THE QUALITATIVE THESIS*

This paper is about:

> **The Qualitative Thesis**. If you are not sure that $\neg\varphi$, then you are sure of the indicative conditional $\varphi > \psi$ just in case you are sure of the material conditional $\varphi \supset \psi$.

The Qualitative Thesis occupies a central place in contemporary theories of indicative conditionals. In §1 of this paper, we will show that the Qualitative Thesis follows from widely-accepted principles about how we should reason with indicative conditionals.

In the rest of the paper, we explore the consequences of the Qualitative Thesis for the semantics of indicative conditionals. Specifically, we will argue that the Qualitative Thesis provides two new and compelling reasons to accept a thesis that we call *Conditional Locality*. Roughly, Conditional Locality says that the interpretation of an indicative conditional depends not just on the *global* context—the context of the conversation—but also on the conditional's local embedding environment. On this view, the contribution that the indicative conditional, *if Matt is in London, then he is in Oxford*, makes to the meaning of (1) is not the same as the contribution it makes to the meaning of (2).

(1) Alice is sure that if Matt is not in London, then he is in Oxford.

(2) Milo is sure that if Matt is not in London, then he is in Oxford.

Here is how the paper will proceed. In §1, we motivate the Qualitative Thesis by showing that it follows from widely-accepted principles. In §2, we introduce what we call the *Local Qualitative Thesis*. We take this to be the weakest plausible precisification of the Qualitative Thesis. The Local Qualitative Thesis is a *context-sensitive* version of the the Qualitative Thesis. It says if the speaker of a context leaves open φ, then she is sure of the material conditional $\varphi \supset \psi$ just in case she is sure of the proposition expressed by the indicative conditional $\varphi > \psi$ *in her context.*

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In §3-§4, we present an argument—due to Ben Holguín—showing that, without Conditional Locality, the Local Qualitative Thesis is in tension with a plausible margin for error principle on rational sureness.[1] However, as we show in §5-§6, if we accept Conditional Locality, it is possible to reconcile the Qualitative Thesis and the margin for error principle. This is our first argument for Conditional Locality.

In §6, we develop a theory of indicative conditionals, which we call the *Local, Shifty Theory* of indicatives. Our theory respects Conditional Locality. We give a *selection semantics* for the indicative conditional, according to which $\varphi > \psi$ is true just in case ψ is true in the selected φ-worlds. But importantly, the selection function is partly determined by the local linguistic environment of the conditional. As a result, the contribution of the indicative conditional, *If Matt is not in London, then he's in Oxford* to (1) differs from its contribution to (2). We show that this gives the Local, Shifty theory the resources to validate the Local Qualitative Thesis while also accepting a margin for error constraint on rational sureness.

In §7, we turn to our second argument for Conditional Locality. We begin with the observation that the Local Qualitative Thesis is simply too weak—the full range of data supports what we call the *Strong Qualitative Thesis*. Specifically, we will show that the Local Qualitative Thesis does not predict that the following inference valid.

(1) Alice is sure that Matt is either in London or Oxford.

(3) So, Alice is sure that if Matt is not London, then he is in Oxford.

The Strong Qualitative Thesis, by contrast, does predict that the inference from (1) to (3) is valid. Since this inference does indeed seem valid, this gives us strong reason to adopt the Strong Qualitative Thesis. We argue that, without Conditional Locality the Strong Qualitative Thesis has unacceptable trivializing consequences. But with Conditional Locality, the Strong Qualitative Thesis is tenable.

I. MOTIVATING THE QUALITATIVE THESIS

In this section we present two arguments for the Qualitative Thesis.

The first argument is that the Qualitative Thesis follows from the conjunction of two plausible claims about reasoning with conditionals. The first claim is that Modus Ponens is valid. This entails one half of the Qualitative Thesis—if you are sure of the indicative conditional $\varphi > \psi$, then you are sure of the corresponding material conditional

[1] Ben Holguín, "Indicative Conditionals Without Iterative Epistemology," *Noûs*, LV, 3 (September 2021): 560-80.

$\varphi \supset \psi$ (regardless of whether you are sure of $\neg\varphi$). The second claim is that Stalnaker's *Direct Argument* is a reasonable inference. This entails the second half of the Qualitative Thesis, namely, that if you are not sure that $\neg\varphi$ and you're sure of the material conditional $\varphi \supset \psi$, then you are also sure of the indicative conditional $\varphi > \psi$.

Modus Ponens is the principle that from an indicative conditional $\varphi > \psi$, together with its antecedent φ, one can infer the conditional's consequent ψ. It goes without saying that there are strong reasons to accept Modus Ponens.[2] Assuming a classical consequence relation, Modus Ponens stands or falls with the principle that from an indicative conditional $\varphi > \psi$, one can infer the material conditional $\varphi \supset \psi$. And if this principle is valid, then one half of the Qualitative Thesis is true: if you are sure of the indicative conditional $\varphi > \psi$, then you are also sure of the material conditional $\varphi \supset \psi$.

The Direct Argument is the argument from the disjunction $\varphi \vee \psi$ to the indicative conditional $\neg\varphi > \psi$. The argument is compelling, as the following example shows.

(4) Matt is either in Los Angeles or London.

(5) So, if Matt is not in Los Angeles, he is in London.

We should not say that the Direct Argument is a *classically valid* inference. For (4) is equivalent to the material conditional *Matt's not in Los Angeles* $\supset$ *Matt's in London*. So to say that (4) classically entails (5) would be to say that the material conditional entails the indicative conditional, a notoriously unacceptable consequence. Following Stalnaker, we should instead say that Direct Argument is a *reasonable inference*—roughly, if you are sure of the disjunction $\varphi \vee \psi$, and are not sure that φ, then you are sure that $\neg\varphi > \psi$.[3] This claim is equivalent to the second half of the Qualitative Thesis: if you are sure of the material conditional $\varphi \supset \psi$ and you are not sure that $\neg\varphi$, then you are sure of the indicative conditional $\varphi > \psi$.[4]

[2] This is not to say that the principle is entirely uncontroversial; see footnote 20 for a discussion of purported counterexamples.

[3] See Robert Stalnaker, "Indicative Conditionals," *Philosophia*, V, 3 (1975): 269-86.

[4] Note that it doesn't follow from the Qualitative Thesis that *whenever* the you are sure of (4), you are in position to infer (5). You might be sure of (4) without leaving open that Matt is in Los Angeles, and the Qualitative Thesis is silent about that case. But, as Stalnaker points out, it is felicitous to assert (4) only if the context leaves open that Matt is not in Los Angeles, and so whenever (4) is felicitously asserted, the posterior context will entail that Matt is in Los Angeles or London, but leave open that Matt is in Los Angeles. This means that The Qualitative Thesis predicts that the speakers can infer (5) from (4) whenever they have become sure of (4) on the basis of a successful assertion of (4).

The second argument for The Qualitative Thesis is that, given plausible assumptions, it follows from Stalnaker's Thesis, stated informally below.

> **Stalnaker's Thesis**. The probability of $\varphi > \psi$ is equal to the probability of ψ conditional on φ whenever the probability of φ is greater than zero.

The case for Stalnaker's Thesis is strong. To see this, take an example. You are holding a standard 52-card deck of cards, and you draw one at random. Ask yourself how confident you are in the following conditional.

(6) The selected card is a jack if it's a red card.

If you are like most, you will judge the probability of (6) to be 1/13: there are 26 red cards, and 2 of them are jacks. 1/13 is also the probability that the selected card is a jack given that it is red. That is exactly what Stalnaker's Thesis predicts; and it is easy to multiply examples like this. In general, we calculate the probability of a conditional $\varphi > \psi$ by calculating the probability of ψ conditional on φ.

If we assume that one is sure of some proposition just in case one assigns credence 1 to that proposition, then Stalnaker's Thesis entails The Qualitative Thesis. To see why, suppose Stalnaker's Thesis is true. If Stalnaker's Thesis is true, so is the following corollary of Stalnaker's Thesis.

> **The Probability 1 Thesis**. If $P(\varphi) > 0$, then $P(\varphi > \psi) = 1$ if and only if $P(\psi|\varphi) = 1$

It can be easily checked that if $P(\varphi) > 0$, then $P(\varphi \supset \psi) = 1$ if and only if $P(\psi|\varphi) = 1$. So The Probability 1 Thesis entails (7).

(7) If $P(\varphi) > 0$, then $P(\varphi > \psi) = 1$ if and only if $P(\varphi \supset \psi) = 1$

If being sure is having credence one, then (7) is equivalent to the Qualitative Thesis.

In this section, we have explained why one should care about the Qualitative Thesis. The first argument is that the Qualitative Thesis follows from the conjunction of two standard claims about indicative conditionals—the claim that Modus Ponens is valid, and the claim that the Direct Argument is a reasonable inference, respectively. The second argument is that, given a plausible probabilistic account of being sure, the Qualitative Thesis follows from Stalnaker's Thesis.

