habits, which may help us understand a little better Frege's connection to Hirzel and to Stoic logic as well as Frege's failure to acknowledge the Stoics. This paper is a *purely historical* offshoot of my essay 'Frege plagiarized the Stoics'. Zero direct insights into either Frege's or Stoic philosophy are offered.

#### **ARTICLE HISTORY**

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#### **KEYWORDS**

Frege; Stoic logic; Rudolf Hirzel; Prantl; Eucken

### 1. Introduction

Philosophers and historians of philosophy have long noticed that there are several significant similarities between Frege's work on logic and language and Stoic logic. On several occasions it has been suggested that the Stoics anticipated some of Frege's views. Gabriel, Hülser and Schlotter offered a novel explanation for the similarities between Stoic logic and Frege's philosophy. They argued that 'for semantics, philosophy of language, and their consequences for logic' there was a direct 'causal connection', namely that 'Frege was actually influenced by Stoic logic'. They proposed 'to provide detailed evidence for the existence of this connection' centred around their main thesis that it is '[t]he classical philologist Rudolf Hirzel' – according to them a 'renowned expert on Stoic philosophy' – who is 'the one who links Frege with Stoicism' (*Gabriel, Hülser and Schlotter 2009*, 369). Frege, they suggest, learned what he adopted from Stoic logic from his philosophical discussions or conversations with Hirzel (e.g. 374, also 384).

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<sup>&</sup>lt;sup>1</sup> Gabriel, Hülser and Schlotter (2009, 370–71) mention the following: propositional logic: Łukasiewicz 1934, 112; philosophy of language: Mates 1962, 20, with Bocheński 1962, 124 & 127, and Kneale and Kneale 1962, 138–158, es149, 153–155 u. 500, Gaskin 1997b; conditionals: Mates 1962, 74; Kneale and Kneale 1962, 170; Frede 1974, 105–6; Ebert 1991, 148–50. To these we can add: predicates as functions, besides Gabriel, Hülser and Schlotter 2009, 78, Gaskin 1997a, 94–101, and Egli 1986, and now also Bobzien and Shogry 2020; philosophical logic: Bobzien 2006, 85. For the ontological status of propositions: Nuchelmans 1973, 85. I do not suggest that this list is complete.

<sup>&</sup>lt;sup>2</sup> Łukasiewicz 1934, Barnes 2007, Gaskin 1997b, 131.

As my 2021 essay 'Frege plagiarized the Stoics' (Bobzien 2021) attests, I wholeheartedly agree with the fact that there are multiple parallels between Frege and Stoic logic. More precisely, I believe that most of the parallels suggested by Gabriel, Hülser and Schlotter exist, or at least something quite similar, and that the parallels are actually more numerous than Gabriel, Hülser and Schlotter indicate.<sup>3</sup> I also wholeheartedly agree with Gabriel, Hülser and Schlotter's novel claim that there is a causal connection. In Bobzien 2021 I briefly presented the points on which I disagree with Gabriel, Hülser and Schlotter and noted that I would provide details of my disagreement elsewhere (*Bobzien 2021*, 151–52). I do so in this paper. I explain here why I believe that Gabriel, Hülser and Schlotter are unsuccessful regarding their main thesis that Rudolf Hirzel was the qualified middleman through philosophical discussion with whom Frege learned what he knew and adopted from Stoic logic. More specifically, I argue that Hirzel was not qualified in matters of Stoic logic proper and that there is insufficient evidence for the claims (i) that Hirzel and Frege had cordial neighbourly relations and (ii) that there were lively philosophical discussions between Frege and Hirzel. I also argue (iii) that the one point of evidence adduced for the supposed discussions - that both Frege and Hirzel knew the same part of Plato's Hippias Major – can be explained without postulating a causal connection. In addition, I take issue with a few smaller points made in the article. That said, it would be entirely wrong to see Gabriel, Hülser and Schlotter 2009 and my Bobzien 2021 as wholly incompatible or as opposed in their endeavours. In fact, they complement each other in various important respects, as I set out in my conclusion, and my Bobzien 2021 builds on certain points made by Gabriel, Hülser and Schlotter 2009.

As a byproduct of my discussion of *Gabriel*, *Hülser and Schlotter* 2009, I offer some very modest findings about some aspects of Frege's and Hirzel's lives and work habits, which may help us understand a little better Frege's connection to Hirzel and to Stoic logic as well as Frege's failure to acknowledge the Stoics. I note here also that this paper is in scope purely historical. It is just an offshoot from my Bobzien 2021. No direct insights into either Frege's or Stoics philosophy are offered.

A note on the structure of the paper. Gabriel, Hülser and Schlotter present their case in their sections 2–6. Below, I work through these sections, mostly in order, to assess the argument for their main claim that the causal relation between Frege and Stoic logic is due to Hirzel acting as the qualified middleman from whom Frege learned in philosophical discussion what he came to know about Stoic logic. Generally, I first present the facts adduced by Gabriel, Hülser and Schlotter, followed by their inferences from these facts, and then my own assessment of the inferences. (The facts are individuated merely for purposes of convenience; there is no presumption that the numbering singles out individual facts according to some ontological principle.)

<sup>&</sup>lt;sup>3</sup> I emphasize here that the fact that Frege took elements from the Stoics on a multitude of points of course doesn't mean that everything Frege ever wrote he took from the Stoics, or even that he took all the most important things he ever wrote from the Stoics. There are many substantial differences between Frege's and Stoic philosophy, including on topics where Frege helped himself to Stoic logic. I mention a few: For the Stoics, as for Aristotle, propositions can change their truth-value over time. Fregean thoughts cannot. Where the Stoics focus on the thinker and speaker, Frege often focuses on the audience in considerations about communication. Unlike Frege's (early) Frege-Hilbert-style axiomatic system, Stoic deduction was a Gentzen-style proof-theoretic system. Frege's logic is classical, Stoic logic is substructural and a kind of connexive logic. Frege used the Begriffsschrift notation as formal language, while the Stoics introduced formal elements similar to Łukasiewicz' notation. (See *Bobzien 2019* on the last four points.)

# 2. Frege and his Tenant Rudolf Hirzel: Cordial Neighbourly Relations?

This section is about *Gabriel*, *Hülser and Schlotter* 2009, 371–74.

Here are the facts about Frege and Hirzel presented by Gabriel, Hülser and Schlotter in their Section 2:

Fact 1: From 1889–1913 Hirzel lived on the upper floor of Frege's house as a tenant (371–2).

Fact 2: Hirzel is not ever mentioned in Frege's work (372).

Fact 3: In his work, Frege repeatedly discusses the - written - work of contemporaries without mentioning their names. Examples are works by F. A. Trendelenburg and Bruno Bauch (372).4

Fact 4: Rudolf Hirzel's father Salomon Hirzel was an important nineteenth century publisher. He published the works of Hermann Lotze and was close friends with Lotze (372-73).

I here add a fact not mentioned in *Gabriel, Hülser and Schlotter 2009*, but known to them: Salomon Hirzel published Prantl's history of logic (Die Geschichte der Logik im Abendlande). Gabriel, Hülser and Schlotter mention that Rudolf Hirzel knew vol. I of Prantl's history.

Fact 5: Hirzel was taciturn and not sociable. Lotze describes Hirzel to his father Salomon as 'most silent of all philologists'. Gabriel, Hülser and Schlotter note that he was 'unable to involve himself in conversation' (2009, 373).<sup>5</sup>

Fact 6: Moriz Haupt supervised Hirzel's doctoral dissertation on Plato's Philebus. Hirzel's Habilitationsschrift was on rhetoric in Plato. Hirzel authored three volumes on Cicero's philosophical writings (Untersuchungen zu Cicero's philosophischen Schriften II. Theil.). This work includes 500 pages on the Stoics, mainly on the history of Stoic ethics (373). From 1888, Hirzel was 'ordentlicher' professor (in Classical Philology) at Jena.

Fact 7: Hirzel 'stayed within his father's social network' (372–73).

Fact 8: Frege was not sociable either.<sup>6</sup> (373).

From these facts, Gabriel, Hülser and Schlotter draw several conclusions.

