BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

LUC BOVENS

ON ARGUMENTS FROM SELF-INTEREST FOR THE NASH SOLUTION AND THE KALAI EGALITARIAN SOLUTION TO THE BARGAINING PROBLEM *

ABSTRACT. I argue in this paper that there are two considerations which govern the dynamics of a two-person bargaining game, viz. relative proportionate utility loss from conceding to one's opponent's proposal and relative non-proportionate utility loss from not conceding to one's opponent's proposal, if she were not to concede as well. The first consideration can adequately be captured by the information contained in vNM utilities. The second requires measures of utility which allow for an interpersonal comparison of utility differences. These considerations respectively provide for a justification of the Nash solution and the Kalai egalitarian solution. However, none of these solutions taken by themselves can provide for a full story of bargaining, since, if within a context of bargaining one such consideration is overriding, the solution which does not match this consideration will yield unreasonable results. I systematically present arguments to the effect that each justification from self-interest for respectively the Nash and the Kalai egalitarian solution is vulnerable to this kind of objection. I suggest that the search for an integrative model may be a promising line of research.

INTRODUCTION

It is deceptive to talk of *the* bargaining problem. Bargaining theory encompasses a family of models, which each come along with a set of conditions that specify the scope of the model. For our purposes it is sufficient to keep in mind a very simple example. Two rational persons are asked to divide a set of goods amongst them. If they cannot reach agreement, both of them will go home with empty hands. We have some information about the relevant preferences of each player and the question we are asked to solve is what constitutes a rational agreement between them. The Nash solution (Nash, 1950) is one of the earliest attempts to solve this problem and it still is the most prominent one. A more recent attempt is the Kalai egalitarian solution (the KE solution) (Kalai, 1977, 1983, Kalai *et al.*, 1985). In this paper I intend to discuss a particular set of arguments for each of these solutions.

Bargaining solutions come along with a jungle of justifications. I believe it is clarifying to keep in mind the following three dichotomies:

Theory and Decision 23 (1987) 231–260.
© 1987 *by D. Reidel Publishing Company.*

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem*** , Theory and Decision, 23:3 (1987:Nov.) p.231

(a) A bargaining solution can be justified by claiming that it is the sole solution which satisfies a set of minimal axioms concerning how bargainers should act. Alternatively it can be argued that a bargaining solution *directly* captures our intuitions of how bargainers should act, i.e. without reference to a set of minimal axioms.

(b) A bargaining solution can be defended in reference to our intuitions concerning a *fair* bargain. Alternatively, it has been argued that a solution matches our intuitions concerning the actions of rational and *self-interested* bargainers, i.e. bargainers who are solely interested in maximizing their own utility payoffs (Harsanyi, p. 14).[1]

(c) A bargaining solution is a static model, i.e. it solely presents us with the *outcome* of a bargaining procedure. An actual bargaining process, however, most often has a *dynamic* character: a solution is an outcome of the subtle interplay between the two parties. For the Nash solution, there exists a model, viz. the Zeuthen model, which attempts to capture this interplay that is involved in bargaining, and which yields an outcome that matches the Nash solution. I believe it is possible to construct a similar model for the KE solution. Justification for these solutions can thus be advanced on their own standards, as well as in reference to their respective dynamic models, which yield identical results.

In this paper I will solely be concerned with the type of justification for the solutions in question that involves purely *self-interested* bargainers. Why this choice? In the first place, the task I have set before me is still sufficiently complex to discuss within the limits of this paper. Secondly, I believe that arguments from self-interest are much more promising than arguments from fairness[2] for the Nash and the KE solution. I intend to show in a later paper that arguments from fairness either supervene on arguments from self-interest or are obviously flawed. On the other hand, arguments from self-interest for the Nash and the KE solution can be advanced which respectively do capture the two relevant considerations in the dynamics of rational and self-interested bargaining, as I will argue in this paper. Taken by themselves however, these solutions cannot provide for a comprehensive account of bargaining.

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

1. THE NASH SOLUTION

1.1. Elementary Mechanics and Types of Arguments

The Nash solution to the two-person bargaining game requires that we express both players' attitudes towards the goods in question in von Neumann–Morgenstern utilities (vNM utilities). Subsequently we are asked to calculate which feasible outcome to the bargaining game maximizes the product of utilities. Nash predicts this outcome of the bargaining game to be the solution which rational and self-interested players will come to agree upon (for an example of this procedure, see Section 1.2).

It is possible to distinguish between the following types of arguments from self-interest for the Nash solution:

(i) The Nash solution is correct, because it *directly* matches our intuitions concerning the actions of rational and self-interested bargainers.

(ii) The Nash solution is correct, because it is the unique outcome of the *Zeuthen model* of bargaining; the latter model captures our intuitions of the bargaining procedure which rational and self-interested bargainers adopt.

(iii) The Nash solution is correct, because it is the sole solution that satisfies *a set of axioms* which capture our intuitions concerning the actions of rational and self-interested bargainers.

In this section I will systematically present and critically assess these arguments.

1.2. The Direct Justification for the Nash Solution

I will at first present the direct argument for the Nash solution. Let us assume that players 1 and 2 need to divide $ 100 between them. If they cannot reach agreement, neither 1, nor 2 will get any money. Let us furthermore assume that for both players utility is linear with money. In this case, the argument goes, they have both equal bargaining strength, and thus will refuse to settle for less than a (1: $ 50; 2: $ 50) split, i.e. the outcome which maximizes the product of utilities. But let us now change the story slightly. Assume that 1 is a rich person and 2 is a poor person. 1's utilities are still linear with money, but 2 is much more risk-averse than

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

1 and this caution is reflected in 2's vNM utilities. Consider the following assignment of utilities (cf. Luce and Raiffa, p. 129):

Monetary Value	*Utility* 1	2
$ 0	0.00	0.00
$ 25	0.25	0.73
$ 50	0.50	0.90
$ 75	0.75	0.98
$ 100	1.00	1.00

It is easy to see that the unequal division (1: $ 75; 2: $ 25) maximizes the product of utilities and thus is the Nash solution to this game.

Here is how the argument goes. We assume that 1 and 2 both have knowledge of each other's preferences. 1 thus knows that 2 is in need of money and this puts her in a stronger bargaining position. She can thus insist on a split that is favorable to herself, since she can reasonably expect that 2 will yield of need. 2 would of course prefer an equal split, but he realizes that, given 1's attitudes towards money, she would rather forego any division than settle on an equal split. And since 2 is hard pressed for money, it is a rational move from his side to accept a split which is not favorable to himself. 2 is in a poor bargaining position.

One might object that this justification only goes part way. The argument from bargaining strength justifies *some* unequal split in favor of 1, but it does not justify why *the* unequal split which maximizes the product of utilities is a correct assessment of this difference in bargaining strength. Furthermore, there exist some alternative solutions which also predict an unequal split in favor of 1, and thus, among these alternative solutions, the argument from bargaining strength cannot grant favorable status to the Nash solution (e.g. Kalai *et al.*, 1975). The obvious response is that the argument from bargaining strength can be refined in reference to the Zeuthen model. I will return to this argument later on in my paper.

I now intend to provide for a more radical objection to the argument from bargaining strength for the Nash solution. Consider the following story. Person 1 and 2 visit an antique shop. The antiquaire, who happens to be in a generous mood, offers 10 glasses to both of them, provided they can reach an agreement about how to divide up this offer. She

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem*, Theory and Decision, 23:3 (1987:Nov.) p.231

furthermore stipulates that no side payments are allowed, i.e. the agreement should only involve a particular division of the glasses and no form of compensation (say, in monetary terms) between the parties is allowed. Among these 10 glasses, there are 8 glasses, which for some reason form a set that an art collector would take an interest in (say they are the complete set of designs of the first year of production of some famous company). The other 2 glasses are simply unrelated pieces.

