Change and contradiction: a criticism of the Hegelian account of motion

Emiliano Boccardi[1]

Abstract

In his *In Contradiction* (1987), Priest levelled three powerful arguments against the received Russellian view of change and motion. He argued that his preferred paraconsistent theory of change, the Hegelian account, is immune from these objections. Here I argue that these three arguments are sound, but that the Hegelian account falls pray to them too. I conclude, however, that the Hegelian account is in a better position to tackle these challenges.

1. Introduction: the received view of motion and change

The idea that things change and that time passes is deeply entrenched in our cognitive and emotional lives. Yet, as soon as we try to explicate these notions in clear conceptual terms, we are immediately faced with great difficulties. Of all the ways in which this worry can be expressed, perhaps the most famous, and the most perspicuous for our purposes, are Zeno‘s celebrated arguments against motion.

Consider an object in uniform motion from location x to location y (the tip of Zeno‘s arrow, for example). And consider any particular instant during its journey, t_0, during which the tip is located exactly in the position x_0, and only at that position. At, or during t_0, the arrow advances not at all away from x_0, in its journey to y. Any such advancement, in fact, would take a finite amount of time, however small. The duration involved in this advancement, therefore, would not be an instant, contrary to the hypothesis. Now, the whole interval of time during which the journey of the object supposedly occurs is made up of such instants. Since no advancement is achieved during any of those constituting instants, it seems to follow that no advancement can be made in any number of them, however great this may be. If no advancement can be made during any time interval, then no advancement is made by the object, contrary to the hypothesis.

At any time t_0 during its journey, according to this premise, the tip of the arrow is located exactly in the position x_0, and only at that position. As Aristotle put it in his famous reconstruction, at any time during its journey, the arrow "occupies a space equal to its size". What makes this premise nearly compelling is that denying it seems contradictory. How could the tip of the arrow be and at the same time be not located at position x_0? How could the arrow occupy a space greater than its size?

According to an often-recounted philosophical gossip, one that is reported in any introductory physics textbook, this dilemma was finally solved by the invention of calculus. As they struggled to come up with a suitable mathematical description of continuity and continuous change, Leibniz and Newton, independently from one another, developed the fundamental notions that constitute what we now know as calculus. Despite its immense fruitfulness in the development of modern science, and its wide spread use among physicists and mathematicians, it was not until the end of the 19th Century that these

[1] Department of Philosophy, Universidade Federal da Bahia. E-mail: emiliano.boccardi@gmail.com

DOI: 10.21452/LnA_serie_n_v01_book_seminario-logica-no-aviao-2013-2018_emiliano-boccardi_p.135-148

notions found a coherent systematization in what is known as the "classical foundation of calculus". As we shall see, however, such coherentization was obtained at the price of a radical reconceptualization of the relevant notions, one that arguably deprives them of their intuitive appeal in the dissolution of Zeno's paradoxes. The central idea involved in this reconceptualization is the quantificational understanding of limits: rather than treating derivatives as ratios of infinitesimal quantities, according to this (now familiar) understanding, limits should be understood as quantifiers.

Let us consider how this idea applies to the notion of instantaneous velocity. Instantaneous velocity, according to the classical view, should be understood as the limit of a sequence of ratios of finite quantities:

$$v(t_0) = \frac{dx}{dt}(t_0) = \lim_{\Delta t \to 0} \frac{x(t_0 + \Delta t) - x(t_0)}{\Delta t}$$

This formula, in turn, should be understood as expressing the fact that the values of the finite ratios $\frac{x(t_0+\Delta t)-x(t_0)}{\Delta t}$ become indefinitely close to $v(t_0)$, the limit, as we chose smaller and smaller values of Δt: for all $\epsilon > 0$, there exists a $\delta > 0$ such that for all $\Delta t < \delta$: $|\frac{x(t_0+\Delta t)-x(t_0)}{\Delta t} - v(t_0)| < \epsilon$.

It should be stressed that the expression "become indefinitely close" ought to be thought of as a dead metaphor, reminiscent of a time when the continuum was conceived as the product of actual motion, and variables as denoting varying quantities: "infinitesimals are an attempt to extend to the values of a variable the variability which belongs to it alone. When once it is firmly realized that all the values of a variable are constants, it becomes easy to see, by taking any two such values, that their difference is always finite, and hence that there are no infinitesimal differences." ([17], p. 357).

Russell enthusiastically took the classical coherentization of calculus as providing us with a coherent response to Zeno's challenge: "people used to think that when a thing changes, it must be in a state of change, and when a thing moves, it is in a state of motion. This is now known to be a mistake. When a body moves, all that can be said is that it is in one place at one time and in another at another." ([16], p. 69).

