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OXFORD STUDIES IN ANCIENT PHILOSOPHY

EDITOR: C. C. W. TAYLOR

VOLUME XIV

1996

CLARENDON PRESS · OXFORD
1996

STOIC SYLLOGISTIC

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FOR the Stoics, a syllogism is a formally valid argument, and the primary function of their syllogistic is to establish the formal validity of arguments. Stoic syllogistic can be understood as a system of formal logic that relies on two types of argumental rules:[1] first, five rules (the accounts of the indemonstrables) which were used to determine whether any given argument is an indemonstrable argument (ἀναπόδεικτος λόγος), i.e. an elementary syllogism the validity of which is not in need of further demonstration (D.L. 7. 79), since its validity is evident in itself (Sextus, *M.* 2. 223);[2] second, one unary and three presumably binary argumental rules, called *themata* (θέματα), which allow one to establish the formal validity of non-indemonstrable arguments by analysing them in one or more steps into one or more indemonstrable arguments (D.L. 7. 78). The function of these rules is not to generate non-indemonstrable syllogisms from indemonstrable ones, but rather to reduce given non-indemonstrable arguments to indemonstrable syllogisms. Moreover, the Stoic method of deduction differs from standard modern ones in that the direction is reversed. The Stoic system may hence be called an 'argumental reductive system of deduction'.

In the following I present a reconstruction of this system of logic. The rules or accounts used for establishing that an argument is indemonstrable have all survived, and the indemonstrables are among the best-known elements of Stoic logic. However, their exact

I am indebted to Jonathan Barnes, Michael Frede, Helmut Linneweber-Lammerskitten, and Mario Mignucci for valuable comments on earlier versions of this paper.

[1] By an argumental rule I mean a rule that produces arguments from (zero or more) arguments, as opposed to a rule that produces propositions from (zero or more) propositions.

[2] The accounts of the indemonstrables, when interpreted as rules, are nullary argumental rules.

role and logical status in Stoic syllogistic are usually neglected. I expound how they are integrated in the system of deduction. The state of evidence for the *themata* is dismal—although perhaps not hopeless. I suggest a reconstruction of the *themata*, based on a fresh look at some of the sources, and then offer a reconstruction of the general method of reduction of arguments and some general remarks on Stoic syllogistic as a whole and on the question of its completeness (much of which will not depend on the particular formulation of the *themata* I propose, but on more general considerations for a reconstruction).

Stoic logic is a propositional logic, and Stoic negation and conjunction are truthfunctional. This has, naturally, led to comparisons with the 'classical' propositional calculus (as e.g. presented in *Principia Mathematica*), including repeated examinations of Stoic syllogistic on completeness in the modern sense. The Stoic theory of deduction invariably comes out as deficient, inferior, or simply outlandish in such comparisons, which has evoked adjusting additions and modifications—tacit or explicit—in previous reconstructions of the system. I suggest that this is the wrong approach; that the classical propositional calculus is the wrong paradigm; that Stoic logic has to be considered first of all in its own light; and that, if one looks for comparisons with contemporary logic, one can find some rather more interesting parallels when turning one's attention to non-truthfunctional propositional logics.

1. The indemonstrable arguments

I begin with an outline of the Stoic theory of indemonstrables. Here it is vital to distinguish between:

(1) the indemonstrables themselves;
(2) the modes (τρόποι) of the indemonstrables;
(3) the five (or more) types, kinds, or classes of indemonstrables;
(4) the general accounts or definitions of the indemonstrables of the different types.

(1) An *indemonstrable* is a particular argument, not an argument form or an argument schema.[3] It is composed of propositions

[3] This has been shown by M. Frede, *Die stoische Logik* (Göttingen, 1974), 167–

(ἀξιώματα). An example of an argument that is a first indemonstrable is

> If it is day, it is light. It is day. Therefore it is light.

(2) A *mode* is 'a sort of scheme of an argument' (D.L. 7. 76; Sextus, *M.* 8. 236) in which ordinal numbers are put in place of propositions in the sequence of their occurrence, the same number being used for all occurrences of the same proposition. There can be modes of simple as well as of complex arguments (Sextus, *M.* 8. 234–6). An example of a mode of a fourth indemonstrable is

> Either the first or the second. The first. Therefore not the second.

The modes seem to have fulfilled various functions in Stoic logic, and where they are used it is often unclear what their exact logical status is.[4]

(3) The *types* or *kinds* or *classes of indemonstrables* lead a rather shadowy existence in Stoic logic. Stoic sources never talk explicitly about five types, kinds, or classes of indemonstrables. Instead, we very occasionally find the phrase 'five indemonstrables' (e.g. D.L. 7. 79; Sextus, *PH* 2. 157). Since there can be no doubt that the indemonstrables are particular arguments (see (1)), so that there is a plurality of first indemonstrables, second indemonstrables, etc., the talk of 'five indemonstrables' must—at least originally—have been an indirect way of speech, being short for something like 'the five kinds of indemonstrables'. However, when the Stoics wanted to point out that an indemonstrable is of a particular kind, they commonly did this by establishing by means of the accounts of the indemonstrables that the argument at issue is *a* first (etc.) indemonstrable.

71. Cf. also Sextus, *PH* 2. 157–9, 198–200; *M.* 8. 224–6, 228–30; Galen, *Inst. log.* 14. 4; [Galen,] *Hist. phil.* 15 = Diels, *Doxographi Graeci*, 607. 7–608. 2.

[4] In later authors τρόπος and the Latin translation *modus* are frequently used as a synonym for 'type of indemonstrable' or 'basic kind of syllogism' (e.g. Philop. *In An. Pr.* 244. 9, 12, 3; 245. 23, 26, 33; Cic. *Top.* 54–7; Martianus Capella 4. 414–21). In these texts the term 'indemonstrable' is not employed at all. However, this use of 'mode' together with the complete absence of the term 'indemonstrable' is not early Stoic. Chrysippus clearly distinguished between indemonstrables and τρόποι, and wrote about τρόποι for arguments that are not indemonstrables (D.L. 7. 194–5). The alteration in meaning of τρόπος presumably sprang from some Peripatetic use of the term. I surmise that it goes hand in hand with a change of the conception of syllogistic in later antiquity. Since I am concerned with early Stoic syllogistic and analysis, and these undoubtedly involved the term 'indemonstrable', I shall focus primarily on those sources that talk about indemonstrables.

(4) The five *accounts* or *definitions of the indemonstrables* determine whether something is an indemonstrable, and at the same time which type of indemonstrable it is of. The accounts are:

A first indemonstrable is an argument that is composed of a conditional and its antecedent (as its premisses), having the consequent of the conditional as conclusion. (Sextus, *M.* 8. 224; D.L. 7. 80)

A second indemonstrable is an argument that is composed of a conditional and the contradictory of its consequent as premisses, having the contradictory of its antecedent as conclusion. (Sextus, *M.* 8. 225; D.L. 7. 80)

A third indemonstrable is an argument that is composed of a negated conjunction and one of its conjuncts (as premisses), having the contradictory of the remaining conjunct as conclusion. (Sextus, *M.* 8. 226; D.L. 7. 80)

A fourth indemonstrable is an argument that is composed of a disjunctive proposition[5] and one of its disjuncts (as premisses), having the contradictory of the remaining disjunct as conclusion. (D.L. 7. 81)

A fifth indemonstrable is an argument that is composed of a disjunctive proposition and the contradictory of one of its disjuncts (as premisses), having the remaining disjunct as conclusion. (D.L. 7. 81)

These accounts have obviously been formulated with great care. They all follow the same pattern, consist largely of set phrases, and use Stoic technical terms throughout. Presumably they were to be memorized by the students. According to the accounts, any indemonstrable argument consists—in the simplest case—of a non-simple proposition as leading premiss (ἡγεμονικὸν λῆμμα) and a simple proposition as co-assumption (πρόσληψις),[6] having another simple proposition as conclusion. (The leading premisses of the indemonstrables were also called 'mode-forming propositions' (τροπικὰ ἀξιώματα): cf. Gal. *Inst. log.* 7. 1.) The accounts are phrased in such a way that each encompasses various subtypes of indemonstrables, which can be classified with respect to the following points:

First, in the case of the third, fourth, and fifth indemonstrables the accounts provide for 'commutativity', in the sense that each time it is left open which constituent proposition or contradictory of a constituent proposition of the leading premiss is taken as co-

[5] Remember that Stoic disjunction (διεζευγμένον), as it occurs in the fourth and fifth indemonstrables, is exhaustive and exclusive and presumably not truthfunctional (Gell. *NA* 16. 8. 12–14; D.L. 7. 72).

[6] These are the Stoic technical terms for the kinds of premisses. Note that in the accounts nothing is said about the order of the premisses in the indemonstrables.

assumption. For instance, in the case of the fourth indemonstrable, the two subtypes covered are of the forms

Either D_1 or D_2; D_1; therefore not D_2
Either D_1 or D_2; D_2; therefore not D_1

(where D_1 and D_2 stand for disjuncts 1 and 2).

Secondly, the descriptions of the indemonstrables are all in terms of propositions and their contradictories, *not* in terms of *affirmative* propositions and their contradictories (which would all be negative propositions). Therefore, in all five cases, the leading premiss can have any of the four possible combinations of affirmative and negative propositions. For instance, in the case of the first and second indemonstrables they can be of the forms (with *A*, *B* for propositions)[7]

If *A*, *B*.
If not *A*, *B*.
If *A*, not *B*.
If not *A*, not *B*.

Putting together these two points, we obtain four subtypes of first and second indemonstrables and eight in the cases of the third, fourth, and fifth. These thirty-two subclasses of indemonstrable arguments may have had thirty-two different modes corresponding to them. Alternatively, it is possible that one mode (or a smaller number of modes) only was used for each basic type of indemonstrable.

A third point that should be mentioned is the fact that the accounts of the indemonstrables permit that the constituent propositions of the leading premisses, and hence the co-assumptions, are themselves complex propositions, or non-simple (οὐχ ἁπλᾶ), as the Stoics call them.[8] This is confirmed by an example in Sextus which is called a second indemonstrable and which has the form

[7] A consequence of this is that negation signs turn up in two different functions: once as part of the constituent proposition, once as the indicator of contradictoriness and hence as a part that is essential to the form of the argument (so in indemonstrables of the last four types). In the second premisses and conclusions this can lead to double negations (If *A*, not *B*; not not *B*; therefore not *A*). The formulation in Diogenes Laertius' account of 'contradictories' (7. 73) may have been chosen to avoid such occurrences of double negations. For this passage implies that a proposition contradictory to another proposition is one of a pair of propositions in which one is the negation of the other. On the other hand, double negations do not pose a problem in Stoic logic (cf. D.L. 7. 69).

[8] Note, though, that for the Stoics negations of simple propositions are themselves

If both *A* and *B*, *C*; not *C* ⊦ not ⟨both *A* and⟩ *B*

(Sextus, *M*. 8. 237; cf. 236). Possibly, one argument could also be assigned various modes of different complexity (D.L. 7. 194).

These are the main characteristics of and differences between the indemonstrables, their modes, their kinds, and their accounts. It remains to determine which of these four items featured in Stoic syllogistic, and what their functions were in Stoic analysis. According to the Stoics, syllogisms are either themselves indemonstrables or reducible to indemonstrables (D.L. 7. 78). This reduction of arguments is what they called 'analysis' (ἀνάλυσις).

Only one illustration of Stoic analysis has been handed down to us, by Sextus Empiricus (*M*. 8. 229–38). In this passage the *themata* are not used, but a logical principle of a similar kind (see below, p. 165). Apart from that, the analysis is fairly similar to that using the *themata*, and there is also a wide overlap with Stoic logical terminology as we find it in the passages on the *themata*. In the absence of any evidence to the contrary, I therefore assume that as far as the indemonstrables and their identification are concerned, the general method of analysis by *themata* worked roughly in the same way as the analysis by means of this logical principle. If we incorporate this example of analysis in our fund of evidence on indemonstrables and syllogistic, the following picture evolves:

(1) In Stoic syllogistic the indemonstrables are the end-products of a successful analysis; they are the basic valid arguments, of which the argument to be analysed is understood to be composed, and into which it has been split up and/or converted in the analysis (Sextus, *M*. 8. 230–3, 237–8).

(2) The function of the modes in Stoic analysis is also clearly illustrated in the passage: they function as abbreviations of particular arguments, in order to facilitate analysis. That is, instead of the whole argument, 'its' mode (a mode of it) can be used. Correspondingly, in such an 'analysis by proxy', the complex mode is reduced to one or more modes of indemonstrables (Sextus, *M*. 8. 234–6). It is of major importance to note that when in the course of an analysis one obtains a simpler mode (say 'If the first, the second. The first. Therefore the second') one still has to establish that it is the mode of an indemonstrable (cf. Sextus, *M*. 8. 236). Moreover,

simple (D.L. 7. 63); for example, 'It is day', 'Not: it is day' (or, in better English, 'It is not the case that it is day'), and 'Not: not: it is day' all count as simple.

there is no evidence whatsoever that the modes played any other role in Stoic analysis than the one just described. In particular, it is not the case that the particular arguments at issue were tested against the modes, as to whether they were indemonstrables.

(3) The five types, kinds, or classes of indemonstrables as such (as determined by the five accounts) are not mentioned at all in the analysis in Sextus or in any other source on Stoic analysis—neither directly, nor by reference to 'the five indemonstrables', 'the first indemonstrable', etc.

(4) There can on the other hand be no doubt that the accounts of the indemonstrables feature prominently in our examples of analysis in Sextus. Particular arguments (or their modes) were tested for 'indemonstrablehood' by checking bit by bit against one of the accounts whether the argument at issue is an indemonstrable (or whether the mode at issue is a mode of an indemonstrable). The correspondence between the accounts themselves and the formulations in the examples of analysis in Sextus is striking. There are parallels, with identical wording, for basically every phrase of the respective account.[9] It appears that the testing for indemonstrablehood was done semi-mechanically, in the manner in which one works one's way through a checklist, making sure that for every and only the parts of the accounts there were corresponding bits in the argument (or mode) at issue.

We can conclude that in a Stoic analysis either indemonstrables and their accounts make an appearance, or alternatively modes of indemonstrables and accounts of indemonstrables, and that indemonstrablehood was established by testing either the indemonstrables or their modes against the accounts. There is no evidence whatsoever that the Stoics used the types or kinds of indemonstrables in the analysis of arguments, e.g. as argument forms or axiom schemata. Attempts at reconstructing Stoic syllogistic by means of the *themata* and 'the five indemonstrables', interpreted

[9] Cf. the account of the second indemonstrable in Sextus, *PH* 2. 157 *δεύτερον* [i.e. *ἀναπόδεικτον*] *τὸν ἐκ συνημμένου καὶ τοῦ ἀντικειμένου τοῦ λήγοντος τὸ ἀντικείμενον τοῦ ἡγουμένου συνάγοντα*, with its application in analysis in Sextus, *M*. 8. 236 *ἔχομεν συνημμένον ἐν ᾧ ἡγεῖται* A, *λήγει δὲ* B, *ἔχομεν δὲ καὶ τὸ ἀντικείμενον τοῦ λήγοντος τὸ* not B, *συναχθήσεται ἡμῖν καὶ τὸ ἀντικείμενον τοῦ ἡγουμένου, τὸ* not A (I put in '*A*', '*B*', 'not *B*', 'not *A*' for the parts of the mode that is tested). The *only* bit that has no direct parallel in the account is the phrase *ἐν ᾧ ἡγεῖται* A, *λήγει δὲ* B, which is necessary for the identification of the two *ἀντικείμενα*. A similar parallel can be found between the account of the first indemonstrable in Sextus, *M*. 8. 224 and its application at ibid. 232.

in this way, are hence historically unjustified. Equally, there are no extant texts that corroborate the view that in Stoic analysis the modes in general functioned as argument forms and those of the indemonstrables as something like logical axiom schemata. Again, reconstructions that proceed in this way are historically inaccurate. In addition, the above-mentioned indeterminacy regarding the logical function(s) of the modes in different contexts and the uncertainty of how many modes correspond to one indemonstrable, together with some resulting difficulties, make the modes unsuitable as a basis for the presentation of Stoic analysis.

I shall hence ground my reconstruction of the Stoic system of deduction on the accounts of the indemonstrables and the *themata*. In this system the accounts perform a function similar to that which argumental logical axiom schemata or nullary argumental rules have in some modern systems. I shall treat the five accounts as so many (nullary) rules for testing arguments for syllogismhood: an argument that 'fits' any of the accounts is a syllogism—and, if one so wants, an axiom of the system.[10] For reasons of convenience, the

[10] Some logicians do not distinguish between logical axiom schemata and nullary rules (e.g. J. Corcoran, 'Remarks on Stoic Deduction', in J. Corcoran (ed.), *Ancient Logic and its Modern Interpretation* [Dordrecht and Boston, 1974], 169–81 at 175). However, I believe it can be useful to make such a distinction. The Stoics themselves never comment on the logical status of the accounts of the indemonstrables (or of the *themata*, for that matter), and they probably lacked the concepts of both axiom schemata and nullary rules. Still, if one wants to compare the accounts to modern logical items, (nullary) rules appear to be better candidates than axiom schemata. An argumental axiom schema typically consists of (1) bits of the object-language, namely the logical constants, (2) meta-variables, and (3) a sign for logical consequence; and no matter what well-formed propositions are put in for the meta-variables, the resulting well-formed argument is an axiom of the system. In Stoic logic, then, one would expect an argumental axiom schema to look as follows:

If *A*, *B*. Not *B*. Therefore not *A*.

