

(3) the proposition 'It has been the case (at some time) that I will never be in Corinth' is necessary.

But since necessity of a proposition is equivalent to the impossibility of its contradictory, from (3) it follows that

(4) the proposition 'It has always been the case that I will be in Corinth (at some time)' is impossible.<sup>43</sup>

Now, according to principle (V), it holds that

(5) if I am in Corinth, then it has always been the case that I will be in Corinth (at some time).

This is equivalent to

(5') the proposition 'It has always been the case that I will be in Corinth (at some time)' follows from the (initial) proposition 'I am in Corinth'.

This makes it possible to apply principle (11), that the impossible follows from the impossible, to (4) and (5'), so that one obtains as a result that

(C) the proposition 'I am in Corinth' is impossible.

And this is precisely what the Master argument was meant to show. Moreover, this argument appears indeed to be valid.

Where does the argument go wrong? The ancients went in for criticizing principles (1) and (11), and one may indeed wonder whether (1) covers cases of the kind to which it has been applied above. But there are also a couple of things questionable with principles (V) and (V1). With a certain continuum theory of time, one could state that (V1) does not hold for those (rather few) cases in which the proposition at issue has started to be false only at the present moment.<sup>44</sup> More importantly, (V) and its variants seem to smuggle in a deterministic assumption.

### III The Stoics

If Aristotelian logic is essentially a logic of terms, Stoic logic is in its core a propositional logic. Stoic inference concerns the relations between items having the structure of propositions. These items are the assertibles (ἀξιωματὸν) which are the primary bearers of truth-value.<sup>45</sup> Accordingly,

<sup>43</sup> Assuming that the proposition 'It has always been the case that I will be in Corinth (at some time)' in (4) is at least equivalent to the contradictory of the proposition 'It has been the case (at some time) that I will never be in Corinth' from (3).

<sup>44</sup> Cf. Demyer 1981b, 43 and 45.

<sup>45</sup> In a derivative sense, presentations (παραστροφῆς) can be said to be true and false: Chrysippus

Stoic logic falls into two main parts: the theory of arguments (λόγοι) and the theory of assertibles, which are the components from which the arguments are built.

#### 1: Assertibles

What is an assertible? In order to answer this, it is best to look at the various definitions or accounts of 'assertible' that have survived. What appears to be the standard definition states that

(1) an assertible is a self-complete sayable that can be stated as far as itself is concerned.<sup>46</sup>

This definition places the assertible in the genus of self-complete sayables,<sup>47</sup> and so everything that holds in general for sayables and for self-complete sayables holds equally for assertibles. According to the definition, what marks off assertibles from other self-complete sayables is 'that (1) they can be stated (if) as far as they themselves are concerned'.

Assertibles can be asserted or stated, but they are not themselves assertions or statements. They subsist independently of their being stated, in a similar way in which sayables in general subsist independently of their being said. This notwithstanding, it is the characteristic primary function of assertibles to be stated. On the one hand, they are the only entities which we can use for making statements: there are no statements without assertibles. On the other, assertibles have no other function than their being stated.<sup>48</sup>

There is a second account of 'assertible' which fits in well with this. It determines an 'assertible' as

(2) that by saying which we make a statement<sup>49</sup> (D.L. VII.66; cf. S.E. M VII.73; 74).

'Saying' here betokens the primary function of the assertible: one cannot genuinely say an assertible without stating it. To say an assertible is more than just to utter a sentence that expresses it. For instance, 'If it is day, it is light' is a complex assertible, more precisely a conditional, that is composed of the two simple assertibles, 'It is day', which comes in as

*Logika Zēnēmatā, Pherc.* 307, III.13-14 (Hülser 1987-8, 818; revised text in Martone 1993); S.E. M VII.244-51; and in a different sense so can arguments (see below, p. 126).

<sup>46</sup> Το πᾶν ἀξιωματὸν . . . εἶναι λεκτὸν ἀντὶ τοῦ ἀξιοματῶν ὅσον ἐπὶ ἑαυτῶν (S.E. PH II.104; cf. D.L. VII.65).

<sup>47</sup> For self-complete sayables see below, pp. 202-3.

<sup>48</sup> In that respect, assertibles differ from propositional content or the common content of different sentences in different moods. For a propositional content is as it were multifunctional: it can not only be stated, but also asked, commanded etc. In contrast, assertibles are unifunctional: one cannot ask or command them etc.

<sup>49</sup> ὁ λέγωντες ἀποφαινόμεθα.

antecedent, and 'It is light', which comes in as consequent. Now, when I utter the sentence 'If it is day, it is light' I make use of all three assertibles. However, the only one I actually assert is the conditional, and the only thing I genuinely say is that if it is day, it is light.

This suggests that the phrase 'can be stated' is sufficient to delimit assertibles from the other kinds of self-complete sayables. But what then is the function of the remaining part of definition (1), the phrase (ii) 'as far as itself is concerned'? In fact it does not serve to narrow down the class of assertibles any further. Rather it is meant to pre-empt a misinterpretation: the locution 'can be asserted' could have been understood as too strict a requirement, that is, as potentially throwing out some things which for the Stoics were assertibles. For there are two things that are needed for a statement of an assertible: first the assertible itself, secondly someone who can state it. According to Stoic doctrine, that someone would have to have a rational presentation in accordance with which the assertible subsists.<sup>50</sup> But there are any number of assertibles that subsist even though no one has a suitable presentation.<sup>51</sup> In such cases, one of the necessary conditions for the 'assertibility' of an assertible is unfulfilled. Here the qualification 'as far as the assertible itself is concerned' comes in. It cuts out this external, additional condition. For something's being an assertible it is irrelevant whether there actually is someone who could state the assertible.

In the two accounts of 'assertible' presented so far, the expression 'to state' (*ἀποφαίνεσθαι*) has been taken as basic and has not been explicated; nor do we find an explication of it elsewhere. But there are two further Stoic accounts of 'assertible', and they suggest that 'statability' was associated with another essential property of assertibles, namely that of having a truth-value. In a parallel formulation to account (2), we learn that (3) assertibles are those things saying which we either speak true or speak false (S.E. *M* VIII.73)

and several times we find the explication that

(4) an assertible is that which is either true or false (e.g. D.L. VII.65; cf. 66).<sup>52</sup>

From (3) and (4) we can infer that truth and falsehood are properties of assertibles, and that being true or false – in a non-derivative sense – is both

<sup>50</sup> Cf. below, pp. 211–13.

<sup>51</sup> See above, p. 93 and below, p. 211.

<sup>52</sup> This account (4) also occurs in the form of a logical principle, '(5) every assertible is either true or false' (Cic. *Fin.* 2.9; [Plu.] *Fin.* 574e). This is a logical metatheorem which is usually called 'principle of bivalence'.

a necessary and a sufficient condition for something's being an assertible. The exact relation between truth, falsehood and 'statability' we are not told. But it seems safe to assume first that one can only state something that has a truth-value, and second that one can only speak true or false if one 'says' something that is itself either true or false, that is, only if one 'says' an assertible.

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From what has been said so far, one can see that assertibles resemble modern propositions in various respects. There are however essential differences. For instance, true and false assertibles differ in their ontological status. According to a passage in Sextus Empiricus, a true assertible is opposed to something – i.e. something false – and is real (*ὑπάρχειν*), whereas a false assertible is opposed to something – i.e. something true – but is not real (S.E. *M* VIII.10). A difficulty here is what is meant by 'being real'. Perhaps assertibles that are real serve at the same time both as true propositions and as states of affairs that obtain, whereas there is no corresponding identity between false propositions and states of affairs that do not obtain, since the Stoics did not allow anything like 'states of affairs that do not obtain'.

The most far-reaching difference is that truth and falsehood are temporal properties of assertibles. They can belong to an assertible at one time and not belong to it at another time. This becomes obvious for instance by the way in which the truth-conditions are determined: the assertible 'It is day' is true when it is day (D.L. VII.65). This understanding of 'true' is certainly close to everyday use: we might say that it is true now that it is raining, implying that it might be false later. So, when the Stoics say 'p is true' we have to understand 'p is true now'.

A modern proposition is often taken as containing no indexicals. Examples of such propositions, say 'Two plus two equals four' or 'Rain occurs in England on 5/6/94' given that they are true, do not allow a serious question: and will they be true? (The present tense used in them is the a-temporal present.) With an assertible like 'Dio is walking' on the other hand, such a question does make sense: as with Hellenistic examples for propositions generally, it contains no definite time. This assertible now concerns Dio's walking now; uttered tomorrow it will concern Dio's walking tomorrow, etc. This 'temporality' of (the truth-values of) assertibles has a number of consequences for Stoic logic.

In particular, assertibles can in principle change their truth-value: the assertible 'It is day' is true now, false later, true again tomorrow, and so

forth. In fact it changes its truth-value twice a day. Assertibles which do (or can) change their truth-value the Stoics called 'changing assertibles' ( $\mu\epsilon\tau\alpha\tau\iota\tau\tau\epsilon\upsilon\sigma\iota\sigma\iota$ ). The majority of Stoic examples belong to this kind.

A temporal concept of truth raises questions about the status of tense and time in relation to assertibles. The Stoics standardly distinguished past, present, and future assertibles. They were expressed in past, present, or future tenses. Examples are 'Socrates walked', 'It is night', 'Dio will be alive.' A passage in Sextus (S.E. *M VIII* 255) makes the distinction between something being in the past or in the future and a statement being made about something past or about something future, and makes clear that past and future assertibles are not themselves past or future, but about past or future. They subsist in the present just as present assertibles do. For they have their truth-value in the present, when they are asserted. 'Being (about the) past', 'being (about the) future' etc. were hence considered as properties of the assertibles themselves and not merely as context-dependent parts of the linguistic structure of the sentences which express these assertibles.<sup>53</sup>

## 2: Simple assertibles

The most fundamental distinction of assertibles was that between simple ( $\alpha\pi\lambda\alpha\kappa\acute{\alpha}$ ) and non-simple ( $\sigma\upsilon\chi$   $\delta\tau\iota\lambda\alpha\kappa\acute{\alpha}$ ) ones (D.L. VII.68-9, S.E. *M VIII* 93, 95, 108). Non-simple assertibles are those composed of more than one assertible, which are linked by connective particles, like 'either . . . or . . .'; 'both . . . and . . .'.<sup>54</sup> Simple assertibles are defined negatively as those assertibles which are not non-simple. There were various kinds of simple and of non-simple assertibles. Before I turn to them, a few preliminary remarks are in order.

