

# COMPOSITION AS IDENTITY AND PLURAL CANTOR'S THEOREM

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Let *Composition as Identity* (CAI) be the thesis that a whole is identical with all its parts collectively, not individually.<sup>1</sup> For a toy example, let my body be a whole composed of some parts, say my arms, legs, head and torso. Then, by CAI, my body is identical with my arms, legs, head and torso collectively, but not with any one of them individually.

*Plural Cantor's Theorem* (PCT) is the proposition that for any plurality containing two or more members, there are more sub-pluralities of it than members.<sup>2</sup> For a toy example, consider you and me. That plurality has 2 members: you and me, but  $2^2-1$  sub-pluralities: you, me, and you-and-me. The point generalizes: for any plurality with  $n$  members, it has  $2^n-1$  sub-pluralities, which is strictly greater than  $n$ , provided  $n>1$ .

It seems to be a well-known fact among philosophers working on the topic that CAI blocks PCT, but, unfortunately, it has so far been neither formally shown nor fully appreciated in print.<sup>3</sup> So, in what follows, I first show in some detail how my favorite version of CAI blocks PCT (section 1). Second, to see some

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<sup>1</sup> See especially Lewis (1991:3.6); Sider (2007); Wallace (2009, 2011); Bohn (2009, 2011, 2014); Cotnoir (2013); and Bricker (forthcoming); as well as the various essays in Baxter & Cotnoir (2014). For criticisms of CAI, see especially van Inwagen (1994), Yi (1999), Merricks (2003:20-28), and Sider (2007, 2014).

<sup>2</sup> See especially Florio (forthcoming).

<sup>3</sup> The closest we get are Sider (2007, 2014), Cotnoir (2013) and Hovda (2014); but see also Saucedo (ms). Sider seems to think it's a problem for CAI, rather than a virtue, but, arguably, Sider's problems for CAI are avoided by the relational units employed below; cf. Bohn (2011, 2014) and Cotnoir (2013).

of its philosophical importance, I show how this in turn blocks a recent argument against both modal realism and necessitism, and how this latter fact can be turned into an abductive argument *for* CAI, *given* modal realism or necessitism (section 2). All in all, I thus hope to show that and how my favorite version of CAI blocks PCT, and that it's a philosophically important fact we need to recognize, a fact that can be put to some interesting philosophical work.

To avoid some potential confusion from the outset, three things are worth noting at this point already. First, I am *not* defending CAI. I'm here simply assuming that CAI is a coherent view, in order to show that (i) *if* CAI is true, *then* PCT is not a universal truth, and that (ii) this fact has important philosophical consequences, which I illustrate by the examples with respect to modal realism and necessitism (presumably there are other examples too). Hence, it's a non-starter to object to the thesis of this paper by objecting to CAI as such. Second, since I am not here defending CAI, neither will I here attempt to develop a version of CAI in full details. I only develop a version to the extent needed to see that it will block PCT, and how. I intentionally leave the various directions of further developments of it open. Third, to the extent I do develop CAI, I only develop one version of it (the version I find the most plausible). Now, there are other versions of it as well, some of them with the same consequences as the ones I show below, but, of these other versions, I say nothing. These assumptions and omissions are justified by the fact that CAI is an ongoing research program,<sup>4</sup> arguably at a stage of maturity that allows taking this program, or at least some aspects of it for granted in order to explore its consequences.

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<sup>4</sup> Witnessed by the growing amount of work on CAI over the last 5-8 years. For some references, see fn.1.

## 1. CAI BLOCKS PCT

Let CAI first and foremost amount to the following stipulative definition of the mereological term ‘compose’:<sup>5</sup>

**(CAI):**  $xx \text{ compose } y =_{df} xx \text{ are (collectively) identical with } y$

where the semantics of the right-hand side is as expected, namely ‘ $\alpha=\beta$ ’ is true iff  $v(\alpha)$  is the same as  $v(\beta)$ , where  $\alpha$  and  $\beta$  are schematic meta-variables for either singular or plural object-variables, and  $v$  is the assignment of a referent to the object-variables.<sup>6</sup> The corresponding laws of identity are as expected, namely Reflexivity:  $\forall\alpha(\alpha=\alpha)$ , and Leibniz’s Law:  $\forall\alpha\forall\beta(\alpha=\beta\rightarrow(\Phi(\alpha)\leftrightarrow\Phi(\beta)))$ , from which we can easily derive Symmetry and Transitivity, where again  $\alpha$  and  $\beta$  are schematic meta-variables for singular or plural object-variables. Mereological composition is thus intended to be just one among four possible forms of (informative) identity: one-one, one-many, many-one and many-many (‘ $x=y$ ’, ‘ $x=yy$ ’, ‘ $xx=y$ ’ and ‘ $xx=yy$ ’). CAI is thus committed to a generalized concept of identity, of which the ordinary one-one (‘ $x=y$ ’) is just one among four possible

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<sup>5</sup> As such we no longer need any mereological term as a primitive, since all classical mereological predicates can be defined in terms of ‘compose’, which we here define in terms of a primitive generalized notion of identity. For the kind of irreducibly plural logic used throughout this paper, see Yi (2005, 2006) and Oliver & Smiley (2013).