This admittedly paradoxical idea that things could change without ever being changing, also known as the at-at theory of change, appears to be uniquely capable of making sense of change without postulating true contradictions, and quickly became the received view.[2]

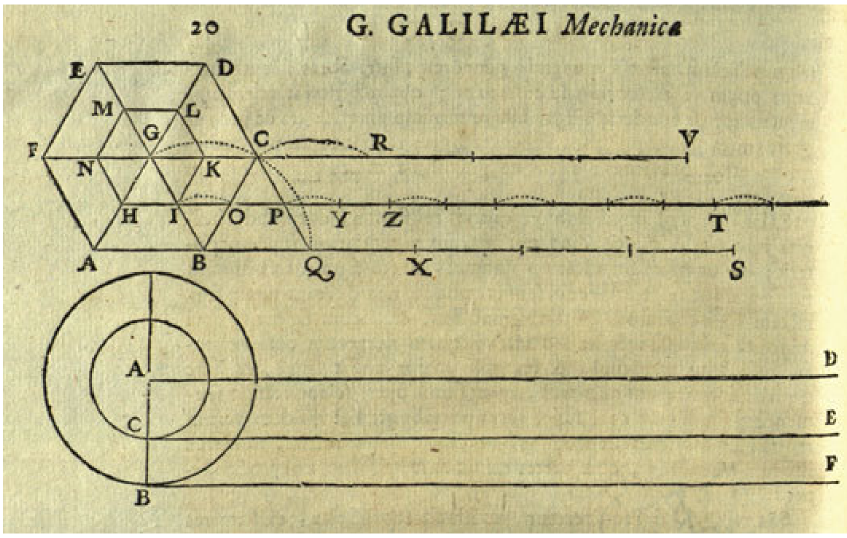
2. Priest's three objections to the Russellian view

In his book *In Contradiction* ([14]), Priest put forward three lines of criticism to the at-at theory of change, with which I fully agree. In this section I will present these criticisms and defend them from some possible objections.

[2] This idea was not new, and a debate over its virtues can be traced back at least to medieval times. William of Ockham and his followers advocated a view according to which change is nothing over and above a sequence of different properties had at different times. It was known as the doctrine of changing form (*forma fluens*), and it opposed the dual-fact account known as the doctrine of change of form (*fluxa formae*). According to the latter, when a leaf (for example) changes its color from green to yellow, not only does it have a determinate shade of (say) green at any given time, but, over and above, it also has a changing shade of green (the dual fact). According to the former, on the contrary, there is no further fact over and above the leaf having different shades of green at different times.

2.1 The problem of incremental accretion. As we have seen, the source of the discomfort one feels when confronted with the arrow paradox is that one finds it hard to intuit how the arrow could manage to get anywhere by a series of going-no-where's. Part of the appeal of the original (incoherent) understanding of infinitesimals was precisely that it appears to alleviate this discomfort, since it allows us to conceive of any stretch of time or space as composed of infinitesimal 'line elements', whose individual contribution to the total advancement of the arrow, while infinitesimal, is not null.

A way to illustrate the intuitive appeal of the original conceptualization is to contrast the case of a polygon revolving on a plane without sliding, with the case of a circle so advancing (see fig. 1).[3]



As the polygon advances, each of its sides touches the plane in its due turn, so that when a total rotation is completed the total stretch of space touched by the polygon, equal in size to its perimeter, can be seen to consist of a finite sum of discrete "jumps", each equal in size to one of its sides. If we try to replicate the same reasoning to the case of a circle revolving on the plane, we are instantly met with a great difficulty. Here too, the figure touches the ground with one element at a time, each in its due turn. However, unlike what happens in the case of the polygon, the element of the circle that touches the ground at each time is a dimensionless point. Since there is no sliding, moreover, just as it happens in the case of the polygon, the velocity of each of these point-elements relative to the ground is a 0.

How could the circle manage to advance, if its advancement consisted of a series of null contributions, each of which in its turn consists of a dimensionless point of the circle touching the ground without moving with respect to it? Prima facie, construing of the circle as a limit case of a polygon with infinitely many "sides" or "line elements", partly alleviates this discomfort, allowing us to describe the two situations as (at least partly) analogous. But this understanding of continuous motion is precisely what the classical foundation of calculus is supposed have banished:

> These days no one worth his salt thinks instants "add up to" periods this way. If there are instants, periods are instants with distance-relations between them. The relations, not their relata, account for periods' extension:

[3] This paradoxical scenario, known as the problem of the Rota Aristotelis, was first discussed in the Greek work Mechanica, traditionally attributed to Aristotle. Upon completing a full rotation, the centre of the greater wheel will be displaced by a distance equal to its circumference. However, as the greater circle completes a full rotation, the smaller wheel, which is attached to it, completes a full rotation too. So, by an analogous reasoning, we ought to conclude that the centre of the smaller wheel (which is the same as that of the greater one), will have moved by a distance equal to its smaller circumference. A contradiction. The scenario was famously discussed by Galileo in his Two New Sciences.

> that is why (in the paradox) without putting distance-relations between the points, we don't get an extension. [Leftow 2014 [12], p. 239].

