

Dharmakīrtian Inference

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Abstract

Dharmakīrti argues that there is no *pramāṇa* (valid means of cognition or source of knowledge) for a thesis that is a self-contradiction (*svavacanavirodha*). That is, self-contradictions such as ‘everything said is false’ and ‘my mother is barren’ cannot be known to be true or false. The contemporary scholar Tillemans challenges Dharmakīrti by arguing that we can know that self-contradictions are false by means of a formal logical inference. The aims of the paper are to answer Tillemans’ challenge from what we take to be Dharmakīrti’s or Dharmakīrtian (someone who is like Dharmakīrti) perspective and to demonstrate the unique features of Dharmakīrti’s view of inference. By so doing, we show that the epistemology in relation to the formal conception of logic that underlies Tillemans’ challenge is problematic from Dharmakīrti’s or Dharmakīrtian perspective. The paper, thus, presents Dharmakīrti’s view of inference and logical reasoning as well as a Dharmakīrtian challenge to the formal conception of logic that is the dominant contemporary conception.

1. Introduction

According to Dharmakīrti, self-contradictions (*svavacanavirodha*)—for example, ‘everything said is false’ and ‘my mother is barren’—express too many things and, consequently, cannot be known to be true or false. In commenting on the relevant passage of Dharmakīrti’s *Pramāṇavārttika*, Tillemans remarks that ‘there is an oddity’ (Tillemans (2000: 140)) in Dharmakīrti’s view about self-contradictions. In effect, he challenges Dharmakīrti and argues that we can know that self-contradictions are false, contrary to Dharmakīrti’s claim that they cannot be known to be true or false.

The aims of the paper are to answer Tillemans’ challenge from what we take to be Dharmakīrti’s or Dharmakīrtian (someone who is like Dharmakīrti) perspective and to

demonstrate the unique features of Dharmakīrti's view of inference. First, we present Dharmakīrti's view about self-contradictions. As we have no dispute with Tillemans about Dharmakīrti's view in this respect, we largely follow Tillemans' textual analysis. Second, we present what is involved in Tillemans' challenge. We show that, behind Tillemans' challenge, there lies a contemporary conception of logic according to which logic is *formal*. Third, we analyse Dharmakīrti's view of inference and show that his view of logic is distinct from the formal conception of logic prevailing in contemporary philosophical literature. Fourth, we show that the epistemology in relation to the formal conception of logic that underlies Tillemans' challenge is problematic from Dharmakīrti's or Dharmakīrtian perspective. The paper, thus, presents Dharmakīrti's view of inference and logical reasoning as well as a Dharmakīrtian challenge to the formal conception of logic that is the dominant contemporary conception.

2. Dharmakīrti on Self-Contradictions

Dharmakīrti in his *Pramānavārttika* IV.98-99 examines the nature of self-contradictions or contradictions with one's own words (*svavacanavirodha*) such as 'my mother is barren'. For him, a speech that expresses self-contradictions is ambiguous.¹ On the one hand, when we say 'my mother is barren', we imply that a person we talk about cannot have children. On the other hand, in saying 'my mother is barren', we imply that the same person has a child since 'mother' means a person with a child. This means that when we say 'my mother is barren', we express both that a certain person referred to by 'my mother' is barren and that the person is not barren. So, to assert 'my mother is barren' is to say too much. For Dharmakīrti, there is an *impediment* (*pratibandhaka*) between saying that someone is barren and saying that the same person is not barren. However, even though we cannot knowingly assert that someone is barren and not barren at the very same time, these two assertions do not 'invalidate' each other, meaning that neither the assertion that this person is barren nor the assertion of its opposite by itself can show the other to be false. There is no valid means to know (i.e., there is no *pramāṇa*) that the saying 'My mother is barren' is true or false. This means that we cannot know that the assertion or the assertion of its opposite is true or false, so Dharmakīrti argues.²

3. Tillemans' Challenge

In commenting on *Pramānavārttika* IV.98-99, Tillemans says that 'there is an oddity' in Dharmakīrti's view about self-contradictions. In effect, he challenges Dharmakīrti about the lack of knowledge in the context of self-contradictions. He writes:

¹ We think that Dharmakīrti understands 'speech' as the act of asserting or saying. However, we don't take any stance on the question of whether Dharmakīrti is primarily talking about statements, sentences, propositions, words, asserting or saying. How to answer this question doesn't affect the main point of this paper.

² For this interpretation of the passage, see Tillemans (2000: 139–40) and Bogacz (2023).

Logically speaking, we might remark, however, that there is an oddity in saying that both “My mother is barren” and “My mother is not barren” are just statements of which we do not know the truth-value. What does not seem to have entered into consideration for Dharmakīrti is the following inference: granted that if P then not- P , we can then infer not- P . Arguably, what Dharmakīrti could have said is that a self-refuting statement is indeed false simply because if it were true, it would be false. This inference would be valid because the statement $((P \rightarrow \text{not-}P) \rightarrow \text{not-}P)$ is a theorem; in fact it is a frequently invoked tautology of propositional calculus. To put things in Dharmakīrtian terms, then, one could well maintain there is a *pramāṇa* [i.e., a source of knowledge] for the opposite of a self-contradictory thesis. (Tillemans (2000: 140))³

In this passage, Tillemans takes Dharmakīrti to be concerned with statements rather than the act of putting them forward as true (asserting or saying). We put aside the question of whether or not Tillemans is entitled to do so. This is because the real issue here is epistemological.

As we saw above, Tillemans analyses *Pramānavārttika* IV.98-99 as presenting the view that self-contradictions cannot be known to be true or false. In other words, he takes Dharmakīrti to be saying that there is no *pramāṇa* for a thesis that is a self-contradiction. In *Pramānavārttika* IV.98-99, Dharmakīrti argues for his position about self-contradiction in the context of scripturally based inference (*āgamāśritānumāna*). Tillemans does not seem to have any problem with Dharmakīrti’s reasoning in that context. However, he rejects Dharmakīrti’s claim that self-contradictions cannot be known to be true or false and claims, instead, that we *can* know and, thus, there is a *pramāṇa* for the thesis that a self-contradiction is false.

Tillemans’ reasoning is as follows. It is a theorem of (classical) propositional logic (i.e., it is a logical truth according to classical logic) that $(P \rightarrow \neg P) \rightarrow \neg P$ where \neg is a negation (not) and \rightarrow is a conditional (if..., then...). This means that if P implies its negation, then the negation must be the case. If we assume the deduction theorem which holds in classical propositional logic ($\models P \rightarrow Q$ iff $P \models Q$, meaning that if $P \rightarrow Q$ is a logical truth, we can validly infer Q from P , and *vice versa*), then we can infer the negation of P . Now, a self-contradiction implies its negation. So, by the theorem, its negation must be the case. If we think of $\neg P$ to mean that P is false, then the self-contradiction is false. By this reasoning, thus, we can know that a self-contradiction is false, so Tillemans argues.