II. THE LOCAL QUALITATIVE THESIS

Here is how we stated The Qualitative Thesis in the introduction.

> **The Qualitative Thesis**. If you are not sure that $\neg\varphi$, then you are sure of the indicative conditional $\varphi > \psi$ just in case you are sure of the material conditional $\varphi \supset \psi$.

There are different ways to make this informal thesis more precise—some stronger, some weaker. In the first part of this paper, we focus on what we take to be the weakest plausible precisification, which, we think, all theorists about conditionals should want to accept. We call this thesis *The Local Qualitative Thesis.*

The Local Qualitative Thesis is inspired by Andrew Bacon's defense of a context-sensitive version of Stalnaker's Thesis.[5] Bacon argues that Stalnaker's Thesis is in tension with *contextualism* about indicative conditionals. To see why, assume φ and ψ are not context-sensitive expressions. According to contextualists, there are still many propositions corresponding to the sentence $\varphi > \psi$, since the proposition expressed by an indicative can vary from context to context.[6] There will be contexts c_1 and c_2 such that $[\![\varphi > \psi]\!]^{c_1}$, the proposition expressed by an utterance of the conditional in c_1, is distinct from $[\![\varphi > \psi]\!]^{c_2}$, the proposition expressed in c_2. Stalnaker's Thesis says that the probability of each of these propositions is equal to the probability of ψ given φ. It follows that the probability of $[\![\varphi > \psi]\!]^{c_1}$ is equal to the probability of $[\![\varphi > \psi]\!]^{c_2}$. But the contextualist cannot accept this consequence. For the contextualist, $[\![\varphi > \psi]\!]^{c_1}$ and $[\![\varphi > \psi]\!]^{c_2}$ are simply different propositions and so the attitude one takes to the first may be very different from the attitude one takes to the second.

Bacon advances a thesis that is similar to Stalnaker's Thesis, but one that is friendly to contextualist accounts of indicatives. Before we state this thesis, it will be helpful to say something about the specific form of contextualism at issue. Contextualists say that indicative conditionals are *information sensitive.* They talk about what's true in antecedent worlds compatible with a contextually-determined body of information. The version of Stalnaker's Thesis that Bacon endorses takes

[5] Andrew Bacon, "Stalnaker's Thesis in Context," *Review of Symbolic Logic*, VIII, 1 (September 2014):131-63.

[6] Henceforth, we will use Greek letters for variables ranging over sentences; so, for example, the variable φ takes a sentence as its value. We will use bold upper case Romans for variables ranging over sets of worlds (i.e. propositions); so, for example, $\mathbf{A}$ takes a set of worlds as its value. $[\![\varphi]\!]^c$ will denote the proposition expressed by φ in context c. We will be loose with use and mention when it comes to the connectives: the accompanying variables will make clear whether, for example, $\supset$ picks out an expression or the material conditional operator.

this information-sensitivity into account. It says, roughly, that if the total evidence available to the speaker in a given context is E, then the probability that she assigns to $\llbracket\varphi > \psi\rrbracket^{c_E}$—the proposition expressed by the indicative conditional in *her context*, relative to *her information*—is equal to the probability that she assign to ψ conditional on φ.

Bacon and other contextualists will want to take the same strategy with respect to the Qualitative Thesis. They will endorse a contextualist version of the Qualitative Thesis, which says, roughly, that you are sure of the proposition expressed by the indicative conditional relative to *your information* just in case you are sure of the corresponding material conditional. To state this thesis, we introduce a new operator to talk about what the speaker in a given context is sure of. Let $S^{c,w}(\llbracket\psi\rrbracket^c)$ mean that the speakers in c are *sure that* $\llbracket\psi\rrbracket^c$ in w. Here is the Local Qualitative Thesis.

> **The Local Qualitative Thesis**. For any world w and context c, if $\neg S^{c,w}(\llbracket\neg\varphi\rrbracket^c)$, then: $S^{c,w}(\llbracket\varphi > \psi\rrbracket^c)$ if and only if $S^{c,w}\llbracket\varphi \supset \psi\rrbracket^c$.

This says: if the speaker in c is sure of $\llbracket\neg\varphi\rrbracket^c$, then she is sure of $\llbracket\varphi > \psi\rrbracket^c$ just in case she is sure of the material conditional $\llbracket\varphi \supset \psi\rrbracket^c$. Note that, for the sake of brevity, we will often refer to the Local Qualitative Thesis simply as 'The Qualitative Thesis'.

We think that everyone, contextualist or not, should endorse the Local Qualitative Thesis. Contextualists should accept it for the reason Bacon gives. If $\llbracket\varphi > \psi\rrbracket^{c_1}$ and $\llbracket\varphi > \psi\rrbracket^{c_2}$ are distinct propositions, then the attitude one takes to the first need not be the same as the attitude one takes to the second. And for non-contextualists, the Local Qualitative Thesis is simply a roundabout way of saying what the original Qualitative Thesis did. For these theorists, the proposition expressed conditional expresses does not depend on the context, so the indicative conditional expresses the same thing relative to one body of information as it does relative to any other body of information.

III. THE QUALITATIVE THESIS IN THE STANDARD FRAMEWORK

Here we present a standard formal framework for thinking about The Qualitative Thesis. This framework gives sureness ascriptions a Hintikka semantics. And it gives the conditional a variably strict semantics, where $\varphi > \psi$ says, roughly, that ψ is true in the closest φ-worlds.[7][8]

[7] In Appendix A.2, we prove that analogous results hold in a strict conditional framework, where $\varphi > \psi$ says that $\varphi \supset \psi$ holds throughout some fixed set of closest worlds.

[8] See Robert Stalnaker, "A Theory of Conditionals," in Nicholas Rescher, ed., *Studies in Logical Theory* (Oxford: Blackwell, 1968): pp. 98-112. See also David Lewis, *Counterfactuals* (Oxford: Blackwell, 1973).

We characterize the Qualitative Thesis in this framework and then use this result to show that the Qualitative Thesis puts a significant constraint on the logic of sureness, entailing a principle we call *No Opposite Materials.*

III.1. A Standard Framework. We begin by constructing a propositional modal language that we can use to describe what a subject is sure of. The set of sentences of the language $\mathcal{L}$ is the set of sentences generated by the following grammar:

- $\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi > \psi \mid S\varphi$

The propositional connectives $\supset$, $\equiv$, and $\vee$ are defined as usual; $>$ is our conditional operator. We read $S\varphi$ as *the subject is sure of* φ.

Next, the interpretation of the language. We assume that we are in some fixed arbitrary context with some relevant speaker who determines the particular interpretation of the conditional; that is, our semantic evaluation function, $[\![\cdot]\!]$, specifies only the *content* of the sentences in our language in this context.

We interpret the logical connectives in the standard way. To give the truth-conditions of the conditional, we use a *selection function*, which we assume is supplied by the background context. Where $f(\mathbf{A}, w)$ is the set of selected $\mathbf{A}$-worlds at w and $[\![\varphi]\!] = \{w : [\![\varphi]\!]^w = 1\}$, we say:

Standard Variably Strict Semantics. $[\![\varphi > \psi]\!]^w = 1$ iff $f(w, [\![\varphi]\!]) \subseteq [\![\psi]\!]$

This clause says that $\varphi > \psi$ is true at a world w just in case all of the selected φ-worlds at w are ψ-worlds. We stipulate that the selection function has the following natural properties:

Success. $f(w, \mathbf{A}) \subseteq \mathbf{A}$

Minimality. If $w \in \mathbf{A}$, then $w \in f(w, \mathbf{A})$

Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $f(w, \mathbf{A}) \neq \emptyset$

Success and Minimality are standard assumptions. Success says that the selected $\mathbf{A}$-worlds at w must be $\mathbf{A}$-worlds. Minimality says that if w is an $\mathbf{A}$-world, then it must be among the selected $\mathbf{A}$-worlds at w; it's needed to validate Modus Ponens. Non-Vacuity says that if there are accessible $\mathbf{A}$-worlds at w, then the set of selected $\mathbf{A}$-worlds at w isn't empty. Non-Vacuity is needed to validate a form of Conditional Non-Contradiction, specifically:

Weak Conditional Non-Contradiction. $\neg S\neg\varphi \supset \neg((\varphi > \psi) \wedge (\varphi > \neg\psi))$

Weak Conditional Non-Contradiction says that if φ is a live possibility, then $\varphi > \psi$ and $\varphi > \neg\psi$ are not consistent. This is a standard—and desirable—principle in conditional logic.[9] In general, there is something very wrong with asserting both $\varphi > \psi$ and $\varphi > \neg\psi$.

Truth for the sureness operator S is defined in terms of an accessibility relation R: wRw' means that w' is compatible with what the subject is sure of in w.[10]

> **Standard Hintikka Semantics**. $[\![S\varphi]\!]^w = 1$ iff $\forall w' \in R(w) : [\![\varphi]\!]^{w'} = 1$

We assume only that R is serial: at every world the subject has consistent beliefs. We assume that the accessibility relation R is that of the relevant agent in the arbitrary context we interpret our language in.

