Foundations of Metaphysical Cosmology: Type System and Computational Experimentation

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Abstract

The ambition of this paper is extensive: to bring about a new paradigm and firm mathematical foundations to Metaphysics, to aid its progress from the realm of mystical speculation to the realm of scientific scrutiny. More precisely, this paper aims to introduce the field of Metaphysical Cosmology. The Metaphysical Cosmos here refers to the complete structure containing all entities, both existent and non-existent, with the physical universe as a subset. Through this paradigm, future endeavours in Metaphysical Science could thus analyse non-physical parts of the Metaphysical Cosmos. New logical notions are displayed as tools for Metaphysical Cosmology, such as a Metric-Space-based predicate system, as well as a revised version of the Existential Quantifier. A type system is presented to derive a construction of the Metaphysical Cosmos. The system is structured through semantic and syntactic definitions in a coherent way and holds only one proper axiom: the existence of (at least) one entity. This paper itself serves as an empirical proof for this axiom. Formulae and equations that depict a clear logical and mathematical structure of the Metaphysical Cosmos are derived from the definitions of the system and this axiom. This culminates in the "Sixth Theorem", whose proof displays logically that there must be something rather than nothing. A computational simulation of the Sixth Theorem is also provided, alongside with methods for a future Metaphysical Science. Thus, this paper does not aim to provide a traditional philosophical argument but rather a mathematical foundation and new paradigm for the science of Metaphysics.

Contents

1 Introduction

1.1 Aim and Scope

Disciplines known nowadays as sciences were once part of philosophy and were inducted into the scientific realm once concrete methods of analysis were found in order to serve as theoretical foundations for them (Frank, 1952, [21]) (Gare, 2018, [36]). This evolution is apparent in the natural sciences, including physics, chemistry, and biology, formerly known as ”natural philosophy” (Newton, 1687, [2]) (Kelvin & Tait, 1867, [10]) (Dalton, 1808, [6]). Similarly, the behavioural sciences such as sociology and psychology, reveal early roots in philosophical explorations of the mind, as seen in Aristotle’s philosophy of mind (Aristotle, 350 B.C., [50]), and experimental psychology as a scientific discipline emerged largely due to Gustav Fechner’s mathematical methods for psychophysics (Fechner, 1860, [9]). Finally, this also applies to the abstract sciences[1] such as mathematics, as mathematical studies conducted by pre-socratic philosophers Pythagoras and Thales laid the foundations for the science (Diogenes Laertius, 3rd century CE, [53]). Although Mathematics followed a path that differs from most sciences, its philosophical roots are undeniable. It is only through philosophical investigations (Frege, 1884,[11]) that a development arose and its formal foundations were laid (Zeremelo, 1908, [15]).

This paper builds upon a meta-philosophical foundation positing that philosophy represents a ”proto-science”, i.e. an evolutionary precursor to science, and that its teleological aim is a self-destructive process, wherein various branches of philosophy are gradually relinquished to the scientific domain. Thus, not only does this paper maintain that philosophy can make progress, going against some recent meta-philosophical analysis (Dietrich, 2023,[48]), but that philosophy has made progress in the past, and that this paper will be a first step towards progress, in the field of Metaphysics. The research provided in this paper aims to open a possibility for Metaphysics, and more specifically Metaphysical Cosmology, to step out of the realm of mystical speculation and into the realm of scientific certainty. Indeed, this paper wishes to establish an initial scientific paradigm in the field, acknowledging its susceptibility to modification and enhancement through ongoing scientific development, akin to Thomas Kuhn’s conceptualisation of paradigm shifts (Kuhn,1962,[24]).

Fundamentally, what is Metaphysical Cosmology? Physical cosmology is an analysis of the overall structure of the physical universe (Peebles,1993,[27]). Metaphysics, on the other hand, could be defined as the study of entities potentially present beyond mankind’s perceptual spectrum[2]. Therefore, Metaphysical Cosmology could be defined as a ”synthetic cosmology” of Metaphysics and Physical Cosmology as it regroups both the physical and the Metaphysical into one complete system. Indeed, Metaphysical and Physical Cosmology are both the study of their respective and complete mathematical system. However, the study of Physical Cosmology is limited to the perceivable physical universe whereas Metaphysical Cosmology is the study of all that exists and beyond as one complete mathematical system, including both perceivable and non-perceivable truths. Thus, Physical Cosmology is fundamentally a subset of Metaphysical Cosmology, as Metaphysical Cosmology operates on a higher scale and thus includes physical realities too.

[1]Computer Science follows a similar path, as its historical roots reside in philosophical investigations of Logic (Aristotle, 350 B.C.,[51]) (Boole,1854,[8]).

[2]Despite the debate surrounding the definition and aim of Metaphysics, this paper will not address Metaphysics as the study of ”the fundamental nature of things”, rather, we will define it literally: as the study of the ”Meta-Physical”, i.e. what lies beyond the physical. I maintain that any sub-fields of Metaphysics exploring notions/entities that are part of the physical world are, in fact, not Metaphysics.

The Metaphysical Cosmos, later defined as "The Absolute"[3] denoted "$\mathfrak{A}$", represents the amalgamation of everything, nothingness, and their interactions. Its structure is a syntactical derivation from the system provided in this paper and aims to serve as an initial scientific paradigm for Metaphysical Cosmology. In more formal terms, it is a comprehensive set encapsulating the following: the "Metaphysical Void", "$\varnothing$", which bears similarity to the "empty set"[4]; the "Totality of Things", "Ψ", which contains all existing entities; and their intersection, "$\mathfrak{X}$". This construction arises from pure logical reasoning, resting on the foundation of a singular axiom and definitions. Consequently, these concepts serve as the utmost level of set-theoretical abstraction in Metaphysical Science, forming the foundation upon which other ideas within the discipline are constructed.

The science of Metaphysical Cosmology, therefore, is closer to the abstract sciences than to the natural ones. However, it can be argued that its object of study remains Nature if we consider the realms beyond perception as also falling under the definition of "Nature".

Nonetheless, it cannot be described as a purely abstract science as its abstract study can only be successful if the foundation from which the abstraction is derived is grounded in apparent reality. This empirical grounding is here provided by our sole assumption, later defined as $\mathfrak{P}$, read "p"[5], the only axiom of our system. Such foundation is required as the object of study of Metaphysical Cosmology remains the world and its actuality. However, it is concerned with the entirety of the world, beyond merely the physical. Metaphysical Cosmology can be stated as displaying the limits and extensions of the world and as depicting logically and mathematically its entirety. Methods for Metaphysical Science will be elucidated in **section 5**.

Finally, even though Metaphysical Cosmology lacks the possibility of direct experimentation in the physical world, it can be tested through computational experimentation, a method rapidly growing in the natural sciences(Karniadakis, 2021,[42]) (Sweeney, 2020, [41])(Zgarbova, 2022,[46]). Computational Experimentation has been conducted to test the paradigm provided in this paper, and the result of this experimentation can be found in **section 4.2**.

The use of firm logical systems and mathematics is the first step towards the complete development of Metaphysical science. In recent years, the emergence of formal Metaphysics has paved the way for a more scrutinised exploration of metaphysical concepts, exemplified by Kurt Gödel's ontological proof (Gödel, posthumous 1986, [25]), David Lewis's modal Metaphysics (Lewis, 1986, [26]) and the ongoing formal ontology studies of Edward N. Zalta and The Metaphysics Research Lab at Stanford University (Zalta, 2023, [49]). These efforts have been fruitful and foreshadow the upcoming Metaphysical Science. However, the study of formal Metaphysics remains scattered and lacks an initial scientific paradigm, which I endeavour to provide here by presenting a new system[6].

[3] The concept of "The Absolute", here is not the Hegelian one (Hegel,1812,[7]), even though it might bear some similarity as they both refer to an exhaustive higher metaphysical entity.

[4] Though the Metaphysical Void does differ from the Empty Set, in so far as it is the set of all non-existent things.

[5] "P" here stands for "phenomenological" as it is through phenomenological/empirical analysis that we can assert that there exists (at least) one entity.

[6] In addition, as the new paradigm we wish to introduce in Metaphysical Cosmology encompasses both the physical and the metaphysical, it could serve as a fundamental foundation for all sciences, not just Metaphysics. Indeed, all other sciences could be seen as studies of smaller subsets of the fundamental sets we will derive from our system.

In the thousands of years of philosophical debate surrounding the foundations and justifications of Metaphysics as a science, one of the main arguments against it was and still remains its lack of mathematical formalism and/or empirical basis. In the words of David Hume:

"If we take in our hand any volume; of divinity or school Metaphysics, for instance; let us ask, Does it contain any abstract reasoning concerning quantity or number? No. Does it contain any experimental reasoning concerning matter of fact and existence? No. Commit it then to the flames: for it can contain nothing but sophistry and illusion." - (Hume, 1748, [4])

The system put forward here will rely on the most basic phenomenological and experimental claim, of which one cannot doubt, namely that: "Something exists" represented by the axiom "$\mathfrak{P}$". The very existence of this paper serves as empirical justification for the claim of the axiom "$\mathfrak{P}$". Moreover, this system makes use of precise logical formalism and mathematical reasoning, as well as computational experimentation.

Thus, in the words of Hume, does it contain any abstract reasoning concerning quantity or number? Yes, as the system is a logical and mathematical modelling of the Metaphysical Cosmos.

Does it contain any experimental reasoning concerning matter of fact and existence? Yes, as the system relies fundamentally on the empirical fact of $\mathfrak{P}$ and is tested experimentally through computation.

Then, hold on tight to it, as it might be a first step towards enlightenment.

1.2 New logical notions

The addition of new logical concepts generates an expansion of our thinking field and gives rise to new possibilities towards understanding Metaphysical Reality. Indeed, just as the invention of infinitesimal calculus was needed by Isaac Newton for the formulation of his mechanical laws (Newton,1687,[2]), the addition of new logical notions is essential for an acute and complete mathematical/logical representation of the Metaphysical Cosmos (referred to as "The Absolute").

Indeed, I will here introduce two new tools for Metaphysical Cosmology. Firstly, a new metric-space-based infinitary predicate system, and then a revised use of the existential quantifier, enabling further studies on the fundamental notion of existence.

1.2.1 A New Metric-Space Based Infinitary Predicate System

Before delving into the mathematical intricacies of our new metric-space-based infinitary predicate system, let us first explore its historical context.

In the late 19th century, Gottlob Frege's axiomatisation of mathematics, outlined in *The Foundations of Arithmetic* (Frege, 1884,[11]) contained a law defining sets as collections that contain all entities that possess a certain property, the Basic Law V[7].
Here is its formal definition:
Where "S" is any given set, "x" is any individual variable and "φ" is any predicate:

$$\forall x_1,...,x_n \exists S[x_n \in S \equiv \varphi(x_n)]$$

[7] also known as the axiom schema of unrestricted comprehension

In response to it, philosopher and mathematician Bertrand Russell demonstrated that from Frege's Basic Law V, we could construct the following set (Russell, 1903, [13]):

$$R = \{x| \, x \notin x\}$$

However, this set leads to the following seemingly paradoxical conclusion:

$$R \in R \rightarrow R \notin R \rightarrow R \in R \rightarrow R \notin R ...$$

And thus $R \in R \equiv R \notin R$

Indeed, the set appears to, simultaneously, be a member of itself and not be a member of itself. This seemingly paradoxical consequence of Frege's axioms started a logical revolution that ended in the formulation of the ZF and ZFC set theories (Zermelo,1908,[15]), which are now the current fundamental building blocks of mathematics. But what if this set was not fundamentally paradoxical? Could it be possible for a logical entity to possess multiple properties, even if contradictory, simultaneously? This is the fundamental idea behind this new metric-space-based infinitary predicate system, bearing similarity to paraconsistent and/or dialetheist systems (Priest, 2006,[30])[8] . This new Logic is not a system in and of itself. Rather, it is an extension that can be applied to many systems of logic. In addition to a given system's axioms, rules or definitions, the following conception of predicates can be added to allow for an extension in possible predicate values:

A mandatory requirement for this Logic is that all properties yield a value in the interval $[0,1]$.