The Stoic modes of indemonstrables come closest to this, e.g.:

If the first, the second. Not the first. Therefore not the second.

However, there is not even a hint in our sources that modes of indemonstrables functioned as axiom schemata in Stoic analysis (see above). The *accounts* of the indemonstrables, on the other hand, are not good candidates for axiom schemata because they (1) neither contain any (Stoic) logical constants, (2) nor any meta-variables for propositions, (3) nor a sign for logical consequence (like e.g. 'therefore'). Instead, the accounts have the following characteristics. They are formulated in ordinary language (with great care and precision), larded with Stoic *meta*logical expressions ('conjunction' instead of 'and'; 'having . . . as a conclusion' instead of 'therefore', etc.). They are worded in a general way such that the five accounts cover thirty-two logically distinct subclasses of indemonstrables, none of which has a designated position or is in need of proof. (Similarly, the wording makes a structural rule concerning the order of the premisses superfluous.) These characteristics point in the direction of rules rather than axiom schemata. Furthermore, for most of these characteristics there are parallels in the *themata* (which are commonly interpreted as rules), and the accounts seem to have been applied roughly in the same (semi-mechanical) way as the latter (see below, pp. 152–60).

accounts or definitions of the indemonstrables may be 'formalized' as follows:

(A1) $$\overline{\text{cond}\ (A_{\text{-ant}}/A_{\text{-cons}}),\ A_{\text{-ant}} \vdash A_{\text{-cons}}}$$

(A2) $$\overline{\text{cond}\ (A_{\text{-ant}}/A_{\text{-cons}}),\ \text{ctrd}\ (A_{\text{-cons}}) \vdash \text{ctrd}\ (A_{\text{-ant}})}$$

(A3) $$\overline{\text{neg}\ (\text{conj}\ (A_{\text{-conj-1}}/A_{\text{-conj-2}})),\ A_{\text{-conj-}i} \vdash \text{ctrd}\ (A_{\text{-conj-}j})}$$

(A4) $$\overline{\text{disj}\ (A_{\text{-disj-1}}/A_{\text{-disj-2}}),\ A_{\text{-disj-}i} \vdash \text{ctrd}\ (A_{\text{-disj-}j})}$$

(A5) $$\overline{\text{disj}\ (A_{\text{-disj-1}}/A_{\text{-disj-2}}),\ \text{ctrd}\ (A_{\text{-disj-}i}) \vdash (A_{\text{-disj-}j})}$$

where $i, j = 1, 2$ and $i \neq j$.

The following are two examples of the application of these rules to arguments (note that I use $p, q, r \ldots$ as *abbreviations* of (Stoic) affirmative simple propositions, so that 'If p, q; $p \vdash q$' is an abbreviated argument, and *not* an argument form or an argument schema):[11]

$$\overline{\text{If } p,\ q;\ p \vdash q} \qquad \text{(A1)}$$

$$\overline{\text{If } (p \text{ and } q),\ r;\ \text{not } r \vdash \text{not } (p \text{ and } q)} \qquad \text{(A2)}$$

Both arguments are hence syllogisms.

[11] This is in line with one use the Stoics made of their modes, where 'the first', 'the second', etc. take the place of my p, q, etc. (You may take p, q, r, and s as abbreviations of 'It is day', 'It is night', 'It is dark', and 'It is light'.)

2. The *themata*

All syllogisms were considered as either indemonstrables themselves or as reducible to or—as the Stoics would have said—analysable into indemonstrables (D.L. 7. 78). This analysis or reduction was achieved by use of the four *themata*.[12] According to Diogenes, the *themata* were regarded as sufficient for the analysis of all non-indemonstrable syllogisms into indemonstrables (ibid.). We know further that the Stoics had some logical meta-rules called 'theorems' (θεωρήματα) (D.L. 7. 95; Sextus, *M.* 8. 231; *PH* 2. 3; Orig. *Cels.* 7. 15, p. 166 Koetschau). Some of these seem to have had a function comparable to that of the *themata*. Others seem to have had the status of meta-rules.[13]

Since the surviving information about Stoic syllogistic and the *themata* and their application is so sparse, recently doubt has been voiced whether we shall ever recover the missing *themata* and be able to reconstruct the Stoic method of analysis.[14] However, in the first instance, all our task can be is to do the best possible with the passages that have come down to us, i.e. to find an interpretation that satisfies all the requirements, philosophical and philological, that the extant testimonies present to us. And, as far as I can see, even this more modest goal has not yet been reached.[15] I propose

[12] The sources suggest that there were four *themata*, and that Chrysippus had them already (Gal. *De Hipp. et Plat. dogm.* 2. 3. 18, p. 114 DeLacy; Alex. *In An. Pr.* 284. 13–15); but it cannot be ruled out completely that there were more.

[13] Analysis by means of the *themata* was carried out with the arguments themselves (not with argument forms), although, of course, the analysis is grounded precisely on the form of the arguments. This may strike one as odd, since it appears to imply that analysis had to be done again and again from scratch, each time the validity of a non-indemonstrable argument was in question. But this need not have been so: the Stoics seem to have introduced certain time-saving meta-rules, which would basically state that if an argument is of such and such a form, it is a syllogism. (Cf. e.g. Frede, *Die stoische Logik*, 170–2.) The modes, *qua* abbreviations of particular arguments, were employed in order to facilitate the reduction, too (Sextus, *M.* 8. 216, 235–7; cf. also Frede, *Die stoische Logik*, 138–40).

[14] M. Mignucci, 'The Stoic *themata*', in K. Döring and T. Ebert (eds.), *Dialektiker und Stoiker: Zur Logik der Stoa und ihrer Vorläufer* (Stuttgart, 1993), 217–38 at 238.

[15] Attempts at reconstructing the theory we find in O. Becker, 'Über die vier "Themata" der stoischen Logik', in O. Becker (ed.), *Zwei Untersuchungen zur antiken Logik* (Klassisch-philologische Studien, 17; Wiesbaden, 1957), 27–49; W. Kneale and M. Kneale, *The Development of Logic*, 4th edn. (Oxford, 1984), 164–74; B.

to provide in the following one such reconstruction of the Stoic *themata* and method of reduction.

The first *thema* has survived, in Latin only, in Apuleius (*Int.* 209. 12–14). The third *thema* we find in Alexander (*In An. Pr.* 274. 11–14) and in Simplicius (*In Cael.* 237. 2–4). The second and fourth have not been handed down at all. But if one takes into account all the conditions the *themata* have to fulfil according to the surviving evidence, a plausible reconstruction is possible. These requirements[16] include first a number of trivial points:

- The *themata* have to be applicable, in the sense that by using them one can actually find out, by trying to reduce an argument to indemonstrables, whether it is a syllogism.
- They have to be simple enough to be formulated in ordinary (Greek) language.
- The particular wording of the *themata* that survived has to make sense in the interpretation.

Various passages suggest further conditions:

- The second *thema*, possibly in tandem with the first, must be used to reduce the indifferently concluding arguments and the arguments with two mode-premisses[17] (Gal. *De Hipp. et Plat. dogm.* 2. 3.18, p. 114 DeLacy).
- It would be a plus of a reconstruction if the second *thema* could also be used in the case of duplicated arguments and infinite content (Alex. *In An. Pr.* 164).
- The second *thema* should be such that one could see why some Peripatetics thought of it as useless (Alex. *In An. Pr.* 164).
- The third and fourth *themata* should show some similarity or at least should be used together in the case of certain types of arguments (Gal. *De Hipp. et Plat. dogm.* 2. 3.18, p. 114 DeLacy).

Mates, *Stoic Logic*, 2nd edn. (Berkeley, 1961), 77–82; Corcoran, 'Remarks on Stoic Deduction', 176–81; Frede, *Die stoische Logik*, 167–96; K. Ierodiakonou, *Analysis in Stoic Logic* (diss. London School of Economics; London, 1990). For a convincing critique of the first three cf. Frede, *Die stoische Logik*, 190–5; for discussion of Frede see Ierodiakonou, *Analysis*, 71–5 and my nn. 32, 45, 49, 50, 57, 87, and for Ierodiakonou see my nn. 30, 54, and 87 below. Also relevant are J. Gould, 'Deduction in Stoic Logic', in J. Corcoran (ed.), *Ancient Logic and its Modern Interpretation* (Dordrecht and Boston, 1974), 151–68; I. Mueller, 'The Completeness of Stoic Propositional Logic', *Notre Dame Journal of Formal Logic*, 20 (1979), 201–15; and Mignucci, 'The Stoic *themata*'.

[16] Most of these requirements have already been set out by Frede, *Die stoische Logik*, 172 ff.

[17] For these kinds of arguments see below, pp. 167–70.

- As to their deductive power,[18] the second, third, and fourth *themata* together should cover the same ground as the dialectical theorem and the synthetic theorem (Alex. *In An. Pr.* 284).
- It should be possible to 'simplify' the method of reduction which uses the four *themata* (Gal. *De Hipp. et Plat. dogm.* 2. 3. 19, p. 114 DeLacy).
- And of course, all Stoic non-indemonstrable syllogisms of which we know should be analysable into indemonstrables by the *themata* (D.L. 7. 78).

The first *thema* is a rule of contraposition for arguments. It ran:

> If from two [propositions] a third follows, then from either one of them together with the contradictory of the conclusion the contradictory of the remaining one follows.[19] (Apul. *Int.* 209. 12–14)

Note that—as in the case of the indemonstrables—the order of the premisses is irrelevant, and the wording is in terms of contradictoriness and not of negation. Both the premisses and the conclusion of the argument to be analysed can be affirmative or negative propositions; this has as a consequence that no double negations are introduced by use of the *thema* (and no rule for their elimination is required).[20]

For the analysis of some arguments with more than two premisses a more general version of the rule is required; Galen, *Inst. log.* 6. 5, suggests that there was such a rule[21] and that it should have run like this:

> If from two or more propositions something follows, then from all but one of them together with the contradictory of the conclusion, the contradictory of the remaining one follows.

The third *thema* is a kind of chain-inference rule which allows one to break up a complex argument into two component arguments. The two formulations of it that have survived in Alexander and in Simplicius present, in fact, two different versions. Alexander has:

> When from two [propositions] a third follows, and external assumptions syllogize one of the two, then the same [i.e. third] one follows from the remaining one and the external ones that syllogize the other.[22] (Alex. *In An. Pr.* 278. 12–14)

And Simplicius reads:

> When from two [propositions] a third follows, and from the one that follows [i.e. the third] together with another, external, assumption, another follows, then this other follows from the first two and the externally co-assumed one.[23] (Simpl. *In Cael.* 237. 2–4)

There are two logically significant differences between the two versions. First, in Alexander the external assumptions deduce one of the premisses of an argument that has as its conclusion the conclusion of the argument to be analysed; whereas in Simplicius the external assumption comes in, together with the conclusion of another argument, in order to deduce the conclusion of the argument to be analysed. This is the *main* structural difference between the two versions, a difference to which two different methods of analysis correspond.[24] And it seems that—in whichever way one interprets the two versions otherwise—the Stoics originally either had the external assumptions come in to deduce a premiss of the argument to be analysed or in order to deduce the conclusion of the argument to be analysed, but not both. Hence, in this respect either Alexander or Simplicius reports inaccurately, or at least does not give the early Stoic version of the *thema*. Note that this difference is such that it cannot be explained simply by a scribal error. Rather it suggests a—more or less—deliberate change or adaptation of the *thema* for some purpose. The second relevant difference is that according to Alexander the argument to be analysed can have three or

[18] By 'deductive power' of a *thema* or theorem I mean nothing but how many types of syllogisms can be reduced by them: the more can be reduced, the greater the power.

[19] Si ex duobus tertium quid colligitur, alterum eorum cum contrario illationis colligit contrarium reliquio.

[20] The first *thema* resembles the Peripatetic conversion rule for syllogisms. Only that the Peripatetic rule seems to have covered not only contradictory opposites, but contrary opposites as well (cf. Philop. *In An. Pr.* 423. 4–9; Apul. *Int.* 209. 10–210. 11). Mignucci, 'The Stoic *themata*', 227–9, favours the interpretation that the Peripatetic rule covered contradictories only, and the Stoic rule had 'negation' instead of 'contradiction'. This seems less convincing an interpretation to me, first because of the Philoponus passage cited, then because—as Mignucci concedes—Apuleius' Stoic rule is formulated with 'contradictory', and because this has a parallel in the wording of the accounts of the indemonstrables.

[21] The use of λῆμμα instead of πρότασις for 'premiss' and the reference to the τρόποι suggests Stoic rather than Peripatetic origin.

[22] *ὅταν ἐκ δυεῖν τρίτον τι συνάγηται, ἑνὸς δὲ αὐτῶν ἔξωθεν ληφθῇ συλλογιστικά, ἐκ τοῦ λοιποῦ καὶ ἐκ τῶν ἔξωθεν τοῦ ἑτέρου συλλογιστικῶν τὸ αὐτὸ συναχθήσεται.*

[23] *ἐὰν ἐκ δυεῖν τρίτον τι συνάγηται, τὸ δὲ συναγόμενον μετ' ἄλλου τινὸς ἔξωθεν συνάγῃ τι, καὶ ἐκ τῶν πρώτων δυεῖν καὶ τοῦ ἔξωθεν προσληφθέντος συναχθήσεται τὸ αὐτό.*

[24] See below, pp. 152–60 and n. 54.

more premisses—that is, in principle, indefinitely many—while in Simplicius' version it has exactly three. A third difference, which is a consequence of the first two, is that in Alexander's version the second component argument has two or more external premisses, whereas according to Simplicius, there is precisely one such premiss.

Formally the differences between Alexander's and Simplicius' versions can be made clear as follows: Take $A_1, A_2, A_3, \ldots$ for (non-external) premisses, $E, E_1, E_2, \ldots$ for external premisses, C for the conclusion of the argument to be analysed (to which I shall refer also as 'original conclusion'). The argument to be analysed always takes its place in the bottom line, while the component arguments into which it is analysed have their place in the top line.[25]

Alexander's version

$$(\mathrm{T3_A}) \qquad \frac{A_1, A_2 \vdash C \qquad E_1, \ldots, E_n \vdash A_i}{A_j, E_1, \ldots, E_n \vdash C}$$

(where $i, j = 1, 2$ and $i \neq j$; $n \geq 2$).

Simplicius' version

$$(\mathrm{T3_S}) \qquad \frac{A_1, A_2 \vdash A_3 \qquad A_3, E \vdash C}{A_1, A_2, E \vdash C}$$

This formalization reveals that—from the point of view of modern logic—both versions would count as cut-rules. Furthermore, it appears that most probably the premisses of the second component argument were called 'external' in order to indicate that the assumptions made differ from those of the first component argument.

Any attempt at reconstructing Stoic analysis has to decide on which of the two versions of the third *thema* the reconstruction should be based: that is, which version (if any) it assumes to be the

[25] The *themata* and theorems have been formalized by scholars in slightly different ways, but the differences (inasmuch as they are relevant here) are trivial. However, the external assumptions have not usually been marked out. (The exception is Becker, 'Über die vier "Themata"', 32–3.)

original Stoic one. Most modern authors present Alexander's version as authoritative,[26] but only two of them justify their choice.[27] No author takes Simplicius' version as historically more adequate.[28] I shall now discuss the reasons that have been put forward in favour of Alexander's version.

First, it has been suggested that Alexander's version is preferable since he gives the full version of the *thema*, whereas Simplicius gives a basic version only.[29] But it is most important to realize that

[26] Mates, *Stoic Logic*, 77; Kneale and Kneale, *The Development of Logic*, 169; Corcoran, 'Remarks on Stoic Deduction', 178; Gould, 'Deduction', 163; Frede, *Die stoische Logik*, 181, 193; A. A. Long and D. N. Sedley, *The Hellenistic Philosophers* (Cambridge, 1987), i. 218–19, ii. 221; Ierodiakonou, *Analysis*, 62–6.

[27] Frede and, by and large following him, Ierodiakonou.

[28] Mignucci, 'The Stoic *themata*', 221–4, seems not to commit himself to either version. Mueller, 'Completeness', 201, 203, 211, calls the two versions 'roughly equivalent', but then formalizes the third *thema* as

$$\frac{\Gamma \vdash A \qquad A, \Delta \vdash B}{\Gamma, \Delta \vdash B}$$

i.e. in a way that differs from both texts and is historically unfounded.

Alexander is sometimes understood as giving a version for arguments to be analysed with three premisses only. (Cf. Becker, 'Über die vier "Themata"', 31–4; J. Barnes's suggestion as reported in Mignucci, 'The Stoic *themata*', 223, implies a similar view.) Formally, this interpretation comes out like this:

$$(\mathrm{T3_{A\text{-}basic}}) \qquad \frac{A_1, A_2 \vdash C \qquad E_1, E_2 \vdash A_i}{A_j, E_1, E_2 \vdash C}$$

(where $i, j = 1, 2$ and $i \neq j$).

