

4 Stoic Logic¹

Stoic logic is in its core a propositional logic. Stoic inference concerns the relations between items that have the structure of propositions. These items are the assertibles (*axiômata*). They are the primary bearers of truth-values. Accordingly, Stoic logic falls into two main parts: the theory of arguments and the theory of assertibles, which are the components from which the arguments are built.

I. SAYABLES AND ASSERTIBLES

What is an assertible? According to the Stoic standard definition, it is

a self-complete sayable that can be stated as far as itself is concerned (S. E. *PH* II 104).

This definition places the assertible in the genus of self-complete sayables, and so everything that holds in general for sayables and for self-complete sayables holds equally for assertibles. Sayables (*lekta*) are items placed between mere vocal sounds on the one hand and the world on the other. They are, very roughly, meanings: ‘what we say are things, which in fact are sayables’ (DL VII 57). Sayables are the underlying meanings in everything we say or think; they underlie

¹ This chapter is a modified and much shortened version of Bobzien (1999b), where more details and more textual evidence on all the topics treated here can be found, accessible for readers without Greek or Latin. Other useful and fairly comprehensive treatments of Stoic logic are Frede (1974) and Mates (1953) (although the latter is outdated in part). Still worth reading are also Kneale and Kneale (1962), Ch. 3. The surviving textual evidence on Stoic logic is collected in FDS. There are two collections of articles: Brunschwig (1978) and Döring and Ebert (1993).

any rational presentation we have (S. E. *M* VIII 70). But they generally also subsist when no one actually says or thinks them.² The Stoics hold further that

of sayables some are self-complete (*autotelê*), others deficient (*ellipê*). Deficient are those which have an unfinished expression, e.g.: 'writes', for we ask: who? Self-complete are those which have a finished expression, e.g.: 'Socrates writes' (DL VII 63).

Self-complete sayables include assertibles, questions, inquiries, imperatives, oaths, invocations, assertible-likes, puzzlements, curses, and hypotheses (DL VII 65–8). Of these, besides the assertibles, only the hypotheses and imperatives seem to have been considered in the context of logic in the narrow sense; that is, the logic of inference.³

What marks off assertibles from other self-complete sayables is that (i) they can be stated (ii) as far as they themselves are concerned. Assertibles can be stated, but they are not themselves statements. They subsist independently of their being stated, in a similar way in which sayables in general subsist independently of their being said. This notwithstanding, it is the characteristic primary function of assertibles to be stated. On the one hand, they are the only entities we can use for making statements: no statements without assertibles; on the other, assertibles have no other primary function than their being stated. A second account determines an assertible as

that by saying which we make a statement (DL VII 66).

'Saying' here signifies the primary function of the assertible: one cannot genuinely say an assertible without stating it. To say an assertible is more than just to utter a sentence that expresses it. For instance, 'If Dio walks, Dio moves' is a complex assertible, more precisely a conditional, that is composed of two simple assertibles, 'Dio walks' and 'Dio moves'. Now, when I utter the sentence, 'If Dio walks, Dio moves', I make use of all three assertibles. However, the only one I actually assert is the conditional, and the only thing I genuinely say is that if Dio walks, Dio moves.

² Cf. Barnes (1993), (1999), M. Frede (1994a), Schubert (1994). For an alternative view, see LS.

³ Cf. Barnes (1986) on Stoic logic of imperatives and Bobzien (1997) on Stoic logic of hypotheses.

Thus understood, phrase (i) of the definition ('can be stated') suffices to delimit assertibles from the other kinds of self-complete sayables. What is the function of phrase (ii) 'as far as itself is concerned'? It isn't meant to narrow down the class of assertibles further, but to preempt a misinterpretation: the locution 'can be asserted' could have been understood as potentially excluding some items which for the Stoics were assertibles. For two things are needed for stating an assertible: first, the assertible itself, and second, someone to state it. According to Stoic doctrine, that someone would need to have a rational presentation in accordance with which the assertible subsists. But many assertibles subsist without anyone having a corresponding presentation. In such cases, one of the necessary conditions for the 'stability' of an assertible is unfulfilled. Here the qualification 'as far as the assertible itself is concerned' comes in. It cuts out this external condition. For something's being an assertible it is irrelevant whether there actually is someone who could state it.

There is a further Stoic account of 'assertible'; it suggests that their 'stability' was associated with their having a truth-value:

an assertible is that which is either true or false (DL VII 65).

Thus truth and falsehood are properties of assertibles, and being true or false – in a nonderivative sense – is both a necessary and a sufficient condition for something's being an assertible. Moreover, we can assume that one can only state something that has a truth-value.

Assertibles resemble Fregean propositions in various respects. There are, however, important differences. The most far-reaching one is that truth and falsehood are temporal properties of assertibles. They can belong to an assertible at one time but not at another. This is exemplified by the way in which the truth-conditions are given: the assertible 'It is day' is true when it is day (DL VII 65). Thus, when the Stoics say, "'Dio walks" is true', we have to understand '... is true now', and that it makes sense to ask: 'Will it still be true later?' For the assertible now concerns Dio's walking now; but uttered tomorrow, it will concern Dio's walking tomorrow, and so on. This 'temporality' of (the truth-values of) assertibles has a number of consequences for Stoic logic. In particular, assertibles can in principle change their truth-value: the assertible 'It is day' is true now, false

later, and true again tomorrow. The Stoics called assertibles that (can) change their truth-value 'changing assertibles' (*metapiptonta*). Most Stoic examples belong to this kind.

2. SIMPLE ASSERTIBLES

The most fundamental distinction among assertibles (analogous to the modern one between atomic and molecular propositions) was that between simple and non-simple ones. Non-simple assertibles are composed of more than one assertible (see Section 3). Simple assertibles are defined negatively as those assertibles which are not non-simple. There were various kinds of simple and non-simple assertibles. We are nowhere told the ultimate criteria for the distinctions. But we should remember that the Stoics weren't after giving a grammatical classification of sentences. Rather, the classification is of assertibles, and the criteria for their types are at heart logical. This leads to the following complication: The only access there is to assertibles is via language; but there is no one-to-one correspondence between assertibles and declarative sentences. One and the same sentence (of a certain type) may express self-complete sayables that belong to different classes. Equally, two sentences of different grammatical structure may express the same assertible. How then can we know which assertible a sentence expresses? Here the Stoics seem to have proceeded as follows: Aiming at the elimination of (structural) ambiguities, they embarked upon a programme of regimentation of language such that the form of a sentence would unambiguously determine the type of assertible expressed by it. The advantage of such a procedure is that once one has agreed to stick to certain standardizations of language use, it becomes possible to discern logical properties of assertibles and their compounds by examining the linguistic expressions used.

Now to the various types of simple assertibles.⁴ Our sources provide us (i) with three affirmative types: predicative or middle ones, catagoreutical or definite ones, and indefinite ones; and (ii) with three negative types: negations, denials, and privations (DL VII 69–70, S. E. M VIII 96–100). Each time the first word of the sentence indicates to what type a simple assertible belongs.

⁴ Cf. also Ebert (1993), Brunschwig (1994).

Examples of the predicative (*katêgorika*) or middle assertibles are of two kinds: 'Socrates sits' and '(A) man walks'. They are defined as assertibles that consist of a nominative 'case', like 'Dio', and a predicate, like 'walks' (DL VII 70). The name 'middle' is based on the fact that these assertibles are neither indefinite (they define their object) nor definite (they are not deictic) (S. E. *M* VII 97). Assertibles of the type '(A) man walks' are extremely rare in Stoic logic.

The definite (*hôrismena*) or catagoreutical (*katagoreutika*) assertibles have in their standard linguistic form a demonstrative pronoun as subject expression.⁵ A typical example is 'This one walks'. They are defined as assertibles uttered along with *deixis* (S. E. *M* VIII 96). What do the Stoics mean by '*deixis*'? In one place, Chrysippus talks about the *deixis* with which we accompany our saying 'I', which can be either a pointing at the object of *deixis* (ourselves in this case) or a gesture with one's head in its direction (Galen *PHP* II 2.9–11). So ordinary *deixis* seems to be a non-verbal, physical act of indicating something, simultaneous with the utterance of the sentence with the pronoun.

How are definite assertibles individuated? The sentence (type) by which a definite assertible is expressed does clearly not suffice for its identification: Someone who utters the sentence 'This one walks' pointing at Theo expresses a different assertible from the one they would assert pointing at Dio. However, when I now utter 'This one walks', pointing at Dio, and then utter the same sentence again tomorrow, again pointing at Dio, the Stoics regarded these as two statements of the same assertible. Thus, one way to understand the individuation of definite assertibles is to conceive of a distinction between, as it were, *deixis* type and *deixis* token: a *deixis* type is determined by the object of the *deixis* (and is independent of who performs an act of *deixis* when and where): same object, same *deixis*. By contrast, *deixis* tokens are the particular utterances of 'this one' accompanied by the physical acts of pointing at the object. Hence, there is one assertible 'This one walks' for Theo (with the *deixis* type pointing-at-Theo), one for Dio (with the *deixis* type pointing-at-Dio), and so forth.