In this system, a property is independent of its negation, which will require truth values, to be defined in terms of provability, as outlined in **section 2.2** and **section 2.4**. There is space associated with any given property P and its negation, labelled "P^*_{space}", is given by the Cartesian product of P_{space}, the set of all values of x for $P(x)$, and $\neg P_{space}$, the set of all values of x for $\neg P(x)$.

The sets P_{space} and $\neg P_{space}$ are defined as follows:

$$P_{space} = \{x \in [0,1] | x = P(y) \wedge y \in D\}$$

$$\neg P_{space} = \{x \in [0,1] | x = \neg P(y) \wedge y \in D\}$$

P^*_{space} is thus given by:

$$P^*_{space} = (P_{space} \times \neg P_{space}, d)$$

Such that D is the domain, and where the distance function d is given by the Manhattan distance (Minkowski,1896,[12])such that[9]:

$$d((x_1,y_1),(x_2,y_2)) = |x_2 - x_1| + |y_2 - y_1|$$

[8]This, will however, not forbid systems implementing this new logic from being sound, as displayed in the proof of soundness of the system presented in this paper in **section 2.10**.

[9]Other distance functions, such as the Euclidean one (Euclid, 300 BC, [52]), could also be appropriate.

For a ”classical” entity, the sum of its predicate value and its value for the negation of the predicate must equal 1:

$$x \text{ is ”classical” if and only if } (\neg P(x) = 1 - P(x))$$

Thus, ”classical” entities’s predicate values are all positioned on the same line in a graph representing P^*_{space} for a given property.

For example, if a ”classical” x has a predicate value of $P(x) = 0.7$, due to the properties of the metric space, it will also have a value of: $\neg P(x) = 0.3$.
This can be visualised using the following diagram, where the red line is the ”Classical line” and the point is such that $P(x) = 0.7 \wedge \neg P(x) = 0.3$:

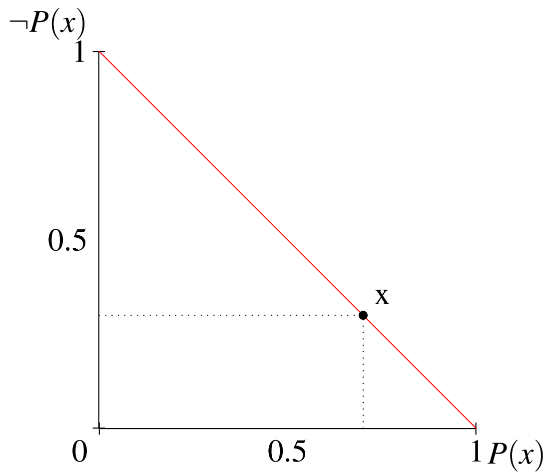

Figure 1: ”Classical” point in the Metric space of a predicate and its negation

Values of predicates and their negation can therefore be represented by a pair $(P(x), \neg P(x))$ such that if that pair is complementary, i.e. $P(x) + P(x) = 1$, the entity can be labelled ”classical”, and the entity is labelled ”non-classical” if not.

In the case of entities that exhibit a paradoxical behaviour, non-classical entities such as Russell's set are positioned outside the "Classical line".

If we define the property $\mathscr{R}$ as: $\mathscr{R}(x) \equiv x \notin x$

Then Russell's set, R, that we defined above, simultaneously holds the value of 1 for both $\mathscr{R}(R)$ and $\neg\mathscr{R}(R)$ as $\mathscr{R}(R) \equiv \neg\mathscr{R}(R)$. This, however, cannot happen if R can only hold a position on the "Classical" line in the property metric space.

The position of Russell's set can be visualised in the following diagram:

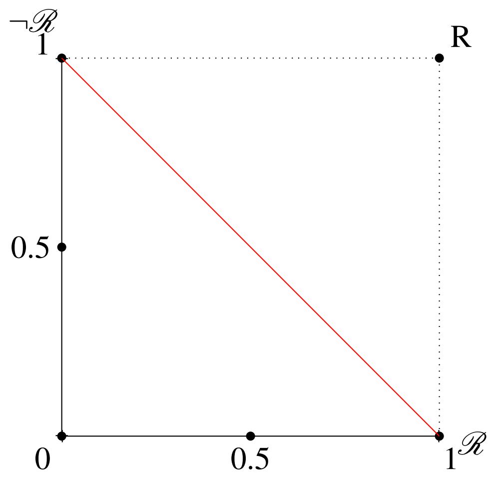

Figure 2: Russell's set position in $\mathscr{R}^*_{\text{space}}$

Thus, in such a case, the paradoxical nature of R is captured without rendering it inconsistent, as $(\mathscr{R}(R), \neg\mathscr{R}(R)) = (1,1) \in \mathscr{R}^*_{\text{space}}$

With the use of this new metric-space based logic, which is (grounded in an Infinitary Predicate System, upon which all predicates are functions that map the domain of a system to [0,1] $\in \mathbb{R}$), we can now use an unrestricted set theory, without having to succumb to the axiom of restricted comprehension (Zeremelo, 1908, [15]) or to a ramified type theory (Russell, 1908, [14]).

Leveraging this new system of predicates expands our intellectual reach, enabling the analysis of entities beyond classical limits. This progressive approach contributes to a more nuanced understanding of the intricate fabric of Metaphysical Reality. This notion, of relying on metric space-based structures in logical systems, is not entirely foreign to recent academic endeavours (Ortiz, 1997, [29]) (Stojanovic, 2018, [37]). A similar conception can be found in logical systems implementing superposition, which has been applied to propositional logic (Tzouvaras, 2018,[38]) as well as higher-order logic (Bentkamp, Blanchette, Tourret & Vukmirović, 2023, [47]).

1.3 Existential Quantifier "$\exists$" Revisited

Finally, after having outlined this new metric-space based predicate system, I will now explain my revisited use of the existential quantifier. Indeed, existence is a fundamental notion in Metaphysical Cosmology and it will be necessary to speak about entities existing without having to specify any characteristic of this entity.

Therefore, our syntax will not be (Where x is any individual and is P any predicate): $\exists x P(x)$. Instead, our syntax will simply be: $\exists x$, as we need an operator to simply denote existence, which will then be defined as a meta-predicate. However, trails of the "classical" existential quantifier remain as we define our existential quantifier as signifying that $\exists x$ if and if only if the sum of all predicate values of x is non-zero:

$$\exists x \equiv \sum_{P_{i.j} \in \mathscr{P}} P_{i.j}(x) > 0$$

And

$$\nexists x \equiv \sum_{P_{i.j} \in \mathscr{P}} P_{i.j}(x) = 0$$

Where $\mathscr{P}$ is the set of all predicates.

And where $P_{i.j}$ denotes any predicate, classified such that $P_{i.j}$ is the j-th property at level i. This is outlined in more detail in **section 2.4.1**.

Through this conception, the classical relationship between existence and properties remains and is simply made more precise and more tailored towards a scientific study of existence. Indeed, more precise as it relies on an extended model of properties capable of describing more intricate notions than a classical model of predicates, and more tailored towards a scientific study of existence, as we can now dive into the concept of existence head-on, by changing the syntax involved.

In cases where a traditional existential quantifier would have been used syntactically, one could simply start with x and describe it using conjunction: $x \wedge P(x)$, or use: $x \in D$, where D is the domain. This means that in the syntax of systems adopting this new use of the existential quantifier, there is no necessity to bind an entity to the quantifier for the formula to be syntactically meaningful and well-formed.

2 Inner structure and definitions of the Type system

2.1 Presentation and aim of the system

The system I will introduce in this paper is constructed through ”types”, which can be considered as ”valid inputs”, and their definitions. This system’s foundational mechanism is closely related to one of Kurt Gödel’s systems, ”P”, outlined in his paper *On Undecidable Propositions of Principia Mathematica and Related Systems*, displaying his ”Incompleteness Theorems” (Gödel,1931,[20]). Indeed, Gödel structured the basic signs of his system ”P” through different types (constants, first type variables: individuals, second type variables: classes of individuals...) and our system follows a similar scheme. Essentially, this system follows a mechanism that is also comparable to Russell’s ramified theory of types (Russell, 1908, [14]). However, our system is made to be unrestricted to follow reason with no assumption other than the existence of one entity. Therefore, the type system that I am presenting here does not have type restrictions, i.e. nothing forbids a universal set of all types for instances.

The *only* axiom of the system is ”$\mathfrak{P}$”, which is proved by the very existence of this paper. The rest of the ”laws” are syntactic definitions of the uses of the allowed inputs in the system. Indeed, Types in this system are the natural language equivalent of defining categories of words such as verbs, adverbs, subjects, nouns...

This system is epistemologically structured in a coherent and fundamentalist way. Types and valid inputs are justified together as a part of a consistent whole and are not defined on an axiomatic basis. This is the usual approach for semantics of natural language and is therefore in line with generating formal syntax and semantics for this system. Words in natural language are defined through the use of other words, types and operators are here defined and structured similarly. The only fundamentalist/axiomatic instance in the foundational structure of the system is our singular axiom, $\mathfrak{P}$, giving rise to a fundamentalist-coherent justification system.

The decision to use this specific carefully crafted system through the use of types and definitions finds its roots in the necessity for a system with axioms that cannot be called into doubt.

The system is both free (through our revised existence quantifier) and paraconsistent (through the possibility of values beyond the classical line of metric-space of properties and their negation). This is due to the will to capture the full extent of human reason to model the Metaphysical Cosmos and be able to analyse entities beyond classical logic.

It is crucial to remember that this system presents itself as a model of the world at the highest metaphysical scale, and is not a study in pure logic and mathematics. This is why the sole assumption, beyond definitions, of this system is ”$\mathfrak{P}$”, which is true by virtue of the very existence of this work itself. Indeed, even if this paper was an illusion, it would still follow that it exists, this axiom is justified by immediate experience. This follows a Cartesian-like reasoning which aims to use undeniable claims as foundations for systematic reasoning (Descartes, 1637,[1]).

”$\mathfrak{P}$” represents the set of all phenomenologically justified claims that cannot be doubted by reason. Its content is nothing more than ”I exist”, ”I am having a perception” and ”something exists”. For the sake of this paper at least, even though it is slightly more complex, we will define logically "$\mathfrak{P}$" the following way[10]:

$$\mathfrak{P} \equiv \exists \mathrm{t}_n$$

All "t_n" refers to any valid input present in a given Type (n): "$\mathbb{T}_n$".

2.2 Type 0: Formulas:

Sentences in this system are of the form: [/justification/] ⊢: /formula/

Such that /formula/ is structured through all the other types, organised according to their definitions. A well-formed formula is a formula that uses all its structural elements: set, numbers, operators, individuals... according to their definitions. To clarify the formula's structure and the scope of its operators, the use of brackets "(", ")" and "[", "]" is syntactically authorised, and often needed. Brackets such as "{" and "}" will however be reserved from sets, as per definitions of Type 3.

When "↓" is written in the [/justification/] box, it means that the formula is justified by the previous one. And when a symbol or a Type number is written in it, it means that the formula is justified by the definition of the Type or symbol in the box.

The justification of a formula works in the following way:
if a formula "α"is justified by the previous formulae/definitions $X_0, ..., X_n$ the justification scheme of the formula is the following:

$$([X_0]...[X_n] \vdash: \alpha) \equiv^{def} ((X_0 \wedge ... \wedge X_n) \rightarrow \alpha)$$

Moreover, the previous formula is the equivalent of the following proof tree:

$$\frac{\text{x y z}}{\alpha}$$

Thus, a formula can be derived from the system if it can be derived from our axiom $\mathfrak{P}$, our definitions or previous formula(s).

Finally, the truth values of formulas are given by the following: A formula X is true if and only if it can derived from the system (either directly from the axiom $\mathfrak{P}$ and definitions or from formulas derived from them):

- $(X \text{ is True}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X) \vee (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0...X_n \vdash X)$.

- $(X \text{ is False}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \nvdash X) \vee (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0...X_n \nvdash X)$.

This truth-value function will itself be considered as a predicate, which will be elucidated in Type 2.