<sup>6</sup> See Cotnoir (2013), Bohn (2014) and Bricker (forthcoming). This stipulative definition of composition will of course not convince anyone of the general coherency of CAI unless already convinced of the coherency of the underlying primitive generalized notion of identity and its corresponding semantics, but, recall, CAI is an ongoing research program, and convincing people of that program is not our present aim. We here only explore some things that follow from its supposed success.

cases, another which is composition (' $xx=y$ '). The general idea is just that a whole and all its parts collectively is one and the same ontological constituent, or "portion of reality", just conceptualized in two different ways.<sup>7</sup> We thus see a first sense in which CAI is committed to a revisionary language, namely a language allowing each side of its identity sign to be flanked by either a singular or a plural term, independently of each other.

But consider my body. Let  $a$  be my body and  $bb$  be my arms, legs, head and torso, and assume  $bb$  compose  $a$ . Then, according to CAI,  $bb=a$ . But  $a$  has the cardinal property *one*, which  $bb$  does not; and  $bb$  has the cardinal property *six*, which  $a$  does not. So, by Leibniz's Law (and the assumption that the cardinal properties *one* and *six* exclude each other), we get a contradiction. Likewise,  $a$  forms a singleton set, but  $bb$  does not, so, by Leibniz's Law, we get another contradiction, assuming  $bb=a$ . Also, my left arm is one of  $bb$ , but not one of  $a$ , so, by Leibniz's Law, we again get a contradiction, assuming  $bb=a$ . What such cases have in common is that the properties in question (e.g. cardinality, set-formation, and being one of) only hold relative to a unique kind of "division" of their subject. For obvious reasons, Sider (2007) calls such properties *set-like*.

To solve for the kind of contradictions we get from such set-like properties, we let CAI be committed to all such properties being, contra what we might have initially thought, *relational* properties, i.e. properties that hold only relative to a unit, which I henceforth (non-essentially) assume is a *concept*.<sup>8</sup> So,

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<sup>7</sup> For a discussion of the notion of "portion of reality", see especially Hawley (2013) and Bricker (forthcoming).

<sup>8</sup> Though I will henceforth take concepts to be my relational units, note that for logical purposes, any kind of relational unit will do; e.g. modes of presentation, or perhaps just contexts. The general idea is of course a modification of Frege's

for example, the property of being one in number is relative to a concept C, and the property of being some number larger than one in number, say six, is relative to some other concept C\*. We then get that *a* (and *bb*) is one relative to C (being a body), but *bb* (and *a*) is not one relative to C\* (being arms, legs, head and torso), which resolves the contradiction. The solution generalizes to all other such contradictions that are due to set-like properties, e.g. those due to forming a set and being one of some things mentioned above.<sup>9</sup> We thus see a second sense in which CAI is committed to a revisionary language, namely a language in which a predicate we might have initially thought was n-place, is really m-place, for some  $m > n$ , with concepts filling the “new” places (assuming the predicates are to match the structure of the properties they express).<sup>10</sup>

The cardinality of something (as well as any other set-like property) is thus always relative to a concept used to present it with, a concept that provides us with a certain “division” of the referent of the subject term. For example, using the concept of being a deck of cards, what’s in my hand has the cardinality one,

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famous idea in his *Grundlagen* (1884) that cardinality is a property attaching to a concept. Cf. Bohn (2009, 2011); Wallace (2009, 2011); Cotnoir (2013).

<sup>9</sup> In short, and in general, while  $F(x) \ \& \ \sim F(x)$  is a contradiction,  $F(x,c) \ \& \ \sim F(x,c^*)$  is not, provided  $c \neq c^*$ . For the general strategy, see Bohn (2009, 2011, 2014) and Cotnoir (2013); see also Appendix. Wallace (2009, 2011) suggests a similar strategy, but it is unclear to me how much her suggestion generalizes beyond purely numerical predication. McDaniel (2013) suggests that a proponent of CAI should not relativize numerical predication as above, but just accept that one and the same thing can have two different cardinal numbers. But McDaniel’s solution is insufficiently general; it becomes simply incoherent in other cases, e.g. in the cases of forming a set and being one of some things.

