ARISTOTELIAN INFINITY

JOHN BOWIN

ARISTOTLE begins his treatment of the infinite in book 3 of the *Physics* in an overtly systematic fashion. The *Physics*, he says, is a study of nature, and nature has been defined as a principle of change and rest. Change, in turn, is thought to be something continuous, and what is continuous is thought to be infinitely divisible. So the topic of the infinite falls neatly out of the topic of nature. It follows, then, that the student of nature must first investigate whether the infinite exists or not, and then, if it exists, enquire how it exists. It is clear that the infinite must exist in *some* sense, because if it did not, 'many impossible consequences' would result, such as a beginning and an end of time, and the existence of indivisible lines. It remains, then, to determine in what sense the infinite does exist, and in what sense it does not. Aristotle reminds us, at the beginning of his positive account of infinity in chapter 6, that to exist means either to exist actually or to exist potentially, so if the infinite exists, it must exist in one of these senses. The previous two chapters have established that the infinite cannot exist actually, so, by disjunctive syllogism, the infinite must exist potentially. In Aristotle's words, 'The alternative then remains that the infinite has a potential existence' (*Phys.* 3. 6, 206^{a}18–19).

Jonathan Lear is right to point out that what is at stake, for Aristotle, in the rejection of the actual infinite is 'the possibility of philosophy—of man's ability to comprehend the world—[which] depends on the fact that the world is a finite place containing objects that are themselves finite'. In Aristotle's view, our ability to understand the world amounts to our ability to comprehend substances or actualities, and we could not do this if the definitions of

I would like to thank R. James Hankinson, Alexander Mourelatos, Paul Roth, David Sedley, Stephen White, Paul Woodruff, and especially Richard Sorabji for their comments on drafts of this paper.

these things were infinitely complex (*Post. An.* 82^{b}37–9). What I wish to emphasize for the purpose of this paper, however, is that the rejection of the actual infinite also positions the concept of potentiality at the core of Aristotle's positive account of infinity in chapter 6. The only alternative in sight, at the beginning of this chapter, is that the infinite has a potential existence.

Of course, the force of this conclusion is that the infinite has *only* a potential existence, and never an actual one, since actual infinities have been categorically ruled out. Aristotle emphasizes this point in the following passage:

> But we must not construe potential existence in the way we do when we say that it is possible for this to be a statue—this will be a statue, but something infinite will not be in actuality. (*Phys.* 206^{a}18–21)[1]

As Aristotle proceeds to explain how the infinite exists potentially, however, his train of thought becomes less clear. The text continues as follows:

> Being is spoken of in many ways, and we say that the infinite is in the sense in which we say it is day or it is the games, because one thing after another is always coming into existence. For of these things too the distinction between potential and actual existence holds. We say that there are Olympic Games, both in the sense that they may occur and that they are actually occurring. (*Phys.* 206^{a}21–5)

At first sight, Aristotle seems to be telling us that the potential infinite exists as a process exists which may occur but is not yet occurring. Infinity, we are told, exists as a process does, and processes, like substances, are either potential or actual: they are potential when they may occur and actual when they do occur. We already know that infinity exists potentially, not actually, so infinity must exist potentially in the sense that a process exists potentially, viz. as a process which may occur but is not yet occurring. Twenty-one lines later at 206^{b}13–14, however, we are told that infinity exists in actuality as a process that *is* now occurring—it 'exists in actuality [ἐντελεχείᾳ] in the sense in which we say "it is day" or "it is the games"'. If we are to save Aristotle from contradiction, we must revise our hypothesis of how the infinite's status as potential relates to its existence as a process. Clearly, if the ban on actual infinities is

[1] Translations of Aristotle in this paper are, with minor modifications, from *The Complete Works of Aristotle: The Revised Oxford Translation*, ed. J. Barnes (Princeton, 1984).

to hold, then there needs to be a sense in which existing in actuality as a presently occurring process is also existing potentially.

Unfortunately, there is no explicit solution to this problem in the text. Instead, we get an account of how the infinite exists as matter exists, which seems to compete with the notion that the potential infinite exists as a process, rather than illuminate it. This, I believe, is the chief difficulty that we face in interpreting Aristotle's notion of infinity, and the chief reason why so much ink has been spilt on the subject by commentators. In the sequel, I shall give an account of how existing as processes do relates to existing potentially. I shall also show how, so far from conflicting or competing with each other, the concepts of existing as matter exists and existing as processes exist actually complement one another in Aristotle's account. Finally, I shall make use of Aristotle's claim that infinity is a *per se* accident of number and magnitude (*συμβεβηκὸς καθ' αὑτό*) at *Physics* $203^{b}33$ and $204^{a}18$–19 in order to reconcile the notion of potentially infinite processes with a passage in *Physics* 6. 10 that seems to rule them out, and with Aristotle's definition of change in *Physics* 3. 1 that conflicts with their lack of a *telos*. But first I need to deal with what I regard as a red herring.