Owing to the first difference between Alexander and Simplicius, there would still be two external premisses, instead of Simplicius' one. Comparing ($\mathrm{T3_{A\text{-}basic}}$) and ($\mathrm{T3_S}$), one can see that they have the same deductive power, and differ only with respect to the premisses that are marked out as 'external'. (For example, you can get from ($\mathrm{T3_S}$) to ($\mathrm{T3_{A\text{-}basic}}$) by reformulating the latter as

$$(\mathrm{T3_{A\text{-}basic'}}) \qquad \frac{E_1, E_2 \vdash A_1 \qquad A_1, A_2 \vdash C}{A_2, E_1, E_2 \vdash C}$$

and then substituting E_1 for A_1, E_2 for A_2, A_1 for A_3, and A_2 for E.) This reading of the Alexander passage could perhaps be backed up as follows. Alexander cites the third *thema* in the context of an example that requires only a 'basic' version of the *thema* (like ($\mathrm{T3_{A\text{-}basic}}$)) (*In An. Pr.* 278. 3–14). Moreover, he uses the expression 'to syllogize something' only in cases where a conclusion follows from two premisses (cf. *In An. Pr.* 275. 5, 278. 5, 278. 20–1, 278. 33–4, 280. 3–4). However, these reasons are not compelling, and at any rate, even if Alexander's version was 'basic', the first-mentioned difference between Alexander and Simplicius would still remain in need of explanation.

[29] Frede, *Die stoische Logik*, 193. Frede tries to explain Simplicius' restriction to the basic version by pointing out that the example he adduces has only one

when one expands Simplicius' version to a 'full version' by changing 'external assumption' to 'external assumption or assumptions' (thus allowing for arguments to be analysed with three or more premisses), one obtains a rule which is categorically different from Alexander's. A formal representation of this expansion of Simplicius' version would be

$$(\mathrm{T3s_{-exp}}) \qquad \frac{A_1, A_2 \vdash A_3 \qquad A_3, E_1, \ldots, E_n \vdash C}{A_1, A_2, E_1, \ldots, E_n \vdash C}$$

This rule is deductively much more powerful in that—by way of repeated application—it allows the reduction of many more syllogisms. The reason for this is the first difference (see above), which would of course still remain. Thus Simplicius' version is not a basic version *of Alexander's* and the 'basic-vs.-full-version' model hence does not really explain the difference between the two sources. Moreover, a 'full' version of the third *thema* may not even be required in Stoic syllogistic (see below), and it is not beyond all doubt that Alexander's version really was a 'full' version (see n. 28).

As a second reason it has been stated that the fact that Simplicius introduces the *thema* with 'the account of which is this, according to the ancients' (οὗ λόγος κατὰ τοὺς παλαιοὺς τοιοῦτος) suggests that in Simplicius' time it was no longer in use, and that *because of that* Alexander has more authority in that point.[30] It seems to me, on the contrary, that someone who does not fully understand something is more likely to quote verbatim; whereas people who do, and perhaps use it in an atypical context, are more likely to change it accordingly to make it fit, because they can distinguish between what is relevant and what is not. Remember that the discrepancy in question between Alexander and Simplicius is very unlikely to have happened incidentally, e.g. as a scribal error.

A third reason given is that Simplicius' rule has a scope of application that is too limited. The assumption here is that in the original version of the third *thema* at least one component of the arguments must admit more than two assumptions—a condition

external assumption. But note that Alexander's example is no more complex than Simplicius'.

[30] Frede, *Die stoische Logik*, 193. I agree with Ierodiakonou, *Analysis*, 64, that 'the ancients' here presumably refers to some Peripatetics. I am, however, not convinced by her proposal that what Simplicius reports is in fact the synthetic theorem (for which see below, p. 164). As far as I can see, the passage implies no more than that Simplicius (or his source) copied the *thema* from a Peripatetic text.

satisfied in Alexander, but not in Simplicius.[31] However, one has to keep in mind that we have only two of the four *themata*, and it is in fact easily possible to construct the missing two *themata* in such a way that they and the third in its *basic* version, with twice two premisses only, cover all the cases to be covered.[32]

Finally, it has been argued that it is hard to see why a limited version such as Simplicius' should be a *thema*, if a further rule is required for the more general case.[33] However, one cannot rule out the possibility that there was one *thema* (e.g. the third) for three-premiss arguments to be analysed and another *thema* for four- (and more) premiss arguments, so that their scope of application does not overlap.[34]

The last two reasons for Alexander's version can also be countered in another way: it is certainly possible that, just as there was a basic version of the first *thema* for two-premiss arguments and an expanded version for two- or more premiss arguments, so there was a basic version of the third *thema* for three-premiss arguments (the one we have in Simplicius) and an expanded version for three- or more premiss arguments like the one given above (p. 148). As has been shown, such an expansion differs significantly from Alexander's version. And it would then be quite likely that in cases of three-premiss arguments to be analysed standardly only the basic form was cited—analogously to the first *thema* when used for two-premiss arguments.

Thus it turns out that none of the grounds brought forward for Alexander's version is compelling or decisive in favour of Alexander, and that there is hence so far no reason why a reconstruction should start with his rather than with Simplicius' version. In the absence of any further reasons either way,[35] I shall ground my re-

[31] Frede, *Die stoische Logik*, 193. Behind this assumption lies Frede's belief that there is hardly any prospect of finding a pair of rules which together with it has the deductive power of the synthetic theorem. (For this condition for a reconstruction of the *themata* see below, pp. 164–7.)

[32] *Pace* Frede, *Die stoische Logik*, 193. For instance, one obtains such a reconstruction by combining my second and fourth *themata* (see below) into the second and splitting up my third into a basic case as the third and a full version (minus the basic case) as the fourth; or, alternatively, by combining the full version (minus the basic case) of the third with my fourth into the fourth. Both these alternatives can easily be formulated in ordinary language.

[33] Frede, *Die stoische Logik*, 193.

[34] Mignucci, 'The Stoic *themata*', 224, contemplates a comparable idea.

[35] The differences in wording of the two versions (cf. above, nn. 20–1) are of no help. The προσληφθέντος in Simplicius calls to mind the Stoic technical term

construction on a version of the third *thema* based on Simplicius. I shall assume that there was a basic and an expanded version of the third *thema*, that we have the basic one in Simplicius, and that the expanded version is the one formalized above as ($T3_{S\text{-}exp}$). But it is also easily possible to construct variants on my reconstruction that are based only on Simplicius' basic version as the third *thema* (cf. n. 32).[36]

My main concern is not so much the point that the original Stoic third *thema* is an *expansion* of Simplicius' version, but that it—and the second and fourth *themata*—share with Simplicius' version the property that it is the second component argument from which the conclusion of the argument to be analysed is inferred (cf. the first-mentioned difference between Alexander and Simplicius above). As will become clear in the following, starting from a third *thema* that has this property leads to a family of reconstructions with fewer shortcomings than those that have been proposed, starting from Alexander. It allows a reconstruction of the second and fourth *themata* which has considerably fewer shortcomings than those reconstructions suggested on the basis of Alexander's version. In particular, the latter imply a method of 'reduction' or analysis that is rather difficult to apply (see below, n. 49), whereas reconstructions based on Simplicius' version are easily workable.

But before I present my reconstruction of the *themata*, let me make a proposal about how one could readily explain the fact that in Alexander we find a version of the third *thema* in which the first component argument (instead of the second) infers the conclusion of the argument to be analysed. For when one considers the context

πρόσληψις. But the verb was used in the same sense in Peripatetic logic at least from the time of Alexander (see e.g. Alex. *In An. Pr.* 134. 5; 197. 17; 214. 14). Then, there is a parallel in one of Chrysippus' book-titles (quoted in n. 36) to the expression συλλογιστικά τινος from Alexander's formulation. But we also find τὰ συλλογιστικά τινος quite regularly in Alexander, e.g. *In An. Pr.* 275. 5, 278. 5, 278. 20–1, 278. 33–4, 280. 3–4.

[36] We have the title of a book by Chrysippus in which he dealt with *syllogisms* with three or more premisses. This, together with the wording of the title, suggests that the book may have discussed the third and/or fourth *thema*. The title is Περὶ τοῦ τίνα συλλογιστικά τινος μετ' ἄλλου τε καὶ μετ' ἄλλων α' (D.L. 7. 194). This can be translated as 'On the question of which [propositions] syllogize something together with another [assumption] or with other [assumptions]'. If the title refers to the third *thema*, this speaks in favour of the expansion of Simplicius' version. If it refers to the third and fourth, it speaks for Simplicius' version as the third *thema*. Either way, it would not fit Alexander's version, which requires a minimum of *two* external assumptions, so that Chrysippus' μετ' ἄλλου in the singular would make no sense.

in which Alexander introduces the third *thema*, his version of it can quite plausibly be understood as an alteration of Simplicius' version for the purpose of making it fit the example Alexander is in the middle of discussing.

When one compares Alexander's version with the example and the context in which it occurs, one realizes that the context is not the *analysis* of complex arguments (as it is in Stoic syllogistic); rather the context is Aristotle's *Prior Analytics* 41^b36–42^a19, and roughly the question of how it can be that one conclusion can be derived from more than two premisses. In the case at issue,[37] Alexander (like Aristotle) starts out with the syllogism $A, B \vdash E$. He then considers the possibility that one of the premisses (A, B) of the component argument which deduces the original conclusion (E) is in turn deduced from two further premisses, C, D. (In the case discussed immediately before, Alex. *In An. Pr.* 277. 33–7, both the pair A, B and the pair C, D separately deduce E.) This procedure which starts with the 'component' argument that deduces the original conclusion is not that of someone who intends to analyse a complex syllogism into simpler ones: in the latter case the premisses first at issue would be C, D, and A[38] but not A and B. B would be 'hidden' and come to light only in the course of the analysis. On the other hand, from the perspective of someone who constructs or *synthesizes* syllogisms, no premisses are hidden; thus the construction can equally work its way 'up' from the component argument that has the original conclusion as conclusion to the remotest premisses. Only from such a perspective of synthesis (starting with $A, B \vdash E$), but not from the Stoic *analytic* perspective, do C and D naturally appear as 'externals', added as premisses that deduce one of the premisses of the argument at issue in Aristotle, i.e. the final component syllogism.[39] So, the reason why Alexander's version of the third *thema* differs structurally from Simplicius' (in the way it does) could be that Alexander or some earlier Peripatetic modified it in order to adjust it to the Aristotelian case under discussion.[40]

[37] Arist. *Pr. An.* 42^a15–19, the case τῶν *A B* θάτερον; cf. 41^b38–9, the case (διὰ τῶν) *Α Γ Δ*.

[38] Or C, D, and B, of course.

[39] That Alexander has this 'un-Stoic' understanding of the third *thema* is confirmed by the immediately following observation on prosyllogisms (*In An. Pr.* 278. 14–20), and by his remarks about the dialectic and sophistic use of arguments of the kind at issue (*In An. Pr.* 278. 32–279. 3), where C and D are referred to as ἔξωθεν ἐπιβαλλόμεναι.

[40] An obvious conjecture about how this change from Simplicius' to Alexander's

3. My reconstruction of the *themata* and how it works

Having set out the various requirements a proper reconstruction has to meet, and having discussed some of the extant evidence, I shall now present my suggestion, first in ordinary language, then formalized, juxtaposing basic and expanded versions:

The first thema (*basic version*)
When from two propositions a third follows, then from either one of them together with the contradictory of the conclusion the contradictory of the remaining one follows.

The first thema (*expanded version*)
When from two or more propositions another follows, then from all but one of them together with the contradictory of the conclusion the contradictory of the remaining one follows.

The second thema
When from two propositions a third follows and from the third and one (or both) of the two another follows, then this other follows from the first two.

The third thema
When from two propositions a third follows and from the third and one (or more) external proposition(s) another follows, then this other follows from the first two and the external(s).

The fourth thema
When from two propositions a third follows and from the third and one (or both) of the two and one (or more) external proposition(s) another follows, then this other follows from the first two and the external(s).

version of the third *thema* may have happened is the following. Alexander knew two things: firstly, that the Stoic third *thema* could somehow be applied to arguments of the type under discussion at Arist. *Pr. An.* 42a15–19; secondly, that the third *thema* (like the first, and presumably all four) started with the clause 'When from two a third follows'. Alexander then identified 'the two' from the beginning of the *thema* with A and B, and 'the third' with E, from Aristotle's argument (see above), for the simple reason that $A, B \vdash E$ is the 'component' argument *Aristotle* started out from. Once this misidentification has taken place, one almost automatically ends up with a version of the *thema* of Alexander's type: Aristotle's C and D conveniently become the external premisses that deduce either A or B, so that C and D together with the remaining premiss of the first argument, B or A, too, deduce 'the third', i.e. E.

(T1)

$$\frac{A_1, A_2 \vdash A_3}{A_i,\ \text{ctrd}\,(A_3) \vdash \text{ctrd}\,(A_j)} \qquad (\text{with } i, j = 1, 2 \text{ and } i \neq j)$$

(T1⁺)

$$\frac{A_1, A_2, \ldots, A_n \quad \vdash A_{n+1}}{A_i, \ldots, A_{i-1}, A_{i+1}, \ldots, A_n,\ \text{ctrd}\,(A_{n+1}) \vdash \text{ctrd}\,(A_i)}$$

(with $n \geq 2$ and $1 \geq i \geq n$)

(T2)

$$\frac{A_1, A_2 \vdash A_3 \qquad A_i, A_3 \vdash C}{A_1, A_2 \vdash C}$$

(with $i = 1, 2$)

(T2⁺)

$$\frac{A_1, A_2 \vdash A_3 \qquad A_i, A_j, A_3 \vdash C}{A_1, A_2 \vdash C}$$

(with $i, j = 1, 2$)

(T3)

$$\frac{A_1, A_2 \vdash A_3 \qquad A_3, E \vdash C}{A_1, A_2, E \vdash C}$$

(T3⁺)

$$\frac{A_1, A_2 \vdash A_3 \quad A_3, E_1, \ldots, E_n \vdash C}{A_1, A_2, E_1, \ldots, E_n \vdash C}$$

(T4)

$$\frac{A_1, A_2 \vdash A_3 \qquad A_3, A_i, E \vdash C}{A_1, A_2, E \vdash C}$$

(with $i = 1, 2$)

(T4⁺)

$$\frac{A_1, A_2 \vdash A_3 \quad A_3, A_i, A_j, E_1, \ldots, E_n \vdash C}{A_1, A_2, E_1, \ldots, E_n \vdash C}$$

(with $i, j = 1, 2$)

Each *thema* has a 'typical' kind of argument to which it applies; but they can also be used in combination with each other (cf. D.L. 7.78) and can be used more than once in one reduction. Taken together, they cover precisely the range of arguments which we expect the early Stoics to want to be reducible to indemonstrables.

The first *thema*, in its basic form, can firstly be used on its own to reduce certain simple arguments[41] to indemonstrables. For instance, if we have a non-indemonstrable argument

$$p;\ \text{not } q \vdash \text{not (if } p, q)$$

this can be reduced to the first indemonstrable

[41] Arguments that are not chain-inferences, or that are not compounded from more than one argument.

$$\text{If } p, q; p \vdash q$$

by employing the *thema*, as follows:

> When from 'p' and 'if p, q' 'q' follows (this being the indemonstrable), then from 'p' and 'not q' 'not (if p, q)' follows (this being the non-indemonstrable argument to be analysed).

In all cases in which such a one-step procedure leads to one of the five indemonstrables, the original argument is a syllogism. We can formalize the example, adding the application of the rule for first indemonstrables:

$$\frac{}{\text{If } p, q; p \vdash q} \qquad \text{(A1)}$$

$$\frac{}{p; \text{ not } q \vdash \text{not (if } p, q)} \qquad \text{(T1)}$$

In general, by using the *thema* on all possible simple non-indemonstrable arguments, one can show of four types of arguments that they are syllogisms, namely of those of the forms:[42]

A, not $B \vdash$ not (if A, B)	(first or second)
A, $B \vdash$ both A and B	(third)
A, $B \vdash$ not (A or B)	(fourth)
not A, not $B \vdash$ not (A or B)	(fifth)

(The indemonstrables to which they reduce are given in brackets after the argument.)

Secondly, the first *thema* can be used in combination with other *themata* and can also be employed several times in the same reduction. When the first *thema* is used in combination with others and *before* those others, its function is to make possible the application of one of the other *themata* (see below, p. 169, for an example).

Thirdly, for certain three-premiss arguments the full version of the first *thema* is needed, together with at least one other *thema*: so, for instance, in the case of those with three mode-forming premisses and with a simple proposition as conclusion. For example, in the case of the argument

$$\text{If } p, q; \text{ if } r, q; p \text{ or } r \vdash q,$$

[42] Both affirmative and negative propositions can be inserted for A, B.

the only possible first step is of that kind, e.g.:[43]

$$\frac{\text{If } r, q; p \text{ or } r; \text{ not } q \vdash \text{not (if } p, q)}{\text{If } p, q; \text{ if } r, q; p \text{ or } r \vdash q} \qquad \text{(T1}^{+}\text{)}$$

The second, third, and fourth *themata* are chain-inference rules. They all function by splitting off an indemonstrable from the rest of the argument each time they are employed, by means of the clause 'when from two a third follows'.[44]

The basic function of the second *thema* is to analyse two-premiss arguments in which one or both premisses have implicitly been used twice (or even more often). The simplest case, of the form A_1, $A_2 \vdash C$, has the following 'hidden structure' (every 'triangle' gives the form of a simple two-premiss argument with the letter at the bottom indicating the place of the conclusion. An asterisk refers to a premiss that is 'implicit' only):

type (i)

$$\begin{array}{ccccc} A_1 & & A_2 & & \\ & \backslash & / & & \\ & & A_3^* & & A_i^* \\ & & & \backslash \; / & \\ & & & C & \end{array}$$

(with $i = 1, 2$)

Arguments of this form take the second *thema* once. In the analysis, the 'hidden structure' would be reflected,[45] as one can see when one adds the asterisk to the formalized version of the *thema*:[46]

$$\frac{A_1, A_2 \vdash A_3^*; A_i^*, A_3^* \vdash C}{A_1, A_2 \vdash C}$$

(with $i = 1, 2$)

[43] For the rest of this particular reduction see below, pp. 169–70.