[10] The unusual syntax of the existential quantifier in this definition is explained in section *1.3* as well as within the definitions of operators in section **4.1**

2.3 Type 1: Individuals:

2.3.1 Type 1.1: individual variables and constant:

These can be numerical and/or logical and are represented by lowercase letters: “x”, “y”, “z”… or “φ”, “ϕ”, “θ”… or “$\mathfrak{a}$”, “$\mathfrak{b}$”, “$\mathfrak{c}$”…

And can be of the form: /lowercase letter/ or /lowercase letter/ /lowercase letter/ and so on… Such that: “a”, “ab”, or “abc”… represent Type 1.1 variables.

Any given x is an individual if and only if it is equal to itself: ”x is an individual $\equiv x = x$”.

Individuals can become constants if they are defined. They follow the same syntactical schema as individual variables and are bound to a specific value or entity after definition.

Note ”n”: refers to a variable of type 1.2 (a number)

2.3.2 Type 1.2: numbers:

These can be represented by individual numbers and/or fractions. they are divided in different categories (sets, *type 3*), such as:

- natural ”$\mathbb{N}$”: $0, 1, 2, 3, 4, 5...$,
- integers ”$\mathbb{Z}$”: $-3, -2, -1, 0, 1, 2, 3...$,
- rationals”$\mathbb{Q}$”: $\frac{-4}{6}, 0.777, 1, 2.4888....$,
- real ”$\mathbb{R}$”: , $\sqrt{-52}$, 0, $\sqrt{2}, \pi$ ...,
- complex ”$\mathbb{C}$”:-17.9567, 0, $2+3i$....

Such that:

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

As well as Transfinite Numbers as developed by Georg Cantor, which offers a framework for understanding different sizes of infinities (Cantor, 1915, [17]):
Cardinals: $\aleph_0$, $\aleph_1$, $\aleph_2$, $\aleph_3$... such that the cardinality (size) of ”$\mathbb{N}$” is equal to $\aleph_0$
And Ordinals: $\omega_0, \omega_1, \omega_2$... such that the ordinality (order type) of ”$\mathbb{N}$” is equal to ω_0.

There is also ”∞” such that ∞ is equal to any $\mathfrak{t}_{1.2}$ input (any number) larger than the cardinality of the natural numbers: $(\mathfrak{t}_{1.2} = \infty) \equiv \mathfrak{t}_{1.2} \geq |\mathbb{N}|$

We will also refer to the cardinality of the real numbers, $|\mathbb{R}|$, as ”$\mathfrak{c}$”, using the notation which was also introduced by Georg Cantor (Cantor, 1915, [17]).

2.4 Type 2: predicates or attributes

2.4.1 Type 2.1 Predicates

These can be represented by the form: "/uppercase letter/ t_n" :
"$X(\mathrm{t}_n)$", "$Y(\mathrm{t}_n)$", "$\mathrm{Z}(\mathrm{t}_n)$" ... or "$\Theta(\mathrm{t}_n)$", "$\Upsilon(\mathrm{t}_n)$", "$\Phi(\mathrm{t}_n)$"... or "$\mathfrak{Z}(\mathrm{t}_n)$", "$\mathfrak{T}(\mathrm{t}_n)$", "$\mathfrak{R}(\mathrm{t}_n)$"...

Properties possess a rank and a position in the ordered set of properties. They are more formally denoted $P_{i.j}$ such that $P_{i.j}$ is the j-th property at level i.

And the set of all properties, $\mathscr{P}$, is defined as the set of properties, which are functions that map the domain of the system ($\mathbb{T}_{1-4.3}$), to $[0,1] \in \mathbb{R}$:

$$\mathscr{P} = \{P_{i.j} | P_{i.j} : \mathbb{T}_{1-4.3} \mapsto [0,1] \wedge i \in \mathbb{N} \wedge j \in \mathbb{N}\}$$

Where a total order is defined on this set such that:

$$P_{i.j} \leq P_{n.m} \equiv i < n \vee (i = n \wedge j \leq m)$$

Indeed, properties are built from a set of lower-level properties, such that:

$$P_{i.j} = \circledast(P_{i-1.k}, ..., P_{i.k_m})$$

Where $\circledast$ represents the property construction operator, which takes lower level (i-1) properties and uses any Type 4 operator to construct a higher level (i) property $P_{i.j}$. It is worth noting that not all properties from the lower level are required for the construction of a $P_{i.j}$, only those necessary for $P_{i.j}$. Moreover, though the series of lower-level properties upon which a $P_{i.j}$ can be classified in order, as per the equation above, it is not required nor necessary that all properties that build $P_{i.j}$ come from an "uninterrupted series". For example, for a $P_{i.j}$ that is formed of three lower-level properties, it is not necessary (and even quite rare) that: $P_{i.j} = \circledast(P_{i-1.1}, P_{i-1.2}, P_{i-1.3})$, indeed most properties can have a structure such as: $P_{i.j} = \circledast(P_{i-1.18}, P_{i-1.94}, P_{i-1.347})$.

Thus, through the use of the $\circledast$, the form of all predicates can be represented as :

$$P_i.j(x) \equiv t_0 \text{ where } x \text{ is a term of } t_0$$

Where t_0 refers to a Type 0, i.e. a formula.

Now from this how do we calculate the value of a $P_{i.j}(x)$?

The value of a $P_{i.j}(x)$ is given by:

$$P_{i.j}(x) = \left(\sum_{P_{i-1.k} \in B_{i.j}} w_{i-1.k}^{P_{i.j}} \right) \cdot \frac{|S_x^{P_{i.j}}|}{|B_{i.j}|}$$

Where $B_{i.j}$ is the set of all the lower-level properties used to "build" $P_{i.j}$:
$B_{i.j} = \{P_i - 1.k | \circledast (P_{i-1.k}, ..., P_{i.k_m}) = P_{i.j}\}$

Where $S_x^{P_{i.j}}$: $S_x^{P_{i.j}} = \{P_{i-1.k} \in B_{i.j} | P_{i-1.k}(x) = 1\}$

And where $\sum_{P_{i-1.k} \in B_{i.j}} w_{i-1.k}^{P_{i.j}}$ is the sum of all the weights $w_{i-1.k}^{P_{i.j}}$ for all the properties $P_{i-1.k}$

used to build a property $P_{i.j}$, through the property generator $\circledast$. The weights allow for multiple properties formed from the same lower-level properties but connected in different ways through different logical operators to have different values.

Therefore the property value of an x is given by the sum of all the weights of lower-level properties used to build $P_{i.j}$, multiplied by the number of lower-level properties used to build $P_{i.j}$ that x fully satisfies, divided the total number of lower-level properties used to build $P_{i.j}$.

This equation therefore yields a value between $[0,1] \in \mathbb{R}$. If a predicate or meta-predicate is written without a value, such as $P_{i.j}(x)$, then it is a shorthand notation for: $P_{i.j}(x) = 1$

Finally, some properties do not have the same functioning as they are atomic properties. Some empirically verifiable properties can be considered as atomic if they cannot be built for logic itself. The most fundamental property of all is identity, given the rank of $P_{1.1}$: $P_{1.1}(x) \equiv x = x$.

However, this most fundamental property is relying on the identity sign "=", defined through Leibniz's law in Type 4.1. Therefore, $P_{1.1}(x) \equiv \forall P_{i.j}(P_{i.j}(x) \equiv P_{i.j}(x))$.

Predicates are the skeleton of the system. Indeed, each entity of each type possesses its own "logical genome", Z, a matrix indicating the entity's value for all properties:

$$Z(x) = \begin{bmatrix} P_{1.1}(x) & 0 & 0 & \cdots & 0 \\ P_{2.1}(x) & P_{2.2}(x) & P_{2.3}(x) & \cdots & P_{2.m_2}(x) \\ P_{3.1}(x) & P_{3.2}(x) & P_{3.3}(x) & \cdots & P_{3.m_3}(x) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{n.1}(x) & P_{n.2}(x) & P_{n.3}(x) & \cdots & P_{n.m_n}(x) \end{bmatrix}, \quad \text{where } \forall i \forall m_i (m_i \in \mathbb{N} \wedge m_{i+1} > m_i)$$

As well as the complete value of all their properties:

$$Z^+(x) = \sum_{P_{i.j} \in \mathscr{P}} P_{i.j}(x)$$

There is only one property of i=1, namely identity "$P_{1.1}$", hence the values of 0 in the matrix for all others of the i=1.

Moreover, there is a predicate, an element of $\mathscr{P}$, that is used to determine truth values. We will refer to it as Truth(x) as the property that an x belongs to Type 0, i.e. is a formula, and is derivable from the system:

$$\text{Truth}(x) \equiv x : \mathbb{T}_0 \wedge \mathbb{T}_{1-4} \vdash x$$

It can thus be formed using our "$\circledast$" property generator, where $x : \mathbb{T}_0$ and $\mathbb{T}_{1-4} \vdash x$ represent lower properties:

$$\text{Truth}(x) = \circledast(x : \mathbb{T}_0, \mathbb{T}_{1-4.3} \vdash x)$$

Therefore, this predicate yields:

$$\text{Truth}(x) = \begin{cases} 1 \text{ if } x : \mathbb{T}_0 \wedge \mathbb{T}_{1-4.3} \vdash x \\ n, \text{ such that } n < 1 \text{ otherwise} \end{cases}$$

Moreover, all types can serve as input for properties, even if this might sound unintuitive. For example, a quite unusual type to conceive as an input of a predicate function would be an operator, such as "$\vee$". However, "$\vee$" does satisfy a predicate. Indeed, we can define a $P_\vee$ such that:

$$P_\vee(x) \equiv [x(y,z) \wedge x(y,z) \equiv \neg(\neg y \wedge \neg z)]$$

Indeed, $P_\vee$ is the property of being a relation that binds two types x and y if and only if they cannot both be negated. The operator "$\vee$" thus satisfy this property fully, and $P_\vee(\vee) = 1$.

It must be noted, however, that no Type 0, formulas, can be used to form a predicate if one of its terms contains $Z(x)$, $Z^+(x)$ or any other meta-predicate. These do not serve as valid types of input and their definition will be further elucidated in **section 2.4.2**. However, any higher-order predicate, i.e. predicates applied to predicates, would still be a predicate within the system, as they can be used by the $\circledast$ operator to form new ones.

A metric space $\mathscr{P}_{space}$ is defined for all properties as the Cartesian product of all property values for an x, paired with a distance function:

$$\mathscr{P}_{space} = \left(\prod_{P_i.j \in \mathscr{P}} P_{i.j}(x), d \right)$$

This metric space allows for unrestricted infinitary predicate values, yielding values for both classical and non-classical entities, as outlined in **section 1.2.1**. This metric space is, therefore, infinite-dimensional, as $|\mathscr{P}| \geq |\mathbb{N}|$.

For example, in a reduced model where Z(x) is composed of solely three properties, the metric space can be represented the following way:

Where:

$$Z(x) = \begin{bmatrix} P_1(x) \\ P_2(x) \\ P_3(x) \end{bmatrix}$$

Such that:

$$(P_2(x) = \circledast(P_1) = \neg P_1(x)) \wedge (P_3(x) = \circledast(P_2) = \neg P_2(x))$$

This reduced model, with only three properties and where the only operator usable by the property generator $\circledast$ is negation $\neg$, can be represented visually by the following diagram:

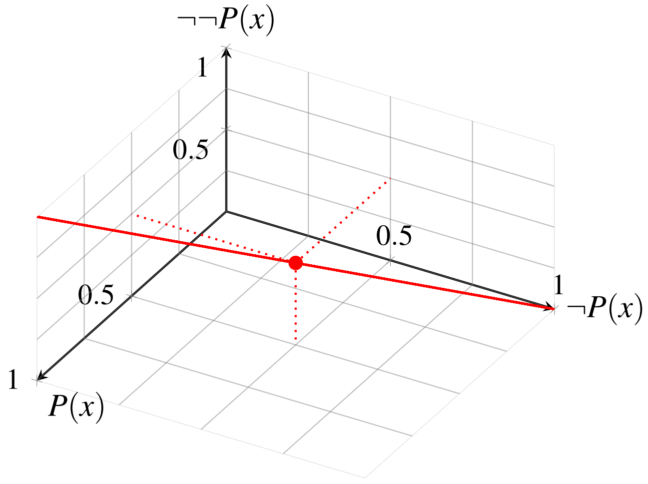

Figure 3: Metric Space for a reduced 3-properties model based on negation

Here the red line represents the "classical line", given by values that satisfy:

$$(P_{i.j}(x) + \neg P_{i.j}(x) = 1) \wedge P_{i.j}(x) = \neg\neg P_{i.j}(x)$$

Finally, as displayed in **section 1.2.1**, one can form a metric space "P^*_{space}" for each property and its negation. This tool can be useful to clarify non-classical instances. The formal definitions P^*_{space}, P_{space} and $\neg P_{space}$, apply to all Type 2 in the system.