However, the mere definition of a mathematical function, called "distance", whose values are totally independent from the individual advancements achieved at each instant during the journey, can at best be seen as a restatement of the problem, rather than its solution. As Priest rightly points out:

> That one can prove a small mathematical theorem or two is one thing; but it does not ease the discomfort that one finds (or at least, that I find) when one tries to understand what is going on physically, when one tries to understand how the arrow actually achieves its motion. At any point in its motion it advances not at all. Yet in some apparently magical way, in a collection of these it advances. Now a sum of nothings, even infinitely many nothings, is nothing. So how does it do it? [ibid., p. 175]

The problem we are discussing should be distinguished from the mere problem of understanding how a static infinite aggregate of dimensionless elements could 'add up' to a finite quantity. In the dynamic setting that we are considering, this problem is further aggravated by the fact that these elements enter the scene one at the time, so that it is hard to understand how we could deny that these solitary facts should make a "contribution" to the total feat. In the case of a static, or standing variety of infinity, all the elements are already there, waiting to be named and counted, as it were, and so are all the relations that obtain between them. Two points of a line have the distance they have quite independently from the existence of any intervening points. These standing elements need not be *reached* in any sense, nor should their distance count as an *achievement* of any kind. The situation is crucially different in the case of a growing variety of infinity, which is what is at issue here. To be told that the distance between the bow and the target is some finite quantity only proves that *if* the target could be reached, then the total advancement of the arrow would be equal to that quantity. It tells us nothing, however, about whether such feat can be accomplished, since it tells us nothing about the physical formation of this path during the journey. To use Bergson's apt words, according to the Russellian view, "movement is composed of immobilities" ([4], p. 308).

2.2 The problem of the non-intrinsicality of motion. Standard physics textbooks tell us that the state of the universe at any time is determined by the state at any previous time plus the laws of physics. The state of the universe at a time t, we are told, is given by the positions of all the particles at t, plus their velocities at t. However, according to the Russellian view, the instantaneous velocities of the particles supervene solely on their positions at various times preceding and succeeding t, hence they add nothing to the state of the universe at exactly the time t. Now, the positions of particles at t have clearly no effect whatsoever on the positions in the immediate future of t. How could their velocities have any greater explanatory role, if they supervene solely on these positions themselves? As Priest put it:

> It follows from the definition that there is no such thing as an intrinsic state of motion. If one had a body in motion and took, as it were, a logical "picture" of it at an instant, the picture obtained would be no different from one of the same body at the same place, but at rest. Of course, an object in motion can have an instantaneous non-zero velocity, but it would be wrong to think that this differentiates it intrinsically from a static body. [ibid., p. 173]

Some authors tried to derive less dramatic consequences from the application of calculus in response to Zeno's argument, denying that the quantificational understanding of limits and derivatives renders instantaneous velocity a relational property (cf. Smith 2003 [18]). I have argued elsewhere that this view, known as "the modified at-at theory of change", is untenable.[4] Here I shall content myself with presenting a symptom which afflicts the Russellian account, and which I think lends indirect support to Priest's observation.

It has been often noted that it is hard to understand how Russellian instantaneous velocities could fulfill the explanatory role that is standardly ascribed to them in physics. This is well illustrated by the following argument by Frank Arntzenius:

> Consider a ball moving from right to left through some region in space, and a qualitatively identical ball, perhaps the same one, some time later, moving from left to right through that very same region. If the full state at a time of a ball does not include an instantaneous velocity then the full state of the two balls is exactly the same when they occupy the same region. [...] Why does the one ball subsequently move to the left and the other ball subsequently move to the right? Surely it is conservation of velocity, or something like that, which determines that the one ball will keep moving to the left and the other will keep moving to the right. But if there is no such thing as instantaneous velocity, as there is not in the at-at theory, then why do the balls continue their motions in different directions? [[2]: § 2.]

This objection has been raised several times under different guises in the recent literature.[5] It appears to expose a fatal shortcoming of the received view, one that derives directly from the non-intrinsicality of change. The general problem appears to be that Russellian velocities make the state of the universe at an instant conceptually dependent on its states at other instants, since the velocity objects have at a time is a property of their trajectory around that time, not an instantaneous intrinsic property.[6] This deprives these "states" of all their explanatory power.

2.3 The problem of the direction of time. The third and last criticism raised by Priest against the standard view is that it appears to be incapable of accounting for the intrinsic directedness of time and change, the so-called anisotropy of time. Time and change always happen in one direction only, from earlier to later, from past to future. Here then is the difficulty, in Priest's words:

> What accounts for the anisotropy of time? This again has been a thorny problem, particularly for those who have denied the reality of the flow of time. They have had to locate the anisotropy of time not in time itself, but in processes in time - a tall order, since apparently all causal laws are time-symmetric. [Ibid., p. 116].

Now, while Priest is surely right that reconciling the apparent irreversibility of physical processes with the time-reversal invariance of fundamental laws is a "thorny problem", I think that mentioning this in that context obscures the full force of his objection. To see why, it is worth discussing briefly a common fallacy. It has often been suggested that the second law of thermodynamics, which implies that the entropy of a closed system never decreases, could be used to explain and reduce the directionality of time itself. According

[4] [6], pp. 164-168.

[5] See for example [1], [11], [2], [19], [5], [9].