<sup>10</sup> Note that by thus relativizing the set-like properties, there is no need to put a restriction on Leibniz’s Law, and, as pointed out in Bohn (2009, 2011, 2014) and Cotnoir (2013), we also block the devastating results of CAI for plural logic shown in Yi (1999) and Sider (2007, 2014). Yi and Sider’s results rest on the derivation of the principle Sider (2007) calls *Collapse* ( $x$  is part of the fusion of  $yy$  iff  $x$  is one of  $yy$ ), but the derivation of that principle equivocates on the relational aspects of set-like predications. In my mind, any satisfactory version of CAI must block Collapse. See also Bricker (forthcoming).

but using the concept of being cards, it, the very same thing, has the cardinality fifty-two. None of them is privileged in the sense of being *the* cardinality of it. It has both cardinalities, but relative to *different* concepts providing different “divisions” of it.<sup>11</sup> Examples of properties that are *not* thus relative (i.e. are not set-like) are *mass*, *spatial location*, and *identity*.<sup>12</sup>

A legitimate worry at this point is how to individuate the set-like properties, as opposed to the non-set-like properties. For example, why is cardinality (or forming a set, or being one of) a relational property, but self-identity (or mass, or spatial location) not? The obvious, but perhaps not too informative answer is that the former holds of a subject only relative to a particular kind of “division” of the subject, but the latter holds independently of any such particular kind of “division”. For example, the deck of cards in my hand counts fifty-two only relative to a particular kind of “division” (divide it differently and you get a different count), but it is self-identical relative to any kind of “division” (divide it however you want, self-identity holds no matter what<sup>13</sup>).

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<sup>11</sup> Cf. Frege (1884). One might of course also appeal to the idea of some properties being more natural than others (Lewis, 1986:59-69), and hence argue that it has one of the cardinalities more fundamentally (in some sense or other) than the other. Though I am sympathetic to this idea, I ignore it for present purposes.

<sup>12</sup> Note that CAI is *not* committed to the thesis of relative identity. Cf. Geach (1967).

<sup>13</sup> More precisely, “divide” it into one thing, *a*, and *a* is self-identical; “divide” it into three things, *bcd*, and *bcd* are self-identical, both individually and collectively. The point generalizes to any kind of “division”, so self-identity is not set-like in the relevant sense. Note also that if  $a=bcd$ , and *a* is one self-identical object and *bcd* are three self-identical objects, then, assuming cardinalities exclude each other, one might be tempted to conclude that *a* both is and is not one self-identical object, which is a contradiction. But this contradiction is solved for by the fact that the cardinalities are relative, not the self-identity.

Now, it would take us too far afield to fully explore the individuation of set-like properties here, but note that, plausibly, there might not be any fully satisfactory such criterion of individuation of set-like properties. They might in the end have to be individuated simply by our linguistic intuitions concerning the relevant truth-conditions: does the truth of this or that predication depend on a particular kind of “division” of the subject of predication? If yes, it’s set-like; if no, it’s not set-like. But note also that, assuming CAI, the fact that we get a contradiction with respect to some properties, but not with respect to others should be taken to be a good indicator of the former being set-like, but the latter being non-set-like.<sup>14</sup>

Note finally, and importantly for what’s to come, that if I counted what’s in my hand as being fifty-two cards *and* one deck of cards and from that concluded that I have fifty-three things in my hand, then, given CAI, I would have *double counted* the content in my hand. That is, I would have counted the same thing under two different concepts, summed up both counts, ignored that each count is of one and the same thing, and as a result falsely concluded that there are fifty-three things in my hand. It is as if I count the morning star and the evening star and conclude that there are two different things there. Given the identity between the morning star and the evening star, that conclusion is just

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<sup>14</sup> Bricker (forthcoming) denies that CAI needs such relational properties, so he avoids the above individuation problem altogether. But, in return, he gets a much weaker thesis, which is, in my mind, harder to see as a coherent picture. In any case, I take the individuation problem to be one of many interesting problems to be further explored in the ongoing research program at hand.

false; likewise in the case of a deck of cards and its cards, as well as in the case of my body and its arms, legs, head and torso, given CAI.<sup>15</sup>

Of course, CAI has many problems yet to be resolved, but I take it we now have a sufficient characterization of it to see how it blocks PCT. As just shown, CAI comes with a revisionary language in the sense that (i) it contains a primitive generalized identity-predicate, and (ii) many predicates only hold of something, or some things, relative to a concept that “divides” up its subject in a certain way. (See Appendix.) I now present a simple counterexample to PCT, *as re-interpreted in this revisionary language of CAI*. That should suffice to show that given CAI, PCT fails to be a universal truth.<sup>16</sup>

The basic idea behind the counterexample is simply that given CAI, we need to be careful when we count our ontology. By CAI, two overlapping things are not wholly distinct things, so counting them both amounts to at least partly double counting ones ontology. Of course, for many purposes, such double counting is harmless, but not so for the purposes of what’s in one’s ontology, in which case it is harmful to the truth. In counting one’s ontology, one must therefore count by concepts with *disjoint* extensions on pain of harmful double counting. It’s not that, according to CAI, we *cannot* count by concepts with

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<sup>15</sup> I here ignore in my mind exotic metaphysical positions according to which fusions *constitute* (but not compose) further objects, or substances. Such structures of constitution can be added on top of CAI’s mereological structures, if wanted.