Their first conclusion is derived mainly from Facts 2 and 3: The fact that Hirzel is not mentioned in Frege's works is not in itself evidence that Frege was not influenced in his work by philosophical discussions with Hirzel.<sup>7</sup> My assessment of this inference: It is true that in his work Frege discussed the views of many philosophers and scholars without providing direct bibliographical references. These, including the cases adduced by Gabriel, Hülser and Schlotter, all appear to have been cases in which Frege read someone's work and cited neither author nor work. De facto, Gabriel, Hülser and Schlotter provide evidence that Frege did not – always – cite author or work when he worked from and discussed some contemporary's written work. I note that, if anything, this would suggest that Frege's source for Stoic logic was a written work.

<sup>&</sup>lt;sup>4</sup> '[D]ie durch Rudolf Eucken angeregte Beschäftigung mit den Schriften F. A. Trendelenburgs zur Idee einer Begriffsschrift (vgl. Gabriel 2008) sowie die Auseinandersetzung mit dem Begriff der Verneinung seines Jenaer Kollegen Bruno Bauch (Schlotter 2006).' Possibly Frege also made such use of works by Lotze and Herbart (372).

<sup>&</sup>lt;sup>5</sup> Lotze referred to him as '[S]chweigsamste[r] aller Philologen', 'nicht fähig sich auf Konversation einzulassen' (Falckenberg 1901, 74). Gabriel, Hülser and Schlotter also mention Hirzel's contact with Zeller, Sauppe.

<sup>&</sup>lt;sup>6</sup> See also *Kreiser 2001*, 485.

<sup>&</sup>lt;sup>7</sup> 'Wir haben inzwischen allerdings hinreichend viele Belege dafür, dass man auf eine Fehlanzeige in Sachen Namensnennung im Falle Freges gar nichts geben kann. Vielmehr finden sich in dessen Werk zahlreiche indirekte Bezüge ohne direkten Nachweis, und dies gerade dann, wenn Frege Anregungen aus seinem persönlichen Umfeld aufgegriffen hat.'

Their second inference is drawn mainly from Fact 3: Frege was part of the [academic] social microcosmos in Jena and obtained in conversations many suggestions and hints (372). My assessment: Fact 3 does not support this second inference. Since Frege was not a sociable person and all of Gabriel, Hülser and Schlotter's examples appear to involve Frege's use of written works, we cannot infer that he had regular oral discussions with his colleagues in his academic microcosmos.<sup>9</sup>

Their third inference is drawn from Facts 1-8: There existed a cordial neighbourly relationship between Frege and Hirzel. My assessment: Gabriel, Hülser and Schlotter's conclusion does not seem to follow from the evidence. What follows is that until 1913, their neighbourly relationship was good enough to prevent the Hirzels from moving out.

Here it may be helpful to consider the Hirzels' family and social background. Rudolf Hirzel married Dorothea Hirzel, née Springer, on 14 March 1887. They had no children. Both grew up in an environment of free thinkers, humanists, political activists, and academics who socialized with political activists. See e.g. Debes 1979. Although Hans Sluga suggests that Frege indicated in his diaries that he once thought of himself as 'a liberal' (Sluga 1993, 99ff.), there is no evidence that Frege was a Freidenker or humanist or leftleaning liberal. Rather we find a devout Lutheran and admirer of Bismarck (Kreiser 2001, chapters 6 and 7). A stark difference in political and ideological orientation between the Frege and Hirzel families seems evident.

The fact that both Frege and Hirzel were introverts is no guarantee that they had cordial neighbourly relations. Consider the alternative possibility that the social glue between the households was rather the women. These were Frege's mother, Auguste Wilhelmine Sophie Frege (1815–1898); Frege's wife, Margarete Katharina Sophia Anna Lieseberg (1856–1904); and Dorothea Hirzel. It appears that during the Hirzels' tenancy, none of these women worked outside the house. There would have been multiple occasions for them to run into each other. Nor are they known for being unsociable. There is evidence that Dorothea Hirzel was quite aware of her husband's reluctance to talk to people, that she was herself more outgoing, and that she learned to live with his silence. It is most plausible that there would have been conversations between Frege's wife and Dorothea Hirzel.

<sup>&</sup>lt;sup>8</sup> 'Angesichts solcher und anderer Beispiele können wir inzwischen davon ausgehen, dass Frege in den kommunikativen Mikrokosmos Jena eingebunden war und in Gesprächen vielfältige Hinweise und Anregungen erhalten hat [...]' (372).

<sup>&</sup>lt;sup>9</sup> Thus, accidentally, Gabriel, Hülser and Schlotter's Facts 3 and 8 provide better support for my thesis that Prantl's chapter on Stoic logic was Frege's source of Stoic logic than for their own claim that the source was oral discussions with Hirzel. Tappenden (forthcoming in this volume) notes that Frege seems to have conversed with several mathematicians and scientists. In correspondence, Tappenden also notes that, since Frege hoped that philosophers would take note of his work, he may have had an interest in talking with some philosophers, so that he would understand how philosophers talk (e.g. Eucken). Since Frege was presumably not aiming at classical philologists as an audience for his work, this seems not to be sufficient to infer that Frege would have been interested in conversing with a classical philologist like Hirzel. Note here also that Wittgenstein is on the books as saying that Frege was only interested in talking about logic and mathematics with him, and if the topic strayed from logic, Frege would quickly steer it back (Anscombe and Geach 1961, 129-130).

<sup>&</sup>lt;sup>10</sup> Tappenden in correspondence, on Sluga's suggestion that Frege says he was a liberal in his career, notes that this is at best misleading and that one cannot get this from Frege's diary. He notes that 'at Frege's time it was clearly possible to count as a 'liberal' in the sense of the 'Nationalliberale Partei' while counting as extremely conservative by modern standards. For example, the NLP supported Bismark's anti-Socialist law in 1878. A shared desire for German unification could paper over a lot of other political differences.' Tappenden also notes that Frege's regular scriptural allusions and other remarks in his diary leave no doubt that he was a devout Lutheran, and that his negative remarks about the Centre Party (Zentrumspartei) paired with positive remarks about Bismarck's distrust of this party suggest that Frege 'may have counted himself a (National) 'Liberal' at least in part because of enthusiasm for Bismarck's Kulturkampf.'

There is a further point, not mentioned by Gabriel, Hülser and Schlotter, that may have interfered with warm and cordial relationships. This is the fact that Rudolf Hirzel was ethnically Jewish (see e.g. Woodward 2015, 9). While his father's family had already converted to Reform Protestantism, with the rise of -ethnically based-antisemitism (as opposed to the religiously based anti-Judaism) in Germany in the second half of the nineteenth century, 11 for late 19th and early twentieth century Germans the Hirzels would have counted as Jewish. We also know that Frege was, at least later in life, clearly antisemitic and reactionary. He can hardly have been unaware that Hirzel was - ethnically - Jewish. Consider what this meant. There would have been both a cultural divide and an ideological divide which may have made anything like a warm friendship problematic.

In his twenty-four-page 1919 obituary of Hirzel, Benno von Hagen confirms Hirzel's taciturnity, adds that he was of a friendly disposition, and mentions that he had two close friends (Otto Immisch and Otto Crusius, both classical scholars). We also learn that Hirzel would work throughout the week, dedicate Sundays to reading for pleasure, and generally stayed away from social gatherings. Hagen notes Hirzel's address in Jena as the upper floor of Forstweg 29. Frege is not mentioned once.<sup>12</sup>

There are thus many reasons to doubt that the fact that Hirzel was Frege's tenant is sufficient evidence to conclude that they had cordial neighbourly relations and, relatedly, that Hirzel was an important interlocutor for Frege on philosophical issues.

# 3. Frege, Hirzel, Eucken: A Lively Academic Philosophical Discussion Circle?

Be this as it may. I believe that, given the facts presented by Gabriel, Hülser and Schlotter, it is highly plausible that Frege and Hirzel occasionally talked with each other. It is not so clear whether they would ever have discussed philosophical matters. For Gabriel, Hülser and Schlotter's argument, as the authors are well aware, it is not just relevant whether Frege and Hirzel cordially and regularly talked to each other. It is relevant whether they conversed about philosophy and about Stoic logic (in Frege's sense of logic). In addition to Facts 1-8 from their Section 2, Gabriel, Hülser and Schlotter 2009 present as relevant a range of further facts in their Section 3.<sup>13</sup>

Their argument appears to have two parts. In the first, based on the facts from Section 2 and a few additional pieces of information, they argue that there was a lively philosophical discussion circle comprising Frege, Hirzel, and Eucken. The second argument produces one piece of evidence in – further – support of the claim that there were conversations and academic exchanges between Frege and Hirzel. I take these points in turn.