2.4.2 Type 2.2 Meta-predicates

Meta-predicates are predicates constructed with the use of the property generator $\circledast$ applied to Type 0, i.e. formulas, with include $Z(x)$ or $Z^+(x)$ as terms.

Therefore, the form of a meta-predicate $M_{i.j}$ is:

$$M_{i.j} \equiv t_0 \text{ where } Z(x) \text{ or } Z^+(x) \text{ are terms within } t_0$$

Meta predicates obey the same rules as predicates but are however in a different category. Indeed, Meta-predicates map the system to the [0,1] interval on the real numbers, i.e. $M_{i.j} :\mapsto [0,1]$. Moreover, $M_{1.1}$, similarly to $P_{1.1}$ is based on identity and given by: $M_{1.1} \equiv Z(x) = Z(x)$. Moreover, there is a set of all Meta-predicates $\mathscr{M}$ such that:

$$\mathscr{M} = \{M_{i.j} | M_{i.j} : \mathbb{T}_{1-4.3} \mapsto [0,1] \wedge i \in \mathbb{N} \wedge j \in \mathbb{N}\}$$

Finally, higher-level versions of $Z(x)$ and $Z^+(x)$ exist for these properties and will be denoted as $Z_1(x)$ and $Z_1^+(x)$, where the "usual" $Z(x)$ and $Z^+(x)$ can have as alternative notation: $Z_0(x)$ and $Z_0^+(x)$.

Indeed, formulas containing $Z_1(x)$ and $Z_1^+(x)$ as terms cannot be used to form meta-predicates $M_{i.j}$.

The higher levels of meta-predicates go on for infinity, where the $Z_n(x)$ and $Z_n^+(x)$ cannot serve as valid terms to form predicates as the n-th level and therefore generate predicates of the level n+1.

All the rules for the foundational level of predicates Type 2.1, apply to the higher ones, and therefore there is an order set of meta-predicates for each level.

2.5 Type 3: Sets

These can be represented by Upper Case letters: "X", "Y", "Z"... or "$\mathfrak{Z}$", "$\mathfrak{B}$", or "$\mathfrak{C}$"... or "Θ", "Λ", "Ω"...

Note: Uppercase variables can also represent the function operator on sets defined in Type 4.2 in the form: "/uppercase letter/: $\mathrm{t}_{3(a)} \longmapsto \mathrm{t}_{3(b)}$"

Sets are of the general form: "/upper case letter/ $= \{\mathrm{t}_n | \mathrm{t}_2(\mathrm{t}_n)\}$"

When including individuals, sets are of the form: "/upper case letter/ $= \{\mathrm{t}_1 | \mathrm{t}_2(\mathrm{t}_1)\}$".

Thus, when including sets, sets are of the form:"/upper case letter/ $= \{\mathrm{t}_3 | \mathrm{t}_2(\mathrm{t}_3)\}$".

Moreover, some sets can be formed out of meta-predicates and would be of the logical form: "/upper case letter/ $= \{\mathrm{t}_n | \mathrm{t}_{2.2}(\mathrm{t}_n)\}$"

There can be sets of any type whatsoever.

2.6 Type 4: Operators and Constants:

2.6.1 Type 4.1: Operators on all types (except type 4):

The following definitions are of the form: "/operator/ ^{def}/use of the operator/≡ /meaning of the operator/" **(these definitions are for individuals, but the same applies for sets and formulae, by replacing "t_1" by "t_3" or "t_0")**

i. "Implies": "→" $:^{def}$ $(\mathrm{t}_{1(a)} \rightarrow \mathrm{t}_{1(b)}) \equiv \neg \mathrm{t}_{1(a)} \vee \mathrm{t}_{1(b)}$

ii. "For all": "∀" $:^{def}$ $\forall \mathrm{t}_1[\mathrm{t}_2(\mathrm{t}_1)] \equiv [(\mathrm{t}_1 = \mathrm{t}_1) \rightarrow \mathrm{t}_2(\mathrm{t}_1)]$

iii. "Not": "¬" or "$\not\chi$" $:^{def}$ $\neg \mathrm{t}_{1(a)} \equiv \mathbb{T}_{1-4.3}, \mathfrak{P} \nvdash \mathrm{t}_{1(a)}$

iv. "There does not exist": "∄" $:^{def}$ $\nexists \mathrm{t}_1 \equiv Z^+(x) > 0$

v. "There exists": "∃" $:^{def}$ $\exists \mathrm{t}_1 \equiv Z^+(x) = 0$

vi. "And": "∧" $:^{def}$ $\mathrm{t}_{1(a)} \wedge \mathrm{t}_{1(b)} \equiv \neg(\neg \mathrm{t}_{1(a)} \vee \neg \mathrm{t}_{1(b)})$

vii. "Or": "∨" $:^{def}$ $\mathrm{t}_{1(a)} \vee \mathrm{t}_{1(b)} \equiv \neg(\neg \mathrm{t}_{1(a)} \wedge \neg \mathrm{t}_{1(b)})$

viii. "Equivalent to": "≡" $:^{def}$ $(\mathrm{t}_{1(a)} \equiv \mathrm{t}_{1(b)}) \equiv (\mathrm{t}_{1(a)} \rightarrow \mathrm{t}_{1(b)}) \wedge (\mathrm{t}_{1(b)} \rightarrow \mathrm{t}_{1(a)})$

ix. "Equal to:" "=" $:^{def}$ $(t_{1(a)} = \mathrm{t}_{1(b)}) \equiv \forall \mathrm{t}_2 \forall n(n \in \mathbb{R} \wedge \mathrm{t}_2(\mathrm{t}_{1(a)}) = n \equiv \mathrm{t}_2(\mathrm{t}_{1(b)}) = n) \equiv Z(x) = Z(y)$

x. "Necessarily": "□" $:^{def}$ $\Box \mathrm{t}_1 \equiv \neg \Diamond \neg \mathrm{t}_1$

xi. "Possibly": "◇" $:^{def}$ $\Diamond \mathrm{t}_1 \equiv \neg \Box \neg \mathrm{t}_1$

2.6.2 Type 4.2: Operators on type 3 (sets):

The following definitions are of the form: "/operator/ $:^{def}$ /use of the operator/$\equiv$ /meaning of the operator/"

(these definitions are for sets of individuals, but the same applies for sets of sets or sets of numbers, by replacing "t_1" by "t_3" or "$t_{1.2}$")

i. "Union": "$\cup$" $:^{def}$ $t_{3(a)} \cup t_{3(b)} \equiv \{t_1 | (t_1 \in t_{3(a)}) \vee (t_1 \in t_{3(b)})\}$

ii. "Intersection": "$\cap$" $^{def} t_{3(a)} \cap t_{3(b)} \equiv \{t_1 | (t_1 \in t_{3(a)}) \wedge (t_1 \in t_{3(b)})\}$

iii. "Subset": "$\subseteq$" $:^{def}$ $t_{3(a)} \subseteq t_{3(b)} \equiv \forall t_1 (t_1 \in t_{3(a)} \rightarrow t_1 \in t_{3(b)})$

iv. "Proper Subset": $:^{def}$ $t_{3(a)} \subsetneq t_{3(b)} \equiv (t_{3(a)} \subseteq t_{3(b)}) \wedge (t_{3(a)} \neq t_{3(b)})$

v. "Cartesian product": "$\times$" $:^{def}$ $t_{3(a)} \times t_{3(b)} \equiv \{(t_{1(a)}, t_{1(B)}) | t_{1(a)} \in t_{3(a)} \wedge t_{1(b)} \in t_{3(b)}\}$

vi. "Set minus": "$-$"$:^{def}$ $t_{3(a)} - t_{3(b)} \equiv \{t_1 | (t_1 \in t_{3(a)}) \wedge (t_1 \notin t_{3(b)})\}$

vii. "Complement set": "t_3^C" $:^{def}$ $t_3{}^C \equiv \{t_1 | t_1 \notin t_3\}$

viii. "Cardinality (Size)": "$|t_{3(a)}|$" $:^{def}$ $|t_{3(a)}| \equiv |t_{3(b)}| \wedge [(\exists F : t_{3(a)} \longmapsto t_{3(b)}) \wedge (\exists F : t_{3(b)} \longmapsto t_{3(a)})]$

ix. "Function": "F" $:^{def}$ $F : t_{3(a)} \longmapsto t_{3(b)} \equiv \{(t_{1(a)}, t_{1(b)}) | (t_{1(a)} \in t_{3(a)} \wedge t_{1(b)} \in t_{3(b)}) \wedge t_{1(b)} = F(t_{1(a)})\}$

x. "Composite Function": "$\circ$"$:^{def}$ $(F_{(a)} \circ F_{(b)})(x) \equiv F_{(a)}(F_{(b)}(x))$

xi. "Power set" "$\wp(t_{3(a)})$" $:^{def}$ $\wp(t_{3(a)}) \equiv \{t_{3(x)} | t_{3(x)} \subseteq t_{3(a)}\}$

xii. "Element of": "$\in$" $:^{def}$ $t_1 \in t_3 \equiv t_3 = \{t_1 | t_2(t_1) = n\} \wedge t_2(t_1) = n$

xiii. " Large Cartesian product": "$\prod_{i=1}^{n} t_{3(a)_i}$" $:^{def}$ $\prod_{i=1}^{n} t_{3(a)_i} \equiv \{(t_{3(a)_1}, t_{3(a)_2}, \ldots, t_{3(a)_n}) | a_i \in A_i\}$

2.6.3 Type 4.3: Operators on type 1.2 (numbers)

The following definitions are of the form: "/operator/ $:^{def}$ /use of the operator/$\equiv$ /meaning of the operator/"

- i. - "Addition" "$+$" $:^{def}$ $t_{1.2(a)} + t_{1.2(b)} \equiv t_{3(a)} \cup t_{3(b)} \wedge [(t_{1.2(a)} = t_{3(a)}) \wedge (t_{1.2(a)} = t_{3(a)})]$
- ii. - "Subtraction" "$-$" $:^{def}$ $t_{1.2(a)} - t_{1.2(b)} \equiv t_{3(a)} - t_{3(b)} \wedge [(t_{1.2(a)} = t_{3(a)}) \wedge (t_{1.2(a)} = t_{3(a)})]$
- iii. - "Multiplication": "$*$" $:^{def}$ $t_{1.2(a)} t_{1.2(b)} \equiv t_{3(a)} \times t_{3(b)} \wedge [(t_{1.2(a)} = t_{3(a)}) \wedge (t_{1.2(a)} = t_{3(a)})]$
- iv. - "Division": "/" $:^{def}$ $\frac{\mathrm{t}_{1.2(a)}}{\mathrm{t}_{1.2(b)}} \equiv \{(\mathrm{t}_{1.2(x)}, \mathrm{t}_{1.2(y)}) | (\mathrm{t}_{1.2(x)} = \mathrm{t}_{1.2(a)} \times t_{1.2(y)}) \wedge (t_{1.2(y)} \neq 0) \wedge (\mathrm{t}_{1.2(x)} < \mathrm{t}_{1.2(b)})\}$
- v. "Change in": "Δ" $:^{def}$ $\Delta \mathrm{t}_{1.2} \equiv \mathrm{t}_{1.2(1)} - \mathrm{t}_{1.2(0)}$
- vi. "Larger or Equal to": "$\geq$" $:^{def}$ $\mathrm{t}_{1.2(a)} \geq \mathrm{t}_{1.2(b)} \equiv (\mathrm{t}_{1.2(a)} > \mathrm{t}_{1.2(b)}) \vee (\mathrm{t}_{1.2(a)} = \mathrm{t}_{1.2(b)})$
- vii. "Smaller or Equal to": "$\leq$" $:^{def}$ $\mathrm{t}_{1.2(a)} \leq \mathrm{t}_{1.2(b)} \equiv (\mathrm{t}_{1.2(a)} < \mathrm{t}_{1.2(b)}) \vee (\mathrm{t}_{1.2(a)} = \mathrm{t}_{1.2(b)})$
- viii. "Sum" : "$\sum_{i=a}^{b} \mathrm{t}_2(\mathrm{t}_{n_i})$" $:^{def}$ $(\sum_{i=a}^{b} \mathrm{t}_2(\mathrm{t}_{n_i}) = n) \equiv (n = \mathrm{t}_2(\mathrm{t}_{n_a}) + \mathrm{t}_2(\mathrm{t}_{n_{a+1}}) + \cdots + \mathrm{t}_2(\mathrm{t}_{n_b}))$