[44] In the case of a single or a 'final' application of the first *thema*, this phrase in the first *thema* refers to an indemonstrable too; see above.

[45] Frede's reconstruction of the second *thema* is

$$\frac{P_1, P_1, P_2 \ldots \vdash C}{P_1, P_2 \ldots \vdash C}$$

(Frede, *Die stoische Logik*, 180; cf. 178–80; 185–90). It is not a chain-inference rule and hence does not reflect the arguments from which the argument to be analysed is composed (cf. also Ierodiakonou, *Analysis*, 71–3). Used by itself, it cannot reduce an argument to indemonstrables.

[46] In terms of the—presumably Stoic—pair of expressions ἐπιβάλλων–ἐπιβαλλόμενον (cf. Alex. *In An. Pr.* 283–4) $A_1, A_2 \vdash A_3^*$ takes the ἐπιβαλλόμενον argument, $A_i^*, A_3^* \vdash C$ the ἐπιβάλλων argument.

The cases in which *one* of the premisses is used more than twice require the second *thema* twice or more times, etc.[47]

The third *thema* differs from the second in that in the second component argument of its analysis the premiss or premisses taken in addition to the conclusion of the first component argument[48] are external or taken from outside (ἔξωθεν). That is, these premisses are external only in relation to the premisses of the first component argument, namely in so far as they are distinct from them.[49] Accordingly, the third *thema* (used on its own, one or more times) takes all three- or more premiss arguments which in their 'hidden structure' do not use any of their premisses more than once. The simplest type is a three-premiss argument, i.e. of the form $A_1, A_2, A_3 \vdash C$; it takes the third *thema* once and its hidden structure is:

```
type (ii)     A1   A2
                \  /
                A4*   A3
                  \  /
                   C
```

[47] The form of these arguments is again $A_1, A_2 \vdash C$; their 'hidden' structure is:

```
type (i)'     A1   A2
                \  /
                A3*  Ai*
                  \  /
                  A4*  Ai*
                    \  /
                    etc. ...
                          \  /
                           C
```

(with $i = 1, 2$)

Two-premiss arguments that take both premisses (implicitly) twice would require one application of the second and one of the third *thema*; in this case the second *thema* is the 'full version'. Some two-premiss arguments might require the fourth after the second has been used. (No examples of either type have survived.)

[48] By 'first component argument' I mean the one that corresponds to the first clause of the respective *thema* or theorem.

[49] Frede, *Die stoische Logik*, 193, and Ierodiakonou, *Analysis*, 73–4, too, assume that this is the meaning of ἔξωθεν in this context. However, this meaning is not preserved in Frede's reconstruction: Frede's second *thema* always has to be followed by an application of his third. In these cases, when the third is applied, ἔξωθεν refers to a premiss identical to one that has been taken in the first component argument. The fact that Frede suggests renumbering them $P_1, P_2 \ldots P_{n+1}$ for $P_1, P_i \ldots P_n$ (*Die stoische Logik*, 191) does not change this: analysis can always be done with the arguments themselves, and there the premisses are identical, whichever number one assigns to them.

There are two possibilities for four-premiss arguments, i.e. arguments of the form $A_1, A_2, A_3, A_4 \vdash C$, which do not use any of their premisses more than once. Their hidden structure is of one of the following two types:

```
type (iii)  A1   A2              type (iv)  A1   A2     A3   A4
              \  /                            \  /       \  /
              A5*  A3                         A5*        A6*
                \  /                             \       /
                A6*  A4                             C
                  \  /
                   C
```

Arguments of both types require two applications of the third *thema*.[50] More complex arguments (without implicitly doubled premisses) require more applications. More precisely, in cases in which the third *thema* only is used, if there are n premisses ($n \geq 3$) then the third *thema* is applied $n - 2$ times; there will be $n - 2$ external premisses in the first step, $n - 1$ in the second, down to 1 in the last step.

The fourth *thema* is simply a combination of the second and third. It covers the cases in which there are both at least one external assumption and at least one premiss implicitly taken more than

[50] In Frede's reconstruction his third and fourth *themata* together do the work of my third *thema*: his third *thema* is an equivalent to ($T3_A$); his fourth *thema* is

$$\frac{P_1 \ldots P_i \vdash Q_1 \qquad P_j \ldots P_n \vdash Q_2 \qquad Q_1, Q_2 \vdash C}{P_1 \ldots P_n \vdash C}$$

(*Die stoische Logik*, 180–1; 193–6). In Frede's reconstruction arguments of type (iii) would take (his) third *thema* twice, arguments of type (iv) (his) fourth once. That is, for Frede it is neither the number of premisses of the argument to be analysed, nor the status of premisses of the component arguments as external or not, but the hidden structures of the arguments to be analysed which determine the introduction of the fourth *thema*. However, it is not quite clear to me why for the Stoics four-premiss arguments of type (iv) should be allotted such a prominent place and an extra *thema*, whereas four-premiss arguments of type (iii) have to make do with two applications of the third *thema*—especially since arguments of type (iv) are by no means prominent in Stoic logic or among Stoic arguments in general. In fact, so far I have not located a single instance of them.

Rather it is Aristotle and, following him, Alexander who consider this type (iv) of argument (Arist. *Pr. An.* 42ᵃ1–3; Alex. *In An. Pr.* 274. 7–19), again with the perspective that starts from the final syllogism ($A, B \vdash E$); cf. my remarks above, p. 151. Note in particular that arguments of type (iv) require two applications of the synthetic theorem, which was in Peripatetic use. (Cf. below, p. 164, and see Alex. *In An. Pr.* 275. 38–276. 8 for something like an application of it.). So I cannot see what motivation the Stoics could have had to introduce a special *thema* that analyses these arguments in only *one* step.

once. Accordingly, the simplest cases are three-premiss arguments, of the form $A_1, A_2, A_3 \vdash C$, with a hidden structure like:

$$\text{type (v)}\quad \begin{array}{ccccccc} A_1 & & A_2 & & & & \\ & \backslash & / & & & & \\ & & A_4^* & & A_i & & \\ & & & \backslash & / & & \\ & & & A_5^* & & A_3 & \\ & & & & \backslash & / & \\ & & & & C & & \end{array} \qquad \text{type (vi)}\quad \begin{array}{ccccccc} A_1 & & A_2 & & A_i^* & & A_3 \\ & \backslash & / & & & \backslash & / \\ & & A_4^* & & & A_5^* & \\ & & & \backslash & / & & \\ & & & & C & & \end{array}$$

(with $i = 1, 2$)

(or a variation of (v) in which A_i and A_3 are interchanged). Application of the fourth *thema* has always to be followed by that of a third or a fourth (or of a full version of the first); the last application of the fourth in any reduction has to be followed by an application of the third. This, of course, is so because the fourth *thema* is the only one that has always an argument with at least three premisses left after an indemonstrable has been split off; and the only *thema* that cuts down the number of the (remaining) premisses is the third.

One can conceive of more complex arguments that take combinations of the second, third, and fourth *themata*, and even of arguments that take all four. Thus, with the four *themata* in my reconstruction, all non-indemonstrable arguments that are composed of indemonstrable arguments can actually be reduced.[51]

Before I describe the general method of analysis of arguments into indemonstrables, I shall illustrate the application of the *themata* by way of two examples. I have chosen examples of the same forms as those used by Sextus Empiricus (*M.* 8. 230–8) to demonstrate Stoic analysis by means of a dialectical theorem (for which see below, p. 165). There, the second argument to be analysed is a three-premiss argument of the same form as this one:

If (p and q), r; not r; $p \vdash$ not q.

It is of type (ii), i.e. it takes the third *thema* once. By simply 'inserting' this argument into the *thema* we obtain:

> When from two propositions (i.e. If (p and q), r; not r) a third follows (i.e. not (p and q), according to a second indemonstrable) and from the third and an external one (i.e. p) another follows (i.e. not q, according to a third indemonstrable), then this other one (i.e. not q) also follows from the first two propositions and the external one.[52]

[51] For one special case that cannot be reduced cf. below, pp. 175–9.

Sextus' first example is a two-premiss argument of type (i). It has the same form as this argument:

If p, (if p, q); $p \vdash q$,

and it takes the second *thema* once:

> When from two propositions (If p, if p, q; p) a third (if p, q) follows, and from the third and one of the two (p) another follows (q), then this other follows from the first two.[53]

In general then, the method of analysis into indemonstrables by means of the *themata* appears to have worked as follows: In a very first step, you check whether the argument to be analysed, or original argument, happens to be an indemonstrable, i.e. whether any of the rules (A1)–(A5) applies. If so, the argument is a syllogism. If not, the next thing to do is always to try to pick from the set of premisses of the argument to be analysed two from which a conclusion can be deduced by forming an indemonstrable together with them.

If the original argument is a syllogism, this conclusion, together with the remaining premiss(es) (if there are any), and/or one or both of the premisses that have been used already, implies the original conclusion—either by forming an indemonstrable or by forming an argument that by use of any of the four *themata* can be analysed into one or more indemonstrables. Hence you look to see whether one of the remaining premisses plus this conclusion yields the premisses to another indemonstrable (in which case you apply the third *thema*); if there are no premisses left, or none of them works, you look to see

[52] Or, formalized:

$$\text{(A2)}\ \dfrac{\dfrac{}{\text{If } (p \text{ and } q),\ r;\ \text{not } r \vdash \text{not } (p \text{ and } q);} \qquad \dfrac{}{\text{not } (p \text{ and } q);\ p \vdash \text{not } q}\ \text{(A3)}}{\text{If } (p \text{ and } q),\ r;\ \text{not } r;\ p \vdash \text{not } q}\ \text{(T3)}$$

[53] Again, formalized:

$$\text{(A1)}\ \dfrac{\dfrac{}{\text{If } p, (\text{if } p, q);\ p \vdash \text{if } p, q} \qquad \dfrac{}{\text{if } p, q;\ p \vdash q}\ \text{(A1)}}{\text{If } p, (\text{if } p, q);\ p \vdash q}\ \text{(T2)}$$

whether one of the premisses used already in the first step is such a premiss (in which case you apply the second or the fourth *thema*).

If the second component argument thus formed is an indemonstrable too, and all premisses have been used at least once and the last conclusion is the original conclusion, the analysis is completed, the original argument a syllogism. If not, the same procedure is repeated with the argument which is not an indemonstrable (i.e. the second component argument, which has the original conclusion as conclusion); and so forth until the premisses of the second component argument imply the original conclusion by forming an indemonstrable with it.

If at any point in the analysis no indemonstrable can be formed, there is still the chance that the first *thema* might help: namely, if the negation of the conclusion would produce a premiss you need, i.e. a premiss which together with one of the available premisses makes up a pair of premisses for an indemonstrable. If at any step the application of none of the *themata* leads to two premisses that can be used in an indemonstrable, the argument is not a syllogism.

This method of reduction is practicable and easy. All one has to know is the *themata* and the five types of indemonstrables, plus those four types of simple arguments which can be reduced to indemonstrables (see above, p. 154). (One can always reobtain these four types by using the first *thema* on all simple arguments.) The number of steps one has to go through is finite; they are not very many, even in complex cases. The method appears to be effective, the system decidable.[54]

[54] Note that we have no examples of how the reductions work in the cases in which more than one *thema* is used. We hence do not know whether the reductions worked from the argument to be analysed to the indemonstrables ('backwards') or from the indemonstrables to the argument to be analysed ('forwards'), and we have to rely on conjecture. (Note further that my reconstruction of the *themata* is completely independent of how this question is answered.) The reason why I am quite confident that the method worked 'backwards', as described in the main text, is that in the Stoic system this method is much more straightforward and appears to be decidable, and that its application is so much easier. In contrast, the method implied by Frede's reconstruction (which works forwards) is rather difficult to handle. Frede outlines the procedure in *Die stoische Logik*, 191. He assumes that the component argument split off in the cases of application of the third and fourth *themata* is that which has the final conclusion as conclusion. Accordingly, the task in each step is not:

> Find among the premisses of the argument to be analysed two premisses which could be the premisses of an indemonstrable,

but:

> Find among those premisses one which, together with some other proposition, would form an indemonstrable with the final conclusion as conclusion (in the case of the third *thema*). Or, if that does not work, it might be a case of the fourth *thema*: then find two premisses which are not premisses of the argument to be analysed which imply the final conclusion as conclusion (and are conclusions of arguments the premisses of which taken together imply the final conclusion).

Try out this method with the following two four-premiss arguments:

not (p and q)	if p, q
if not r, q	if ((if r, s) and t), p
s or p	not q
if r, s	t
not p	not (If r, s)

You will almost certainly obtain a successful analysis only after you have worked through the complete hidden structure of the argument in your head (or on a piece of paper), and then, having worked 'backwards' from there, applied the *themata*. And this would in a way be a bit like cheating: for it is in fact not much different from using the dialectical theorem (see below, p. 165). One problem with Frede's method is that for the first steps there is an enormous number of possibilities, which makes the method, if not ineffective, at least impracticable. The same holds for Ierodiakonou's suggestion (*Analysis*, 72–5). In my reconstruction the number of possibilities in each step is clearly limited.

It is perhaps not out of place briefly to compare Stoic analysis, which is basically a 'backward' or 'reductive' method of deduction, with ordinary 'forward' methods of deduction as we find them in many modern systems of logic. I shall restrict myself here to argumental (as opposed to sentential) axiomatic deductive systems.

A 'forward' method of deduction attempts to give a proof of a particular argument by starting from the axioms of the system and deriving from them (and/or other arguments already proved) in one or more steps, via the deduction rules of the system, the argument to be proved. The method works *from* the axioms etc. *to* that argument. Typically, such a derivational approach involves an 'imaginative process', since there is no mechanical procedure one can follow. Backward deduction, like the Stoic one, on the other hand, starts from the argument to be proved and, by using the deduction rules of the system, attempts to *end up* with nothing but axioms (i.e. indemonstrables). In this case, the deduction rules (i.e. the *themata*) are as it were used 'backwards'[55] and are applied in the first step on the argument to be proved. This method works *from* the argument to be proved *to* the axioms, and the Stoic procedure is at least semi-mechanical.

This difference between forward and backward method extends to the 'axioms': In ordinary 'forward' deduction, it is the function

[55] So tentatively suggested by Corcoran, 'Remarks on Stoic Deduction', 180.

of logical axiom schemata or of *ab initio* rules to generate axioms or formally valid arguments that are not in need of a proof. Such schemata or rules would always be used *at the beginning* of a deduction.

As indicated above (pp. 139–40), in Stoic logic the five accounts of the indemonstrables have roughly the function of axiom schemata or of nullary rules. However, just as the *themata* are best understood as deduction rules 'used backwards', so the accounts can be interpreted as *ab initio* rules 'used backwards': they are used not to generate syllogisms, but to test particular arguments, which evolve in the course of an analysis for syllogismhood. Accordingly, they are not used first in a deduction, but only after at least one of the (other) deduction rules has been used, and they are always used in the last step of a particular deduction. And once an account has been applied successfully to an argument, no other deduction rule can be used on that 'branch' of the reduction. When on all branches of a reduction an account has been used, the reduction is successfully completed. Hence the accounts of the indemonstrables can perhaps best be characterized as argumental end-rules.

Schematically, a successful Stoic reductive deduction or analysis can be presented like this (Greek capital letters stand for sets of propositions with at least two elements):[56]

$$
\begin{array}{lccc}
(a) & & \Gamma \vdash A & \\
(b) & \underline{\Delta \vdash B} & & \mathrm{E} \vdash C \\
(c) & & \mathrm{Z} \vdash D & \underline{\mathrm{H} \vdash E} \\
(d) & & \underline{\Theta \vdash F} &
\end{array}
$$

For example, in the first step (from (*a*) to (*b*)) the third *thema* could have been used; in the second step (from (*b*) to (*c*)) the third *thema* could have been used again, and in the third step (from (*b*) to (*c*)) the first *thema*; in addition, in each step one account of an indemonstrable would have been used to close off a 'branch' of the reduction.

[56] Note that, while I present particular deductions as working from the bottom line upwards, I present this schema the other way around in order to emphasize the 'backwards' character of the deductions.

A 'forward' deduction, on the other hand, would have run from (*d*) to (*a*), generating an indemonstrable in its first step. In the case of a successful (completed) proof there is little difference between the two types of methods: on paper, the resulting deductions may look just the same—only that they were started at opposite ends. The difference becomes relevant and visible, however, in cases in which one does not succeed in deducing an argument (for an illustration see below, p. 174).

4. The requirements a reconstruction of the *themata* has to satisfy

It remains to show that my suggestion for the reconstruction of the *themata* satisfies the requirements listed above, pp. 143–4.

(*a*) *General points*

It has just been demonstrated that the *themata* are easily applicable. They are also easily formulable in ordinary language, as a look at my formulations above will confirm.[57] Moreover, all the rules have a fairly similar overall structure and are applied in comparable ways. What is more, the second, third, and fourth *themata* are all proper chain-inference rules which mirror the structure of the arguments from which the argument to be analysed has been composed. They also form a relatively natural sequence, from the simplest case that reduces non-indemonstrable two-premiss arguments, to that of ordinary three- (or more) premiss arguments, to the more complex case which takes arguments that combine the distinctive features of the arguments reduced by the second and those reduced by the third *thema*.