But how then does a definite assertible differ from the corresponding predicative one – for example, 'This one walks' (pointing at Dio)

⁵ On definite assertibles, see also Denyer (1988).

from 'Dio walks'? Are they not rather two ways of expressing the same assertible? Not for the Stoics. We know from a passage on Chrysippus' modal theory that in the case of the assertibles, 'Dio is dead' and 'This one is dead' (pointing at Dio) uttered at the same time one could be true, the other not (Alex. *In Ar. An. pr.* 177.25–178.4). For the latter assertible is said to be false while Dio is alive but destroyed once Dio is dead, whereas the former simply changes its truth-value from false to true at the moment of Dio's death. The reason given for the destruction of the definite assertible is that once Dio is dead the object of the *deixis*, Dio, no longer exists. Now, for an assertible destruction can only mean that it ceases to subsist, and hence no longer satisfies all the conditions for being an assertible. And this should have something to do with the *deixis*. So perhaps in the case of definite assertibles, statability becomes in part point-at-ability, and Stoic point-at-ability requires intrinsically the existence of the object pointed at. This is not only a condition of actual statability in particular situations – as is the presence of an asserter; rather, it is a condition of identifiability of the assertible, of its being this assertible.

The indefinite (*aorista*) assertibles are defined as assertibles that are governed by an indefinite particle (S. E. *M VIII* 97).⁶ They are composed of one or more indefinite particles and a predicate (DL VII 70). Such particles are 'someone' or 'something'. An example is 'Someone sits'. This assertible is said to be true when a corresponding definite assertible ('This one sits') is true, since if no particular person is sitting, it isn't the case that someone is sitting (S. E. *M VIII* 98).

The most important kind of negative assertible is the negation (*apophatikon*). For the Stoics, a negation is formed by prefixing to an assertible the negation particle 'not:', as for instance in 'Not: Diotima walks'. In this way an ambiguity is avoided regarding existential import in ordinary language formulations, such as 'Diotima doesn't walk': 'Diotima doesn't walk' counts as an affirmation, which – unlike 'Not: Diotima walks' – presupposes for its truth Diotima's existence (Apul. *De int.* 177.22–31, Alex. *In Ar. An. pr.* 402.8–12).⁷ Stoic negation is truth-functional: the negation particle, if added to true

⁶ On indefinite assertibles, see also Crivelli (1994).

⁷ Cf. A. C. Lloyd (1978a).

assertibles, makes them false; if added to false ones makes them true (S. E. *M* VIII 103). Every negation is the negation of an assertible; namely, of the assertible from which it has been constructed by prefixing 'not:'. Thus 'Not: it is day' is the negation of 'It is day'. An assertible and its negation form a pair of contradictories (*antikeimena*):

Contradictories are those (assertibles) of which the one exceeds the other by a negation particle, such as 'It is day' – 'Not: it is day'. (S. E. *M* VIII 89)

This implies that an assertible is the contradictory of another if it is one of a pair of assertibles in which one is the negation of the other (cf. DL VII 73). Of contradictory assertibles, precisely one is true and the other false.

The Stoics also prefixed the negation particle to non-simple assertibles in order to form complex negations. The negation of a simple assertible is itself simple; that of a non-simple assertible non-simple. Thus, the addition of the negative doesn't make a simple assertible non-simple. The negation particle 'not:' isn't a Stoic connective (*syndesmos*), for such connectives bind together parts of speech and the negation particle doesn't do that.

A special case of the negation is the so-called super-negation (*hyperapophatikon*) or, as we would say, 'double negation'. This is the negation of a negation, for instance, 'Not: not: it is day'; it is still a simple assertible. Its truth-conditions are the same as those for 'It is day' (DL VII 69).

The second type of negative assertible, the denial (*arnêtikon*), consists of a denying particle and a predicate. An example is 'No-one walks' (DL VII 70). This type of assertible has a compound negative as subject term. Unlike the negation particle, this negative can form a complete assertible if combined with a predicate. The truth-conditions of denials have not been handed down, but they seem obvious: 'No-one φ 's' should be true precisely if it isn't the case that someone φ 's. Denials must have been the contradictories of simple indefinite assertibles of the kind 'Someone φ 's'. Finally, the privative (*sterêtikon*) assertible is determined as a simple assertible composed of a privative particle and a potential assertible, like 'This one is unkind' (DL VII 70, literally 'Unkind is this one', a word order presumably chosen to have the negative element at the front of the sentence). The privative particle is the alpha privativum ' α ' ('un-').

3. NON-SIMPLE ASSERTIBLES

Non-simple assertibles are those that are composed of more than one assertible or of one assertible taken twice (DL VII 68–9) or more often. These constituent assertibles are combined by one or more propositional connectives. A connective is an indeclinable part of speech that connects parts of speech (DL VII 58). An example of the first type of non-simple assertibles is ‘Either it is day, or it is night’; one of the second type is ‘If it is day, it is day.’

Concerning the identification of non-simple assertibles of a particular kind, the Stoics took what one may call a ‘formalistic’ approach. In their definitions of the different kinds of non-simple assertibles they mention the characteristic propositional connectives, which can have one or more parts, and determine their position in (the sentence that expresses) the non-simple assertibles. The place of the connectives relative to (the sentences expressing) the constituent assertibles is strictly regulated in such a way that the first word of the assertible is indicative of the type of non-simple assertible it belongs to, and – mostly – the scope of the connectives is disambiguated.

Non-simple assertibles can be composed of more than two simple constituent assertibles (Plut. *St. rep.* 1047c–e). This is possible in two ways. The first has a parallel in modern logic: the definition of the non-simple assertible allows that its constituent assertibles are themselves non-simple. An example of such an assertible is ‘If both it is day and the sun is above the earth, it is light.’ The type of non-simple assertible to which such a complex assertible belongs is determined by the overall form of the assertible. Thus the above example is a conditional. The second type of assertible with more than two constituent assertibles is quite different. Conjunctive and disjunctive connectives were conceived of not as two-place functors, but – in line with ordinary language – as two-or-more-place functors. So we find disjunctions with three disjuncts: ‘Either wealth is good or (wealth) is evil or (wealth is) indifferent’ (S. E. *M* VIII 434).

All non-simple assertibles have their connective, or one part of it, prefixed to the first constituent assertible. As in the case of the negation, the primary ground for this must have been to avoid ambiguity. Consider the statement

p and q or r.

In Stoic 'regimented' formulation, this becomes either

Both p and either q or r.

or

Either both p and q or r.

The ambiguity of the original statement is thus removed. Moreover, like Polish notation, the Stoic method of prefixing connectives can in general perform the function that brackets have in modern logic. Avoidance of ambiguity may also have been behind the Stoic practice of eliminating cross-references in non-simple assertibles. Thus, where ordinary discourse has 'If Plato walks, he moves', the Stoics repeated the subject term: '... Plato moves'.

The truth-conditions for non-simple assertibles suggest that the Stoics weren't aiming at fully covering the connotations of the connective particles in ordinary language. Rather, it seems, the Stoics attempted to filter out the essential formal characteristics of the connectives. Leaving aside the negation – which can be simple – only one type of non-simple assertible, the conjunction, is truth-functional. In the remaining cases, modal relations (like incompatibility), partial truth-functionality, and basic relations like symmetry and asymmetry, in various combinations, serve as truth-criteria.

For Chrysippus we know of only three types of non-simple assertibles: conditionals, conjunctions, and exclusive-cum-exhaustive disjunctive assertibles. Later Stoics added further kinds of non-simple assertibles: a pseudo-conditional and a causal assertible, two types of pseudo-disjunctions, and two types of comparative assertibles. Possibly, the main reason for adding these was logical, in the sense that they would allow the formulation of valid inferences which Chrysippus' system couldn't accommodate. A certain grammatical interest may also have entered in.

The conjunction (*sumpeplegmenon*, *sumplokê*) was defined as 'an assertible that is conjoined by certain conjunctive connective particles; for example, 'Both it is day and it is light'' (DL VII 72). Like modern conjunction, the Stoic one connects whole assertibles: it is 'Both Plato walks and Plato talks', not 'Plato walks and talks'. Unlike modern conjunction, the conjunctive assertible is defined in such a way that more than two conjuncts can be put together on a par (cf. Gellius XVI 8.10). The standard form has a two-or-more part

connective: ‘both...and...and... ..’. The truth-conditions, too, are formulated in such a way as to include conjunctions with two or more conjuncts: a Stoic conjunction is true when all its constituent assertibles are true, and otherwise false (S. E. *M* VIII 125, 128); it is thus truth-functional.

The conditional (*sunêmmenon*) was defined as the assertible that is formed with the linking connective ‘if’ (DL VII 71). Its standardized form is ‘If p, q’. In Chrysippus’ time, the debate about the truth-conditions of the conditional – which had been initiated by the logicians Philo and Diodorus – was still going on.⁸ There was agreement that a conditional ‘announces’ a relation of consequence; namely, that its consequent follows (from) its antecedent (*ibid.*). Under debate were what it is to ‘follow’ and the associated truth-conditions. A minimal consensus seems to have been this: the ‘announcement’ of following suggests that a true conditional, if its antecedent is true, has a true consequent. Given the acceptance of the principle of bivalence, this amounts to the minimal requirement for the truth of a conditional that it must not be the case that the antecedent is true and the consequent false – a requirement we find also explicitly in our sources (DL VII 81). It is equivalent to Philo’s criterion.