Reasoning in this way, Tillemans assumes certain things about logic and epistemology. First, he assumes that logic is essentially *formal* in the sense that valid inferences can be expressed formally by the statements such as $(P \rightarrow \neg P) \rightarrow \neg P$ (via deduction theorem). He thus assumes that inference according to formal validity (in terms of theorems or formally valid inferences) is a *pramāṇa* (a source of knowledge). Second, he assumes that if an inference is valid, it is valid in virtue of its *form*. We will show that the first assumption is problematic as inference according to formal validity cannot be a *pramāṇa* and the second assumption is incoherent from Dharmakīrti’s or Dharmakīrtian perspective. This is the case whether or not Dharmakīrti would subscribe to the principle $(P \rightarrow \neg P) \rightarrow \neg P$. But, before that, we will unpack

³ For another formulation of the same point, see Tillemans (2016: 103–7).

these two assumptions by articulating what is involved in the idea that logic is formal in order to show that the two assumptions are problematic.

4. Formal Logic and Its Place in Epistemology

It is commonly assumed that logic is essentially formal. This assumption is so prevalent these days that any explanation may be thought to be redundant. In contrast, Dharmakīrti does not understand inference to be a formal matter. This contrast has been pointed out in various places;⁴ however, the exact difference between the formal conception and Dharmakīrti's or Dharmakīrtian conception has yet to be articulated. We will show this difference by first unpacking what it means for logic to be formal and second articulating the Dharmakīrtian conception of inference.

The thought that logic is essentially formal probably occurred to many people throughout the history of logic. However, the thought that logic is essentially formal was clearly expressed in the writings of Immanuel Kant. While several conceptions of formality have been introduced since the time of Kant,⁵ we will briefly look at how Kant conceived of logic as formal as he presented an early account of formal logic and a clear articulation of the formal conception of logic.

In his *Critique of Pure Reason* (KrV), Kant provides a complex taxonomy of logics.⁶ In his *Lectures on Logic*, in particular *Jäsche Logic* (JL) (as well as some places in the *Critique*), however, he identifies *logic* with what he calls 'pure general logic'. Kant held that (pure general) logic is a maximally general science. By this, he meant that logic deals with necessary *rules* of the understanding. These are the rules 'without which no use of the understanding would be possible at all' (JL 12). These necessary rules of the understanding are contrasted with the contingent rules 'which depend upon a determinate object of cognition' (JL 12). Thus, for Kant, logic is concerned with the rules that 'contain merely the conditions for the use of the understanding in general' (JL 12) independently of the object of cognition. Hence, logic is *general* in the sense that it is applicable to the understanding as such without any dependence upon the objects of cognitions. For Kant, logic is 'the science of the rules of the understanding in general' (KrV A52/B76).⁷

From the *generality* of logic, Kant inferred that logic is *formal*. Since logic is general in the sense that it contains only the conditions for the use of the understanding as such, the necessary rules of logic are without qualification, '*without distinction among its objects*' (JL 12). Logic is independent of the discrimination between the objects of our cognitions. Hence, logic is abstracted from the objects themselves which provide the contents of thought.⁸ Thus, as Kant argues, 'the universal and necessary rules of thought in general can concern merely its *form* and not in any way its *matter*' (JL 12). In this way, Kant infers, from generality, that 'logic deals with nothing but the mere form of thought' (KrV A54/B78), 'abstrac[ted] from all content

⁴ Balcerowicz (2019); Gillon (2016); Siderits (2003).

⁵ See MacFarlane (2000); Dutilh Novaes (2011).

⁶ See for example KrV A50/B74-A64/B88.

⁷ See also KrV A53/B77-A54/B78.

⁸ See for example KrV A54/B78, A55/B79 and JL 51.

of knowledge, that is, from all relation of knowledge to the object' (KrV A55/B79).⁹ This is Kant's *formal* conception of logic:

[T]his science of the necessary laws of the understanding and of reason in general, or what is one and the same, of the mere form of thought as such, we call *logic*. (JL 13)

Now, Kant's conception of logic can be better appreciated by examining it in his critical framework as presented, especially, in his *Critique of Pure Reason*, as it is all couched in his critical philosophy. As is famously known, Kant held that '[t]houghts without content are empty, intuitions without concepts are blind' (KrV A51/B75). For Kant, an object is given to us through (sensible) intuition which receives impressions in the form of representations. Without our capacity to receive sensible impressions, we are unable to acquire any knowledge of objects: we are not given any object to know anything about. However, these impressions themselves are undetermined as to what they represent. In order to have a determinate knowledge of the object, the given representations must be judged and thus thought by means of concepts (ultimately the categories). But concepts are not directly linked to objects. They are concerned with representations: they are functions that bring 'various representations under one common representation' (KrV A68/B93). For Kant, this capacity to unify various representations together is the understanding. Hence, '[t]he understanding can intuit nothing, the senses can think nothing. Only through their union can knowledge arise' (KrV A51/B75). Logic, for Kant, is then concerned with the forms in which representations are unified by means of concepts in the understanding in order to make a judgement.

An important point to note is that, for Kant, a judgement has an objective significance only in relation to the objects given through intuition. A concept has objective contents only if the representations that are brought together by the concept are 'combined *in the object*' in a judgement (KrV B142) by relating representations immediately to the objects (KrV A68/B93). In other words, judgements must be the judgements of objects in order to be objective. Otherwise, a judgement is not about any object and thus doesn't have any objective significance at all. Kant writes:

Knowledge involves two factors: first, the concept, through which an object in general is thought (the category); and secondly, the intuition, through which it is given. For if no intuition could be given corresponding to the concept, the concept would still indeed be a thought, so far as its form is concerned, but would be without any object, and no knowledge of anything would be possible by means of it. So far as I could know, there would be nothing, and could be nothing, to which my thought could be applied. (KrV B146)

This means that, for Kant, logic (i.e., pure general logic) has no objective significance since it is abstracted from the objects of cognition. It is concerned with mere forms in which the

⁹ See also JL 94. Kant also infers from generality the *a priori* nature of logic. See for example KrV A53/B77, A54/B78 and JL 12.

representations of the intuition are unified in judgements without having any objective significance of their own.¹⁰

This analysis of Kant's formal conception of logic is done using a Kantian language. But there are features of his formal conception of logic that can be extracted from it. These features that Kant attributes to logic have been presented and discussed in different ways and sometimes in different contexts; however, the idea that logic has those features has largely remained since then.¹¹ In order to show that the formal conception of logic is not applicable to Dharmakīrti and that it is problematic from Dharmakīrti's or Dharmakīrtian perspective, it is important that these features of the formal conception of logic are shed some light on.