<sup>16</sup> More specifically, I do this by providing a domain over which we can derive a contradiction from the conjunction of CAI and PCT (as understood in the slightly revisionary language of CAI), which suffices to show that if CAI is true, then PCT fails to hold for all domain. I here intentionally stay neutral on the more constructive side of things, e.g. the exact plural logic that should accompany CAI (though see fn.10).

overlapping extensions (we often truly do); it's just that when it comes to purposes of what's in ones ontology, it would give us a false answer: we would count as distinct what's not distinct. As we'll now see, given CAI, PCT is guilty of such harmful double counting; so PCT fails as a universal truth.

We write ' $f(xx)$ ' for the fusion of  $xx$  (i.e. the unique thing  $xx$  composes<sup>17</sup>); and ' $\langle x,y \rangle$ ' for the ordered pair of  $x$  and  $y$ .<sup>18</sup> Letting  $bb$  be a plurality of ordered pairs, we define the *domain* of  $bb$  –  $dom(bb)$  – as the plurality of all and only the first members of the pairs in  $bb$ . We say that some  $x$  in the domain of  $bb$  *codes* the plurality of all and only the second members of the ordered pairs of which  $x$  is the first member, and  $bb$  codes a plurality  $xx$  iff some  $x$  in the domain of  $bb$  codes  $xx$ . We define the predicate 'among':  $xx$  are *among*  $yy$  iff for any  $z$ , if  $z$  is one of  $xx$ , then  $z$  is one of  $yy$ ; where 'is one of' is understood as expected:  $x$  is one of  $y_1y_2\dots$  iff  $x=y_1$  or  $x=y_2$  or.... Call this definition of 'among', D1. Note that all the pluralities among  $yy$  are all and only the *sub-pluralities* of  $yy$ .

PCT can then be more precisely formulated as follows: *there is no plurality of pairs that codes every sub-plurality of its domain, if the domain is larger than one:*

(PCT):  $\sim \exists xx(|dom(xx)| > 1 \ \& \ \forall yy(yy \text{ are among } dom(xx) \rightarrow \exists x \forall y(\langle x,y \rangle \text{ is one of } xx \leftrightarrow y \text{ is one of } yy)))$

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<sup>17</sup> I here assume uniqueness of composition, though it follows from CAI; cf. Sider (2007).

<sup>18</sup> The following terminology, as well as the more precise formulation of PCT below, is adopted from Florio (forthcoming), which is, as far as I know, the only place that gives the proof of PCT in full details. The sketch in Hawthorne & Uzquiano (2011) is too rough to use to show exactly how CAI blocks PCT.

Let  $cc$  be the three-membered plurality of ordered pairs  $\langle f, f \rangle$ ,  $\langle a, a \rangle$  and  $\langle b, b \rangle$ , where  $f$  is short for  $f(ab)$ , i.e. the fusion of  $ab$ . We officially state CAI as before:  $xx$  compose  $y =_{df} xx=y$ . We know from our earlier definitions that  $dom(cc)$  is the three (and only three) membered plurality  $fab$ . So, we also know that there are seven (and only seven) sub-pluralities of  $dom(cc)$ :  $fab, fa, fb, ab, a, b$ , and  $f$ . Since we know that  $dom(cc)$  contains more than one thing, we universally instantiate PCT by our case  $cc$ , drop the first conjunct, and perform the following derivation:

1.  $\sim \forall yy (yy \text{ are among } dom(cc) \rightarrow \exists x \forall y (\langle x, y \rangle \text{ is one of } cc \leftrightarrow y \text{ is one of } yy))$
2.  $\exists yy \sim (yy \text{ are among } dom(cc) \rightarrow \exists x \forall y (\langle x, y \rangle \text{ is one of } cc \leftrightarrow y \text{ is one of } yy))$
3.  $\sim (aa \text{ are among } dom(cc) \rightarrow \exists x \forall y (\langle x, y \rangle \text{ is one of } cc \leftrightarrow y \text{ is one of } aa))$
4.  $aa \text{ are among } dom(cc) \ \& \ \sim \exists x \forall y (\langle x, y \rangle \text{ is one of } cc \leftrightarrow y \text{ is one of } aa)$
5.  $aa \text{ are among } dom(cc)$
6.  $aa=fab \vee aa=fa \vee aa=fb \vee aa=ab \vee aa=a \vee aa=b \vee aa=f$

Lines 1-5 are obtained by standard plural and singular quantificational logic, and line 6 follows from 5 by D1 plus our knowledge of the seven (and only seven) sub-pluralities of  $dom(cc)$ . Tediously running through each one of the seven cases of line 6, and instantiate the second conjunct of line 4 appropriately, we find a direct contradiction in each of the cases  $aa=a$ ,  $aa=b$  and  $aa=f$ , but not in any of the cases  $aa=fab$ ,  $aa=fa$ ,  $aa=fb$ ,  $aa=ab$ . I here only show the two cases of  $aa=f$  and  $aa=ab$ .