We are nowhere told what the ultimate criteria for the distinctions are. But it is important to keep in mind that the Stoics were not trying to give a grammatical classification of *sentences*. Rather, the classification is of *assertibles*. So the criteria for the distinctions are not merely grammatical, but at heart logical or 'ontological'. This leads to the following complication: the only access there is to assertibles is by way of language. But there is no one-to-one correspondence between assertibles and declarative sentences. For Chrysippus one and the same sentence (of a certain type) may express an assertible or a self-complete sayable that belongs to different

<sup>53</sup> This view that time is - in some way - a property intrinsic to the assertibles leads to several difficulties; one being the problem of the status of time-indicals in assertibles (e.g. in 'I will be alive tomorrow'); another the relation between a future assertible stated now and a corresponding present assertible, stated at the relevant future time. <sup>54</sup> See below, p. 103.

classes - although one may 'reveal itself' more readily than the other.<sup>55</sup> Equally, two sentences of different grammatical structure may express the same assertible.

This view of the relation between assertibles and sentences offers a gloomy prospect for the development of a logic of assertibles. How can we know which assertible a sentence expresses? Here the Stoics seem to have proceeded as follows: aiming at the elimination of (structural) ambiguities they embarked upon a programme of standardization of language such that it became possible (or easier) to read off from the form of a sentence the type of assertible expressed by it.

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I now turn to the various types of simple assertibles. Our main sources for them are two lists, one in Diogenes Laertius (D.L. VII.69-70), the other in Sextus Empiricus (S.E. *M VIII* 96-100), and a handful of titles of works by Chrysippus. At first glance, the lists in Sextus and Diogenes do not match very well: Diogenes enumerates six kinds of simple assertibles, three affirmative, three seemingly negative; Sextus gives only three kinds, which show strong parallels to Diogenes' affirmative ones. But the names differ in two of the three cases, and their accounts differ, to various degrees, in all three cases. Diogenes lists negations ( $\epsilon\pi\iota\sigma\theta\epsilon\tau\iota\kappa\acute{\alpha}$ ), denials ( $\acute{\alpha}\nu\theta\eta\tau\iota\kappa\acute{\alpha}$ ), privations ( $\sigma\tau\epsilon\pi\eta\tau\iota\kappa\acute{\alpha}$ ), categorical ( $\kappa\alpha\tau\eta\gamma\omicron\upsilon\pi\iota\kappa\acute{\alpha}$ ), categorical ( $\kappa\alpha\tau\omicron\gamma\omicron\upsilon\sigma\theta\epsilon\tau\iota\kappa\acute{\alpha}$ ), and indefinite ( $\acute{\alpha}\sigma\iota\sigma\tau\omicron\tau\alpha$ ) assertibles. Sextus lists indefinite ( $\acute{\alpha}\sigma\iota\sigma\tau\omicron\tau\alpha$ ), definite ( $\acute{\alpha}\sigma\iota\sigma\tau\omicron\tau\alpha$ ), and middle ( $\mu\epsilon\sigma\tau\alpha$ ) ones.

The accounts in Diogenes show a greater degree of uniformity and are more grammatically orientated. They apply exclusively to simple assertibles. The accounts in Sextus on the other hand are rather 'philosophical' and do not necessarily apply to simple assertibles only. There are good reasons for assuming that the list in Sextus is earlier than that in Diogenes. But it is likely that both lists were developed in the third and second centuries BC, and the two sets of concepts are in fact perfectly compatible.<sup>56</sup> Chrysippus wrote three books about negations and seven

<sup>55</sup> Cf. e.g. Chrysippus *Logika Zēnēmatā*, *Pherc.* 307, xi.19-30 (Hülster 1987-8, 826; revised text in Marrone 1993).

<sup>56</sup> First, neither of the lists is introduced as an exhaustive disjunction: Sextus introduces the classification with  $\tau\iota\upsilon\delta\ \mu\epsilon\upsilon\ \dots\ \tau\iota\upsilon\delta\ \acute{\alpha}\delta$  instead of  $\tau\acute{\alpha}\ \mu\epsilon\upsilon\ \dots\ \tau\acute{\alpha}\ \acute{\alpha}\delta$  (S.E. *M VIII* 96); Diogenes starts with  $\epsilon\upsilon\ \acute{\delta}\epsilon\ \tau\omicron\iota\varsigma\ \dots\ \tau\iota\upsilon\delta\ \acute{\alpha}\delta$  (D.L. VII.69). Then, although Sextus does not list different types of 'negative' assertibles, he discusses Stoic negation ( $\epsilon\pi\iota\sigma\theta\epsilon\tau\iota\sigma\iota$ ) in S.E. *M VIII* 103-7, that is immediately after his threefold distinction and immediately before talking about non-simple assertibles, to which he turns with the words 'now that we have touched upon the simple assertibles to some degree. . . .' (S.E. *M VIII* 108). Finally, we find traces of most of the kinds of assertibles in Chrysippus, with names from the sets of both authors.

about indefinite assertibles, one book on catagoreuticals and at least one book on privations (D.L. VII.190, *Simp. Cat.* 396.19–21); he wrote about definite assertibles (D.L. VII.197) and used in his writings the term 'definite' in the way it is determined in Sextus.<sup>57</sup> There are no traces of the terms 'categorical' and 'middle'; and there is no evidence that Chrysippus discussed denials.

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Examples of Sextus' middle and Diogenes' categorical assertibles are of two kinds: 'Socrates is sitting' and '(A) man is walking'. The rather unhelpful name 'middle' is based on the fact that these assertibles are neither indefinite (since they define their object) nor definite (since they are not deictic) (S.E. *M* VII.97). Why in Diogenes the assertibles are called 'categorical' remains in the dark. They are defined as those that consist of a nominative case (ὀρθὴ πρὸς ὁρισί), like 'Dio' and a predicate, like 'is walking' (D.L. VII.70). It is noteworthy that assertibles of the type '(A) man is walking' are extremely rare in Stoic logic: besides the example in Sextus there seems to be only one other example, namely the second premiss and the conclusion in the No-one fallacy (D.L. VII.187).

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The next class of simple assertibles, that is, definite and catagoreutical ones, have in their standard linguistic form a demonstrative pronoun as subject expression. A typical example is 'This one is walking'. According to Sextus Empiricus, a definite assertible is defined as one that is uttered along with reference or *deixis* (δείξις) (S.E. *M* VIII.96). What do the Stoics mean by '*deixis*' here? Galen (*PHP* II.2.9–11) cites Chrysippus talking about the *deixis* with which we accompany our saying 'I', which here can be either a pointing at the object of *deixis* (ourselves in this case) with one's hand or finger, or making a gesture with one's head in its direction. So, ordinary *deixis* seems to be a non-verbal, physical act of pointing at something or someone, simultaneous to the utterance of the sentence with the pronoun. Further information is provided by a *scholium* to Dionysius Thrax, namely that

every pronoun is fully defined either through *deixis* or through *anaphora*, for a pronoun either signifies a *deixis*, like 'I', 'you', 'this one', or an *anaphora*, as in the case of 'he' (αὐτός)

<sup>57</sup> Cf. Chrysippus *Logika Zētimata*, *Phron.* 307, v.1.4.17 (Hülster 1987–8, 820; revised text in Marone 1993).

and that

every *deixis* is primary<sup>58</sup> knowledge and knowledge of a person that is present. (ΣDThrax 518–19)

Here *deixis* and *anaphora* are contrasted with each other, from which one may infer that definite assertibles, which require *deixis*, do not include those in which a pronoun is used anaphorically.<sup>59</sup> Moreover, we learn that besides 'this one' and 'I', 'you' can be used along with *deixis*. And thirdly, we learn that *deixis* is always direct reference to an object that is present. This suggests that if, say, I point at a statue of Hipparchia and utter 'This one is a philosopher', I have not performed a genuine *deixis*. (Whereas, if I had said 'This one is a statue of a philosopher' I would have.) '*Deixis* by proxy' seems to be excluded.

Despite these clarifications of the Stoic concept of *deixis*, there remain difficulties with definite assertibles: first, how does one identify a particular definite assertible? Certainly, the sentence (type) by which a definite assertible is expressed does not suffice for its identification. For example if I have my eyes closed while someone utters the sentence 'This one is walking', thereby pointing at someone, I do not know which assertible was stated. For the sentence 'This one is walking' uttered, say, while pointing at Theo expresses a different assertible than when uttered while pointing at Dio.<sup>60</sup>

However, we have every reason to believe that when now I utter 'This one is walking' pointing at Dio, and when I utter the same sentence tomorrow, again pointing at Dio, the Stoics regarded these as two assertions of the same assertible. Thus, regarding the individuation of definite assertibles, the easiest way to understand the Stoic position is to conceive of a distinction between '*deixis* type' and '*deixis* token': a '*deixis* type' would be determined by the object of the *deixis* (and is independent of who performs an act of *deixis* when and where): whenever the object is the same, the *deixis* is of the same type; and the tokens are the particular utterances of 'this one' accompanied by the physical acts of pointing at that object. Hence we should imagine there to be one assertible 'This one is walking' for Theo (namely with the *deixis* type pointing-at-Theo), and one for Dio (with the *deixis* type pointing-at-Dio).

But now the question arises: how does a definite differ from the corresponding middle assertible – e.g. 'This one is walking (pointing at Dio)'

<sup>58</sup> 'first-hand' or 'prior' or 'primary' – the Greek word is πρῶτος.

<sup>59</sup> For the Stoic treatment of cases of anaphora see below, pp. 104–5.

<sup>60</sup> Our texts suggest that the Stoics identified a particular definite assertible when it is used, by the accompanying act of pointing; and when it is mentioned, by the addition of a phrase like 'pointing at Dio' just as I did above (cf. *Alex. AP*, 177.28–9).

from 'Dio is walking'? Are they not rather two different ways of expressing the *same* assertible? Not for the Stoics. For we know that in the case of the assertibles 'Dio is dead' and 'This one is dead (pointing at Dio)' at the same time one could be true, but the other not (see below p. 116). What is it then that distinguishes them? Some information about the difference between middle and definite assertibles can be obtained by scrutinizing the case in which their truth-values differ.

In a passage that reports Chrysippus' rejection of the modal theorem that from the possible only the possible follows<sup>61</sup> we learn that the assertible 'This one is dead (pointing at Dio)' cannot ever become true, since so long as Dio is alive it is false, and thereafter, once Dio is dead, instead of becoming true, it is destroyed. The corresponding assertible 'Dio is dead', on the other hand, does – as expected – simply change its truth-value from false to true at the moment of Dio's death. The reason given for the destruction of the definite assertible is that once Dio is dead the object of the *deixis*, i.e. Dio, no longer exists.