2.7 Type 5: Meta-Language Operators

When referring to the system itself and its mechanisms, we will make use of Meta-Language operators. These operators are part of the language used when analysing the system. They can, however, also be used to form a formula, t_0, enabling the system to reference and analyse its own structure. Their definitions are of the form: definitions are of the form: "/operator/ $:^{def}$ /use of the operator/$\equiv$ /meaning of the operator/".

i. "Proves": "$\vdash$" $:^{def}$ $X \vdash Y \equiv$ There is a proof of Y from X.

ii. "Models": "$\vDash$" $:^{def}$ $X \vDash Y \equiv$ Y is true in every model where X is true.

iii. "Does not Prove": "$\nvdash$" $:^{def}$ $X \nvdash Y \equiv$ There is not a proof of Y from X.

iv. "Does not Model": "$\nvDash$" $:^{def}$ $X \vDash Y \equiv$ Y is not true in every model where X is true.

v. "Belongs a Type n": " : " $:^{def}$ $x : \mathbb{T}_n \equiv$ x is of Type n.

2.8 Rules of Inference

From the definitions of Type 4, rules of inference are implied for the derivations of Type 0 (formulas).

Indeed, all definitions of Types are of the form $X \equiv Y$, where X is the operator and Y is its definition.

Moreover, "$\equiv$" itself is an operator of the system, defined as a Type 4.1, in the definition *viii.* as: $(\mathrm{t}_{n(a)} \rightarrow \mathrm{t}_{n(b)}) \wedge (\mathrm{t}_{n(b)} \rightarrow \mathrm{t}_{n(a)})$, where "n" could be 1, 3 or 0.

Now the following principle can be applied, where if the operator X is defined as $X \equiv Y$:

1. $X \equiv Y \vdash Y \rightarrow X$

2. $X \equiv Y, Y \vdash X$

3. $X \equiv Y \vdash X \rightarrow Y$

4. $X \equiv Y, X \vdash Y$

Moreover, as highlighted in the definition of Type 0, we will use the notation "[X]" to refer to the definition "$X \equiv Y$" of an operator or type X.

If a formula Z derived from the system, alongside a definition "$X \equiv Y$" imply a formula G then:

5. $Z \wedge (X \equiv Y) \vdash G$

This will be simplified under the notation "$[Z][X] \vdash G$". This can be done for any number of definitions present in the system and/or formulas derived from it.

The same applies to multiple formulas derived from the system implying a formula G, as well as multiple definitions and both, where X refers to definitions and Z formulas:

6. $[Z_1]...[Z_n] \vdash G$

7. $[X_1]...[X_n] \vdash G$

8. $[Z_1]...[Z_n][X_1]...[X_n] \vdash G$

Any definition or formula that has been derived, can be repeated within the system, deriving itself:

9. $X \vdash X$

Definitions using identity "=" instead of logical equivalence "≡", follow similar rules. The following can be demonstrated as per the identity definition through the use of the "≡" operator, for $X = Y$ we get:

10. $\mathrm{t}_2(X), X = Y \vdash \mathrm{t}_2(Y)$

11. $\mathrm{t}_2(Y), X = Y \vdash \mathrm{t}_2(X)$

Finally, when a formula X cannot be derived from the system, then it is false, which leads to its negation being derivable from the system:

12. $[(\mathbb{T}_{1-4.3}, \mathfrak{P} \nvdash X) \wedge (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0...X_n \nvdash X)] \rightarrow [(\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash \neg X) \wedge (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0...X_n \vdash \neg X)]$

Establishing these rules does not violate our principle of building a system simply from a single axiom and definitions as these rules of inference are a direct consequence of our definitions.

2.9 Soundness of the system

As truth is system defined as: $(X \text{ is True}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X) \vee (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0 ... X_n \vdash X)$; and falsity is defined within the system as $(X \text{ is False}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \nvdash X) \vee (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0 ... X_n \nvdash X)$.

It follows that $\forall X[(\text{X is True}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X)]$, as "$\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0 ... X_n \vdash X$" can be reduced to "$\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X$" if "$X_0 ... X_n$" are represented as steps in the proof of X from the system.

Which then implies that $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X \rightarrow \mathbb{T}_{1-4.3}, \mathfrak{P} \vDash X$. As if all true statements are provable, all provable statements are true in all models.

More formally:

1. $(X \text{ is True}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X) \vee (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0 ... X_n \vdash X)$ By [Definition, in Type 0]
2. $(X \text{ is False}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \nvdash X) \vee (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0 ... X_n \nvdash X)$ By [Definition, in Type 0]
3. $(\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X_0 ... X_n \vdash X) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X)$ By [$\vdash$]
4. $\forall X[(\text{X is True}) \equiv (\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X)]$ By [1-3]
5. $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X \rightarrow$ X is True By [4]
6. $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash X \rightarrow \mathbb{T}_{1-4.3}, \mathfrak{P} \vDash X$ By [5][$\vdash$] [$\vDash$]

Once again, the notation "By [x]" in the proofs, maintains that "x" or the definition of "x" implies the formula.

Therefore, the system is sound, as the soundness of a given system S is defined as $S \vdash X \rightarrow S \vDash X$.

3 Outline of The Absolute (from Type system)

3.1 Derivation of ”$\mathfrak{A}$” from Types and ”$\mathfrak{P}$”

The following proof gives a clear outline of The Absolute “$\mathfrak{A}$” (Metaphysical Cosmos), starting from the phenomenological fact that there exists (at least) something, and deriving “$\mathfrak{A}$” from this premise and the definitions of symbols and their Types. I will here provide two central theorems, namely that the Absolute is the set of all entities that exist or does not exist, and that all things are members of the Absolute. A more formal proof sketch of these theorems is provided in **section 6**, as the First and Second Theorems.

From empirical analysis (perceiving this text) we can derive that there exists something[11]:

$$[\mathfrak{P}] \vdash: \exists x \tag{1}$$

such that:

$$x = \mathfrak{t}_{n(x)} \tag{2}$$

Where $\mathfrak{t}_{n(x)}$ refers to any entity, of any type.

Note: throughout the paper, x always refers to any entity, of any type, unless specified.

From the definition of ”$\exists$”, we can derive that this entity has a Z^+ value larger than zero:

$$[\downarrow][\exists] \vdash: Z^+(x) > 0 \tag{3}$$

From $\mathbb{T}_3$ we can derive that this entity exists in a set Ψ, such that Ψ is the set of all things that satisfy at least one property to a degree larger than 0, which is the inherent form of sets. Thus, it is the set of all the things that exist:

$$[\downarrow][\mathbb{T}_3] \vdash: x \in \Psi \tag{4}$$

such that:

$$[\downarrow] \vdash: \Psi = \{x|\exists x\} = \{x|Z^+(x) > 0\} \tag{5}$$

It is essential to emphasise that Ψ extends beyond a mere collection of entities; it encompasses not only the entirety of things but also manifests as the comprehensive realm of facts, as Ψ also contains all types 0. This conceptualisation does not preclude interpretations aligning with philosophical perspectives such as the one articulated by Ludwig Wittgenstein in his statement: ”The world is the totality of facts, not of things” (Wittgenstein, 1921, [18]). As a matter of fact, Ψ is the totality of things and of facts.

From the definition of the complement-set operator $\mathfrak{t}_3^C$, we can derive Ψ^C is the set of all things that do not satisfy any properties, the Void: ”$\varnothing$”.Thus, it is the set of all things that do not exist.

$$[\downarrow][\mathfrak{t}_3^C] \vdash: \Psi^C = \varnothing \tag{6}$$

such that:

$$[\downarrow] \vdash: \varnothing = \{x|\nexists x\} = \{x|Z^+(x) = 0\} \tag{7}$$

As the Metaphysical Void is here defined as the complement set of the Totality of things ”Ψ” and concerns objects that do not possess any properties and thus, using our definition

[11] Even if the perception of this paper were to be deemed illusory, it would still possess the property of being an illusion, thereby affirming its existence.

of existence do not exist, we can tie our representation to current research on the onto-epistemological status of the empty set (Birgül, 2022,[35]). Indeed, Birgül ties the empty set to the Kantian concept of nihil privativum, describing the empty object of a concept and relating to our definition of non-existence as the absence of properties.

From the definition of "∩" we can derive that the intersection of Ψ and $\varnothing$ is $\mathfrak{X}$, the set of all things that both exist and not exist.

$$[5][6][\cap] \vdash: \Psi \cap \varnothing = \mathfrak{X} \tag{8}$$

such that:

$$[\downarrow] \vdash: \mathfrak{X} = \{x | \exists x \wedge \nexists x\} = \{x | Z^{+}(x) > 0 \wedge Z^{+}(x) = 0\} \tag{9}$$

As the condition for this set is inherently paradoxical, any member of $\mathfrak{X}$ would be non-classical, this is however allowed by our system and will be elucidated furthermore within **Section 3.2**, describing the existential metric space.

From the definition of "∪", we can derive that the union of Ψ and $\varnothing$ is the set of all things that exist and/or do not exist, have a property and/or no property. This set is the Metaphysical Cosmos itself: The Absolute ("$\mathfrak{A}$").

$$[5][6][\cup] \vdash: \Psi \cup \varnothing = \mathfrak{A} \tag{10}$$

such that

$$[\downarrow] \vdash: \mathfrak{A} = \{x | \exists x \vee \nexists x\} = \{x | Z^{+}(x) > 0 \vee Z^{+}(x) = 0\} \tag{11}$$

The Absolute can be considered as the Universal set of this system, as it contains all possible types[12]:

x	$x \in \mathfrak{A}$
$\exists(x)$	True
$\nexists(x)$	True
$\exists(x) \wedge \nexists(x)$	True
$\exists(x) \vee \nexists(x)$	True

Table 1: Truth Values for $x \in \mathfrak{A}$

(12)

[12]The notion of power-sets of The Absolute, that seems to counter the idea of it being the Universal Set will be discussed in part **4.3** and used to represent the expansion of the Metaphysical Cosmos.

The concept of The Absolute in our type system is similar to the concept of the universal set "**V**" in Bertrand Russell and Alfred North Whitehead's Principia Mathematica (Whitehead & Russell, 1910, [16]). However, there are three major differences between the two concepts:

1. The universal set "**V**" in Principia Mathematica is purely mathematical whereas The Absolute "$\mathfrak{A}$" has metaphysical implications.

2. The universal set "**V**" in Principia Mathematica is the opposite of its respective empty set "Λ" whereas The Absolute "$\mathfrak{A}$" is the merging of the Metaphysical Void "$\varnothing$" and the totality of things "$\boldsymbol{\Psi}$". In that regard, "**V**" bears similarity with "$\boldsymbol{\Psi}$".

3. The universal set "**V**" in Principia Mathematica is restricted to inputs of a specific type whereas The Absolute "$\mathfrak{A}$" includes elements of all types.