[6] See for example [1], p. 10.

to these suggestions, the future direction of time is nothing but the direction in which global entropy increases. The passage of time would be grounded by the fact that the entropy gradient is not zero. But these reductive accounts cannot succeed. The second law of thermodynamics, in fact, can only establish a contingent correlation between the entropy gradient and the direction of time. If the intrinsic direction of time was not fixed independently from the entropy gradient, how could we even meaningfully express the empirical content of the law? What does it even mean to say that entropy never "increases", if not that it never increases in the direction of the future?

Imagine a world where the second law does not hold. It would be a world where broken glasses would spontaneously recombine to end up on the tables from which they fell. But In order to even represent this possibility, we have to assume that the broken-glass states in that world would be instantiated *before* the on-the-table states. If the direction of time itself were reversed in such a scenario, then that world would be just indistinguishable from ours! In fact, the second law of thermodynamics would hold in that world too, and by necessity, contrary to the hypothesis. As Tim Maudlin quite aptly put it, if the future-directed arrow of time amounted to nothing but the direction in which entropy increases, "[entropy] could go up and down like the stock market, but since the 'direction of time' would obligingly flip along with the entropy changes, entropy would still never decrease" (2007, p. 129).

What was said about the second law applies equally well to any other time-asymmetric law postulated for grounding the direction of time, regardless of whether it be a fundamental law or a statistical one (as is the case with the second law). The general lesson to be learned by the failure of these reductive attempts is this. Mere asymmetric relations are never enough, by themselves, to fix a direction. At best, asymmetric relations can determine an order among the elements of their domains. But mere order is "blind" to directionality, so to speak. Three points on a line, for example, have an order. B lies between A and C, etc. However, these betweenness relations are insensitive to the direction in which the series might be transversed (from A to C, or from C to A). Asymmetric relations determine an order, but not a direction. Unfortunately, this direction, or sense, is precisely the feature that any account of time and change should explain, and that the Russellian account struggles to accommodate.

3. Priest's Hegelian account of motion

This is where we stand. Zeno's arguments appear to show that time and change are contradictory. The main attempt to remove these contradictions without denying time and change altogether, the now widely accepted Russellian account, proves to be at best a restatement of the problem. We are thus faced with a dilemma. Either accept the phenomenological datum that things change and time passes, at the cost of accepting that at least some contradictions are true; or hold fast to the principle of contradiction, and repudiate time and change as mere appearances. Heraclitus and Hegel (according to some interpretations) opted for the first horn of this dilemma. Parmenides, Zeno and McTaggart, instead, opted for the second one.

To make matters worse, as I have argued in my Boccardi and Perelda 2018 ([8]), the claim that time and change are mere appearances runs the risk of being itself self-contradictory, lest it be carefully qualified. In claiming that our experience represents the world as instantiating dynamic features absent from reality, the denier of change must be careful not to concede too much, as she would if her account required that dynamic experiences themselves (or their contents) instantiate dynamic properties incompatible with the B-theory. It is not enough to claim that passage is mind dependent in the sense that it requires the interaction of a conscious subject with external reality (as is the case, for example, in [3]). The strict denier of passage wants to further claim that the content of the experience of passage is never true of the actual world, not even of that part of it consisting of the interactions between subjects and their objects: 'the dynamic character of our immediate experience does not require time itself to be dynamic' ([10]: p. 391).]

This view, I argue, is self-undermining. If we concede that change and time are mere appearances, doesn't it follow necessarily from this that at least these appearances really change? If every time that things appear to us to change we make a mistake, in that the things that appear to change don't really change, isn't it still true that we make different such mistakes at different times, and hence that at least appearances themselves really do change?

Everything suggests that time and change are real and contradictory. Very few philosophers, however, took seriously this option, since questioning the principle of contradiction threatens to blow a fatal strike at the heart of rational thinking itself. It is unfortunate that the few philosophers who took up the challenge of denying the principle, most notably Heraclitus and Hegel, are among the most obscure and cryptic thinkers of all times. The philosopher Graham Priest, in more recent times, devoted a good part of his career to the herculean task of raising the vague contention that (some) contradictions might be true to the highest standards of clarity and analytic thinking.

This unprecedented accomplishment was made possible, among other things, by the discovery and development of logical systems that block the Principle of Explosion (ex contraditione quodlibet). These systems, known as paraconsistent logics, opened the way for a non-trivial treatment of true contradictions, or dialetheia. One paraconsistent logic that is particularly apt for this purpose is the so-called logic of paradox (LP), introduced by Priest in his [13]. Using this logic as background, Priest was able to put forward a dialetheist account of motion that promises to overcome the three deficiencies of the standard account that we discussed above.

Let me introduce this account. I will then proceed to illustrate how the account is supposed to overcome the three difficulties mentioned above. Finally, I shall argue that the Hegelian account suffers from the same difficulties as the Russellian account (sec. 4).