I

The passage at the beginning of chapter 6 that I have been quoting continues as follows:

> The infinite exhibits itself in different ways—in time, in the generations of man, and in the division of magnitudes. For generally the infinite has this mode of existence: one thing is always being taken after another, and each thing that is taken is always finite, but always different. Again, 'being' is spoken of in several ways, so that we must not regard the infinite like a 'this', such as a man or a horse, but must suppose it to exist in the sense in which we speak of the day or the games as existing—things whose being has not come to them like that of a substance, but consists in a process of coming to be or passing away, finite, yet always different. But in spatial magnitudes, what is taken persists, while in the succession of time and of men it takes place by the passing away of these in such a way that the source of supply never gives out. (*Phys*. $206^{a}25$–$^{b}3$)

This passage claims that in the case of the infinite division of a magnitude, 'what is taken' persists, while in the succession of time

and of men it does not, a point that is reiterated at the end of chapter 8, in answer to an argument claiming that the infinite exists not only potentially but also actually as a 'separate thing':

> It remains to go through the arguments which are supposed to support the view that the infinite exists not only potentially but as a separate thing. Some have no cogency; others can be met by fresh objections that are true. . . . Time indeed and movement are infinite, and also thinking; but the parts that are taken do not persist. (*Phys*. 208[a]5–23)

This passage is elliptical, but the main points are clear: someone (perhaps even Aristotle himself anticipating an objection) has suggested that time, movement, and thought are actually infinite (exist 'as a separate thing' rather than in potentiality). The reason Aristotle gives for these things not being actually infinite is that their *past* parts have passed out of existence. Hence, the reason why they were originally taken to be actually infinite must have turned on the number of their *past* parts. We know Aristotle held that there was no beginning of time, so, on this assumption, the past parts of time must have been infinitely numerous. We also know Aristotle held that the present cosmological order had existed in its present state for an eternity of past time, which requires, among other things, the existence of infinitely many past rotations of the celestial spheres. Hence, in the case of time and movement at any rate, it must have been these or similar infinities which were alleged to be actual.[2] The argument just reconstructed is the same, in substance, as the one raised by John Philoponus against the pagan Neoplatonist Proclus in the sixth century AD. Philoponus, in order to vindicate the Old Testament account of the creation of the universe, was trying to disprove the Aristotelian doctrine, adopted by the pagan Neoplatonists, that time and the world had no beginning. Philoponus argued that if the present cosmological order had existed in its present state for an eternity of past time, then time and the past generations of men and plants and the other individuals in each species would be actually infinite:

> If the world *had* existed from everlasting, it would be absolutely necessary for the number of things that have come into existence in the world from the beginning up until now—I mean men and plants and the other individuals in each species—to have become actually infinite as well. For should one

[2] Past thoughts may be infinitely numerous too, presumably, in the case of an immortal being.

hypothesize that the number of men or plants or of individuals of any other kind that have come to be is finite, since each of them has had its existence in a finite time, it would also be necessary for the whole of time to be finite; for that which consists of finite [parts] is finite. So since, if the world is ungenerated, the time that has elapsed is also actually infinite, the individual things that have come to be in this infinite time must, I imagine, also be actually infinite in number. And so it will follow that, if the world is ungenerated, an infinite number actually exists and has occurred. (*De aeternitate mundi contra Proclum*, 9. 4–20 Rabe)[3]

Simplicius, a contemporary of Philoponus, answers this objection on behalf of the pagan Neoplatonists by drawing attention to the fact that Aristotle had already addressed this problem in the *Physics*: that is, past generations of men, for instance, escape being an actual infinity because they do not persist. Simplicius' preoccupation with this issue, however, causes him to offer a misleading gloss on Aristotle's distinction between existing as a substance and existing as a process. In *Physics* $206^{a}25$–$^{b}3$ Aristotle tells us that the latter mode of existence 'consists in a process of coming to be or passing away', while the former mode does not, and that processes involve 'one thing . . . always being taken after another'. Simplicius, however, claims that the gist of this distinction is that the parts of a substance exist 'all at once' (*ἀθρόως* or *ἅμα*) while the parts of a process do not, and that time and the past generations of men escape being actual infinities *because* they do not all exist simultaneously.[4] This turned out to be a very influential reading. Aquinas adopts it without modification, as well as the claim that it solves the problem of past infinities,[5] and Kant argues for a similar distinction (without mentioning Aristotle) in his First Antinomy.[6] Clearly, it is true that the parts of a process do not exist all at once, in part because the past parts of a process do not persist. And by glossing the distinction in this way, Simplicius handily combines the notion of existing as a process with the claim that past times and generations of men do not persist. But there is a good reason to keep these issues distinct: for Aristotle, at least, the status of the past parts of a

[3] Trans. M. Share, *Philoponus:* Against Proclus, *On the Eternity of the World* 1–5 (London, 2004), 23–4.