Additionally, there is another similarity between our system and Principia Mathematica. Indeed, our derivations of the Totality of Things "$\boldsymbol{\Psi}$" and of the Void "$\varnothing$" from our axiom "$\mathfrak{P}$" bears strong similarity to the following commentary made on "**V**" and "Λ" :

*If the monistic philosophers were right in maintaining that only one individual exists, there would be only two classes, Λ and **V**, **V** being (in that case) the class whose only member is the one individual. Our primitive propositions do not require the existence of more than one individual.* - (Whitehead & Russell, 1910, [16])

In our description, the "one individual" existing, namely this paper itself, is brought upon by our axiom $\mathfrak{P}$. Just as Λ and **V** are two fundamental sets that are formed with the existence of (at least) one individual, we form $\boldsymbol{\Psi}$ and $\varnothing$ from $\mathfrak{P}$. However, our derivation leads us to the necessary existence of a third set: $\mathfrak{X}$, due to the paraconsistent nature of the system, and this allows us to go beyond the everything/nothing duality of Λ and **V** in Principia Mathematica, $\boldsymbol{\Psi}$ and $\varnothing$ in our system.

Through these primordial derivations, we have now set clear boundaries to the domain of the Metaphysical Cosmos. Indeed, it is logically impossible for any entity to transcend, surpass or go beyond The Absolute. I am now going to provide a proof for the second theorem, namely that all entities are members of the Absolute.

From Type 2, we can derive that all entities x have a $Z^+(x)$ associated with a real number:

$$[\mathbb{T}_2] \vdash: \forall x(Z^+(x) = n \land n \in \mathbb{R}) \tag{13}$$

From the definition of the real numbers, we can derive that all real numbers are equal or not equal to 0:

$$[\downarrow][\mathbb{T}_2][\mathbb{T}_{1.2}] \vdash: \forall n(n \in \mathbb{R} \rightarrow n = 0 \lor n \neq 0) \tag{14}$$

From the previous formula, Type 2 and the definitions of real numbers, we can derive that all entities have a value of $Z^+(x)$ equal or not equal to 0:

$$[\downarrow][\mathbb{T}_2][\mathbb{T}_{1.2}] \vdash: \forall x(Z^+(x) = 0 \lor Z^+(x) \neq 0) \tag{15}$$

From the previous formula and the definition of existence and non-existence, we can derive that:

$$[\downarrow][\exists][\nexists] \vdash: \forall x(\exists x \lor \nexists x) \tag{16}$$

Therefore, from the previous formula and the definition of The Absolute, we can derive that:

$$[\downarrow][11] \vdash: \forall x(x \in \mathfrak{A}) \tag{17}$$

3.2 Existential Metric-Space

I will here elaborate a metric space for existence and non-existence which are meta-predicates, according to predicate definitions given in the outline of Type 2 as well as in **section 1.2.1**.

From the outline of Type 2 and the definitions of existence and non-existence, we can define existence as $M_{\exists}$[13] and non-existence as $M_{\nexists}$:

$$[\mathbb{T}_2][\exists][\nexists] \vdash: M_{\exists}(x) \equiv (Z^{+}(x) \neq 0) \wedge M_{\nexists}(x) \equiv (Z^{+}(x) = 0) \tag{18}$$

From the outline of Type 2, and the previous formula, we can generate $M_{\exists_{Space}}$ and $M_{\nexists_{Space}}$, as the sets of all possible values of $M_{\exists}$ and $M_{\nexists}$:

$$[\downarrow][\mathbb{T}_2] \vdash: M_{\exists_{Space}} = \{x \in [0,1] | x = M_{\exists}(y) \wedge y \in \mathfrak{A}\} \wedge M_{\nexists_{Space}} = \{x \in [0,1] | x = M_{\nexists}(y) \wedge y \in \mathfrak{A}\} \tag{19}$$

From the outline of Type and the previous formula, we can generate the metric space for existence and non-existence, $M^{*}_{\exists_{Space}}$, as the Cartesian product of $M_{\exists_{Space}}$ and $M_{\nexists_{Space}}$, paired with a distance function:

$$[\downarrow][\mathbb{T}_2] \vdash: M^{*}_{\exists_{Space}} = (M_{\exists_{Space}} \times M_{\nexists_{Space}}, d) \tag{20}$$

Here the distance function d can vary, but can be interpreted either through the Manhattan distance (Minkowski,1896, [12]) or the Euclidean (Euclid, 300 B.C., [52]).

Now how does this existential metric space help us understand the foundations of the Metaphysical Cosmos?

For each of the meta-predicates associated with the fundamental sets of the absolute, there is a range of possible coordinates for an object that satisfy these predicates in the Existential Metric-Space.

From Type 2, the definition of The Absolute, and the definition of $M^{*}_{\exists_{Space}}$, we can derive that members of The Absolute can have every coordinate in $M^{*}_{\exists_{Space}}$, except $(0,0)$:

$$[\downarrow][\mathbb{T}_2][11] \vdash: x \in \mathfrak{A} \equiv \neg(M_{\exists}(x) = 0 \wedge M_{\exists}(x) = 0) \wedge \equiv (M_{\exists}(x), M_{\nexists}(x)) \neq (0,0) \tag{21}$$

From Type 2, the definition of The Totality of Things, and the definition of $M^{*}_{\exists_{Space}}$, we can derive that members of The Totality of Things satisfy the membership criterion if they are in the range of possible existential coordinates of $(1, n \in [0,1])$:

$$[\downarrow][\mathbb{T}_2][5] \vdash: x \in \Psi \equiv (M_{\exists}(x) = 1) \equiv (M_{\exists}(x), M_{\nexists}(x)) = (1, n \in [0,1]) \tag{22}$$

[13] As there cannot be negative values of $Z^{+}(x)$,$M_{\exists}$ can be defined as both $Z^{+}(x) \neq 0$ and $Z^{+}(x) > 0$.

From Type 2, the definition of The Void, and the definition of $M^*_{\exists_{Space}}$, we can derive that members of The Void satisfy the membership criterion if they are in the range of possible existential coordinates of $(n \in [0,1], 1)$:

$$[\downarrow][\mathbb{T}_2][7] \vdash: x \in \varnothing \equiv (M_{\nexists}(x) = 1) \equiv (M_{\exists}(x), M_{\nexists}(x)) = (n \in [0,1], 1) \tag{23}$$

Finally, from Type 2, the definition of The Portal, and the definition of $M^*_{\exists_{Space}}$, we can derive that members of The Portal satisfy the membership criterion if they are at the point (1,1) in the Existential Metric Space:

$$[\downarrow][\mathbb{T}_2][9] \vdash: x \in \mathfrak{X} \equiv (M_{\exists}(x) = 1 \wedge M_{\nexists}(x) = 1) \equiv (M_{\exists}(x), M_{\nexists}(x)) = (1,1) \tag{24}$$

Thus, we can visually represent the Existential Metric Space in the following way:

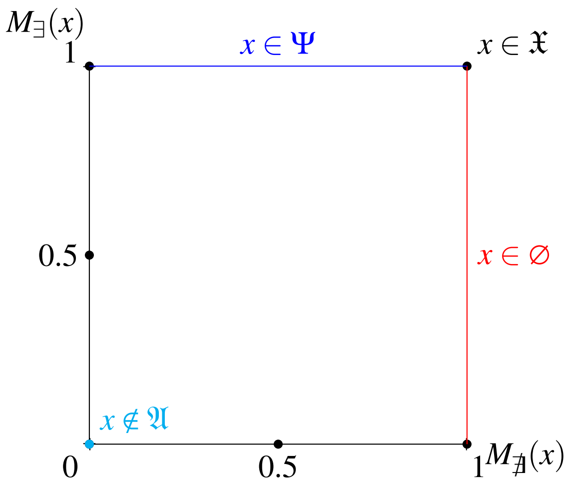

Figure 4: Diagram of The Existential Metric Space

Where the red line represents the range of possible values that satisfy the membership criteria of The Void and the dark blue line represents the range of possible values that satisfy the membership criteria of The Totality of Things. Every point in this diagram except $(0,0)$, represented in light blue, is part of the Absolute. The reason that $(0,0)$ is not in the Absolute is that this point would imply that an x is not associated with a Z or Z^+, which is impossible as all types are associated with one of each, i.e. Z and Z^+[14].

3.3 Inner Structure of The Metaphysical Cosmos

3.3.1 Analysis of The Totality of Things "Ψ"

I will here demonstrate that the "Totality of Things" denoted by "Ψ" is of infinite size. A moral formal proof can be found in **section 6**, as the third theorem.

From the definition of the set $\mathbb{R}$ and the definition of Ψ, we can derive that all elements belonging to the set $\mathbb{R}$, also belong to Ψ:

$$[5][\mathbb{R}] \vdash: \forall x (x \in \mathbb{R} \rightarrow x \in \Psi) \tag{25}$$

From the previous formula, and the definition of subset-hood, we can derive that the Totality of Things contains by essence the set of all real numbers:

[14]This is showed by formula (13)

$$[\mathbb{T}_{1.2}][5] \vdash: \mathbb{R} \subseteq \Psi \tag{26}$$

From the previous formula, we can derive that the cardinality of the set of all real numbers "$\mathbb{R}$" is smaller or equal to the cardinality of Ψ:

$$[\downarrow] \vdash: |\mathbb{R}| \leq |\Psi| \tag{27}$$

From $\mathbb{T}_{1.2}$, we can derive that the cardinality of $\mathbb{R}$ equal to $\mathfrak{c}$.

$$[\mathbb{T}_{1.2}] \vdash: |\mathbb{R}| = \mathfrak{c} \tag{28}$$

From (26) and the previous formula, we can derive that the cardinality of the Totality of Things is larger or equal to the infinite size of the real numbers.

$$[26][\downarrow] \vdash: |\Psi| \geq \mathfrak{c} \tag{29}$$

It is worth noting that the totality of things is far larger than just the physical universe. Indeed, it would, for instance, contain all non-empty logically possible worlds. Under modal realism, the belief that all logically possible worlds are metaphysically real, expressed by David Lewis (Lewis, 1986, [26]), these possible worlds would be located in the Totality of Things within the Metaphysical Cosmos. Contemplating this concept aids in grasping the immense expanse of The Absolute, given that the totality of things transcends the cumulative vastness of all possible worlds. Moreover, The Absolute itself surpasses even this staggering magnitude, accentuating the vastness of Metaphysical reality. This is also one of the occurrences where the introduction of a cosmological paradigm, namely The Absolute, can guide metaphysical research as it gives a clear mapping of cosmological the location of possible worlds, as it is a currently popular area of study in formal Metaphysics (Fouché,2022,[45]) (Longenecker,2019,[40]).

One could potentially represent The Absolute and its constituents the following way, where "w" represents any given world:

$$\mathfrak{A} = \{w | (\Diamond w \vee \neg \Diamond w) \wedge (Z^{+}(w) = 0 \vee Z^{+}(w) \neq 0)\} \tag{30}$$

$$\varnothing = \{w | (\Diamond w \vee \neg \Diamond w) \wedge Z^{+}(w) = 0\} \tag{31}$$

$$\Psi = \{w | (\Diamond w \vee \neg \Diamond w) \wedge Z^{+}(w) \neq 0\} \tag{32}$$

$$\mathfrak{X} = \{w | (\Diamond w \vee \neg \Diamond w) \wedge (Z^{+}(w) = 0 \vee Z^{+}(w) \neq 0)\} \tag{33}$$

It is worth noting that the Totality of Things also contains impossible worlds insofar as they are existent, as there can be some non-classical worlds in our system of properties whose Z^{+} value is non-zero.

Moreover, the study of Physics and Physical Cosmology, as mentioned in the introduction, is a subset of Metaphysical Cosmology as the Physical Universe is a subset of "Ψ".

Indeed, the Physical Universe can be represented as the following set:

$$\Omega = \{x | \mathscr{L}(x) = 1\} \tag{34}$$

Where $\mathscr{L}(x)$ is the property of being perceivable, through the senses or any enhancement of them through experimental tools.