Given how we understand the interpretation of our language, we can characterize the Local Qualitative Thesis by characterizing the following object language principle:

> **QT**. $\neg S \neg\varphi \supset (S(\varphi > \psi) \equiv S(\varphi \supset \psi))$

Our interpretation of the language forces us to understand QT *locally*—specifically, as saying that if the speaker of a given context c leaves open $[\![\varphi]\!]$, then she is sure of the proposition expressed by $\varphi > \psi$ relative to the information in *her* context just in case she is sure of $[\![\varphi \supset \psi]\!]$.

III.2. Characterizing the Qualitative Thesis. We will now characterize QT and derive from it an important constraint on the accessibility relation. Consider Stalnaker's *Indicative Constraint*:

> **Indicative Constraint**. If $R(w) \cap \mathbf{A} \neq \emptyset$, then if $w' \in R(w)$, then $f(w', \mathbf{A}) \subseteq R(w)$.[11]

The Indicative Constraint says that if $\mathbf{A}$ is left open, then the selected $\mathbf{A}$-worlds at any accessible world are themselves accessible worlds. More precisely, if $\mathbf{A}$ is compatible with what the speaker is sure of in a world w, then for any world w' that is compatible with what the speaker is sure of in w, the selected $\mathbf{A}$-worlds at w' are a subset of the worlds compatible with what the subject is sure of at w.

[9] Why not a stronger version of Conditional Non-Contradiction that just says $\varphi > \psi$ and $\varphi > \neg\psi$ are not consistent? This stronger principle is inconsistent with Logical Implication, which says that $\varphi > \psi$ is always true when φ entails ψ. Weak Conditional Non-Contradiction, by contrast, is consistent with Logical Implication.

[10] We use the term *doxastic accessibility* to mean compatibility with what the subject *is sure of*, not what she believes.

[11] See Robert Stalnaker, "Indicative Conditionals," *Philosophia*, *op. cit.*.

In Appendix A.1, we prove that the Indicative Constraint characterizes QT.[12]

> **Fact 1**. QT is valid on a normal variably strict frame $\mathcal{F}$ just in case $\mathcal{F}$ meets the Indicative Constraint.

We can use Fact 1 to show that the Qualitative Thesis has important epistemological upshots. Consider:

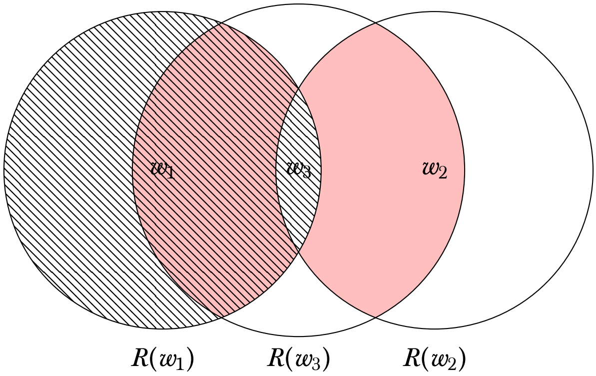
> **No Opposite Materials**. For any two worlds w_1, w_2, if there's some w_3 such that $w_1 R w_3$ and $w_2 R w_3$, then, for any $\mathbf{A} \subseteq W$: if $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$, then there's no $\mathbf{C} \subseteq W$ such that $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$.

No Opposite Materials says that for certain pairs of worlds, and certain propositions $\mathbf{A}$, you can't be sure of a material conditional $\mathbf{A} \supset \mathbf{C}$ at the first world and sure of the 'opposite' material conditional, $\mathbf{A} \supset \neg\mathbf{C}$, at the second. Which pairs of worlds? Any two worlds that see a world in common. And for which propositions? Any proposition that is consistent with what you're sure of at all three worlds.

One can easily show that that No Opposite Materials is a consequence of QT. Specifically:

> **Fact 2**. A normal variably strict frame $\mathcal{F}$ validates QT only if No Opposite Materials holds on $\mathcal{F}$.

We can give an informal explanation of why this holds. If No Opposite Materials fails we are left with a situation like below:



[12] The numbering of the facts here corresponds to that in the appendices.

There must be some w_1 and some w_2 such that both w_1 and w_2 see some w_3. And all three worlds must see worlds where some proposition $\mathbf{A}$ is true. In the diagram above, the pink region above represents $\mathbf{A}$. Finally, for some $\mathbf{C}$, $\mathbf{A} \supset \mathbf{C}$ is true throughout $R(w_1)$ and $\mathbf{A} \supset \neg\mathbf{C}$ is true throughout $R(w_2)$. This is secured in the diagram above by letting $\mathbf{C}$, the shaded region, coincide with $R(w_1)$. We can now show that QT must fail at either w_1 and w_2. For suppose it held at both. Since $\neg S \neg \mathbf{A}$ and $S(\mathbf{A} \supset \mathbf{C})$ are true at w_1, $S(\mathbf{A} > \mathbf{C})$ must be too and so $\mathbf{A} > \mathbf{C}$ is true at all worlds seen by w_1. For similar reasons, $S(\mathbf{A} \supset \neg\mathbf{C})$ must be true at w_2 and so $\mathbf{A} > \neg\mathbf{C}$ is true at all worlds seen by w_2. But w_3 is seen by both; so it follows that both $\mathbf{A} > \mathbf{C}$ and $\mathbf{A} > \neg\mathbf{C}$ are true at w_3. But this contradicts Weak Conditional Non-Contradiction.

In the next section we develop an argument due to Ben Holguín showing that No Opposite Materials is inconsistent with a plausible margin for error requirement on rational sureness.[13] Fact 2 tells us that QT entails No Opposite Materials. It follows that QT is itself inconsistent with the margin for error requirement.

IV. NO OPPOSITE MATERIALS AND MARGIN FOR ERROR PRINCIPLES

To illustrate the margin for error requirement, we begin with a case from Timothy Williamson.[14] Mr. Magoo is staring out the window at a tree some distance off, wondering how tall it is. Assuming his only sources of information are reflection and present perception of the tree, what should he believe? That depends on how tall the tree actually is. If the tree is 100 inches tall, Mr. Magoo's visual information rules out possibilities in which the tree is 200 inches tall, or so we can imagine. So it would be reasonable for Magoo to be sure that the tree is not 200 inches tall. On the other hand, Magoo's visual information does not rule out possibilities in which the tree is 101 inches tall; his eyesight is simply nowhere near that good. It would not be reasonable for Magoo to be sure that the tree is not 101 inches tall.

Mr. Magoo's beliefs about the height of the tree are rational only if they leave a *margin for error*.[15] If the tree is n inches tall, a belief that

[13] See Holguín, "Indicative Conditionals Without Iterative Epistemology," *op. cit.*. Note that Holguín draws a very different moral from his argument, concluding that if you accept the margin for error principle you should reject The Qualitative Thesis. We think these can be reconciled.

[14] Timothy Williamson, *Knowledge and its Limits* (Oxford: Oxford University Press, 2000).

[15] We will often use the term 'belief' because neither 'surety' nor 'sureness' sounds quite right (and 'surenessess' is even worse). But when we say 'Magoo's belief' we should be understood as talking about the state of being sure; and when we talk about 'Magoo's belief set' we should be understood as talking about the set of worlds compatible with what Magoo is sure of.

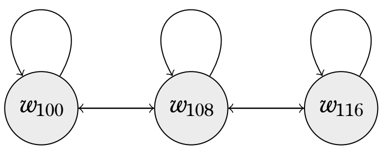
the tree is not $n+1$ inches tall does not leave a sufficiently wide margin for error; that belief is false in nearby worlds where the tree is slightly taller. On the other hand, a belief that the tree is not $n+100$ inches tall does leave a sufficiently wide margin for error; that is true in nearby worlds where the tree is a bit taller.

To state the margin for error requirement, we introduce a *margin for error frame* $\langle W, R\rangle$. W is a set of worlds representing possible tree heights. Where i is the height in inches of the tree in w, $W = \{w_i : i \in \mathbb{R}$ and $i > 0\}$. R is a binary doxastic accessibility relation on W: $w_i R w_j$ means that, in a world where the tree is i inches tall, it is compatible with everything Magoo is rationally sure of that the tree is j inches tall. R is defined as follows, relative to an arbitrarily chosen positive constant h.

Magoo's Margin. $w_i R w_j$ if and only if $|j - i| < h$.

h is Magoo's margin for error; h is positive, for otherwise his discrimination would be perfect.

No Opposite Materials fails on every margin for error frame. To see this, suppose that $h = 10$, and consider three worlds in W: w_{100}, w_{108}, and w_{116}. Here is a diagram depicting Mr. Magoo's beliefs in these three worlds.