[4] *In Phys.* 494. 14–495. 5 Diels; cf. 492. 26, 493. 10, 497. 15.

[5] *In Phys.*, lib. 3, l. 10, nn. 4–6.

[6] 'Demnach kann ein unendliches Aggregat wirklicher Dinge nicht als ein gegebenes Ganzes, mithin auch nicht als zugleich gegeben, angesehen werden' (Immanuel Kant, *Kritik der reinen Vernunft*, B 456).

process can have nothing to do with the way in which a process exists *potentially*, and, therefore, with the way in which infinity exists *potentially* as a process exists.

In *Nicomachean Ethics* 6. 2 Aristotle tells us: 'What is past is not capable of not having taken place; hence Agathon is right in saying "For this alone is lacking even to God, to make undone things that have once been done"' (*NE* 1139^{b}7–9); and in *De caelo* 1. 12 he says: 'No potentiality relates to being in the past, but always being in the present or future' (283^{b}13–14). I infer from these passages that no potentiality attaches to what has already happened, including the past phases of a process. Since this is the case, the status of these phases can have no bearing on the sense in which processes exist potentially and, therefore, on the sense in which the infinite exists potentially as a process does. Thus, the claim that the past generations of men escape being an actual infinity because they do not persist can have no relation to the claim that the infinite exists potentially, and if the central tenet of Aristotle's theory is that the infinite exists potentially, then the former claim must be wholly extraneous to that theory. This is one reason why invoking the non-persistence of past men and time sits poorly with the rest of Aristotle's discussion. Another is recognized by Simplicius himself, who notes that if the fact that men perish saves their number from being an actual infinity, it also saves them from being *any* sort of infinity, whether actual or potential, because the number of men at any time does not even tend towards infinity (*In Phys.* 506. 5 ff. Diels). But one of the motivations for positing a 'potential' infinity was to account for the prima facie infinity of the generations of men.

Clearly, what Aristotle has in mind is just that the process of generation and the advance of time go on and on without limit. Chapter 6 shows Aristotle entirely focused on this aspect of processes. He speaks of 'dividing *ad infinitum* [*εἰς ἄπειρον*]' (206^{b}5–6), which with the accusative implies a movement towards but not an arrival at infinity. When he claims that the infinite exists as processes do, such as days and Olympic Games, he says: 'one thing after another [*ἄλλο καὶ ἄλλο*] is always coming into existence' (206^{a}22). The examples of infinities Aristotle is trying to account for are disparate (e.g. time, generations of men, divisions of a magnitude), but he emphasizes that, even so, they all exist in the sense that 'one thing is always being taken after [*ἄλλο καὶ ἄλλο*] another' (206^{a}27–8). The problem that Philoponus raises, however, and which Aristotle re-

cognizes himself in chapter 8, is not a problem about 'one thing always being taken after another'. It is a problem, rather, of one thing always preceding another. In other words, it is a problem of infinite *pre*cession rather than infinite *suc*cession. And as such, I think it caught Aristotle completely off guard, since his theory of the potential infinite was clearly devised to explain the latter. When faced with the prospect of admitting an actual infinity of past days and men, Aristotle had no choice but to opt for the *ad hoc* expedient of claiming that the past times and generations of men escape being an actual infinity because they do not persist. As we attempt to interpret Aristotle's notion of potential infinity, then, and in particular as we attempt to explain how existing as processes do relates to existing potentially, we can only regard this issue of persistence as a red herring, and must, at any rate, not conflate it with the relevant sense in which processes exist, which I shall now endeavour to describe.

II

To avoid saddling Aristotle with a contradiction, we must find a sense in which existing actually as a presently occurring process exists is also existing potentially, since the infinite was said to exist actually in this way. Sentences such as 'one thing after another [ἄλλο καὶ ἄλλο] is always coming into existence' (*Phys.* 206^{a}22) seem to suggest the existence of an inexhaustible store of unfulfilled *future* possibilities for dividing, counting, etc. Perhaps presently occurring processes also exist potentially because they have these unfulfilled future potentialities. Charlton objects that this cannot be so.[7] If all there is to existing potentially (δυνάμει) is to have unfulfilled potentialities, 'we should all exist δυνάμει'. But since we are told that infinity exists as a process exists, this objection ignores a relevant difference in the way in which substances and processes have potentialities. Simplicius highlights this point in his commentary on chapter 6:

> Just as the actuality of the changeable preserving the potential is change, so is the actuality of the unlimited. Just as things having their being in becoming lose their being in losing their becoming, so things whose being

[7] W. Charlton, 'Aristotle's Potential Infinites', in L. Judson (ed.), *Aristotle's Physics: A Collection of Essays* (Oxford, 1995), 129–49 at 145.

is in potentiality exist just so long as their potentiality exists. (*In Phys*. 493. 24–7 Diels)[8]

As Simplicius points out, Aristotle tells us at the beginning of book 3 of the *Physics* that a motion exists only in so far as, and as long as, it has the unfulfilled potentiality of being completed by the arrival of the moving thing at a goal state that is intrinsic to the motion. A motion is an actuality of a potentiality for a moving thing to be en route to a goal, but as long as the moving thing is en route, the motion is an actuality *qua* existing potentially since it is potentially, but not actually, completed.[9] Once this potentiality is realized, the motion no longer exists, but as long as it does exist, the motion has this unfulfilled potentiality. So one could say that a motion always has an unfulfilled potentiality, but this is different from saying that someone always has the potential to be a concert violinist. Substances, of course, always have unfulfilled potentialities, including potentialities to achieve states that are uniquely determined by their natures, but they do not exist in so far as, and as long as, they have these potentialities, as changes do.

Moreover, substances are not incomplete by virtue of having unrealized potentialities, whereas processes are. 'Change', says Aristotle, 'is thought to be a sort of actuality, but incomplete, the reason for this view being that the potential whose actuality it is is incomplete' (*Phys*. $201^{b}31$–3). Aristotle refers back to this conclusion a number of times, at *Physics* $257^{b}6$–9, *De anima* $417^{a}16$–17 and $431^{a}6$–7, and *Metaphysics* $1048^{b}29$–30, but at *Nicomachean Ethics* $1174^{b}2$–5 he gives the reason for this incompleteness, viz. 'the whence and whither give [changes] their form'. Change is not simply the actuality of a potentiality to be in any chance state, but rather, it is the actuality of a potentiality to be in a state that is the incomplete realization of a particular goal (i.e. the 'whither'). If manhood is the goal, for instance, change is the actuality of the potentiality to be en route to manhood. It is the actuality of the potentiality to be a teenager, for instance. But while the process of growth may be said to be incomplete, the teenager himself is not. He has the same form or species essence as the grown man, and none of his constitutive parts is missing.

A thing is incomplete if it has some of its constitutive parts miss-

[8] Trans. J. O. Urmson, *Simplicius:* On Aristotle, *Physics* 3 (London, 2002), 116.

[9] As Aristotle puts it, 'change is the actuality of what exists potentially, *qua* existing potentially' (*Phys*. $201^{a}10$–11).

ing, and indeed there are always some temporal parts of a process that are not present. Commentators have often assumed that this is what Aristotle means when he says that the potential infinite exists as processes do and then describes it as incomplete, as he does in the following passage:

> Thus something is infinite if, taking it quantity by quantity, we can always take something outside. On the other hand, what has nothing outside it is complete and whole. For thus we define the whole—that from which nothing is wanting, as a whole man or box. What is true of each particular is true of the whole properly speaking—the whole is that of which nothing is outside. On the other hand, that from which something is absent and outside, however small that may be, is not 'all'. Whole and complete are either quite identical or closely akin. Nothing is complete [*τέλειον*] which has no end [*τέλος*] and the end is a limit. (*Phys*. 207^{a}7–15)

But processes are not incomplete simply because some of their temporal parts are not present. Processes are incomplete because they are actualities that fall short of a goal state. What is unusual about processes that go on and on indefinitely is that they have no goal state. In the passage just quoted, Aristotle argues that what is incomplete always has something outside it, and 'nothing is complete [*τέλειον*] which has no end [*τέλος*]; and the end is a limit' (*Phys*. 207^{a}14–15). The absence of a goal or end, then, is a sufficient condition for a process always to have something outside of itself, which is, in turn, a sufficient condition for the infinite to be suspended in a perpetual state of potentiality. Thus, as Aristotle says about the process of dividing a continuous magnitude, 'the fact that the indefinitely extendable process of dividing never comes to an end ensures that this activity exists potentially' (*Metaph*. *Θ* 6, 1048^{b}14–17).