Furthermore, as has been shown above, the expression ἔξωθεν in the third and fourth *themata* makes proper sense. In addition, it

[57] For comparison, try to formulate Frede's suggestions for the fourth and second *themata*. Cf. nn. 40 and 45 above for the formalized versions. Long and Sedley, *The Hellenistic Philosophers*, i. 219, suggest as a formulation of Frede's fourth *thema*:

> When from two propositions a third is deduced, and extra propositions are found from which one of those two follows syllogistically, and other extra propositions are found from which the other of those two follows syllogistically, the same conclusion will be deduced from all the extra propositions taken together.

becomes quite clear why the third *thema* expressly mentions that premisses have to be external. It is the feature that distinguishes the third *thema* from the second. In the second *thema* the premisses used in the second component argument (in addition to the conclusion of the first component argument) are not external to the premisses of the first component argument.[58] That this distinction is pertinent to Stoic syllogistic—but would hardly be so to a modern system with argumental deduction rules—has its ground in the fact that the Stoic method is reductive. They start from a given argument, searching for the component arguments from which it is made up. And from this perspective it does matter that one knows whence or in which ways one can obtain the premisses for the second component argument, and that the ways differ, depending on whether one has an original argument with two, three, or more premisses.

(b) *The four* themata *and the dialectical and synthetic theorems*

Apart from the Stoic *themata*, there were two further rules known and used for the reduction of syllogisms in antiquity; both seem to have performed by and large the same task as the *themata*. In Alexander (and Themistius) we several times encounter a rule with the name 'synthetic theorem (*συνθετικὸν θεώρημα*)'.[59] It runs:

> When from some propositions something (A) follows, and from that which follows (A) together with one or more propositions something (B) follows, then, too, from those propositions from which it (A) follows together with the one or more propositions from which together with it (A) something (B) follows, the same thing (B) follows.[60] (Alex. *In An. Pr.* 278. 8–11)

This may be tentatively and provisionally formalized as:

$$\frac{A_1, \ldots, A_n \vdash A_{n+1} \qquad A_{n+1}, \ldots, A_m \vdash C}{A_1, \ldots, A_n, A_{n+2}, \ldots, A_m \vdash C}$$

Alexander reports that the Stoics took the (Peripatetic) synthetic theorem, divided it (up), and made from it the so-called second,

[58] I first became aware of this point through Ierodiakonou's reconstruction, in which this holds as well. Cf. Ierodiakonou, *Analysis*, 72–4.

[59] Alex. *In An. Pr.* 274. 21–4, 278. 8–11, 283. 15–17; Them. *In An. Pr.* 81. 7–9; 84. 16–18.

[60] *ὅταν ἔκ τινων συνάγηταί τι, τὸ δὲ συναγόμενον μετά τινὸς ἢ τινῶν συνάγῃ τι, καὶ τὰ συνακτικὰ αὐτοῦ, μεθ' οὗ ἢ μεθ' ὧν συνῆγέ τι ἐκεῖνο, καὶ αὐτὰ τὸ αὐτὸ συνάξει.*

third, and fourth *themata*.[61] We should be sceptical of the historical claim that the Stoics 'divided the synthetic theorem up', i.e. developed their *themata* from it. But the passage certainly suggests that the deductive power of the synthetic theorem was considered the same as that of the three *themata* taken together[62] or at least that Alexander thought this, or wanted his readers to believe it. Sextus Empiricus presents us with a dialectical theorem for analysis (*M.* 8. 231), and illustrates its application with two examples (ibid. 232–8). This theorem runs:[63]

> When we have the conclusive premisses of some conclusion, then we have potentially in those premisses that conclusion as well, even if it is not explicitly stated.[64]

It has been argued that the synthetic and dialectical theorems allow the reduction of the same syllogisms or that they have the same deductive power.[65] This is correct (when they are interpreted in a certain way). However, the dialectical theorem is considerably less precise. For instance, it would in principle allow the second component argument to have one premiss only, whereas the synthetic theorem clearly requires a minimum of two premisses. It also leaves it open how many applications of the rule are required for certain arguments, i.e. whether certain reductions are carried out in one, two, or more steps. (Sextus' illustration leaves no doubt about how it is applied in the case of the paradigm arguments of types (i) and (ii); but for more complex ones this remains unclear.) In the case of the synthetic theorem it is clear that certain arguments (e.g. of types (iii) and (iv)) need repeated application of the theorem. The dialectical theorem states, as it were, the general idea of the reduc-

[61] Alex. *In An. Pr.* 284. 10–17; a few pages earlier (*In An. Pr.* 278. 6–8) he has said already that the third *thema* falls under the synthetic theorem.

[62] This has been questioned by Mignucci, 'The Stoic *themata*', 225, his reason being that this would be at odds with the claim of the Peripatetics that the Stoic second *thema* is useless (Alex. *In An. Pr.* 164. 27–31). But I do not think that this is a real difficulty: at *In An. Pr.* 164. 27–31 Alexander says only that Aristotle did not explicitly deal with arguments which can be reduced with the second *thema*, and justifies him by pointing out that those arguments are demonstratively useless. This claim is quite compatible with the fact that such arguments could be reduced by the synthetic theorem, since they are implicitly covered by it: only that—in contrast to Stoic syllogistic—such arguments would be mere uninteresting by-products.

[63] Alex. *In An. Pr.* 274. 12–14 states a rule that is almost identical to this dialectical theorem.

[64] *ὅταν τά τινος συμπεράσματος συνακτικὰ λήμματα ἔχωμεν, δυνάμει κἀκεῖνο ἐν τούτοις ἔχομεν τὸ συμπέρασμα, κἂν κατ' ἐκφορὰν μὴ λέγηται.*

[65] Mignucci, 'The Stoic *themata*', 218–20.

tion of chain inferences; the synthetic theorem provides a proper rule that can be applied almost mechanically.[66]

How, then, do the synthetic and dialectical theorems incorporate the second, third, and fourth *themata*? The answer is simply: the distinction between implicitly repeated 'internal' premisses and external premisses was dropped (or not made in the first place). The synthetic theorem begins: 'When from some propositions ($A_1, A_2, \ldots$) something follows, and from that which follows together with one or more propositions ($A_{n+2}, A_{n+3}\ldots$) . . .'. Here it is nowhere stated that the second group of premisses ($A_{n+2}, A_{n+3}\ldots$) have to differ from the first ($A_1, A_2\ldots$). They are certainly never referred to as 'taken from outside'.[67] Hence, one should think, the theorem covers cases in which in the second component argument

- all we have are one or more 'external premisses' ($A_{n+2}, A_{n+3}\ldots \neq A_1, A_2\ldots$);
- we have the same ones only (all $A_{n+2}, A_{n+3}\ldots$-premisses are identical with $A_1, A_2\ldots$-premisses);
- we have both at least one premiss that is the same and at least one that is different or 'external'.

Thus, the synthetic theorem covers precisely the same ground as the second, third, and fourth *themata* together.

The dialectical theorem starts 'When we have the conclusive premisses of some conclusion . . .', and the second or last component argument is not even mentioned. Evidently, there is no distinction made in this theorem between implicitly repeated premisses and external ones. More importantly, Sextus' illustration of the theorem with examples makes clear beyond all doubt that, regarding the second component argument, it is possible both that a premiss from the first argument is repeated (so in his first example, *M.* 8. 230; 232–3) and that the premiss(es) are 'external' (so in his second example, *M.* 8. 234–8). It should be a safe guess that more complex arguments which have both repeated and external premisses were also covered by the theorem. Hence, the dialectical theorem too appears to cover precisely the same ground as my second, third, and fourth *themata* together.[68]

[66] A third author, Galen, in *De Hipp. et Plat. dogm.* 2. 3. 19, p. 114 DeLacy, says about the arguments which can be analysed by using the four *themata* that 'as Antipater wrote, most of these [syllogisms] can be analysed in a different and shorter way'. This has led some people to assume that Antipater might have developed the dialectical or the synthetic theorem. The 'most' in the quotation would then refer to all those syllogisms that do not require *themata* other than the first in their analysis. At least the passage suggests that a subgroup of the *themata* can be simplified; and the most obvious way to simplify my second, third, and fourth *themata* is to make them into something like the synthetic theorem.

[67] Note that Chrysippus in the title of his books on syllogisms to which I referred earlier has συλλογιστικὰ μετ' ἄλλου τε καὶ μετ' ἄλλων (D.L. 7. 194), which should refer to those premisses taken from outside; the synthetic theorem in the corresponding places, on the other hand, only has μετά τινὸς ἢ τινῶν.

(c) *Requirements concerning specific* themata

A reconstruction of the *themata* has to square with two further testimonies which provide information about the *themata* and their application. First, the Galen passage *De Hipp. et Plat. dogm.* 2. 3. 18, p. 114 DeLacy,[69] implies that the Stoics distinguished types of syllogisms that were analysed by way of the first and/or[70] second *thema*, and types of syllogisms that were analysed by using the third or[71] fourth *thema*.[72] This claim is borne out by my reconstruction: the third and fourth *themata* have something in common which the first and second lack, namely that they require 'external premisses', and accordingly, that they can be applied only to arguments with at least three premisses. As to the application of the first and/or second *thema*, Galen mentions two types of arguments: those with two mode-premisses and the so-called 'indifferently concluding' ar-

[68] The fact that the synthetic theorem starts '$A_1 \ldots A_n \vdash \ldots$' instead of simply '$A_1, A_2 \vdash \ldots$', as the *themata* do (in my reconstruction), does not increase its deductive power; nor does it reduce the number of applications of the rule needed in the analysis. In fact, as long as the function of the rule is to *reduce* complex arguments, the addition of the possibility of more than two premisses for the first component arguments seems altogether superfluous: all it allows in certain cases is an alternative 'route' to the basic syllogisms.

[69] νυνὶ δὲ πῶς μὲν οἱ διὰ δύο τροπικῶν ἀναλύονται συλλογισμοὶ καὶ πῶς οἱ ἀδιαφόρως περαίνοντες ἤ τινες ἄλλοι τοιοῦτοι τῷ πρώτῳ καὶ δευτέρῳ θέματι προσχρώμενοι, πολλοῖς ἔστι συντυχεῖν ἀκριβῶς ἠσκημένοις, ὥσπερ ἀμέλει καὶ ἄλλοις ἐφ' ὅσους διὰ τοῦ τρίτου θέματος ἢ τετάρτου συλλογισμοὺς ἀναλύουσι (Gal. *De Hipp. et Plat. dogm.* 2. 3. 18, p. 114 DeLacy).

[70] I say 'and/or' since the 'and' (καί) in the text is ambiguous in this respect. Cf. 'The pupils of the first and second classes who use the first and second volumes of Johnson's *Human Biology* . . .', where it is open which group of pupils uses which volume(s).

[71] The 'or' (ἤ) could be inclusive or exclusive; that is, the formulation of the text does not preclude the possibility that certain arguments are analysed by use of both the third and the fourth *themata*.

[72] The structure of the sentence in Galen is somewhat unclear, but whichever way one reads it, the distinctions I have given seem to be implied. Perhaps there were 'books' or chapters on analysis which dealt with typical or standard cases of the application of the first and second *themata*, and others that dealt with the application of the third and fourth *themata*.

guments or syllogisms. The second passage, Alex. *In An. Pr.* 164. 27–31, provides us with the additional information that the Peripatetics considered duplicated arguments, indifferently concluding arguments, the infinite content, and in general the second *thema* as useless.[73] This listing suggests that if arguments of any of the types enumerated need reduction, they require the second *thema*. In addition to those mentioned by Galen, such arguments could be the duplicated arguments and what is called 'the infinite content'.

I shall now briefly demonstrate how my reconstruction fits in with these claims concerning the first and second *themata* and the uselessness of the latter. For the indifferently concluding arguments ((ἀ)διαφόρως περαίνοντες)[74] we obtain as example: 'Either it is day or it is night. Now it is day. Therefore it is day.'[75] Their general form (or at least that of one type of them) is hence:

$$A \text{ or } B; A \vdash A$$

Their name is presumably based on the fact that it is irrelevant for the argument's validity what comes in as second disjunct. Arguments of this type take the second *thema* once, e.g.:

$$\begin{array}{ll} (b) & \\ (a) & \end{array} \quad \dfrac{\text{(A4)}\ \overline{p \text{ or } q; p \vdash \text{not } q} \qquad \overline{p \text{ or } q; \text{not } q \vdash p}\ \text{(A5)}}{p \text{ or } q; p \vdash p}\ \text{(T2)}$$

The duplicated (δι(α)φορούμενοι) arguments or syllogisms are commonly mentioned in tandem with the indifferently concluding ones.[76] Their name appears to rest on the fact that their leading premiss (or one of their non-simple premisses) is a so-called 'duplicated proposition' (D.L. 7. 68–9; Sextus, *M.* 8. 108), i.e. is composed of the same simple proposition used twice or several times. The standard and only example that has survived is: 'If it is day, it is day. Now it is day. Therefore it is day.' It has the general form

$$\text{If } A, A; A \vdash A.$$

[73] ὅθεν δῆλον, ὅτι καὶ ταῦτα, περὶ ὧν αὐτὸς μὲν οὐκ εἴρηκε, λέγουσι δὲ οἱ νεώτεροι ἀχρήστων ὄντων πρὸς ἀπόδειξιν, δι' ἀχρηστίαν οὐ δι' ἄγνοιαν παρέλιπεν, οἷοί εἰσι διαφορούμενοι οἱ λόγοι ἢ διαφόρως περαίνοντες ἢ ἡ ἄπειρος ὕλη λεγομένη καὶ καθόλου τὸ θέμα τὸ δεύτερον καλούμενον παρὰ τοῖς νεωτέροις (Alex. *In An. Pr.* 164. 27–31).

[74] I shall discuss the indifferently concluding syllogisms and the duplicated syllogisms in more detail elsewhere.

[75] Apul. *Int.* 201. 4–6; cf. Alex. *In Top.* 10. 10–12.

[76] Alex. *In Top.* 10. 7–10; *In An. Pr.* 18. 17–18; Apul. *Int.* 201. 6–7.

Being a special case of the first indemonstrable, it needs no reduction. Non-indemonstrable duplicated arguments occur nowhere in our texts, but arguments of the following form could be examples:

$$A \text{ or } A; A \vdash A$$
$$\text{If } A \text{ (if } A, A); A \vdash A.$$

They would take the second *thema* once, the first with an implicitly doubled premiss of the form '*A* or *A*', the second with one of the form '*A*'.

The second type of syllogism mentioned by Galen as taking the first and/or second *thema* are those with two mode-premisses (οἱ διὰ δύο τροπικῶν), that is, it seems, two-premiss arguments composed of two mode-premisses[77] which have a simple proposition as conclusion. The examples we get are of the form (Orig. *Cels.* 7. 15, pp. 166–7):

$$\text{If } A, B; \text{ if } A, \text{ not } B \vdash \text{not } A.$$

Syllogisms of this form take the first *thema* twice, the second once,[78] e.g.:

$$\begin{array}{l} (d) \\ (c) \\ (b) \\ (a) \end{array} \quad \dfrac{\dfrac{\text{(A1)}\ \overline{\text{If } p, q; p \vdash q} \qquad \dfrac{\overline{p; \text{if } p, \text{ not } q \vdash \text{not } q}\ \text{(A1)}}{p, q; p \vdash \text{not (if } p, \text{ not } q)}\ \text{(T1)}}{\text{If } p, q; p \vdash \text{not (if } p, \text{ not } q)}\ \text{(T2)}}{\text{If } p, q; \text{ if } p, \text{ not } q \vdash \text{not } p}\ \text{(T1)}$$

A type of syllogism related to that with two mode-premisses is that consisting of three mode-premisses and a simple conclusion. We know that some Stoics employed arguments of such a kind, in particular of the form of a simple constructive dilemma, i.e.

$$\text{If } A, B; \text{ if } C, B; A \text{ or } C \vdash B.$$

Their reduction requires the first *thema*, the fourth, the third, and the first again,[79] e.g.:

[77] Premisses which are a disjunction or a conditional or a (negated) conjunction (cf. above, p. 136); i.e. 'hypothetical premisses' in the Peripatetic jargon (e.g. Gal. *Inst. log.* 3. 4).

[78] Arguments of the form 'not (*A* and *B*); not (*A* and not *B*) ⊢ not *A*' can be reduced in the same way.

[79] In some manuscripts of Gal. *De Hipp. et Plat. dogm.* 2. 3. 18, p. 114 DeLacy, we

(A1)
(e) (A5) p; if p, $q \vdash q$ (T1)
(d) p or r; not $r \vdash p$ p; not $q \vdash$ not (if p, q) (T3)
(c) (A2) If r, q; not $q \vdash$ not r p or r; not r; not $q \vdash$ not (if p, q) (T4)
(b) If r, q; p or r; not $q \vdash$ not (if p, q) (T1$^+$)
(a) If p, q; if r, q; p or $r \vdash q$

A special case of this type is of the form:

If A, A; if not A, A; A or not $A \vdash A$.

We have many instances of this kind in Sextus (*M.* 8. 281–2; 292; 466–7; 9. 205–6; *PH* 2. 186). Such arguments could be reduced in the same way as the general type. Unlike the general case, they might well be thought to be useless; and indeed, they could also be reduced by using the first *thema*, the third, the *second*, and the first again.[80]

Finally, Alexander is the only source that mentions the arguments with infinite content (ἡ ἄπειρος ὕλη).[81] The most plausible guess about their structure (and it cannot be more than a guess) is that these are arguments of the following forms:[82]

(i) If A, (if A, B); $A \vdash B$
(ii) If A, (if A, (if A, B)); $A \vdash B$
(iii) If A, (if A, (if A, (if A, B))); $A \vdash B$
etc.

have τριῶν after οἱ διὰ δύο τροπικῶν (cf. the quotation of the passage in n. 68), which is commonly emended to ἢ τριῶν. I do not assume that ἢ τριῶν was part of the original text. If it was, this would suggest that these arguments required the second *thema*. However, according to the passage of Alexander quoted in n. 73, arguments that use the second *thema* appear useless from a Peripatetic point of view; but we have no reason to assume that the Peripatetics considered simple constructive dilemmas as useless.