Now, for an assertible destruction can only mean ceasing to subsist. When an assertible ceases to subsist, that implies that it no longer satisfies all the conditions for being an assertible. And this should have something to do with its being definite, that is, with its being related to *deixis*. So one could say that in the case of definite assertibles, assertibility or stability (being *δτιοποντόν*) becomes in part point-at-ability, and Stoic point-at-ability requires intrinsically the existence of the object pointed at. This is not only a condition of actual stability in particular situations – as is the presence of an asserter; rather it is a condition of identifiability of the assertible; of its being this assertible.<sup>62</sup>

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Next the indefinite assertibles. In Sextus, they are defined as assertibles that are governed by an indefinite part of speech or 'particle' (S.E. *M* VIII.97). According to Diogenes they are composed of one or more indefinite particles and a predicate (D.L. VII.70). Such indefinite particles are 'someone' (*τις*) or 'something' (*τι*). Examples are of the type 'Someone is sitting'.<sup>63</sup> Again, Sextus presents special truth-conditions: the indefinite

<sup>61</sup> The passage is quoted in full, below, p. 116.

<sup>62</sup> Diogenes Laertius' categoretical assertibles are defined as being composed of a definite nominative case and a predicate (D.L. VII.70). This implies that the object of the *deixis* must be referred to by a definite pronoun in the nominative. The categoricals might then simply have formed a subclass of Sextus' definite assertibles.

<sup>63</sup> The Stoics had both simple and non-simple indefinite assertibles and Sextus' account of indefinite assertibles seems to cover both kinds. For the latter see below, pp. 111–14.

assertible 'Someone is sitting' is true when a corresponding definite assertible ('This one is sitting') is true, since if no particular person is sitting, it is not the case that someone is sitting (S.E. *M* VIII.98). This truth condition, in connection with the requirement of existence of the object of *deixis* for the subsistence of a definite assertible, gets the Stoics into trouble. Assertibles like 'Someone is dead', it seems, can never be true – since no assertible 'This one is dead' can ever be true. The Stoics could have easily mended this by expanding the truth condition to: '... if a corresponding definite or middle assertible is true'. The indefinites are the Stoic counterpart to our existential propositions, and their classification on a par with the other simple assertibles leads to some complications when it comes to the construction of non-simple assertibles.<sup>64</sup>

We do not know how the Stoics classified simple assertibles that are expressed by sentences with more than one noun expression, like 'This one loves Theo', 'Aspasia loves this one', 'Leontion loves someone' etc. The accounts in Diogenes make one think that the criterion for classification may have been always the subject expression. At any rate, examples of these kinds are extremely rare in Stoic texts.<sup>65</sup>

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The most important kind of negative assertible is the negation. For the Stoics, a negation is formed by prefixing to an assertible the negation particle 'not' (*οὐχι*, *οὐκ* etc.) as for instance in 'Not: it is day'. The negation is truth-functional: the negation particle, if added to true assertibles, makes them false, if added to false ones makes them true (S.E. *M* VIII.103).

Every negation is the negation of an assertible, namely of the assertible from which it has been constructed by prefixing 'not'. The assertible 'Not: it is day' is the negation of the assertible 'It is day'. An assertible and its negation form a pair of contradictories (*ἀντιρρήσιμα*):

Contradictories are those <assertibles> of which the one exceeds the other by a negation particle, such as 'It is day' – 'Not: it is day' (S.E. *M* VIII.89)

This implies that an assertible is the contradictory of another if it is one of a pair of assertibles in which one is the negation of the other (cf. D.L. VII.73). Of contradictory assertibles precisely one is true and the other false.<sup>66</sup>

<sup>64</sup> See below, pp. 111–14.

<sup>65</sup> Cf. also Brunschwig 1994b, 66–7; Ebert 1991, 117–18.

<sup>66</sup> The concept of contradictoriness is pertinent to various parts of Stoic logic, e.g. to the truth-conditions for the conditional and to the accounts of the indemonstrable arguments; see below, p. 106 and p. 127.

Why did the Stoics insist on having the negation particle prefixed to the assertible? If we assume that they were looking for a standardized formulation for the negation of an assertible which expresses its contradictoriness, this becomes readily comprehensible. In order to obtain an assertible's contradictory, the scope of the negation particle has to encompass the whole assertible which it negates. This is achieved with a minimum of structural ambiguity if one places it right in front of this assertible. A negation particle elsewhere in a sentence – especially in its common place before the predicate – can easily be understood as forming a negative assertible that is not contradictory to the original assertible. For instance, in the view of some Stoics, 'Callias is walking' and 'Callias is not walking' could both be false at the same time: namely in the case that Callias does not exist (Alex. *APr.* 402.3–19).<sup>67</sup> This explains why the Stoics did not call negative assertibles of this kind negations, but rather affirmations (ApuL. *Int.* 191.6–15; Alex. *APr.* 402.8–12); for them in the above example it is affirmed of Callias that he is not walking.

Although in Diogenes Laertius the negation is introduced as one of the types of simple assertibles, the Stoics equally prefixed the negation particle to non-simple assertibles in order to form complex negations. The negation of a simple assertible is itself simple, that of a non-simple assertible non-simple. Thus, differently from modern logic, the addition of the negative does not make a simple assertible non-simple. The negation particle 'not' is not a propositional connective (*συνδεσμος*), for such connectives bind together parts of speech and the negation particle does not do that.

An interesting special case of the negation is the 'super-negation' (*ὑπερnegation*) or, as we would say, 'double negation': This is the negation of a negation, for instance 'Not: not: it is day'. This is still a simple assertible. Its truth-conditions are the same as those for 'It is day'. It posits 'It is day', as Diogenes puts it (D.L. VII.69).

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Diogenes' list contains two further types of negative assertibles: denials and privations. A denial consists of a denying particle and a predicate, the example given is 'No one is walking' (D.L. VII.70). That is, this type of assertible has a compound negative as subject term. Unlike the negation particle, this negative can form a complete assertible if put together with a predicate.

The truth-conditions of denials have not been handed down, but they seem obvious: 'No one φ's', should be true precisely if it is not the case

that someone φ's. Denials must have been the contradictories of simple indefinite assertibles of the kind 'Someone φ's'. But why do they form an extra class? Would not the negations of indefinite assertibles, like 'Not: someone φ's' have sufficed? Possibly the Stoics who introduced denials pursued a chiefly grammatical interest in classifying assertibles. Alternatively, they may have aimed at differentiating them from categorical assertibles: although grammatically they could be seen as consisting of a nominative and a predicate, they do not have existential import.<sup>68</sup>

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Finally, the privative assertible is determined as a simple assertible composed of a privative particle and a potential assertible, like 'This one is unkind' (D.L. VII.70, literally 'Unkind is this one', a word order presumably chosen in order to have the negative element at the front of the sentence). The privative particle is the *alpha privativum* 'α-' ('un-'). It is unclear why the rest of the assertible, '(-)kind is this one' is regarded merely as a potential assertible. For e.g. in the case of the negation proper, in 'Not: it is day', 'It is day' is referred to simply as an assertible.

### 3: Non-simple assertibles

The analogue to the modern distinction between atomic and molecular propositions is the Stoic distinction between simple and non-simple assertibles. Non-simple assertibles are those that are composed of more than one assertible or of one assertible taken twice (D.L. VII.68–9; S.E. *M* VIII.93–4) or more often. These constituent assertibles of a non-simple assertible are put together by one or more propositional connectives. A connective is an indeclinable part of speech that connects parts of speech (D.L. VII.58). An example of the first type of non-simple assertible is 'Either it is day, or it is light'; one of the second type 'If it is day, it is day'.

Concerning the identification of non-simple assertibles of a particular kind, the Stoics took what one may call a 'formalistic' approach, for which they were often – and wrongly – rebuked in antiquity.<sup>69</sup> In their definitions of the different kinds of non-simple assertibles they provide the characteristic propositional connectives, which can have one or more parts, and determine their position in (the sentence that expresses) the non-simple assertibles. In this way it is shown how the connectives are syntactically combined with the constituent assertibles; their place relative to (the

<sup>67</sup> Cf. Lloyd 1978b.

<sup>68</sup> Cf. Ebert 1991, 122. The Stoics can thus express all four basic types of general statements, e.g. 'Someone φ's', 'No one φ's' and 'No one does not φ'.  
<sup>69</sup> Cf. e.g. Gal. *Inst. Log.* IV.6; III.5.

sentences expressing) the constituent assertibles is strictly regulated. The advantage of such a procedure is that once one has agreed to stick to certain standardizations of language use, it becomes possible to discern logical properties of assertibles and their compounds by looking at the linguistic expressions used.<sup>70</sup>

Non-simple assertibles can be composed of more than two simple constituent assertibles (Plu. *Stoic. Rep.* 1047c-e). This is possible in two ways. The first has a parallel in modern propositional logic: the definition of the non-simple assertible allows that its constituent assertibles are themselves non-simple. An example of such a non-simple assertible is 'If both it is day and the sun is above the earth, it is light'.<sup>71</sup> The type of non-simple assertible to which such a complex assertible belongs is determined by the overall form of the assertible. The above example, for instance, is a conditional. The second type of assertible with more than two constituent assertibles is quite different. Conjunctive and disjunctive connectives were conceived of not as two-place functors, but – in line with ordinary language – as two-or-more-place functors. So we find disjunctions with three disjuncts: 'Either wealth is good or <wealth> is evil or <wealth is> indifferent' (S.E. *M VIII*.434; S.E. *PH II*.150).

It is worth noting that all non-simple assertibles have their connective or one part of it prefixed to the first constituent assertible. As in the case of the negation, the primary ground for this must have been to avoid ambiguity. Consider the statement

'*p* and *q* or *r*'.

In Stoic 'standardized' formulation this would become either

Both *p* and either *q* or *r*

or

Either both *p* and *q* or *r*.

The ambiguity of the original statement is thus removed. More than that, the Stoic method of pre-fixing connectives can in general perform the function brackets have in modern logic.<sup>72</sup>

The avoidance of ambiguity must also have been one reason behind the Stoic practice of eliminating cross-references in non-simple assertibles.

<sup>70</sup> See Frede 1974a, 198-201.

<sup>71</sup> Cf. S.E. *M VIII*.230 and 232, 'If it is day, <if it is day> it is light.'