Chrysippus offered a truth-criterion that differed from Philo’s and Diodorus’ (Cic. *Acad.* II 143, DL VII 73, Cic. *Fat.* 12). It was also described as the criterion of those who introduce a connection (*sunartêsis*) (S. E. *PH* II 111); this connection can only be that which holds between the antecedent and the consequent. The requirement of some such connection must have been introduced to avoid the ‘paradoxes’ that arose from Philo’s and Diodorus’ positions. In the truth-criterion itself, the connection in question is determined indirectly, based on the notion of conflict or incompatibility (*machê*): a conditional is true precisely if its antecedent and the contradictory of its consequent conflict (DL VII 73). Consequently, the example ‘If the earth flies, Axiothea philosophises’ – which would be true for both Philo and Diodorus – is no longer true. It is perfectly possible that both the earth flies and Axiothea doesn’t philosophise. For a full understanding of Chrysippus’ criterion, we need to know what sort of conflict he had in mind. But here our sources offer little

⁸ For Philo’s and Diodorus’ logic, see Bobzien (1999b).

information. Some later texts state that two assertibles conflict if they cannot be true together. This confirms that the conflict is some sort of incompatibility.

It is historically inappropriate to ask whether Chrysippus intended empirical, analytical, or formal logical conflict, given that a conceptual framework which could accommodate such distinctions is absent in Hellenistic logic. Still, we can be confident that what we may call formal incompatibility would have counted as conflict for Chrysippus: Assertibles like 'If it is light, it is light' were regarded as true (Cic. *Acad.* II 98) – presumably because contradictoriness was the strongest possible conflict between two assertibles. Equally, some cases that some may describe as analytical incompatibility were covered: for instance 'If Plato walks, Plato moves' was regarded as true. And it seems that some instances of cases of what we might label 'empirical incompatibility' were accepted by some Stoics: so conditionals with causal connections of the kind 'If Theognis has a wound in the heart, Theognis will die' were probably considered true (S. E. *M* VIII 254–5). On the other hand, the connection expressed in divinatory theorems ('If you are born under the Dog-star, you won't die at sea') seems to have been an exception. Chrysippus denied that such theorems would make true conditionals, but held that they would make true (indefinite) negations of conjunctions with a negated second conjunct (Cic. *Fat.* I I–I5).⁹

Some Stoics introduced two further kinds of non-simple assertibles, grounded on the concept of the conditional (DL VII 71–4). Both were probably added only after Chrysippus. The first, called 'pseudo-conditional' (*parasunêmmenon*), is testified at the earliest for Crinis and has the standardized form 'Since p, q'. The truth-criterion for such assertibles is that (i) the 'consequent' must follow (from) the 'antecedent', and (ii) the 'antecedent' must be true. The second kind is entitled 'causal assertible' (*aitiôdes*) and has the standard form 'Because p, q'. The name is explained by the remark that p is, as it were, the cause/ground (*aition*) of q. The truth-condition for the causal assertible adds simply a further condition to those for the pseudo-conditional; namely (iii), that if p is the ground/cause for q, q cannot be the ground/cause for p, which in particular implies that 'Because p, p' is false.

⁹ Cf. Bobzien (1998), Ch. 4.2.

The Greek word for 'or' ($\epsilon\acute{\nu}$) has several different functions as a connective particle, which are distinct in other languages. It covers both the Latin *aut* and the Latin *vel*, and also both the English 'or' and the English 'than'. It plays a role as a connective in at least three different types of non-simple assertibles.

The early Stoics seem to have concentrated on one type of disjunctive relation only: the exhaustive and exclusive disjunctive relation, called '*diezeugmenon*', here rendered 'disjunction'. This is the only disjunctive that figures in Chrysippus' syllogistic. It is defined as 'an assertible that is disjoined by the disjunctive connective "either", like "Either it is day or it is night"' (DL VII 72). The disjunctive connective could take more than two disjuncts, and there are examples of such disjunctions (S. E. *PH* I 69). Thus, the connective was 'either... or... or... . . . ' with its first part ('either') prefixed to the first disjunct. One source presents the truth-conditions for disjunctions as follows:

... (i) all the disjuncts must be in conflict with each other and (ii) their contradictories... must be contrary to each other. (iii) Of all the disjuncts one must be true, the remaining ones false. (Gellius XVI 8.13)

Here, first a non-truth-functional criterion is given ((i) and (ii)); this is followed by a truth-functional criterion (iii). I take (iii) to be an uncontested minimal requirement as we had it in the case of the conditional. For it certainly was a necessary condition for the truth of a disjunction that precisely one of its disjuncts had to be true, but most sources imply that this was not sufficient. The truth-condition they state is stricter and typically involves the term 'conflict' already familiar from the conditionals. It is a conjunction of the two conditions (i) and (ii). First, the disjuncts must conflict with each other; this entails that, at most, one is true. Second, the contradictories of the disjuncts must all be contrary to each other; this ensures that not all of the contradictories are true, and hence that at least one of the original disjuncts is true. The two conditions combined mean that 'necessarily precisely one of the disjuncts must be true'. As in the case of the conditional, a full understanding of the truth-criterion would require one to know what kind of conflict the Stoics had in mind.

Some Stoics distinguished two kinds of a so-called pseudo-disjunction (*paradiezeugmenon*) (Gellius XVI 8.13–14). Regarding

their standard form, most examples are formed with ‘either . . . or . . .’ or, occasionally, just with ‘. . . or . . .’; some have more than two pseudo-disjuncts. Thus, the two types of pseudo-disjunctions seem indistinguishable in their linguistic form from disjunctions (and from each other). Their truth-criteria are simply the two halves of the truth-condition for the genuine disjunction. One kind is true if its pseudo-disjuncts conflict with each other, which entails that, at most, one of them is true. The other is true if the contradictories of its pseudo-disjuncts are contrary to each other, which entails that at least one of the pseudo-disjuncts is true.

As mentioned previously, the Greek word for ‘or’ serves another purpose: that of the English word ‘than’. Accordingly, we sometimes find a further kind of non-simple assertible discussed in the context of the disjunctives, the comparative assertible, formed by using a comparative (*diasaphêtikos*) connective.¹⁰ Two types are known (DL VII 72–73), with the connectives ‘It’s rather that . . . than that . . .’ and ‘It’s less than . . . than that . . .’. These are two-part connectives, again with the characteristic part prefixed to the first constituent assertible, thus allowing the identification of the type of assertible. The truth-conditions have not survived.

The definition of the non-simple assertibles implies that they take any kind of simple assertibles as constituents, and that by combining connectives and simple assertibles in a correct, ‘well-formed’ way, all Stoic non-simple assertibles can be generated. But apparently this isn’t so: non-simple assertibles that are composed of simple indefinite ones raise special problems. Unlike the case of definite and middle assertibles, one can conceive of two different ways of linking indefinite ones.

First, following Stoic formation rules to the letter, by combining two simple indefinite assertibles into a conjunction or a conditional, one obtains assertibles like the following:

If someone breathes, someone is alive.
Both someone walks and someone talks.

According to Stoic criteria these would be true, respectively, if ‘Someone is breathing’ and ‘Not: someone is alive’ are incompatible and if ‘Someone (e.g., Diotima) walks’ is true and ‘Someone (e.g.,

¹⁰ Cf. Sluiter (1988).

Theognis) talks' is true. However, complex assertibles with indefinite pronouns as grammatical subject more commonly tend to be of the following kind:

If someone breathes, that one (he, she) is alive.
Someone walks and that one talks.

Here the truth-conditions are different, since the second 'constituent assertible' isn't independent of the first. In fact, we find no Stoic examples of the first type of combinations of indefinite assertibles but quite a few of the second (e.g., DL VII 75; 82). It was explicitly dealt with by the Stoics and it seems that the terms 'indefinite conjunction' and 'indefinite conditional' were reserved for it. In order to express the cross-reference in the second 'constituent assertible' to the indefinite particle of the first, 'that one' (*ekeinos*) was standardly used.

The Stoics were right to single out these types of assertibles as a special category. Plainly, the general problem they are confronted with is that of quantification. The modern way of wording and formalizing such statements, which brings out the fact that their grammatical subject expressions do not have a reference ('For anything, if it is F, it is G') didn't occur to the Stoics. We do not know how far they 'understood' such quantification as lying behind their standard formulation; but we know that they suggested that sentences of the kind 'All S are P' be reformulated as 'If something is S, that thing is P' (S. E. M XI 8–9).