III

At *Physics* 206^{b}14–16 Aristotle says: 'the infinite . . . exists . . . potentially as matter exists, not *per se* [*καθ' αὑτό*] as what is finite does'. He expands on this claim in the following three passages:

> [The infinite] is in fact the matter of the completeness which belongs to magnitude, and what is potentially a whole, though not in actuality. It is divisible both in the direction of reduction and of the inverse addition. It

is a whole and limited; not, however, *per se*, but in virtue of something else. It does not contain, but, in so far as it is infinite, is contained. Consequently, also, it is unknowable, *qua* infinite; for the matter has no form. (Hence it is plain that the infinite stands in the relation of part more than of whole. For the matter is part of the whole, as the bronze is of the bronze statue.) (*Phys*. $207^{a}21$–32)

The matter and the infinite are contained inside what contains them, while it is the form which contains. (*Phys*. $207^{a}35$–$^{b}1$)

In the fourfold scheme of causes, it is plain that the infinite is a cause in the sense of matter, and that its essence is privation, the subject *per se* being what is continuous and sensible. All the other thinkers, too, evidently treat the infinite as matter—that is why it is ridiculous of them to make it what contains, and not what is contained. (*Phys*. $207^{b}34$–$208^{a}4$)

In the discussion of place in *Physics* 4. 2 ($209^{b}5$–8) Aristotle identifies the spatial extension of a magnitude with its matter, and the magnitude itself with a form/matter composite consisting of a bounding surface and a spatial extension respectively. If this is the definition of a magnitude, and the infinite '*is* the matter of the completeness which belongs to magnitude', then the point of the passages just quoted appears to be that the infinite, understood as *what is* infinite, is to be identified with the material element or material cause of a form/matter composite. As matter, what is infinite is potentially but not actually complete and whole. It is complete and whole in so far as it is limited and contained by form, but only potentially so, because, as matter, form does not belong to it *per se*. Thus, what is infinite is unbounded, or ἄπειρος, in the sense that it is unlimited by anything *intrinsic* to it. The first passage also tells us that what is infinite, i.e. matter, is 'divisible both in the direction of reduction and of the inverse addition' ($207^{a}22$–3), which I take to mean *infinite* divisibility. Aristotle has told us at $206^{b}16$–20 that divisibility in the direction of reduction, or the infinite in respect of division, is 'in a sense the same' as divisibility in the direction of the inverse addition, or the infinite in respect of addition. This amounts to the claim that the division of a magnitude according to an infinite geometric sequence such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, . . ., $\frac{1}{2^n}$, . . . (n = 1, 2, 3, . . .), for instance, can also be viewed as the summation of the parts divided according to the infinite series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}+\ldots$ (n = 1, 2, 3, . . .), or rather, according to the sequence of partial sums $\frac{1}{2}$, $(\frac{1}{2}+\frac{1}{4})$, $(\frac{1}{2}+\frac{1}{4}+\frac{1}{8})$, . . ., $(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n})$, . . . ($n$ = 1, 2, 3, . . .). The infinite divisibility of a magnitude by reduction

and addition relates to the potential completeness and wholeness of a magnitude in respect of the status of the material parts produced and added in this way. A magnitude is complete and whole if it has all of its material parts. But a magnitude has all of its material parts only potentially since new material parts may always be produced by additional divisions and added to the collection of material parts already produced. Thus, the infinite divisibility of a magnitude is the cause of its being complete and whole only potentially, and the material element, in turn, is the source of this infinite divisibility. Thus, potential infinity is a property that the material element of a form/matter composite (i.e. 'the infinite' understood as 'what is infinite') contributes to the form/matter composite, viz. its infinite divisibility, or conversely, its being filled out by a potentially infinite number of material parts.

At first sight, we seem to have competing accounts of potential infinity: one where infinity exists as a process, and one where it exists as matter. Recent commentators have tried to promote one of these accounts at the expense of the other. Hintikka, who favours the notion that infinity exists as a process, claims that the assimilation of infinity to matter is a remnant of a superseded earlier line of thinking.[10] Jonathan Lear takes the opposite extreme by locating potential infinity entirely in the 'structure of the magnitude', and demoting process to the role of merely 'bearing witness' to the potential infinite.[11]

Hintikka's chief reason for preferring the notion that infinity exists as a process exists is that it saves the 'principle of plenitude', or the doctrine that every genuine possibility is actualized in the fullness of time. In accordance with the principle of plenitude, claims Hintikka, actually infinite sets of objects *do* come to be in the fullness of time, just not simultaneously. They come to be successively as processes do. Lear points out that, while this might have some plausibility in the case of time, it loses all plausibility in the case of the infinite division of a magnitude because it seems to commit Aristotle to the view that there *will* be a magnitude that is endlessly divided.[12] But we need not feel obliged to accommodate the principle of plenitude at all costs since there is evidence that Aris-

[10] J. Hintikka, 'Aristotelian Infinity', *Philosophical Review*, 75 (1966), 197–218 at 207.

[11] J. Lear, 'Aristotelian Infinity', *Proceedings of the Aristotelian Society*, 80 (1979), 187–210 at 191.