<sup>72</sup> In this respect one might rightly consider the Stoic formulation as a forerunner of Polish notation; cf. Ebert 1991, 115-16.

Instead of formulations of ordinary discourse like 'If Plato walks, he moves' and 'Plato walks and (he) moves', the subject term is repeated in full: 'If Plato walks, Plato moves', 'Plato walks and Plato moves'.<sup>73</sup> This practice of elimination is not reflected upon in our sources, but its regular occurrence leaves no doubt that it was exercised intentionally – perhaps to simplify the application of formal logical procedures.

Truth-conditions for the non-simple assertibles are given separately from their definitions. They suggest that the Stoics were not aiming at fully covering the connotations of the connective particles in ordinary language. Rather they lend themselves to the interpretation that the Stoics attempted to filter out the essential formal characteristics of the connectives. Leaving aside the negation – which can be simple – only one type of non-simple assertible, the conjunction, is truth-functional. In the remaining cases modal relations (like incompatibility), partial truth-functionality and basic relations like symmetry and asymmetry, in various combinations, serve as truth-criteria.<sup>74</sup>

For Chrysippus we know of only three types of non-simple assertibles: conditionals, conjunctions, and exclusive-cum-exhaustive disjunctive assertibles. Later Stoics added further kinds of non-simple assertibles, although the number seems always to have been fairly small. Besides the three Chrysippean kinds, we find a pseudo-conditional and a causal assertible, two types of pseudo-disjunctions, and two types of disjunctive assertibles. It is quite possible that the main reason for adding these was logical, in the sense that they would allow the formulation of valid inferences which Chrysippus' system could not accommodate. A certain grammatical interest may have entered into it, but this alone could not account for all the choices and omissions made.<sup>75</sup>

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The conjunction (συντηρηγυμένον or συντηρόκη) seems generally to have been regarded as unproblematic. One account runs 'A conjunction is

<sup>73</sup> Cf. D.L. VII.77, 78, 80; S.E. *M VIII*.246, 252, 254, 305, 308, 423; S.E. *PH II*.105, 106, 141; Gell. *XVI* 89; Gal. *Inst. Log.* IV.1; Simp. *Phys.* 3300; Alex. *APr.* 345; Cic. *Fin.* 12-13; see also below, pp. 111-14 on indefinite non-simple assertibles.

<sup>74</sup> In addition to the direct determination of the truth-conditions, we frequently find another way of providing truth-conditions, namely by stating what a particular non-simple assertible 'announces' (ἀρκεύει/ἀρκεύει) (D.L. VII.72; Epict. *Dis.* 11.9-8). Occasionally, as in the case of the conditional, this 'announcement' covers only the unconnected main features of the truth-criterion (D.L. VII.71).

<sup>75</sup> The most comprehensive list of Stoic non-simple assertibles is provided in D.L. VII.71-74. Other important texts are Gell. *XVI*.8-9-14; Gal. *Inst. Log.* III, IV and V, and various passages in Sextus *Empiricus*.

an assertible that is conjoined by certain conjunctive connective particles, like "Both it is day and it is light." (D.L. VII.72). Like a modern conjunction, the Stoic one connects whole assertibles: it is 'Both Plato walks and Plato talks', not 'Plato walks and talks'. Unlike modern conjunction, the conjunctive assertible is defined and understood in such a way that more than two conjuncts can be put together on a par (cf. Gell. XVI.8.10). The standard form has a two-or-more part connective: 'both . . . and . . . and . . . .' (καί . . . καί . . .). The truth-conditions, too, are formulated in such a way as to include conjunctions with two or more conjuncts. A conjunction is true when all its constituent assertibles are true, false if one or more are false (S.E. *M* VIII.125,128; Gell. XVI.8.11). The Stoic conjunction is therefore truth-functional.<sup>76</sup>

For Stoic syllogistic the negated conjunction (ἀποφωτιστή συζυγία) (D.L. VII.80) is of chief importance, since only when negated is the conjunction suitable as a 'leading' premiss.<sup>77</sup> Typically of the kind 'Not: both  $p$  and  $q$ '. Some more complex arguments have conjunctions with negated conjuncts as minor premiss.<sup>78</sup>

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The conditional (συντημύεον) was defined as the assertible that is formed by means of the linking connective 'if' (εἰ) (D.L. VII.71). Its standardized form is 'if  $p$ ,  $q$ ', with  $p$  as the antecedent and  $q$  as the consequent.

In Chrysippus' time the debate about the truth-conditions of the conditional – which had been initiated by the logicians Philo and Diodorus – was still going on. There was agreement that a conditional 'announces' a relation of consequence, namely that its consequent follows (from) its antecedent (D.L. VII.71). It was what it is to 'follow' and the associated truth-conditions that were under debate. A minimal consensus seems to have been this: the 'announcement' of following suggests that a true conditional, if its antecedent is true, has a true consequent. Given the acceptance of the principle of bivalence, this amounts to the minimal requirement for the truth of a conditional that it must not be the case that the antecedent is true and the consequent false – a requirement we find also explicitly in our sources (D.L. VII.81). It is equivalent to the Philonian criterion.

We know that Chrysippus offered a truth-criterion that differed from Philo's and Diodorus' (Cic. *Acad.* II.143), and we can infer that

Chrysippus' alternative was the one which Sextus reports, without naming an originator, as third in his list which starts with Philo and Diodorus. For it is presented as Stoic in D.L. VII.73 and alluded to as Chrysippean in Cic. *Fat.* 12.<sup>79</sup>

Sextus ascribes the Chrysippean criterion to those who introduce a connection (συνδέτησις) (S.E. *PH* II.111); this connection can only be that which holds between the antecedent and the consequent. The requirement of some such connection must have been introduced to avoid the paradoxes that arise from Philo's and Diodorus' positions. A look at the criterion itself shows that the connection in question is determined indirectly, based on the concept of conflict or incompatibility (ἕξις): it states that a conditional is true precisely if its antecedent and the contradictory of its consequent conflict (D.L. VII.73). According to this criterion, for example, 'if the earth flies, Axiothea philosophizes' – which came out as true for both Philo and Diodorus – is no longer true. It is perfectly compatible that the earth flies and that it is not the case that Axiothea philosophizes.

For a full understanding of Chrysippus' criterion, one has to know what sort of conflict he had in mind. But here our sources offer little information. We find the shift to a modal expression in some later texts, according to which two assertibles conflict if they *cannot* be true together. This confirms that the conflict is some sort of incompatibility. Then there is a brief passage in Alexander (Alex. *Top.* 93.9–10) which has been interpreted as saying that two assertibles  $p$ ,  $q$  conflict precisely if, assuming that  $p$  holds,  $q$  fails to hold *because*  $p$  holds.<sup>80</sup> However, the passage need not be of Stoic origin, and due to the condensed form of the text the interpretation inevitably remains speculative.

It is inappropriate to ask whether Chrysippus intended empirical, analytical or formal logical conflict: a conceptual framework which could accommodate such a distinction is absent in Hellenistic logic. Still, we can ask whether kinds of conflict that we would place in one or the other of those categories would have counted as conflict for Chrysippus. We can be confident that formal incompatibility would count. Assertibles like 'If it is light, it is light' were regarded as true (Cic. *Acad.* II.98; S.E. *PH* II.111) – presumably because 'It is light' and 'It is not the case that it is light' are incompatible, contradictoriness being the strongest possible conflict between

<sup>76</sup> See also Brunschwig 1994c, 72–9. <sup>77</sup> See below, p. 121. Cf. also Sedley 1984.

<sup>78</sup> For the indefinite conjunction see below, p. 113.

<sup>79</sup> Cf. Frede 1974a, 82–3. Sextus mentions a fourth type of conditional, which is based on the concept of *emphatic* (S.E. *PH* II.112). Its truth-conditions are that the consequent has to be potentially included in the antecedent. It is unclear who introduced this conditional. Cf. Frede 1974a, 90–3; Croissant 1984. <sup>80</sup> Cf. Barnes 1980, 170.



two assertibles. Equally, some cases that some may describe as analytical incompatibility were covered: for instance 'If Plato is walking, Plato is moving' and 'If Plato is breathing, Plato is alive' were regarded as true.

The question then boils down to the point whether any cases of what one might label 'empirical' incompatibility would count as conflict for Chrysippus.<sup>81</sup> There are a number of true conditionals where it is hard to decide whether the connection is empirical or logical, for instance, 'If it is day, it is light' and 'If there is sweat, there are invisible pores'. But some instances of empirical incompatibility were accepted by some Stoics: so conditionals with causal connections of the kind 'If someone has a wound in the heart, that one will die' were considered true.<sup>82</sup>

The connection expressed in theorems of divination on the other hand seems to have been an exception (Cic. *Fat.* 11-15). Such theorems are general statements which give in their 'consequent' the predicted future type of event or state, and in their 'antecedent' a sign of the event, to which the diviner has access. Chrysippus accepted that such theorems, if genuine, held without exception, and hence that in all instantiations the 'consequent' is true when the 'antecedent' is true. Nevertheless, he claimed that they would not make true conditionals.<sup>83</sup> Instead, he maintained that the diviners would formulate their theorems adequately if they phrased them as negated conjunctions with a negated second conjunct, i.e. if instead of 'If  $p$ ,  $q$ ' they said 'Not: both  $p$  and not  $q$ '.<sup>84</sup> Given that the conjunction and the negation are truth-functional, the resulting non-simple assertible is equivalent to a Philonian conditional.

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Grounded on the concept of the conditional, the Stoics introduced two further kinds of non-simple assertibles (D.L. VII.71, 72). Both were probably added only after Chrysippus. Their accounts and truth criteria are in principle open to interpretation with Philo's, Diodorus', or Chrysippus' truth criterion for the conditional as their basis. Yet the presentation of Chrysippus' conditional in the same section in Diogenes suggests that it was his conditional these later Stoics had in mind.

<sup>81</sup> Scholars are divided on this point. Cf. e.g. Frede 1974a, 84-9; Donini 1974-5; Bobzien 1998, 156-70.

<sup>82</sup> S.E. *M* VII.24-5; cf. *M* V.104, where the heartwound is referred to as cause ( $\alpha\rho\tau\omicron\nu\nu$ ); cf. also *Alex. Aph.* 404.21-4.

<sup>83</sup> This implies that Chrysippus thought that there was no conflict in the required sense between the sign of a future event and the non-occurrence of that event. Perhaps he assumed that there was no causal connection, either direct or indirect, between sign and future event, let alone any logical link.

<sup>84</sup> Cicero's example for a theorem of the diviners is in fact a negated *indifferitiz* conjunction. For this, see below, pp. 113-14.