The Stoic accounts of assertibles reveal many similarities to modern propositional logic, and there can be little doubt that the Stoics attempted to systematize their logic. However, their system is quite different from the propositional calculus. In particular, Stoic logic is a logic of the validity of arguments, not a system of logical theorems or logical truths. Of course, the Stoics did recognise some logical principles which correspond to theorems of the propositional calculus. But, although they had a clear notion of the difference between meta- and object language, logical principles that express logical truths were apparently not assigned a special status, different from logical meta-principles. A survey of the principles concerning assertibles may be useful. First, there is the principle of bivalence (Cic. *Fat.* 20), which is a logical meta-principle. Then, corresponding

to logical truths, we find:

- a principle of double negation, expressed by saying that a double-negation (Not: not: p) is equivalent to the assertible that is doubly negated (p) (DL VII 69)
- the principle that all conditionals that are formed by using the same assertible twice (like 'If p, p') are true (Cic. *Acad.* II 98)
- the principle that all disjunctions formed by a contradiction (like 'Either p or not: p') are true (S. E. *M* VIII 282)

Moreover, some Stoics may have dealt with relations like commutativity and contraposition via the concepts of inversion (*anastrophê*) and conversion (*antistrophê*) of assertibles (Galen *Institutio logica* VI 4). Inversion is the change of place of the constituent assertibles in a non-simple assertible with two constituents. Commutativity could thus have been expressed by saying that for conjunctions and disjunctions, inversion is sound. In a conversion, the two constituent assertibles are not simply exchanged, but each is also replaced by the contradictory of the other. The Stoics seem to have recognized that conversion holds for conditionals; that is, they seem to have accepted the principle of contraposition (cf. DL VII 194).

Finally, regarding the interdefinability of connectives, there is no evidence that the Stoics took an interest in reducing the connectives to a minimal number. For the early Stoics, we also have no evidence that they attempted to give an account of one connective in terms of other connectives, or that they stated logical equivalences of that kind.

4. MODALITY¹¹

As the previous sections have illustrated, the Stoics distinguished many different types of assertibles, which were generally identifiable by their linguistic form. In addition, the Stoics classified assertibles with respect to certain of their properties which weren't part of their form. The most prominent ones, after truth and falsehood, were the modal properties possibility, necessity, impossibility, and non-necessity. Two further such properties were plausibility and

¹¹ Cf. Bobzien (1986), (1993), and (1998), Ch. 3.1.

probability (DL VII 75–6): An assertible is plausible (*pithanon*) if it induces assent to it (even if it is false); an assertible is probable or reasonable (*eulogon*) if it has higher chances of being true than false.

Stoic modal logic is not a logic of modal propositions (e.g., propositions of the type ‘It is possible that it is day’ or ‘It is possibly true that it is day’) formed with modal operators which qualify states of affairs or propositions. Instead, their modal theory was about non-modalized propositions like ‘It is day’, insofar as they are possible, necessary, and so forth. The modalities were considered – primarily – as properties of assertibles and, like truth and falsehood, they belonged to the assertibles at a time; consequently, an assertible can in principle change its modal value. Like his precursors in Hellenistic logic, Philo and Diodorus, Chrysippus distinguished four modal concepts: possibility, impossibility, necessity, and non-necessity.

The Stoic set of modal definitions can be restored with some plausibility from several incomplete passages (DL VII 75, Boeth. *Int.* II 234.27–235.4). We can be confident that these definitions were Chrysippus’ (cf. Plut. *St. rep.* 1055df). Like the modal notions of Philo and Diodorus, they fit the four requirements of normal modal logic that (1) every necessary proposition is true and every true proposition possible; every impossible proposition is false and every false proposition non-necessary; (2) the accounts of possibility and impossibility and those of necessity and non-necessity are contradictory to each other; (3) necessity and possibility are interdefinable in the sense that a proposition is necessary precisely if its contradictory is not possible; and (4) every proposition is either necessary or impossible or both possible and non-necessary:

A possible assertible is one which (A) is capable of being true and (B) is not hindered by external things from being true;

an impossible assertible is one which (A') is not capable of being true (or (B') is capable of being true, but hindered by external things from being true);

a necessary assertible is one which (A'), being true, is not capable of being false or (B') is capable of being false, but hindered by external things from being false;

a non-necessary assertible is one which (A) is capable of being false and (B) is not hindered by external things (from being false).

In the cases of possibility and non-necessity, two conditions (A and B) have to be fulfilled. In the cases of necessity and impossibility, one

of two alternative conditions has to be satisfied (A' or B'), leading to two types of necessity and impossibility. The first parts of the definitions (A, A') are almost identical with Philo's modal definitions. The second parts (B, B') feature 'external things' which must or must not prevent the assertibles from having a certain truth-value. We have no examples of such external things, but they should be external to the logical subject of the assertible. For instance, things that prevent truth should include ordinary, physical hindrances: a storm or a wall or chains that prevent you from getting somewhere.

The accounts leave us in the dark about another aspect of the hindrances; namely, *when* they need to be present (or absent). Knowledge of this is essential for an adequate understanding of the modalities. One text (Alex. *In Ar. An. pr.* 177–178) suggests that for the possibility of an assertible, the requirement of absence of hindrances covers present-plus-future time – relative to the utterance of the assertion. For we learn that for Chrysippus 'Dio is dead' is possible (now) if it can be true at some time; equally, that 'this one is dead [pointing at Dio]', which is impossible, wouldn't be impossible (now) if, although being false now, it could be true at some later time. If one reads 'can be true' as short for Chrysippus' requirement 'is capable of being true and not prevented from being true', it seems that an assertible is possible for Chrysippus if (A) it is capable of truth, and (B) there is some time later than now when it will not be hindered from being true. For instance, 'Sappho is reading' is Chrysippean possible, as long as Sappho isn't continuously prevented from reading from now on. Correspondingly, an assertible falls under the second part of the definiens of the impossible if (B') it is capable of being true, but is from now on prevented from being true – as in the above example, if Sappho were suddenly struck by incurable blindness or died. Chrysippean necessity of the second type (B') would require continuous prevention of falsehood; non-necessity, at least temporary absence of such prevention.

5. ARGUMENTS

The second main part of Stoic logic is their theory of arguments. Arguments (*logoi*) form another subclass of complete sayables (DL VII 63); they are neither thought processes nor beliefs, nor linguistic expressions; rather, like assertibles, they are meaningful, incorporeal

entities (S. E. *PH* III 52). However, they are not assertibles, but compounds of them.

An argument is defined as a compound or system of premisses and a conclusion (DL VII 45). These are self-complete sayables, standardly assertibles, which I shall call the ‘component assertibles’ of the argument. The following is a typical Stoic argument:

- P₁ If it is day, it is light.
 P₂ But it is day.
 C Therefore, it is light.

It has a non-simple assertible (P₁) as one premiss and a simple assertible (P₂) as the other. The non-simple premiss, usually put first, was referred to as ‘leading premiss’ (*hêgemonikon lêmma*). The other premiss was called the ‘co-assumption’ (*proslêpsis*). It is usually simple; when it is non-simple, it contains fewer constituent assertibles than the leading premiss. It was introduced by ‘but’ or ‘now’, and the conclusion by ‘therefore’. It was the orthodox Stoic view that an argument must have more than one premiss.

A passage in Sextus defines ‘premisses’ and ‘conclusion’: the premisses of an argument are the assertibles that are adopted by agreement for the establishing of the conclusion; the conclusion is the assertible established by the premisses (S. E. *M* VIII 302). A difficulty with this account is that it seems that something only counts as an argument if the premisses – at the very least – appear true to the discussants. This rules out arguments with evidently false premisses such as reductions to the absurd and arguments with premisses the truth of which isn’t (yet) known, such as arguments concerning future courses of actions.

Difficulties like these may have given rise to the development of the Stoic device of hypothesis and hypothetical arguments: the Stoics thought that occasionally one must postulate some hypothesis as a sort of stepping-stone for the subsequent argument (Epict. *Diss.* I 7.22). Thus, one or more premisses of an argument could be such a hypothesis in lieu of an assertible; and it seems that hypothetical arguments were arguments with such hypotheses among their premisses. These were apparently phrased as ‘Suppose it is night’ instead of ‘It is night’ (Epict. *Diss.* I 25.11–13). They could be agreed upon *qua* hypotheses; that is, the interlocutors agree – as it were – to enter a non-actual ‘world’ built on the relevant assumption, but they

remain aware of the fact that this assumption and any conclusions drawn hold only relative to the fact that this assumption has been made.¹²

The most important distinction among arguments is that between valid and invalid ones. The Stoic general criterion was that an argument is valid if the corresponding conditional formed with the conjunction of the premisses as antecedent and the conclusion as consequent is correct (S. E. *PH* II 137). If the assertible 'If (both P_1 and...and P_n), then C ' is true, then the argument ' $P_1; \dots P_n$; therefore C ' is valid. It seems that the criterion for the correctness of the conditional was the Chrysippean one: An argument is valid provided that the contradictory of the conclusion is incompatible with the conjunction of the premisses (DL VII 77). Thus, the Stoic concept of validity resembles our modern one (see also the end of Section 6). But one should recall that the conditional has to be true according to Chrysippus' criterion, which isn't necessarily restricted to logical consequence. This brings out a shortcoming of the Stoic concept of validity, since what is needed is precisely *logical* consequence. It is unfortunate to have the same concept of consequence for both the antecedent-consequent relation in a conditional and the premisses-conclusion relation in an argument. In any event, the concept of conflict seems too vague to suffice as a proper criterion for validity.

In addition to validity, the Stoics assumed that arguments had the properties of truth and falsehood. An argument is true (we would say 'sound') if, besides being valid, it has true premisses; it is false if it is invalid or has a false premiss (DL VII 79). The predicates of truth and falsehood are here based on the truth of assertibles but are used in a derivative sense. The relevance of truth and falsehood of arguments is epistemic: Only a true argument warrants the truth of the conclusion.