According to Priest, what prevented us from developing a suitable treatment of time and motion, vis a vis the problems that we have been discussing, is the assumption that bodies in motion occupy coherently one exact position at the time, at the expense of all others:

> Consider a body in motion-say, a point particle. At a certain instant of time, t, it occupies a certain point of space, x', and, since it is there, it is not anywhere else. But now consider a time very, very close to t, t'. Let us suppose that over such small intervals of time as that between t and t' it is impossible to localize a body. Thus, the body is equally at the place it occupies at t', x' (Hence, at this instant the body is both at x and at x' and, equally, not at either). [Ibid., p. 176]

The idea can be spelled out as follows. Let the motion of the body be represented by the equation $x = f(t)$. According to the Russellian view, the proposition that the body is located at position r at time t is true if and only if $r = f(t)$. It is false if and only if $r \neq f(t)$. According to Priest's account, instead, for each time t during the motion of the body, there is an interval containing t, θ_t, called a "spread", such that, if $t' \in \theta_t$, the occupation of the body's location at t' is "reproduced" at t. In other words, the proposition that the body is located at position r at time t is true if and only if, for some $t' \in \theta_t$, $r = f(t')$. It is false if and only if for some $t' \in \theta_t$, $r \neq f(t')$.

To see why this view is dialetheist, it is perhaps better to characterize it in terms of state descriptions. Let q denote any quantity, ranging over a set of values V (for example, the real numbers). Let $v(t) = q$ be a function of time, and $Q(r)$ be the predicate that q has value r. Finally, let $S_t = \{Q(r) : r = v(t)\} \cup \{Q(r) : r \neq v(t)\}$. The propositions contained in S_t state that at t the value of q is $v(t)$ and not any other value. S_t can then be thought of as the state description of the system at time t. The spread hypothesis amounts to the claim that, for every time t, there is an interval containing t, θ_t, such that the complete state description at t is the set theoretic union of all the consistent state descriptions $S_{(t')}$ for all the $t' \in \theta_t : \cup_{(t' \in \theta_t)} S_{(t')}$. Clearly, if the state of the system is changing in a neighbourhood of t, such union will be inconsistent.

3.1 Priest's account of temporal passage. One of the most interesting features of Priest's treatment of change is that it promises to be applicable to all sorts of change, including the peculiar kind of change - if it is a kind of change at all - which the passage of time consists of.

When change is conceptualized in a Russellian fashion, as the instantiation of different states at different times, it becomes conceptually absurd to claim that time itself changes in any sense. And yet there is something impermanent about time. We make a gesture towards this feature when we refer to the incessant âadvancementâ of the present towards the future. It is perhaps not a chance that all the metaphors that we use to express the impermanence of time are metaphors of motion. We say that time passes, that it flows, that it flies. We approach the end of the semester, etc. What does the passage of time have in common with change in general, and with motion in particular? What is the cash value of these metaphors?

In this regard, the intrinsicality of change in Priest's account constitutes a great advantage. We have discussed the account as it applies to motion, but we might as well have applied it to any other magnitude as well, charge, mass, or whatever. According to the Hegelian account, for a quantity, any quantity, to be in a state of change is for it to be in a certain contradictory state.[7] This very general idea of change lends itself immediately to an application to the case of time itself. Here is how this is supposed to work.

Let q denote the state of time, whatever this may be, and V the set of the reals. The proposition $Q(t)$, for example, could mean that time t is present. Let $v(t)$, finally, be the identity function. Applying the spread principle as we did above, one can see that, for each time t, there exists a interval θ_t such that, for all $t' \in \theta_t$, $Q(t')$ is true at t. The claim that time is passing, or that the time that is present is constantly "changing", can then be seen to rest on the fact that the spread θ_t is non degenerate, or equivalently that the state description of time is always inconsistent.

Let us now turn to Priest's application of the Hegelian account to the three challenges presented above. In what follows I illustrate the proposed application of the Hegelian account to tackle the three challenges discussed above. I argue that the account falls pray to all of them.[8].

[7] Priest of course is not denying that if a quantity assumes different values at different times then it has changed. The claim is rather that this is not what change could consist of.

[8] Some of the arguments against the Hegelian account presented here can be found in my Boccardi and Bustos 2017 ([7])

4. The Hegelian account and Priest's three objections

4.1 How the Hegelian account is supposed to tackle the problem of incremental accretion, and why it fails. Priest argues that the Hegelian account of motion ought to be preferred to its rival, the standard Russellian view, since the spread hypothesis solves at one stroke all the difficulties mentioned above. Let us see how this is supposed to work in each case, starting from the problem of incremental accretion. The problem, remember, is that of explaining how a series of non-advancements might produce a finite displacement, a positive advancement. Here is how the spread hypothesis is supposed to overcome this difficulty:

> The Hegelian account of motion may be taken to locate a fault in the [arrow] argument, but at a point different from that upon which Russell lights. For, according to Zeno's argument, at a particular point in time the object occupies only a single point in space, whence it follows that it advances not on its journey during that instant, i.e. that the measure of the set of points occupied at that instant is zero. Given the spread hypothesis, however, it is not true that the moving body occupies only a single point. At an instant, t, it occupies all the points in Σ_t, which is, in general, not a singleton. Indeed, provided the function of motion, f, is continuous, Σ_t is an interval, and therefore has non-zero measure. Thus, advance is made during a single instant, and hence during the aggregate of instants. [Ibid., p. 180]

This response, I argue, squeezes out of the Hegelian account more than it can deliver. The unwarranted assumption, crucial for giving the appearance of a solution to Zeno's paradox, is that the mere fact that the instantaneous location of the object be of non-zero measure would suffice to make of it an "advancement". A way to expose the role of this subtle but crucial assumption is to note that Zeno's argument can be easily generalized to apply to the Hegelian account as well. Here is a Hegelian friendly version of Zeno's arrow argument.