The first, called 'pseudo-conditional' ( $\tau\epsilon\pi\alpha\sigma\sigma\upsilon\nu\eta\mu\acute{\epsilon}\nu\omicron\nu$ ), is testified at the earliest for Crinis and has the standardized form 'Since ( $\acute{\epsilon}\tau\tau\epsilon\iota$ )  $p$ ,  $q$ ' (D.L. VII.71). The truth-criterion for such assertibles is that (i) the 'consequent' must follow (from) the 'antecedent' and (ii) the 'antecedent' must be true (D.L. VII.74).<sup>85</sup>

The second kind is entitled 'causal assertible' ( $\alpha\rho\tau\iota\omega\delta\epsilon\varsigma$ ) and has the standard form 'Because ( $\delta\iota\acute{\omicron}\tau\iota$ )  $p$ ,  $q$ '. The name is explained by the remark that  $p$  is, as it were, the cause/ground ( $\alpha\rho\tau\iota\omega$ ) of  $q$ .<sup>86</sup> The truth-condition for the causal assertible adds simply a further condition to those for the pseudo-conditional ((i) and (ii)). It is the element of symmetry that is ruled out for causal assertibles: the extra condition is (iii) that if  $p$  is the ground/cause for  $q$ ,  $q$  cannot be the ground/cause for  $p$ , which in particular implies that 'Because  $p$ ,  $q$ ' is false. This condition makes some sense: 'being a cause of' and 'being a reason for' are usually considered as asymmetrical relations. In contrast, assertibles of the kind 'Since  $p$ ,  $q$ ' are true pseudo-conditionals, and it is possible that both 'Since  $p$ ,  $q$ ' and 'Since  $q$ ,  $p$ ' are true.

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The Greek word for 'or' ( $\acute{\eta}$ ) has several different functions as a connective particle, which are distinct in other languages. It covers both the Latin *aut* and the Latin *vel* and also both the English 'or' and the English 'that'. Not surprisingly, it plays a role as a connective in at least three different types of non-simple assertibles.

Chrysippus and the early Stoics seem to have concentrated on one type of disjunctive relation only: the exhaustive and exclusive disjunctive relation, called  $\delta\iota\acute{\epsilon}\xi\epsilon\nu/\mu\acute{\epsilon}\nu\omicron\nu$ , here rendered 'disjunction'. This is the only disjunctive that figures in Chrysippus' syllogistic. In Diogenes it is defined as 'an assertible that is disjoined by the disjunctive connective "either", like "Either it is day or it is night"' (D.L. VII.72). As in the case of the conjunction, the disjunctive connective was understood as being able to have more than two disjuncts, and there are examples of such disjunctions (Gell. *XV* 8.12; S.E. *PH* 1.69; S.E. *M* VIII.434). Thus the connective was 'either ... or ... or ...' ( $\acute{\eta}\tau\omicron\iota\ \dots\ \acute{\eta}\ \dots\ \acute{\eta}\ \dots$ ) with its first part ('either') prefixed to the first disjunct.

<sup>85</sup> It has been suggested that 'Since  $p$ ,  $q$ ' is an economical and appropriate way of expressing Stoic sign-inference (so Burnyeat, 1982c, 218-24; Sedley, 1982c, 242-3). For a sign is the antecedent in a sound conditional which both begins and ends with truth, and is revelatory of its consequent (cf. S.E. *PH* II.101.104.106).

<sup>86</sup> The Greek  $\alpha\rho\tau\iota\omega$  covers both physical causes and grounds, reasons, explanations.

The formulation of the truth-conditions raised some difficulties, not least due to the fact that more than two disjuncts were allowed. Gellius presents them as follows:

But (i) all the disjuncts must be in conflict with each other and (ii) their contradictories . . . must be contrary to each other. (iii) Of all the disjuncts one must be true, the remaining ones false. (Gell. xv.8.13)

First a non-truth-functional criterion is given ((i)-(ii)), which is then immediately followed by something like a truth-functional criterion (iii). This could be either an alternative truth-criterion; or – similar to the case of the conditional – just an uncontested minimal requirement, perhaps to permit one to single out some false disjunctions more readily.<sup>87</sup>

It certainly was a necessary condition for the truth of the disjunction that precisely one of its disjuncts had to be true and all the others false. But most sources imply that this was not sufficient. The criterion they state is stricter and typically involves the term 'conflict', which is already familiar from the conditionals. The criterion is in fact a conjunction of two conditions ((i) and (ii)). First, the disjuncts must be in conflict with each other; this entails that at most one is true. Secondly, the contradictories of the disjuncts must all be contrary to each other; this ensures that not all of the contradictories are true, and hence that at least one of the original disjuncts is true. The two conditions may be contracted into one as 'necessarily precisely one of the disjuncts must be true'. As in the case of the conditional, a full understanding of the truth-criterion would require one to know what kind of conflict the Stoics had in mind.

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According to Gellius the Stoics distinguished two kinds of the so-called 'pseudo-disjunction' (παραδιεζευγμένον).<sup>88</sup> Regarding their standard form, most examples are formed with 'either . . . or . . .' or, occasionally, just with '. . . or . . .' and some have more than two pseudo-disjuncts. Thus apparently the two types of pseudo-disjunctions are indistinguishable in their linguistic form from disjunctions (and from each other). Thus – unlike the case of the other non-simple assertibles – it becomes impossible to tell from the language whether an assertible is a pseudo-disjunction or a disjunction, and hence which truth-conditions it has to satisfy.

<sup>87</sup> It is unclear whether the disjunction was ever understood as truth-functional. Reference to the truth-values of the constituent disjuncts is made repeatedly (Gell. xv.8.13; S.E. PH II.191; D.L. VII.72).

<sup>88</sup> Gell. xv.8.14. Cf. Proclus (*Procli ad 1st century AD*) in the *Digesta Justiniani Augusti* 50.16.124-125.

Other sources mention just one kind, e.g. Ap. *Dysc. Conf.* 219.12-24.

If we follow Gellius (xv.8.13-14), the truth-criteria for the two types of pseudo-disjunctions are simply the two halves of the truth-condition for the genuine disjunction: One kind is true if its pseudo-disjuncts conflict with each other, which entails that at most one of them is true. The other kind is true if the contradictories of its pseudo-disjuncts are contrary to each other, which entails that at least one of the pseudo-disjuncts is true.<sup>89</sup>

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As mentioned above, the Greek word for 'or' (ή) serves another purpose: that of the English word 'than'. Accordingly, we find a further kind of non-simple assertible which is sometimes discussed in the context of the disjunctives. This is the comparative or disjunctive<sup>90</sup> assertible, formed by using a comparative or disjunctive connective (διασφατητικός σύνδεσμος). Diogenes reports two types (D.L. vii.72-3), with the connectives 'more (or rather) . . . than . . .' (μᾶλλον . . . ἢ . . .) and 'less . . . than . . .' (ἥττον . . . ἢ . . .). These are two-part connectives, again with the characteristic part prefixed to the first constituent assertible, thus allowing the identification of the type of assertible.

The truth-conditions for these two types have not survived in Diogenes, but the treatment of such assertibles by the grammarians offers some help. One text describes the comparative statements as 'when two are posited and one of them is stated', another the connective as 'as if it became the umpire of the disjunctive'.<sup>91</sup> This suggests that the comparative assertibles stand to the disjunction as the pseudo-conditional to the conditional: the truth-conditions would be equivalent to those of 'Both either *p* or *q* and *p*' (*ψ* μᾶλλον *q*) and 'Both either *p* or *q* and *q*' (*ψ* ἥττον *q*).

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The definition of the non-simple assertibles implies that they take any kind of simple assertibles as constituents, and that by combining connectives and simple assertibles in a correct, 'well-formed' way, all Stoic non-simple assertibles can be generated. But apparently this is not so: non-simple assertibles that are composed of simple indefinite ones raise

<sup>89</sup> This latter type has the modern inclusive disjunction with the connective 'v' as its truth-functional counterpart.

<sup>90</sup> The Greek names for these assertibles are διασφατῶν τὸ μᾶλλον ἔξῆρα and διασφατῶν τὸ ἥττον ἔξῆρα (D.L. vii.72-3). Cf. Sluiter 1988.

<sup>91</sup> Ὅταν τῶν δύο ποσθηθέντων τὸ ἐν εἰρηται (*Ephemerismi ad Rom.* 189) ὁμοί ἐμπρηται ἔνοσηται τῆς διαδιεζεύξας (Ap. *Dysc. Conf.* 222.25-6). Moreover, the disjunctive statement is said to ἐπιτροπυ/δίαεται . . . τοῦτο . . . οὐ τοῦτο' (*ibid.* 223.1).

some special problems. Unlike the case of definite and middle assertibles, one can conceive of two different ways of putting together indefinite ones.

First, following Stoic formation rules to the letter, by combining two simple indefinite assertibles into a conjunction or a conditional, one obtains assertibles like

If someone is breathing, someone is alive.

Both someone is walking and someone is talking.

According to Stoic criteria these would be true, respectively, if 'Someone is breathing' and 'Not: someone is alive' are incompatible and if 'Someone (e.g. Diocima) is walking' is true and 'Someone (e.g. Theognis) is talking' is true. However, complex assertibles with indefinite pronouns as grammatical subject more commonly tend to be of the following kind:

If someone is breathing, that one (he, she) is alive.

Someone is walking and that one is talking.

Here the truth-conditions are different from those in the previous case. For the second 'constituent assertible' is not independent of the first. As a matter of fact, we find no Stoic examples of the first type of combinations of indefinite assertibles but quite a few of the second (D.L. VII.75; 82; Cic. *Fat.* 15; S.E. *M.xi.* 8, 10, 11; cf. 1.86). The second type was explicitly dealt with by the Stoics and it seems that the terms 'indefinite conjunction' and 'indefinite conditional' were reserved for it.<sup>92</sup> In order to express the cross-reference in the second constituent assertible to the indefinite particle of the first, 'that one' (ἐκεῖνος) was standardly used (D.L. VII.75; 82; Cic. *Fat.* 15; S.E. *M.xi.* 8, 10, 11).<sup>93</sup>

The Stoics were certainly right to single out these types of assertibles as a special category. Plainly, the general problem they are confronted with is that of quantification. The modern way of wording and formalizing such statements, which brings out the fact that their grammatical subject expressions do not have a reference ('For anything, if it is F, it is G', (x) (Fx → Gx)), did not occur to the Stoics. We do not know how far they 'understood' such quantification as lying behind their standard formulation. Three things suggest that at least they were on the right track.

