

## Logic

JONATHAN BARNES (I), SUSANNE BOBZIEN (II AND  
III.1-7) AND MARIO MIGNUCCI (III.8)

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*Edited by*

KEIMPE ALGRA

LECTURER IN ANCIENT PHILOSOPHY,  
UNIVERSITY OF UTRECHT

JONATHAN BARNES

PROFESSOR OF ANCIENT PHILOSOPHY,  
UNIVERSITY OF GENEVA

JAAP MANSFELD

PROFESSOR OF ANCIENT AND MEDIEVAL PHILOSOPHY,  
UNIVERSITY OF UTRECHT

MALCOLM SCHOFIELD

PROFESSOR OF ANCIENT PHILOSOPHY,  
UNIVERSITY OF CAMBRIDGE



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If we allow that Theophrastus said *something* about such arguments, two questions arise. First, what exactly did he say about them? To this question our texts offer no response; but if he discussed them in detail, and if his discussion (like his account of wholly hypothetical syllogisms) was modelled on Aristotle's categorical syllogistic, then we can make a romantic guess.<sup>21</sup>

Secondly, did Theophrastus thereby invent Chrysippean logic?<sup>22</sup> After all, the argument-forms (1)–(5) are identical with, or at least intimately related to, the five 'indemonstrables' on which Chrysippean logic was grounded,<sup>23</sup> and Theophrastus flirted with (1)–(5) long before Chrysippus courted his indemonstrables. Nevertheless, it is plain that even if Theophrastus discussed (1)–(5), he did not anticipate Chrysippus' achievement. There are several minor reasons for this answer. (Thus it is not clear that Alexander wishes to ascribe (A3) to Theophrastus at all.) And there is a major reason: even if Theophrastus had explicitly recognized all of (1)–(5), his Aristotelian approach to the study and organization of argument-forms would have given his discussion of mixed hypothetical syllogisms an utterly unStoical aspect.<sup>24</sup>

## II The 'Megarics'

Apart from the various logical puzzles and sophisms, there are only two topics on which we can be sure of a positive contribution to logic<sup>25</sup> by the 'Megarics'.<sup>26</sup> These are the positions of Diodorus Cronus and of Philo on the theory of conditionals and on modal logic. Why the discussion of these topics came down to us, we can only divine. Certainly both involve notorious difficulties. Again, they were topics which were extensively and intensely discussed in Hellenistic logic; so much so that the disputes became part of the general knowledge of the intelligentsia of the time (e.g. S.E. *M* 1.309–10). In addition, the theory of modalities was believed to have far-reaching results for other areas of philosophy.

The passages on the conditional and on modal logic, together with some scattered testimonies, allow one to draw some conclusions about 'Megaric' logic in general: The treatment of conditionals and modalities implies that – like most Hellenistic philosophers – the 'Megarics' worked

<sup>21</sup> See Barnes 1985, 571–3.

<sup>22</sup> So Prantl 1855, 1.379; and see also Bochenski 1947, 9; Graeser 1975, 42, 46. *Contra* e.g. Sandbach 1985, 18. <sup>23</sup> See below, pp. 127–31. <sup>24</sup> See Barnes 1985, 574–6.

<sup>25</sup> Logic in the narrow sense, i.e. not including contributions to the study of ambiguity.

<sup>26</sup> On the extent to which it is legitimate to speak of a 'Megaric' (or Dialectical), 'school', see above, p. 47 n. 105.

with a concept of proposition that differs from ours in that it allows truth-values to change over time. We may also conjecture that Philo and Diodorus distinguished between simple propositions, like 'It is day', and complex or non-simple propositions which are composed from simple ones, for instance disjunctions and conditionals. But although we can confidently assume that the truth-conditions of non-simple propositions were examined, we know the 'Megaric' views only in the case of the conditional.

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In the debate about the conditional (*συνημμένον*) the point of disagreement concerned the question of what the right truth-conditions of a conditional were (Cic. *Acad.* II.143). This controversy was played out against the background of a common acceptance of what counts as a conditional, and what its function is. Conditionals were understood to be non-simple propositions containing one proposition as antecedent and one as consequent. The antecedent has the particle 'if' prefixed to it; the standard form is 'If  $p$ ,  $q$ '. A conditional serves to manifest the relation of consequence (*ἀκολουθία*): it announces that its 'consequent follows from (*ἀκολουθεῖν*) its antecedent (S.E. *M* VIII.110-12).<sup>27</sup>

Philo's criterion for the truth of a conditional is truth-functional. It was later generally accepted as a minimal condition for the truth of a conditional. Philo maintained that a conditional is false precisely when its antecedent is true and its consequent false, and true in the three remaining cases: whenever the antecedent is false, and when both antecedent and consequent are true (S.E. *M* VIII.113-17; *PH* II.110). Thus this concept of a conditional comes very close to that of modern material implication. (It is not quite the same, since Hellenistic truth is relativized to times.) Philo's suggestion is remarkable in that it deviates noticeably from the common understanding of conditional sentences and requires abstraction on the basis of the concept of truth-functionality.