Let us now take a close look at both 1 and 2's preferences. 1 is an enthusiastic art-collector, who has a strong interest in owning complete sets. She would thus simply be thrilled if she could own the set of 8 glasses. But, she also knows that each of these glasses is valuable by itself and would be quite happy to own an incomplete set. Furthermore, if she could own the complete set, she would have some interest as well in owning the unrelated pieces in addition. 2 does not care about collecting art. What comes to his mind is that it would be rather nice to own two of the glasses, just for daily usage in his (two-person) household. He is not indifferent about getting more glasses, but this is not such a big deal for him. Remember also that both persons are solely interested in maximizing their own utilities and are not motivated by any conception of fairness.

Now, let us establish the vNM utilities for both 1 and 2. I believe that the following assignment matches my story fairly accurately:

Glasses	*Utility*	
	1	2
0	0	0
1	0.05	0.05
2	0.10	0.80
3	0.15	0.825
4	0.20	0.850
5	0.25	0.875
6	0.30	0.900
7	0.35	0.925
8	0.80	0.950
9	0.90	0.975
10	1	1

It is easy to calculate that the Nash solution for this game predicts a (1: 8G; 2: 2G) split.[3] But, is this an accurate reflection of the bargaining-strength of both players? It is assumed that both players have complete

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

knowledge of each other's preferences. It seems to me that, given that 2 does not care too much whether he gets *these* glasses or, say, an equal amount from some cheap contemporary glass collection, and given that 1 would already be quite happy to own one of these glasses, 2 could win his case by holding out for a (1: 1G; 2: 9G) split.[4] 2 is in a strong bargaining position. He knows that 1, though she would be thrilled to own a complete set, already craves to own one single glass, while he himself is sufficiently indifferent to take his chances and hold out for a split that is favorable to himself.

It might be objected that, within normal circumstances, there is a good chance that 1 and 2 will settle on a (1: 8G; 2: 2G) split. I agree. But this is so only because people in general do take some interest in each other's interests, or have some moral sense. Keep in mind, however, our stipulation that both 1 and 2 are purely self-interested players. And on this stipulation 2 has no scruples to hold out for a division that is slightly more favorable to himself, even if this is extremely costly to 1, provided that he sees a good chance of winning his case.

So, what has gone wrong? Well, let us look carefully in what aspects my own story differs from Luce and Raiffa's game of dividing $ 100. What is striking about my own example is that, if we were to express 1 and 2's preferences in terms of interpersonally comparable utilities (IC utilities) 1's utility of 1 glass would be higher than 2's utility of 10 glasses and 1's utilities would also cover a much broader range (since every additional glass drastically increases her want-satisfaction). Thus, on my own example, the person who gets favorable treatment on the Nash solution, has a range of IC utilities which is genuinely much broader than her opponent's range. Luce and Raiffa provide for an example in which utility is linear with money for both 1 and 2 and an example in which 2 has diminishing marginal utility, due to his risk aversion as a poor person for betting smaller sums of money. 1, on the other hand, is well off, and her utility is linear with money.

This first example is uninteresting for our purposes, because IC utility functions for players 1 and 2 can take any form, due to the lack of background information. In the second case, however, there is good reason to assume that the first $ 25 for 2 yield a higher utility to him than $ 100 to 1 and that *each* increase of $ 25 yields a higher utility increase to 2 than to 1. So, in this particular case, the person who gets favorable

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

treatment on the Nash solution has a range of IC utilities which is a lot more narrow than her opponent's range. Now, I find it extremely suspicious that arguments from bargaining strength for the Nash solution are always supported by stories of the latter type (see also Hamburger's story, 1979, pp. 136–137).

What I want to argue is something that I believe comes natural to somebody who has never heard of vNM utilities. And that is that in bargaining over a division of goods it is a good strategy not to show too much enthusiasm for the goods in question. My bargaining strength is determined by how much I really care for the goods in question, relative to how much my opponent cares. Any argument from bargaining strength thus requires an interpersonal comparison of utilities. And since this information is not contained in the vNM utilities of the Nash solution, this solution cannot be justified by means of an argument from bargaining strength.

But how come that one has been tempted to think that such argument for the Nash solution *did* hold so far? There is peculiar connection between interpersonally comparable utilities and vNM utilities. If a set of goods yields high want-satisfaction, it is quite often the case that its marginal utility will be strongly diminishing and it is the latter information which is captured by vNM utilities. If I am strongly in need of money, I will prefer to go from $ 0 to $ 1000 twice as much as from $ 1000 to $ 2000, to go from $ 1000 to $ 2000 twice as much as from $ 2000 to $ 3000 etc. When I do not have this strong need for money, I will just as much prefer each separate increase. In cases of strong need, the first unit of the good in question may provide for an increase in expected want-satisfaction that is so overwhelming that all additional units are forgotten (cf. 'If I could just have *one* cigarette'). Now as long as we choose examples in which this generalization holds, the argument from bargaining strength will *per accidens* also hold for vNM utilities. Thus, a pattern of strongly diminishing marginal utility is indeed an indicator of a weak bargaining position, but only if this pattern is itself a consequence of the fact that the good in question yields high want-satisfaction. *But the connection between high want-satisfaction and strongly diminishing marginal utility is neither a conceptual, nor an empirical truth.* We can imagine special circumstances in which a particular good yields high want-satisfaction to a person, but for some reason this person prefers a

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

shift from the second to the third unit twice as much as a shift from the first to the second etc. In other words, it is possible to think of examples in which low want-satisfaction goes hand in hand with strongly diminishing marginal utility and high want-satisfaction goes hand in hand with strongly increasing marginal utility. And when such examples are brought to the fore it becomes clear that bargaining strength is actually a function of the relative level of want-satisfaction which both persons derive from the good in question and *not* a function of diminishing versus increasing marginal utility, as is assumed in the Nash solution. And this is precisely the morale of the story of the generous antiquaire.

1.3. The Justification for the Zeuthen Model

There exists a more ambitious variant of the argument from self-interest, which attempts to provide for a justification of the Zeuthen model. Any argument in favor of the Zeuthen model is also an argument for the Nash solution, since the Zeuthen model precisely has the Nash solution as its outcome. The argument for the Zeuthen model is more ambitious, because it not only attempts to explain why rational and self-interested bargainers settle for an unequal split in favor of one person, but also why there is good reason for them to settle on *the* unequal split which maximizes the product of utilities (i.e. the Nash solution). Let us now turn to this argument.

Assume that at some moment in the process of negotiation two proposals for a solution are on the table. Person 1 does not accept person 2's proposal and vice versa. 1's proposal yields utility payoffs (u_1', u_2') and 2's proposal yields utility payoffs (u_1'', u_2''). Now Zeuthen's idea is that person 1 will make a concession if and only if

$$\frac{u_1' - u_1''}{u_1'} \leqslant \frac{u_2'' - u_2'}{u_2''}$$

and person 2 will make one if and only if this inequality is reversed. Now it can be proven [5] that for all $u_i' \geqslant 0$ and $u_i'' \geqslant 0$ this inequality is equivalent to $u_1' u_2' \leqslant u_1'' u_2''$ (Luce and Raiffa, p. 135).

The Nash solution yields of all feasible solutions the highest product of utilities. Since every proposal that is countered to it will yield a lower

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem*** , Theory and Decision, 23:3 (1987:Nov.) p.231

product, and given my proof of the equivalence of inequalities, the Zeuthen model will demand of the proponent of this counter to make the next concession. Thus, the game of alternating concessions will go on until somebody hits on the Nash solution. At that time, whatever move the opponent makes, the Zeuthen model will point to her to make the next concession, until she simply agrees to the Nash solution.

Let us now turn to the justification for the Zeuthen model. Consider some point in time during the bargaining procedure at which both players 1 and 2 have made a proposal for a solution. The Zeuthen model measures the proportionate utility loss of 1 if she were to concede to 2's proposal and the proportionate utility loss of 2 if he were to concede to 1's proposal. The argument is that smaller proportionate utility loss provides for the motivation to keep negotiations going and thus rationally commits one to make the next move in the game of mutual concessions.