The study of Physical Cosmology, no matter how vast it could be (Multiverse Cosmology for example), will always be subject to Metaphysical Cosmology and represent a portion of its study. Max Tegmark, a physical cosmologist, gave a classification of the potential extensions to the Physical Universe that can be quite useful in order to picture the vastness of Ψ (Tegmark,2007,[33]):

1. Level 1: Regions beyond our (physical) cosmic horizon: Possible direct extensions of our physical universe beyond the currently visible one due to cosmic inflation. (Tegmark,2007,[33])
2. Level 2: Other post-inflation bubbles: Possible multiverses, formed during the hypothetical process of eternal cosmic inflation. (Guth,2007,[32])
3. Level 3: Many-Worlds interpretation of Quantum Mechanics: An interpretation of quantum mechanics that claims that the wave function never collapses, instead other possible outcomes are realised in other universes. It thus posits that there is an infinite number of alternative universes. (Everett,1957,[22])
4. Level 4: Other mathematical structures: Possible mathematical structures that differ from our Physical Universe. (Tegmark,2007, [33])

All extensions of all levels are fundamentally part of Ψ. Extensions to and including level 3 within this scheme are categorised under the domain of physical cosmology. However, the level 4 extensions, which describe mathematical realms of a fundamentally different nature than our physical reality, could fall into the realm of Metaphysics, if there is no direct physical path from our universe to them.

Nonetheless, one must not misunderstand the derivation above. Indeed, saying Ψ is larger or equal to the largest possible infinity simply means that because all numbers have properties, they exist and are therefore part of Ψ. The derivation does not claim in any way the existence of an infinity of parallel worlds or realities, nor does it deny it. Moreover, our conception of Ψ does not deny nor confirm solipsism, nor does it rely on a materialist/naturalistic conception of the world. It is a paradigm within which to investigate Metaphysics, that does not, in and of itself, contain assumptions about the nature of the physical world.

3.3.2 The Void $\varnothing$ / Portal $\mathfrak{X}$ Equivalence

I will here demonstrate that $\varnothing \subseteq \mathfrak{X}$, the formal proof sketch for this theorem can be found in **section 6**, as the fourth theorem.

From the definition of $\varnothing$, we can derive that all members of The Void have a Z^+ value of 0:

$$[7] \vdash : \forall x[(x \in \varnothing) \rightarrow (Z^+(x) = 0)] \tag{35}$$

From the previous formula and the definition of $Z^+(x)$, we can derive that all members of The Void have 0 for all entries in their $Z(x)$ Matrix:

$$[\downarrow][\mathbb{T}_2] \vdash: \forall x[(x \in \varnothing) \to Z(x) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}] \tag{36}$$

From the definition of identity, we can derive that the Z of an element of The Void is equal to itself, which implies that an element of The Void would satisfy $P_{1.1}$, i.e. identity:

$$[=][\mathbb{T}_2] \vdash: \forall x[(x \in \varnothing) \to (Z(x) = Z(x)) \to P_{1.1}(x) = 1] \tag{37}$$

From the previous formula, we can derive that the Z^+ of a member of the Void is equal to or larger than one:

$$[\downarrow][\mathbb{T}_2] \vdash: \forall x[(x \in \varnothing) \to Z^+(x) \geq 1] \tag{38}$$

This thus yields that any member of the void, would be non-classical and hold two opposite values for Z^+ simultaneously :

$$[35][\downarrow] \vdash: \forall x[(x \in \varnothing) \to Z^+(x) = 0 \wedge Z^+(x) > 0] \tag{39}$$

From the previous formula and the definition of The Portal $\mathfrak{X}$ we can now derive that all members of the Void, satisfy the membership condition for the Portal:

$$[7][9][\downarrow] \vdash: \forall x[(x \in \varnothing) \to (x \in \mathfrak{X})] \tag{40}$$

Thus, from the previous formula we can derive:

$$[\subseteq][\downarrow] \vdash: \varnothing \subseteq \mathfrak{X} \tag{41}$$

From the definition of intersection, we can derive that members of The Portal also exist in The Void:

$$[\subseteq][\mathfrak{X}] \vdash: \forall x[(x \in \mathfrak{X}) \to (x \in \varnothing)] \tag{42}$$

From the previous formula, we can derive that The Portal is also a subset of The Void:

$$[\subseteq][\downarrow] \vdash: \mathfrak{X} \subseteq \varnothing \tag{43}$$

Therefore, The Void and The Portal are equal:

$$[\subseteq][41][43] \vdash: \varnothing = \mathfrak{X} \tag{44}$$

This conclusion reveals a fundamental truth of nothingness, which we will investigate in the next section, through the proof of ”why there is something rather than nothing”.

3.3.3 The Absolute $\mathfrak{A}$ as The Totality of Things Ψ

In this section, will show that the Absolute $\mathfrak{A}$ is the Totality of Things Ψ. The formal proof sketch for this theorem can be found in **section 6**, as the fourth theorem.

From the definition of The Absolute, we can derive that it is equal to the union of The Void and the Totality of Things:

$$[10] \vdash: \mathfrak{A} = \varnothing \cup \Psi \tag{45}$$

From the previous formula and the fourth theorem, we can derive that The Absolute is equal to the union of The Portal and the Totality of Things:

$$[\downarrow][44] \vdash: \mathfrak{A} = \mathfrak{X} \cup \Psi \tag{46}$$

From the definitions of subset-hood, union, The Portal and The Totality of Things, we can derive that the Union of The Portal and the Totality of Things is equal to the Totality of Things:

$$[\cup][\subseteq][5][9] \vdash: \mathfrak{X} \cup \Psi = \Psi \tag{47}$$

Thus, from the two previous formulas, we can derive that The Absolute is equal to the Totality of things:

$$[\downarrow][46] \vdash: \mathfrak{A} = \Psi \tag{48}$$

Now how to make sense of this refined structure of the Metaphysical Cosmos?

Within this newly found conception of the Metaphysical Cosmos, we can still find a similar structure. Indeed, the Absolute, i.e. the entirety of the Metaphysical Cosmos is now confined to Ψ. However, within Ψ, remains the fundamental contrast between the realm of the non-existent, i.e. the Void/Portal, and the realm of the truly existent, its complement.

It is worth noting again that these sets yield a structure that describes Metaphysical Nature, as they are a reflection of the highest, most unrestricted form of structure, and that this system yields not just a logical abstract truth, but a genuine description of the world at the highest level.

To make better sense of these ”places” in the Metaphysical cosmos, we will need to define more formally, the complement set of The Portal/Void, the realm of the purely existent entities, which we will denote as $\mathfrak{A}_\Psi$:

$$[\mathfrak{X}][t_n^C] \vdash: \mathfrak{A}_\Psi = \mathfrak{X}^C = \{x | x \in \mathfrak{A} \land x \notin \mathfrak{X}\} \tag{49}$$

Thus, the Metaphysical Cosmos can be visualised in the following way:

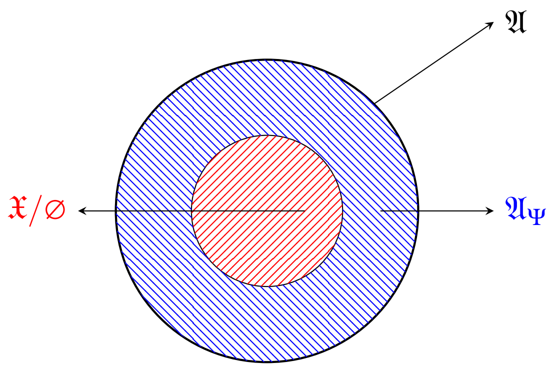

Figure 5: Diagram of the Metaphysical Cosmos

4 Proof That There Must Be Something Rather Than Nothing

4.1 Non-existence Must Imply Existence

I will here thus demonstrate a proof for theorem[15] "$\nexists x \rightarrow \exists x$", that non-existence implies existence, and thus "why there is something rather than nothing". This will be displayed in the formal proof sketch as the sixth theorem in **section 6**. Indeed, this proof differentiates itself from the rest of the derivations made from the system in this paper as it will rely solely on the Type definitions, and not our foundational axiom $\mathfrak{P}$[16].

From the definition of non-existence, we can derive:

$$[\nexists] \vdash: \nexists x \equiv Z^{+}(x) = 0 \tag{50}$$

From the Type 2, and the definition of $Z^{+}(x) = 0$, we can derive:

$$[\downarrow][\mathbb{T}_2] \vdash: Z^{+}(x) = 0 \rightarrow Z(x) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{51}$$

From identity, The matrix of properties Z of this non-entity x, is equal to itself:

$$[\downarrow][=][\mathbb{T}_2] \vdash: \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{52}$$

Thus, x is equal to itself:

$$[\downarrow][=] \vdash: x = x \tag{53}$$

Therefore $P_1.1(x) = 1$:

$$[\downarrow][\mathbb{T}_2] \vdash: P_{1.1}(x) = 1 \tag{54}$$

Which yields that $Z^{+}(x)$ is not equal to zero:

$$[\downarrow][\mathbb{T}_2] \vdash: Z^{+}(x) \neq 0 \tag{55}$$

Therefore, a Z^{+} value of zero implies a Z^{+} of non-zero:

$$[50-55] \vdash: Z^{+}(x) = 0 \rightarrow Z^{+}(x) \neq 0 \tag{56}$$

Therefore, non-existence implies existence, hence why there is something rather than nothing:

$$[\downarrow][\nexists][\exists] \vdash: \nexists x \rightarrow \exists x \tag{57}$$

[15]The proof, I will here demonstrate will be a more abstract form of the Fourth theorem, displayed in section **3.2.2**.

[16]Indeed, as we are here demonstrating the logical impossibility of nothing without something, we must get rid of our assumption of the existence of something, i.e. $\mathfrak{P}$

Thus, when starting with an empty Metaphysical Cosmos, we end with the existence of at least one entity, which is that very same empty and non-existent entity. There is therefore a necessity for "something" within the Metaphysical cosmos.

This theorem is here derived from this specific system, but it could work in classical logic, if the system defined non-existence and existence as we did here, based on properties. Indeed, as the proof for this theorem yields a contradiction, the theorem in classical logic would thus be "$\forall x(\exists x)$". Which would still prove the necessity of something rather than nothing.

This proof here considers "nothing" in its purest form: not as empty space, not as an entity devoid of certain characteristics, but as pure nothingness, devoid of all properties. Moreover, it must be not that this system extends beyond classical logic, and this proof therefore reflects the true nature of nothingness in an unbounded matter and treats the essence of the problem, i.e. the consequences yielded by a truly empty cosmos.

This perplexing question has been the topic of thousands of years of enquiry and debate (Leibniz, 1714, [3]) (Heidegger, 1929,[19]). Recently, some formal approaches have been given to answer this problem (Phillips, 2021, [44]) (Inwagen & Lowe, 1996, [28]) (Heylen, 2016, [35]), but though they are thorough, answers largely focus on dismal of the question, and once again, they remain fragmented and lacking the background of an initial scientific paradigm, which we aim to provide here.

4.2 Experimentation: Computational Metaphysics

Listing 1: Experimental Computation for Sixth Theorem ”$\nexists x \to \exists x$”

```
(* Step 1: Define an entity *)

(* Declare Properties to ensure they are recognized properly *)
ClearAll[P11, P12, P13];

(* Define Properties as 0*)
properties[x_] := {P12[x] -> 0, P13[x] -> 0};

(* Define the sum of properties, Z^+(x) *)
ZPlus[x_] := P11[x] + P12[x] + P13[x];

(* Define Identity Property P11 *)
identityRule[x_] := P11[x] -> If[ZPlus[x] === ZPlus[x], 1, 0];

(* Check for logical consistency of Z^+(x) = 0 *)
CheckContradiction[x_] := Module[{rules, result},
  rules = properties[x] ~Join~ {identityRule[x]};
  result = Simplify[ZPlus[x] == 0 /. rules];
  If[result === False, "Contradiction: ZPlus[x] != 0", "No Contradiction"]
];

(* Run the contradiction check for a specific entity x *)
CheckContradiction[x]


OUTPUT:
Contradiction: ZPlus[x] != 0
```

This here is a simulation of the proof for the Sixth Theorem run on the software *Mathematica*. The proof occurs in a reduced model with only three properties, it defines one property as identity, $P_{1.1}$, and the other two as 0. This is thus to evaluate the statement $Z^{+}(x) = 0$, in a reduced framework that still contains identity as a property. After, evaluating the statement, the software found the contradiction underpinning the Sixth Theorem, namely that when evaluating $Z^{+}(x) = 0$ it yields $Z^{+}(x) \neq 0$.