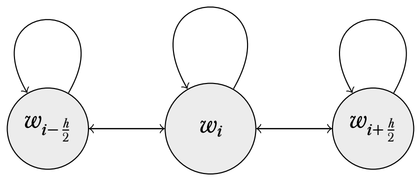


Mr. Magoo's belief worlds at w_{116} overlap with his belief worlds at w_{100}: w_{108} is consistent with what he is sure of in w_{116} and consistent with what he is sure of in w_{100}. Moreover, it's consistent with what Magoo is sure of at each world that the tree is either 100 inches tall or 116 inches tall. This means that the antecedent of No Opposite Materials is satisfied. The right and left worlds see a world in common, w_{108}. And the proposition that the tree is either 100 inches tall or 116 inches tall is consistent with what Magoo is sure of at all three worlds. But the consequent of No Opposite Materials is not satisfied. Since Magoo's margin for error is 10, w_{100} does not see w_{116} and w_{116} does not see w_{100}. As a result, Mr. Magoo is sure of 'opposite' material conditionals at w_{100} and w_{116}. At w_{100}, Mr. Magoo is sure that (8) is true; at w_{116}, Mr. Magoo is sure that (9) is true:

(8) $(116 \vee 100) \supset 100$

(9) $(116 \vee 100) \supset 116$

This shows that No Opposite Materials fails on every margin for error frame when $h = 10$. But the choice of 10 inches for h was arbitrary. It is not hard to see that No Opposite Materials will fail on every margin for error frame, regardless of the value of h.[16] Any such frame will contain, for some positive real number i, three worlds: w_i, $w_{i+\frac{h}{2}}$, and $w_{i-\frac{h}{2}}$.



The right and left worlds see a world in common, w_i. The proposition that the tree is either $i + \frac{h}{2}$ inches tall or $i - \frac{h}{2}$ inches tall is consistent with what Magoo is sure of at each world. So the antecedent of No Opposite Materials is satisfied. But the consequent is not. $w_{i-\frac{h}{2}}$ does not see $w_{i+\frac{h}{2}}$ and $w_{i+\frac{h}{2}}$ does not see $w_{i-\frac{h}{2}}$. This means that Mr. Magoo is sure of 'opposite' material conditionals at $w_{i-\frac{h}{2}}$ and $w_{i+\frac{h}{2}}$. At $w_{i-\frac{h}{2}}$, Magoo is sure of (10) and at $w_{i+\frac{h}{2}}$ he is sure of (11):

(10) $((i + \frac{h}{2}) \vee (i - \frac{h}{2})) \supset (i - \frac{h}{2})$

(11) $((i + \frac{h}{2}) \vee (i - \frac{h}{2})) \supset (i + \frac{h}{2})$

[16] In Timothy Williamson, "Gettier Cases in Epistemic Logic," *Inquiry*, LVI, 1 (2013): 1-14, more complex frames are used to model the margin for error requirement. These frames treat worlds as ordered pairs $\langle j, k\rangle$, where j is the real height of the tree and k is the apparent height of the tree. Williamson defines R as follows: $\langle j, k\rangle R\langle j', k'\rangle$ just in case (1) $k = k'$ and (2) $|j' - k| \leq |j - k| + h$. These frames validate No Opposite Materials. However, we think that this is merely an artifact of Williamson's simplifying assumption that appearances are luminous, which proponents of margin for requirements should ultimately reject: (1) says that one world sees another only if the apparent height of the tree is the same in the two worlds. If we replace (1) with a constraint (1′), which says that $\langle j, k\rangle R\langle j', k'\rangle$ only if $|k' - k| \leq c$, for some positive constant c, No Opposite Materials will no longer be valid. To see this, suppose $c = 6$ and $h = 10$. Then $\langle 108, 108\rangle$ and $\langle 114, 114\rangle$ see each other, and $\langle 108, 108\rangle$ and $\langle 102, 102\rangle$ see each other, but $\langle 102, 102\rangle$ does not see $\langle 114, 114\rangle$ and $\langle 114, 114\rangle$ does not see $\langle 102, 102\rangle$. In this model, the antecedent of No Opposite Materials is satisfied: $\langle 102, 102\rangle$ and $\langle 114, 114\rangle$ see a world in common, namely $\langle 108, 108\rangle$. And the proposition that the tree is either 102 inches tall or 114 inches tall is consistent with what Magoo is sure of at all three worlds. But the consequent of No Opposite Materials is not satisfied. At $\langle 102, 102\rangle$, Magoo is sure of the material conditional $(102 \vee 114) \supset 102$; at $\langle 114, 114\rangle$, Magoo is sure of the 'opposite' material conditional $(102 \vee 114) \supset 114$. Thanks to Simon Goldstein and Bernhard Salow for discussion.

V. ENRICHING THE FRAMEWORK

In §4 we showed that The Qualitative Thesis entails No Opposite Materials in the standard variably strict framework. In §5 we showed that if we accept a margin-for-error requirement on rational sureness, we must reject No Opposite Materials. Putting these two things together, we conclude that if we accept the margin for error principle, we must reject The Qualitative Thesis.

We want to take a moment to explain the tension between No Opposite Materials and The Qualitative Thesis in a less formal, and hopefully more intuitive, way. Recall our three-world partial model of Mr. Magoo's sureness state. Call this model *Williamson's Tree*.

$$\begin{tikzcd} w_{100} \arrow[loop above] \arrow[r, leftrightarrow] & w_{108} \arrow[loop above] \arrow[r, leftrightarrow] & w_{116} \arrow[loop above] \end{tikzcd}$$

Consider the proposition that the tree is either 100 inches tall or 116 inches tall ($100 \vee 116$). At w_{100}, Magoo is sure that (8) is true, and at w_{116} Magoo is sure that (9) is true.

(8) $(100 \vee 116) \supset 100$

(9) $(100 \vee 116) \supset 116$

Suppose The Qualitative Thesis holds at both w_{100} and w_{116}. Then Magoo is sure of the indicative conditionals (12) and (13) at w_{100} and w_{116}, respectively.

(12) $(100 \vee 116) > 100$

(13) $(100 \vee 116) > 116$

Remember that w_{108} is consistent with what Magoo is sure of at w_{100} and it is consistent with what he is sure of at w_{116}. So, (12) and (13) are both true at w_{108}. This is where we run into trouble. If (12) is true at w_{108}, the the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{100}\}$. If (13) it true at w_{108}, the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{116}\}$.

But the selection function cannot meet both of these demands. The set of selected $(100 \vee 116)$-worlds at w_{108} can be a subset of $\{w_{100}\}$ and $\{w_{116}\}$ only if there are *no* selected $(100 \vee 116)$-worlds at w_{108}. But that would violate Non-Vacuity, which says that if an antecedent φ is doxastically possible at w, then the set of selected φ-worlds at w is not empty. We know that ($100 \vee 116$) is consistent with what Magoo is sure of w_{108}: w_{108} sees w_{100} and w_{116}. So, by Non-Vacuity, the set of selected ($100 \vee 116$)-worlds at w_{108} is not empty.

In models that violate No Opposite Materials, The Qualitative Thesis places inconsistent demands on the selection function. Putting the problem this way suggests a solution. Instead of just one selection function, which we use to evaluate an indicative relative to just any belief state, we have multiple selection functions, indexed to different belief states. This will allow us to validate The Qualitative Thesis in models like Williamson's Tree. Instead of placing incompatible demands on one selection function, we place different demands on different selection functions. The selection function indexed to Magoo's belief state at w_{100} will satisfy a version of the Indicative Constraint stated in terms of Magoo's belief worlds at w_{100}. The selection function indexed to what Magoo is sure of at w_{116} will satisfy a version of the Indicative Constraint stated in terms of Magoo's belief worlds at w_{116}.

VI. LOCAL CONTEXTS AND SHIFTY SELECTION FUNCTIONS

In this section, we develop the idea just sketched by making the conditional's contribution sensitive to its *local context.* We say that when a conditional occurs under an attitude verb, the conditional is evaluated relative to the local context introduced by the attitude verb. We validate The Qualitative Thesis using a version of the Indicative Constraint. But importantly, our account is not subject to the problem of conflicting demands. That is because the selection function used to interpret the conditional is indexed to the conditional's local context. When the local context changes, the selection function does, too. In the rest of this section, we develop our theory. In §7.1, we say more about what local contexts are, describing how they have been used in theories of presupposition and epistemic modality. In §7.2, we present our account: the *local, shifty* theory. In §7.3, we show how the theory validates The Qualitative Thesis while escaping the problem of conflicting demands.