[12] Ibid. 190.

totle meant it to apply only within a restricted domain of objects. Sorabji has pointed out that in all clear cases where Aristotle accepts the principle of plenitude, it concerns everlasting properties of everlasting objects.[13]

Lear makes his case for favouring the notion that infinity exists as matter exists by emphasizing how Aristotle reduces the infinite by addition as well as the infinite extendability of the natural numbers to the infinite divisibility of magnitudes. Since Aristotle thinks the universe is only finitely large, he must say that magnitude is not indefinitely extendable by the addition of unit lengths (and, thus, is not even infinite potentially: *Phys.* 206^{b}12–13). Rather, a magnitude is indefinitely extendable only by the addition of parts according to a convergent infinite series—that is, only by adding parts resulting from the infinite division of a magnitude. Hence, Aristotle says, 'In a way the infinite by addition is the same thing as the infinite by division' (*Phys.* 206^{b}3–4). He then takes the further step of claiming that our ability to think of ever larger natural numbers also depends upon the infinite divisibility of magnitudes:

> In the direction of largeness it is always possible to think of a larger number; *for* [γάρ] the number of times a magnitude can be bisected is infinite. Hence this infinite is potential, not actual: the number of parts that can be taken always surpasses any definite amount. (*Phys.* 207^{b}10–13)

All of this emphasis on the infinite divisibility of magnitudes does seem to focus Aristotle's account away from the case of time and towards the structure of magnitudes. But even so, Aristotle still talks about *processes* of division. For example, he says: 'For just as we see division going on *ad infinitum* [διαιρούμενον εἰς ἄπειρον], so we see addition being made in the same proportion to what is already marked off' (*Phys.* 206^{b}5–6). What Lear needs to do, in order to make his case that these processes only 'bear witness' to potential infinity, is to establish that infinite divisibility is a property that a magnitude can possess independently of any process of division. Lear thinks he has found his evidence in *Physics* 8. 8, where Aristotle claims that a magnitude has an infinite number of potential parts (*Phys.* 263^{a}28–9, 263^{b}3–9). Aristotle tells us at *De*

[13] R. Sorabji, *Necessity, Cause, and Blame: Perspectives on Aristotle's Theory* (London, 1980), 128–35. For a refutation of the claim that *Metaph.* 1047^{b}4–6 endorses the principle of plenitude without restriction see R. T. McClelland, 'Time and Modality in Aristotle, Metaphysics IX, 3–4', *Archiv für Geschichte der Philosophie*, 63 (1981), 130–49.

anima 430^{b}11 that an 'object has no actual parts until it has been divided', so clearly, the existence of a process is required for a potentially infinite (i.e. infinitely increasable) collection of *actual* parts. No process seems to be required, however, for the existence of *potential* parts, and indeed, it does not seem possible to change the number of potential parts of a magnitude through any operation. If we can equate having an infinite number of potential parts with being infinitely divisible, then, perhaps, it might seem plausible to say that processes just 'bear witness' to potential infinity.

The first problem I see with this account is that dividing a magnitude would bear witness to an *actual* infinity rather than a potential infinity. If a potential infinity is a quantity that can be increased without limit, and, as we just said, it is not possible to change the quantity of the potential parts of a magnitude, then the quantity of the potential parts of a magnitude will not be potentially infinite. Aristotle says that these parts constitute *some* sort of infinity, so it remains for them to be actually infinite. The process of division, then, would 'bear witness' to an *actual* infinity of potential parts. And if having an infinite number of potential parts is the same as being infinitely divisible, this process would bear witness to a magnitude being actually infinite by division. But Lear himself says: 'No actual process of division could bear witness to a length being actually infinite by division.' This may be a problem for Aristotle, however, as well as for Lear. Sorabji thinks Aristotle's admission of an actual infinity of potential parts in *Physics* 8. 8 is a mistake,[14] and I tend to agree, since it conflicts with his prohibition on actual infinities in book 3 and weakens his claim that past time is not an actual infinity. One can see, though, how this result might have seemed unavoidable: once one allows magnitudes to have potential parts, the question naturally arises of how many potential parts they have. It seems implausible that they should have a finite number of them, so they must have an infinite number of them, and they cannot have a potentially infinite number of them for the reason just stated. So the only alternative seems to be that they have an actually infinite number of potential parts. One way out of this difficulty might be to deny that having an actually infinite number of potential parts is equivalent to being actually infinite by division. Another alternative might have been to claim that the number of

[14] R. Sorabji, 'Infinity and the Creation', in id. (ed.), *Philoponus and the Rejection of Aristotelian Science* (London, 1987), 164–78 at 170.

potential parts is *indefinite* or *indeterminate* rather than infinite, but Aristotle chose neither of these options.