Assume  $aa=f$ . We universally instantiate the second conjunct of line 4:  $\exists y \sim (\langle f, y \rangle \text{ is one of } cc \leftrightarrow y \text{ is one of } aa)$ . There are three and only three cases to consider:  $a$ ,  $b$  and  $f$ . Both  $a$  and  $b$  make both sides of the latter bi-conditional false,

and hence the entire bi-conditional true, and hence its negation false, and hence contradicts the second conjunct of line 4. But  $f$  makes both sides of the latter bi-conditional true, and hence the entire bi-conditional true too, and hence its negation false, and hence contradicts the second conjunct of line 4. But there are no other possible instantiations. So,  $\sim\exists y\sim(\langle f,y \rangle \text{ is one of } cc \leftrightarrow y \text{ is one of } aa)$ , which contradicts the second conjunct of line 4. So, if  $aa=f$ , we get a contradiction. We get the same kind of contradiction if  $aa=a$  or  $aa=b$ .

Assume  $aa=ab$ , and universally instantiate the second conjunct of line 4:  $\exists y\sim(\langle f,y \rangle \text{ is one of } cc \leftrightarrow y \text{ is one of } aa)$ . Again, there are three and only three cases to consider as possible instantiations (we just need one of course, but let's go for all three):  $a$ ,  $b$  and  $f$ . Both  $a$  and  $b$  make the latter bi-conditional false (by making its left-hand side false, but its right-hand side true), and hence its negation true; hence no contradiction. But  $f$  makes the bi-conditional false as well (by making its left-hand side true, but its right-hand side false), and hence its negation true; hence no contradiction. Neither is a contradiction found if  $aa=fb$ ,  $aa=fa$  or  $aa=fab$ .

So, all in all, a contradiction is found in the cases  $aa=a$ ,  $aa=b$  and  $aa=f$ , but not in any of the cases  $aa=fab$ ,  $aa=fa$ ,  $aa=fb$  or  $aa=ab$ . But then, so far our case  $cc$  satisfies PCT by at least one of the disjuncts in line 6 being true, and hence the whole disjunction being true. But by CAI, together with the laws of (generalized) identity and collapse of redundant plural listing,<sup>19</sup> line 6 collapses<sup>20</sup> into:

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<sup>19</sup> By 'the laws of (generalized) identity' I mean as before the appropriately generalized versions of both Leibniz's Law:  $\forall\alpha\forall\beta(\alpha=\beta\rightarrow(\Phi(\alpha)\leftrightarrow\Phi(\beta)))$ , and Reflexivity:  $\forall\alpha(\alpha=\alpha)$ , where each one of  $\alpha$  and  $\beta$  is a singular or plural term, independently of each other. From those two laws, we can derive Symmetry and Transitivity as expected. Note again that it's not Leibniz's Law as such that needs to be relativized or restricted because the relational units are built into the

$$7. \quad aa=a \vee aa=b \vee aa=f$$

But, as we saw above, the cases  $aa=a$ ,  $aa=b$  and  $aa=f$  are all and only the cases in which we get a contradiction with the second conjunct of line 4, so, since we get a contradiction from each one of the disjuncts of line 7, we also get a contradiction from the entire disjunction, i.e. line 7. We have thus established our counterexample to PCT, within the revisionary language of CAI.

The acute reader will have noticed that in providing the above counterexample we never appealed to relational predicates. However, by CAI together with the standard laws of (plural) identity and collapse of redundant plural listing again, line 6 *also* collapses into:

$$7.* \quad aa=a \vee aa=b \vee aa=ab$$

And by CAI alone there is *no* reason to accept 7 over 7\* because  $ab=f$ . Interestingly, there is no contradiction arising from 7\*, because, as we have seen,  $aa=ab$  verifies the entire disjunction, i.e. line 7\*. So, by collapsing 6 into 7\* instead of into 7, we don't get our counterexample to PCT on the basis of CAI. But, of course, according to CAI, the difference between 7 and 7\* is a mere change of conceptualization of one and the same thing. That is, we have merely changed

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substitutions for  $\Phi$ . So, since everything in  $\Phi$  except  $\alpha$  and  $\beta$  is to be constant across both sides of the biconditional, LL holds as expected. By collapse of redundant plural listing, I mean that any plural list containing the same term more than once, collapses into a list that contains that term only once, e.g.  $abcb$  collapses into  $abc$ .

<sup>20</sup> Because:  $fab=fa=fb=f=ab$ .

the relational units hidden in the underlying predicates involved in the proof (e.g. in the predicates ' $| > 1$ ', 'is one of', 'are among' and ' $< , >$ '). By tediously unpacking definitions based on the official language (cf. Appendix), and filling in the relational units, we can see that CAI blocks PCT relative to one set of concepts, namely one according to which we double count the whole in addition to its parts (cf. the third disjunct of line 7), but not relative to another, namely one according to which we don't double count the whole in addition to its parts (cf. the third disjunct of line 7\*). In fact, we can see this almost directly from lines 7 and 7\*. More generally, it can be shown that by accepting CAI, we can accept PCT just in case we count our ontology on the basis of a partitioning of it into *disjoint* members; that's the only way to avoid double-counting. Given CAI, as soon as we let overlapping members into our ontology, PCT no longer holds due to illegitimate double counting, i.e. counting the same twice over.<sup>21</sup>

In sum, assuming CAI, there will be pluralities such that there is a map from its members onto all its sub-pluralities, and this is so because by CAI those pluralities are such that some of their members are identical with some of their sub-pluralities such that we get that map. PCT thus holds only if we either ignore those identities and double count (which is ontologically misleading) or we don't ignore those identities but only consider pluralities with no overlapping members.