VI.1. What are Local Contexts? Here's a standard idea. The interpretation of a sentence at a certain point in a conversation depends on the common commitments of the speakers at that point in the conversation. Starting in the early 1970s, theorists noticed that a sentence's *local* informational environment can also influence its interpretation. Specifically, how we interpret an expression in a sentence is partly determined by the information contained in the rest of the sentence.[17] The phenomenon of *presupposition projection* provides an illustration of

[17] See Robert Stalnaker, "Pragmatic Presuppositions," in Munitz, M. K. and Unger, P., eds, *Semantics and Philosophy* (New York: New York University Press, 1974): pp. 197-213.

this. Much contemporary research starts from the idea that a presupposition must be satisfied in the context in which it is uttered. But this won't do if by 'context' we mean the *global context*—the context of the conversation—modeled by a set of worlds representing the common commitments of the speakers. For consider (14):

(14) If Suzie used to smoke, she stopped smoking.

The consequent of (14) presupposes that Suzie used to smoke. But (14) can be felicitously uttered even when the speakers don't know that she used to smoke. This shows that the presupposition of the consequent of (14) need not be satisfied by the global context representing the common commitments of the speakers; instead, it only has to be satisfied relative to a kind of *local context* that also includes information present in the sentence but not in the global context — here that being the antecedent of the conditional, that Suzie used to smoke.

The notion of a local context has been taken up both by both *dynamic* and *static* approaches to meaning. In a dynamic semantics, the semantic value of a sentence is its *context change potential*, that is, the information it tends to *add* to the context. Formally, the meaning of a sentence is taken to be a function from a set of worlds to another set of worlds. Dynamic semantics offers a particular way to understand what a local context is. Roughly, we can think of the local context for a clause within a sentence as the context which that clause updates in the course of performing the update of the sentence as a whole. But local contexts are also intelligible on a static approach to meaning. Philippe Schlenker has developed a theory of local contexts in a static framework:[18] the local context for an expression is just whatever information would be redundant, if it were explicitly added to the embedding environment.

The notion of a local context has also been put to work in the semantics of epistemic modals. In particular, they have been used to explain the infelicity, both unembedded and embedded, of *epistemic contradictions*, sentences like:[19]

(15) # It's raining and it might not be raining.

See also Lauri Kartunnen, "Presupposition and Linguistic Context," *Theoretical Linguistics*, I, 1 (1974): 181-194. Finally, see Irene Heim, *The Semantics of Definite and Indefinite Noun Phrases*, PhD diss., University of Massachusetts, Amherst, 1982.

[18] See Philippe Schlenker, "Local Contexts," *Semantics and Pragmatics*, II, 3 (2009): 1 -78.

[19] See Seth Yalcin, "Epistemic Modals," *Mind*, CXVI, 464 (November 2007): 983-1026.

Epistemic contradictions are invariably defective. A natural explanation is that the epistemic modal conjunct takes for granted the information in its local context: (15) sounds bad because the local context for *it might not be raining* already entails that it *is* raining.[20]

This thought has been implemented in both traditions for thinking about local contexts. The dynamic approach to epistemic modality, defended by Veltman and Gillies among others, favors a *test semantics* for epistemic modals.[21] On this view, epistemic *might* tests whether its local context is consistent with its prejacent. This correctly predicts that epistemic contradictions are marked both outside and inside of embeddings. A similar idea has been developed within static frameworks. For example, Seth Yalcin's *domain semantics* adds an information state parameter to the index and stipulates that this parameter shifts under attitudes in much the way that Schlenker's local contexts do.[22] Matthew Mandelkern's *bounded theory* of epistemic modality draws an even tighter connection between epistemic modals and local contexts, arguing that the domain of quantification for an epistemic modal is determined by the modal's local context.[23] More specifically, Mandelkern claims that an epistemic modal must be interpreted relative to a modal base that takes any world w to a subset of the modal's local context. This constraint, which Mandelkern calls the *Locality Constraint*, accounts for the infelicity of epistemic contradictions, embedded and unembedded.

We will follow this precedent of employing local contexts in the semantics for epistemic vocabulary and advocate for the following thesis:

> **Conditional Locality**. The semantic contribution of a conditional under an attitude is determined by the local context supplied by the attitude.

Our first argument for this thesis will be that it resolves the tension between the Qualitative Thesis and margin for error principles. We will outline a static implementation of Conditional Locality that takes its cue from Mandelkern's approach to epistemic modals: we add to the index a local context parameter—a set of worlds representing the local

[20] This high-level description doesn't exactly capture what's going on with *all* theories employing something like the notion of a local context. Still, we think it is fair to say that this accurately characterizes a wide range of views.

[21] See Frank Veltman, "Defaults in Update Semantics," *Journal of Philosophical Logic*, XXV, 3 (June 1996): 221-61. Compare Anthony Gillies, "On the Truth-Conditions for *If* (but Not Quite Only *If*)," *Philosophical Review*, CXVIII, 3 (July 2009): 325-49.

[22] See Yalcin, "Epistemic Modals," *op. cit.*

[23] See Matthew Mandelkern, "Bounded Modality," *The Philosophical Review*, CXXVIII, 1 (January 2019): 1-61.

context—that shifts under embedding operators in the way Schlenker's algorithm predicts. Notice that Mandelkern's Locality Constraint bears a close similarity to the Indicative Constraint. The difference is that the Indicative Constraint is stated in terms of the *global context*—the set of worlds compatible with what the speakers believe—whereas the Locality Constraint is stated in terms of local contexts. In the next section, we propose to modify the Indicative Constraint so that it bears an even closer similarity to Mandelkern's Locality Constraint. However, we should emphasise that Conditional Locality is equally compatible with a dynamic approach to conditionals. We suspect similar results can be achieved by combining a test semantics for conditionals with a standard dynamic semantics for attitudes.[24]

VI.2. The Localized Indicative Contraint and Shifty Selection Functions. We will assume a variably strict theory of the indicative conditional. Where κ is the conditional's local context, here's our semantic entry.

> **Local, Shifty Variably Strict Semantics**. $[\![\varphi > \psi]\!]^{\kappa,w} = 1$ if and only if: $\forall w' \in f_\kappa(w, [\![\varphi]\!]^\kappa) : [\![\psi]\!]^{\kappa,w'} = 1$

The Local, Shifty Variably Strict Semantics is similar to the Standard Variably Strict Semantics. The difference is that there is a new parameter—a local context parameter—and the selection function is indexed to that parameter.[25] Since selection functions are indexed to

[24] We think it is worth showing how to achieve the goals of this paper in a static and classical variably strict framework and do not intend to argue at length here for this view over dynamic accounts. However, we will note one difference between the accounts. They differ over the validity of Or-to-if, which we formulate with an epistemic modal $\Diamond$ as:

> **Or-to-if**. $\Diamond\neg\varphi, \varphi \vee \psi \models \neg\varphi > \psi$

Dynamic accounts validate Or-to-if because they use non-classical consequence relations. Since we use a classical consequence relation, we do not validate or-to-if. (However, we can account for the reliability of Or-to-if *reasoning* using Stalnaker's notion of reasonable inference.)

We are inclined to think our account is right to *in*validate Or-to-if. Or-to-if is not probabilistically valid: it is easy to create counterexamples by focusing on cases where $[\![\varphi]\!]$ by itself accounts for most of the probability of $[\![\varphi \vee \psi]\!]$. If one is inclined to think, as we do, that valid inferences should preserve probability, then this feature of our account favors it over dynamic and informational theories. Thanks to Sam Carter and an anonymous referee for discussion.

[25] In this statement, the variably strict conditional does not shift the local context for the consequent. However, it is generally accepted in the literature on presupposition that the local context for the consequent of the consequent includes the information in the antecedent. To reflect this, we could modify the entry in the text as follows:

> **Local, Fully Shifty Variably Strict Semantics**. $[\![\varphi > \psi]\!]^{\kappa,w} = 1$ if and only if: $\forall w' \in f_\kappa(w, [\![\varphi]\!]^\kappa) : [\![\psi]\!]^{\kappa \cap [\![\varphi]\!]^\kappa, w'} = 1$

local contexts, we can impose constraints on selection functions that make reference to local contexts. We propose to replace Stalnaker's Indicative Constraint with the following *Localized Indicative Constraint*:[26]

> **Localized Indicative Constraint**. If $\mathbf{A} \cap \kappa \neq \emptyset$, then $\forall w' \in \kappa$: $f_\kappa(w', \mathbf{A}) \subseteq \kappa$

The Localized Indicative Constraint tells us that the selected antecedent worlds relative to a world w in the local context for the conditional must be a subset of the local context (so long as the antecedent is compatible with the local context).

With this new parameter, we restate the remaining constraints on the selection function.

> **Success**. $f_\kappa(w, \mathbf{A}) \subseteq \mathbf{A}$
>
> **Minimality**. If $w \in \mathbf{A}$, then $w \in f_\kappa(w, \mathbf{A})$.
>
> **Non-Vacuity**. If $\kappa \cap \mathbf{A} \neq \emptyset$ then $f_\kappa(w, \mathbf{A}) \neq \emptyset$.