This proof displays once again the viability of this new paradigm for Metaphysical Cosmology as it not only is epistemological sound and does not rely on unfounded assumptions ($\mathbb{T}_{1-4.3}$), finds ground in empirical proof ($\mathfrak{P}$), is structured through mathematics and formal logic, has a possibility for experimentation (Computational Metaphysics), but it also provides a logico-mathematical answer to, arguably, the biggest problem in the History of Metaphysics (Heidegger, 1929,[19]) (Wittgenstein, 1921,[18]).

5 Methods For Metaphysical Science

In this section, I will here display the methods that can be used in the upcoming Metaphysical Science.

Metaphysical Science can be divided into two sections: Metaphysical Cosmology, as studied in this paper, and Noumenology, in reference to the Kantian ”noumenon” (Kant, 1783,[5]), as the study of worlds beyond perception.

Discipline	Modelling Tools	Experimental Tools	Domain of Study
Metaphysical Cosmology	Type system presented in this paper, enhanced versions of it, or other logico-mathematical systems founded on definitions and undeniable axiom(s)	Experimental Computation, Simulations of Theorems and of Models of the entire Metaphysical Cosmos	$\mathfrak{A}$, The Metaphysical Cosmos
Noumenology	Sub-systems of systems of Metaphysical Cosmology and various other mathematical and logical systems	Experimental Computation, Experimental Psychology and Neuroscience	Ω^C, the set of all things that are not in the Physical Universe.

Table 2: Table of Methods for Metaphysical Science

The Type system presented in this paper thus serves as an initial paradigm in Metaphysical Cosmology, upon which more research can be conducted, such as additional computational experimentation and simulations of the system and its derivations, or even enhancements of the system and the type definitions. The falsifiability of Metaphysical Cosmology thus comes from experimental computation, which can confirm or deny results (Popper, 1959,[23]). Computational tools are already starting to be used in formal Metaphysics (Fitelson & Zalta,2007,[31]) (Kirchner & Benzmüller & Zalta,2019,[39]). Moreover, it still stands on empirical grounds through the $\mathfrak{P}$ axiom. This thus makes Metaphysical Cosmology a hybrid science in between formal sciences, i.e. logic, computation and pure mathematics, and natural sciences, such as physics, chemistry and biology. Indeed, the object of study of Metaphysical Cosmology remains the world, or ”Nature”, as broadly understood, and it possesses some experimental tools and empirical foundations. However, it relies mainly on the formalism and systematic rendition and modelling of its paradigm.

The case of the advent of Noumenology is harder to illustrate as it is not the object of this paper. However, its fundamental aim is to discern realms beyond perception and analyse their structure. Noumenology, however, does have an advantage over Metaphysical Cosmology, as it might benefit from a wider range of experimental tools and empirical evidence. Indeed, experimentation in neuroscience and psychology could serve as empirical foundations for Noumenological modelling. In fact, a realm beyond the physical world, though arguably dependent on it, can be analysed empirically through experimental psychology and neuroscience: dreams. The inner structure of dreams is not made of matter, nor is it directly perceived by the senses, though it is arguably dependent on it. Thus, some studies already conducted in the realm of experimental psychology and neuroscience, could serve as empirical evidence for Noumenonlogical modelling (Konkoly, 2021, [43]) (Nir & Tononi, 2010, [34]).

These are thus the frameworks and methodologies upon which Metaphysical Science would rely, and they could also serve as foundations for other sciences, as Ω is a subset of the Metaphysical Cosmos $\mathfrak{A}$. Indeed, Ω, the Physical Universe, thus represents the domain of study of fundamental Physics, and other natural sciences study subsets of Ω.

6 Proof Sketch

In this section, I will provide proof sketches of the derivations made from our system in this paper. All proofs are made according to the derivation rules expressed in the foundations of the system, and constructed through the definitions. Whenever ”x” occurs in these proofs, ”x” will refer to a variable of any type, not just an individual, it is therefore a shorthand for ”$\mathrm{t}_{n(x)}$”.

6.1 First Theorem : $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash \Psi \cup \varnothing = \mathfrak{A}$

$$
\begin{array}{lll}
 & | \quad \mathbb{T}_{1-4.3} & \text{Def} \\
 & | \quad \mathfrak{P} & \text{Axiom, by empirical justification} \\
\vdots & & \\
1. & | \quad \exists x & \text{Law 9, By } [\mathfrak{P}] \\
2. & | \quad Z^+(x) > 0 & \text{Law 4, By } [\exists] \\
3. & | \quad x \in \Psi \wedge \Psi = \{x|\exists x\} = \{x|Z^+(x) > 0\} & \text{Law 5, By } [\downarrow]\ [\mathbb{T}_3] \\
4. & | \quad \Psi^C = \varnothing = \{x|\nexists x\} = \{x|Z^+(x) = 0\} & \text{Law 5, By } [\downarrow]\ [\mathrm{t}_3^C] \\
5. & | \quad \Psi \cap \varnothing = \mathfrak{X} = \{x|\exists x \wedge \nexists x\} = \{x|Z^+(x) > 0 \wedge Z^+(x) = 0\} & \text{Law 8, By } [\downarrow]\ [3]\ [\cap] \\
6. & | \quad \Psi \cup \varnothing = \mathfrak{A} = \{x|\exists x \vee \nexists x\} = \{x|Z^+(x) > 0 \vee Z^+(x) = 0\} & \text{Law 8, By } [\cup]\ [3]\ [4]
\end{array}
$$

6.2 Second Theorem: $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash \forall x(x \in \mathfrak{A})$

$$
\begin{array}{lll}
 & | \quad \mathbb{T}_{1-4.3} & \text{Def} \\
 & | \quad \mathfrak{P} & \text{Axiom, by empirical justification} \\
\vdots & & \\
1. & | \quad \forall x(Z^+(x) = n \wedge n \in \mathbb{R}) & \text{Law 7, By } [\mathbb{T}_2]\ [x : \mathbb{T}_n] \\
2. & | \quad \forall n(n \in \mathbb{R} \rightarrow n = 0 \vee n \neq 0) & \text{Law 7, By } [\mathbb{R}]\ [\vee] \\
3. & | \quad \forall x(Z^+(x) > 0 \vee Z^+(x) = 0) & \text{Law 5, By } [\downarrow]\ [\mathbb{T}_2] \\
4. & | \quad \forall x(\exists x \vee \neg\nexists x) & \text{Law 5, By } [\downarrow]\ [\exists] \\
5. & | \quad \forall x((\exists x \vee \neg\nexists x)) \equiv x \in \mathfrak{A}) & \text{Law 6, By } [\downarrow]\ [\text{6 in First Theorem}] \\
6. & | \quad \forall x(x \in \mathfrak{A}) & \text{Law 6, By } [4]\ [5]
\end{array}
$$

6.3 Third Theorem: $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash |\Psi| \geq \mathfrak{c}$

| $\mathbb{T}_{1-4.3}$ Def
| $\mathfrak{P}$ Axiom, by empirical justification
⋮

1. | $\forall x(x \in \mathbb{R} \to x \in \Psi)$ Law 5, By [ℝ] and [3 in First Theorem]
2. | $\mathbb{R} \subseteq \Psi$ Law 5, By [⊆] and [↓]
3. | $|\mathbb{R}| \leq |\Psi|$ Law 5, By [$|\mathfrak{t}_3|$] and [↓]
4. | $|\mathbb{R}| = \mathfrak{c}$ Law 9, By [ℝ]
5. | $|\Psi| \geq \mathfrak{c}$ Law 8, By [3-4] [≥]

6.4 Fourth Theorem: $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash \varnothing = \mathfrak{X}$

| $\mathbb{T}_{1-4.3}$ Def
| $\mathfrak{P}$ Axiom, by empirical justification
⋮

1. | $\forall x[(x \in \varnothing) \to (Z^{+}(x) = 0)]$ Law 5, By [∈] and [4 in First Theorem]

2. | $\forall x[(x \in \varnothing) \to Z(x) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}]$ Law 5, By [$\mathbb{T}_2$] and [↓]

3. | $\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$ Law 2, By [=]

4. | $\forall x[(x \in \varnothing) \to (Z(x) = Z(x))]$ Law 5, By [↓] and [$\mathbb{T}_2$]
5. | $\forall x[(x \in \varnothing) \to P_{1.1}(x) = 1]$ Law 5, By [↓] and [$\mathbb{T}_2$]
6. | $\forall x[(x \in \varnothing) \to Z^{+}(x) \geq 1]$ Law 5, By [↓] and [$\mathbb{T}_2$]
7. | $\forall x[(x \in \varnothing) \to Z^{+}(x) = 0 \wedge Z^{+}(x) > 0]$ Law 6, By [1] and [↓]
8. | $\forall x[(x \in \varnothing) \to (x \in \mathfrak{X})]$ Law 6, By [↓] [5 in First Theorem]
9. | $\varnothing \subseteq \mathfrak{X}$ Law 5, By [⊆] and [↓]
10. | $\forall x[(x \in \mathfrak{X}) \to (x \in \varnothing)]$ Law 5, By [⊆] [5 in First Theorem]
11. | $\mathfrak{X} \subseteq \varnothing$ Law 5, By [⊆] and [↓]
12. | $\varnothing = \mathfrak{X}$ Law 8, By [9] [11][=]

6.5 Fifth Theorem: $\mathbb{T}_{1-4.3}, \mathfrak{P} \vdash \mathfrak{A} = \Psi$

$\mid \; \mathbb{T}_{1-4.3}$	Def
$\mid \; \mathfrak{P}$	Axiom, by empirical justification
$\vdots$	
1. $\mid \; \mathfrak{A} = \varnothing \cup \Psi$	Law 9, By [6 in First Theorem]
2. $\mid \; \mathfrak{A} = \mathfrak{X} \cup \Psi$	Law 6, By [12 in Fourth Theorem] and [$\downarrow$]
3. $\mid \; \mathfrak{X} \cup \Psi = \Psi$	Law 8, By [$\subseteq$] [$\cup$] [3 and 5 in First Theorem]
4. $\mid \; \mathfrak{A} = \Psi$	Law 6, By [2 and 3]

6.6 Sixth Theorem: $\mathbb{T}_{1-4.3} \vdash \nexists x \to \exists x$

$\mid \; \mathbb{T}_{1-4.3}$	Def
$\vdots$	
1. $\mid \; \nexists x \equiv Z^{+}(x) = 0$	Law 9, [$\nexists$]
2. $\mid \; Z^{+}(x) = 0 \to Z(x) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$	Law 5, By [$\downarrow$] and [$\mathbb{T}_2$]
2. $\mid \; \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$	Law 2, By [$=$]
3. $\mid \; x = x$	Law 5, By [$\downarrow$] and [$=$]
4. $\mid \; P_{1.1}(x) = 1$	Law 5, By [$\downarrow$] and [$\mathbb{T}_2$]
5. $\mid \; Z^{+}(x) \neq 0$	Law 5, By [$\downarrow$] and [$\mathbb{T}_2$]
6. $\mid \; Z^{+}(x) = 0 \to Z^{+}(x) \neq 0$	Law 6, By [2-4]
7. $\mid \; \nexists x \to \exists x$	Law 8, By [$\downarrow$] [$\exists$] [$\nexists$]

7 Conclusion

In conclusion, we have here demonstrated, through meticulous logical systematisation the inner skeleton of the Metaphysical Cosmos. From a type system built on semantic definitions and holding for sole assumption the existence of an entity, we were able to analyse the fundamental structure of the Metaphysical Cosmos. This research aims to play a pivotal role in advancing Metaphysics as a science by laying the foundations for an initial scientific paradigm in the field. I strongly encourage fellow researchers to build upon the foundation laid here, propelling us further in our collective quest for understanding the intricacies of the Metaphysical Cosmos. Just as physics studies instances within the physical universe, after having established a new paradigm for research in this paper, Metaphysics can now have the task of exploring instances within The Absolute.

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