At each moment during its journey, the arrow occupies (inconsistently) a region of space the same size as its spread Σ_t. During each instant t, the spread Σ_t of locations occupied by the arrow advances not at all. Then how does it manage to advance over a finite interval of time, given that this is constituted by nothing but a sequence of such spreads?

The inconsistency of the Hegelian locational schema resides in the fact that the spread Σ_t which the arrow occupies at time t is larger than the size of the arrow itself. But why should we think that this larger space occupied by the arrow constitutes an advancement of the arrow? If at each moment during the journey the arrow occupies the spread Σ_t, then the motion of the arrow can only consist of its subsequently occupying different spreads, each in its due turn. How this feat is achieved, the account does not say.

Another way to expose this difficulty is by considering Priest's intriguing hypothesis that quantum mechanical indeterminacies might constitute the physical ground of the spreads:

> Perhaps the measure of Σ_t, $\sigma(\Sigma_t)$, just is the uncertainty in the location of the object at t. Perhaps quantum mechanical indeterminacies are fundamentally the result of inconsistencies in motion, and in particular in the spread postulated by the spread hypothesis. This suggestion at least allows us to give physical significance to the spread. (Ibid.)

The difficulty under discussion here can be exposed by constructing a quantum version of Zeno's arrow, and then testing the Hegelian interpretation against it. Consider then a particle whose locational state at time t is given by the wave function $\Psi(x,t)$. The Zenonian argument then goes as follows. Over the finite interval of time $[t_0, t_1]$, the state of the particle changes from $\Psi(x,t_0)$ to $\Psi(x,t_1)$. But at each time during its journey the state of the particle changes not at all. How is it, then, that in a sum of changing-not-at-all's the particle manages to instantiate a different state?[9]

On the Hegelian interpretation, the instantaneous states represented by $\Psi(x,t)$ should be construed as representing inconsistent superpositions of exact locations. But notice that the inconsistency of the account is never used by Priest to solve the problem of incremental accretion. As Priest noted in objecting to my criticism, the only feature of it that is relevant is the non-zero length of the location occupied by the particle at the instant.[10] However, it should be noted that whether the particle occupies a single dimensionless point at an instant, as the standard view has it, or an inconsistent spread of locations, it does so consistently: on the Russellian account the particle occupies a point of space at the expense of all others, while on the Hegelian account it occupies a spread of locations at the expense of all others. The challenge is to explain how the location occupied might change without ever being changing. It seems to me that with regard to this, the Hegelian account faces exactly the same difficulty as the Russellian one.

Of course, the discomfort would be alleviated if the view was that the arrow proceeds by "jumping" discretely from one spread of locations to the adjacent one. In this case, its advancement would be analogous to that of the revolving polygon, where each side touches the ground all at once, contributing to the whole journey with an advancement equal to its finite length. But this is not Priest's view. The edges of the spread, as well as all the other points inconsistently occupied by the tip of the arrow at an instant, are all durationless points. During each instant of time, all these locations advance not at all. Yet, if the arrow is to reach its target, it has to occupy all of these points, one by one, in succession. How does it do it?

One can also express this worry by noting that the whole arrow, unlike its tip, *does* occupy a finite amount of space at each instant during its journey. Yet, nobody is tempted to consider the extension of the arrow as an "advancement" towards its target. The spreads occupied by the tip of the arrow at each instant of time, I argue, are no different from the whole arrow, when it comes to accounting for the advancement of the arrow. Each position inconsistently occupied by the tip of the arrow has no element of movement, just like the positions consistently occupied by it in the Russellian account don't.

4.2 How the Hegelian account is supposed to tackle the problem of non-intrinsicality of motion, and why it fails. As we have seen, one of the problems with the standard account is that it makes intrinsic velocities - and states of change in general - relational or neighborhood properties. The Hegelian account promises to solve this problem rather well:

> This is obviously no problem for the Hegelian account. For it, there is an intrinsic state of motion: a certain inconsistent state. The difference between a body genuinely in motion and one changing place but at rest

[9] In my Boccardi and Bustos 2017 ([7]) we argue that the quantum interpretation of the spread hypothesis is problematic, for a technical reason that I shall not repeat here.

[10] See Priest's response at minute 41:30 of https://www.youtube.com/watch?v=iR58cnYyviw&t=2014s.

> each instant is exactly that between a Hegelian state description and the corresponding Russellian one. [ibid., p. 180]

Priest is surely right that his account renders motion an intrinsic feature of objects. And this is prima facie an advantage. The problem, remember, was that Russellian intrinsic velocities are properties of trajectories, rather than intrinsic temporary properties of objects. This rules them out as potential explanations for these trajectories themselves. Now, the positions that the arrow may (inconsistently) occupy at any given time according to the Hegelian account are totally unrelated to the locations it may occupy at any other time. So, the Hegelian account at least does not have this problem.