Since the concept of truth of arguments is based on that of truth of assertibles, and the latter can change their truth-value, so can arguments. For instance, the argument given above will be true at day-time but false at night. It seems that arguments with premisses that did (or could) change truth-value were called 'changing arguments' (*metapiptontes logoi*) (Epict. *Diss.* I 7.1).

¹² Cf. Bobzien (1997).

The Stoics also assumed that arguments could be possible, impossible, necessary, and non-necessary (DL VII 79). These modal predicates, too, would be used in a derivative sense. With Chrysippus' modal accounts, a necessary argument would then be one that either cannot be false or can be false but is hindered by external circumstances from being false, and similarly for the three remaining modalities.

6. SYLLOGISTIC¹³

More important for logic proper are the divisions of valid arguments. These are based primarily on the form of the arguments. The most general distinction is that between syllogistic arguments or syllogisms and those called 'valid in the specific sense' (*perantikoi eidikôs*). The latter are concludent (i.e., they satisfy the general criterion of validity), but not syllogistically so (DL VII 78). Syllogisms are, first, the indemonstrable arguments; and second, those arguments that can be reduced to indemonstrable arguments.

The indemonstrable syllogisms are called 'indemonstrable' (*anapodeiktoi*) because they are not in need of proof or demonstration (DL VII 79), given that their validity is obvious in itself (S. E. M II 223). The talk of five indemonstrables alludes to classes of argument, each class characterized by a particular basic argument form in virtue of which the arguments of that class are understood to be valid. Chrysippus distinguished five such classes; later Stoics, up to seven.

The Stoics defined the different kinds of indemonstrables by describing the form of an argument of that kind. The five Chrysippean types were described as follows (S. E. M VIII 224–5; DL VII 80–1). A first indemonstrable is an argument that is composed of a conditional and its antecedent as premisses, having the consequent of the conditional as conclusion. The following is an example:

If it is day, it is light.
It is day.
Therefore it is light.

A second indemonstrable is an argument composed of a conditional and the contradictory of its consequent as premisses, having

¹³ For a detailed discussion of Stoic syllogistic, see Bobzien (1996).

the contradictory of its antecedent as conclusion; for example:

If it is day, it is light.
 Not: it is day.
 Therefore not: it is light.

A third indemonstrable is an argument composed of a negated conjunction and one of its conjuncts as premisses, having the contradictory of the other conjunct as conclusion; for example:

Not: both Plato is dead and Plato is alive.
 Plato is dead.
 Therefore not: Plato is alive.

A fourth indemonstrable is an argument composed of a disjunctive assertible and one of its disjuncts as premisses, having the contradictory of the remaining disjunct as conclusion; for example:

Either it is day or it is night.
 It is day.
 Therefore not: it is night.

A fifth indemonstrable, finally, is an argument composed of a disjunctive assertible and the contradictory of one of its disjuncts as premisses, having the remaining disjunct as conclusion; for example:

Either it is day or it is night.
 Not: it is day.
 Therefore it is night.

Each of the five types of indemonstrables thus consists – in the simplest case – of a non-simple assertible as leading premiss and a simple assertible as co-assumption, having another simple assertible as conclusion. The leading premisses use all and only the connectives that Chrysippus distinguished.

The descriptions of the indemonstrables encompass many more arguments than the examples suggest, and this for three reasons. First, in the case of the third, fourth, and fifth indemonstrables, the descriptions of the argument form provide for ‘commutativity’ in the sense that it is left open which constituent assertible or contradictory of a constituent assertible is taken as co-assumption.

Second, the descriptions are all given in terms of assertibles and their contradictories, not in terms of affirmative and negative

assertibles. In all five cases, the first premiss can have any of the four combinations of affirmative and negative assertibles: for instance, in the case of the first and second indemonstrable (if we symbolize affirmative assertibles by p , q , negative ones by 'not: p ', 'not: q ')

if p , q if not: p , q if p , not: q if not: p , not: q .

Combining these two points, we obtain four subtypes under the first and second descriptions of indemonstrables and eight in the case of the third, fourth, and fifth (i.e., thirty-two subtypes in all).

The third reason for the multitude of kinds of indemonstrables is the fact that the descriptions, as formulated, permit the constituent assertibles of the leading premisses to be themselves non-simple. And indeed, we have an example that is called a second indemonstrable and that is of the kind:

If both p and q , r ; now not: r ; therefore not: (both p and) q .

In addition to describing the five types of indemonstrables at the meta-level, the Stoics employed another way of determining their basic forms; namely, by virtue of modes (*tropoi*). A mode is defined as 'a sort of scheme of an argument' (DL VII 76). An example of the (or a) mode of the first indemonstrable would be:

If the first, the second; now the first; therefore the second.

It differs from a first indemonstrable in that ordinal numbers have taken the place of the antecedent and consequent of the leading premiss, and the same ordinals are re-used where the antecedent and consequent assertibles recur in co-assumption and conclusion. A mode is syllogistic when a corresponding argument with the same form is a syllogism. It seems that the modes, and parts of modes, performed at least three functions in the Stoic theory of arguments.

First, the modes functioned as forms in which the different indemonstrables – and other arguments – were propounded (S. E. *M* VIII 227). If, for instance, one wants to propound a first indemonstrable, the mode provides a syntactic standard form in which one has (ideally) to couch it. When employed in this way, the modes resemble argument forms: the ordinals do not stand in for particular assertibles; rather, their function resembles that of schematic letters. So, any argument that is propounded in a particular syllogistic mode is a valid argument, but the mode itself isn't an argument. The logical

form presented by a syllogistic mode is the reason for the particular argument's formal validity. In this function, the modes can be used to check the validity of arguments.

In the two other ways in which modes and ordinal numbers are employed, the ordinals seem to stand in for assertibles and the modes are used as abbreviations of particular arguments rather than as argument forms. Thus, in the analysis of complex syllogisms (discussed later in this section), for purposes of simplicity and lucidity, ordinals may stand in for simple assertibles, in the sequence of their occurrence in the argument (S. E. *M* VIII 235–7). And in the so-called mode-arguments (*logotropoi*), the constituent assertibles are given in full when first occurring, but are then replaced by ordinal numbers, as in the following:

If it is day, it is light.

Now the first.

Therefore the second (DL VII 77).

In which respects then are all and only the indemonstrables basic and evident? We can infer from the presentation of the types of indemonstrables that their validity is grounded on their form. We can also list some ways of being basic and evident which Chrysippus cannot have had in mind. First, it seems that Chrysippus was not entertaining the idea of minimizing connectives (see Section 3, p. 99). Second, Chrysippus cannot have been concerned to minimize the number of types of indemonstrables: for, with the help of the first *thema*, second indemonstrables can be reduced to first ones (and vice versa), and fifth to fourth ones (and vice versa), and this can hardly have escaped his attention. Third, Chrysippus seems not to have aimed at deducing the conclusions from premisses of the minimum possible strength. For any conclusion one can draw from a first or second indemonstrable (with a leading premiss 'If p, q'), one could also draw from a corresponding third indemonstrable (with a leading premiss 'Not: both p and not:q'). The extra requirement in the truth-criterion for the conditional – compared with the negated conjunction – i.e., the element of conflict, seems irrelevant to the conclusions one can draw.

What could have been Chrysippus' positive criteria for choosing the indemonstrables? In the indemonstrables – and consequently in all syllogisms – all and only the Chrysippean connectives ('and', 'if',

'or') and the negation ('not') are used to construct non-simple assertibles. Among these non-simple assertibles, Chrysippus distinguished a particular class entitled 'mode-forming assertibles' (*tropika axiômata*). These were apparently conditionals, disjunctions, and negations of conjunctions. All indemonstrables have as leading premiss such a 'mode-forming assertible', and perhaps the deductive power of the indemonstrables was thought to be somehow grounded on these. Perhaps the thought was that the validity of the indemonstrables could not reasonably be doubted, because understanding the mode-forming premisses implies knowing the validity of the corresponding forms of the indemonstrables. (Understanding 'Not: both p and q' implies knowing that if one of them holds, the other doesn't; understanding 'If p, q' implies knowing that (i) if p holds, so does q, and (ii) if q doesn't hold, neither does p; and so on.) This kind of criterion would, for instance, fail the following candidate for indemonstrability, although it is simple and evident in some way:

p, q, therefore p and q.

It wouldn't rank as an indemonstrable since understanding p doesn't imply knowing that if q then 'p and q'.

The situation is complicated by the fact that Chrysippus also recognized fifth indemonstrables with several (disjuncts) (S. E. *PHI* 69). They are of the following kind:

Either p or q or r
 Now, neither p nor q
 Therefore r.

Their form obviously differs from that of the fifth indemonstrables as given above. Such arguments cannot be reduced to some combination of indemonstrables, and this could be why Chrysippus regarded them as indemonstrables. However, as the name implies, he did not introduce them as 'sixth indemonstrables'; rather, they are a special version of the fifth – that is, they are fifth indemonstrables. If we take this seriously, we have to revise our understanding of the fifth indemonstrable. We should assume that the leading premiss in a fifth indemonstrable has two *or more* disjuncts, and that the 'basic idea' which one grasps when one understands the disjunctive connective is 'necessarily precisely one out of several' rather than '... out of two'. As a consequence, one also has to modify one's

understanding of the co-assumption: its description ‘the contradictory of one of its disjuncts’ becomes a special case of ‘the contradictory of one *or more* of its disjuncts’, the added possibility coming down to ‘the conjunction of the negation of all but one of them’. Such co-assumptions were standardly expressed with ‘neither... nor...’ (e.g., S. E. *PH* I 69).