Why is proportionate utility loss considered to be a correct indicator for a person's motivation not to break off negotiations? If a person feels that she has got little to lose even if she would simply accept her opponent's proposal, it is rational for her to make the next concession, rather than taking the risk that negotiations will break off because of the obstinacy of her opponent. The closer she gets to her goal , the less she is willing to take chances. But why *proportionate* loss of utility? Let us assume that for person 1 and 2 utilities are linear with money. They are involved in a negotiation for a home and for a car. Now it seems reasonable that conflicting offers of \$ 100 000 vs. \$ 110 000 for a house provide for the same motivational force to keep negotiations going to each respective player as conflicting offers of \$ 10 000 and \$ 11 000 for a car. Thus, at a high level of aspiration, the motivation for rational concession remains constant with higher loss of utility. And this is why the Zeuthen model measures *proportionate* loss of utility.

Let us now zoom in on the first round of proposals in the antique shop. 2 unsuspectingly proposes an equal split, but 1, obsessed by the idea of owning a complete set, claims 8 glasses for herself. Furthermore, she is a firm believer in the Zeuthen bargaining model, and knowing that she can count on 2's gametheoretical rationality, she already has some perverse pleasure in contemplating the prospect that 2 will be forced to make a row of concessions until he will come to agree to the Nash solution, i.e. her own initial proposal.

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

But if 2 knows 1's preferences, knows that 1 knows his (i.e. 2's) preferences, and acts purely from self-interest, will it be rational for him to make any concessions whatsoever? I do not think so. 2 knows that 1 knows that he will simply shrug his shoulders if they cannot come to an agreement. 2 also knows that, though 1 may, due to her preference for complete sets, act quite risk-taking when it comes to getting less than either 8 glasses for sure or a lottery with a chance to win 10 glasses or nothing, she does care a great deal to get at least some glasses. Now it seems to me that under these circumstances 2 can feel fairly comfortable to hold out and wait for 1 to come down to his own proposal.[6] And 1 faces a choice between losing her faith in the Zeuthen model or losing the chance to get an incomplete set of glasses.

What has gone wrong? Well, let us think carefully about what precisely determines a person's motivation to concede. Consider the following proposal:

A person is rationally motivated to make a partial concession in a two person bargaining game if and only if

(a) her proportionate utility loss from conceding to her opponent's proposal is relatively – i.e. in comparison to the other person's proportionate utility loss from conceding – *small* and

(b) her non-proportionate utility loss from not conceding to her opponent's proposal if her opponent were not to make any concession, is relatively – i.e. in comparison to the other person's non-proportionate utility loss from not conceding if she herself were not to make any concessions – *large*

or one of these conditions is fulfilled and overrides the other condition.

This proposal needs some clarification. *In the first place*, I make a distinction between 'partially conceding' and 'conceding to one's opponent's proposal'. The idea is that if I decide to make a concession, I need not simply accept my opponent's proposal. However, in assessing whether it is worth for me to make this partial concession, the questions 'What if I were to concede to my opponent's proposal?' and 'What if I were not to concede to my opponent's proposal?' play a central role.

Let us consider the first question. The motivation to make a partial concession is determined by how close (in proportional terms) I am to my goal. If my opponent can offer me a utility level which is sufficiently close to my own proposal, I have good reason for not breaking off negotiations. But this does not imply that I will simply accept my opponent's offer.

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

Let us now turn to the second question. The second question is relevant because it gives me an idea of what is at stake by my obstinacy. If my opponent is not willing to make any concessions, the utility to me of her proposal is what I have foregone by holding out. I will be more motivated to make at least partial concessions if this loss of utility is relatively large.

Secondly, why is it that I take proportionate utility loss to be relevant in condition (a), as opposed to non-proportionate utility loss in condition (b)? In condition (a), what is at stake is a loss of utilities from one proposal for division to another. Now I think it is reasonable that an identical change of utilities at a low and at a high level of want-satisfaction has more motivational force in the former than in the latter case (cf. my example of bargaining about the price of a house vs. about the price of a car). In condition (b) the question is how much utility is foregone by not accepting one's opponent's proposal. I propose to consider the bargaining problem as an isolated one, such that both players can be assigned a utility function with baseline at zero.[7] Now the question is how much utility would be foregone if one were to fall from one's opponent's proposal to this baseline. I take it to be meaningful to say that 2 would lose less utility from rejecting 1's proposal than 1 would lose from rejecting 2's proposal, if these rejections were to result in no agreement. But in order to make such claims, we need to compare non-proportionate utility losses, since the proportionate utility loss from not conceding if one's opponent were not to concede as well, would be identical for each player.[8]

Thirdly, so far, I have loosely been making interpersonal comparisons of utility in both condition (a) and (b). Condition (a), however, seems to match the justification I advanced for the Zeuthen model quite closely, though the latter model solely involves reference to vNM utilities. It is easy to see though, that in measuring *proportionate* loss of utility, vNM utilities yield results which are identical to IC utilities.[9]

The same cannot be said, however, for condition (b). In condition (b) interpersonal comparisons of utility are being made, and we are in need of IC utilities to make the relevant measurements. So once again, the episode in the antique shop has pointed out that a comprehensive model of bargaining requires an interpersonal comparison of utility that is beyond the information that is contained in vNM utilities.

Finally, I need to say something about the disjunction in the right half

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

of the biconditional. I claim that a person's motivation to make partial concessions is a function of both conditions (a) and (b). Now if these conditions are either both fulfilled or both not fulfilled for one player, it is clear who is to make the next concession. But then of course there is the case in which one condition is fulfilled for each player. In this case we are left to assess the relative strength of these conditions for both players in order to decide who is rationally committed to making the next concession.

I will anticipate now on some ideas which I will discuss further in Section 2 of this paper. I believe it is an argument in favor of my analysis, that both considerations find their match in actual gametheoretical models. The first condition provides for a good rationale for a solution which equalizes proportionate utility loss, i.e. the Zeuthen model and the Nash solution. In the same way the second condition provides for a good rationale for a solution which equalizes IC utility gain above the no agreement point. It is worth noticing that this is precisely the egalitarian solution which Kalai is defending in his latest work. (Kalai, 1977, 1983; Kalai *et al.*, 1985) I have argued in the last section that the argument from bargaining strength as a direct justification of the Nash solution is flawed because it rests on a spurious connection between the level of want-satisfaction and the marginal utility rate. This argument from bargaining strength, however, does match condition (b): a person who has a strong preference for the goods in question, will be tempted to make quick concessions, and thus is in a poor bargaining position. It is fascinating that the classical argument from bargaining strength thus proves to be flawed for the Nash solution, but actually does provide for a justification for a more recent model, viz. the Kalai egalitarian solution. I will present a more careful argument to this effect in Section 2.

For now, however, let us return to my initial question, viz. why is it that the Zeuthen model did not yield intuitively plausible results in my own story? The answer should be clear now. 2's motivation to concede is not solely a (negative) function of his relative proportionate utility loss from conceding, but also a (positive) function of his relative non-proportionate utility loss from not conceding (if 1 were not to concede as well). The Zeuthen model solely captures this first condition. But this is not sufficient. I have precisely chosen my example such that (i) the difference in non-proportionate utility loss from not conceding for both

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

players is considerable and (ii) this difference is set in such a direction that it counteracts the effects of condition (a), as captured by the Zeuthen model. And this is the reason why in this particular case we intuitively assign the bargaining advantage to person 2: it may well be the case that 2 has proportionately less to lose from conceding than 1 – and this is certainly relevant information – but what is decisive in this particular case is that 2 has so much less to lose from not conceding, if 1 were not to concede, than 1 has to lose from not conceding, if 2 were not to concede, that he can comfortably hold out for this equal split proposal. I think 1 would do better to set her faith in the Zeuthen model aside if she does not want to forego her chances to enrich her collection of antiques, though with an incomplete set.