As we can see in the way that Kant articulates his formal conception of logic, to think that logic is formal is to think that logic is not concerned with the objects of knowledge; rather, it is concerned only with the structure or the form of knowledge. For Kant, this means that logic applies to knowledge in general rather than knowledge of any particular objects. An implication of the formal conception of logic is then that logic is *topic-neutral* in the sense that it is not concerned with any objects or 'topics' of knowledge but only with knowledge in general.¹² This implies that, under the formal conception, each logical operation (e.g., each step of a proof or an argument) is insensitive to what it is about. For instance, a logical operation in a proof may eliminate a conjunction ($P \& Q \vDash P$). According to the formal conception, such a step does not require any specific information about either of the conjuncts because of topic-neutrality. Thus, it is a step that is conceived to be 'safe' at any stage of a proof and in any proof.

Thus, there is a consequence of the formal conception of logic that we should note as it becomes relevant to an examination of Dharmakīrti's or Dharmakīrtian view. Given that logic, under the formal conception, is not concerned with objects, logical operations by themselves make no contribution to illuminating the subject matter in question. From the perspective of logical operations, it does not matter whether the operations are performed in a proof or reasoning about numbers or geometric figures,¹³ or even about mathematics or politics, for instance. This means that logical operations, whether they are the steps in proofs or inferences drawn in reasoning, do not apply to the objects of cognitions nor are they licensed by those objects of investigation. This feature of logical operations under the formal conception are at odds with how Dharmakīrti understands logical reasoning or inference (*anumāna*).

5. Dharmakīrtian Inference

Dharmakīrti understands logical reasoning or inference very differently from the formal conception of logic. It is not just the notion of form that is absent from Dharmakīrti's or Dharmakīrtian view, but all of the main elements of the formal conception are not applicable to him. In other words, the whole package under the umbrella of formal conception does not

¹⁰ Transcendental logic is importantly different in this very respect. See, for instance, Rödl (2012).

¹¹ See, for instance, Lapointe (2018).

¹² Note that this is not how Kant expressed his thought about logic; rather, it is an implication of his view.

¹³ This is exactly what Hilbert shows in his *Grundlagen* Hilbert (2015).

make much sense to him. Moreover, from Dharmakīrti's or Dharmakīrtian perspective, the formal conception of inference is problematic.

In order to show this, we identify three main elements of Dharmakīrtian inference: cognitive focus, knowledge expansion and existential commitment. We will articulate what they are in Dharmakīrti's or Dharmakīrtian contexts and show that they are not present in the formal conception. This will show that Dharmakīrtian inference is different in kind from formal inference (inference understood under the formal conception of logic). We will then use this Dharmakīrtian perspective to argue against the formal conception of logic.

5.1. Cognitive Focus

One feature of Dharmakīrtian inference that is importantly distinct from formal inference has to do with the fact that Dharmakīrti focuses on cognitions that arise in acquiring knowledge by inference rather than the understanding or knowledge *in general*. As we saw in the way that Kant articulates his formal conception of logic, logical relations are concerned with the understanding as such without being dependent on the objects of cognitions. How exactly to make sense of this should be left for Kant scholars. However, one thing is clear: it does *not* mean that, for Kant (as well as most contemporary logicians who accept the formal conception), logic is concerned with a series of particular cognitions that arise in making inferences.

According to the formal conception, logic is not about any particular cognitions that arise as we reason and, thus, that logical relations are *abstracted* from inferential cognitions. This means that logical relations are not about the relations that hold between cognitions. This makes it difficult to account for the possibility of applying logical principles to the inference we perform as our inference involves a series of cognitive states.

For Dharmakīrti, inference is a source of knowledge. However, he does not consider the understanding or knowledge as such to be what inference is for. When Dharmakīrti (or any Buddhist as well as Indian logicians generally) talks about inference, he is talking about a series of cognitions that arise in making an inference. If the inference is 'valid', these cognitions lead to a cognition that can be characterised as a knowledge state. The focus is, thus, on inferential *cognitions* that arise in producing knowledge rather than on the understanding or knowledge as such which does not depend on any particular cognitions.¹⁴ That is, when Dharmakīrti talks about inference, what is talked about are the cognitions that arise in reasoning.

This point of Dharmakīrtian inference is significant in the context of comparing it to the contemporary logical orthodoxy. Because of the anti-psychologistic climate, the contemporary logical orthodoxy shies away from referencing cognitions. We do not have enough space to defend psychologism or reject anti-psychologism here. However, it is important to examine why it is crucial to focus on cognitions for Dharmakīrti.

The importance can be brought out by examining the two contexts in which inference is said to be a source of knowledge. (1) It is an instrument for becoming aware of some truth such as the Four Truths of the Noble (*catvāri āryasatyāni*—often translated (or, perhaps,

¹⁴ Whether or not such knowledge can be understood outside of the context of Kant or the formal conception of logic is another matter. We let Kant scholars or formalists about logic to answer such a question.

mistranslated) as Four Noble Truths (see Pecchia (2015: 6-7)) by ourselves. Inference used in this context is called inference for the sake of oneself (*svārthānumāna*). (2) It is an instrument for showing that the opponents' views are mistaken and for demonstrating that one's own views are correct in dialectical situations. Inference used in this context is called inference for the sake of others (*parārthānumāna*). The proponent verbally expresses the inference to the opponent with an intention to make the opponent reproduce the same inference in their mind. For example, the proponent may try to make the opponent to infer for themselves that there is fire on a mountain because they (the opponent) saw smoke. With an intention to make the opponent reproduce the same inference in their mind, the proponent says: 'the mountain has fire because of smoke and wherever there's smoke, there is fire.' This saying is the inference for the sake of others.

We can see that Dharmakīrti's focus is on a series of cognitions rather than the forms that knowledge in general should have when he argues that inference for the sake of oneself is primary. Dharmakīrti claims that inference for the sake of others (*parārthānumāna*) is not really an inference and it is called an inference only metaphorically. The inference for the sake of others is just a speech (*vacana*) and speech is not a source of knowledge.¹⁵ For Dharmakīrti, speech is only a pantomime act.¹⁶ Merely hearing the words expressing an inference is not enough to produce knowledge in the opponent's (or dialectical partner's) mind. To inferentially learn that there is fire on the mountain, the opponent must think for themselves—infer for themselves—that there is fire on the mountain. This shows that Dharmakīrtian inference is about the cognitions that are involved in making an inference and how to bring about the cognitions that can be identified as knowledge states rather than about propositions or what words express that are abstracted from cognitions.

5.2. Expanding Knowledge

Our discussion of cognitive focus spotlights inferential *cognitions* and the role they play in producing knowledge. The difference between Dharmakīrtian inference and formal inference comes to the fore *not* when we consider the nature of logic and logical principles as such but when we consider the cognitions that arise in making inference. Dharmakīrtian conception takes the nature of inferential *cognitions* differently from the formal conception.