Remarkable as it is, Philo's view has the following two drawbacks: first, as in the case of material implication, for the truth of the conditional no connection at all between antecedent and consequent is required. Thus, for example, during the day 'If virtue benefits, it is day' is Philonian true. This introduces a variant of the so-called 'paradoxes of material implication'. Sextus' presentation shows that the ancients were aware of this

<sup>27</sup> The term *ἀκολουθία* was also commonly used for the relation between premisses and conclusion in a valid argument.

problem. Secondly, due to the time-dependency of Hellenistic propositions, Philo's criterion implies that conditionals can change their truth-value over time: for instance, 'If it is day, it is night' is true at night, but false in the daytime. This is counter-intuitive as regards the ordinary use of if-sentences. Moreover, if the concept of a conditional is meant to provide for logical consequence between premisses and conclusion, this leads to the result that arguments could in principle change from being valid to being invalid and vice versa.

For Diodorus, a conditional proposition is true if it neither was nor is possible that its antecedent is true and its consequent false (S.E. *M* VIII.115-17; *PH* II.110-11). The reference to time in this account ('was... is possible') suggests that the possibility of a truth-value change in Philo's truth-condition was one of the things to be improved on.

We do not know whether Diodorus had his own modal notions in mind when talking about possibility in his criterion, or just some pre-technical, general concept of possibility,<sup>28</sup> or whether he perhaps even intended to cover both.<sup>29</sup> If one assumes that he had his own modal notions in mind when giving this account, the truth-criterion for the conditional stands in the following relation to Philo's: a conditional is Diodorean true now precisely if it is Philonian true at all times. Diodorus has, as it were, quantified the Philonian criterion over time. The conditional 'If I walk, I move' is now true because at no time is the antecedent true and the consequent false. Thus for Diodorus, a conditional cannot change its truth-value. If it is true (false) at one time, it is true (false) at all times. If on the other hand one presumes that Diodorus had some unspecified general concept of possibility in mind when producing his account, the criterion would be correspondingly less specific. However, it would presumably still be a minimal requirement that it is never the case that the antecedent is true and the consequent false.

Diodorus' criterion bears some resemblance to the modern concept of strict implication. In particular, it shares some of its disadvantages in that we encounter a parallel to the 'paradoxes of strict implication'. As in Philo's case, no connection is required between antecedent and consequent. This time, whenever either the antecedent is impossible or the consequent necessary, the conditional will be true, regardless of whether there is any relevant connection between the two constituent propositions. So for instance 'If the earth flies, Axiothea philosophizes' would be

<sup>28</sup> For Diodorus' modal concepts see below. The verb used here for being possible, ἐνδέχασθαι differs from the word used for possibility in Diodorus' modal theory, which is δυνατόν.

<sup>29</sup> The latter is argued for in Denyer 1981b, 39-41; cf. also Sedley 1977, 101-2.

Diodorean true, since the antecedent was considered impossible (D.L. VII.75). Again, Sextus' example for the Diodorean criterion (S.E. PH II.111) suggests that there was some awareness of these paradoxes.

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Modal logic is the second topic where we have evidence about the positions of Philo and Diodorus and their influence on the Stoics.<sup>30</sup> Although the modalities were discussed under the heading of 'On things possible', the Hellenistic modal systems were each built on a set of four modalities: possibility, impossibility, necessity and non-necessity. The matter of dispute was which system was the right one, that is, which one adequately described the modalities inherent in the world. In connection with this, an extra-logical concern provided additional fuel to the debate: the belief that if propositions about future events that will not happen turn out to be impossible, the freedom and choices of individuals would be curtailed.<sup>31</sup>

For the 'Megarics' the modalities were primarily properties of propositions or of states of affairs. There is no discussion of modal propositions, i.e. of propositions of the type 'It is possible/possibly true that it is day'. Philo's concept of possibility has survived in four sources<sup>32</sup> but only in Boethius are the accounts of all four modal notions reported:

Possible is that which is capable of being true by the proposition's own nature . . . necessary is that which is true, and which, as far as it is in itself, is not capable of being wrong. Non-necessary is that which as far as it is in itself, is capable of being false, and impossible is that which by its own nature is not capable of being true. (Boeth. *Int.* 2.II.234)

So according to Boethius the basic feature of Philonian modalities is some intrinsic capability of the propositions to be or not to be true or false. That this feature is intrinsic is plain from the phrases 'own nature' and 'in itself'. In Simplicius both phrases are used to characterize Philonian possibility (*Simp. Cat.* 195); hence both phrases may have originally applied to all four accounts.