1.4. An Axiomatic Justification of the Nash solution

I will now turn to Nash's axiomatic justification of his bargaining solution. Nash proves that solely his own solution can satisfy the axioms (a) of invariance with respect to linear utility transformations, (b) of Pareto optimality, (c) of independence of irrelevant alternatives (IIA) and (d) of symmetry. Most objections to Nash's axiomatic approach focus on the axiom of IIA, and some alternative models have been advanced in which all but this axiom are preserved. Though I am sure that there is room for interesting debate in this area, I would like to direct my attention to a particular issue which fits in with the general argument of this paper. Nash believes that vNM utilities contain sufficient information to decide on a bargaining solution. If a particular vNM utility function correctly assesses a person's preferences, then any linear transformation of this function does so as well and thus, the Nash solution must remain constant under any such transformation (axiom (a)). The same commitment to vNM utilities is also expressed in the axiom of symmetry. The axiom of symmetry asserts that, if two persons have the same vNM utilities, then their position is interchangeable, and thus the Nash solution must assign equal utility to each. In other words, it follows from the axiom of symmetry that, given both players are rational and self-interested, nothing concerning the *identities* of both players but their preferences as expressed in vNM utilities matters in reaching a bargaining solution. I have argued that there is some information concerning both players which

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

goes beyond vNM utility functions, though actually *does* matter, viz. the information from interpersonal comparisons between both persons' preferences. If this claim is correct, then it must be possible to set up a case in which both persons have identical vNM utilities, while rational bargaining from self-interest will not yield a solution with equal utilities. In other words, it must be possible to refute Nash's symmetry axiom.

Well, let us return to the antique shop. Both 1 and 2 have the same interests as before. In the set of 10 glasses, however, there are in this case solely 5 glasses (instead of 8 glasses) which constitute a complete set. Furthermore, person 2's household contains 5 persons (instead of 2 persons). Now I think it is not unreasonable under these circumstances to assign the same vNM utility function to both 1 and 2:

Glasses	*Utility* 1	2
0	0	0
1	0.05	0.05
2	0.10	0.10
3	0.15	0.15
4	0.20	0.20
5	0.80	0.80
6	0.84	0.84
7	0.88	0.88
8	0.92	0.92
9	0.96	0.96
10	1	1

Since 1 and 2 are assigned the same vNM utility function, the bargaining set is symmetric, and thus, on the symmetry axiom, both parties should get equal utility. And indeed, the Nash solution prescribes an equal split of the glasses, which provides for equal utility to each party. But, does this outcome match our intuitions concerning rational and self-interested bargaining? I do not think so. If the whole bargain is not such a big deal for 2 as it is for 1 and if both 1 and 2 know so, then 2 can exploit this difference in attitude by holding out for a better deal than equal split and 1 better go along with it.[10]

This argument brings us to the same conclusion as before. In bargaining there is something more that is relevant than the information contained in vNM utilities, viz. the interpersonal comparison of utilities between

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

both players. And this is why the axiom of symmetry cannot hold for vNM utilities. The axiom is readily refuted by any example in which 1 and 2's preferences can be characterized by the same vNM utility function, though both players' preferences do not have the same intensity. In bargaining, *being 1* or *being 2* seems to matter a great deal indeed, if one person happens to be less thrilled in general by the good in question than the other.

2. THE KALAI EGALITARIAN SOLUTION

2.1. Elementary Mechanics and Types of Arguments

In Section 1.3 I have suggested that there exists a gametheoretical solution which adequately captures consideration (b) in the game of alternating concessions, viz. the Kalai egalitarian solution. Let us now turn to the elementary mechanics of this model.

Kalai argues that the solution to the two-person bargaining game is the Pareto optimal point which equalizes weighted utility gain for both players. Formally, for every bargaining pair (d,S), $f(d,S)$ is Pareto optimal and there exists a pair of weights (λ_1, λ_2) such that

$$\lambda_1(f_1(d,S)-d_1)=\lambda_2(f_2(d,S)-d_2)$$

The weights λ_i are set such that utility gains for player 1 and for player 2 are rendered interpersonally comparable. It is easy to see that this procedure is equivalent to a procedure which first normalizes the utility function of each player such that utility differences are interpersonally comparable and then equalizes utility gain.[11]

How can we obtain the weights which are supposed to render the vNM utility differences interpersonally comparable? Kalai first imposes an additional requirement on vNM utility functions viz. that they are comparable across games for each player. Formally, for each (d,S) and (d,W) in the set of all bargaining games, if $u \in S$ and $w \in W$, then for each player i, $u_i > w_i$ if and only if i prefers her payoff in u over her payoff in w. Now, since the utility function of a person i remains constant across games, it is clear that the weight λ_i which normalizes this function will remain constant across games as well. In other words, λ_i will hold good for i in each bargaining game that i is involved in.

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

Kalai has a few proposals for determining the weights λ_i for each player i. I will present a slightly modified version of one such proposal.[12] Engage player 1 and 2 in pairs of lotteries $L^i = (L^i{}_1, L^i{}_2)$. The set of lotteries $L^i{}_1$ is of the form $(p\$x^i,(1-p)\$0)$ and the set of lotteries $L^i{}_2$ is of the form $(p\$0,(1-p)\$y^i)$. Now for each pair of lotteries L^i let 1 and 2 bargain over the value of p. Now consider some pair of lotteries $(L^t{}_1, L^t{}_2)$ for which the outcome of the bargain is $p = 0.50$. Assuming the Kalai egalitarian solution is correct, on any utility function u which allows for interpersonal comparison of utility differences, $u_1(L^t{}_1) - u_1(\$0) = u_2(L^t{}_2) - u_2(\$0)$. And thus given the vNM utilities $v_1(L^t{}_1)$, $v_2(L^t{}_2)$ and stipulating that for both players i, $v_i(\$0) = 0$, λ_1 and λ_2 can be set at respectively

$$\frac{1}{v_1(L^t{}_1)} \quad \text{and} \quad \frac{1}{v_2(L^t{}_2)}$$

The weight λ_i thus normalizes v_i (which is constant across games)[13] in any two-person bargaining game (d, S) such that utility differences in (d, S) are rendered interpersonally comparable between both players.

What justification can be advanced for the KE solution? Once again I will restrict myself to arguments from self-interest. I can see the following possibilities:

(a) The Kalai egalitarian solution is a correct bargaining solution, because it *directly* captures our intuitions of how rational and self-interested bargainers act.

(b) the Kalai egalitarian solution is a correct bargaining solution, because it is the sole solution that (in the presence of a set of weak axioms) satisfies *the axiom of monotonicity*, which accurately captures our intuitions of rational bargaining.

(c) The Kalai egalitarian solution is a correct bargaining solution, because it is the sole solution that (in the presence of a set of weak axioms) satisfies *the axiom of path-independence*, which accurately captures our intuitions of rational bargaining.

I will now systematically present and critically assess these arguments.

2.2. The Direct Justification for the Kalai Egalitarian Solution

In Section 1.3 I proposed that a rational and self-interested person is motivated to make a concession in a bargaining game if and only if (a)

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

her relative proportionate utility loss from conceding is small and (b) her relative non-proportionate utility loss from not conceding (if the other person were not to concede as well) is large, or one of these conditions is fulfilled and overrides the other. Consideration (a) provides for a good reason for the Nash solution, since this solution equalizes the proportionate utility loss for both players. In exactly the same way it can be argued that consideration (b) provides for a good reason for the Kalai egalitarian solution, because it equalizes the non-proportionate utility loss for both players, if the other person were not to concede to that solution. In the same way as the Zeuthen model provides for a dynamic model of bargaining which has the Nash solution as its unique outcome. I believe it is possible to formulate a dynamic model of bargaining for the Kalai egalitarian solution, viz. player 1 is rationally motivated to make a partial concession if and only if

$$\lambda_1(u_1'' - d_1) \geqslant \lambda_2(u_2' - d_2)$$

in which u_1'' stands for player 1's payoff on player 2's proposal and u_2' stands for player 2's payoff on player 1's proposal. Thus, this analogue of the Zeuthen model for the Kalai egalitarian solution states that player 1 is rationally motivated to concede if and only if the non-proportionate utility loss from not conceding which 1 foregoes if 2 were not to concede as well is larger than or equal to 2's non-proportionate utility loss from not conceding if 1 were not to concede.