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<sup>21</sup> Arguably, our denial of PCT also amounts to a denial of *Plural Comprehension* (PC) as a universal truth, the proposition that for any non-empty predicate, there are some things that are all and only the things that satisfy that predicate. PC is formulated in terms of the predicate 'is one of', which is one of the predicates that, according to CAI, need to be appropriately relativized. Unfortunately, a full discussion of this must wait for another time; though see fn.23. On CAI and PC, see Sider (2014); though note that Sider's conclusions look very different, much less worrisome, when we invoke relativized predicates as above.

I take it this much suffices to show that if CAI is true, then PCT is blocked as a universal truth. We now turn to two examples of interesting philosophical upshots of this fact (presumably there are other such examples as well).

## 2. BLOCKING ONE THING BLOCKS ANOTHER

Hawthorne and Uzquiano (2011) present us with the following puzzle. Assume there can be at least two co-located point-sized concrete objects in a point-sized region of space.<sup>22</sup> *How many* such co-located points can there be? Given that there can be at least two, any particular number above two seems objectionably *ad hoc*. For any such particular number, finite or transfinite, the question immediately arises: why not more? But then, since we grant at least two, but accept no particular number above two, the following two answers seem to be the only viable options:

(P): at least as many as the alephs

(IE): not as many as the alephs, but for each aleph there can be at least as many as that

where the alephs is assumed to be the entire series of all the cardinal numbers, having the absolute size  $\Omega$ , into which all things can be 1-1 mapped, i.e. the

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<sup>22</sup> By 'can' I here mean metaphysically possible; by 'co-located' I mean exactly co-located; by 'point-sized' I mean zero-dimensional (though this is inessential; we only need that it is mereologically atomic); 'concrete object' I take to be a primitive, but opposed to 'abstract object'; I have no idea what a region of space is, nor what more exactly it is to be located in one. But let's not quarrel about any of this here. I henceforth use 'point' to mean zero-dimensional concrete object. The less heretic among us could think of these zero-dimensional objects as concrete angels dancing on the point of a needle, instead of as concrete points.

alephs are assumed to be that than which nothing larger is or can be.<sup>23</sup> P (plenitude) then says that there can be as many co-located points as there are alephs, while IE (indefinite extensibility) says that is not the case, but that there can nonetheless be indefinitely many.

Hawthorne and Uzquiano (H&U) presents two different arguments to the effect that P is false, so, given that P and IE are the only viable options, IE is true; but IE in turn contradicts modal realism and necessitism, so modal realism and necessitism must be false.<sup>24</sup> In what follows I show how to block one (but not the other) of their two arguments against modal realism and necessitism by virtue of the results from section 1 above.<sup>25,26</sup>

We assume both that composition is *unrestricted*: any plurality *xx* compose something, and that composition is *unique*: if *xx* compose *y*, then *xx* compose nothing but *y*.<sup>27</sup> If *xx* compose *y*, we also say that *y* is the *fusion* of *xx*.

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<sup>23</sup> I assume that  $\Omega$  is an *amodal* matter of metaphysical necessity in the sense of the alephs not being tied to any possible world, but rather being “outside” all possible worlds (hence ‘amodal’), but nonetheless holding true in all possible worlds (hence ‘metaphysical necessity’). Perhaps contra H&U, I intentionally avoid thinking of them in terms of set theory. I am also not comfortable with talking as if  $\Omega$  is a definite size, or a mathematical object in its own right (because then it seems something could be larger), but H&U talk this way (though presumably without any particular commitments), so for present purposes we can and do too. In any case,  $\Omega$  is an absolute limit on size, what so ever.

<sup>24</sup> For modal realism, see Lewis (1986); for necessitism, see Williamson (2013).

<sup>25</sup> Their other argument rests on wholly different (set-theoretical rather than mereological) premises, and so demands separate treatment, and so must be left for another time. But just to put my cards on the table, I reject their second argument too on the basis of accepting unrestricted set-formation, but denying unrestricted plural comprehension.

<sup>26</sup> Note that the argument is basically a generalization of the Russell-Myhill Paradox. See Klement (1995). The general problem is also lurking in Lewis (1991), Rosen (1995) and Nolan (1996).

<sup>27</sup> Arguably, both these assumptions follow from CAI. See Sider (2007) and Bohn (2014). McDaniel (2010) and Cameron (2012) argue that the first assumption does not thus follow; Bohn (2014) replies.