However, it could be argued that the inconsistent spread of locations at a time is not the right kind of intrinsic state. Notice, in fact, that in a Russellian world the positions (consistently) occupied by all objects at a time do constitute a temporary intrinsic state of the universe at that time. Yet, nobody would be tempted to consider that state as the ontological ground of instantaneous motion. Why? Presumably, because the position of an object at a time is totally, not merely conceptually, independent from its position at other times. Two identical locational states of the universe may yet diverge radically as to their dynamic properties.

Now, how is the inconsistent spread of positions postulated by the Hegelian account different in this respect? The spread occupied by the arrow at a time is totally, not merely conceptually, independent from the spread it occupies at other times. So why should we think that it has anything to do with motion? Why should the instantiation of inconsistent states of affairs as to the location of the ball at a time "force" it, or "induce it" or "dispose it" to be elsewhere at other times?

It will be objected that this criticism asks too much from a theory of motion. After all, the complaint against the Russellian view was that it renders the velocities of objects at a time t (and hence the instantaneous states of the universe at that time) conceptually dependent on the positions of the objects at all times around t, thus depriving them of their explanatory power. Isn't it preposterous, then, to complain that on Priest's account intrinsic states of motion do not entail the correlated trajectories? The spread hypothesis, by itself, does not explain the trajectories precisely because it complies with the requirement that it be conceptually independent from them! At best, one could require that there be a causal, or law-like connection between intrinsic states of change and trajectories. This is exactly what Priest does (Ibid., p. 179).

I think that this line of defense is fair. However, I still think that my objection points at a difficulty with the Hegelian account, for two reasons. First, the ambition of the theory is to provide an account of change in general, not of motion only. We have seen, for example, how this generality allows us to apply the Hegelian account to the passage of time itself. If Priest were right, true contradictions would not just contingently correlate with trajectories: they would constitute the ontological ground of change in general. They would be the essence of change, as it were. This is one of the chief allures of the Hegelian account. Given the generality of the account, however, I think it is not unreasonable to demand an elucidation of this systematic connection. Of course, one could postulate a different causal connection for each kind of change. However, the general pattern of connections would still cry for an explanation, lest it be a bizarre fluke.

The second worry that I have with the "just so" response that we are considering, is that it is in tension with Priest's use of the spread hypothesis in tackling the problem of incremental accretion. The problem, as James put it, was that the "conceived positions, however numerously multiplied, contain no element of movement, so Zeno, using nothing but them in his discussion, has no alternative but to say that our intellect repudiates motion as a non-reality." [James 1987: 735.]. If the spread hypothesis is to tackle this problem, the contradictory states must not only provide us with a cause for the trajectories. They must further be the basic instantaneous elements of motion itself. Each spread occupied during the journey of the arrow must make a contribution to the whole journey. Therefore, each spread must be, and not merely cause, a kind of displacement. If things are so, the total lack of conceptual dependence between contradictory state descriptions and trajectories in the Hegelian account is just as problematic as the total conceptual dependence between them in the Russellian account.

4.3 How the Hegelian account is supposed to tackle the problem of the direction of time, and why it fails. It is undoubtedly an interesting advantage of Priest's treatment, as we have seen, that it applies to the passage of time just as well as it applies to any other kind of change. Priest also claims that the Hegelian view can account for the intrinsic directionality of passage, something that standard accounts are at great difficulty to explain. On this latter point, I dissent from Priest. The solution proposed rests on the observation that: "θ_t will not necessarily be distributed symmetrically about t. In fact, there are reasons to suppose that t is the leading edge of θ_t, so that the interval is skewed all to the past of t, at least normally." [Ibid., p. 214]

This asymmetry in the distribution of the spread, according to Priest, would solve the problem of grounding the directionality of time:

> Again, the solution to the problem on the present approach is obvious. Since the identity function is monotonically increasing, $v(t)$ is always the upper bound of the spread of q at t. Thus, the direction of the flow of time is perpetually from past to future, which seems just about right. [ibid.]

The problem with this solution, just as for the Russellian account, is that it tries to ground directionality on mere asymmetry. We have already seen how this feat is impossible. Priest surreptitiously avails himself of a primitive notion of directedness when he claims that the identity function is monotonically "increasing". The identity function, by itself, is neither increasing nor decreasing, just like the great pyramid of Giza is neither ascending nor descending.

The problem can be aptly illustrated by considering the order induced on the real numbers by the relation smaller-than. Like any relation that is asymmetric, connected and transitive, the relation smaller-than induces an order on the reals. In addition, the reals present a structural asymmetry. This can be seen by noting that the positive reals have a square root, while the negatives don't. As Reichembach ([15], p. 26) noted, this asymmetry can be exploited to differentiate structurally the relation smaller-than from its converse, the relation greater-than. This can be done, for example, by saying that any number which is the square of some number is greater than any number which is not the square of some number. This feature of the reals provides the series with a kind of "directionality". However, this anemic kind of directionality is not the kind of directionality that Priest is seeking to ground. Mere order, even embellished by the perfection of structural asymmetry, does not determine any directionality. To see this, it suffices to observe that, despite the structural asymmetry of the relation smaller-than, it simply makes no sense to ask whether the reals are increasing or decreasing, or whether they "run" in one direction as opposed to the other.