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<sup>92</sup> Cf. Cic. *Fat.* 15. On the other hand, 'indefinite disjunctions', i.e. assertibles of the kind 'Either someone φ's or that one ψ's' are not recorded.

<sup>93</sup> Note that in non-simple assertibles composed of two definite assertibles we have οὐτως twice, and not a cross-reference with ἐκεῖνος or any other pronoun; see e.g. S.E. *M.viii.* 246.

First, in the context of the Stoic theory of definition and division we are told that the assertible 'Every human-being is a rational, mortal animal' was reformulated in standardized form as 'If something is a human being, that thing is a rational, mortal animal' (S.E. *M.xi.* 8-9). That is, in general,

All S are P

became

If something is S, that thing is P.

The term used for such universal assertibles seems to have been 'universal' (καθολικόν) (S.E. *M.xi.* 8-11; Epict. *Diss.* II.20.2-3).

Secondly, the same passage also tells us something about what the Stoics regarded as the truth-conditions of such statements: indefinite assertibles have non-indefinite ones 'subordinated' to them. These are all those definite and middle assertibles that differ from the indefinite only with respect to their subject. The indefinite conditional is false if (at least) one of the subordinated conditional assertibles is false (S.E. *M.xi.* 9, 11) and a sufficient condition for falsehood is that at least one of the subordinated conditionals has a true antecedent and a false consequent (*ibid.* 10). From this we can infer that indefinite conditionals are true if all their subordinated conditionals are true. There is some evidence that negative universals were subordinated to a corresponding reformulation. A passage in Epictetus (*Dis.* II.20.2-3) implies that the negative universal 'No S is P' became 'If something is S, not: that thing is P'.

Cic. *Fat.* 11-15 suggests that, parallel to the distinction between conditionals and negated conjunctions, the Stoics distinguished a weaker type of universal statement, namely of the kind

Not: both something is S and that thing is not P.

Such negated conjunctions would cover mere universality, as in 'All cars in this street are rabble'.

We have seen that in their classification of simple assertibles the Stoics could fit in all four types of general statement without specific subject expression.<sup>94</sup> But we learn of no standard formulations for Aristotelian particular statements of the kinds

Some S are P

Some S are not P.

<sup>94</sup> See above, pp. 100-3 and esp. n. 68.

Still, since the *negated* indefinite conjunction stands in for a universal, it is possible that the Stoics thought of such particulars as indefinite conjunctions. In that case we only have to remove the prefixed negation in the negated indefinite conjunction and we obtain the two types of particulars

Both something is P and not: that thing is S

Both something is P and that thing is S.

Here the standard truth-conditions for indefinites apply: there must be at least one subordinated assertible that is true – for instance, 'Both Diodotima is P and not: Diodotima is S'.

Thirdly, one reason for the importance of indefinite conditionals and indefinite negated conjunctions was no doubt the need to obtain certain types of valid arguments by means of which one can infer a singular case from a universal. For instance, the Stoics used arguments of the kind

If someone  $\phi$ 's, that one  $\psi$ 's.

Now Dio  $\phi$ 's.

Therefore Dio  $\psi$ 's.

These arguments will be discussed later.

The Stoic accounts of assertibles, simple and non-simple, reveal many similarities to modern propositional logic. It is tempting to draw further parallels with the modern propositional calculus, but one can easily go too far. There can be little doubt that the Stoics attempted to systematize their logic. But theirs is a system quite different from the propositional calculus. In particular, Stoic logic is a logic of the validity of arguments, not a system of logical theorems or tautologies, or of logical truths.<sup>95</sup> Of course, the Stoics did have logical principles, many of them parallel to theorems of the propositional calculus. But, although they had a clear notion of the difference between meta- and object language,<sup>96</sup> apparently logical principles that express logical truths were not assigned a special status or dealt with any differently from logical meta-principles. A survey of the principles concerning assertibles may be useful. First, there is the principle of bivalence (Cic. *Fat.* 20), which is a logical meta-principle. Then, corresponding to logical truths we find:

– the principle of double negation, expressed by saying that a double-negation (Not: not:  $\phi$ ) is equivalent (ισοδυναμείν) to the assertible that is doubly negated ( $\phi$ ) (D.L. VII.69);

– the principle that all conditionals that are formed by using the same assertible twice (Στασσοποιύμεν) (like 'If  $\phi$ ,  $\phi$ ') are true (Cic. *Acad.* II.98; S.E. *M* VIII.281, 466);

– the principle that all disjunctions formed by a contradiction (like 'Either  $\phi$  or not:  $\phi$ ') are true (S.E. *M* VIII.282, 467).

No principles of commutativity have survived in explicit formulations, and they may not have been expressed as principles. However, the accounts of the indemonstrable arguments tend to have symmetry of conjunction and disjunction 'built in' so that no extra rules are required.<sup>97</sup> Moreover, at least some later Stoics may have dealt with relations like commutativity and contraposition via the concepts of inversion (ἄναστροπή) and conversion (ἀντιστροφή) of assertibles (Gal. *Inst. Log.* VI.4). Inversion is the change of place of the constituent assertibles in a non-simple assertible with two constituents. Thus, with  $\sigma_1$ ,  $\sigma_2$  standing for the parts of the connective, ' $\sigma_1 \phi$ ,  $\sigma_2 \psi$ ' is inverted to ' $\sigma_1 \psi$ ,  $\sigma_2 \phi$ '. Commutativity could thus have been expressed by saying that in the case of conjunction and disjunction inversion is sound or valid. In a conversion the two constituent assertibles are not simply exchanged, but each is replaced by the contradictory of the other. So ' $\sigma_1 \phi$ ,  $\sigma_2 \psi$ ' is converted to ' $\sigma_1$  not:  $\phi$ ,  $\sigma_2$  not:  $\psi$ '. The Stoics seem to have recognized that conversion holds for the conditional; that is, they seem to have accepted the principle of contraposition (cf. D.L. VII.194). Moreover, a passage in Philodemus suggests that some Stoics may have explicitly stated the principle (Phil. *Sigm.*, *Pherc.* 1065, XI.26–XII.14).

A final question concerns principles regarding the interdefinability of connectives. There is no evidence that the Stoics took an interest in reducing the connectives to a minimal number. For the early Stoics we also have no evidence that they ever attempted to give an account of one connective in terms of other connectives, or that they stated logical equivalences of that kind.<sup>98</sup>

#### 4: Modality

As the previous sections have illustrated, the Stoics distinguished many different types of assertibles: simple and non-simple, definite and indefinite, negative, conjunctive etc.; these were generally identifiable by their form. In addition, the Stoics classified assertibles with respect to certain

<sup>95</sup> See below, p. 128.

<sup>96</sup> The passage Cic. *Fat.* 15 that is sometimes cited in this context states that there is a logical difference between a conditional and a negated conjunction with a negated second conjunct (cf. also Frede 1974a, 103–4).

<sup>95</sup> See below, pp. 121–57.

<sup>96</sup> See below, p. 132.

of their properties which were not part of their form. The most prominent ones, after truth and falsehood, were the modal properties possibility, necessity etc.

Two further such properties were plausibility and probability: an assertible is plausible ( $\tau\rho\iota\beta\omicron\upsilon\upsilon\upsilon$ ) if it induces assent to it (even if it is false), like 'If someone has given birth to ( $\tau\rho\iota\kappa\rho\epsilon\upsilon\iota$ ) something, she is its mother'; for a bird who lays or gives birth to ( $\tau\rho\iota\kappa\rho\epsilon\upsilon\iota$ ) an egg is not its mother (D.L. VII.75). We would expect this rather to be discussed in the context of epistemology: An assertible is probable or reasonable ( $\epsilon\upsilon\lambda\omicron\gamma\omicron\upsilon\upsilon$ ) if it has higher chances of being true, like 'I shall be alive tomorrow' (D.L. VII.76; cf. *ibid.* 177).

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Stoic modal logic<sup>99</sup> developed out of the debate over the 'Megaric' modalities, in particular over Diodorus Cronus' Master argument and the threat of logical determinism.<sup>100</sup> Cleanthes, Chrysippus and Antipater wrote about possibility and all three attacked the Master argument: Cleanthes rejected its first premiss, that true past propositions are necessary. Chrysippus had a go at the second premiss, i.e. the principle that from the possible only the possible follows (Epict. *Diss.* II.19.1-5, 9). A passage in Alexander gives us the details:

But Chrysippus says that nothing precludes that something impossible follows something possible. . . . For he says that in the conditional 'If Dio is dead, this one is dead', which is true when Dio is pointed at, the antecedent 'Dio is dead' is possible, since it can at some time become true that Dio is dead; but 'This one is dead' is impossible; for once Dio has died, the assertible 'This one is dead' is destroyed, since the object of the *deixis* no longer exists. For <in the present case> the *deixis* is of a living being and in accordance with <is being a> living being. Now, if - him being dead - the 'this one' is no longer possible, and if Dio does not come into existence again so that it is possible to say of him 'This one is dead'; then 'This one is dead' is impossible. This assertible would not be impossible, if at some later time, after the death of that Dio about whom the antecedent was said when Dio was alive, one could say of him again 'this one' (Alex. *APr.* 177.25-178.4)

Chrysippus' argumentation is in short as follows: First, the assertible 'Dio is dead' is possible, since it will be true at some time. Secondly, the assert-

<sup>99</sup> For a detailed discussion of Stoic modal logic see Frede 1974a, 107-17, Bobzien 1986, 40-120, Bobzien 1993.

<sup>100</sup> For the 'Megaric' modalities and the Master Argument see above, pp. 86-92.

ible 'This one is dead (pointing at Dio)' is impossible. For any assertible that neither is nor can ever be true is impossible. And 'This one is dead (pointing at Dio)' is necessarily either false (namely as long as Dio is alive) or destroyed (namely when Dio is dead, since then there is nothing to point at any more). Thirdly, the conditional 'If Dio is dead, this one is dead' (pointing at Dio) - as long as it subsists - is true:<sup>101</sup> any conditional of the form 'If  $x$  is  $\phi$ -ing, this one (pointing at  $x$ ) is  $\phi$ -ing' is a true conditional, according to all three Hellenistic truth criteria for conditionals.<sup>102</sup> Finally, if a conditional is true, its consequent follows from its antecedent. Hence Chrysippus provided a case of a conditional in which the consequent assertible, which is impossible, follows from the antecedent assertible, which is possible. Whatever we may think about it, Chrysippus must have been sufficiently content with his rejection of the Master argument; for he developed his own system of modal notions, which soon became the Stoic one.