Before going on to elaborate on this point, we should note that the difference we are about to expand on is not something that Dharmakīrti himself entertained. It is an implication of cognitive focus we have investigated rather than something that comes directly from Dharmakīrti's texts. Nevertheless, considering this implication is important to understand the nature of Dharmakīrtian inference in contrast to formal inference.

Under the formal conception, we may be able to validly infer that there is fire on a mountain from that there is smoke on the mountain (under certain assumptions, especially the

¹⁵ For the view that the inference for the sake of oneself is primary, see PS III.1 and PSV 40b8-12, NB 3.1-2; PV IV.17 and PVin II.1. See also Iwata (1995: 156 fn. 21); Dunne (2004: 147). For the view that speech is not a source of knowledge, see PVSV ad PV I.213-217 and PV IV.48-108. See also Tillemans (1990: 24-35; 1999: Ch. 1, 2, 3; 2000: 78 ff); Eltschinger (2013: Ch. 3).

¹⁶ Thanks go to Danielle Macbeth for the expression 'pantomime act'.

crucial conditional that connects the presence of smoke and fire (wherever there is smoke, there is fire) as well as the formal validity of the inference). This is the case even if the cognitive state that there is fire on the mountain did not ‘grow out of’ the cognitive state that there is smoke on the mountain. Under the formal conception, if the proposition that there is fire on the mountain is true whenever the premises are true, then the cognitive state that expresses the presence of fire on a mountain may be considered to be validly inferred from the cognitive state that expresses the presence of smoke on the mountain. Importantly, this is the case even if the cognitive state about the presence of fire is one that spontaneously arises so long as truth is preserved from the propositions that there is smoke on a mountain to the proposition that there is fire on the mountain. In other words, inference from the presence of smoke to the presence of fire may be formally valid (under certain assumptions) even if a particular cognition that there is fire on a mountain does not arise from the particular cognition that there is smoke on the mountain. This shows that the formal conception does not necessarily treat inference as unified in cognition. In terms of the cognition involved in making inference, thus, the formal conception allows a cognitive state expressing a conclusion to count as a valid consequence of the cognitive state expressing the premises (premises-cognition) even when the cognition about the conclusion (conclusion-cognition) does not arise *because* of the premises-cognition. Hence, according to the formal conception, the conclusion-cognition does not have to be an extension or expansion of the premises-cognition.

In contrast, Dharmakīrti can be described as taking the cognitive state about fire to be an *extension* or *expansion* of the cognitive state about smoke. According to him, for there to be an inference, there must be a causal relation between the premises-cognition and the conclusion-cognition. Just as everything that is (really) existent is causally efficacious,¹⁷ if a series of cognitions arises to generate knowledge, those cognitions are causally efficacious. Thus, inferential cognition must be causally unified and the conclusion-cognition must arise *because* of the premises-cognition.

For Dharmakīrti, then, knowledge that one acquires through inference is an extension or expansion of the cognitive state that triggers the inference. Unlike the formal conception, Dharmakīrtian inference does not allow the cognitive state expressing the conclusion of the inference to be a mere concatenation of the cognitive state expressing the premises.¹⁸ It must be a state that arises *because* of the state that prompts the inference.

It is in this sense that inference is a *reliable* means of producing knowledge for Dharmakīrti. ‘Inferring’ that there is fire on the mountain where there is smoke by guessing, for instance, is not a *pramāṇa* for him even when the conclusion happens to be true. Our analysis can provide an explanation for this. Such an ‘inference’ is concerned only with the truths of the propositions expressing the premises and the conclusion and *not* concerned with the cognitive states and the changes of the states. This means that, even though the conclusion may happen to be true, inference, according to the formal conception, is *not* a reliable means of acquiring knowledge for Dharmakīrti.

¹⁷ PV III.3ab and PV I.166ab (Yoshimizu (1999: 233); Eltschinger (2010: 406); Franco and Notake (2014: 35)).

¹⁸ This concatenation is most obvious in the case of disjunction introduction: $P \vDash P \vee Q$.

5.3. Existential Commitment

The third point of departure for Dharmakīrtian inference from formal inference is that a formal inference is not concerned with any object and, thus, topic-neutral, whereas, for Dharmakīrti, an inference is about an object or objects and, thus, topic-specific. This can be highlighted by noting the following two strands of thought in Dharmakīrti's views on inference.

The first strand of thought is that, following Dignāga, Dharmakīrti holds that there are two *pramāṇas*: perception (*pratyakṣa*) and inference (*anumāna*). Dharmakīrti holds that there are two aspects of or modes in which objects are presented to cognition: particulars or particular characteristics (*svalakṣaṇa*) and universals or universal characteristics (*sāmānyalakṣaṇa*).¹⁹ For Dharmakīrti, particulars are causally efficacious and universals are conventionally existent. Particulars are objects of perception and universals are objects of inference.²⁰

Even though the object of inference (i.e., some universal characteristic) is only conventionally existent (and not ultimately existent because it is not causally efficacious), inference depends on particulars. This aspect of Dharmakīrtian inference can be brought out in two ways.

(1) Conventionally existing objects of inference are nothing like the universal characteristics of non-existent objects. For instance, consider rabbit's horns. Because there are no rabbit's horns, there are no particular characteristics of rabbit's horns. So the universal 'rabbit's horns' does not depend on any particulars. The cognition of rabbit's horns that we may have while reading about rabbit's horns in a children's book, cannot successfully guide action: we fail to actually decorate rabbit's horns with colorful ribbons every time we try, for instance. This is because actions can be performed only on causally efficacious objects and, thus, ultimately existent objects (particulars). On the other hand, the universal characteristics of an object of inference must depend on particular characteristics of that object so that the cognition of universal characteristics can successfully guide actions involving this object.²¹ The object of inference is a conventionally existing object as it is a universal. However, it is dependent on the particular characteristics of the object and, thus, dependent on the ultimately existent object.

(2) We as humans have desires and needs and it is the particular things that can satisfy them. For instance, it is particular chemical compounds contained in the vaccines rather than vaccines in general that can help the body fight against COVID-19. Since different vaccines are conducive to help reduce the effects of the virus, they are all treated as similar and categorised as 'COVID-19 vaccines' to universally characterise all the vaccines. However, what satisfies human desires and needs is not how the particular vaccines are conceived but their causal capacities. Likewise, the desire or need to know about the presence of fire faced with smoke on a mountain can only be met by the causal capacities of particular smoke and

¹⁹ PV I.166, PVSV 24,16-93,5, PV III.3 (Kellner (2004); Franco and Notake (2014: 35–37); Eltschinger *et al.* (2018)).

²⁰ PV III.54cd, cf. PS I.1 (Franco and Notake (2014: 140)).

²¹ PV III.11-2, III. 55-58 (Franco and Notake (2014: 57–74, 141–44)).

fire. In other words, it is up to the world, and not up to us, what should count as valid inference. Hence, inference must depend on individual particulars and is, thus, existentially committing.²²

This feature of inferential knowledge has an important implication for Dharmakīrti (and other Buddhist logicians). Inferential knowledge is about some object—a universal aspect of some causally efficacious object—and inference is a means to bring about knowledge of that object. This means that there is an object (*artha*) which inference is about and inference brings about insight about this very object.