In some Latin authors we find lists of seven basic syllogisms which may be of Stoic origin (e.g., Cic. *Topics* 53–57; Martianus Capella IV 414–421). The lists vary slightly from one source to another, but the first five types always correspond closely to Chrysippus’ indemonstrables. Perhaps the sixth and seventh types were intended to have pseudo-disjunctions as leading premisses, but the texts are unclear on this point.

Not all Stoic syllogisms are indemonstrables. Non-indemonstrable syllogisms can be more complex than indemonstrables in that they have more than two premisses, but they can also have just two premisses. For example, in our sources we find Stoic non-indemonstrable syllogisms of the following kinds:

If both p and q, r; not r; p; therefore not:q.
 If p, p; if not:p, p; either p or not:p; therefore p.
 If p, if p, q; p; therefore q.

The Stoics distinguished and discussed several special cases of syllogisms, both indemonstrable and non-indemonstrable. First, there are the indifferently concluding arguments (*adiaphorôs perainontes*), such as:

Either it is day or it is light.
 Now it is day.
 Therefore it is day. (Alex. *In Ar. Top.* 10.10–12)

This argument is of the kind:

Either p or q; p; therefore p.

The name of these arguments is presumably based on the fact that it is irrelevant for their validity what comes in as second disjunct. Often mentioned in tandem with the indifferently concluding arguments are the so-called duplicated arguments (*diaphoroumenoi logoi*) (Alex. *In Ar. Top.* 10.7–10). It seems that their name rests on the fact that their leading premiss is a ‘duplicated assertible’; that is,

composed of the same simple assertible, used twice or several times (cf. DL VII 68–9). The standard example is:

If it is day, it is day.
Now it is day.
Therefore it is day.

It is a special case of the first indemonstrable.

A third type of syllogism was those with two mode-forming premisses; that is, arguments composed of two mode-forming assertibles as premisses and a simple assertible as conclusion: our examples are of this kind:

If p, q; if p, not:q; therefore not:p.

The following is a Stoic example:

If you know you are dead, you are dead.
If you know you are dead, not: you are dead.
Therefore not: you know you are dead. (Orig. *Contra Celsum* VII 15)

It is likely that the Stoics distinguished further types of syllogisms (Alex. *In Ar. An. pr.* 164.27–31).

Arguments of all these kinds were syllogisms. And, since all syllogisms are either indemonstrable or can be reduced to indemonstrables, these arguments, too – if they are not indemonstrables themselves – should be reducible to indemonstrables. The Stoic expression for reducing arguments was to analyze them into indemonstrables (DL VII 195). What is the purpose of such an analysis? It is a method of proving that certain arguments are formally valid by showing how they stand in a certain relation to indemonstrables. This relation between the argument-to-be-analyzed and the indemonstrables is basically either that the argument is a composite of several indemonstrables, or that it is a conversion of an indemonstrable, or that it is a mixture of both. The analysis was carried out with certain logical meta-rules, called '*themata*', which determined these relations. They were argumental rules; that is, rules that can only be applied to arguments. They reduce arguments to arguments, not (say) assertibles to assertibles. Our sources suggest that there were four of them (Alex. *In Ar. An. pr.* 284.13–17; Galen *PHP* II 3.188). We know further that the Stoics had some logical meta-rules, called 'theorems', which were relevant for the analysis of arguments (DL VII 195; S. E. *M* VIII 231). Since the *themata* were regarded as sufficient

for the analysis of all non-indemonstrable syllogisms, the function of some of the theorems was presumably to facilitate the analysis.

Stoic analysis is strictly an upwards method (to the indemonstrables) rather than a downwards method (from the indemonstrables). Analysis always starts with a given non-indemonstrable argument, and with the question whether it can be analyzed into indemonstrables by means of the *themata*. There are no signs that the Stoics ever tried to establish systematically what kinds of formally valid non-indemonstrable arguments could be deduced or derived from their set of indemonstrables with the *themata*.

Related to this point is the fact that Stoic analysis was carried through with the arguments themselves, not with argument forms – although, of course, the analysis depends precisely on the form of the arguments. This appears to imply that analysis had to be carried out again and again from scratch, each time the (formal) validity of a non-indemonstrable argument was in question. But this need not have been so: the Stoics seem to have introduced certain meta-rules, which would state that if an argument is of such and such a form, it is a syllogism or can be analysed into indemonstrables in such and such a way (S. E. *PH* II 3 together with Orig. *Contra Celsum* VII 15.166–7). Moreover, sometimes the modes were employed in order to facilitate the reduction; that is, ordinal numbers were used as abbreviations for constituent assertibles (S. E. *M* VIII 234–6). Such abbreviation brings out the form of the argument and makes it easier to recognize which *thema* can be used.

How did Stoic analysis work in detail?¹⁴ How were the *themata* and theorems applied to arguments? Let us look first at the first *thema*:

When from two (assertibles) a third follows, then from either of them together with the contradictory of the conclusion the contradictory of the other follows (Apul. *De int.* 191.6–10).

The wording of the rule leaves the premiss order undetermined. It can be presented formally as:

$$(T1) \quad \frac{P_1, P_2 \mid\!-\! P_3}{P_1, \text{ctrd } P_3 \mid\!-\! \text{ctrd } P_2}$$

¹⁴ Warning: On the following pages the discussion gets a little more technical.

'ctrd' stands for 'contradictory', '|-' for 'therefore'; $P_1, P_2 \dots$ mark places for assertibles. In an application of the rule, the argument-to-be-analysed would occupy the bottom line, the syllogism into which it is analysed the top line. For instance, if we have a non-indemonstrable argument of the kind

$p; \text{not:}q; \text{therefore not: if } p, q$

this can be reduced to a first indemonstrable of the kind

If $p, q; p; \text{therefore } q$

by employing the first *thema* as follows: When from 'p' and 'if p, q' 'q' follows (this being the indemonstrable), then from 'p' and 'not: q' 'not: if p, q' follows (this being the non-indemonstrable argument). Or formalized:

$$\frac{\text{If } p, q; p \mid- q}{\text{p; not:}q \mid- \text{not: if } p, q} \text{ (T1)}$$

Whenever this procedure leads to one of the five indemonstrables, the argument-to-be-analysed is a syllogism. Application of the rule to all possible kinds of simple non-indemonstrable arguments leads thus to the reduction of syllogisms of four further types. As we will see, the first *thema* can also be employed several times in the same reduction, or in combination with one or more of the other rules of analysis.

It is helpful to consider the meta-rule known as a 'dialectical theorem' before discussing the remaining three *themata*:

When we have (the) premisses which deduce some conclusion, we potentially have that conclusion too in those premisses, even if it isn't expressly stated. (S. E. *M* VIII 231)

This theorem presumably did the same work as the second, third, and fourth *themata* together. Plainly, as it stands, it doesn't fully determine a method of analysis. It is only a general presentation of a principle. But a passage in Sextus (S. E. *M* VIII 230–8) illustrates how the analysis works, by applying it to two arguments. In the second example, the analysis is carried out first with the mode of the argument, then by employing the argument itself. Let us look at the former, which begins by presenting the mode of the argument-to-be-analysed:

For this type of argument is composed of a second and a third indemonstrable, as one can learn from its analysis, which will become clearer if we use the mode for our exposition, which runs as follows:

If the first and the second, the third.

But not the third.

Moreover, the first.

Therefore not: the second.

For since we have a conditional with the conjunction of the first and the second as antecedent and with the third as consequent, and we also have the contradictory of the consequent, 'Not: the third', we will also deduce the contradictory of the antecedent, 'Therefore not: the first and the second', by a second indemonstrable. But in fact, this very proposition is contained potentially in the argument, since we have the premisses from which it can be deduced, although in the presentation of the argument it is omitted. By combining it with the remaining premiss, the first, we will have deduced the conclusion 'Therefore not: the second' by a third indemonstrable. Hence there are two indemonstrables, one of this kind

If the first and the second, the third.

But not: the third.

Therefore not: the first and the second.

which is a second indemonstrable; the other, which is a third indemonstrable, runs like this:

Not: the first and the second.

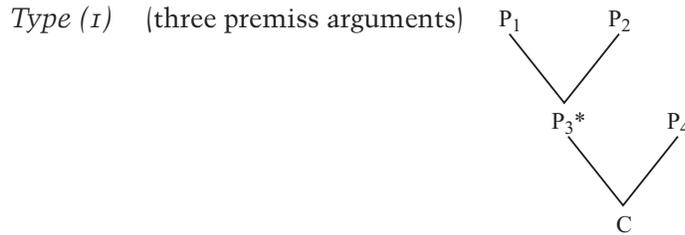
But the first.

Therefore not: the second.

Such is the analysis in the case of the mode, and there is an analogous analysis in the case of the argument (S. E. *M* VIII 235–7).