The second problem relates to how Aristotle construes the modal force of divisibility. Specifically, one must ask whether Aristotle distinguishes the divisibility of a magnitude from the possibility of its being divided. If Aristotle *does* distinguish these notions, then there is room for a type of divisibility within his philosophy that is conceptually separate from processes. If not, then we must say that divisibility requires at least the possible existence of a process of division. The test for this question is to ask whether, if no processes were available to divide a magnitude (e.g. in a motionless universe), it would still be infinitely divisible. The answer to this question, for Aristotle, would probably be 'No'. Aristotle does not distinguish countability from the possibility of being counted at *Physics* 4. 14, $223^{a}21$–9, where he claims that if there were no one to count the before and after in change, then there would be no time, because time is change *qua* countable. And at *Physics* $263^{a}25$–6 he makes the process of counting tantamount to the process of division in so far as they each mark off parts, so the cases of dividing and counting should be essentially equivalent. In any event, Sorabji points out that Aristotle conflates ϕ-ability and the possibility of being ϕ-ed in other instances as well and cites the fact that Aristotle does not distinguish perceptibility from the possibility of being perceived at *Metaphysics* 4. 5, $1010^{b}30$–$1011^{a}2$, and *De anima* 3. 2, $426^{a}15$–26.[15]

At least in Aristotle's mind, then, it does not appear that the infinite divisibility of a magnitude was conceptually separable from the process that divides it. Indeed, it appears that for Aristotle, the infinite divisibility of a magnitude is *defined* in terms of a possible process of division. What is more, existing as matter exists and existing as processes exist seem to complement each other in Aristotle's account, since the infinite divisibility of a magnitude depends on the possibility of an infinitely extendable process of division, and the possibility of an infinitely extendable process of division depends on the structure of the magnitude to supply it with an infinite number of points at which it can be divided. Thus, potential infinity is not predicated solely of a process or solely of a magnitude. It does not pertain only to the structure of a magnitude, or only to the nature of a process that divides it. There is a

[15] R. Sorabji, *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages* (London, 1983), 90.

potential infinity of material parts and a potential infinity of acts of division. Or, perhaps a more accurate way to say it is that there can be an ever larger number of divided parts as well as an ever larger number of acts of division.

IV

I have suggested that potential infinity exists as a process exists, and in particular, as a process that has no goal or end. But this would seem to conflict with certain other texts. Aristotle argues in *Physics* 6. 10, for instance, that there can be no infinite or goalless changes (cf. *Phys.* 8. 2, 252^{b}7–12). In generation and corruption, alteration, and increase and decrease, contradictories or contraries form the natural limits of a change. The case of locomotion is more difficult, since 'it is not always between contraries', but

> since that which cannot be cut (in the sense that it is not possible that it should be cut, the term 'cannot' being used in several ways)—since it is not possible that that which in this sense cannot be cut should be being cut, and generally that that which cannot come to be should be coming to be, it follows that it is not possible that that which cannot have changed should be changing to that to which it cannot have changed. If, then, that which is in locomotion is to be changing to something, it must be capable of having changed. (*Phys.* 241^{b}3–11)

The point seems to be that the definition of change includes completability within it, so saying that a change is not completable is to state a contradiction. This is consistent with *Physics* 3. 1, which defines change as such as something that involves a goal or *telos*, viz. as the actuality of a potentiality to be in a state that is the incomplete realization of a particular goal. Thus, the potentiality associated with a process is the potentiality to reach some specific goal, not the potentiality to go on and on indefinitely.

But perhaps the infinite 'processes' that do go on and on indefinitely can be classed as accidental changes and, as such, will fall outside the class of teleological change considered in the rest of the *Physics*.[16] Perhaps the procession of time owes its lack of teleology to the fact that time is at once a property of all teleological change,

[16] I take all natural change discussed in the *Physics*, even motions of the simple bodies, to be in some sense teleological, even though simple bodies are not said to move 'for the sake of' anything. Since nature is a principle of motion and rest, and

and thus lacks any teleology of its own. It is an accidental and goalless composite property of all of the changes in the universe. Likewise, the propagation of mankind as a species is perhaps the accidental sum of the teleological strivings of all individual men and women, and as such, it has no ultimate goal of its own either.