That an inference produces knowledge only if it depends on particulars is clearly visible in Dharmakīrti's discussions of essential connection (*svabhāvapratibandha*) and inference functioning due to the force of real entities (*vastubalapravṛttānumāna*). According to Dharmakīrti, when we infer the presence of fire on the mountain from the presence of smoke, our cognition that there is fire counts as knowledge because the concepts of fire and smoke are essentially connected (*svabhāvapratibandha*). What underlies this essential connection is the *causal* connection between particular smoke and particular fire (Steinkellner (1984); Katsura (1986); Hugon (2011)). This is why Dharmakīrti calls inference like this 'an inference functioning due to the force of real entities' (*vastubalapravṛttānumāna*); that is, an inference functioning due to the force of particulars. The knowledge status of a cognition that there is fire on the mountain is a consequence of the fact that particular fire causes particular smoke (PV I.215; PV IV.48; Tillemans (1999:28-29; 2000: 78-79)).

For the second strand of Dharmakīrti's thought about inference, consider *Pramāṇaviniścaya* III.1ab where Dharmakīrti defines inference for the sake of others (*parārthānumāna*) as an explanation (*prakāśana*) of an object (*artha*) that a person who provides this explanation (the proponent) came to know by themselves.²³ And, in *Pramāṇaviniścaya* III.1cd and the *Pramāṇavārttika* IV.13-14, following Dignāga, Dharmakīrti argues that the object of an inference cannot be merely imagined. For example, the object of an inference cannot be just a universal characteristic that doesn't depend on particulars, or an object known only from scriptures like a Buddhist heaven or details of karmic consequences. Instead, it must be a real entity (*vastu*). Thus, Dharmakīrti holds that inference must have objects.

Hence, for Dharmakīrti, inference as a *pramāṇa* does not operate on an empty content; it is about some object and it brings about 'insight' about that object. In making an inference, we are not completely ignorant of what the inference is about as inference is about an object or objects from Dharmakīrtian perspective. In fact, according to Dharmakīrti, inference is a process of acquiring knowledge specifically about objects.

²² Many thanks go to Mark Siderits who encouraged us to emphasize this aspect of Dharmakīrtian inference. See also Tanaka (202x).

²³ In our translation: 'Inference for the sake of others (*parārthānumāna*) is an explanation (*prakāśana*) of the object (*artha*) which is understood by [a proponent] himself (*svadṛṣṭa*). [Auto-commentary:] Just like the cognition (*jñāna*) of an indicated thing (*liṅgin*) arises for oneself on the basis of the mark which possesses the triple characteristic (*trirūpaliṅga*), in the same way the cognition of an indicated thing arises for the other on the basis of the intention to explain the mark which possesses the triple characteristic to the other. For, there being a cause [i.e., explanation], there is a metaphorical transfer to the effect [i.e., cognition].' Skt. *parārtham anumānam tu svadṛṣṭārthaparakāśanam | yathaiiva hi svayaṃ trirūpāl liṅgāl liṅginī jñānam utpannam, tathā paratra liṅgijñānotpipādayiṣayā trirūpaliṅgākhyānam parārtham anumānam, kāraṇe kāryopacārāt* (Hugon and Tomabeche 2011: 1). For translation and discussion of this fragment see Tani (1987: 3); Iwata (1995: 155–56). See also PS III.1, PSV ad III.1ab (Watanabe (2011: 466)) and PV IV.1ff (Tillemans (2000: 9ff)).

This is the case even when we argue about non-existents.²⁴ Dharmakīrti (and Buddhists generally) has (and have) difficulty in reasoning about non-existents. It is (or was) commonly accepted in India that a thesis whose basis (*āśraya*) or subject (*dharmin*) is non-existent cannot be established or proved. Buddhists and their philosophical opponents both accepted the fallacious nature of *āśrayāsiddha* (unestablished basis) (Tanaka (2021); Tillemans and Lopez (1998); Tillemans (1999)). This means that, for anyone who does not believe that God (Īśvara) exists (like Buddhists), it is fallacious to show that God (who does not exist) does not exist. Despite this, a Buddhist can consider with *prasaṅga* (a form of counterfactual reasoning or *reductio ad absurdum*) the claim that God has to produce effects successively or all at once because he (really) exists. Given that we as atheists know that God does not actually do anything in the world, we might contrapose this claim to: God does not (really) exist because he does not produce effects successively or all at once. Even though this reasoning is not about *anything* (since there is no God), both the *prasaṅga* and its contraposition (*prasaṅgaviparyaya*) involve the same necessary connection (*sambandha*) between existents and the reasoning must depend on the particulars as they are the objects of the *prasaṅgaviparyaya* (PV IV.12; PV in III.1-2; Tillemans (2000: 2ff); Iwata (1997)). So, this reasoning about non-existents is essentially a limit case. Inferring about non-existents is intelligible only because the primary application of inference is to real existence.²⁵ Thus, even in the context of reasoning about non-existents, inference involves existential commitment.²⁶

6. Formal Validity and *Pramāṇa*

Now that we have Dharmakīrti (or Dharmakīrtian) view of logic on the table, we can examine Tillemans' assumptions that reasoning according to formal validity is a *pramāṇa* and that logic is essentially formal. We will show that these assumptions are problematic from a Dharmakīrtian perspective. (1) For the first assumption, principles of logic articulated formally may be referenced in judging the validity of the forms of inferences one may make. However, formal principles are inessential for whether logical reasoning results in knowledge. (2) For the second assumption, from a Dharmakīrtian perspective, the formal conception of logic is incoherent. We will articulate these two points in this section.

(1) There are two reasons for why formal principles are inessential for reasoning resulting in knowledge. First, valid forms of inferences are neither sufficient nor necessary for cognitions to result in knowledge.²⁷ To see that they are not sufficient, consider the inference we have been considering:

There is smoke on a mountain.

Where there is smoke, there is fire.

²⁴ Many thanks go to Tom Tillemans for pointing this out. The following explanation largely comes from him.

²⁵ Again, thanks go to Danielle Macbeth for putting the point in this way.

²⁶ Thanks to the anonymous referee for their suggestions about how to improve our discussion of reasoning about non-existents.

²⁷ Many thanks to Danielle Macbeth for her insightful suggestion about how this point should be made. For a slightly different discussion to make the same point, see Tanaka (2013).

∴ There is fire on that mountain.