Here, in an abbreviated fashion, are the additional facts adduced in support of the first point (375–376). Besides Hirzel, they are mostly concerned with the German philosopher Rudolf Christoph Eucken (1846–1926).

Fact 9: Hirzel was a classicist with philosophical interests and special knowledge of the Stoa (372).

<sup>&</sup>lt;sup>11</sup> Cf. e.g. Kreiser 2001. For reasons of space, of the vast literature on 19th century antisemitism I mention here only Rose

<sup>12</sup> Hagen 1919, Gabriel, Hülser and Schlotter adduce Hirzel's presence at the funeral of Frege's mother as a further indicator that there existed a close relationship between the two professors. Note, however, that in the relevant evidence, written by Frege in his diary (printed in Kreiser 2001, 509) he refers to his friends Else and Ernst Abbe as Abbes, to Hirzel and Eucken by contrast as Prof. Hirzel and Prof. Eucken. On Abbe see also *Tappenden 2011*.

<sup>&</sup>lt;sup>13</sup> This section is about *Gabriel, Hülser and Schlotter's* Section 3 (374–76): Evidence for academic ('wissenschaftlich') exchange between Hirzel and Frege.



Fact 10: Until 1910, Eucken lived right across the street from Frege and Hirzel (375).

Fact 11: Eucken's Geschichte der philosophischen Terminologie (Eucken 1879) is quoted in Hirzel's 1913 paper (375).

Fact 12: Frege quotes the same book in paragraph 32 of his Eucken 1884 (Grundlagen der *Arithmetik, Foundations of Arithmetic, GA) (375).* 

Fact 13: Eucken suggested Frege for promotion at Jena for his book(let) Begriffsschrift (BS).

I add two facts (not in Gabriel, Hülser and Schlotter 2009) that provide further support for some connection between Frege and Eucken: Eucken appears to have incorporated into his logic lectures at least one element from Frege's BS, namely the notion of function and argument for predicates (so Kreiser 2001, 291), and Eucken's 1880–1882 logic lectures contained a substantial component on the history of logic which included some remarks on Stoic logic (Kreiser 2001, 289–90, 292).

From Facts 1–13, Gabriel, Hülser and Schlotter draw the following conclusions:

- I. Hirzel and Frege had shared interests in philosophy and logic. (Hirzel was a philosopher rather than philologist; an 'excellent expert of Stoic philosophy' (372)<sup>14</sup>)
- II. From Facts 1-7: Hirzel became an important interlocutor for Frege from circa  $1890.^{15}$
- III. The [three] inhabitants of the Forstweg were connected by lively discussions with each other. (More literally: 'There was an ongoing discussion (Diskussionszusammenhang) among the [three] inhabitants of Forstweg' (375 bottom)).
- There existed a discussion circle (Diskussionskreis) consisting of Frege, Eucken, and Hirzel (375 fn18).

The combination of the above-stated facts, including the two I added, is the *only* evidence for Gabriel, Hülser and Schlotter's conjecture that there existed a lively discussion circle consisting of Hirzel, Frege, and Eucken. For Gabriel, Hülser and Schlotter's Hippias point, see below.

My problem lies with how Gabriel, Hülser and Schlotter arrive at their conclusion that there was a lively discussion group consisting of Hirzel, Frege, and Eucken. Let me make clearer why I believe this to be problematic by offering an alternative for the conjunctive conjecture I/II/III/IV.

Ad I & II: I am not convinced that Frege and Hirzel had shared interests in philosophy and logic of the kind that would make them want to talk to each other. Before coming to Jena, Hirzel had had a strong interest in Plato: in the *Philebus* and in Platonic rhetoric. The vast majority of Hirzel's output on the Stoics is (i) Stoic ethics; (ii) 'Begriffsgeschichte', i.e. the tracing of a philosophical concept and its development over time; (iii) related historical investigations, e.g. showing that the Stoics were the first to use 'logikê' as a term for a philosophical discipline (Gabriel, Hülser and Schlotter 2009, Section 5, 381). From what we have

<sup>14 &#</sup>x27;[A]usgezeichneter Kenner der stoischen Philosophie'. In Leipzig, Hirzel lectured mostly on ancient philosophy and occasionally on Aristophanes (https://histvv.uni-leipzig.de/dozenten/hirzel\_r.html). There seems to be no evidence that he worked or lectured on philosophy other than ancient philosophy. His doctor-father Moriz Haupt was a philologist.

<sup>&</sup>lt;sup>15</sup> 'Angesichts der geschilderten Gegebenheiten liegt es nahe zu vermuten, dass Hirzel als philosophisch interessierter Altphilologe mit besonderen Kenntnissen im Bereich der Stoa für Frege ab etwa 1890 zu einem wichtigen Gesprächspartner

of his writings, it appears that Hirzel wrote almost nothing on Stoic logic (in the contemporary and Fregean sense) and, moreover, that what he did write was simply a by-product of his interests (i)–(iii).

Notably, Hirzel's academic degrees and positions were without exception in classical philology (Klassische Philologie), not in philosophy. 16

Regarding epistemology, we know that Frege read other eminent philosophical authors and he could have drawn from these. 17 Thus, without much overlap in interests, and without evidence that Hirzel and Frege had cordial neighbourly relations, Inference II - that Hirzel became an important interlocutor on philosophical issues for Frege from roughly 1890 – is not compelling.

Ad III & IV: Now let's look at Eucken. Eucken's 1879 work on philosophical terminology (Eucken 1879) was a standard work for a long time. It is a dictionary of philosophical concepts of a sort common in Germany until the late twentieth century, and few academics would have read it from cover to cover. Eucken, who moved away from Frege's street in 1910, was the 1908 winner of the Nobel Prize in Literature. He was something of a wunderkind, a bit of a celebrity, and a gregarious host of large social gatherings. 18 So the fact that both Hirzel and Frege cite Eucken's work on philosophical terminology would not have been unusual, even if they had not all lived within spitting distance of each other. I find it hard to imagine that the extroverted, outgoing, pompous person about town Eucken selected the unsociable Frege and Hirzel for a philosophical ménage à trois. 19 More likely, Eucken may occasionally have given Frege ideas about what to read. Recall that, by Facts 11 and 12, Frege and Hirzel each refer to a published work by Eucken. Recall also that Gabriel, Hülser and Schlotter themselves suggest the possibility that Eucken alerted Frege to Trendelenburg's work (Trendelenburg 1840), which Frege then read, Gabriel, Hülser and Schlotter 2009, 372, with reference to Gabriel 2008.<sup>20</sup>

So, I suggest the following as a more plausible set of inferences from Facts 10–13. Frege and Hirzel each knew of, and acquired or borrowed or consulted, Eucken's book, and had (at least partially) read it, and had done so independently of each other. Hirzel did so because philosophical terminology was one of his main interests, as among other things his work on the term *ousia* and on the Stoic use of *logikê* illustrates. Frege consulted Eucken's

<sup>16</sup> http://research.uni-leipzig.de/catalogus-professorum-lipsiensium/leipzig/Hirzel\_854/markiere:Hirzel/

<sup>&</sup>lt;sup>17</sup> In this paper I do not further discuss similarities between Frege and the Stoics regarding epistemology. There are 19<sup>th</sup> century, possibly earlier, modern sources (and among ancient sources possibly Plato and later ancient Plato reception) from which Frege could have taken most of the epistemological elements that Gabriel, Hülser and Schlotter attribute to the Stoics via Hirzel. There is also a wealth of 19<sup>th</sup> and early 20<sup>th</sup> century literature on epistemological, as opposed to logical, topics that I have not sufficiently studied. Hence I cannot in good conscience claim that the Stoics were a, or the, source from which Frege drew for epistemology. I leave these historical pursuits to others.