Another, and perhaps more promising, strategy might be to claim that this accidental character applies to the *goallessness* of certain processes, rather than to the processes themselves. The division of a magnitude according to a geometric sequence such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, . . ., $\frac{1}{2^n}$, . . . ($n = 1, 2, 3, \ldots$), for instance, is clearly a genuine, non-accidental change. But one will find, as a matter of fact, that any process of division will end after a finite number of steps. Perhaps, then, the potential infinity of the task is reflected in the *counterfactual* possibility that the task could have gone on longer. And perhaps one might generalize this to say that all genuine changes are, in fact, finite, but some of them are accidentally goalless, viz. it is just an accidental property of some of them that they could, counterfactually, have gone on indefinitely. This view finds support in *Physics* 3. 4 and 5, where Aristotle refers to the potential infinite first as a *per se* accident of number and magnitude (*συμβεβηκὸς καθ' αὑτό*) at $203^{b}33$, and then, equivalently, as a *per se* affection of number and magnitude (*καθ' αὑτὸ πάθος τι*) at $204^{a}18$–19 (cf. $204^{b}30$). The concept of a *per se* accident or affection is introduced at *Metaphysics* $1025^{a}30$–1, as 'what attaches to each thing *per se* but is not in its substance, as having its angles equal to two right angles attaches to the triangle'. A triangle is presumably defined as 'a three-angled figure', and this is its essence. But certain other properties not in the definition of a triangle may be deduced from this, which hold eternally and necessarily, e.g. that its angles sum to two right angles, and these properties are *per se* accidents or affections. Likewise, since Aristotle defines change in terms of definite goal states, the processes of dividing a magnitude and counting its divisions are, like all processes, essentially finite. But perhaps it is a *per se* accident or affection of some of these processes that they could, counterfactually, have gone on indefinitely. And perhaps it is the structure of the magnitude being divided that gives these processes this peculiar property.

Aristotle says that potential infinity is a *per se* affection of number

since motion is defined teleologically, I do not see how one can avoid ascribing at least some form of teleology, however etiolated, to everything that has a nature.

and magnitude in a manner analogous to the way in which speech is incidentally invisible (*Phys.* 204[a]14–17). Just as 'the invisible is not an element in speech, though the voice is invisible', so the infinite is not an element in number and magnitude, though number and magnitude are infinite. The invisible is not an element in speech in the sense that invisibility is not part of the definition of speech, yet speech is, of course, invisible, and necessarily invisible. Likewise, if it is in the essence of a magnitude to be a bounded extension, or a form/matter composite consisting of a bounding surface (form) and a spatial extension (matter), perhaps it is a *per se* accident or affection of a magnitude to have a potentially infinite number of material parts, i.e. a property which is not specified in the definition of magnitude, but is deducible from the mention of matter in its definition. Similarly, if it is in the essence of each and every number to be some finite and countable plurality of units, perhaps it is a *per se* accident or affection of each such number to be a member of the class of numbers (referred to generically as 'number', instead of 'a number') that can be increased indefinitely.

Thus, taking potential infinity as a *per se* accident or affection seems to explain the infinite increasability of number and the infinite extendability of certain types of changes. But given that infinity is a property of number and magnitude, and given the way that Aristotle defines these things, it is inevitable that infinity *must* be this sort of property. Aristotle clearly thinks that infinity exists as a property. *Physics* 3. 5 proceeds as a disjunctive syllogism: the infinite either exists as a substance or as a property; the infinite does not exist as a substance; therefore the infinite exists as a property (*Phys.* 204[a]29–30), and, indeed, as a property of number and magnitude (*Phys.* 204[a]18–19). Since number and magnitude are the sole members of the genus quantity (*Metaph.* 1020[a]7–14), infinity is a property of quantity. Quantity is, by nature, measurable or countable because it is divisible into units by which it is measured or counted, viz. parts which are 'by nature a "unit" and a "this"' (*Metaph.* 1020[a]7–32, 1057[a]2–4). This is why Aristotle says that a quantity is always 'a particular quantity, e.g. two or three cubits; quantity just means these' (*Phys.* 206[a]3–5). Since it is impossible to count an infinity of units, and since quantities are, by definition, measurable or countable, there is no infinite quantity (*Phys.* 204[b]8–10, [a]28–9). But how can Aristotle claim that infinity is a property of number and magnitude if number and magnitude

cannot be infinite? To say that there is no infinite quantity is also to say that there is no quantity in whose definition the term 'infinity' appears, or as Aristotle puts it, 'if the infinite is not a substance, but an accident, then it cannot be, *qua* infinite, an element in things' (*Phys.* 204[a]14–16). Infinity, then, must be a property of number and magnitude which does not appear in the definitions of number and magnitude, and it must be a necessary and eternal property, since it is a necessary and eternal fact that number does not give out in thought, and that continuous magnitudes are infinitely divisible. Infinity must be, in other words, a *per se* accident or affection of number and magnitude (*Phys.* 204[a]29–30; cf. 204[a]14, 28–9).[17]

University of California, Santa Cruz

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[17] In the case of number, this also follows from the fact that numbers are either odd or even and that infinity is neither odd nor even (*Metaph.* 1084[a]2–4). In the case of magnitude, this also follows from the alleged fact that there is no infinitely large body or collection of bodies of which an infinite magnitude can be a property (*Phys.* 204[a]34–206[a]7).