I have argued that since the Nash solution and the Zeuthen model solely capture consideration (a), they do not provide for a comprehensive model of bargaining. My strategy has been to present a case in which a person's relative proportionate loss from conceding was small, but her relative non-proportionate loss from not conceding was so small that the latter consideration clearly overruled the former, and thus, this person had good reasons *not* to make concessions. Since the Zeuthen model and the Nash solution solely capture the former condition, it is clear that they will yield counterintuitive results in a case of this sort. Since the KE dynamic model and the KE solution solely capture consideration (b), I will argue that they cannot provide for a comprehensive model of bargaining since they entirely overlook a person's relative proportionate utility loss from conceding. My strategy will be a mirror-image of my strategy in assessing the Nash solution. I will present a case in which a person's relative

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

non-proportionate utility loss from not conceding (if her opponent were not to concede as well) is small, but her relative proportionate utility loss from conceding is so small that the latter consideration clearly overrules the former, and thus, in this particular case the KE dynamic model and the KE solution will yield counterintuitive results.

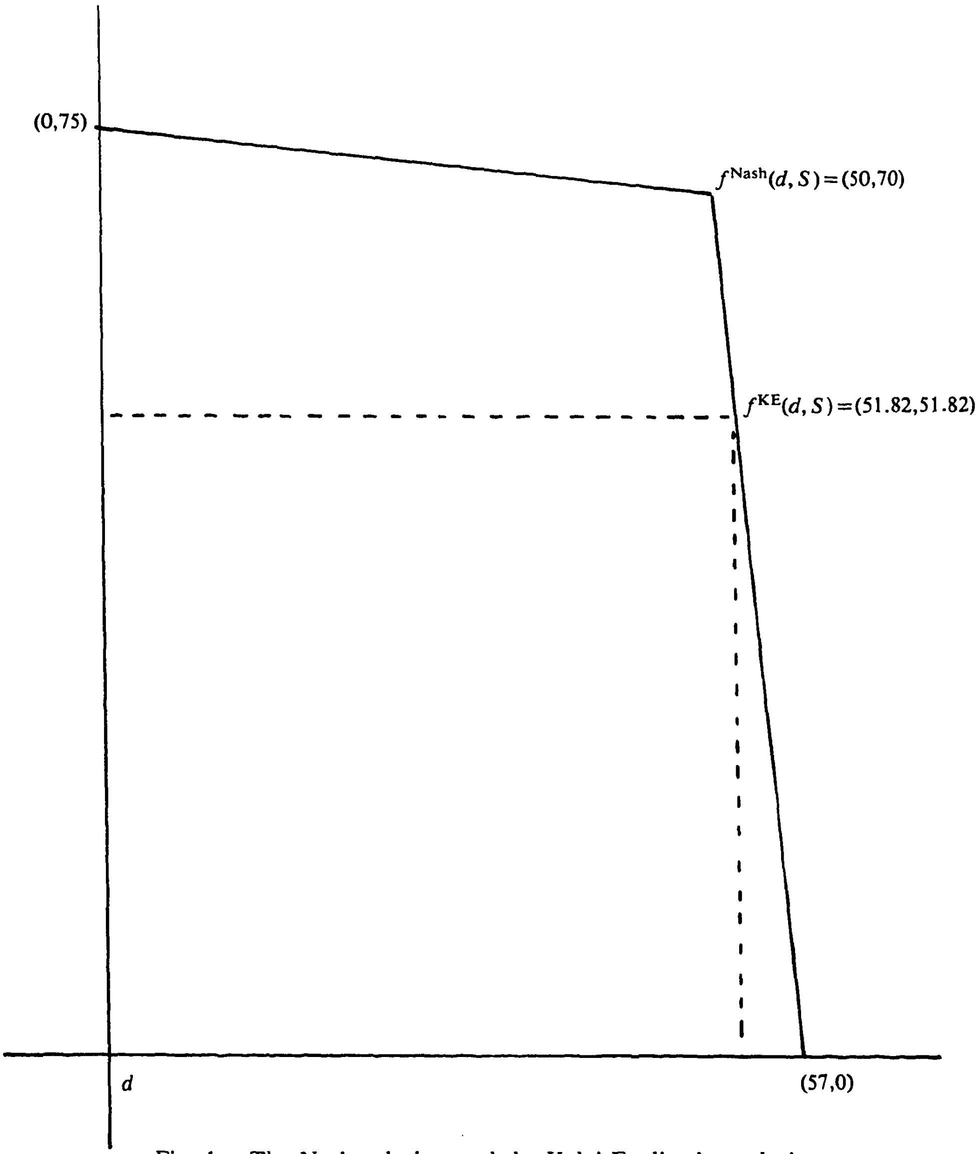

Fig. 1. The Nash solution and the Kalai Egalitarian solution.

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

Let us return to the antique shop with the generous antiquaire. The antiquaire's offer of the week is a set of 50 glasses and 70 cups. 1 and 2 are enthusiastic antique collectors, but 1 really gets a thrill from glasses, 2 from cups. 1 is not entirely indifferent about cups, but they do actually interest her very little. And the same can be said for 2's attitude towards glasses. I propose to assign the following IC utilities to both players:

	Utility 1	2
Glasses		
1	1.00	0.1
Cups		
1	0.1	1.00

The marginal utility for 1 and 2 for both glasses and cups is constant and for both 1 and 2, in whatever combination the cups and glasses come, they provide for a fixed amount of utility. This bargaining game is represented in Figure 1. Let us also assume that the utility functions of both players allow for interpersonal comparisons of utility differences. It is easy to see that the Nash solution and the Kalai egalitarian solution will yield different results in this particular case:

$$f^{\text{Nash}}(d, S) = (50,70)$$
$$f^{\text{KE}}(d, S) = (51.82,51.82)$$

Thus the Nash solution assigns 50 glasses to 1 and 70 cups to 2. The KE solution is less generous to 2. 1 will get 50 glasses and 18 cups, while 2 only gets the remaining 52 cups (approximately). Now let us zoom in on the first round of negotiations. 1 and 2 have just made the following conflicting offers, which respectively happen to match the KE solution and the Nash solution:

1's offer: (1: 50G & 18C; 2: 52C)
2's offer: (1: 50G; 2: 70C)

Let us now consult the dynamic models at our disposal. The Zeuthen model points to 1 to make the next concession, while the dynamic KE model points to 2. Now given that 1 and 2 are rational and self-interested bargainers, what is reasonable to expect in this particular case? It seems

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

to me that if 1 really does not care too much about cups and knows of 2's obstinacy when it comes to getting cups, it would be rational for her to make a concession to 2, in order not to forego her chances to get the set of 50 glasses. And this reasoning, I believe, can be repeated, until 1 simply comes to agree to 2's initial proposal. But is not 1's non-proportionate utility loss from not conceding if 2 were not to concede *smaller* than 2's non-proportionate utility loss from not conceding if 1 were not to concede at the Nash solution? In other words, is 2 not in a stronger bargaining position? Indeed, and this is certainly relevant information, but I believe that in this particular case, the argument from bargaining strength is overruled by the consideration that 1 has proportionately so much less to lose from conceding to 2's proposal than 2 has to lose from conceding to 1's proposal, that it would be silly for 1 to hold out for the KE solution. So in this case the consideration of proportionate utility loss from conceding takes the overhand in rational bargaining over the consideration of non-proportionate utility loss from not conceding. This argument against the Kalai egalitarian solution is thus a mirror-image of my argument against the Nash solution.

2.3. *The Justification for the Axiom of Monotonicity*

There exist several versions of the axiom of monotonicity. Let us take a look at Kalai's version of this axiom. Kalai claims that a bargaining solution must satisfy the following condition: if (d, S) and (d,T) are two bargaining pairs such that S is a subset of T, then the payoff to both players must be larger or equal in T than in S. Formally, for each (d, S) and (d,T), if $S \subseteq T$, then $f_1(d,T) \geqslant f_1(d,S)$ and $f_2(d,T) \geqslant f_2(d,S)$. Kalai proves that (in the presence of a set of weak axioms) solely his own egalitarian solution satisfies this axiom.