Success says that the selected **A**-worlds are a subset of **A**. Minimality says that if w is an **A**-world, then w is one of the selected **A**-worlds at w. We assume Success and Minimality for the same reasons as the standard framework does. Non-Vacuity says that if there are some **A**-worlds in κ, then the set of selected **A**-worlds at w is not empty. This constraint guarantees a *local* version of Weak Conditional Non-Contradiction: whenever there are φ-worlds in κ, at most one of $\varphi > \psi$ and $\varphi > \neg\psi$ can be true at a point of evaluation $\langle \kappa, w \rangle$.

We said that selection functions are indexed to local contexts and obey the Localized Indicative Constraint. The reason this matters, of course, is that local contexts are *shiftable*. In particular, they can be shifted by attitude predicates, such as *believe*, *want*, and, our focus in this paper, *is sure that*. Following Schlenker, we assume that the local context introduced by an attitude predicate like *is sure that* at a world w is the set of worlds compatible with what the subject is sure of in w.[27] Where R is a doxastic accessibility relation representing what an

---

For simplicity, we stick with the formulation in the text. However, see footnote 23 for further discussion of the above entry.

[26] Matthew Mandelkern has independently developed a version of the Localized Indicative Constraint. See Matthew Mandelkern, "If p, then p!," forthcoming in this JOURNAL. There are important differences between our constraint and Mandelkern's, however. The main difference is that Mandelkern's constraint is stated as a definedness condition on the interpretation of the conditional, whereas our constraint is stated as a constraint on the selection function.

[27] See Philippe Schlenker, "Local Contexts," *op. cit.*

arbitrary subject is sure of and $R(w)$ is the set of worlds compatible with what that subject is sure of in w:

> **Shifty Hintikka Semantics**. $[\![S\varphi]\!]^{\kappa,w} = 1$ if and only if: $\forall w' \in R(w) : [\![\varphi]\!]^{R(w),w'} = 1$

Shifty Hintikka Semantics treats 'is sure that' as a necessity operator, just as the standard Hintikka semantics does. But now we've added a new parameter, a local context parameter, to the index. Shifty Hintikka Semantics says that attitude operators shift this parameter to $R(w)$, the set of worlds compatible with what the subject is sure of in w. This means that when we evaluate an attitude ascription like ⌜Magoo is sure that $\varphi > \psi$⌝ at a world w, we evaluate the embedded conditional relative to Magoo's belief state *at w*. As we show in the next section, this is exactly what we need to validate The Qualitative Thesis without falling prey to the problem of conflicting demands.

VI.3. Local, Shifty Indicatives and The Qualitative Thesis. We leave the proof to Appendix B, but here's an informal explanation of why QT, repeated below, is valid on our account.

> **QT**. $\neg S\neg\varphi \supset (S(\varphi \supset \psi) \equiv S(\varphi > \psi))$

It will be useful to divide the thesis into two theses and take them in turn.

> **Indicative-to-Material**. $\neg S\neg\varphi \supset (S(\varphi > \psi) \supset S(\varphi \supset \psi))$
>
> **Material-to-Indicative**. $\neg S\neg\varphi \supset (S(\varphi \supset \psi) \supset S(\varphi > \psi))$

Begin with Indicative-to-Material. Suppose that, in an arbitrary world w, you are not sure of $\neg\varphi$ and you are sure of the indicative $\varphi > \psi$. Consider an arbitrary world w' that is compatible with what you are sure of in w. We know that $\varphi > \psi$ is true at w'. To show that $\varphi \supset \psi$ is true at w', suppose that φ is true at w'. Minimality tells us that if φ is true in w', then w' is among the selected φ-worlds at w'. Since $\varphi > \psi$ is true at w', it follows that ψ is true at w'. So, the material conditional $\varphi \supset \psi$ is true at w'. Since w' was chosen arbitrarily, we conclude that $\varphi \supset \psi$ is true at every world compatible with what you are sure of in w. You are sure of the material conditional $\varphi \supset \psi$ in w.

Turn to Material-to-Indicative. Suppose that, in an arbitrary world w, you are not sure of $\neg\varphi$ and you are sure of the material conditional $\varphi \supset \psi$. Consider an arbitrary world w' that is compatible with what you are sure of in w. We want to show that $\varphi > \psi$ is true in w'. Since you are not sure that $\neg\varphi$ in w, the Localized Indicative Constraint tells us that selected all of the selected φ-worlds at w', *relative to your belief state in* w, are compatible with what you are sure of in w. Since you are sure of the material conditional $\varphi \supset \psi$ in w, all of these selected φ-worlds must be ψ-worlds. It follows that $\varphi > \psi$ is true at w' relative to your belief state in w. Since w' was chosen arbitrarily, we conclude that $\varphi > \psi$ is true, relative to your belief belief state in w, at every world compatible with what you are sure of in w. And that, according to **Shifty Hintikka Semantics**, is just what it takes for you to be sure of $\varphi > \psi$ in w.[28]

[28] Whether QT holds in full generality depends on whether we adopt the simplified variably strict semantics in the main text or the entry in footnote 22. Given the latter, both Indicative-to-Material and Material-to-Indicative fail for right-nested conditionals (though they continue to hold for non-conditional antecedents and consequents). In both cases, this is because the material does not shift the local context for the consequent while the indicative does. For a model where Material-to-Indicative fails, consider the following:

$$R(w_1) = \{w_1, w_2\};$$

$$V(p) = \{w_1\};\ V(q) = V(r) = \{w_2\};$$

$$f_{R(w_1)\cap[\![p]\!]^{R(w_1)}}(V(q), w) = \{w_1, w_2\}$$

Here $\neg S\neg p$ and $S(p \supset (q > r))$ are true at w_1 but $S(p > (q > r))$ is false. (Note that since there simply are no $p \wedge q$-worlds here, the constraints on the selection function are satisfied vacuously.) For a model where Indicative-to-Material fails, consider:

$$R(w_1) = \{w_1, w_2, w_3\};$$

$$V(p) = \{w_2, w_3\};\ V(q) = \{w_2, w_3\};\ V(r) = \{w_3\};$$

$$f_{R(w_1)}(V(q), w_1) = \{w_3\};\ f_{R(w_1)\cap[\![p]\!]^{R(w_1)}}(V(q), w_1) = \{w_2\}.$$

Here $\neg S\neg p$ and $S(p > (q > r))$ are true at w_1 but $S(p \supset (q > r))$ is false.

However, we are in fact inclined to endorse the counterexamples to QT in both directions. To turn the above into an intuitive counterexample to Indicative-to-Material, consider the following situation. We don't know whether Alice came to the party. We know that Alice came iff Billy did not. And we know that either Billy didn't come or Carol did too. Here you should be sure that if Billy came, then so did Carol. So, since you are sure of the right disjunct, you should be sure that

(16) Either Alice didn't come or if Billy came then so did Carol.

However, it does not sound true to say:

(17) If Alice came, then if Billy came then Carol came.

After all if Alice came then we know Billy didn't. (Perhaps this example is be best treated as involving a presupposition failure by adding a presupposition that the local context contains some antecedent worlds; but (17) certainly should not be predicted to be true.)

For a counterexample to Material-to-Indicative, consider the following situation. Imagine I take a normal pack of cards and remove the Jack of Clubs. You must now remove the top card from the deck. Here the following seems true:

(18) If the top card is black, then if the top card is a jack, it's the Jack of Spades.

That concludes our informal explanation of why The Qualitative Thesis is valid. The last thing to do is explain why we do not fall prey to the problem of conflicting demands in models where No Opposite Materials fails. So that we have everything in front of us, here is Williamson's Tree again.

$$
\begin{tikzcd}
w_{100} \arrow[loop above] \arrow[r, leftrightarrow] & w_{108} \arrow[loop above] \arrow[r, leftrightarrow] & w_{116} \arrow[loop above]
\end{tikzcd}
$$

Magoo is sure of the material conditional (8) in w_{100} and he is sure of the material conditional (9) in w_{116}.

(8) $(100 \vee 116) \supset 100$

(9) $(100 \vee 116) \supset 116$

In the standard variably strict framework, there is no way to guarantee that The Qualitative Thesis holds at both w_{100} and w_{116} without placing conflicting demands on the selection function at the overlap world w_{108}. To secure The Qualitative Thesis w_{100}, the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{100}\}$; otherwise $(100 \vee 116) > 100$ would be false at w_{108}, and so Magoo would not be sure of it at w_{100}. To secure The Qualitative Thesis at w_{116}, the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{116}\}$; otherwise $(100 \vee 116) > 116$ would be false at w_{108} Magoo would not be sure of it at w_{116}. The selection function cannot meet both of these demands on pain of violating Non-Vacuity.

On the other hand, the following strikes us as dubious.

(19) Either the top card is not red (i.e. non-black), or if the top card is a jack, it's the Jack of Spades.

It's not obvious to us in assessing the right disjunct we hold fixed that the top card is black.