The first premise is a claim about some actual mountain a . The second premise expresses a rule of inference in a conditional.²⁸ As a rule, it is general: if there is smoke in a location x , then there is fire in x . So, the form of the inference is *universal modus ponens*:

$$\begin{array}{l} Sa \\ (\forall x)(Sx \rightarrow Fx) \\ \therefore Fa \end{array}$$

To make the inference, we must be able to recognize that the mountain a , which the first premise is about, falls within the scope of the universal quantifier of the second premise and then to apply the rule articulated by the second premise to the instance of mountain in question. That is, the second premise must be recognized as being applicable to the first premise. This means that it is not enough to judge the form to be valid; we must recognize the first premise to be an instance of the second premise in order to make the inference. Thus, the form of the argument is not sufficient for making the inference.²⁹

To see that the forms of inferences are not necessary for the production of knowledge, note again that the second premise articulates a rule of inference. The inference from the premises to the conclusion is essentially an application of this rule. In order for the inference to take place resulting in a cognition that is knowledge, thus, the rule cannot just specify what form the cognitive transition must take but it must be a materially valid rule that generates not only a conditionally-true conclusion (conclusion is true *if* the premises are true) but a true conclusion and, thus, that can actually generate knowledge. Hence, the form of the argument is not even necessary for inferring the conclusion.

To see the second reason why formal principles are inessential for acquiring knowledge, we must observe that validity, according to the formal conception, is *relational*. Formal validity is defined in terms of *truth-preservation*: if the premises of an inference are all true, the conclusion must also be true. In other words, a formally valid inference *transfers* truth from the premises to the conclusion. For this to be the case, the conclusion must be already contained in the premises. Thus, a formally valid inference may be said to transfer knowledge in the sense that if one has knowledge of the premises of an inference, one may end up not losing this knowledge by inference. However, such formally valid inference is not conceived to be a means of *generating* knowledge. A *pramāṇa*, on the other hand, is a source of acquiring or producing knowledge. As we can see in the inference about smoke and fire on a mountain, one must apply the rule articulated by the second premise to the claim about the mountain expressed by the first premise. The conclusion is generated only as a result of this application. Hence,

²⁸ We note that this and similar conditionals are not expressions of *vyāpti* (pervasion) or *svabhāvapratibandha* (essential connection) between smoke and fire, contrary to what is often assumed, e.g., Staal (1960); Chi (1969); Hayes (1988); Oetke (1996); Galloway (1996). The *vyāpti* may serve as a ‘truth condition’ for the conditional but it itself is not what the conditional expresses.

²⁹ A similar point was raised in recent discussions about the so-called ‘adoption problem’ (Padró (2015); Finn (2019)). See also Carroll (1895) as well as Besson (202x) for an excellent analysis of Carroll’s discussion.

Tillemans' first assumption is problematic. Inference according to formal validity cannot be a *pramāṇa* (valid source of knowledge).

(2) For the second assumption, the reason why the formal conception of logic is incoherent from a Dharmakīrti's (or Dharmakīrtian) perspective is that the formal conception places *norms* in a wrong place. A formal inference is valid if it comes from logical principles (as in an axiomatic system) or it is in accordance with them (as in natural deduction). But, these logical principles which serve as norms for inference are abstracted from cognitions involved in making inferences. It is then not clear what they have got to do with, for instance, the cognitions of smoke and fire on a mountain. From the formal perspective, logical principles do not govern the cognitions that arise as we reason or even the cognitions of the ideal agent since they are not concerned with cognitions at all. So, it is hard to understand the normative import of formal logical principles.³⁰ But, the reason why the cognition of fire *should* follow the cognition of smoke is because it is a cognition of *fire* based on a cognition of *smoke*. It is because the cognition is about smoke that it puts constraints on what should follow. For instance, it is because a cognition is about smoke that the subsequent cognition should not be about, say, water. That is, what cognition we should have following the occurrence of the cognition of smoke should depend on what the world is like. If the cognition of fire on a mountain is really the result of an inference, it is because the world is cooperating with our way of conceptualising what we think of as fire and smoke. Inference should, thus, be grounded in what it is about rather than the principles that float over and above cognitions. Hence, from a Dharmakīrtian perspective, when we consider the cognitions that arise in making inference, the formal conception cannot make sense of those cognitions.

7. Dharmakīrtian Inference vs. Formal Inference

There are mainly three differences between Dharmakīrtian inference (inference understood by Dharmakīrti or a Dharmakīrtian) and formal inference (inference understood according to the formal conception). First, according to the formal conception, logical relations do not concern inferential cognitions. They are 'structural' in the sense that they are relations between propositions or thoughts that are not instantiated by particular cognitions. In contrast, Dharmakīrti focuses on inferential cognitions themselves. Dharmakīrtian inference is, thus, not abstracted from the cognitions involved in making inferences.

Second, according to the formal conception, an inference counts as valid if the conclusion is true whenever the premises are true. This is not the case for Dharmakīrtian inference. The conclusion of a Dharmakīrtian inference must arise *because* of its premises. Or, to put it in cognitive terms, the conclusion-cognition must arise because of the premises-cognition. Basically, the conclusion has to grow out of the premises. One way to understand this is to think of the cognition that arises in inference as an extension or expansion of the cognition that prompts the inference. And, because the conclusion-cognition is an expansion of the premises-cognition, inferential cognition which encompasses both of these cognitions

³⁰ See Fitelson (2008); Steinberger (2016).

must be unified. This is not necessarily the case under the formal conception, since it does not consider cognitions to be causally unified.

Third, while formal inference is conceived not to be about anything and, thus, topic-neutral, Dharmakīrtian inference has an object or objects and, thus, it is topic-specific. Unlike its counterpart, Dharmakīrtian inference is about *something* and inference brings out some insight about this thing.

Once these differences are articulated, we can see that Tillemans' challenge to Dharmakīrti's epistemological position on self-contradictions (*svavacanavirodha*) is misguided. Tillemans is wrong to suggest that the formal principle $(P \rightarrow \neg P) \rightarrow \neg P$ allows us to see that we *can* come to know that self-contradictions are false as a way of rejecting Dharmakīrti's claim that we *cannot* know that self-contradictions are true or false. Formal inference is not *pramāṇa* because formal principles by themselves do not generate knowledge. Moreover, Tillemans' challenge attributes a wrong conception of logic as the formal conception is not applicable to Dharmakīrti. In arguing that we cannot know the truth value of self-contradictions, Dharmakīrti does not rely on formal principles. In fact, the very notion of formal principles is foreign to him. Instead, Dharmakīrti relies on a conception of logic that is different from the formal conception which is prevalent in the contemporary philosophical literature. In fact, the formal conception of logic is incoherent for Dharmakīrti.