The general procedure of reduction with the dialectical theorem is then as follows: take any two of the premisses of the argument-to-be-analysed and try to deduce a conclusion from them, by forming with them an indemonstrable. Then take that 'potential' conclusion and look whether by adding any of the premisses, you can deduce another conclusion, again by forming an indemonstrable. (The old premisses are still in the game and can be taken again, if required, as is plain from Sextus' first example: S. E. *M* VIII 232–3.) Proceed in this manner until all premisses have been used at least once and the last assertible deduced is the original conclusion. In that case, you have shown that the argument-to-be-analysed is a syllogism.

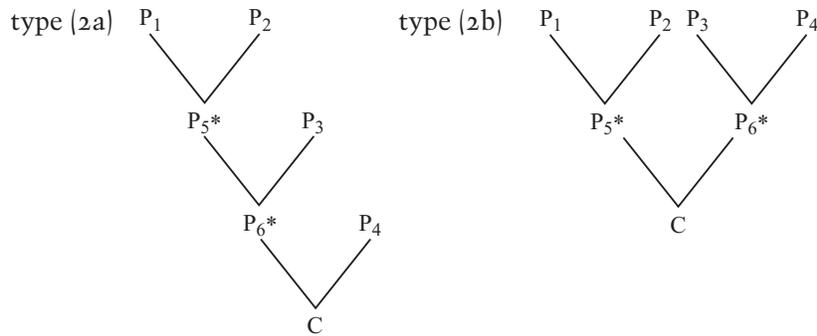
Thus, the dialectical theorem turns out to be a rule for chain-arguments by which a complex non-indemonstrable is split into two component arguments. The theorem should suffice to analyse all composite arguments; that is, all arguments with any of the following as underlying or ‘hidden’ structures. (A triangle gives the form of a simple two-premiss argument with the letter at the bottom giving the place of the conclusion. $P_1 \dots P_n$ give the places of the premisses; C that of the conclusion of the argument-to-be-analysed; P_n^* that of a premiss that is a ‘potential conclusion’ and hence doesn’t show in the argument-to-be-analysed. The type of argument-to-be-analysed has been added underneath each time.)



$$P_1, P_2, P_4 \vdash C$$

The argument in the above quotation, for instance, is of this type.

Type (2) (four premiss arguments)

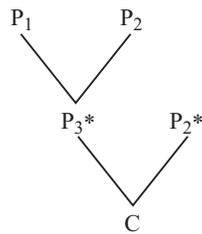


$$P_1, P_2, P_3, P_4 \vdash C$$

Expansions of these types are gained by inserting two-premiss arguments into the original argument in such a way that their conclusion is one of the formerly unasterisked premisses. These

conclusions then count as 'potential'; that is, do not appear in the argument-to-be-analysed; they accordingly get an '*'. As is clear from Sextus' first example of analysis (S. E. *M* VIII 232–3), the dialectical theorem also covers inferences in which the same premiss is implicitly used more than once, but occurs only once in the original argument. The most basic type of these is:

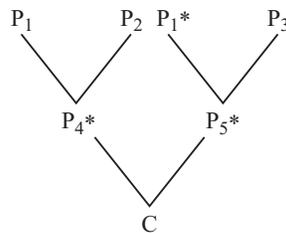
Type (3)



$P_1, P_2 \vdash C$

Sextus' first example, which is of the kind 'If p, if p, q; p ⊢ q', is of this type. A more complex case is:

Type (4)



$P_1, P_2, P_3 \vdash C$

Again, all expansions and variations of these types, and moreover all their combinations with Type (1), can be analysed by repeated use of the theorem. If one takes together the first *thema* and the dialectical theorem, with their help all non-indemonstrable Stoic syllogisms of which we know can be analysed into Stoic indemonstrables.

Next are the second, third, and fourth Stoic *themata*. Formulations of the third *thema* have survived in two sources (Simp. *Cael.* 237.2–4; Alex. *In Ar. An. pr.* 278.12–14). The second and fourth are not handed down. However, a tentative reconstruction of them and of the general method of analysis with the *themata* is possible since

there are a number of requirements that these three *themata* have to satisfy:

- The second, third, and fourth *themata* together should cover the same ground as the dialectical theorem.
- The *themata* have to be applicable, in the sense that by using them one can find out whether an argument is a syllogism.
- They have to be simple enough to be formulated in ordinary Greek.
- The second *thema*, possibly in tandem with the first, must reduce the indifferently concluding arguments and the arguments with two mode-premisses.
- The third and fourth *themata* should show some similarity or should be used together in some analyses (Galen *PHP* II 3.188).

The following is a reconstruction that satisfies these requirements.¹⁵ One source presents the third *thema* thus:

When from two (assertibles) a third follows, and from the one that follows (i.e., the third) together with another, external assumption, another follows, then this other follows from the first two and the externally co-assumed one. (Simp. *Cael.* 237.2–4)

Thus, like the dialectical theorem, the third *thema* is a kind of chain-argument rule which allows one to break up a complex argument into two component arguments. Or formally: (P_1, P_2, \dots give the places for non-external premisses; E, E_1, E_2, \dots for external premisses; C for the conclusion of the argument-to-be-analysed).

$$\frac{P_1, P_2 \vdash P_3 \quad P_3, E \vdash C}{P_1, P_2, E \vdash C}$$

For the analysis of arguments with more than three premisses, one needs an expanded version of the third *thema* in which one of the component arguments has more than two premisses. One obtains this if one modifies Simplicius' version in such a way that the second component argument can have more than one 'external premiss'. The expanded version then runs:

¹⁵ This reconstruction is based on Bobzien (1996). For alternative reconstructions, see Mueller (1979), Ierodiakonou (1990), Mignucci (1993).

When from two assertibles a third follows, and from the third and one or more external assertibles another follows, then this other follows from the first two and those external(s).

$$\text{Or formalized:} \quad (\text{T3}) \quad \frac{P_1 P_2 \vdash P_3 \quad P_3, E_1 \dots E_n \vdash C}{P_1, P_2, E_1 \dots E_n \vdash C}$$

There are two types of composite arguments the reduction of which isn't covered by the third *thema*: first, those in which there are no 'external' premisses, but instead one of the premisses used in the first component argument is used again in the second component argument; second, those in which both a premiss of the first component argument and one or more external premisses are used in the second component argument. I conjecture that the remaining two *themata* covered these two cases. They hence could have run:

When from two assertibles a third follows, and from the third and one (or both) of the two another follows, then this other follows from the first two.

$$\text{Formalized:} \quad (\text{T2}) \quad \frac{P_1, P_2 \vdash P_3 \quad P_1, (P_2,) P_3 \vdash C}{P_1, P_2 \vdash C}$$

And:

When from two assertibles a third follows, and from the third and one (or both) of the two and one (or more) external assertible(s) another follows, then this other follows from the first two and the external(s).

$$\text{Formalized:} \quad (\text{T4}) \quad \frac{P_1, P_2 \vdash P_3 \quad P_3, P_1, (P_2,) E_1 \dots E_n \vdash C}{P_1, P_2, E_1 \dots E_n \vdash C}$$

Each of the second to fourth *themata* thus has a typical kind of argument to which it applies; but they can also be used in combination or more than once in one reduction. Going back to the types of arguments distinguished when discussing the dialectical theorem, one can see that arguments of Type (1) take the third *thema* once; those of Types (2a) and (2b) take it twice. More complex ones – without implicitly multiplied premisses – take it more often. Arguments of Type (3) take the second *thema* once; those of Type (4) take the fourth and third each once. More complex arguments may take combinations of the second, third, and fourth *themata*. Occasionally, the first

thema is needed in addition. Taken together, the second, third, and fourth *themata* cover precisely the range of the dialectical theorem.

How were the *themata* applied? Before I describe the general method of analysis, here are a few examples. First, take again the second example from the Sextus passage (S. E. *M* VIII 230–8). The argument-to-be-analysed is of the following kind:

If both p and q, r; not:r; p |- not:q.

It has three premisses and takes the third *thema* once. By simply ‘inserting’ this argument into the *thema* we obtain:

When from two assertibles
 [i.e., If both p and q, r; not:r]
 a third follows
 [i.e., not: both p and q (by a second indemonstrable)]
 and from the third and an external one
 [i.e., p]
 another follows
 [i.e., not: q (by a third indemonstrable)]
 then this other
 [i.e., not: q]
 also follows from the two assertibles and the external one.