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Stoic modal logic is not a logic of modal propositions, e.g. propositions of the type 'It is possible that it is day' or 'It is possibly true that it is day', formed with modal operators which qualify states of affairs, or propositions. Instead, their modal theory was about non-modalized propositions like 'It is day', insofar as they are possible, necessary and so on. (This is well illustrated in the Alexander passage, *APr.* 177-8, quoted above.) The modalities were considered - primarily - as properties of assertibles and, like truth and falsehood, they belonged to the assertibles at a time; consequently an assertible can in principle change its modal-value. Like Philo and Diodorus, Chrysippus distinguished four modal concepts: possibility, impossibility, necessity and non-necessity. Although the concept of contingency (in the sense of that which is both possible and non-necessary) was important for the Stoic debate about determinism, we do not find a special term for it. The discussion of contingent assertibles was usually conducted in terms of two sub-groups, assertibles that are both false and possible and assertibles that are both true and non-necessary.<sup>103</sup>

For the Stoic system of modal notions, the situation with the sources is

<sup>101</sup> At the point when Dio dies, the antecedent assertible turns true. The consequent, instead of turning true as well, is destroyed, and together with it - or so Chrysippus must assume - the whole conditional is destroyed, i.e. ceases to subsist.

<sup>102</sup> See above, pp. 84-6 and pp. 106-8.

<sup>103</sup> Cf. Cic. *Frd.* 13. For the relevance to the debate about determinism see Bobzien 1997a, 75-6.

bad but not hopeless; besides several passages that deal with some of the Stoic modalities,<sup>104</sup> there are two reports of a set of Stoic modal definitions, one in Diogenes Laertius (VII.75) and one in Boethius (*Int.* 2. II. 234.27-235.4 Meiser); although the reports differ in various respects, they in fact present the same account. By adding up all the bits and pieces, and making the plausible assumption that the Stoic modal notions, too, fit the four requirements of normal modal logic<sup>105</sup> it becomes possible to restore as follows the definitions given in Diogenes and Boethius:<sup>106</sup>

A possible assertible is one which (A) is capable of being true and (B) is not hindered by external things from being true;

an impossible assertible is one which (A') is not capable of being true <or (B') is capable of being true, but hindered by external things from being true>;

a necessary assertible is one which (A''), being true, is not capable of being false or (B'') is capable of being false, but hindered by external things from being false;

a non-necessary assertible is one which (A) is capable of being false and (B) is not hindered by external things <from being false>.

We can be confident that this set of modal concepts was Chrysippus'; for we know that Chrysippus' modal concepts were meant to improve on Diodorus' (*Cic. Fat.* 12-14) and in Plutarch (*Stoic. Rep.* 1055d-f); we find remnants of Diogenes' accounts, with identical formulations, ascribed to Chrysippus.

The definitions of possibility and non-necessity are conjunctions; in their case, two conditions (A and B) have to be fulfilled. The definitions of necessity and impossibility, on the other hand, are disjunctions; in their case one of two alternative conditions has to be satisfied (A' or B'); in this way in effect two types of necessity and impossibility are distinguished. Diogenes' example, 'Virtue benefits' (D.L. VII.75), most probably illustrates necessity of the first type; his example 'The earth flies?' (*ibid.*) illustrates impossibility of the first type.

The first parts of all four definitions (A, A'), conjuncts and disjuncts

<sup>104</sup> *Plu. Stoic. Rep.* 1055d-f; *Cic. Fat.* 12-15; *Epicur. Dis.* II.19.1-5, 9; *Alex. Fat.* ch. 10.

<sup>105</sup> See p. 87.

<sup>106</sup> This reconstruction is based on Frede 1974a, 107-14, Bobzien 1986, 45-56. The possibility definition (ὀυδενὸν μὲν <εἶναι δέ(ῶ)ν> τὸ ἐπιβεβηκὸν τοῦ ἀληθῆς εἶναι τῶν ἐκτὸς μὴ ἐνυπάρχουσάντων τοῦτ' ὅτι ἀληθῆς εἶναι) could also be translated as 'A possible assertible is one which is capable of being true, when external things do not prevent its being true' (cf. e.g. Mates 1961, 41); the same holds for the non-necessity definition. However, this interpretation is logically and historically less satisfactory (cf. Bobzien 1986, 40-4, 51-3).

alike, very much resemble Philo's modal definitions;<sup>107</sup> this can hardly be a coincidence. Chrysippus must have chosen Philo's accounts as the basis for his own.

In the case of possibility and non-necessity the second parts (B) add a further condition. These conditions feature 'external things' (τὰ ἐκτὸς) that do not prevent the assertibles from having a certain truth-value. The affirmative counterparts to these conditions (B') specify the second type of Chrysippean necessity and impossibility. Here the external things have to prevent the assertibles from having a certain truth-value. We have no examples of such external things, but ἐκτὸς should refer to something external to the logical subject of the assertible. Things that prevent truth should include ordinary, physical hindrances: for example, a storm or a wall or chains that prevent you from getting somewhere; the surrounding ocean that prevents some wood from burning. It is harder to imagine what counted as external hindrances for something's being false. Presumably they were things that externally forced something to be the case. Locked doors might force Dio to be or remain in a certain room; and hence prevent 'Dio is in this room' from being false. The accounts leave us in the dark about another aspect of the external hindrances, namely at what time or times they are taken as being present (or absent). Knowledge of this is essential for an adequate understanding of the modalities. At first blush one might think that the circumstances are meant to hinder just at the time of utterance of the assertible. But that is unlikely. For it would have the curious effect that, say, the assertible 'Sappho is not reading' is necessary at a time at which someone keeps her from reading (e.g. by temporarily hiding all reading material), but three minutes later, that hindrance being removed, the same assertible would no longer be necessary; and a few minutes later it could be necessary again etc.<sup>108</sup>

The passage in Alexander quoted above (*Alex. APr.* 177-8) suggests that for the possibility of an assertible, the requirement of absence of hindrances is not restricted to the time of its utterance; but rather covers present plus future time - relative to the utterance of the assertion. For we learn that for Chrysippus 'Dio is dead' is possible (now) if it can be true at some time (πότε, 177.29-30); equally, that 'this one is dead (pointing at Dio)', which is impossible, would not be impossible (now) if, although being false now, it could be true at some later time (ὕστερον πότε, 178.1-4).

<sup>107</sup> For Philo's modal accounts see above, p. 86.

<sup>108</sup> Certainly, this would clash with the Stoic assumption that that which is necessary is - in some sense at least - always true (*Alex. Fat.* 177.8-9).

If one reads 'can be true' as short for Chrysippus' requirement 'is capable of being true and not prevented from being true', it seems that an assertible is possible for Chrysippus if (A) it is Philonian possible and (B) there is some time from now on at which it is not hindered from being true. For instance, 'Sappho is reading' is Chrysippean possible, as long as Sappho is not continuously prevented from reading. Correspondingly an assertible falls under the second part of the definiens of the impossible if (B) it is capable of being true, but is from now on prevented from being true - as in the above example, if Sappho were suddenly struck by incurable blindness or died. Chrysippean necessity of the second type (B') would require continuous prevention of falsehood; non-necessity at least temporary absence of such prevention. For example, 'Sappho is reading' is non-necessary as long as she is not continuously externally forced to read.<sup>109</sup>

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So we can see that Chrysippus took a middle position between Philo and Diodorus, combining elements of both modal systems. A comparison between Diodorus and Chrysippus shows that all assertibles that are contingent for Diodorus are contingent for Chrysippus as well: 'It is night' was contingent for Diodorus, since there are present-or-future times at which it is true, and present-or-future times at which it is false. The same assertible is contingent for Chrysippus (Alex. *APr.* 178.5-8), since there are both present-or-future times at which it is not hindered from being true, and times at which it is not hindered from being false. But for Chrysippus, in addition, there are assertibles that neither are nor will ever be true, but are still possible; namely all those that are false but are at some present-or-future time not hindered from being true. So, if Hipparchia never read Plato's *Symposium*, 'Hipparchia reads Plato's *Symposium*' would still not have been Chrysippean impossible; but it would always have been Diodorean impossible.

Contrasting Chrysippus with Philo, fewer assertibles are possible: for instance 'This wood is burning' (namely the piece of wood that is and will be at the bottom of the sea until it decomposes) is Philonian possible; but it is not Chrysippean possible, since there is a lasting circumstance (the sea or its wetness) which prevents the assertible from being true.

<sup>109</sup> Some later Stoics seem to have considered the modalities as merely epistemic (Alex. *Met.* 176.14-14b) according to them, possible is that which as far as we know is not externally prevented from being the case.

Why did Chrysippus add the Philonian requirement to his definitions? The answer should be that there are assertibles that are not in any way hindered by external circumstances from having a certain truth-value, but which Chrysippus nevertheless must have wanted not to be contingent: think of assertibles like 'This triangle has three sides' or 'This square is round'. For such assertibles the first disjuncts of the necessity and impossibility accounts were required.

#### 5: Arguments

The second main part of Stoic logic is their theory of arguments. Arguments ( $\lambda\omicron\gamma\omicron\iota$ ) form another subclass of complete sayables (D.L. VII.63); they are neither thought processes nor beliefs, nor are they linguistic expressions; rather, like assertibles, they are meaningful, incorporeal entities (S.E. *PH* III.52; *M* VII.336). However, they are not themselves assertibles, but compounds of them.

An argument is defined as a compound or system of premisses ( $\alpha\eta\mu\alpha\tau\omicron\varsigma$ ) and a conclusion ( $\epsilon\pi\iota\gamma\omicron\phi\acute{\alpha}$ ,  $\sigma\upsilon\mu\pi\acute{\epsilon}\rho\omicron\sigma\iota\mu\alpha$ ) (D.L. VII.45). Premisses and conclusion, in turn, are self-complete sayables, standardly assertibles, which I shall call the 'component assertibles' of the argument. The following is a typical Stoic argument:

- P<sub>1</sub> If it is day, it is light.  
 P<sub>2</sub> But it is day.  
 C Therefore, it is light.