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if p, p; not $p \vdash$ not p (A2)
(A1) If not p, p; not $p \vdash p$ p, not $p \vdash$ not (if p, p) (T1)
(A5) p or not p; not $p \vdash$ not p not p; if not p, p; $\vdash$ not (if p, p) (T2)
p or not p; if not p, p; not $p \vdash$ not (if p, p) (T3)
If p, p; if not p, p; p or not $p \vdash p$. (T1$^+$)

[81] In fact, we cannot even be certain that they were arguments.

[82] Cf. Becker, 'Über die vier "Themata"', 38; Frede, *Die stoische Logik*, 185, 189–90.

This fits the name well; we can also see why Alexander could think of such arguments as futile. The Stoics, we know, did not: we have an example for (i) in Sextus, *M.* 8. 230–3 (see above). Of this type of argument, those of form (i) take the second *thema* once, those of form (ii) take it twice, those of form (iii) three times, etc. Hence, in my reconstruction, this conjecture also squares well with what the Alexander passage implies, namely that they take the second *thema*.

There are two further points which no one who aims to reconstruct Stoic syllogistic can ignore: one concerns the Stoic general concept of argument, the other the Stoic general criterion for validity of arguments.

(d) *The Stoic concept of argument*

For the Stoics an argument has to have at least two premisses and a conclusion, and that has certain consequences for their syllogistic which a reconstruction has to take into account. The Stoics define an argument as being composed of premiss*es* and conclusion (D.L. 7. 45; Sextus, *M.* 8. 301; *PH* 2. 135). Moreover, we have direct evidence that the Stoics (except Antipater) denied the existence of one-premiss arguments (μονολήμματοι).[83] This implies that for them a compound of propositions of the form

[83] Cf. Sextus, *M.* 8. 443 τὸ μὲν γὰρ λέγειν μὴ ἀρέσκειν τῷ Χρυσίππῳ μονολημμάτους εἶναι λόγους, ὃ τάχα τινὲς ἐροῦσι πρὸς τὴν τοιαύτην ἔνστασιν, τελέως ληρῶδες. οὔτε γὰρ ταῖς Χρυσίππου φωναῖς ὡς πυθοχρήστοις παραγγέλμασιν ἀνάγκη πείθεσθαι, οὔτε μαρτυρίᾳ προσέχειν ἀνδρῶν ἔστιν, †εἰς οἰκείαν ἀπόρρησιν† ἐκ μάρτυρος τοῦ τὸ ἐναντίον λέγοντος· Ἀντίπατρος γάρ, τῶν ἐν τῇ Στωϊκῇ αἱρέσει ἐπιφανεστάτων ἀνδρῶν, ἔφη δύνασθαι καὶ μονολημμάτους λόγους συνίστασθαι. See also Sextus, *PH* 2. 167 εἰ δὲ οὐκ ἀρέσκει τισὶ λόγους μονολημμάτους εἶναι, οὐκ εἰσὶν ἀξιοπιστότεροι Ἀντιπάτρου, ὃς οὐδὲ τοὺς τοιούτους λόγους ἀποδοκιμάζει. This is confirmed by Varro, *Sat. Men.* fr. 291 (Macropolis), p. 50 Astburg 'Cui celer δι' ἑνὸς λήμματος λόγος, Antipatri Stoici filius, rutro caput displanat'.

The remaining passages on μονολήμματοι stem from texts on Peripatetic logic. (Two mention Antipater (Apul. *Int.* 200; Alex. *Top.* 8–9), one 'the moderns' (οἱ νεώτεροι). The rest do not mention the Stoics at all (Alex *In An. Pr.* 17–18, 21–2; *Top.* 574; Philop. *In An. Pr.* 35–6, 33; Amm. *In An. Pr.* 27, 32); the last three passages connect the μονολήμματοι with the rhetoricians.) None of these texts says expressly that for the Stoics there are no one-premiss arguments. Equally, they never state that any Stoic (other than Antipater) admits one-premiss arguments. (The context is always Aristotle's claim that *syllogisms* must have at least two premisses. Hence these authors have no interest in making the point that there are no one-premiss *arguments*—although they do not always clearly distinguish between syllogisms and arguments.)

Against the evidence in Sextus and Varro, Mueller, 'Completeness', 203–4, and Mignucci, 'The Stoic *themata*', 229, assume that Chrysippus did accept certain

$$A \vdash B$$

did not count as an argument—even if the corresponding conditional proposition of the form

$$\text{If } A, B$$

was regarded as true; and this although the relation of consequence (*akolouthia*) between premisses and conclusion in an argument appears to have been considered the same as that between antecedent

ferences with a single premiss both as arguments and as valid. Mignucci's and presumably Mueller's view is in short:

- that Chrysippus rejected arguments that (1) have a single premiss and (2) are correct for extralogical reasons (it is unclear whether their existence or their validity is meant to be rejected by Chrysippus);
- that Antipater called arguments which satisfy both these conditions 'one-premiss arguments';
- that Chrysippus accepted arguments that (1) have a single premiss and (2) are correct for logical reasons (like e.g. those of the form 'If not A, $A \vdash A$'), and that these were not called 'one-premiss arguments'.

The motivation for this view is for both authors that in *their* reconstructions of Stoic systems logically valid sequents or inferences with a single premiss can be derived, and that they think they should be derivable (see pp. 184–5).

Mignucci and Mueller back up their view by the claim that according to Alex. *In Top.* 8. 18 ff. Chrysippus rejected one-premiss arguments of the type 'It is day. Therefore it is light'. However, the passage does not say anything at all about Chrysippus (nor do any of the other Peripatetic passages). It only states that *Antipater's* one-premiss arguments were of that type. Nowhere do we hear of any other kinds of Stoic arguments with a single premiss. This suggests that the best conjecture is that the Stoics did not admit other kinds of arguments with a single premiss, and *a fortiori*, that there were no other kinds of Stoic arguments with a single premiss that they considered valid. Moreover, μονολήμματος λόγος, which I translate as 'one-premiss argument', means exactly that: 'argument with one premiss'. So, had there been more types of arguments with one premiss, and the Stoic dispute was only about that type for which examples are given in the texts (see above), it would have been an absurd move for Antipater to call them μονολήμματοι simply 'one-premiss arguments'. To argue that the fact that the Stoics called the one-premiss arguments 'one-premiss *arguments*' suggests that the Stoics considered them to be arguments is implausible: Antipater introduced one-premiss arguments (see [illegible], quoted above), and he hence gave them the obvious name, 'one-premiss arguments'. When used by other philosophers, 'argument' here has to be understood as in a quotational context: 'Those things that Antipater called "one premiss arguments"'. (Think of examples like 'National Socialism': when someone uses the term, we certainly cannot infer from this that this person thinks that National Socialism is a type of socialism.)

A historically more plausible interpretation of the μονολήμματοι is hence:

- that Antipater allowed arguments with one premiss, because he thought that there are some valid arguments with one premiss, of which the examples given or similar ones were some;
- that Antipater's label μονολήμματοι λόγοι denoted all arguments with only one premiss (just as the name suggests);

and consequent of a conditional. From the definition of 'argument' it follows further that the Stoics did not admit arguments with no premisses, or without a conclusion, i.e of the forms

$$\vdash A \quad \text{or} \quad \Delta \vdash$$

Since for the Stoics the class of syllogisms is a subclass of that of arguments (D.L. 7. 78), there can equally be no zero- or one-premiss syllogisms, let alone syllogisms without a conclusion.

Since we are not given the reason why the Stoics accepted only arguments with a plurality of premisses, we may simply have to accept this as a historical fact. However, this constraint on their syllogistic is perhaps easier to understand if one keeps in mind that the Stoics aimed at producing a method to establish the formal validity of given arguments, and not something like a complete list of logical truths in propositional logic, and that the Peripatetics too—even in their 'propositional logic'—insisted on a minimum of two premisses and a conclusion.

My reconstruction of the *themata* bears out this restriction to two- or more premiss arguments. No zero- or one-premiss 'arguments' are reducible, since every indemonstrable has two premisses, and every *thema* can be applied only to arguments with two or more premisses.

The exclusion of one-premiss arguments appears to have repercussions on some types of complex arguments (with two or more premisses) in the Stoic system. For, since the class of arguments encompasses the class of syllogisms (D.L. 7. 78), anything that can be proved to be a syllogism by reduction to indemonstrables must itself be an argument. Moreover, all testimonies on *themata* suggest that at any stage in a reduction the *themata* were applied to arguments only and not to just any combination of propositions. In accord with this, the use of the verb συνάγειν (which in logic is used for arguments only) in the formulation of the third *thema* suggests that both the compound of propositions that is to be analysed, and the compound or compounds into which it is analysed have to be

- and that Chrysippus did not accept the existence of any arguments with a single premiss (just as Sextus states).

Mignucci's worries that this would lead to inconsistencies in Chrysippus' logic can be dismissed as unfounded once one realizes that Chrysippus' system was not an attempt to develop classical propositional calculus (see pp. 181–8), and that the deductive method the Stoics use works from the argument to be analysed to the indemonstrables rather than the other way round.

arguments—at least if the compound of propositions to be analysed is a syllogism. The same holds for the formulation of the first *thema*, given that *colligitur* and *colligit* are translations of the relevant forms of συνάγειν. A consequence of all this is that by way of the *themata* no complex argument can be reduced that has at some point in its analysis a one-premiss 'argument' left over as component 'argument' (and that cannot be reduced into indemonstrables otherwise).

Take, for example, the following argument, which would be valid in classical propositional calculus:

(*a*) If not (p and q), r; not $p \vdash r$.

Stoic (backward) deduction could start only with applying the first *thema*. For no indemonstrable can be formed with the two premisses of the argument, and hence none of the other *themata* can be employed:

$$\begin{array}{ll} (b) & \text{If not } (p \text{ and } q),\ r;\ \text{not } r \vdash p \\ \hline (a) & \text{If not } (p \text{ and } q),\ r;\ \text{not } p \vdash r \end{array} \quad (\mathrm{T1})$$

In a further step (*c*), one can then form a second indemonstrable with the premisses of (*b*):

$$\begin{array}{l} (c)\ \ \dfrac{}{\text{If not } (p \text{ and } q),\ r;\ \text{not } r \vdash (p \text{ and } q)}\,(\mathrm{A2}) \quad p \text{ and } q;\ ? \vdash p \\ \hline (b)\ \ \text{If not } (p \text{ and } q),\ r;\ \text{not } r \vdash p \quad (\mathrm{T1}) \\ \hline (a)\ \ \text{If not } (p \text{ and } q),\ r;\ \text{not } p \vdash r \end{array} \quad (\mathrm{T?})$$

As regards the numbers of premisses, the only *thema* that could be used from (*b*) to (*c*) is the second. The two possibilities for '?' in the second component argument are 'not r' and 'if not (p and q), r'. Neither would produce a second indemonstrable, and neither allows the use of any other *thema* in a further step (*d*)—except the first *thema* again, which leads nowhere. Hence one is stuck, and (since all possible uses of *themata* have been tried out) one can say that the argument to be analysed is not a syllogism.

The Stoics may have concluded that the argument to be analysed is composed from a second indemonstrable and 'p and $q \vdash p$'.[84] And since for the Stoics 'p and $q \vdash p$' is not an argument, it is neither

[84] That is, the Stoics may have generally understood their 'backward' method of deduction as giving some information concerning the compounds of propositions an argument is 'composed' of, even in the case of an invalid argument to be analysed, e.g. as to *why* the argument at issue is not a syllogism. With the 'forward' method, characteristically, one will simply be unable to find a proof of the argument to be analysed. In our example, 'forwards' one would presumably start with the first indemonstrable:

(*a'*) If not (p and q), r; not (p and q) $\vdash r$

and then—as there is no obvious way of transforming 'not (p and q)' into 'not p' by way of the Stoic *themata*—one is probably stuck. One can in this case not claim to have shown the invalidity of the argument.

a syllogism nor analysable into one.[85] Hence the Stoics may have thought that the argument to be analysed is not genuinely 'composed' of indemonstrables, and that this becomes manifest in the fact that they cannot be analysed into indemonstrables. (For other, independent, reasons why the Stoics may not have accepted such arguments as syllogisms see below, pp. 183–7.)

An interesting point that should be mentioned in the context of one-premiss 'arguments' is that neither with the dialectical and synthetic theorems nor in my reconstruction of the *themata* can arguments of the following kind be reduced: arguments which (1) have as part of their hidden structure a component argument in which the same premiss (A_n) taken twice deduces a hidden conclusion (A^*_m), and in which neither occurrence of the premiss is itself the conclusion of a further component argument, and which (2) contain this premiss (A_n) only once. (That is, the component argument would have the structure $A_n, A^*_n \vdash A^*_m$.) Such arguments cannot be reduced to indemonstrables because the two theorems and the *themata* all start with the requirement that one must take two or more premiss*es* of the argument to be analysed; and this does not—in any ordinary reading of the phrases—cover the case in which the same premiss of the argument to be analysed is taken twice.

The dialectical theorem begins 'when we have the conclusive premiss*es* (λήμματα) . . .' and goes on 'then . . . in *those* premisses . . .'. The synthetic theorem begins 'when from some proposition*s* (τινων) something follows . . .' and continues 'then, too, *from those* propositions . . .'. The *themata* begin with 'when from two propositions a third follows . . .', and the second, third, and fourth

[85] On the other hand, in classical propositional calculus 'p and $q \vdash p$' is correct, which is not surprising, since the argument to be analysed was so, too.

go on 'follows from the first two . . .'. Thus, in all these cases arguments of the above-mentioned type[86] cannot be reduced.[87]

It is of course possible that the Stoics and Peripatetics simply overlooked this fact—for instance, because arguments of this kind are fairly unusual and never came up to be analysed. But before one brands the ancients as bad logicians, it may be worthwhile following up the idea that the Stoics, and perhaps the Peripatetics, were aware of this limitation of the *themata* and theorems. As has been seen, the particular kinds of arguments that are excluded from being syllogisms all 'contain' a component of the form

$$A, A \vdash B.$$

It should be evident that there are no indemonstrables that have this form. Hence, if there are component arguments of this form that are syllogisms, they require a further application of the first *thema*, or of the first and second *themata*. Keeping this in mind together with the fact that the Stoic *themata* were applied to arguments only (see above), I now make the tentative suggestion that (1) for the Stoics compounds of propositions of the form $A, A \vdash B$ were not arguments, so that (2) *no* compounds of propositions of the form $A, A \vdash B$ and *no* arguments which contain such compounds of propositions as component 'arguments' could be reduced to indemonstrables.

Point (1). First, recall that the Stoics, except Antipater, denied the existence of one-premiss arguments (see above); that they defined an argument as being composed of premiss*es* and conclusion (see above); and that they introduced the premisses of an argument as propositions (ἀξιώματα) (Sextus, *PH* 2. 136). Now, if compounds of the form $A, A \vdash B$ were arguments, we should expect to have some mention that the premisses of an argument are either (a plurality of) propositions or one proposition taken twice or several times.[88] But this is not what we get—and it is unlikely that formalists like the Stoics would have forgotten to add such a phrase. For comparison, look at the Stoic account of non-simple propositions as 'those propositions that consist either of one proposition taken twice (or 'of a duplicated proposition') or of propositions' (D.L. 7. 68; cf. Sextus, *M.* 8. 93, 112). Here the Stoics talk about 'one proposition taken twice', not about 'two proposition tokens of one type' or 'two propositions of the same kind', etc. Hence I assume that compounds of propositions of the form $A, A \vdash B$ did not count as arguments, since for the Stoics they did not have a genuine plurality of premisses.

From the point of view of Stoic ontology one could explain this by the fact that for them premisses, *qua* propositions, are incorporeal entities, and that for this reason, however often one writes down or utters a sentence that expresses a particular proposition (when stating the premisses of an argument), one only has *assumed one and the same* proposition once, even though one may have stated the sentence expressing it several times. That is, one has made only *one* assumption.

Perhaps then, expressions of the form '$A, A \vdash B$'—if they ever came up—were considered simply a non-standard way of phrasing a compound of propositions of the normal form $A \vdash B$. (More generally, an expression '$\Delta, A, A \vdash B$' may have been considered a non-standard way of formulating $\Delta, A \vdash B$.) This is confirmed by a passage in Alexander:

That is why even if there is a plurality of expressions for what is posited,

[86] Frede's second *thema* covers such cases (he discusses them explicitly: *Die stoische Logik*, 179); his second, third, and fourth *themata* together are hence more powerful in deduction than the synthetic theorem. Ierodiakonou, in her reconstruction, cannot reduce such arguments (*Analysis*, 72–4). However, in her reconstruction it is not possible either to reduce arguments of the form $A_1, A_2, A_3 \vdash C$ with a hidden structure of type (vi).