There is one further complication here. We have been assuming that the material conditional is paraphrasable in terms of disjunction. But many in the literature on presupposition think that in a disjunction the local context for the right disjunct is the negation of the left disjunct. If so, then, assuming the standard truth-table for the material conditional, $\varphi \supset \psi$ will not in general be equivalent to $\neg(\varphi \vee \psi)$; in particular the equivalence will fail in cases where the right disjunct is a conditional. This would make it more difficult to evaluate the empirical consequences of our prediction that QT fails for embedded conditionals, as these are usually assessed by considering the equivalent disjunction. (Though note that the truth of (19) above seems to push against treating disjunction this way.)

In the Local, Shifty framework, by contrast, different belief states correspond to different selection functions. When we evaluate an indicative conditional relative to Magoo's belief state at w_{116}, we use one selection function; when we evaluate a conditional relative to his belief state at w_{100}, we use a different selection function. Consider (20) and (21):

(20) $[\![\text{Magoo is sure that: } 100 \vee 116 > 100]\!]^{\kappa}$

(21) $[\![\text{Magoo is sure that: } 100 \vee 116 > 116]\!]^{\kappa}$

Where R is an accessibility relation representing Magoo's beliefs, (20) is true at w_{100} just in case (22) is true at every world in $R(w_{100})$: w_{100} and w_{108}. (21) is true at w_{116} just in case (23) is true at every world in $R(w_{116})$: w_{108} and w_{116}.

(22) $[\![(100 \vee 116) > 100]\!]^{R(w_{100})}$

(23) $[\![(100 \vee 116) > 116]\!]^{R(w_{116})}$

But (22) and (23) do not place incompatible demands on the selection function at the overlap world w_{108}. (22) is true at w_{108} only if the selected $(100 \vee 116)$-world at w_{108}, *relative to Magoo's belief state at* w_{100}, is w_{100}, whereas (23) is true at w_{108} only if the selected $(100 \vee 116)$-world at w_{108}, *relative to Magoo's belief state at* w_{116}, is w_{116}. These are simply different demands on different selection functions, so there is no inconsistency.

VII. THE STRONG QUALITATIVE THESIS

At the start of this paper we said that we would give two arguments for Conditional Locality. We have completed the first argument: without Conditional Locality, any plausible precisification of the Qualitative Thesis will have unwelcome epistemological consequences. Now we move to our second argument. We will argue that the correct precisification must be stronger than the Local Qualitative Thesis. In this section, we introduce what we call the *Strong Qualitative Thesis*. We then show that, without Conditional Locality, the Strong Qualitative Thesis has unacceptable, trivializing consequences. But with Conditional Locality, it is tenable.

VII.1. Motivating The Strong Qualitative Thesis. Recall the Local Qualitative Thesis, which we stated and characterised in the first half of the paper:

> **The Local Qualitative Thesis**. For any world w and context c, if $\neg S^{c,w}([\![\neg\varphi]\!]^c)$, then: $S^{c,w}([\![\varphi > \psi]\!]^c)$ if and only if $S^{c,w}[\![\varphi \supset \psi]\!]^c$.

To see why this thesis is too weak, let's look at an example. Suppose I'm wondering where Matt is, and you have reason to believe that Alice knows. You check with Alice and report back that Alice is sure that Matt is either in Los Angeles or London, but she's not sure which. So (24) is true in my context.

(24) Alice is sure that Matt is either in Los Angeles or London, but she's not sure which.

From (24), I infer (25).

(25) So, Alice is sure that if Matt is not in Los Angeles, he's in London.

The inference is reasonable, nay obligatory: if I accept (24), I must also accept (25). Observe that (26) seems to attribute to Alice an incoherent state of mind:

(26) # Alice is sure that Matt is either in Los Angeles or London, but she's not sure that he's in London if he's not in Los Angeles.

Just as you can't accept $\varphi \vee \psi$ while denying $\neg\varphi > \psi$, you can't coherently describe *others* as accepting $\varphi \vee \psi$ while denying $\neg\varphi > \psi$.

But that is not what the Local Qualitative Thesis predicts. Suppose that I am the speaker in context c, the context relative to which we are interpreting our conditional operator $>$. Then the Local Qualitative Thesis says that if I am sure that Matt is either in Los Angeles or London, and I'm not sure that Matt is not in Los Angeles, then I am sure of the conditional expressed by *if Matt is not in Los Angeles, he's in London* relative to my information. But the Local Qualitative Thesis does not predict that (25) follows from (24) in my context. If we want to predict that (25) follows from (24), we need a version of the Qualitative Thesis that applies to *any* attitude operator in our language, one that applies to all subjects, regardless of what context they're in, or what information they have. To state such a thesis, we enrich our language from §4. Where $\mathcal{A}$ is a finite set of names, we add the operator S^s to our language, for each $s \in \mathcal{A}$. $S^s\varphi$ says that s is sure that φ; or more precisely, where R_s is an accessibility relation representing what s is sure of, we have:

Generalized Standard Hintikka Semantics. $[\![S^s\varphi]\!]^{c,w} = 1$ iff $\forall w' \in R_s(w) : [\![\varphi]\!]^{c,w'} = 1$

This allows us to state the *Strong QT*, which says that for any s, the following holds:

Strong QT. $\neg S^s\neg\varphi \supset (S^s(\varphi \supset \psi) \equiv S^s(\varphi > \psi))$

The Strong QT goes beyond the Local Qualitative Thesis in just the way we wanted: sentences like (26) are incoherent.

VII.2. Strong QT without Conditional Locality. We've introduced Strong QT and argued that it is desirable. Now we argue that in the standard framework, Strong QT is untenable because it has trivialising consequences.

First compare what Local QT and Strong QT require of the standard framework. Local QT requires coordination between the speaker's material conditional beliefs and their beliefs in their own conditional (i.e. what the indicative expresses in their context). Strong QT requires far more than this: it requires coordination between *any given person*'s material conditional beliefs and their beliefs about the *speaker's* conditional.

Even seen at this high level, we should expect trouble. Each instance of the Strong QT will require that there is coordination between the speaker's selection function for the conditional and the relevant agent's doxastic state. But even within a single world, different people will be sure of lots of different, perhaps conflicting, things. If there is enough divergence in what different people are sure of, it is hard to see how Strong QT could hold in a sensible way — it would require the same conditional to be coordinated with too many diverging doxastic states.

We make this worry precise in the form of a triviality result. Here is what we show. Assuming sureness is probability 1, one particular consequence of Strong QT is the following:

> **The Global Probability 1 Thesis**. For any subject s, context c, and world w: if $P_{s,w}(\llbracket\varphi\rrbracket^c) > 0$, then $P_{s,w}(\llbracket\psi\rrbracket^c|\llbracket\varphi\rrbracket^c) = 1$ iff $P_{s,w}(\llbracket\varphi > \psi\rrbracket^c) = 1$

This says that, when they assign φ non-zero probability, *anyone* should have probability 1 in the speaker's conditional $\varphi > \psi$ just in case they give ψ probability 1, conditional on φ. We then prove that the Global Probability 1 Thesis entails:

> **Triviality**. Any subject who assigns positive probability to $\llbracket\varphi\rrbracket^c$ conditional on $\llbracket\psi\rrbracket^c$ and positive probability to the conditional $\llbracket\varphi > \psi\rrbracket^c$ is certain of $\llbracket\neg\psi\rrbracket^c$ conditional on $\llbracket\neg(\varphi > \psi)\rrbracket^c$.

But this result is absurd. Suppose I am not sure whether it is raining out or not. And I'm not sure whether we will have a picnic if it is sunny out. I should not become sure that there will be no picnic if I simply learn that *it is not the case that if it's sunny out, we're having a picnic*; I can learn the conditional *if it's sunny out, we're having a picnic* is false and still not know whether or not we are going to have a picnic. In general, when you learn the negation of a conditional you do not thereby learn the negation of its consequent.

First, let's derive the Global Probability 1 Thesis. We assume that sureness is probability 1, or more precisely:

(27) $R_s(w) \subseteq \mathbf{A}$ iff $P_{s,w}(\mathbf{A}) = 1$

Given the Generalised Standard Hintikka Semantics, we can rewrite this as the principle we call the Non-Shifty Link:

Non-Shifty Link. $[\![S^s\varphi]\!]^{c,w} = 1$ iff $P_{s,w}([\![\varphi]\!]^c) = 1$

Now note that, as a matter of pure probability, when $[\![\varphi]\!]^c$ has positive probability, then $[\![\varphi \supset \psi]\!]^c$ has probability 1 just in case $[\![\psi]\!]^c$ has probability 1, conditional on $[\![\varphi]\!]^c$:

(28) If $P_{s,w}([\![\varphi]\!]^c) > 0$, then $P_{s,w}([\![\varphi \supset \psi]\!]^c) = 1$ iff $P_{s,w}([\![\psi]\!]^c|[\![\varphi]\!]^c) = 1$

Given this fact, the Non-Shifty Link and Strong QT straightforwardly yield the Global Probability 1 Thesis.