<sup>&</sup>lt;sup>18</sup> German Wikipedia entry 'Eucken' (consulted in April 2022): 'Im Jahr 1910 hatte Eucken eine 1897 erbaute Villa in der Botzstraße 5 in Jena erworben, die seitdem 'Villa Eucken' genannt wird. Eucken war in Jena für seine Gastfreundschaft bekannt. Seine Villa wurde zu einem Treffpunkt für Künstler, Gelehrte und Studenten, hier gingen unter anderen der Komponist Max Reger und Literaten wie Stefan George und Hugo von Hofmannsthal ein und aus. Der schottische Dichter Charles Sorley, der sich als Student im Sommersemester 1914 in Jena aufhielt, beschrieb in einem Brief das rege Treiben beim sonntäglichen Tee in Euckens Villa: 'Eucken war außergewöhnlich freundlich zu mir und sprach sehr nett über Euch. Doch bald fluteten Scharen aller möglichen Völker, Nationen und Sprachen herein – Griechen, Türken, Russen, Amerikaner und Japaner –, und es gestaltete sich zu einem höchst lustvollen Gedränge [...] Nun ja, wir alle wandelten im Garten umher und redeten [...] und die Menge wurde nicht kleiner bis Viertel vor acht. [...] Und Eucken stand zur Verabschiedung an der Tür und versicherte allen Gästen, als sie aufbrachen, daß sie für ihn das Vergnügen des Tages gewesen wären.' In einem anderen Brief resümierte Sorley, Eucken werde in Jena "hauptsächlich als eine gesellschaftliche Erscheinung

<sup>&</sup>lt;sup>19</sup> See the depiction of his parties in the previous footnote.

<sup>&</sup>lt;sup>20</sup> See fn 3 above.

work for the entry 'unit(y)' ('Einheit') in his section III of GA, 'Views on unit(y) and one'. In short, then, we have -so far- no reason to assume the existence of a discussion group connecting the two introverts and their companionable across-the-street neighbour. We also – so far – have no evidence that Frege and Hirzel talked about philosophy regularly and in a lively manner, or that Hirzel was an important interlocutor for Frege.

Next I turn to the second point of evidence, which is the main topic of Gabriel, Hülser and Schlotter 2009's Section 3. There, Gabriel, Hülser and Schlotter offer precisely one case as testimony for an academic ('wissenschaftlich') oral exchange between Hirzel and Frege on philosophical topics. This postulated evidence concerns the Platonic dialogue Hippias Major (hereafter Hippias for brevity). First, again, the facts provided by Gabriel, Hülser and Schlotter:

Fact 14: Hirzel published an article called *Ousia* in the journal *Philologus* in 1913. In that piece he refers to the *Hippias*, a possibly spurious Platonic dialogue.

Fact 15: Frege refers to a passage from the *Hippias* in 1919,<sup>21</sup> namely 301bff.

Fact 16: Frege seems to refer to the content of that same passage (301bff), with reference only to Plato, not to a specific dialogue, in his 'Zahlaussagen über einen Begriff', a piece of writing of uncertain date which Carnap suggests may perhaps have been used in lectures during the summer semester in 1913 (Gabriel, Hülser and Schlotter 2009, fn 16).

Fact 17: No other passages in Frege refer to Plato for Frege's notion of numbers as qualities of concepts (375).<sup>22</sup>

Fact 18: The part of the *Hippias* that Hirzel discusses in his *Philologus* paper includes the passage to which Frege refers (301bff).

Based on these (and some earlier) facts, Gabriel, Hülser and Schlotter offer the following conjecture:

- V. Gabriel, Hülser and Schlotter explain the fact that Frege uses Plato only so late in his life (possibly 1913, definitely 1919) by the assumption that Hirzel was the cause of Frege's familiarity with this passage from the Hippias. (Frege refers to Spinoza instead in a related passage in his earlier work GA (§49), see Gabriel, Hülser and Schlotter 2009, 376, fn20).
- Gabriel, Hülser and Schlotter assume that this presumed casual relation is further evidence that there was a lively discussion and a discussion circle that involved both Frege and Hirzel.

For argument's sake only, I now assume that Frege's reference to Hippias and to Plato (above Facts 15 and 16.) somehow owes its origin to interaction with Hirzel (point V above.)

With this assumption in place, recall that German academics of the nineteenth century generally read a lot. We know for certain that Frege read various philosophical works. We know that the number of annual publications in philosophy, classics, logic, and mathematics was a fraction of what we have today. There would also have been certain works that were considered standard, and Frege would have read some of those. Additionally, there would have been annual publications that printed lists with descriptions of new academic

<sup>&</sup>lt;sup>21</sup> Frege 1919 (reprinted in Frege 1983).

<sup>&</sup>lt;sup>22</sup> 'Die sogenannten Zahladjektive keine Eigenschaften von Gegenständen, sondern von Begriffen zum Ausdruck bringen, selbst also 'Begriffe zweiter Stufe' sind und kategorial als Quantoren fungieren' (375).

work on a given subject. It would have been expected that academics and students took notice of their teachers' and, up to a point, their colleagues' work. We know that there existed offprints of *Philologus* in the early 20<sup>th</sup> (and probably in the late 19<sup>th</sup>) century.

Consequently, no discussion circle or regular lively philosophical conversations are required to explain conjecture V. One gift by Hirzel to Frege of an offprint of his ousia paper from *Philologus*, or of a draft, proofs, or preprint, or one mention of the published article (presumably available in the Jena University library) would be quite sufficient to explain Facts 15-18 and conjecture V. So would have been one mention to Frege of the sentences about numbers from the *Hippias*, since this was logic and maths of sorts. The sentence 'Daraus, dass 'wir beide zwei sind', folge keineswegs, dass 'auch jeder von uns zwei sei" (Hippias Major 301d-e, see below 'From the fact that we both [together] are two it doesn't follow that each of us is two.') would presumably have sufficed.

Whether the connection was made by offprint or mention, this it seems would have to have been towards the very end of Hirzel's tenancy with Frege: published in 1913, probably not written before 1911. Thus Gabriel, Hülser and Schlotter's argument only gives us a post *quem* of 1913 for *one* interaction, whereas their general thesis (of Frege and Hirzel as lively philosophical interlocutors) starts in 1890, based on the time when the Hirzels moved in above the Freges. That is, we are offered one piece of circumstantial evidence in twentythree years.<sup>23</sup> Note also that we only have clear evidence for 1919 as the date when Frege refers to the Hippias, which is five years after Hirzel moved out.

Moreover, even if Frege got one point about Plato's work from conversation with Hirzel around 1911-1913 - which I am not certain of, see below - this is insufficient evidence that Frege obtained any information about Stoic logic from discussion with Hirzel between 1890–1913. Hirzel's paper was published in the year he moved out from Frege's house: from this we cannot infer over twenty years of philosophical discussion.

So far I have assumed for argument's sake that V is correct. Methodologically, Gabriel, Hülser and Schlotter provide only a *partial* argument for V. To strengthen the argument's plausibility, one would have to rule out that there was someone else who made an academic mention of the *Hippias* passage to Frege; and that there were any publications that Frege read, or plausibly would have read, or lectures that he attended, that discussed or referred to the relevant *Hippias* passage in a context of interest to Frege. Gabriel, Hülser and Schlotter do not provide such 'negative' evidence.

Where else might Frege have come across the relevant passage? Gabriel, Hülser and Schlotter provide a summary presentation of the relevant part of the dialogue (374–5, my translation):

HIPPIAS: 'You will never find that a quality that belongs neither to me nor to you nonetheless belongs to us two together' (Hipp. Maj. 300d7-9). — SOCRATES: 'It is possible that a quality that belongs neither to me nor to you, none-the-less belongs to both of us; and other qualities that belong to us both, belong to neither of us alone.' (*Hipp. Maj.* 300e3–6) — HIPPIAS: 'When we are both [together] just, isn't it then the case that also each of us is just?' (*Hipp. Maj.* 300e) — SOCRATES: 'From the fact that we both [together] are two it doesn't follow that each of us is two' (*Hipp. Maj.* 301d).<sup>24</sup>

<sup>&</sup>lt;sup>23</sup> Remember also that we have exclusively circumstantial evidence. *Gabriel, Hülser and Schlotter 2009*'s claim that 'Freges Verweis auf den Hippias maior [belegt] dass es einen solchen [regen?] Austausch [zwischen Frege und Hirzel] auch tatsächlich gab' (376) ('indeed existed') may thus be a tad strong.

<sup>&</sup>lt;sup>24</sup> Following the journal editor's style request, I have replaced the traditional small capital letters to denote the two interlocutors used in the original with large capital letters and have added a space between 'Hipp.' and 'Maj.'.