It has a non-simple assertible (P<sub>1</sub>) as one premiss and a simple assertible (P<sub>2</sub>) as the other. The non-simple premiss, usually put first, was referred to as the 'leading premiss' ( $\eta\gamma\epsilon\iota\omicron\nu\upsilon\kappa\omicron\nu$   $\lambda\eta\eta\mu\alpha$ ). The second or the last premiss was called the 'co-assumption' ( $\pi\rho\acute{o}\sigma\lambda\eta\eta\mu\iota\varsigma$ ). It is usually simple; when it is non-simple, it contains fewer constituent assertibles than the leading premiss. The co-assumption was introduced by 'but' ( $\delta\acute{\epsilon}$ ) or 'now' ( $\delta\alpha\lambda\lambda\acute{\alpha}$   $\mu\eta\nu$ ), and the conclusion by 'therefore' ( $\acute{\alpha}\rho\alpha$ ).

All accounts of 'argument' have in common that they talk about a plurality of premisses - and indeed, it was the orthodox Stoic view that an argument must have more than one premiss.<sup>110</sup> We are not told why. Thus, for the Stoics, compounds of assertibles of the kind

- $p$ ; therefore  $p$   
 $p$  and  $q$ ; therefore  $p$   
 $p$ ; therefore either  $p$  or  $q$

<sup>110</sup> The exception is Antipater who admitted single premiss arguments; see below, p. 155.

are not arguments. On the other hand,

If  $p, p; p$ ; therefore  $p$

counts as an argument – and as valid at that.

A passage in Sextus defines 'premises' and 'conclusion': the premises of an argument are the assertibles that are adopted by agreement for the establishing of the conclusion; the conclusion is the assertible that is established by the premises (S.E. *M* VII.302; cf. *PH* II.136).<sup>111</sup> 'Premises' and 'conclusion' are thus determined as relative terms that depend on each other. The account of 'premises' illustrates clearly that for the Stoics the theory of argument is still embedded in the dialectical practice of conducting arguments by question and answer.

A difficulty with this account is that it seems to imply that something only counts as an argument if the premises – at the very least – appear true to the discussants. This apparently rules out arguments with evidently false premises and with premises the truth of which is not or not yet known. In this way a whole range of arguments seems to be precluded from being recognized as such by the Stoics: indirect proof, theories grounded on hypotheses, 'thought experiments', arguments concerning future courses of actions etc.

Perhaps not all Stoics shared this account of 'premises'. It is also possible that difficulties like the above gave rise to the development of the Stoic device of supposition or hypothesis (ὑπόθεσις)<sup>112</sup> and hypothetical arguments (ἀόγιοι ὑποθετικοί): the Stoics thought that occasionally 'it is necessary to postulate some hypothesis as a sort of stepping-stone for the subsequent argument' (Epiet. *Diss.* 1.7.22 tr. Oldfather).<sup>113</sup> Thus, one or more premises of an argument could be such a hypothesis in lieu of an assertible; and it seems that hypothetical arguments were arguments with such hypotheses among their premises.<sup>114</sup> Hypotheses as premises apparently were phrased as 'Suppose it is night' (ἔστω νύξ) instead of 'It is night', by which an assertible is expressed (Epiet. *Diss.* 1.25.11–13;

<sup>111</sup> 'Established' (κατασκευασμένον) should not mean 'validly derived' here, since that would exclude the existence of invalid arguments.

<sup>112</sup> Hypothesis is one of the kinds of self-complete sayables on which see below, pp. 202–3.

<sup>113</sup> Chrysippus wrote a considerable number of books on hypotheses and hypothetical arguments (D.L. VII.196; 197; cf. D.L. VII.66). The placement of the book-titles after those about changing arguments, which Epictetus repeatedly mentions together with the hypothetical arguments, as well as the book-titles on hypothesis and exposition (ἐκθεσις) in the same section render it likely that Chrysippus' hypothetical arguments were those Epictetus talks about. Cf. also Bobzien 1997b.

<sup>114</sup> The range of examples for Stoic hypotheses fits well the above-mentioned types of arguments in which assertibles would not do as premises; e.g. 'Suppose that the earth is the centre of the solar sphere' (Ammon. *Int.* 2.31–2); 'Suppose it is night' (while it is day) (Epiet. *Diss.* 1.25.11–13).

Ammon. *Int.* 2.31–2). These premises could be agreed upon *quia* hypotheses; that is, the interlocutors agree – as it were – to enter a non-actual 'world' built on the respective assumption, but they remain aware of the fact that this assumption and any conclusions drawn hold only relative to the fact that this assumption has been made.<sup>115</sup>

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The most important distinction of arguments is that between valid and invalid ones. The Stoic general criterion was that an argument is valid (συνωκτικὸς, τεσπρωτικὸς) if the corresponding conditional formed with the conjunction of the premises as antecedent and the conclusion as consequent is correct (S.E. *PH* II.137; cf. S.E. *M* VIII.415; *PH* II.249). If the assertible 'If (both  $P_1$  and ... and  $P_n$ ), then  $C$ ' is true, then the argument ' $P_1; \dots; P_n$ ; therefore  $C$ ' is valid. Diogenes Laertius' report of the criterion for invalidity of arguments (D.L. VII.77) implies that the criterion for the correctness of the conditional was the Chrysippean one: an argument is valid provided that the contradictory of the conclusion is incompatible with the conjunction of the premises. Thus there is some similarity between the Stoic concept of validity and our modern one. But it must be kept in mind that the conditional has to be true according to Chrysippus' criterion, which as we have seen, is not necessarily restricted to logical consequence.<sup>116</sup> This brings out a shortcoming of the Stoic concept of validity: for what is needed is precisely logical consequence. It is thus unfortunate to have the same concept of consequence for both the relation between antecedent and consequent in a conditional, and the relation between premises and conclusion.<sup>117</sup> In any event, the concept of conflict is too vague to serve as a proper criterion for validity.

Perhaps the Stoic classification of invalid arguments may shed some further light on their general concept of validity. Sextus tells us about some Stoics who distinguished four ways in which arguments could be invalid (*M* VIII.429–434; *PH* II.146–51): first, in virtue of disconnectedness (σδιόρησις), that is, when the premises lack communality or connectedness with one another and with the conclusion, as in

<sup>115</sup> Cf. Epiet. *Diss.* 1.25.11–13; on Stoic hypotheses and hypothetical arguments in general see Bobzien 1997b.

<sup>116</sup> See above, p. 106. Note also that the Stoic validity criterion differs in both content and function from the modern principle of conditionalization. For the Stoics, the truth of the conditional is a criterion for the validity of the argument; not vice versa; moreover, the conditional must be constructed from a Stoic argument, which implies that its antecedent must be a conjunction.

<sup>117</sup> One result of this is that true arguments in *modus ponens* inevitably turn out redundant. See S.E. *M* VIII.441–2 and Barnes 1980, 173–5.



If it is day, it is light.  
Now wheat is being sold in the market.  
Therefore it is light.

Secondly, in virtue of surplus or redundancy (τροποικη), that is, when something is added extrinsically and superfluously, as 'virtue benefits' in the following argument:

If it is day, it is light.  
Now it is day.  
And also virtue benefits.  
Therefore it is light.

Thirdly in virtue of being propounded in an incorrect (μοχθηρός) form, as for example, in

If it is day, it is light.  
Now it is light.  
Therefore it is day.

Finally in virtue of omission or deficiency (ἐλλειψις) as in

Either wealth is good or wealth is bad.  
But wealth is not bad.  
Therefore wealth is good.

Here, what is claimed to have been omitted is the disjunct '(or) wealth is indifferent' in the leading premiss, and accordingly the negated conjunct '(and) neither is wealth good' in the co-assumption, such that the proper conclusion would have been 'Therefore wealth is indifferent'.

This fourfold distinction is unsatisfactory from the point of view of modern logic: the examples of redundancy and of omission seem to be perfectly valid;<sup>118</sup> the example of disconnectedness seems to be nothing but a special case of invalidity due to an incorrect form (and so would be examples of omission, say, of a whole premiss). This makes the Stoic authors look rather bad logicians. We could reprove them and leave it at that. Alternatively, if we acknowledge that Hellenistic theory of argument developed out of the practice of dialectical debate, and is still entrenched in that context (recall the account of premisses and conclusion in Sextus), we can at least get an idea of what those Stoics were after. First, one may notice that Sextus reports that 'invalid arguments come

about in four ways' rather than 'they distinguished four kinds of invalid arguments',<sup>119</sup> and they come about 'in virtue of' (κατὰ) disconnectedness etc., which might hence be external to them. So we should expect what follows not to be entirely a matter of formal logic. Indeed, all four ways in which invalid arguments come about seem to be connected with the *intention* of the proponent of the argument. The four ways make most sense if one understands them as four ways of criticizing an argument by indicating how to *mend* it such that the argument that is intended or appropriate in the particular discourse comes out right. We have to assume that in the cases of redundancy, omission and disconnectedness the proponents do not get the form wrong; rather, they envisage the right form, but add something superfluous, leave something out, or put in the wrong assertible or assertibles 'in that form', as it were. Whereas in the case of the incorrect form, leaving out, adding, or replacing something does not help, since the proponent envisages the wrong form and would justify the argument by referring to the validity of arguments of that form: in this case the proponent would have to understand that the form is not correct.

How does Chrysippus' notion of validity square with this conception of invalidity? Tested against his general criterion of validity, incorrect form, disconnectedness and omission (of a straightforward case – one would hope he did not accept the example in Sextus) would turn out as invalid, too. But what about redundancy? One can imagine why redundancy was seen as an obstruction to validity. It is not only that, if one proounds an argument and adds irrelevant premisses, it might obfuscate the deductive structure of the inference; also, one might claim that the conclusion does not in any true sense follow *from* the irrelevant premisses. We know that Chrysippus wrote two books about redundancy; they are listed in the context of his works on syllogistic (D.L. VII. 195). But when we look at his validity criterion, certainly at first sight it would not outlaw redundancy: if a conjunction of assertibles ( $P_1, P_2, \dots, P_n$ ) conflicts with another assertible (not-C), then it will certainly also conflict with it when any further conjunct whatsoever is added. This, however, might not in fact be so, if Chrysippus' concept of consequence resembled the – implicit – concept of conflict we find in Alex. *Top.* 93.9–10.<sup>120</sup> For if 'conflict' means that  $P_1$  and  $P_2$  and  $\dots, P_n$  conflict with not-C since, *because*  $P_1$  and  $P_2$  and  $\dots, P_n$ , not-C fails to hold; the addition of a further conjunct might cancel the

<sup>118</sup> The fourth, illustrating omission, appears to confound the truth of the leading premiss (and the way the proponent got it wrong) with the validity of the argument.

<sup>119</sup> *M. VII. 429*; 'to come about' (γίνεσθαι) recurs three times, and equally in the *PH* passage.

<sup>120</sup> See above, p. 107.