What sort of justification does Kalai advance for this condition? He provides for both an argument from fairness and for an argument from self-interest. In this paper I will solely discuss Kalai's argument from self-interest.[14] Kalai argues that if $S \subseteq T$ and for some player i $f_i(d,T) < f_i(d,S)$ then i "can in effect block the alternatives giving rise to the utilities in $T-S$ and improve his outcome" (Kalai, 1983, p. 22). Thus, if the condition of monotonicity is satisfied, then, Kalai argues, "none of the players will have an incentive to misrepresent his resources

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

or to destroy some of them before coming to the arbitrator'' (Kalai, 1983, p. 22).

Let us expand a little bit on this argument. In order to make sense of the expressions ''blocking the alternatives'' and ''misrepresenting or destroying resources'', we must have Kalai assume that the expansion from the bargaining set S to the bargaining set T is due to the fact that an extra set of goods came into play. Thus Kalai justifies the axiom of monotonicity in reference to a bargaining game characterized by (d,T) over a set of goods X and a bargaining game characterized by (d, S) over a set of goods $Y \subseteq X$. Kalai thus *claims* that *a solution f can solely be a genuine solution to the bargaining game if for all $f(d,T)$ over X there does not exist some $f(d, S)$ over $Y \subseteq X$ such that some player i gets a more favorable outcome in $f(d, S)$ than in $f(d,T)$*. The *argument* is that *if some $f(d, S)$ were larger for some player i, i would be rationally committed to vetoing $f(d,T)$ and would demand that the bargaining game $f(d, S)$ were played over $Y \subseteq X$.*

In this section I will make the following arguments:

(a) Kalai's *argument* supporting his *claim* is incorrect.

(b) Even if it were possible to justify his *claim*, this would still be insufficient justification for the axiom of monotonicity.

Let us turn to my first objection. Let us assume that two self-interested bargainers – whether rational or not – actually did reach an agreement $f(d, S)$ in a bargaining game over Y and an agreement $f(d,T)$ in a bargaining game over X such that for some i $f_i(d, S) > d_i(d,T)$. Now, according to Kalai this agreement cannot be a genuine solution, since it is irrational for 1 to agree to $f(d,T)$ over X rather than demand the bargaining game (d, S) over $Y \subseteq X$. But is it? Let us zoom in on the point at which 1 and 2 reached a tentative agreement $f(d,T)$ over X. 1 comes to realize that if she could move 2 to bargain over Y, she would get a better deal. So is it rational for her to make this move? Let us assume that both 1 and 2 receive a considerable degree of want-satisfaction from $f(d,T)$ and 1 receives a minor increase in utility from $f(d, S)$ while 2 falls back almost to d. Now what would 2 be rationally motivated to do, if 1 were to propose to bargain over Y? It seems to me that obstinacy about the initial bargain over X is 2's only rational response. Certainly if 2 were to yield to 1's proposal he would agree to $f(d, S)$, but there is nothing that forces 2 to move away from $f(d,T)$. And 1 does well to refrain from

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

tampering with the set of goods that are under discussion, if he does not want to forego his chances to obtain $f_1(d,T)$.[15]

I need to emphasize that I have *not* presented an argument to the effect that there exists some bargaining pair (d,T) over X such that for some player i $f_i(d,T) < f_i(d,S)$ over $Y \subseteq X$. I have solely argued that Kalai's *argument* to the effect that such bargaining pair cannot exist does not hold. It might well be the case that *for some reason* each tentative solution between self-interested players $f(d,T)$ over X such that there exists some $f(d,S)$ over $Y \subseteq X$ and for some players i $f_i(d,T) < f_i(d,S)$ is genuinely irrational. But this is not because it is the sole rational course of action for i to demand a bargaining game over $Y \subseteq X$ with absolute obstinacy.

Let us now turn to my second objection. I intend to argue that even if Kalai's *claim* were to hold true, this still would not provide for sufficient support to the axiom of monotonicity. I will thus *grant* Kalai that for each pair of bargaining pairs $\{(d,S), (d,T)\}$ such that $S \subseteq T$ it is possible to assign an interpretation such that (d,T) is a bargaining game over a set of goods X, (d,S) is a bargaining game over a set of goods $Y \subseteq X$, and for all such interpretations it fits our intuitions of rational and self-interested bargaining that for all players i $f_i(d,T) \geqslant f_i(d,S)$. But let us now assign an interpretation such that (d,T) is a bargaining game over a set of goods X, (d,S) is a bargaining game over a set of goods Y and it is not the case that $Y \subseteq X$. Figure 2 represents a pair of bargaining pairs $\{(d,S), (d,T)\}$. For this pair I propose the following interpretation. The set of goods Y consists of 50 glasses and 71 cups and the set of goods X consists of 60 glasses and 70 cups. Player 1 and 2 are characterized by the following IC-utilities:

	Utility 1	2
Glasses		
1	1.00	0.1
Cups		
1	0.1	1.00

Furthermore we assume that the marginal utility for both glasses and cups is constant for each player and in whatever combination the cups and glasses come, they provide for a fixed amount of utility. Now, following my argument in Section 2.2, the solution to the bargaining games (d,S)

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

and (d, T) can – at least under the assignment of this particular interpretation – be set at resp. (50, 71) and (60, 70). Our intuitions concerning rational and self-interested bargaining thus lead us to a pair of solutions which do not satisfy the condition of monotonicity.

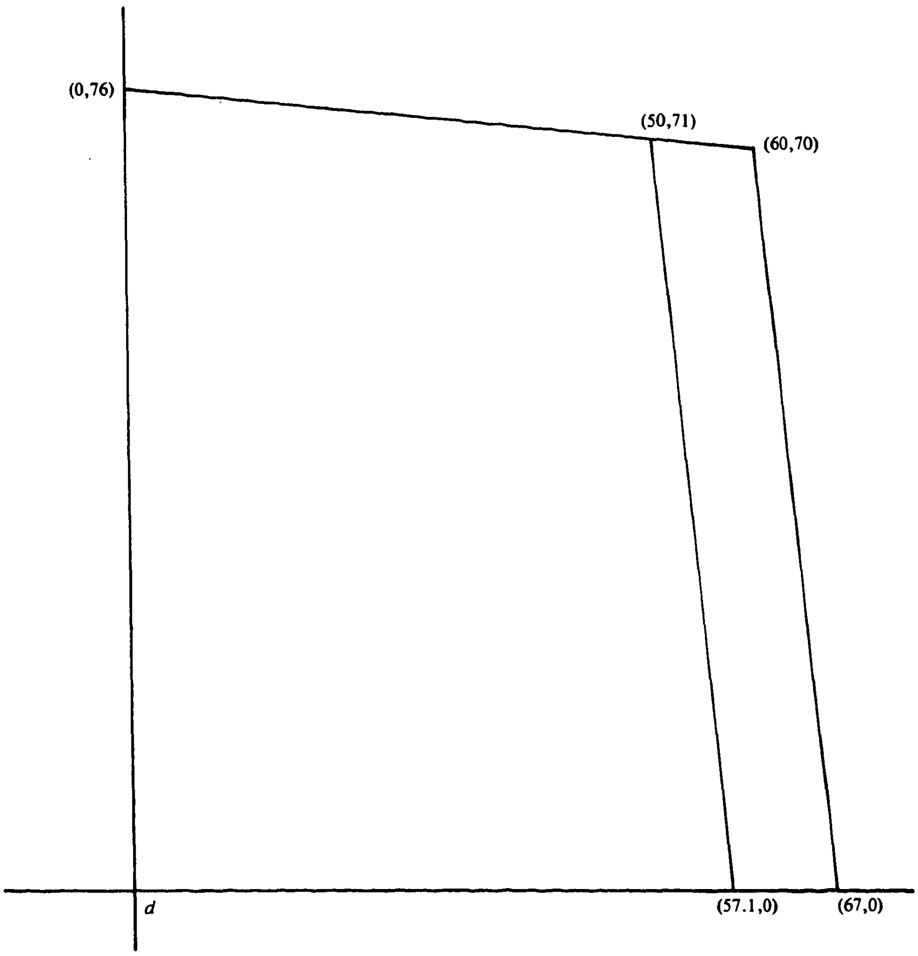

Fig. 2. Two bargaining sets and the question of monotonicity.