[87] That is, there are no reductions of the form

$$\frac{A_1, A_1 \vdash A_3 \qquad A_3, A_2 \vdash C}{A_1, A_2 \vdash C} \text{ (not reducible)}$$

whereas, it seems, there could be reductions of the forms

$$\frac{A_1, A_2 \vdash A_2 \qquad A_2, A_2 \vdash C}{A_1, A_2 \vdash C} \qquad \text{(T2?)}$$

$$\frac{A_1, A_2 \vdash A_3 \qquad A_3, A_3 \vdash C}{A_1, A_2, A_3 \vdash C} \qquad \text{(T3?)}$$

$$\frac{\text{ctrd } C, A_1 \vdash \text{ctrd } A_1}{A_1, A_1 \vdash C} \qquad \text{(T1?)}$$

$$\frac{A_1, A_1 \vdash A_2 \qquad A_2, A_1 \vdash C}{A_1, A_1 \vdash C} \qquad \text{(T2?)}$$

etc.

[88] Accordingly, since the initial clause of the *themata*, 'When from two a third follows', admits only the implicit addition 'proposition', and not 'premiss', one would expect 'When from two propositions or from one proposition taken twice a third follows . . .'.

still, if the rest signify the same as the first, there will be no syllogism from them in such a case. For an argument of this sort is virtually a one-premiss argument, e.g.

> It is day.
> But it is not the case that it is not day.
> Therefore, it is light.

For 'It is not the case that it is not day' differs from 'It is day' only in expression.[89] (Alex. *In An. Pr.* 18. 2–7)

This passage implies that—whatever the view about the example in question—there was agreement that a compound of propositions of the form A, $A \vdash B$ was a 'one-premiss argument', i.e. an 'argument' the normal form of which would be $A \vdash B$. And hence, for the Stoics, it would not be an argument at all.

Point (2). If this *was* the Stoic view, then arguments of the type mentioned above were not the only ones that could not be reduced to indemonstrables. In addition, the realm of application of the *themata* would have been restricted: they would not be applicable to compounds of propositions of the type A, $A \vdash B$ at all.[90] Hence—as there are no indemonstrables of this form—no arguments that contain as constituent 'arguments' compounds of the form A, $A \vdash B$,

[89] *διὸ οὐδ' ἂν αἱ μὲν λέξεις ὦσι πλείους τῶν τιθεμένων, ταὐτὸν δὲ τἆλλα* σημαίνῃ τῷ πρώτῳ, οὐδ' οὕτω συλλογισμὸς ἐκ τῶν τοιούτων ἔσται· καὶ γὰρ ὁ οὕτως ἔχων λόγος τῇ δυνάμει μονολήμματος, ὡς ὁ λέγων "ἡμέρα ἐστίν· ἀλλὰ καὶ οὐχὶ οὐχ ἡμέρα ἐστίν· φῶς ἄρα ἐστίν"· τὸ γὰρ "οὐχὶ οὐχ ἡμέρα ἐστί" τοῦ "ἡμέρα ἐστί" μόνῃ τῇ λέξει διαφέρει.*

* reading *τἆλλα* for *ταῦτα*

This passage discusses an argument built from standard Stoic examples; it uses terms and elements of Stoic logic, like 'one-premiss argument' and double-negation; and it stands in the vicinity of the discussion of a piece of Stoic logic.

[90] In this context it matters that the Stoic method of deduction works backwards. If the first *thema* worked 'forwards' *from* arguments of the form

$$A \text{ or } A, \text{ not } A \vdash A$$

(i.e. a special case of the fifth indemonstrable) *to* those of the form

$$\text{not } A, \text{ not } A \vdash \text{not } (A \text{ or } A)$$

or their contractions of the form

$$\text{not } A \vdash \text{not } (A \text{ or } A)$$

it would be strange indeed if 'arguments' of the latter forms had not been regarded as syllogisms. However, as the direction of Stoic deduction is the reverse, there is no absurdity. For a compound of propositions of the form

$$\text{not } A \vdash \text{not } (A \text{ or } A)$$

does simply not fit the first (or any other) *thema*, since the *thema* requires a minimum of two premisses.

and no such compounds themselves, could be reduced to indemonstrables by the *themata*.[91]

If one wants to formulate this restriction of the *themata* expressly, one can devise a structural rule, e.g.:

(S1) Before any application of a *thema* to an argument all premiss doublets of that argument must be deleted.

However, if the Hellenistic view on the individuation of premisses was as I have just suggested, one would not expect to find any statement of such a rule. It would have been implicit, in so far as it would have been implied by the concepts of premisses and argument.

We have no way of knowing if and how the Stoics and Peripatetics dealt in actual fact with 'arguments' of the form A, $A \vdash B$ in their reductions. In any event, one can see that this restriction of application of the *themata* would not be a great loss from the point of view of Stoic logic, if one looks to see which compounds (and arguments which have such compounds as components) are thus excluded from being syllogisms. The compounds of propositions are of the following four types:

$$A, A \vdash \text{not (if } A, \text{ not } A)$$
$$A, A \vdash A \text{ and } A$$
$$A, A \vdash \text{not } (A \text{ or } A)$$
$$\text{not } A, \text{ not } A \vdash \text{not } (A \text{ or } A)$$

These are the only such compounds that have twice the same premiss and would directly reduce to an indemonstrable—all via the first *thema*.[92] Needless to say, we do not have Stoic examples of any of them, or of compound arguments with any of them as components.

(*e*) *Non-redundancy of premisses*

Finally, a reconstruction of Stoic syllogistic should do justice to the fact that the Stoics denied validity to arguments with redundant premisses. An argument is invalid according to redundancy if it has

[91] Hence, for instance, no reductions of the forms listed in n. 86 would work.

[92] 'Arguments' of the form 'not A, not $A \vdash$ not (if not A, A)' are of the first type, since negative propositions can be put in for A. And some 'arguments', like 'A, $A \vdash$ (A and A) and A', take the second *thema* first, and then the first *thema*.

one or more premisses that are added to it from outside and superfluously (Sextus, *M.* 8. 431).[93] For cases of non-simple arguments one may interpret the clause 'from outside and superfluously' as meaning that there is *no* deduction in which for the derivation of the conclusion this premiss is required in addition to the other premisses of the argument. So the premiss 'p or not p' would not be redundant in the argument

$$\text{If } p, p;\ \text{if not } p, p;\ p \text{ or not } p \vdash p,$$

since there is a deduction of the conclusion in which it is used, namely when one considers the argument as a special case of the simple constructive dilemma (cf. above, pp. 169–70). Similarly, in

$$\text{If } p, p;\ p \vdash p$$

the premiss 'If p, p' would not be redundant, because the argument is a special case of the first indemonstrable. The requirement of non-redundancy leads to the exclusion, for instance, of most arguments of the following forms from being syllogisms

$$A;\ B;\ C \vdash A \text{ and } B$$
$$\text{If } A, B;\ A;\ C \vdash B$$

although they would be valid in all standard propositional calculi.[94]

In my reconstruction, redundant arguments cannot be reduced to indemonstrables: the indemonstrables have no 'redundant' premisses, and the *themata* require that all premisses of the argument to be analysed are components of the indemonstrables into which it is analysed—either as premiss or as negation of a conclusion.

[93] Chrysippus wrote two books on redundancy in arguments: cf. D.L. 7. 195. Why do we have 'from outside' (ἔξωθεν) in the account in Sextus? This expression is neither picked up in the subsequent explication by example, nor does it occur in the parallel passage Sextus, *PH* 2. 147. I take it to mean simply 'external to (and thus not used in) the derivation of the conclusion'.

[94] It is uncertain what the Stoic view on arguments like 'p, $q \vdash p$' was or would have been; 'q' is evidently not required for obtaining the 'p' of the conclusion. However, the remaining compound of propositions '$p \vdash p$' is no longer an argument. At any event, such arguments are neither indemonstrables nor reducible to indemonstrables.

5. The Stoic system of deduction[95]

I shall now add some more general remarks about the Stoic system of deduction as determined by the accounts of the indemonstrables (A1)–(A5) and the *themata* (T1)–(T4) and as expounded in the preceding sections. Taking (1) the standard properties of consequence and (2) the natural deduction rules as guidance, we obtain the following picture:

Reflexivity

$$A \vdash A$$

Reflexivity does not hold in the system. The Stoics considered all conditionals of the form 'If A, A' as true (cf. Sextus, *M.* 8. 281, 466), but 'arguments' of the form '$A \vdash A$' are neither axioms of the system, nor can they be derived; the latter is due to the requirement of a plurality of premisses.

Monotonicity

$$\text{If } \Delta \vdash C, \text{ then } \Delta, A \vdash C$$

Monotonicity does not hold in the system, in accordance with the restriction on redundancy.

Cut (*or transitivity*)

$$\text{If } \Delta \vdash A \text{ and } A, \Theta \vdash B \text{ then } \Delta, \Theta \vdash \mathrm{B}$$

Cut is catered for by (T2)–(T4), however with the restriction that Δ must have at least two elements and Θ at least one, in line with the requirement of a plurality of premisses.

Premiss permutation

$$\text{If } \Delta, A, B, \Theta \vdash C \text{ then } \Delta, B, A, \Theta \vdash C$$

The irrelevance of the order of the premisses is warranted in the ordinary-language formulations of (A1)–(A5) and (T1)–(T4).

[95] Most of what I say in this section and the next is in fact independent of the particular reconstruction of the *themata* that I offer and holds for any reconstruction that satisfies the requirement that the second, third, and fourth *themata* together have the same deductive power as the synthetic theorem.

Contraction

$$\text{If } \Delta, A, A, \Theta \vdash C \text{ then } \Delta, A, \Theta \vdash C$$

Contraction is partly covered by (T2). Owing to the requirement of a plurality of premisses, either Δ or Θ must be non-empty. In addition, there is some restriction on arguments with component arguments of the form A, $A \vdash B$ (see above, pp. 175–6). (From the Stoic perspective of 'backward' deduction contraction is perhaps better thought of as 'expansion'.)

Premiss repetition

$$\text{If } \Delta, A, \Theta \vdash C \text{ then } \Delta, A, A, \Theta \vdash C$$

Premiss repetition is not covered by the *themata*, and we hear of no other rule that would ensure this property. Since premiss repetition is a special case of monotonicity, I suspect it did not hold in the Stoic system, because of the restriction on redundancy.[96] (Again, within the Stoic system premiss repetition would rather appear as 'contraction'.)

Suppression

$$\text{If } \Delta, A \vdash B \text{ and} \vdash A \text{ then } \Delta \vdash B$$

Suppression is a special case of cut. It finds no application in the Stoic system, in line with the exclusion of zero-premiss arguments by the requirement of a plurality of premisses.

Antilogism

'Not-B' follows from A and 'not-C' iff C follows from A and B

Antilogism is taken care of by (T1).

The general notation and layout of the system by the Stoics makes a comparison with natural deduction rules appear more promising than one with the axioms or axiom schemata of some axiomatic presentation of classical propositional calculus (PC). Since the Stoic

[96] If my conjecture about arguments of the form 'A, $A \vdash B$' is correct (above, pp. 176–9), there is some obligation to eliminate premiss doublets before the application of (T1)–(T4).

connectives 'if' and 'or' are non-truthfunctional, obviously in their cases all we get is an analogy to rules in PC. We have then:

(i) 'and'-introduction, since all arguments of the form 'A, $B \vdash A$ and B' can be reduced to a third indemonstrable by way of (T1);
(ii) 'if'-elimination, contained in (A1);
(iii) 'or'-elimination, contained in (A5).

But we do not have (iv) 'and'-elimination; (v) 'if'-introduction; (vi) 'or'-introduction.

The resulting system of deduction is admittedly somewhat unorthodox. Let us enquire what sequents, valid in the classical propositional calculus, have no analogues in the Stoic system. (For the purpose of this comparison I treat Stoic 'if', 'or', and 'therefore' as analogous to '$\rightarrow$', '$\rightarrowtail$' ('exclusive or'), and '$\vdash$' in PC, although obviously the meaning of these logical constants is precisely one of the things at issue.)

There are first all zero- and one-premiss sequents. For example, no compounds of propositions of the following forms would be syllogisms in the Stoic system, although all of them are correct sequents in PC (remember, the 'or' in 5, 12, and 12′ is exclusive):

1 $A \vdash A$
2 $A \vdash$ not not A
3 not not $A \vdash A$
4 $\vdash$ not [A and not A]
5 $\vdash A$ or not A
6 If A, $B \vdash$ not [A and not B]
7 If A, $B \vdash$ if not B, not A

8 $A \vdash A$ and A
9 A and $B \vdash A$
10 A and $B \vdash B$
11 A and not $A \vdash B$
11′ A and not $A \vdash$ not A
12 A or $A \vdash B$
12′ A or $A \vdash$ not A
13 $A \vdash$ if B, A
14 If A, not $A \vdash$ not A

For 1–7 we have some evidence that the Stoics accepted either all corresponding conditionals of a form as true, or a metalogical principle that in some way corresponds to the sequent.[97] For 8–14 I have not found any documentation in the sources. It is interesting in this context that even though 1, 2, 3, 6, and 7 cannot be analysed into indemonstrables, they are nevertheless preserved in Stoic syllogistic, in so far as whenever an argument containing the proposition in front of '$\vdash$' is deducible, so is an argument which has for this proposition uniformly substituted the one after '$\vdash$'. For 8–14 this does not hold (cf. below, pp. 186–7). In addition, no arguments of any of the following forms can be reduced, although all of them have corresponding valid sequents in PC (remember, the 'or' in 19 is exclusive):[98]

15 If [A and A], B; $A \vdash B$
16 If not [A and B], C; not $A \vdash C$
17 If not [A and B], C; not $B \vdash C$
18 If not [A and not A], B; $A \vdash B$
19 If not [A or A], B; $A \vdash B$
20 If not [if A, not A], B; $A \vdash B$
21 If if C, A, then B; $A \vdash B$

To my knowledge there are no examples of arguments of type 15–21 or their variations anywhere in Stoic logic, and independent of the *themata* we learn nothing about whether the Stoics regarded them as syllogisms, as valid in the specific sense,[99] or as invalid.

Modern scholars have often simply propped up Stoic syllogistic by adding some of the properties of consequence or some of the theses in which the Stoic system falls short of PC. In this way

[97] For 1 see Sextus, *M.* 8. 281, 466; for 2 and 3 D.L. 7. 69; for 5 Sextus, *M.* 8. 282, 467; for 6 D.L. 7. 81; and for 7 D.L. 7. 194; 4 follows from the Stoic Principle of Bivalence (e.g. Cic. *Fat.* 20), together with the Stoic definition of negation.

[98] Cf. above, p. 174, for the analysis of an argument of the form 16. Analyses for the rest work in a similar way. For 15–21 variations with second to fifth instead of first indemonstrables as first component arguments are equally irreducible—as are, of course, all arguments that have any of these as components in an analysis, without there being an alternative reduction. Nos. 16–18 and their variants can be derived in Frede's reconstruction of Stoic syllogistic.

[99] For arguments that are not syllogisms but are valid in the specific sense see D.L. 7. 78 and the section in the chapter on Stoic Logic in J. Barnes, T. Mansfeld, and M. Schofield (eds.), *The Cambridge History of Hellenistic Philosophy* (Cambridge, 1996).

reflexivity,[100] monotonicity,[101] contraction,[102] premiss repetition,[103] 'and'-elimination,[104] and 'or'-introduction[105] have been added. By and large, the motivation for these additions to the system is the aspiration to demonstrate (weak) completeness of the system (usually with respect to 'and' and 'not'), combined with the general disbelief that the Stoics could possibly have strayed that far from PC. I suggest that this is the wrong approach; that the classical propositional calculus is the wrong paradigm; that what we need to do instead is first of all take the Stoics seriously in what they say, and look and see what system ensues; and then—if a comparison is desired—look to modern non-truthfunctional alternatives to PC. We can then see that for most of the Stoic straying from classical propositional calculus there are some interesting parallels in twentieth-century logic.

There is good independent evidence that the Stoics, or at least Chrysippus, were aware of paradoxes that correspond to those of material implication and strict implication,[106] and that Chrysippus designed an alternative (non-truthfunctional) concept of consequence (ἀκολουθία) or implication, partly in order to avoid those paradoxes of implication. It was to serve both for logical consequence in arguments and for the truth of conditionals; in the latter case it would be expressed with the help of the propositional connective 'if'. Chrysippus' truth-conditions for the conditional were said to involve a connection (συνάρτησις: Sextus, *PH* 2. 111), which must have been that which holds between the antecedent and the consequent. This connection was determined indirectly, based on the concept of conflict (μάχη), and there are some indications that the resulting concept is that of an implication stronger than strict

[100] Becker, 'Über die vier "Themata"', 41 (in its general form); Mignucci, 'The Stoic *themata*', 236 (as axiom schema).

[101] Mignucci, 'The Stoic *themata*', 236–8, cf. Mueller, 'Completeness', 204.

[102] Mueller, 'Completeness', 203, 214; Mignucci, 'The Stoic *themata*', 220, 229, 230.

[103] Mueller, 'Completeness', 203, 214.

[104] Becker, 'Über die vier "Themata"', 42; Mignucci, 'The Stoic *themata*', 236.

[105] Mueller, 'Completeness', 211. The proposal is 'A, not $B \vdash A$ or B' and 'not A, $B \vdash A$ or B'. However, we should expect that the Stoics would not have accepted Mueller's suggestion, if only because of the fact that their disjunction is not truth-functional (cf. n. 5).

[106] These are the implications introduced by Philo of Megara and Diodorus Cronus respectively; cf. Sextus, *PH* 2. 110–11, and the chapter on Megaric Logic in the *Cambridge History*.

implication.[107] The Stoic requirement of non-redundancy of the premisses of an argument points in the same direction: it suggests that the Stoic concept of logical consequence is stronger than strict implication.