This is a passage about mathematics that occurs in the Hippias in the context of mathematics, in a sub-argument to the main argument in the dialogue (which is about beauty, τὸ καλόν). Now, if Frege was interested in the history of ancient logic (as Gabriel, Hülser and Schlotter agree he was), he may also have been interested in the history of ancient mathematics. And this would not have been just a quirk of Frege's. At his time, knowing the history of philosophy and mathematics was generally taken more seriously than it is today. So, we should look for publications in the history of mathematics that Frege did or may have come across.

We know that in 1882 Frege borrowed Euclid's *Elements* from the Jena university library. Three years earlier in 1879, Frege borrowed Jean-Étienne Montucla's Histoire des mathématiques, vol. I (Montucla 1799) and Hermann Hankel's History of Mathematics in Antiquity and the Middle Ages (Hankel 1874).<sup>25</sup> This supports my assumption that Frege was interested in the history of mathematics, including in antiquity. The then recently published standard work of the day on the history of mathematics in German was Moritz Cantor's mammoth three-volume Vorlesung über Geschichte der Mathematik (Lecture(s) on the History of Mathematics), Cantor 1907 the first volume of which encompassed antiquity and the Middle Ages. The first edition of the first volume was published in 1880, the third in 1907 (Cantor 1907). This work mentions the *Hippias* but does not refer to the relevant passage. It does refer to Nicomachus of Gerasa's famous Introduction to Arithmetic (1st-2nd century CE), which devotes ample space to Plato and mentions the *Hippias* several times. I have not had the opportunity to work through all publications on the history of mathematics that included antiquity and to which Frege may have had access.<sup>26</sup> I do want to mention that in Cantor's first volume (in the 1907 edition on 215) we find a footnote about a work by Benedikt Rothlauf, his 74-page publication Die Mathematik zu Platons Zeiten und seine Beziehungen zu ihr (Mathematics at Plato's time and his relation to it, Rothlauf 1877–1878). On page 23 of this work, we read:

But Plato counts 1 consistently among the odd numbers. Hippias Major 302. 'Socrates: Don't you consider 1 odd?' Socrates must assume that Hippias' answer will be in the affirmative (as it indeed will be), since only thus can he find Hippias guilty of error for his claim that that which two are, also each of the two is.'27

This is precisely the *Hippias* passage to which Gabriel, Hülser and Schlotter refer (301dff: Socrates' argument runs from 301d to 302b) and which they argue Frege obtained from Hirzel (Gabriel, Hülser and Schlotter 2009, 374–5). Now, while Hirzel does not specifically refer to this Platonic argument in his Philologus article 'ousia', Rothlauf does refer to it (see the quotation above). More importantly, Rothlauf's paraphrase from Plato may have been sufficient for Frege to gauge that the text would provide fine historical support for his own view. He may have looked up the *Hippias*. We can assume that he owned Plato's works, possibly inherited from his mother, but perhaps he went and asked Hirzel for his copy ... This is my alternative to Gabriel, Hülser and Schlotter as answer to the question of

 $<sup>^{25}</sup>$  See Kreiser 1984, 25. We do not have Frege's library records for after 1884. We can assume, though, that he did not suddenly stop borrowing books.

<sup>&</sup>lt;sup>26</sup> Hankel discusses Plato at length, if without reference to the *Hippias Major. Montucla 1799–1802* mentions Plato repeatedly and the mathematician Hippias twice but does not mention Plato's dialogue Hippias Major. Zeuthen does not mention Plato's Hippias either (Zeuthen 1912).

<sup>&</sup>lt;sup>27</sup> 'Platon aber rechnet 1 folgerichtig zu den ungeraden Zahlen. 'Hippias der Grössere' 302: 'Socrates: Oder hältst du 1 nicht für ungerade?' die Antwort des Hippias muss Socrates als bejahend voraussetzen (wie sie auch ausfällt), weil er nur so den Hippias des Irrthums überführen kann, der behauptete, dass, was Zwei seien, auch Jeder von Beiden sei. -

how Frege found the Hippias passage that he thought would lend support to his own theory of numbers as qualities of concepts. If I am correct, then Gabriel, Hülser and Schlotter's only piece of evidence for the existence of philosophical discussions between Hirzel and Frege disappears.

Result: Regardless of whether Hirzel mentioned the *Hippias* to Frege, in their Section 3 Gabriel, Hülser and Schlotter do not provide sufficient evidence for their claim that there was a lively discussion circle comprising Eucken, Frege, and Hirzel. This claim is, however, crucial for their main thesis that Frege obtained what he knew about and adopted from Stoic logic via discussion with Hirzel. If Frege did not discuss philosophy and logic with Hirzel, he did not get his presumed knowledge of Stoic logic from discussions with Hirzel. Gabriel, Hülser and Schlotter's remaining Sections 4, 5, and 6 do not provide further evidence for the existence of such discussion. They presuppose it.

# 4. Hirzel, Zeller, Prantl: 'On the Reception of Stoic Semantics up to Eduard Zeller'

This section is about Gabriel, Hülser and Schlotter 2009, Section 4 (376–81).

The methodological function of Gabriel, Hülser and Schlotter's Section 4 appears to be to provide information about the 'state of knowledge of the philologico-philosophical research [on Stoic logic and semantics] of the time', 28 which Hirzel would have known or to which he had access, and to which Frege would have been less likely to have access. The purpose is to underscore the claim that statements in Frege that resemble Stoic logic were the result of Hirzel telling Frege about the corresponding Stoic views.

In Section 4, Gabriel, Hülser and Schlotter introduce the Stoic notion of the sayable (lekton), the Stoic distinction between complete and incomplete lekta, the Stoic notions of a predicate (an incomplete lekton) and a case (ptôsis), and remark on the Stoic deictic propositions. They then partially trace the transmission of the Stoic theory of the *lekton*, mentioning Stanley's mid-seventeenth century The History of Philosophy (Stanley 1701, 379), Fabricius' eighteenth century edition of Sextus Empiricus (380), and Huebner's nineteenth century edition of Diogenes Laertius (380), the latter two of which were present in the Jena library. Towards the end of the section, Gabriel, Hülser and Schlotter provide a long quote from a fourth and final text, Zeller's Die Philosophie der Griechen (Zeller 1909, vol. III, 1, 88–90), which presents the Stoic theory of the *lekton*, its incorporeality, its distinction from corporeal language and from activity in the soul, and the distinction between complete and incomplete lekta, as well as Zeller's translation of 'lekton' as 'Gedanke'.<sup>29</sup> Gabriel, Hülser and Schlotter note that the distinction between complete and incomplete Sinn is used by Frege and imply that so is the word 'Gedanke' (381). They end the section

<sup>&</sup>lt;sup>28</sup> 'Wissensbestand der philologisch-philosophischen Forschung [...] der Zeit'.

<sup>&</sup>lt;sup>29</sup> Gabriel, Hülser and Schlotter make much of Zeller's use of 'Gedanke' for 'lekton' as a parallel to Frege's use of 'Gedanke' for the 'truth-evaluable Sinn of assertoric sentences'. As they themselves note later in their piece, Zeller's use of 'Gedanke' for 'lekton' does not fully correspond to Frege's use of 'Gedanke' in an important respect (385 'unter Einschränkung auf wahrheitsfähige Inhalte'). The Stoic lekton corresponds closely to Frege's Sinn, rather than Frege's Gedanke. 'Gedanke' used by Frege for solely truth-evaluable entities would correspond to the Stoic propositions (axiômata). This is a difference Frege would not have missed. He would have been very clear about a vital logical difference like the one between Stoic lekta and Stoic axiômata. The fact that Zeller translated 'lekton' as 'Gedanke' thus diminishes considerably in relevance, and Gabriel, Hülser and Schlotter's argument is not convincing. Moreover, to make a plausible case that Frege took the word 'Gedanke' from Hirzel, via Hirzel's use of 'Gedanke' for 'lekton' (385), one would have to show that the word 'Gedanke' is not used for incorporeal meaning entities in other philosophical works accessible to Frege.

by drawing the following conclusion (381): 'since for Frege we cannot assume immediate knowledge of the description in Zeller, this suggests Hirzel as middleman.'