We say that a fusion is *based on* a plurality iff it is the fusion of one of its sub-pluralities; and we say that a plurality is *disperse* iff no two of its sub-pluralities have one and the same fusion. For example, a plurality of mereological atoms is disperse; so is the plurality of all and only the cats. A plurality of a single cat and its left and right halves is not disperse, since the plurality of the cat and its left half has one and the same fusion as that of the cat and its right half.

From these assumptions and definitions, we get what H&U calls the *mereological result*:

**(MR)**: there are more fusions based on a disperse plurality of two or more things than there are members of that disperse plurality

*Proof*: consider an arbitrary disperse plurality *xx* of two or more things. By PCT, there are more sub-pluralities of *xx* than there are members of *xx*; but by UC, each of these sub-pluralities has a fusion; so, since *xx* is disperse, there must be more fusions based on *xx* than there are members of *xx* as well. *Q.E.D.*

Based on MR, H&U presents the following argument against P:

**(A)**: assume P, i.e. that there can be at least as many co-located points as there are alephs. By MR, there are strictly more fusions based on those co-located points than there are co-located points. But then there are strictly more things altogether than co-located points, contradicting P or the assumption that all the alephs is that than which nothing larger is or can be.

By virtue of A (and the assumption that all the alephs is that than which nothing larger is or can be), P is false, leaving us with IE as the correct answer to our initial question (given that those two were the only viable options). But, according to H&U, IE contradicts modal realism and necessitism.

It suffices for present purposes to say that *modal realism* is the view according to which all possible worlds and objects exist on a par with (or as concretely as) the actual world and objects, though the possible worlds are presumably spatiotemporally and causally isolated from each other.<sup>28</sup> But if we assume such modal realism together with unrestricted quantification over all possible worlds and objects, i.e. over the entire pluriverse, and the result that IE is the correct answer to our initial question, then a version of argument A arises all over again. The argument rests on the following observation: by IE, for any aleph, there is a possible world having that many co-located points, so by modal realism, there will be as many such points *across the entire pluriverse* as there are alephs (after all, since there is a world for any arbitrary aleph many co-located points, there are as many worlds as alephs!).

(A\*): assume IE and modal realism. Then, by the above observation, there are at least as many points as there are alephs across the entire pluriverse. By MR, there are strictly more fusions based on those points than the points themselves, which means there are strictly more things altogether across the entire pluriverse than alephs, which is impossible on pain of contradicting IE or the assumption that all the alephs is that than which nothing larger is or can be.

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<sup>28</sup> For more details, see Lewis (1986).

It suffices for present purposes to say that *necessitism* is the view that necessarily, everything exists necessarily. In terms of possible worlds, that is to say that for any possible world, everything in it exists in any other possible world as well (though it might switch between being abstract and concrete).<sup>29</sup> But if we assume such necessitism together with unrestricted quantification over all existents, and the result that IE is the correct answer to our initial question, then a version of argument A arises all over again. The argument, rests on the following observation: by IE, for any aleph, there is a possible world having that many co-located points, so by necessitism, there will be as many such (concrete or abstract) points *in actuality* as there are alephs (after all, since there is a world for any arbitrary aleph many co-located points, and all those points in all the worlds must also be something actual, there are actually as many such (abstract or concrete) points as alephs!).

(A\*\*): assume IE and necessitism. Then, by the above observation, there are as many points in actuality as there are alephs. By MR, there are strictly more fusions based on those points than the points themselves, which means there are strictly more things altogether in actuality than alephs, which is impossible on pain of contradicting IE or the assumption that all the alephs is that than which nothing larger is or can be.

So, if everything is correct so far, both modal realism and necessitism are false.

But, of course, given CAI, not everything is correct so far. As we saw in section 1, if CAI is true, then PCT is false; but as we have seen in this section,

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<sup>29</sup> For more details, see Williamson (2013).

H&U's arguments against modal realism and necessitism both rest on MR, which in turn rests on PCT; so, if CAI is true, then H&U's arguments are blocked by virtue of CAI blocking PCT, which in turn blocks MR. (Note that CAI thus not only blocks A\* and A\*\*, but A as well; so P might very well be true after all.) CAI thus provides a way of blocking H&U's arguments against modal realism and necessitism.

But then we in effect have an abductive argument *for* CAI, *given* either modal realism or necessitism. Either composition is identity or it is not. If it is not, then H&U's argument presumably goes through as it is intended, since counting overlapping things don't then amount to double counting; after all, overlapping things are then not in any way identical things, and hence they are distinct, and hence ought to be counted as distinct. If so, both modal realism and necessitism are false on pain of paradox, as argued by H&U. But if composition *is* (an instance of generalized) identity, and we understand that along the lines of CAI as articulated in section 1 above (though presumably there are other ways of understanding it too, which gives the same result), then, as we have seen, PCT is blocked, and hence MR is blocked, and hence H&U's arguments are blocked. So, given modal realism or necessitism (and the two assumptions of the alephs being that than which nothing larger is or can be, and unrestricted composition), one should accept CAI on pain of paradox!