Or, using the formalized *thema*:

If both p and q, r; not:r - not:both p and q	Not:both p and q; p - not:q		(T3)
If both p and q, r; not:r; p - not:q			

We obtain examples of the use of the second *thema* from some of the special types of non-indemonstrable arguments. Indifferently concluding arguments like:

Either p or q; p |- p

use the second *thema* once and reduce to one fourth and one fifth indemonstrable:

Either p or q; p - not:q	Either p or q; not:q - p		(T2)
Either p or q; p - p			

Syllogisms with two mode-premisses like those of the kind:

If p, q; If p, not:q; therefore not:p

take the first *thema* twice, the second once and reduce to two first indemonstrables. The analysis works again step by step from the bottom line (a) to the top line (d):

- (d) $p; \text{if } p, \text{not:}q \mid\text{- not:}q$
 $\frac{\quad}{\quad} \text{(T}_1\text{)}$
- (c) $\text{If } p, q; p \mid\text{- } q \quad p, q \mid\text{- not: if } p, \text{not:}q$
 $\frac{\quad}{\quad} \text{(T}_2\text{)}$
- (b) $\text{If } p, q; p \mid\text{- not: if } p, \text{not:}q$
 $\frac{\quad}{\quad} \text{(T}_1\text{)}$
- (a) $\text{If } p, q; \text{If } p, \text{not:}q \mid\text{- not:}p$

The *general method* of analysis into indemonstrables by *themata* appears then to have worked as follows: In a very first step, you check whether the argument-to-be-analysed is an indemonstrable. If so, it is valid. If not, you next try to choose from the set of premisses of the argument-to-be-analysed two from which a conclusion can be deduced by forming an indemonstrable with them. If the argument-to-be-analysed is a syllogism, this conclusion, together with the remaining premiss(es) (if there are any), and/or one or both of the premisses that have been used already, entails the original conclusion – either by forming an indemonstrable or by forming an argument that by use of the four *themata* can be analysed into one or more indemonstrables. Next you see whether one of the remaining premisses plus this conclusion yields the premisses to another indemonstrable (in which case you apply the third *thema*); if there are no remaining premisses, or none of them works, you find out whether one of the premisses already used in the first step is such a premiss (in which case you apply the second or fourth *thema*). If the second component argument thus formed is an indemonstrable too, and all premisses have been used at least once and the last conclusion is the original conclusion, the analysis is finished, the argument-to-be-analysed a syllogism. If not, the same procedure is repeated with the argument which isn't an indemonstrable (i.e., the second component argument, which has the original conclusion as conclusion); and so forth until the premisses of the second component argument imply the original conclusion by forming an indemonstrable with it. If at any point in the analysis no indemonstrable can be formed, the first *thema* might help: namely, if the negation of the conclusion would produce a premiss you need; that is, a premiss that together with one of the available premisses makes up a pair of premisses for an indemonstrable. If at

any step the application of none of the *themata* leads to two premisses that can be used in an indemonstrable, the argument is not a syllogism.

This method of reduction is practicable and easy. All one has to know is the *themata* and the five types of indemonstrables, plus those four types of simple arguments which can be reduced to indemonstrables by the first *thema*. The number of steps one has to go through is finite; they are not very many, even in complex cases. The method appears to be effective.

Stoic syllogistic is a system consisting of five basic types of syllogisms and four argumental rules by which all other syllogisms can be reduced to those of the basic types (DL VII 78, cf. S. E. *PH* II 156–7; 194). The Stoics didn't explicitly claim any completeness for their system, but their claim of the reducibility of all non-indemonstrable syllogisms can be taken as a statement of completeness of sorts. It is also plausible to assume that the Stoics endorsed some pretechnical notion of syllogismhood, and that the indemonstrables plus *themata* were understood to 'capture' this notion; perhaps also to make it more precise. This leaves us with the problem of how we can find the independent Stoic criteria for syllogismhood; that is, how we can decide which features of the Stoic system preceded their choice of logical rules and which are simply a result of their introducing these rules. However, there is little evidence about what was the Stoic pretechnical notion of syllogismhood, and we cannot hope to decide whether the Stoics achieved completeness on their own terms. All we can do is determine some features of the Stoic system that are relevant to its completeness.

The Stoic system shared the following condition of validity with modern semantic interpretations of formal logic: It is necessary for the validity of an argument that it isn't the case that its premisses are true and its conclusion is false. Accordingly, it is a necessary condition for formal validity (i.e., syllogismhood) that no syllogism or argument of a valid form has true premisses and a false conclusion. To this we can add a couple of necessary conditions for Stoic syllogismhood which are not requirements for formal validity in the modern sense, and which show that the class of Stoic syllogisms can at most be a proper subclass of valid arguments in the modern sense.

First, there is a formal condition which restricts the class of syllogisms not by denying validity to certain arguments, but by denying

the status of argumenthood to certain compounds of assertibles: The Stoic concept of argument is narrower than that of modern logic in that an argument must have a minimum of two premisses and a conclusion. Stoic syllogistic considers only arguments of the form

$$\Delta \vdash A$$

in which Δ is a set of premisses with at least two (distinct) elements. Stoic syllogistic doesn't deal with arguments of the forms

$$\vdash A \quad A \vdash B \quad \text{or} \quad \Delta \vdash \cdot$$

There is also no one-to-one correspondence between valid arguments and logically true conditionals. Such a correspondence exists only between a proper subclass of the latter – those which have the form 'If both A and B and . . . , then C' – and valid arguments.

Second, there is a restriction of validity through the requirement of non-redundancy of the premisses: An argument is invalid owing to redundancy if it has one or more premisses that are added to it from outside and superfluously (S. E. *M* II 431). For cases of non-indemonstrable arguments, one may interpret the clause 'from outside and superfluously' as meaning that there is no deduction in which this premiss, together with the others of the argument, entails the conclusion. The requirement of non-redundancy means that the following kinds of arguments count as invalid:

$p; q; \text{ therefore } p$

If $p, q; p; r; \text{ therefore } q$

although they are valid in all standard propositional calculi.

We can now show that the Stoic system of syllogisms captures the pretechnical elements of syllogismhood as determined by the requirements stated. First, no one- or zero-premiss arguments are reducible, since every indemonstrable has two premisses; and every *thema* can be applied only to arguments with two or more premisses. Second, redundant arguments cannot be reduced: The indemonstrables have no 'redundant' premisses, and the *themata* require that all premisses of the argument-to-be-analysed are components of the indemonstrables into which it is analyzed – either as premiss or as negation of a conclusion. So far then, at least, Stoic syllogistic coincides with what may have been their pretechnical notion of syllogismhood.

7. ARGUMENTS VALID IN THE SPECIFIC SENSE

Finally, we come to the second group of valid arguments distinguished by the Stoics, those called ‘valid in the specific sense’ (DL VII 78–9). The surviving information on these arguments is sparse and many details are under dispute. At least two subclasses were distinguished. One was the subsyllogistic arguments (*hyposyllogistikoi logoi*), another was the arguments named ‘unmethodically concluding’ (*amethodôs perainontes*); there may have been others. The Stoics held that all valid arguments were constructed by means of the indemonstrable syllogisms (*ibid.*). If we take this at face value, the validity of the specifically valid arguments may have been justified by the validity of syllogisms. One would expect this justification to vary from subclass to subclass.

Subsyllogistic arguments differ from the corresponding syllogisms only in that one (or more) of their component assertibles, although being equivalent to those in the syllogism, diverge from them in their linguistic form (Galen *Institutio logica* XIX 6). Examples are of the following type:

‘p’ follows from ‘q’; but p; therefore q

instead of a first indemonstrable. We may assume that the reason why subsyllogistic arguments weren’t syllogisms was that they didn’t share their canonical form. This distinction displays an awareness of the difference between object- and meta-language: A conditional is indeed not the same as a statement that one assertible follows from another. The validity of a subsyllogistic argument may have been established by constructing a corresponding syllogism and pointing out the equivalence.

The following is a Stoic example for an unmethodically concluding argument:

You say that it is day.
But you speak truly.
Therefore it is day. (Galen *Institutio logica* XVII 2)

This isn’t a syllogism. It is neither an indemonstrable nor can it be reduced to one, since it contains no non-simple assertible as component. What was the reason for the validity of such arguments? Perhaps they were dubbed ‘unmethodically concluding’ because there

is no formal method of showing their validity; but even then their validity should have been justified somehow – and if we take the remark at DL VII 79 seriously, these justifications should have involved some suitably related syllogisms. But we have no direct evidence that suggests a way of detecting ‘corresponding syllogisms’, as in the case of the subsyllogisticals.

Several other arguments were considered valid by some Stoics; some of these may have counted as specifically valid arguments. First, the single-premiss arguments (*monolêmmatoi*): The orthodox Stoic view was that arguments must have at least two premisses. However, Antipater admitted single-premiss arguments, and he presumably regarded at least some as valid. If we trust Apuleius, Antipater adduced arguments like the following:

You see.

Therefore you are alive. (Apul. *De int.* 184.16–23)

What reasons he had for admitting these, we are not told. It is unlikely that Antipater proposed that they were syllogisms. For they are not formally valid. Antipater may have regarded them as unmethodically concluding, perhaps with a nonexplicit assumption of the kind ‘If someone sees, that one is alive.’ Second, there are the arguments with an indefinite leading premiss and a definite co-assumption mentioned previously in the context of non-simple assertibles. A typical example is:

If someone walks that one moves.

This person walks.

Therefore this person moves.

Despite the similarity, this isn’t a straightforward first indemonstrable. How did the Stoics justify their validity? Presumably by referring to the truth-conditions of the leading premiss. Since its truth implies the truth of all subordinated assertibles, one can always derive the particular conditional one needs (‘If this one walks, this one moves’) and thus form the needed syllogism – in this case, a first indemonstrable. This relation between the indefinite conditional and the corresponding definite ones may have counted as an implicit assumption by which validity was justified (but which, if added, wouldn’t make the argument formally valid).