Assessment of Gabriel, Hülser and Schlotter's argument: The move from 'we cannot assume immediate knowledge by Frege of Zeller' to 'this suggests Hirzel as middleman [between Frege and the features of Stoic logic that we find in Zeller]' is a big jump which relies on several uncorroborated assumptions. These include:

Assumption 1: Frege would not have had immediate knowledge of Zeller. Frege did not read Zeller. (It is implied that he did not read Stanley, Diogenes, or Sextus either).

Assumption 2: At the time, Zeller was the only available written source that had the relevant information on the Stoics.

Assumption 3: Frege uses an incomplete/complete distinction that corresponds to the Stoic distinction and uses the term 'Gedanke' for the truth-evaluable sense of assertoric sentences. Hence it is obvious ('naheliegend') that he must have gotten this information from the Stoics via Hirzel.

I consider these assumptions in turn. Assumption 1 is conjecture, though somewhat plausible. We do not know whether Frege had immediate knowledge of Zeller, nor whether he read Zeller, but I am willing to agree with Gabriel, Hülser and Schlotter that he did not: It appears much more likely that Frege read books on the history of mathematics, science, and logic than on the history of ancient philosophy in general.<sup>30</sup> For the latter, Zeller's was the German standard work of the time. I agree with Assumption 3, that there exist parallels between Frege and the Stoics on the elements of semantics and logic such as those mentioned by Gabriel, Hülser and Schlotter. For minor details of disagreement, see below and my fn 31 on Zeller on 'Gedanke'.

I take issue with Assumption 2. If it can be shown that Frege had access to all those parallels between Zeller and Frege that Gabriel, Hülser and Schlotter 2009 mention by means of a different path, and possibly via this path to more Stoic logic with parallels in Frege, then Assumption 2 fails, and with it Assumption 3. In that case, the conclusion that Frege must have gotten his information about Stoic logic and semantics via Hirzel collapses.

Now, elsewhere I have shown that Prantl's History of Western Logic contains all this information; that Frege would have known about Prantl's work; that he could easily have obtained a copy (e.g. from Hirzel...), and that all the similarities between Frege and Stoic logic and semantics Gabriel, Hülser and Schlotter mention, and many additional ones that they do not mention, can be explained in this way (Bobzien 2021). Hence, Gabriel, Hülser and Schlotter's argument in Section 4 is not compelling. Here I only briefly add some remarks on why Prantl is more likely than Hirzel to be Frege's source, picking up on Gabriel, Hülser and Schlotter's notes on Prantl.

In his long chapter on the Stoics, Prantl denigrates his subjects on almost every other page. His assessment of Stoic logic is that it is worthless. Gabriel, Hülser and Schlotter mention more than once that Hirzel disagreed with Prantl on this point and that, in response, Hirzel sought to do the Stoics justice (382, 'eine deutliche Spitze gegen Prantl', 384). This all appears to be true. Now, one might think that Frege would not have taken any of the relevant parallels from Prantl, since Prantl paints the Stoics in such a poor light. And hence that Hirzel is a more likely 'missing link'. However, this seems to be an unjustified conclusion.

<sup>&</sup>lt;sup>30</sup> We know, for example, that Frege read and drew on Julius Baumann's Theories of Space, Time and Mathematics in recent Philosophy (Baumann 1868). Erich Reck suggests to me that Prantl's history of logic may have played a similar role for Frege as books like Baumann's in terms of avoiding the study of original texts.

Let me explain. Anyone who is aware of the level of German historical research in the late 19<sup>th</sup> and early twentieth century would easily recognize that Prantl's work displays eminent scholarship, especially in comparison with the work of early to mid-nineteenth century scholars. This is obvious also in the chapter on the Stoics. Despite his regular slanders and the occasional error, Prantl's presentation of Stoic logic, which includes page after page of ancient sources in the original Greek, was an obvious goldmine of information, with nothing of equal breadth and depth up until the 'rediscovery' of Stoic logic by Łukasiewicz, Bocheński, Kneale and Kneale, and Mates. Prantl knew an extraordinarily large number of ancient texts that contained testimony of Stoic logic and by and large he presents the evidence correctly. For 'by and large' see my Bobzien 2021. His volumes were the standard German work on the history of logic for a long time. Frege would have recognized the level of scholarship and the value of the volume as a source of Stoic material. And Frege, who we know had an interest in the history of logic, would have been more likely to look at a history of (ancient) *logic* than at a general history of ancient *philosophy*.<sup>31</sup>

There is then a viable and superior alternative explanation of how Frege may have arrived at the elements of Stoic logic to Gabriel, Hülser and Schlotter's suggestion of philosophical discussions between Frege and Hirzel. Thus, even if we agree that Frege was unlikely to have read Zeller (or Stanley, or Diogenes, or Sextus), the result reached by Gabriel, Hülser and Schlotter 2009 in their Section 4 is not borne out.

# 5. Was Hirzel Qualified and Competent in Stoic Logic?

This Section is about Gabriel, Hülser and Schlotter 2009, Section 5, 'Hirzel's competence as middleman for Stoic logic' (Gabriel, Hülser and Schlotter 2009, 381–384).

The purpose of Gabriel, Hülser and Schlotter's Section 5 is to demonstrate that Hirzel would have been competent and qualified as a middleman between Frege and Stoic logic. If he was not, it would be hard to see how Frege could have obtained what he knew about Stoic logic through philosophical conversations with Hirzel. To show that Hirzel was competent and qualified, Gabriel, Hülser and Schlotter adduce the following facts:

Fact 19: Before coming to Jena, Hirzel published a long paper 'On the logic of the Stoics' (De logica Stoicorum) in an 1879 Festschrift (381).

Fact 20: Hirzel's paper is not about logic. Rather it shows that the Stoics introduced the term 'logikê' for a self-standing philosophical discipline, which is concerned with all questions regarding logos (381).

Fact 21: The last pages of Hirzel's article deal with the order in Diogenes Laertius' list of Chrysippus' (mostly) logical works (381, fn 34).<sup>32</sup>

<sup>&</sup>lt;sup>31</sup> The argument that Frege cannot have taken anything from Prantl, since Prantl depreciates the Stoics constantly is not valid. I have used Prantl to great success when doing research on the Stoics, if usually followed by my looking up relevant passages in the most recent text editions (which would not have been an issue for Frege). Prantl's polemic can almost always be separated guite easily from his paraphrase of Stoic logic. The Aristotelian (mis-)interpretation of the Stoics is generally no worse than that of Prantl's contemporaries. It took someone like Lukasiewicz (or Frege!) to see the essence of Stoic propositional logic. Could one reason why Frege did not cite Prantl be that Frege - rightly - thought that Prantl was too dismissive of the Stoic view and that such citation would not further Frege's goal of incorporating the Stoics' insight about propositions into his own writings? This may be so. It does not answer the question why Frege did not mention the *Stoics*. This is a point I discuss elsewhere.

<sup>&</sup>lt;sup>32</sup> The use of Fact 21 never becomes quite clear. It should be noted here that Prantl quotes the whole Diogenes passage on 404–407, so Hirzel does not have more to offer Frege on this point than Prantl.

Fact 22: Hirzel argues against Prantl who dates the origin of 'logic' to the 1st century BCE: there is reference to Prantl, 535–36, in Hirzel 1879, 63–64 (Gabriel, Hülser and Schlotter 2009, 381, with fn 35).<sup>33</sup>

Fact 23: Prantl did not think highly of Stoic logic (370).

Fact 24: Volume 2 of Hirzel's (1882) contains in IV. Exkurs 'careful critical analysis of the sources of Diogenes' report on Stoic logic' ('sorgfältige quellenkritische Analyse zu dem Bericht des Diogenes Laertius über die Logik der Stoiker') (382).

Fact 25: Hirzel interprets the Stoic theory of assent as follows: the difference between *kataleptic* phantasia and katalêpsis is that although their content is the same, they stand to each other as (Aristotelian) dunamis to energeia (383).

Fact 26: In Hirzel 1882, 542–547 Hirzel writes about Stoic predicates: 'ontologically they are accidents of the underlying bodies and hence their incorporeal effects; epistemologically and logically they are abstractions; ethically the underlying bodies are what is of primary importance. In all these respects there is a basic difference between the predicates and that to which they apply' ( = 'Die Entwicklung der stoischen Philosophie' in the Cicero books) (Gabriel, Hülser and Schlotter 2009, 383 'Gegen Ende [...] zutreffen').