Finally, while the formal conception of inference is foreign to Dharmakīrti, Dharmakīrtian inference is not foreign to the contemporary audience. Poincaré and Brouwer understood mathematical reasoning to expand knowledge in the way that we articulated Dharmakīrtian inference.³¹ With the dominance of the formal conception in mathematical reasoning, their thought might have been forgotten. However, it is common to see mathematicians describing mathematical reasoning to be knowledge expansive. Thurston (1994), for instance, suggests that the most important mathematical activities consist not in presenting 'some collection of "answers"' but 'understanding' (p. 162). For him, the most important question is: 'How do mathematicians advance human understanding of mathematics?' (p. 162) Thus, for Thurston, mathematical reasoning is not about concatenating one truth after another based on formal principles, but it is about mathematics; it is a reflection on mathematics and has the capacity to expand our knowledge of mathematics because it is about mathematics. Thurston's view may have critics but is widely shared by many contemporary mathematicians.³² While fully defending the view that inference is knowledge expansive is beyond the scope of this paper, this brief overview of the contemporary literature on mathematical reasoning suggests that Dharmakīrtian inference is not foreign to the contemporary scholars (though they would not know that their view is similar to Dharmakīrti's) and is, thus, defensible in the context of the contemporary literature even though it may be an oddity in the contemporary philosophical literature on logic.

³¹ See Detlefsen (1990; 1992). See also Macbeth (2005; 2014) who presents Frege as understanding mathematical reasoning in a similar light (although Poincaré and Brouwer seem to be putting forward their views against Frege—see Detlefsen (1990; 1992)) as well as Sundholm (2012).

³² See Atiyah *et al.* (1994).

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Bibliography:

Primary literature:

- JL — Kant, Immanuel (1800) *Jäsche Logic*, G.B. Jäsche (ed.), Königsberg: Friedrich Nicolovius, translation in *Lectures on Logic*, J.M. Young (trans. and ed.), Cambridge: Cambridge University Press, 1992.
- KrV — Kant, Immanuel (1781(A), 1787(B)) *Kritik der reinen Vernunft*, Riga: Johann Friedrich Harknoch, translated as *Critique of Pure Reason* (Revised Second Edition), Norman Kemp Smith (trans.), Houndmills: Palgrave, 2003.
- NB — Dharmakīrti, *Nyāyabindu*, P. Peterson (ed.), *The Nyāyabinduṭīkā of Dharmottara Āchārya: To Which Is Added the Nyāyabindu*. Calcutta: Asiatic Society, 1889.
- PS — Dignāga, *Pramāṇasamuccaya*, T. Tillemans (trans.), *Dharmakīrti's Pramānavārttika: an annotated translation of the fourth chapter (Parārthānumāna)*. Vienna: Österreichische Akademie der Wissenschaften, 2000.
- PSV — Dignāga, *Pramāṇasamuccayavṛtti*, T. Tillemans (trans.), *Dharmakīrti's Pramānavārttika: an annotated translation of the fourth chapter (Parārthānumāna)*. Vienna: Österreichische Akademie der Wissenschaften, 2000.
- PV — Dharmakīrti, *Pramāṇavārttika*, S. D. Shastri (ed.), *Pramāṇavārttika of Acharya Dharmakīrti with the Commentary 'Vṛtti' of Acharya Manorathanandin*. Varanasi: Bauddha Bharati, 1968.
- PV_{in} — Dharmakīrti, *Pramāṇaviniścaya*, T. Vetter (ed.), *Dharmakīrti's Pramānaviniścayah. 1. Kapitel Pratyaksam*. Vienna: Österreichische Akademie der Wissenschaften 1966; E. Steinkellner (ed.), *Dharmakīrti's Pramānaviniścayah Kapitel 2: Svārthānumānam*, 2 vols, Vienna: Österreichische Akademie der Wissenschaften, 1979; P. Hugon and T. Tomabechi (ed.), *Dharmakīrti's Pramāṇaviniścaya: Chapter 3*. Beijing, Vienna: China Tibetology Publishing House and Austrian Academy of Sciences Press, 2011.
- PVSV — Dharmakīrti, *Pramāṇavārttikasvavṛtti*, R. Gnoli (ed.), *The Pramāṇavārttikam of Dharmakīrti: The First Chapter with the Autocommentary*. Rome: Istituto Italiano per il Medio ed Estremo Oriente, 1960.

PVV — Manorathanandin, *Pramāṇavārttikavṛtti*, S. D. Shastri (ed.), *Pramāṇavārttika of Acharya Dharmakīrti with the Commentary 'Vṛtti' of Acharya Manorathanandin*. Varanasi: Bauddha Bharati, 1968.

Secondary literature:

- Atiyah, Michael, Armand Borel, G. J. Chaitin, Daniel Friedan, James Glimm, Jeremy J. Gray, Morris W. Hirsch, et al. 1994. 'Responses to "Theoretical Mathematics: Toward a Cultural Synthesis of Mathematics and Theoretical Physics"', by A. Jaffe and F. Quinn'. *Bulletin of the American Mathematical Society* 30 (2): 178–207.
- Balcerowicz, Piotr. 2019. 'Is There Anything Like Indian Logic? Anumāna, "Inference" and Inference in the Critique of Jayarāśi Bhatta'. *Journal of Indian Philosophy* 47: 917–946.
- Besson, Corine. 202x. *Reasoning and Carroll's Regress: A Defence of Logical Cognitivism*. Oxford: Oxford University Press.
- Bogacz, Szymon. 2023. 'Buddhist Epistemology and the Liar Paradox', *Australasian Journal of Philosophy*. Latest Articles.
- Carroll, Lewis. 1895. 'What the Tortoise Said to Achilles'. *Mind* 4 (14): 278–80.
- Chi, R. S. Y. 1969. *Buddhist Formal Logic*. London: The Royal Asiatic Society of Great Britain.
- Detlefsen, Michael. 1990. 'Brouwerian Intuitionism'. *Mind* 99 (396): 501–34.
- . 1992. 'Poincaré against the Logicians'. *Synthese* 90 (3): 349–78.
- Dutilh Novaes, Catarina. 2011. 'The Different Ways in Which Logic Is (Said to Be) Formal'. *History and Philosophy of Logic* 32: 303–32.
- Dunne, John D. 2004. *Foundations of Dharmakīrti's Philosophy*. Boston: Wisdom Publications.
- Eltschinger, Vincent. 2013. 'Turning Hermeneutics into Apologetics - Reasoning and Rationality under Changing Historical Circumstances'. In *Scriptural Authority, Reason and Action. Proceedings of a Panel at the 14th World Sanskrit Conference, Kyoto, September 1st-5th 2009*, edited by Vincent Eltschinger and Helmut Krasser, 71–147. Vienna: Österreichischen Akademie der Wissenschaften.
- Eltschinger, Vincent, John A. Taber, Michael Torsten Much, and Isabelle Ratié. 2018. *Dharmakīrti's Theory of Exclusion (Apoha). Part I. On Concealing. An Annotated Translation of Pramāṇavārttikasvavṛtti 24,16–45,20 (Pramāṇavārttika 1.40–91)*. Tokyo: International Institute for Buddhist Studies.
- Finn, Suki. 2019. 'The Adoption Problem and Anti-Exceptionalism about Logic'. *The Australasian Journal of Logic* 16 (7): 231–49.
- Fitelson, Branden. 2008. 'Goodman's "New Riddle"'. *Journal of Philosophical Logic* 37 (6): 613–43.
- Franco, Eli, and Miyako Notake. 2014. *Dharmakīrti on the Duality of the Object: Pramanavarttika III 1-63*. Zürich: Lit Verlag.
- Galloway, Brian. 1996. 'The Buddhist Conditional in Set-Theoretic Terms'. *Journal of Indian Philosophy* 24 (6): 649–58.
- Gillon, Brendan S. 2016. 'Inference, Indian Theories Of'. In *Routledge Encyclopedia of Philosophy*, 1st ed. London: Routledge.
- Hayes, Richard P. 1988. *Dignaga on the Interpretation of Signs*. Dordrecht: Kluwer.
- Hilbert, David. 2015. *Grundlagen der Geometrie (Festschrift 1899)*. Edited by Klaus Volkert. Springer-Verlag.