Kalai's reasoning resembles the fallacious argument that a particular invalid pattern of inference is valid because there exists a restricted set of sets of true sentences that all fit the pattern. In order for a pattern of inference to be valid, it must hold true under *any* assignment of an

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

interpretation. Similarly, the axiom of monotonicity does not hold, because for any pair of bargaining pairs $\{(d, S), (d, T)\}$ there exists a restricted set of interpretations – i.e. the set of interpretations in which (d, T) stands for a bargaining game over X and (d, S) stands for a bargaining game over Y and $Y \subseteq X$ – which fits this axiom. For the axiom of monotonicity to hold, it must hold under any interpretation. I have argued that it does not.

I would like to close this section with some tentative remarks. Keep in mind that I have not taken a stand on whether Kalai's *claim* is true or not. A definite answer to this question is beyond the limits of this paper. But if Kalai's claim does hold true, then my second objection is troublesome for the general project of bargaining theory. If there exists a pair of bargaining pairs $\{(d, S), (d, T)\}$ for which there exists an interpretation $\mathscr{I}$ and an interpretation $\mathscr{I}'$ such that under $\mathscr{I}$ the solution to this pair is monotonic, and under $\mathscr{I}'$ the solution is not, then the bargaining pair does not contain sufficient information to determine a definite bargaining solution f. Roemer attempts to show that the information contained in the bargaining pair is insufficient to support any argument from fairness for the axioms of bargaining theory (Roemer, 1984). If Kalai's *claim* holds true, then my argument would provide for an analogue for the axiom of monotonicity *vis-à-vis* arguments from self-interest. If Kalai's claim does *not* hold true, we might as well forget about his version of monotonicity altogether.

2.4. *The Justification for the Axiom of Path-independence*

Kalai's second justification for his egalitarian solution is that it is the sole solution which (in the presence of a set of weak axioms) satisfies the axiom of path-independence.[16] Let us take a closer look at this axiom. A solution f is path-independent if and only if for any bargaining pair (d, T) and for any bargaining pair (d, S) such that $S \subseteq T$, the solution to the bargaining game that is represented by (d, T) is identical to the solution to the bargaining game over the alternatives in $T - S$ with $f(d, S)$ as a baseline.

Now let us take a close look at Kalai's justifications for this axiom. Kalai argues that there are two "appealing features" to a solution which satisfies this axiom viz. that

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

(a) the players can feel secure about dividing up a larger bargaining problem into smaller ones, since the way this division is effected does not affect the final outcome and

(b) if future bargaining problems are unknown at this point in time, this does not keep players from making decisions in present bargaining problems, because the rational solution for the overall package of present and future bargaining problems is *identical* to the rational solution in a step-by-step decision-procedure, i.e. as time presents us with new bargaining problems (Kalai, 1983, pp. 23–24).

Kalai is certainly correct in claiming that these are desirable features for bargainers. Feauture (a) makes bargaining much simpler, since in splitting up a bargaining problem, it becomes useless to consider which particular split could yield a better deal to one player. Feature (b) makes bargaining much less nervous, since it demands less prudential considerations. The rationality of today's bargaining solutions is independent of tomorrow's problems, so as far as bargaining solutions go, there might indeed be no tomorrow. But though these features may be very desirable, they may not characterize the way rational and self-interested bargaining actually goes. It may be very unwise for a self-interested bargainer to be careless about what future bargaining problems they may face, or about the kind of split of the bargaining problem that her opponent is proposing.

Let us consider the example in Section 2.2. I have argued that rational and self-interested bargainers would in this particular example decide upon [1: 50G; 2: 70C] split, i.e. the Nash solution. 1, who has her doubts about the axiom of path-independence, is trying to figure out some split of the bargaining problem which might give her a better deal. She decides to propose to 2 to bargain over the package of 50 cups and 50 glasses at first and subsequently to turn to the remaining 20 cups. 2, who happens to be a firm believer in the axiom of path-independence, shrugs his shoulders, and agrees to the deal. Are 1's efforts – as 2 believes – entirely in vain? I do not think so. It is clear that both players will settle on a [1: 50G; 2: 50C] split in the first round. But now let us focus on the second round. This round resembles the game I have discussed in Section 1.2, except that marginal utility is constant for each player. Player 2 would very much like to have each single cup of the set, while player 1 does not take a very strong interest in them. So player 1 can comfortably

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

hold out for an unequal split that is favorable to her, knowing that 2 will ultimately come down since he hates to forego his chances to obtain even a few cups. Since the absolute utility gain that is at stake is so different for both parties, considerations of non-proportionate utility loss from not conceding if the other party were not to concede as well will play a major role in the game of alternating concessions. And it is thus not unreasonable to suppose that both players will come to agree upon the Pareto-optimal point of equal utility gain, i.e. (1.82, 1.82).

This step-by-step procedure brings 1 to 50G and 18C, while 2 solely gets 52 cups, which is precisely what the Kalai egalitarian solution would assign to both players in bargaining over the overall package as well as in any step-by-step procedure. Now I have argued in Section 2.2 that in bargaining over the overall package the KE solution is not a reasonable outcome and that 2 can secure a much better deal for himself. 1 was thus well-served by her distrust in the axiom of path-independence. And thus, though path-independence may be a *desirable* feature, it does not seem to be an *actual* feature of bargaining between rational and self-interested players. Bargainers may daydream about a world in which they could be negligent about how a bargaining problem is split up, in the actual world of rational and self-interested bargaining they better be on their guards. And thus the axiom of path-independence cannot hold as an axiom of rational and self-interested bargaining: axioms of bargaining theory must capture the limits of rationality, not the limits of what is desirable.

CONCLUSION

In this paper I have argued that there are two considerations in the game of alternating concession which will ultimately determine the outcome of a bargaining problem, viz. the relative proportionate utility loss from conceding and the relative non-proportionate utility loss from not conceding if the other person were not to concede as well. This first consideration is adequately captured by the Zeuthen model and the Nash solution, while the second consideration is adequately captured by the Kalai egalitarian solution. Neither solution tells the full story of bargaining, since, in a bargaining game in which the second consideration is overriding, the Nash solution will yield counterintuitive results, and in a bargain-

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

ing game in which this first consideration is overriding, the KE solution will yield counterintuitive results. In this paper I have systematically presented the arguments from self-interest for both the Nash and the KE solution. I have tried to show that each of these arguments is vulnerable precisely to counterexamples in which the particular consideration which is not captured by the solution in question is overriding. So what is to be done? If it is true that the two considerations in the game of alternating concessions which I have discussed in this paper provide for a full story of bargaining and if I am correct that these conditions are adequately captured respectively by the Nash solution and the Kalai egalitarian solution, it may be a promising direction of research to try to integrate both solutions into one model. This integrative model should thus capture the features of the bargaining pair that determine the relative importance of each consideration in the game of alternating concessions and should on the basis of this information set a unique bargaining solution. But these are only tentative remarks.

NOTES

* I am grateful to Jon Elster, A. Hylland, F. Spinnewijn, J. Verhoeven and the members of the research group for theoretical sociology in the K.U.L. for their comments on an earlier version of this paper. I also thank the Ministry for Foreign Affairs (Norway) for its financial support and the members of the department of sociology in the University of Oslo for their hospitality and interest in my work.