In all sources the concept of possibility stands out, and so it seems likely that Philo built his set of modal notions on a concept of internal

<sup>30</sup> For a detailed discussion of Philo's and Diodorus' modal theory cf. Kneale-Kneale 1962, 117-28; Bobzien 1993.

<sup>31</sup> This is a variation on the problem of logical determinism which is known from Arist. *Int.* IX. Several of the 'Megaric' sophisms touch upon this issue - so the Mower Argument (for which see Seel 1993), the Lazy Argument (for which see Bobzien 1998, 180-233), and the Master Argument.

<sup>32</sup> Alex. *APr.* 184; Philp. *APr.* 169; *Simp. Cat.* 195-6; Boeth. *Int.* 2.II.234-5.

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consistency, as given in his account of possibility. Philo's modal concepts are thus defined by resort to another, perhaps more basic, modal concept. As to the kind of consistency Philo had in mind, we learn nothing more. Notwithstanding this, there can be little doubt that Philo's modal concepts satisfy a number of basic requirements which normal systems of modern modal logic tend to satisfy as well. These requirements are:

(i) Every necessary proposition is true and every true proposition possible; every impossible proposition is false and every false proposition non-necessary.

According to Philo's accounts, a proposition that is not capable of falsehood must be true; one that is true must be capable of being true, etc.

(ii) The accounts of possibility and impossibility and those of necessity and non-necessity are contradictory to each other.

This can be directly read off the definitions.

(iii) Necessity and possibility are interdefinable in the sense that a proposition is necessary precisely if its contradictory is not possible.

This holds for Philo's accounts, if one neglects the difference in the two phrases 'in itself' and 'by its nature' or assumes that originally both were part of all the definitions. Then a proposition is not capable of being false precisely if its contradictory is not capable of being true, etc.

(iv) Every proposition is either necessary or impossible or both possible and non-necessary, that is, contingent.

In Philo's system this amounts to the fact that every proposition is either incapable of falsehood, or incapable of truth, or capable of both. The fact that Philo's modal accounts – and those of Diodorus and the Stoics, as will be seen – satisfy these four requirements is of course no proof that the ancients consciously reflected upon all of them, regarding them as principles with which they had to comply.

We know a little more about Diodorus' modal theory.<sup>33</sup> Still, again only Boethius reports all four definitions of Diodorus' modal notions:

Possible is that which either is or will be <true>; impossible that which is false and will not be true; necessary that which is true and will not be false; non-necessary that which either is false already or will be false.  
(Boeth. *Int.* 2.11.234–5)

<sup>33</sup> Our sources are Epict. *Diss.* 11.19; Cic. *Fat.* 12, 13, 17; *Fam.* 1X, 4; Plu. *Stoic. Rep.* 1055e–f; Alex. *APr.* 183–4; Philp. *APr.* 169; Simp. *Cat.* 195; Boeth. *Int.* 2.11.234, 412.

Two of these modal accounts are disjunctions, the other two are conjunctions. Provided that Diodorus accepted the principle of bivalence, these definitions, too, satisfy the modal requirements (i) to (iv).

Apart from that, Diodorus' modalities are of a very different kind from Philo's. There is no modal expression hidden anywhere in his accounts. Instead, which Diodorean modality a proposition has depends wholly on the range of truth-values it has at present and in the future. For instance, if a proposition is always true from now on, it is now both necessary and possible; if it is, from now on, sometimes true but not always, it is possible, but not necessary. Hence it is not the case that for Diodorus every proposition is either necessary (and possible) or impossible (and non-necessary). There are propositions that are contingent in the sense of being both possible and non-necessary, namely all those which will change their truth-value at some future time. The proposition 'It is day' is such a case.<sup>34</sup>

We do not know what exactly motivated Diodorus to introduce these modal notions.<sup>35</sup> But we know that Hellenistic philosophers generally regarded Diodorus' modal notions as jeopardizing freedom – since they rule out the possibility that something that never happens, or is never true, is nonetheless possible. For example, if 'Dio goes to Corinth' is and will always be false then 'Dio goes to Corinth' is impossible, and then, or so the thought went, it is impossible for Dio to go to Corinth.<sup>36</sup>

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Diodorus' definition of that which is possible can be split into two distinct claims: first that everything that either is or will be true is possible, and second, that everything that is possible either is or will be true. The first statement was not questioned by Hellenistic philosophers. It is the second claim that was and is considered counterintuitive and in need of justification; it is this claim which Diodorus attempted to back up with his Master argument (*Alex. Apr.* 183.34–184.6; *Epict. Diss.* II.19.1).