- Hugon, Pascale. 2011. 'Is Dharmakīrti Grabbing the Rabbit by the Horns? A Reassessment of the Scope of *Prameya* in Dharmakīrtian Epistemology', *Journal of Indian Philosophy* 39 (4/5): 367–89.
- Hugon, Pascale, and Toru Tomabechi. 2011. *Dharmakīrti's Pramāṇaviniścaya: Chapter 3*. Beijing; Vienna: China Tibetology Publishing House and Austrian Academy of Sciences Press.
- Iwata, Takashi. 1995. 'PRAMĀṆAVINIŚCAYA III (1) (Die Definition Des Parārthānumāna)'. *Wiener Zeitschrift Für Die Kunde Südasiens - Vienna Journal of South Asian Studies* 39: 151–79.
- Katsura, Shōryū. 1986. 'Svabhāvapratibandha Revisited', *Journal of Indian and Buddhist Studies (Indogaku Bukkyogaku Kenkyu)* 35: 476–473. doi:10.4259/ibk.35.476.
- Kellner, Birgit. 2004. 'Why Infer and Not Just Look? Dharmakīrti on the Psychology of Inferential Processes'. In *The Role of the Example (DRSTānta) in Classical Indian Logic*, edited by Shōryū Katsura and Ernst Steinkellner, 1–52. Vienna: Österreichische Akademie der Wissenschaften.
- Lapointe, Sandra, ed. 2018. *Logic from Kant to Russell: Laying the Foundations for Analytic Philosophy*. New York: Routledge.
- Macbeth, Danielle. 2005. *Frege's Logic*. Cambridge, MA: Harvard University Press.
- . 2014. *Realizing Reason: A Narrative of Truth and Knowing*. First edition. Oxford ; New York, NY: Oxford University Press.
- Oetke, Claus. 1996. 'Ancient Indian Logic as a Theory of Non-Monotonic Reasoning'. *Journal of Indian Philosophy* 24 (5).
- Padró, Romina. 2015. 'What the Tortoise Said to Kripke: The Adoption Problem and the Epistemology of Logic'. PhD dissertation, City University of New York (CUNY).
- Pecchia, Christina, *Dharmakīrti on the Cessation of Suffering: A Critical Edition with Translation and Comments of Manorathanandin's Vṛtti and Vibhūticandra's Glosses on Pramāṇavārttika II. 190-216*. Leiden ; Boston: Brill, 2015.
- Rödl, Sebastian. 2012. *Categories of the Temporal: An Inquiry into the Forms of the Finite Intellect*. Cambridge, Mass: Harvard University Press.
- Siderits, Mark. 2003. 'Deductive, Inductive, Both or Neither?' *Journal of Indian Philosophy* 31 (1/3): 303–21.
- Staal, J. F. 1960. 'Formal Structures in Indian Logic'. *Synthese* 12 (2/3): 279–86.
- Steinkellner, Ernst. 1984. 'Svabhāvapratibandha Again', *Acta Indologica* 6: 457–76.
- Steinberger, Florian. 2016. 'Explosion and the Normativity of Logic'. *Mind* 125 (498): 385–419.
- Sundholm, Göran. 2012. "'Inference versus Consequence" Revisited: Inference, Consequence, Conditional, Implication'. *Synthese* 187 (3): 943–56.
- Tanaka, Koji. 2013. 'Buddhist Philosophy of Logic'. In *A Companion to Buddhist Philosophy*, edited by Steven M. Emmanuel, 320–30. Chichester: Wiley-Blackwell.
- . 2021. 'How Can Buddhists Prove That Non-Existent Things Do Not Exist?' In *Non-Being*. Oxford: Oxford University Press.
- . 202x. 'Buddhist Logic'. In *Routledge Encyclopaedia of Philosophy*. London: Routledge.
- Tani, Tadashi. 1987. 'The Problem of Interpretation on Pramāṇaviniścaya III Ad vv. 1-3: With the Text and a Translation'. *高知工業高等専門学校学術紀要 - Kōchi Kōgyōkōtōsenmongakkō Gakujutu Kiyō - The Bulletin of Kochi National College of Technology* 26: 1–16.
- Thurston, William P. 1994. 'On Proof and Progress in Mathematics'. *Bulletin of the American Mathematical Society* 30 (2): 161–77.

- Tillemans, Tom J. F. 1990. *Materials for the Study of Āryadeva, Dharmapāla and Candrakīrti*. Vienna: Österreichische Akademie der Wissenschaften.
- . 1999. *Scripture, Logic, Language: Essays on Dharmakīrti and His Tibetan Successors*. Boston: Wisdom Publications.
- . 2000. *Dharmakīrti's Pramānavārttika: an annotated translation of the fourth chapter (Parārthānumāna)*. Vienna: Österreichische Akademie der Wissenschaften.
- . 2016. *How Do Mādhyamikas Think? And Other Essays on the Buddhist Philosophy of the Middle*. Somerville, MA: Wisdom Publications.
- Tillemans, Tom J. F., and Donald S. Lopez. 1998. 'What Can One Reasonably Say About Nonexistence? A Tibetan Work on the Problem of Āśrayāsiddha'. *Journal of Indian Philosophy* 26 (2): 99–129.
- Watanabe, Toshikazu. 2011. 'Dharmakīrti's Interpretation of Pramāṇasamuccaya III 12'. In *Religion and Logic in Buddhist Philosophical Analysis. Proceedings of the Fourth International Dharmakīrti Conference. Vienna, August 23–27, 2005*, edited by Helmut Krasser, Horst Lasic, Eli Franco, and Birgit Kellner, 459–67. Wien: Österreichische Akademie der Wissenschaften.
- Yoshimizu, Chizuko (1999) 'The Development of *Sattvānumāna* from the Refutation of a Permanent Existent in the Sautrāntika Tradition', *Wiener Zeitschrift Für Die Kunde Südasiens / Vienna Journal of South Asian Studies* 43: 231–54.