From these facts (Facts 19-26) and earlier ones, Gabriel, Hülser and Schlotter build by inference an argument to demonstrate that Hirzel was sufficiently competent in Stoic logic that he could function as a 'qualified middleman' between the Stoa and Frege (383–384).

*Inference 1*: Frege and Hirzel had a common interest in logic which probably brought them together. Hirzel was interested in what is serious in the Stoics, since Prantl had defamed them; Frege was interested in Stoic logic, since mathematical problems made a deepened investigation ('vertiefte Fragestellung') necessary; the two approaches overlapped: logic and semantics turned out to be joint topics (Gabriel, Hülser and Schlotter 2009, 384).

My assessment of this inference: I doubt that there was a notable overlap between Frege's interests in logic and those of Hirzel. What Hirzel refers to as logic has little to do with what Frege considers logic. The same holds for semantics. Frege had insight into and knowledge of logic (including the logic that he himself had introduced in BS, which is the logic that closely resembles Stoic logic) and of logical semantics in the contemporary sense. There is no indication that Hirzel had any understanding of, or interest in, logic as Frege and contemporary mathematical logicians understood it. It is also doubtful that Prantl caused Hirzel to be interested in logic thus understood, since Prantl himself, though fairly faithfully reporting Stoic sources, shows not at all that he understands its importance. (Note that Hirzel is said to be interested in Prantl's dating of the term *logikê* as referring to a discipline, rather than in logic itself, see above.) So, I also doubt that a common interest in logic brought Frege and Hirzel together.

Towards the end of their Section 5 (on 384), Gabriel, Hülser and Schlotter additionally provide five specific reasons to believe that Hirzel was a qualified middleman for Stoic logic, based on Hirzel 1882.

Reason 1: 'In addition to the general emphasis on the difference between signifying and signified, Hirzel was able to, first, with reference to the Stoic definitions of the predicate

<sup>&</sup>lt;sup>33</sup> This makes it *very* likely that Hirzel owned Prantl's volume. It certainly makes it possible that Frege borrowed Prantl's work from Hirzel. As someone well read in logic and the history of philosophy, particularly from handbooks and standard works, we can assume that Frege knew of Prantl's work, possibly via Eucken or via Sigwart (see fn 49 and Bobzien forthcoming).

and the explanation in Seneca (Epist. 117.12) lay out that in the Stoa, unlike in the prevalent tradition, predicates, as incomplete lekta, differ categorically from the meanings of nominal phrases and contain place holders (Leerstellen) that need to be filled in order to yield a complete *lekton*, for example a proposition' (384). My assessment: This description of Hirzel's familiarity with Stoic logic is put in semantic terms that are entirely absent in Hirzel's 1882. There is no talk of the signifying and the signified, of nominal phrases, place holders, propositions. Thus, we have no evidence that Hirzel himself was able to understand and convey to the Stoic theory to Frege in a way that would pique Frege's interest in semantics. It is unlikely that Frege would have gained anything from Hirzel's vague and not logically informed writing on Stoic 'logic'. We cannot simply assume that Hirzel knew logic (in Frege's sense) but left out this knowledge in all his publications.

Reason 2: 'He [Hirzel] was able to set out the difference between the lekta and (a) the presentations as psychological entities, (b) the phonetic or written linguistic signs, and (c) the world around us' (382). My assessment: This is true. However, we also find this distinction in Prantl (see below, my Section 6).

Reason 3, based on Fact 25: 'Third, based on his suggestions about the epistemology of the Stoics, [Hirzel] was able to offer a clear conception of how judgement contents can be neatly distinguished from judgement acts (as their acceptance or rejection)' (382–383). This point is picked up later by Gabriel, Hülser and Schlotter: 'Regarding propositional entities, Frege's distinction between judgement content and judgement act corresponds to the Stoic theory of agreement [katalepsis]. That distinction, too, is present already in Begriffsschrift, articulated in more detail in "Function and Concept" (Frege 1891, 21-2), and exists in a more elaborate form in the later doctrine of the judgement as an act of the "acknowledgement of the truth of a thought" (386). My assessment: Gabriel, Hülser and Schlotter's reading of Hirzel's interpretation of assent is implausible. Hirzel believes that the kataleptic impression and katalepsis stand to each other as (Aristotelian) potentiality stands to (Aristotelian) actuality. Both have content, which is in fact the same content. To this content, a true proposition corresponds. When the subject has the impression, they thereby have potential comprehension. Only the 'grasped' kataleptic impression is actual comprehension, that is, *katalêpsis*. This Aristotelian-inspired Hirzelian distinction between dunamis and energeia does not correspond to the Fregean distinction between judgement content and act of judgement. Frege's act of judgement has no content. It is the act of acknowledging the content. Moreover (as Gabriel, Hülser and Schlotter are aware), whereas the content of Stoic kataleptic impressions and katalepsis is true by definition, Fregean judgement content can be false. Thus, Hirzel's understanding of the Stoic distinction does not correspond to Frege's distinction, and so it is unlikely to have been Frege's source for his own distinction. There is a closer and genuinely philosophical parallel between Frege and *Prantl*: Frege's distinction is much closer to the Stoic distinction between *saying* a proposition and asserting the proposition, which we find in Diogenes Laertius (where we also find the quasi-assertion passage that Gabriel, Hülser and Schlotter quote a little later, as well as the Sextus passage referred to in fn 41 (385)). These passages are also quoted in *Prantl* (see below). So, Gabriel, Hülser and Schlotter's assumption that Frege got this distinction from Hirzel (384) is not convincing.

Reason 4: As their fourth reason, Gabriel, Hülser and Schlotter note that Hirzel could appeal to the authority of Zeller (and unnamed others) in conversations with Frege to strengthen his case for his representation of Stoic philosophy (383). My assessment: I am not sure how this strengthens Gabriel, Hülser and Schlotter 2009's argument. How would the fact that Hirzel could refer to Zeller as an authority on Stoic philosophy make Frege more inclined to adopt Stoic thought without acknowledging it as Stoic?

Reason 5: Hirzel knew further components of Stoic logic, components which may overlap with elements present in other authors available in the nineteenth century (383). My assessment: This is speculative. Anything that was available to Frege elsewhere, independently of Hirzel, and turns up in his logic, Frege may have taken from these other sources. If he had philosophical discussions with Hirzel about Stoic logic, he may have taken something from Hirzel. But so far there is no compelling evidence that Frege did have philosophical discussions with Hirzel about Stoic logic.

In sum, none of the reasons adduced by Gabriel, Hülser and Schlotter provides compelling evidence for their thesis that Hirzel was in any way qualified and competent in logic and semantics as Frege understood 'logic' and would have understood 'semantics'; hence there is no evidence that he was qualified in Stoic logic, beyond the epistemological components in point 2 above. It is hard to imagine how Hirzel could have conveyed to Frege the elements of Stoic logic which overlapped, or fit right in, with Frege's way of thinking about logic and with what we would call philosophical logic, without himself having some understanding of those topics.

So, when in the first line of their Section 6 Gabriel, Hülser and Schlotter presuppose that Hirzel was qualified to convey Stoic logic to Frege, this seems highly unlikely. As someone who works both in contemporary and in ancient logic, it is clear to me that it takes academics with a background in contemporary logic and/or linguistics to see what is of (contemporary) interest in Stoic logic. Consider the history of the 'rediscovery' of Stoic logic: Łukasiewicz, Kneale and Kneale, Mates, Egli (note also that Michael Frede, author of the 1974 work Die stoische Logik, was a pupil of Patzig's and discussed ancient logic with Tarski). The ancient texts, Sextus and Diogenes above all, would have given Frege a lot more pertinent information than talking with Hirzel could have provided (as I point out in my Bobzien 2021).

Interims results. There is no conclusive evidence that Frege and Hirzel had cordial neighbourly relations (contra Gabriel, Hülser and Schlotter 2009, Section 2). There is no conclusive evidence of a discussion circle consisting of Frege, Hirzel, and Eucken, nor of regular lively philosophical conversations between Frege and Hirzel (contra Gabriel, Hülser and Schlotter 2009, Sections 2 and 3). For Gabriel, Hülser and Schlotter's one piece of presumed evidence that Hirzel and Frege discussed ancient philosophy, namely that Hirzel alerted Frege to a passage in the *Hippias*, we have a plausible alternative explanation for how Frege may have become acquainted with that passage (contra Gabriel, Hülser and Schlotter 2009, Section 4). We have no reason to believe that Hirzel was qualified in Stoic logic (as Frege would have understood 'logic'), although Hirzel seemed somewhat qualified in Stoic epistemology (contra Gabriel, Hülser and Schlotter 2009, Section 5).<sup>34</sup> Moreover, everything on Stoic logic and epistemology that Frege might have obtained via Hirzel, he could have obtained from reading *Prantl* (see below).