Despite being widely known in antiquity, the argument has not come down to us; all we have is a brief passage in Epictetus:

<sup>34</sup> The dependence of the Diodorean modal concepts on truth-values implies that a proposition can change its modality, from possible to impossible and from non-necessary to necessary. For instance, 'Artemisia is five years old' is now possible, because it is now true. But it will at some future time be impossible, namely once Artemisia has reached the age of six, since from then on it will never be true again.

<sup>35</sup> According to Aristotle, some 'Megarics' maintained that the possibility of an event implies its actuality (*Arist. Metaph.* Θ.3.1046b29–32). Perhaps Diodorus endeavoured to keep the spirit of this concept of possibility.

<sup>36</sup> For a comparison between Philo's, Diodorus' and Chrysippus' modalities, see below, pp. 120–1.

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The Master argument seems to have been developed from the following starting points: There is a general conflict between the following three <statements>: (i) every past true <proposition> is necessary; and (ii) the impossible does not follow from the possible; and (iii) something is possible which neither is true nor will be true. Being aware of this conflict, Diodorus used the plausibility of the first two <statements> in order to show that (iv) nothing is possible that neither is nor will be true. (Epict. *Diss.* II.19.1)<sup>37</sup>

This is usually understood as implying that the argument was grounded on statements (i) and (ii), and had (iv), which is the contradictory of (iii), as conclusion. And this is about as far as the passages lead us. But how did the argument run?<sup>38</sup> A viable reconstruction has to satisfy a number of more or less trivial conditions. It must make use of the principles (i) and (ii) handed down in Epictetus; in addition to these, it must make use solely of premisses plausible to the Stoics; and it must appear valid. For we know that different Stoic philosophers attempted to refute one or other of the principles in Epictetus, but we do not hear of anyone questioning the truth of any other premiss or the validity of the argument. Moreover, the reconstruction must employ only the logical means and concepts available in antiquity; in particular the notions of proposition, consequence, and modalities used must fit in with the logic of the time, and it must be possible to formulate the argument in ordinary language. Finally, the restored argument must not have a complexity which precludes its presentation at a social gathering, since people enjoyed discussing the Master argument over dinner (e.g. Plu. *Quaest. Conv.* 615a; Epict. *Diss.* II.19.8).

In line with Diodorus' modal definition, the general conclusion of the argument (iv) may be reformulated as

(iv') If a proposition neither is nor will be true it is impossible.

The first principle is not so readily comprehensible. It runs

(i) Every past true <proposition> is necessary.

The Greek term used for 'past', παρεληλυθός, is a standard Stoic expression for past propositions, meaning not that the proposition itself subsisted in the past, but that it is in some sense about the past.<sup>39</sup> The principle occurs also in Cicero, together with some explication:

<sup>37</sup> There is some additional information in 19.2-9.

<sup>38</sup> Cf. Giannantoni 1981c and R. Müller 1985, 232-4 for a historical overview and extensive bibliographies. <sup>39</sup> Cf. below, pp. 95-6.



All true <propositions> of the past are necessary . . . since they are unalterable, i.e. since past <propositions> cannot change from true to false. (Cic. *Fat.* 14)

From this passage we may infer that it was a peculiarity of all past true propositions that they cannot change their truth-value to falsehood; and that because of this they are necessary. This suggests that the past true propositions at issue do not include all propositions in the past tense, but that they were those propositions which correspond to some past state or event. For instance, the true past proposition 'I went to Athens' corresponds to the event of my having gone to Athens. It can never become false. Assume that I went to Athens last month. Then the proposition 'I went to Athens' is not only true now, it will also be true tomorrow, the day after, and in fact always from now on. The truth of the proposition is based on the fact that there has been a case of my going to Athens, and - whatever happens from now on - this cannot unhappen. (One may bring out this feature more clearly by reformulating the proposition as 'It has been the case that I went to Athens'.) On the other hand the proposition 'You have not been to Athens' does not correspond to a past state or event. Suppose that up to now you never went to Athens. Then the proposition is true now. Now suppose in addition that you will go to Athens next week. After you have gone there, the proposition 'You have not been to Athens' is no longer true. Hence it is not necessary. We may hence reformulate principle (i) as

(i') Every true proposition that corresponds to a past state or event is necessary.