[1] A. Sen and B. Barry make a distinction to the same effect. (Sen, 1970, pp. 120–121 and Barry, 1979, pp. 164, 175) They claim, however, that the latter justification solely bestows a *predictive* worth, and thus no *normative* worth on the bargaining solution. I believe that this choice of terminology is quite unlucky. In the first place, it can hardly be denied that people (at least sometimes) *actually* do bargain in reference to some conception of fairness. And secondly, I believe an argument can be advanced that within very special circumstances (e.g. under extreme scarcity) morality may be merely a matter of prudence and questions of fairness may not have any application (cf. Rawls, 1971, pp. 126–128). In such cases, a solution which is the outcome of rational and self-interested bargaining may have *moral* worth. Kalai distinguishes between *strategic* and *normative* considerations. (Kalai, 1983, p. 22) My second objection can be repeated *vis-à-vis* this choice of terminology as well.

[2] E.g. Nash (1950, p. 158), Bartos (1967, pp. 257–259) and Kalai (1983, p. 22).

[3] Note that in order to secure the convexity of the bargaining set given 1 and 2's utility functions, the antiquaire must allow for randomized outcomes.

[4] I have chosen this counterproposal for the sake of intuitive clarity. Note however that a [1: 1G; 2: 9G] split corresponds to a non-Pareto optimal point in the convex set. There exists a set of points which jointly dominate this point, and it would thus be more rational for 2 to hold out for any of these points. To this set of points corresponds a set of lotteries

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, *On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem* , Theory and Decision, 23:3 (1987:Nov.) p.231

with prizes [1: 0G; 2: 10G] and [1: 8G; 2: 2G], but in which the odds are very much in favor of 2. The same argument can be made with any such Pareto optimal point.

[5]
$$\frac{u_1' - u_1''}{u_1'} \leqslant \frac{u_2'' - u_2'}{u_2''}$$
$$\Updownarrow$$
$$(u_1' - u_1'')u_2'' \leqslant (u_2'' - u_2')u_1'$$
$$\Updownarrow$$
$$u_1'u_2'' - u_1''u_2'' \leqslant u_2''u_1' - u_2'u_1'$$
$$\Updownarrow$$
$$-u_1''u_2'' \leqslant -u_2'u_1'$$
$$\Updownarrow$$
$$u_2'u_1' \leqslant u_1''u_2''$$
$$\Updownarrow$$
$$u_1'u_2' \leqslant u_1''u_2''$$

[6] Or more correctly, to some randomized Pareto optimal outcome which jointly dominates 2's proposal. (cf. f .4)

[7] I am following Luce and Raiffa's presentation here of the Nash solution and the Zeuthen model. (Luce and Raiffa, pp. 124–128, 135) The same commitment to considering the bargaining problem as an isolated one can also be implemented by solely taking utility *gain* above the baseline (whatever this baseline may be) as a relevant measure. This model may be less restrictive, but it is both mathematically and especially conceptually less elegant. Though all that will be said can also be said in terms of this more realistic model.

[8] Set the utility level for both players at the point of no agreement at 0. Assume now that 1 would forego utility u_1'' from not conceding while 2 would forego utility u_2'. Their proportionate utility loss would thus be

$$\frac{u_1'' - 0}{u_1''} = \frac{u_2' - 0}{u_2'} = 1$$

[9] Since (a) the baseline is fixed at 0 for both players (cf. f. 6) and (b) we are concerned with interpersonal comparisons of proportionate utility *loss*, it solely needs to be argued that the utility function which allows for interpersonal comparisons of utility *differences* with baseline at 0 yields identical results as vNM utility functions with baseline at 0. Let us assume that for each player i, there exists one single IC^{diff} utility function with baseline at 0, such that for any vNM utility function with baseline at 0 of player i there exists a linear transformation solely operating on the a-parameter which yields this function. Now, since for all x such that x is an a-parameter.

$$\frac{xu_1' - xu_1''}{xu_1'} \text{ is constant.}$$

it is clear that this IC^{diff} function does not have any privileged status *vis-à-vis* a measure of proportionate utility loss.

[10] Note that also in this example the antiquare must allow for randomized outcomes to secure the convexity of the bargaining set.

[11] Notice that utility levels need not be interpersonally comparable. Consider the set F_i of vNM utility functions which adequately capture i's preferences. Now I assume that Kalai believes that there exists a single IC^{level} utility function f_i'' of person i such that $f_i'' \in F_i$. And

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem***, Theory and Decision, 23:3 (1987:Nov.) p.231

thus, for all f_i, there exists a constant $a>0$ and a constant b such that for each lottery L $f''_i(L)=af_i(L)+b$. Now consider the subset $F'_i \subset F_i$, such that for all $f'_i \in F'_i$, there exists a constant b such that $f''_i(L)=f'_i(L)+b$. Now since we are solely concerned with interpersonally comparable utility *gain*, it is sufficient to normalize the vNM utility function f_i into some $f'_i \in F'_i$, since all members of F'_i yield identical measures of utility *gain*. And this explains why Kalai's normalization solely operates on the a-parameter.

[12] Kalai assigns the task of determining L for players 1 and 2 to an arbitrator who is motivated by considerations of fairness. In a later paper I intend to argue that the Kalai egalitarian solution is not a *fair* solution, but solely attempts to capture our intuitions of rational and self-interested bargaining.

[13] In other words, for any bargaining game between both players i, λ_i normalizes the vNM utility functions v_i, if and only if v_i reflects i's relative preferences between the outcomes in this game, the outcome \$ 0 and the outcome L'_i, even if the latter two outcomes, for which the values v_i are by now fixed, do not occur in this particular game.

[14] I believe Roemer has presented an adequate rejection of Kalai's argument from fairness in Roemer (1984, pp. 19–20).

[15] Part of Kalai's argument has an even more radical flavor. Kalai believes that $f(d,T)$ cannot even be a genuine solution because 1 would be rationally motivated to destroy $X-Y$ in order to secure a better deal for herself. Now bargaining theory does allow for preplay *communication*, but I do not see why we also need to stipulate that the players have complete access to the goods during the negotiations.

[16] Kalai calls this the step-by-step negotiation condition.

REFERENCES

Barry, B.: 1979, 'Don't Shoot the Trumpeter – He's Doing His Best', *Theory and Decision* **11**, 153–180.

Bartos, I. J.: 1967, *Simple Models of Group Behavior* (Columbia University Press, London).

Hamburger, H.: 1979, *Games as Models of Social Phenomena* (Freeman, San Francisco).

Harsanyi, J.: 1977, *Rational Behavior and Bargaining Equilibrium in Games and Social Situations* (Cambridge University Press, Cambridge).

Kalai, E. and Smorodinsky, M.: 1975, 'Other Solutions to Nash's Bargaining Problem', *Econometrica* **43**, 513–518.

Kalai, E.: 1977, 'Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons', *Econometrica* **45**, 1623–1630.

Kalai, E.: 1983, 'Solutions to the Bargaining Problem', in Hurwicz, Schmeidler and Sonnenschein, *Social Goals and Social Organisation* (forthcoming) (references to: Discussion paper No. 556, Department of Managerial Economics and Decision Sciences. Northwestern University, March 1983).

Kalai, E. and Samet, D.: 1985, 'Monotonic Solutions to General Cooperative Games', *Econometrica* **53**, 307–327.

Luce and Raiffa: 1957, *Games and Decisions* (John Wiley and Sons, New York).

Nash, J.: 1950, 'The Bargaining Problem', *Econometrica* **18**, 155–162.

Rawls, J.: 1971, *A Theory of Justice* (Harvard University Press, Cambridge, MA).

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers

BOVENS, LUC, ***On Arguments from Self-Interest for the Nash Solution and the Kalai Egalitarian Solution to the Bargaining Problem*** , Theory and Decision, 23:3 (1987:Nov.) p.231

Roemer, J. E.: 1984, *The Mismarriage of Bargaining Theory and Distributive Justice*, Working Paper Series, No. 253, Dept. of Economics, University of California, Davis (*Ethics* **79**, (1986)).

Sen, A.: 1970, *Collective Choice and Welfare* (Holden-Day, San Francisco).

National Foundation for Scientific Research (Belgium)
Katholieke Universiteit te Leuven,
Department of Sociology,
E. van Evenstraat 2C,
3000 Leuven,
Belgium.

Copyright (c) 2003 ProQuest Information and Learning Company
Copyright (c) Kluwer Academic Publishers