Of course, there are many possible replies to such an abductive argument, but my point here is only that we now at least have a debate up and running due

to the fact shown in section 1, namely that given CAI, PCT fails to be a universal truth. In general, PCT can no longer be uncritically appealed to.<sup>30</sup>

### **Appendix: a sketch of the language of CAI**

Below is a sketch of the language of CAI. I also provide an intuitive translation-function from an ordinary plural first-order language into the language of CAI, to facilitate understanding.

**Alphabet:**

*Constants:*  $\sim, \wedge, \exists$

*Non-Logical Predicates:*  $F^n_i$

*Logical Predicates:*  $=$

*Object-variables:*  $x_i, xx_i$

*Object-constants:*  $a_i, aa_i$

*Concept-variables:*  $Y_i$

for any  $0 < i$ , where  $n$  indicates the number of places of  $F$ . We call the object-variables and constants (i.e. excluding concept-variables), *terms*. We also have *complex terms*: if  $\alpha$  and  $\beta$  are terms, then so is:  $\alpha\beta$ . Complex terms are associative and commutative.

**Well-formed formulas:**

*Atomic:* (i)  $\Pi^{2n}\alpha_1, \dots, \alpha_n Y_1, \dots, Y_n$  is an atomic wff, where  $\Pi^{2n}$  is a  $2n$ -place non-logical predicate,  $\alpha_i$  is a term, and  $Y_i$  is a concept-variable indexed to  $\alpha_i$ ; and (ii)  $\alpha=\beta$  is an atomic wff, where each one of  $\alpha$  and  $\beta$  can be any term (simple or complex), independently of each other.

*Non-Atomic:* (iii)  $\sim\Phi$ ; (iv)  $\Phi\wedge\Psi$ ; and (v)  $\exists\alpha\Phi$ ; if  $\Phi$  and  $\Psi$  are wffs, and  $\alpha$  is a singular or plural object-variable. (The other logical connectives and quantifiers are defined in the usual way.)

**Truth-conditions:**

Let  $d$  be our denotation-function on predicates and terms,  $d^*$  be our denotation-function on concept-variables, and let  $v$  be our evaluation-function on wffs. Then:

*Atomic:* (i)  $\Pi^{2n}\alpha_1, \dots, \alpha_n Y_1, \dots, Y_n$  is true iff  $d\Pi^{2n}$  is instantiated by  $\langle d\alpha_1, \dots, d\alpha_n, d^*Y_1, \dots, d^*Y_n \rangle$ ; (ii)  $\alpha=\beta$  is true iff  $d\alpha$  is identical with  $d\beta$ .

*Non-Atomic:* (iii)  $\sim\Phi$  is true iff  $v\Phi$  is not true; (iv)  $\Phi\wedge\Psi$  is true iff  $v\Phi$  is true and  $v\Psi$  is true; and (v)  $\exists\alpha\Phi$  is true iff  $\Phi$  is true of some  $d\alpha$ .

**Translation-function from a more standard plural first-order language:**

$Tr(\Pi^n\alpha_1, \dots, \alpha_n) = \Pi^{2n}\alpha_1, \dots, \alpha_n Y_1, \dots, Y_n$

$Tr(\alpha=\beta) = \alpha=\beta$ , where  $=$  is hybrid  $n$ - $m$  identity instead of the usual  $n$ - $n$  identity

$Tr(\sim\Phi) = \sim Tr(\Phi)$

$Tr(\Phi\wedge\Psi) = Tr(\Phi)\wedge Tr(\Psi)$

$Tr(\exists\alpha\Phi) = \exists\alpha Tr(\Phi)$

where, intuitively,  $\Pi^{2n}$  holds of  $\langle \alpha_1, \dots, \alpha_n \rangle$  relative to  $Y_i$  holding of  $\alpha_i$ . (Note that I have been sloppy by, among other things, saying that open wffs are true, but if you are still reading you probably get the point.) In particular, there are, according to CAI, cases where  $\Pi^{2n}$  holds of  $\langle \alpha_1, \dots, \alpha_n \rangle$  *just in case*  $d\alpha_i$  has a unique type of “decomposition”, or “division”. In such cases we can say that  $\Pi^{2n}$  holds of  $\langle \alpha_1, \dots, \alpha_n \rangle$  *essentially* relative to  $Y_i$  holding of  $\alpha_i$ . These include the important relational properties according to CAI, the ones that solve for the paradoxes, and explain the failure of PCT (and PC).

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<sup>30</sup> Thanks to Salvatore Florio, Øystein Linnebo, Sam Roberts, Gabriel Uzquiano, and several anonymous referees (some of which might be identical!).

A critical discussion point, to be pursued in future research, is: how are we to understand  $d^*$ ? Most likely,  $d^*$  is contextually determined. That is, most likely the value of  $Y_i$  is contextually determined. In purely logical formulas we then also most likely need to quantify over such contexts.

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