5. Conclusion

Zeno's arguments appear to show that time and change are contradictory. The main attempt to remove these contradictions without denying time and change altogether, the now widely accepted Russellian account, proves to be at best a restatement of the problem. We are thus faced with a dilemma. Either accept the phenomenological datum that things change and time passes, at the cost of accepting that at least some contradictions are true; or hold fast to the principle of contradiction, and repudiate time and change as mere appearances. Priest has shown that the first option is not as abhorrent as we may think, and that it ought to be seriously explored. He also claimed that the Hegelian account ought to be preferred to the Russellian one on the ground that it is immune from the problems of incremental accretion, of the non-intrinsicality of motion, and of the directionality of time. I have argued that this is not the case. The Hegelian account does not help overcoming these difficulties.

However, I think that Priest's account does present a number of advantages over the Russellian one. The main one is its generality. As we have seen (sec. 3.1), the account can be applied to the case of temporal passage, a problem that is apparently intractable within a Russellian world view. Secondly, even if the spread hypothesis does not solve the three problems, it seems to be in a better position to solve them. For example, even if the hypothesis does not solve the problem related to the non-initrinsicality of change, at least it points in the right direction, by making change an intrinsic state of affairs. For the same reason, I think that Priest's view is also in a better position than its rival when it comes to solving the problem of the directionality of time. As we have seen, the mere asymmetry in the spread could not, by itself, account for the directionality of time. However, whatever does so ground directionality, arguably, must at least supervene on some intrinsic state of affairs.

Finally, Priest's account helps to explain why the issue of contradiction proved so recalcitrant in connection with time and change. Contradictions, qua contradictions, I have argued, don't appear to contain any essentially dynamic element. However, the suspicion that the idea of change might be contradictory has continued to resurface over and over in the history of philosophy, and all attempts to dissolve these paradoxes proved to be unsatisfactory. Thus, it should not come as a surprise if it turned out that change essentially involves true contradictions.

Acknowledgment. I am very grateful to Graham Priest for his incisive comments. I am also indebted to Moises Macias Bustos, with whom I co-authored a paper which contains some of the ideas discussed here. Finally, I wish to thank the editors Rodrigo Freire, Alexandre Costa-Leite and Edgar Almeida.

Bibliography

[1] D. Z. Albert. *Time and Chance*. Cambridge (MA), London: Harvard University Press., London, 2000.

[2] F. Arntzenius. Are there really instantaneous velocities? *The Monist*, 83:187–208, 2000.

[3] L. R. Baker. Temporal reality. In M. OâRourke J. K. Campbell and H. S. Silverstein, editors, *Time and Identity*, pages 27–47. The MIT Press, Cambridge, MA, 2010.

[4] H. Bergson. *Creative Evolution*. Holt, Reinhart and Winston, New York, 1911.

[5] J. Bigelow and R. Pargetter. *Science and Necessity*. Cambridge University Press, Cambridge, 1990.

[6] E. Boccardi. Time as motion. *Metaphysica: International Journal for Ontology and Metaphysics*, 19(1):1–31, 2018.

[7] E. Boccardi and M. Bustos. Contradictions in motions: Why they're not needed and why they wouldn't help. *Humana.Mente, Journal of Philosophical Studies*, 32:195–227, 2017.

[8] E. Boccardi and F. Perelda. The delusive illusion of passage. *Analysis*, 78: 3:387–396, 2018.

[9] J. Carroll. Instantaneous motion. *Philosophical Studies*, 110:49–67, 2002.

[10] B. Dainton. Time, passage, and immediate experience. In C. Callender, editor, *The Oxford Handbook of Philosophy of Time*, pages 382–419. Oxford University Press, Oxford, 2011.

[11] M. Lange. How can instantaneous velocity fulfill its causal role? *The Philosophical Review*, 114 (4):433–468, 2005.

[12] B. Leftow. Instants, events, and god. In N. Oaklander, editor, *Debates in the Metaphysics of Time*. Bloomsbury, London, 2014.

[13] G. Priest. The logic of paradox. *Journal of Philosophical Logic*, 8(1):219–241, 1979.

[14] G. Priest. *In Contradiction: A Study of the Tranconsistent*. Clarendon Press, Oxford, 1987.

[15] H. Reichembach. *The Direction of Time*. Dover Publications, Mineola, NY, 1956.

[16] B. Russell. Mathematics and the metaphysicians. In B. Russell, editor, *Mysticism and Logic*. Barnes and Noble Books, Totawa, NJ:, 1981.

[17] Bertrand Russell. *Principles of Mathematics*. W.W. Norton and Company, inc, New York, 1903.

[18] S. Smith. Are instantaneous velocities real and really instantaneous? *Studies in History and Philosophy of Modern Physics*, 34:261–280, 2003.

[19] M. Tooley. In defense of the existence of states of motion. *Philosophical Topics*, 16(1):225–254, 1988.