<sup>&</sup>lt;sup>34</sup> However, see my assessment of Reason 3.

# 6. The Parallels between Frege and Stoic Logic

This section is about Gabriel, Hülser and Schlotter 2009, Section 6: 'Frege's Umsetzungen des Gehörten' (384–387).<sup>35</sup>

In Section 6, Gabriel, Hülser and Schlotter present passages from and components of Frege's theory (or rather pieces of theory) of logic and semantics and argue that these parts of Frege's works are implementations ('Umsetzungen') of what Hirzel told him in philosophical conversation about Stoic logic. Section 6 presupposes throughout the results from Sections 2–5. Since none of these prior sections was sufficiently successful, Section 6 taken on its own cannot provide evidence of its main claim. Even if we disregard the failure of the previous sections, Section 6 poses a general methodological issue. Its - inherent - argument can be successful only if there are no representations of Stoic logic elsewhere that were accessible to Frege and equally close or closer to Frege's theory than what Gabriel, Hülser and Schlotter suggest Frege received orally from Hirzel. As I have set out in detail in my Bobzien 2021, there is such a representation of Stoic logic that was accessible to Frege, namely in Prantl's History of Western Logic (Prantl 1855).

I briefly list the points of similarity adduced by Gabriel, Hülser and Schlotter and add where the relevant texts for each can be found in Prantl 1855.

- (1) The distinction between what signifies (sign) and what is signified ('Unterscheidung zwischen Bezeichnendem (Zeichen) and Bezeichnetem [...]') (384) (Prantl 1855, 414, 415, 416, with fn 41, 48).
- (2) The Stoic distinction between complete and incomplete lekta (384) (Prantl 1855, 438-440).
- (3) The argument-function structure of language (384–85) (Prantl 1855, 438–439, 443-444).36
- (4) Stoic predicate and Stoic case (ptôsis) corresponding to predicate function and argument (385) (Prantl 1855, 439).
- (5) The Stoic distinction between incorporeal lekton, corporeal language, corporeal psychological thoughts ('Denkgebilde'), presentations (phantasiai), and corporeal objects in the external world (385) (*Prantl 1855*, 415–416 with fn 47).<sup>37</sup>
- (6) Quasi-propositions (385–6) (Prantl 1855, 441–443, with multiple examples from fiction in fn 115).

<sup>&</sup>lt;sup>35</sup> The section title implies that Frege *heard* some things, namely on Stoic logic, from Hirzel. Remember here that we have no direct evidence and very patchy circumstantial evidence for such a conjecture. As I have shown above, for all we know, Frege and Hirzel may never have conversed about Stoic logic.

<sup>&</sup>lt;sup>36</sup> A brief note on page predicates: In *Bobzien 2021*, I did not discuss predicates, but only indicated on 151 that I believe that Frege's notion of predicate as function was not taken from the Stoics. In the meantime, I co-authored a longer piece on Stoic predicates (Bobzien and Shogry 2020), which offers my view on the Stoic theory. I agree with Gabriel, Hülser and Schlotter (2009, 384-85) that Frege had a functional understanding of predicates already in his BS. Likewise, I consider it possible that he adopted the terminology 'incomplete' / 'complete' from his study of Stoic logic (via Prantl 1855, in my view). In BS, in the chapter entitled 'Die Function' (sic), Frege explicitly states that he took his notion of a function from mathematics, more precisely from analysis ('analysis') (p. 19). In his 'Function and Concept' ('Funktion und Begriff'), Frege explicitly states that 'Function' was first used in mathematical analysis, and he develops his notion of the predicate as function in great detail in analogy to mathematical functions, devoting ten pages to the mathematical side and using mathematical examples and analogies throughout the paper (See also Picardi 2022, 12–13.) Thus, even if Frege adopted the terminology 'incomplete' / 'complete' from the Stoics, it seems that the idea of the functionality of predicates is entirely Frege's own, and not the result of his adopting Stoic thought – unless of course, if he studied Prantl before he wrote BS, something for which there is insufficient evidence.

 $<sup>^{37}</sup>$  Here (385) Gabriel, Hülser and Schlotter state expressly that Fregean *Gedanken* are the sense (*Sinn*) of assertoric sentences.



- (7) The distinction between judgement content and act of judgement (386) (Prantl 1855, 441 = Diogenes Laertius 7.65 and 66 on assertibility).
- (8) Erkennen involves metaphor of sense of touch: katalêpsis, to grasp ('fassen') (386) (Prantl, 419–420, 426).

### 7. Conclusion

On pages 386-7, Gabriel, Hülser and Schlotter provide their overall conclusion. This conclusion, which coincides with the main thesis of the article, is that the sum of all evidence shows that Frege's house at Forstweg 29 in Jena was the historical interface that connects Stoic logic with modern logic and semantics. It is implied by earlier sections that this connection was made by Hirzel telling Frege in philosophical conversations what Frege came to know about Stoic logic and then incorporated without acknowledgment into his own writings. I believe that in the present paper, which is a minor spinoff from my Bobzien 2021, I have shown that we have no compelling evidence that this is so.

I want to close by stating that, and in which ways, Gabriel, Hülser and Schlotter's paper is nonetheless an important step towards a better understanding of the countless similarities between Stoic logic and Frege's work on logic, philosophical logic, and philosophy of language. First, it appears that Gabriel, Hülser and Schlotter is the first publication in which a causal connection between Stoic logic and Frege is asserted. I regret that I did not make this point more forcefully in my Bobzien 2021. Second, Gabriel, Hülser and Schlotter assemble in one place a number of similarities between Frege's work and Stoic logic. Third, I believe that there is one specific way in which Hirzel might have been the middleman (if not a middleman qualified in Stoic logic) between Stoic logic and Frege. I argue that Frege knew about Stoic logic through reading the one hundred or so pages on Stoic logic in *Prantl*. The information that Gabriel, Hülser and Schlotter have unearthed about Frege and Hirzel provides one possible and plausible way of understanding how Frege could have gained access to Prantl's volume. He could have borrowed it from Hirzel, who, as the son of its publisher and with an interest in ancient philosophy, would - we can assume - have owned a copy.<sup>38</sup> Fourth, Gabriel, Hülser and Schlotter also provide support for another possible way in which Frege could have had access to Prantl's volume, namely via Rudolf Eucken from across the street, who is also likely to have owned a copy.<sup>39</sup> (During the time when Eucken seems to have been an unofficial mentor of Frege at Jena, he referred to Prantl's volume in his logic lectures.) Fifth, even if Frege did not borrow a copy of Prantl's work from either Hirzel or Eucken, either one of them could have alerted him to its existence.<sup>40</sup>

In conclusion, there can be no doubt that, even if its main thesis does not convince, Gabriel, Hülser and Schlotter 2009 is an important piece of scholarship for our understanding of the relations between Frege, Hirzel, and Eucken and of the connection between Frege and Stoic logic.

<sup>&</sup>lt;sup>38</sup> As mentioned in *Bobzien 2021*, 153.

<sup>&</sup>lt;sup>39</sup> Eucken likely referred to Prantl in the historical part of his logic lectures, as I mention in *Bobzien 2021*. Eucken mentions Prantl at least nineteen times in Eucken 1879 and at least six times in Eucken 1878. Tappenden (in personal correspondence) thinks that Eucken and Frege were not particularly close, since Eucken never mentions Frege in any of his written work, even in places where one might expect him to.

<sup>&</sup>lt;sup>40</sup> As I point out in *Bobzien 2021*, Frege also may have learned about its existence from announcements or reviews in scholarly journals, and he could have borrowed it from the Jena library or bought it. After further research into 19<sup>th</sup> century logic texts that Frege read, I am now more inclined to think that Frege learned about Stoic logic and its presentation in Prantl from Sigwart 1871 and 1873, as I set out in Bobzien forthcoming.

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