The discontent with implications comparable to material and strict implication, the development of an alternative concept of implication, and the requirement of some 'connection' between premisses and conclusion in an argument all have counterparts in several alternative systems of propositional logic of twentieth-century logicians. These are sometimes classified under the name of 'relevance logic'.[108] The various systems of relevance logic differ from PC in that in these systems some theses from PC are not theses: viz. those that are regarded as paradoxical and some others, the acceptance of which would lead to paradoxical theses. Different systems expel different theses.

The shared goal of the Stoics and the 'relevance logicians' should then make us *expect* that certain theses of PC are not reducible in the Stoic system, and, what is more, that the Stoics may not have wanted them to be reducible. With this in mind, let us look again at the kinds of argument that cannot be reduced, and about which we lack documentation, i.e. those of the forms 8–21 and the variants of 15–21. About the one-premiss 'arguments' 8–14 we can say that the corresponding PC sequents have all been classified as paradoxical for one reason or other by modern logicians. Owing to the absence of direct evidence, the question whether the Stoics accepted 8–14 (i.e. took the relation of consequence to hold between its 'premiss' and 'conclusion') can be settled in Stoic syllogistic only in the context of the analysis of complex syllogisms. Arguments of the forms 15–

[107] The criterion states that a conditional is true precisely if its antecedent and the contradictory of its consequent conflict (D.L. 7. 73). For a more detailed exposition of Chrysippus' conditional see the chapter on Stoic Logic in the *Cambridge History*.

[108] To name just some: W. Ackermann, 'Begründung einer strengen Implikation', *Journal of Symbolic Logic*, 21 (1956), 113–28; A. R. Anderson and N. D. Belnap, *Entailment: The Logic of Relevance and Necessity* (Princeton and London, 1975); J. M. Dunn, 'A Modification of Parry's Analytic Implication', *Notre Dame Journal of Formal Logic*, 13 (1972), 195–205; H. Linneweber-Lammerskitten, *Untersuchungen zur Theorie des hypothetischen Urteils* (Münster, 1988); S. McCall, 'Connexive Implication', *Journal of Symbolic Logic*, 31 (1966), 415–33. A useful overview can be found in H. Linneweber-Lammerskitten, 'A Survey of the Derivability of Important Implicative Principles in Alternative Systems of Propositional Logic', in G. Meggle and U. Wessels (eds.), *Analyomen*, i (Proceedings of the First Conference 'Perspectives in Analytical Philosophy'; Berlin and New York, 1994), 76–87.

21, when analysed, each have a one-premiss 'argument' left,[109] and these one-premiss 'arguments' all belong to the group 8–14. The two groups pair up as follows: 8 is 'contained' in 15, 9 in 16, 10 in 17, 11 in 18, 12 in 19, 13 in 20, and 14 in 21. And as said above, all of these one-premiss 'arguments' have been regarded as paradoxical one way or another in modern logic. So the Stoics, too, may have had reasons for considering them as incorrect.

The Stoics may have wanted to do without the 'formulae of simplification' 9 and 10 for reasons of redundancy: the propositions for B in 9 and A in 10 may have been considered as irrelevant for the conclusion; or alternatively, because if arguments with them as components were reducible, this would also be the case for arguments with components of the paradoxical forms 'A and not $A \vdash A$' and 'A and not $A \vdash$ not A'.[110] Modern systems without the formulae of simplification have been developed, for instance, by McCall, 'Connexive Implication', and Linneweber-Lammerskitten, *Untersuchungen* (see n. 108). Nos. 11 and 12 are versions of *ex impossibili quodlibet* or of *ex falso quodlibet*, and may have been viewed as paradoxical by the Stoics because of that. Systems in which 11 is not a thesis include Ackermann, 'Begründung'; McCall; Dunn, 'Modification'; Anderson and Belnap, *Entailment*; Linneweber-Lammerskitten. Systems without 11′ are McCall and Linneweber-Lammerskitten. Interpreting Stoic 'A or B' as '$(A \vee B) \wedge \neg(A \wedge B)$' for the purpose of comparison, 12 and 12′ are not theses in McCall and Linneweber-Lammerskitten. We would expect 13 not to be accepted by Chrysippus because of his concept of consequence (see above). In modern systems it is absent as a thesis in all those named as not containing 11. Nos. 8 and 14, finally, have been given up by McCall, since they are in conflict with some more general ideas of his system.

So, whatever the flaws of Stoic syllogistic, it manages to keep out a number of paradoxes which modern logicians try to avoid, too. The parallel with McCall's system of connexive implication is perhaps most striking. Nos. 8–14 hold neither in his nor in the Stoic system, and equally 16–22 and their variations are not theses in McCall, nor can the Stoics reduce arguments of those forms.

[109] See the paradigm analysis above, p. 174; and so have their variations: see above, n. 98.

[110] For the paradoxicality of the formulae of simplification cf. e.g. Linneweber-Lammerskitten, 'Survey of the Derivability', 82–3.

However, neither system is an extension of the other: in McCall some formulae are theses which cannot be reduced by the Stoics, and vice versa.[111]

In conclusion, the Stoic system (without any undocumented additions) makes a lot more sense to us when not measured against the classical propositional calculus, but compared to modern non-truthfunctional propositional logics, in particular 'relevance logics': that is, when it is understood as an attempt to develop a system without certain paradoxes, as for instance those of material and strict implication.[112] But of course, doing without those paradoxes usually has its price, namely that some arguments one would ordinarily consider valid may not be derivable in the system. This brings us to the question of completeness.

6. Completeness

Is Stoic syllogistic complete? In Sextus Empiricus (*PH* 2. 156, 157, 194) we find Stoic claims that can be understood as the assertion of some kind of completeness in their logical system. We learn that the valid non-indemonstrable arguments have the proof of their validity from the indemonstrables (194), that the indemonstrables are demonstrative of the validity of the other valid arguments (156), and that those other arguments can be reduced (ἀναφέρεσθαι) to the indemonstrables (157).

The implication that the proof of the validity of the non-indemonstrables is given by reduction is confirmed by the above-mentioned passage in Diogenes Laertius (7. 78), which says that all syllogisms are either indemonstrables or can be reduced to indemonstrables by means of the *themata*. We may then assume that the claim of 'completeness' in Sextus is that (at least) all non-indemonstrable syllogisms can be reduced to indemonstrables by the *themata* (or by related theorems). One could take this as the—trivial—claim that the *themata* (or theorems) lay down or determine whether an argument is a syllogism. However, even if one takes the passage in Diogenes as a Stoic definition of syllogismhood, this is unlikely. Rather, we should assume that the Stoics had—independently of the *themata*—some pretechnical notion of syllogismhood, and that the indemonstrables plus *themata* were devised in order to 'capture' this notion; perhaps also to make it more lucid and precise.[113]

This is a plausible assumption. It leaves us with the following problem: how can we find the independent Stoic criteria for syllogismhood? That is, how can we decide which peculiarities of the Stoic system preceded their choice of logical rules and which are simply a result of their introducing these rules? The paucity of evidence does not allow us to answer this question fully. *A fortiori*, we cannot decide whether the Stoics achieved completeness, i.e. were successful in devising their rules in such a way that they adequately covered their pretechnical notion of syllogismhood; and consequently, whether they were successful in demonstrating the completeness of their syllogistic.

Still, it is possible to determine a number of features of the Stoic system that are relevant to its completeness, and thus to narrow down considerably the number of possible interpretations of what completeness they wanted. It seems safe to assume that the Stoic system shared the following condition of validity with modern semantic interpretations of formal logic. It is a necessary condition for the validity of an argument that it is not the case that its premisses are true and its conclusion is false. Accordingly, it is a necessary condition for formal validity (syllogismhood) that no syllogism or argument of a valid form (according to the pretechnical notion of validity) has true premisses and a false conclusion. To this we can add a number of necessary conditions for Stoic syllogismhood which are not requirements for formal validity in the modern sense, and which show that the classical propositional calculus is not contained

[111] Another parallel to systems of relevance logic that is perhaps worth mentioning here concerns Stephen Read's 'bunch-logic'. Read develops an 'intensional conjunction' for which he wants neither monotonicity, nor premiss repetition, nor suppression to hold, and his solution for cutting out suppression is simply not to allow sequents without premisses. Cf. S. Read, *Relevant Logic* (Oxford, 1988), ch. 3.

[112] This result confirms Jonathan Barnes's conjecture that Chrysippus anticipated modern relevance logic, which he based on evidence largely independent of Stoic syllogistic. Cf. Barnes, 'Proof Destroyed', in M. Schofield, M. Burnyeat, and J. Barnes (eds.), *Doubt and Dogmatism* (Oxford, 1980), 161–81.

[113] Most probably, the resulting concept of syllogismhood is the upshot of a development based on mutual influence of the pretechnical notion and the attempted formalization: the pretechnical notion leads to rules of inference, these lead to modification of the informal notion, this in turn leads to revision or refinement of the rules, etc.

in the Stoic system or that the Stoic system is not an extension of classical propositional logic.

First, there is a formal condition, which restricts the class of syllogisms, not by denying validity to certain arguments, but by denying the status of argumenthood to certain compounds of propositions: Stoic syllogistic is interested in formally valid *arguments*, not in *propositions* or *sentences* that are logically or necessarily true. And their concept of argument is narrower than that of modern logic: an argument must have a minimum of two premisses and a conclusion. That is, Stoic syllogistic considers (tests etc.) only arguments of the form

$$\Delta \vdash A,$$

in which Δ is a set of premisses with at least two (distinct) elements. A consequence of this is that there are logically true conditionals to which no formally valid arguments correspond. Such a correspondence exists only between a proper subclass of the former—those which have the form 'If both A and B and . . ., then C'—and valid arguments.

Since the Stoic 'claim to completeness' (see passages above) is made exclusively on the level of arguments, not of propositions, and the *themata* are applicable solely to (Stoic) arguments, it is clear that the Stoics aim at a kind of completeness that differs from that of modern propositional calculus. For instance, '$p \vdash p$' would not count as a syllogism, although the Stoics think that every proposition follows from itself (see above). One could of course enquire whether Stoic propositional logic (as *opposed to* logic of arguments) is complete in the way the classical propositional calculus is. (For this one has to introduce 'translations' of the *themata* into sentential rules.) But since we have no hint that any Stoic ever thought about this kind of completeness, let alone tried to prove it (and how, without a truth-table approach?), such a question is of little historical relevance—quite independently of the fact that the answer would presumably still be 'no'.[114]

Second, there is a restriction of validity through the requirement of non-redundancy of the premisses (see above, pp. 179–80). A third restriction known to us—independently of the *themata*—concerns the wholly hypothetical 'syllogisms'. (Their prototype has the form 'If A, B; if B, $C \vdash$ if A, C'.) There are some hints that the Stoics

[114] Cf. e.g. Frede, *Die stoische Logik*, 197, and see above, pp. 181–8.

considered such arguments as valid but not as syllogisms.[115] We do not know whether this restriction was part of the Stoic pretechnical notion of a syllogism; that is, whether these arguments were excluded because they were not analysable in the system, or whether 'the five indemonstrables' and the *themata* were selected partly in order not to allow the reduction of such arguments.

As we have seen, in addition to these three requirements the Stoics may have maintained that an argument cannot have two identical premisses. That is, compounds of propositions of the form

$$\Delta, A, A \vdash B$$

were, it seems, considered as a non-standard way of putting the argument

$$\Delta, A \vdash B$$

—i.e. as an argument in which the same premiss is stated twice rather than in which two premisses of the same form and content are stated. Hence there can be no 'structural rules' which allow us to eliminate or introduce doublets of premisses *ad libitum* (see above, p. 182).

We are now in a position to examine whether the Stoic system of syllogisms, as containing indemonstrables and *themata*, captures the—at least partly—pretechnical notion of syllogismhood as determined by the three requirements stated. And we can see that their system does not permit reduction of any of the arguments that are precluded from being syllogisms by these requirements. As has been said above, first, no one- or zero-premiss arguments are reducible, since the indemonstrables have two premisses, and the *themata* can be applied to two- or more premiss arguments only; and secondly, redundant arguments cannot be reduced, since the indemonstrables have no redundant premisses, and the *themata* require that all premisses of the argument to be analysed are components of the indemonstrables into which it is analysed (see pp. 136 and 153). Thirdly, no wholly hypothetical 'syllogisms' are indemonstrables, nor can they be reduced to indemonstrables: for the last three *themata* require that one splits off one two-premiss argument each time they are used, and this two-premiss argument must contain at least one simple proposition,[116] because it must

[115] Cf. e.g. M. Frede, 'Stoic vs Aristotelian Syllogistic', *Archiv für Geschichte der Philosophie*, 56 (1974), 1–32 n. 5 (*b*) and (*c*); see also Alex. *In An. Pr.* 262. 28–31.

[116] Or a substitution instance of a simple proposition.

be either an indemonstrable itself or reducible into one by the first *thema*. And any reduction to an indemonstrable by means of a single application of the first *thema* also requires that the argument to be analysed contains at least one simple proposition. So far, then, Stoic syllogistic coincides with what might have been their pretechnical concept of syllogismhood.

However, there are still a number of complex arguments which cannot be reduced (see above, p. 184: those of the forms 15–21), and which are not covered by the constraints on validity mentioned so far. This gap between the pretechnical notion of syllogismhood and the Stoic system of deduction can be filled only by conjecture. Perhaps the pretechnical concept included the idea that the ultimate criterion of syllogismhood is (formal) validity that is self-evident. A syllogism must then either be self-evidently valid itself or it must be—somehow—genuinely and comprehensively composed of nothing but such evidently valid arguments, since they are the only unquestionable guarantee of (formal) validity. In this way, the title 'syllogism' might have been retained for a group of valid arguments whose formal validity was regarded as beyond all doubts. In this context it should be of interest that the Stoics admitted arguments (of specific forms) that they considered as valid but not as syllogisms.[117]

Can we state positively what the claim of completeness could have been? Maximally, the claim could have been that the class of arguments that either are indemonstrables themselves or can be analysed into indemonstrables by means of the *themata* contains precisely all arguments of the form '$\Delta \vdash B$', with $\Delta = \{A_1, \ldots, A_n\}$ and $n \geq 2$, which (1) because of their form can never have true premisses and a false conclusion, (2) contain—as relevant to their form—only the Stoic logical constants 'not . . .', 'either . . . or . . .', 'if . . . then . . .', 'both . . . and . . .', (3) contain no premiss doublets and no redundant premisses, (4) are not wholly hypothetical, and (5) are, or are composed of, nothing but self-evidently valid arguments. Perhaps a proof of this kind of completeness is possible.[118]

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[117] These were the above-mentioned arguments valid in the specific sense.

[118] I have been unable to take account of Peter Milne's article 'On the Completeness of Non-Philonian Stoic Logic' (*History and Philosophy of Logic*, 16 [1995], 39–64), since it appeared after the final version of this paper had been submitted.

EPICTETUS ON HOW THE STOIC SAGE LOVES

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WHILE much excellent work has been done on the Stoic doctrine of the emotions in general,[1] and some work on the Stoic concept of friendship,[2] there has been little systematic study of the Stoic account of that spectrum of emotional dispositions covered by our term 'love'.[3] What kind of love is the Stoic Sage (φρόνιμος) capable of? Cicero's Cato declares that 'Even the passion of love when pure is not thought incompatible with the character of the Stoic Sage'.[4] Seneca reports:

> I think Panaetius gave a charming answer to the youth who asked whether the wise man would fall in love: 'As to the wise man, we shall see. What concerns you and me, who are still a great distance from the wise man, is to ensure that we do not fall into a state of affairs which is disturbed, powerless, subservient to another and worthless to oneself.' (*Ep.* 116. 5, trans. Long and Sedley)[5]

[1] A. M. Ioppolo, 'La dottrina della passione in Crisippo', *Rivista critica di storia della filosofia*, 27 (1972), 251–68; A. C. Lloyd, 'Emotion and Decision in Stoic Psychology', in J. M. Rist (ed.), *The Stoics* (Berkeley and Los Angeles, 1978), 285–46; Michael Frede, 'The Stoic Doctrine of the Affections of the Soul', in M. Schofield and G. Striker (eds.), *The Norms of Nature* (Cambridge, 1986), 93–110; Martha C. Nussbaum, 'The Stoics on the Extirpation of the Passions', *Apeiron*, 20 (1987), 129–77.

[2] J.-C. Fraisse, *Philia: La Notion d'amitié dans la philosophie antique* (Paris, 1974), 348–73; Glenn Lesses, 'Austere Friends: The Stoics and Friendship', *Apeiron*, 26 (1993), 57–75.

[3] D. Babut, 'Les Stoïciens et l'amour', *Revue des études grecques*, 76 (1963), 55–63. Malcolm Schofield, *The Stoic Idea of a City* (Cambridge, 1991), ch. 3, discusses Zeno's account of the place of erotic love in his *Republic*. See also Martha C. Nussbaum, 'Eros and the Wise: The Stoic Response to a Cultural Dilemma', *Oxford Studies in Ancient Philosophy*, 13 (1995), 231–67. I thank Prof. Nussbaum for her helpful comments on an earlier version of this paper.

[4] *Fin.* 3. 68 'Ne amores quidem sanctos a sapiente alienos esse arbitrantur', trans. H. Rackham (Cambridge, 1983), 289.

[5] A. A. Long and D. N. Sedley, *The Hellenistic Philosophers*, i (Cambridge, 1987),