The second principle that functions as a premiss in the argument is

(ii) The impossible does not follow from the possible,

The principle was accepted by Aristotle and by almost all logicians Hellenistic and modern alike.<sup>40</sup> At least by the Stoics it was understood as

(ii') An impossible proposition does not follow from a possible one.

This amounts to the statement that if a proposition is impossible and follows from some other proposition, then this other proposition is impossible, too.

The following reconstruction assumes that the argument rests on a

<sup>40</sup> The exception is Chrysippus, see below, pp. 116-17.

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couple of further principles, which might have been generally understood as valid and thus not worth mentioning, or else which might have been generally accepted by the Stoics, and because of this omitted by Epictetus. The first additional principle is

(v) If something is the case now, then it has always been the case that it will be the case.

For instance, if I am in Athens now, then it has always in the past been the case that I would be in Athens (at some time). This principle gains historical plausibility from the fact that we find a version of it in Aristotle, and that another version of it was accepted by the Stoics.<sup>41</sup>

The second supplementary principle is

(vi) If something neither is nor will be true, then it has been the case (at some time) that it will never be the case.

This theorem is based on the idea that if some proposition presently neither is nor will be true, and you step back in time, as it were, then the formerly present 'not being true' turns into a future 'not going to be true', so that from the point of view of the past, the proposition will never be true, and the corresponding state of affairs will never obtain. This is assumed to hold at least for the past moment that immediately precedes the present. This principle has some plausibility to it. However there is no unambiguous evidence that it was discussed in antiquity.<sup>42</sup>

Fallacies and sophisms were generally presented by means of an example which stands in for the general case, and it is a plausible guess that this was so for the Master argument as well. A suitable example can be found in Alexander – the proposition 'I am in Corinth.' The argument then starts with the assumption that

(1) the proposition 'I am in Corinth' neither is nor will ever be true.

and the conclusion to be demonstrated is that

(C) the proposition 'I am in Corinth' is impossible.

By principle (vi) it follows from (1) that

(2) it has been the case (at some time) that I will never be in Corinth.

Using principle (i), that all past truths are necessary, it follows from (2) that

<sup>41</sup> See Arist. *Int.* IX.18b9–11; Cic. *Div.* 1.125; cf. Cic. *Fat.* 19 and 27.

<sup>42</sup> Becker 1960, 253–5 adduces a few passages in which some ideas that are related to the principle are expressed.

(3) the proposition 'It has been the case (at some time) that I will never be in Corinth' is necessary.

But since necessity of a proposition is equivalent to the impossibility of its contradictory, from (3) it follows that

(4) the proposition 'It has always been the case that I will be in Corinth (at some time)' is impossible.<sup>43</sup>

Now, according to principle (v), it holds that

(5) if I am in Corinth, then it has always been the case that I will be in Corinth (at some time).

This is equivalent to

(5') the proposition 'It has always been the case that I will be in Corinth (at some time)' follows from the (initial) proposition 'I am in Corinth'.

This makes it possible to apply principle (ii), that the impossible follows from the impossible, to (4) and (5'), so that one obtains as a result that

(C) the proposition 'I am in Corinth' is impossible.

And this is precisely what the Master argument was meant to show. Moreover, this argument appears indeed to be valid.

Where does the argument go wrong? The ancients went in for criticizing principles (i) and (ii), and one may indeed wonder whether (i) covers cases of the kind to which it has been applied above. But there are also a couple of things questionable with principles (v) and (vi). With a certain continuum theory of time, one could state that (vi) does not hold for those (rather few) cases in which the proposition at issue has started to be false only at the present moment.<sup>44</sup> More importantly, (v) and its variants seem to smuggle in a deterministic assumption.

### III The Stoics

If Aristotelian logic is essentially a logic of terms, Stoic logic is in its core a propositional logic. Stoic inference concerns the relations between items having the structure of propositions. These items are the assertibles ( $\acute{\alpha}\xi\iota\omega\mu\alpha\tau\alpha$ ) which are the primary bearers of truth-value.<sup>45</sup> Accordingly,

<sup>43</sup> Assuming that the proposition 'It has always been the case that I will be in Corinth (at some time)' in (4) is at least equivalent to the contradictory of the proposition 'It has been the case (at some time) that I will never be in Corinth' from (3). <sup>44</sup> Cf. Denyer 1981b, 43 and 45.

<sup>45</sup> In a derivative sense, presentations ( $\phi\alpha\nu\tau\alpha\sigma\iota\alpha$ ) can be said to be true and false: Chrysippus

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