Now let's see why the Global Probability 1 Thesis entails Triviality. Consider two subjects, s and s'. Suppose that the probability function of s' in w is s's probability function in w conditionalized on $[\![\psi]\!]^c$. Formally:

(29) $P_{s',w} = P_{s,w}(\cdot|[\![\psi]\!]^c)$

We make two assumptions: that s assigns positive probability to $[\![\varphi]\!]^c$ conditional on $[\![\psi]\!]^c$ and that s assigns positive probability to $[\![\varphi > \psi]\!]^c$. Formally:

(30) $P_{s,w}([\![\varphi]\!]^c|[\![\psi]\!]^c) > 0.$

(31) $P_{s,w}([\![\varphi > \psi]\!]^c) > 0$

(29) and (30), together with the probability axioms, yield the following facts:

(32) $P_{s',w}([\![\varphi]\!]^c) = P_{s,w}([\![\varphi]\!]^c|[\![\psi]\!]^c) > 0$

(33) $P_{s',w}([\![\psi]\!]^c) = P_{s,w}([\![\psi]\!]^c|[\![\psi]\!]^c) = 1$

Assuming the Global Probability 1 Thesis, (32) and (33) entail (34):

(34) $P_{s',w}([\![\varphi > \psi]\!]^c) = Pr_{s',w}([\![\psi]\!]^c|[\![\varphi]\!]^c) = 1$

But now remember that the probability function of s' is the probability function that results from conditionalizing s's probability function on $[\![\psi]\!]^c$. This means that (34) entails (35):

(35) $P_{s,w}(\llbracket\varphi > \psi\rrbracket^c | \llbracket\psi\rrbracket^c) = 1$

And finally (31), (35), and the probability axioms give us (36).

(36) $P_{s,w}(\llbracket\neg\psi\rrbracket^c | \llbracket\neg(\varphi > \psi)\rrbracket^c) = 1$

We have now derived our triviality result: any subject who assigns positive probability to $\llbracket\varphi\rrbracket^c$ conditional on $\llbracket\psi\rrbracket^c$ and positive probability to the conditional $\llbracket\varphi > \psi\rrbracket^c$ is certain of $\llbracket\neg\psi\rrbracket^c$ conditional on $\llbracket\neg(\varphi > \psi)\rrbracket^c$.

A simple example showed us that this consequence is absurd. But we can bring this into sharper relief by thinking about what a negated conditional might mean. Some theorists say that negated indicatives are the duals of negated indicative conditionals; that is, they think that a negated conditional like:

(37) It's not the case that if it rains, there will be a picnic.

is equivalent to:

(38) If it rains, there might not be a picnic.

Given our result above, this would have the consequence that upon learning (38), you should become sure that there will not be a picnic. This consequence is to be rejected.

Other theories subscribe to the principle of Conditional Excluded Middle, which says that $(\varphi > \psi) \vee (\psi > \neg\psi)$ is always true. Given Weak Conditional Non-Contradiction, this means that when you leave open φ, $\neg(\varphi > \psi)$ and $\varphi > \neg\psi$ are equivalent. So, on such a theory, (37) is equivalent to:

(39) If it rains, there will not be a picnic.

Given our result above, this would have the consequence that upon learning (39) you should become sure that there will not be a picnic. Again, this consequence is to be rejected.

We claimed that adding the Strong QT to the standard framework would spell trouble and we have now made good on our claim. Strong QT forces coordination between the material conditional beliefs of various subjects and their beliefs in the speaker's indicative conditional. As we might have expected from Lewis's results, this trivialises the standard framework. We need another option if we are to validate the Strong QT.

VII.3. Strong QT with Conditional Locality. We have argued that Strong QT is untenable in the standard framework. With Conditional Locality, on the other hand, Strong QT is tenable. In particular, it is valid and non-trivializing in our Local, Shifty framework.

To see why Strong QT is valid, it's helpful to contrast the Indicative Constraint and the Localized Indicative Constraints:

> **Indicative Constraint**. If $R(w) \cap \mathbf{A} \neq \emptyset$, then if $w' \in R(w)$, then $f(w', \mathbf{A}) \subseteq R(w)$.
>
> **Localized Indicative Constraint**. If $\mathbf{A} \cap \kappa \neq \emptyset$, then if $w' \in \kappa$: $f_\kappa(w', \mathbf{A}) \subseteq \kappa$

On the standard, non-shifty variably strict semantics, which does not accept Conditional Locality, there is just one selection function. The Indicative Constraint coordinates this selection function with a *specific* accessibility relation—the accessibility relation relative to which we interpret S_c, the attitude operator corresponding to the speaker of the context. The selection function remains coordinated with that accessibility relation even when the conditional is embedded under other attitude operators that are interpreted relative to different accessibility relations.

Suppose, for example, that I am the speaker in c. And suppose that Alice, whose information differs from mine, is sure of the material conditional $[\![\varphi \supset \psi]\!]^c$. Does it follow that Alice is sure of the indicative conditional $[\![\varphi > \psi]\!]^c$? It doesn't. Suppose there is some world w that is compatible with what I am sure of and with what Alice is sure of. By the Indicative Constraint, the selected φ-worlds at w are a subset of the worlds compatible with what *I* am sure of, not the set of worlds compatible with what *Alice* is sure of. If there are worlds compatible with what I'm sure of where $[\![\varphi]\!]^c$ is true but $[\![\psi]\!]^c$ is not, then these selected φ-worlds may not be ψ-worlds, and in that case, $[\![\varphi > \psi]\!]^c$ will be false at w. Since w is compatible with what Alice is sure of, it follows that Alice is not sure of the indicative conditional $[\![\varphi > \psi]\!]^c$.

The Localized Indicative Constraint works differently. It picks the selected worlds from *whatever the local context for the conditional is.* More precisely, if there are φ-worlds in the local context, then the selected φ-worlds must be in the local context. The Shifty Hintikka Semantics ensures that for *any* sureness operator S^x, the local context for a conditional embedded under that operator is the set of worlds compatible with what x is sure of in the world of evaluation. The interpretation of the conditional is coordinated with the subject of the attitude clause. The Shifty Hintikka Semantics and the Localized Indicative Constraint combine to guarantee the validity of Strong QT. The precise explanation proceeds in just the same way as the explanation of why QT is valid given in §7.3.

Finally, turn to triviality. Without Conditional Locality, we are forced to reject Strong QT or face trivialization. But if we accept Conditional

Locality, and thus reject the Standard Hintikka Semantics in favor of the Shifty Hintikka Semantics, Strong QT does not enforce the kind of problematic coordination as it did in the standard framework. Compare and contrast what the Strong QT requires of Alice and Billy in this framework: it requires that Alice is sure of the proposition expressed by $\varphi > \psi$ relative to her information just in case she is sure of the corresponding material; and that Billy is sure of the proposition expressed by $\varphi > \psi$ relative to *his* information just in case he is sure of the material conditional. Their two conditionals will be different when they have different information. The only coordination required by Strong QT now is of a safer, more desirable kind, that between a person's material conditional beliefs and their beliefs in their *own* conditional.

Putting it another way, observe that the Shifty Hintikka Semantics does not entail the Non-Shifty Link; instead, it entails the following *shifty* link.

Shifty Link. $[\![S^s\varphi]\!]^{c,\kappa,w} = 1$ iff $P_{s,w}([\![\varphi]\!]^{c,R(w)}) = 1$

The Shifty Link says that the sentence $S^s\varphi$ is true in a context c just in case s has probability 1 in the proposition expressed by φ *relative to the local context introduced by the attitude predicate.*

To see why the Shifty Hintikka Semantics gives us the Shifty Link, note that can rewrite the semantic entry as follows.

(40) $[\![S^s\varphi]\!]^{c,\kappa,w} = 1$ if and only if: $R_s(w) \subseteq [\![\varphi]\!]^{c,R(w)}$

We also assume that an agent assigns probability 1 to a proposition $\mathbf{A}$ in a world w just in case the set of worlds compatible with what she is sure of in w is a subset of $\mathbf{A}$. We repeat this assumption below.

(21) $P_{s,w}(\mathbf{A}) = 1$ iff $R_s(w) \subseteq \mathbf{A}$

The Shifty Link follows from (40) and (27). The Non-Shifty Link, on the other hand, fails.[29]

[29] Here is a simple counterexample. Assume there are just three worlds: w_1, where Matt is in London; w_2, where Matt is in Paris; and w_3, where Matt is in Los Angeles. Suppose w_1 and w_3 are compatible with what Billy is sure of in w_3 and w_2 and w_3 are compatible with what Alice is sure of in w_3. Now take a context where Alice is the speaker; so the global context for the conditional:

(41) If Matt isn't in Los Angeles, he's in London.

is what *Alice* is sure of. In other words, $[\![$If Matt isn't in Los Angeles, he's in London$]\!]^{c,\kappa_c}$ is the same proposition as $[\![$If Matt isn't in Los Angeles, he's in London$]\!]^{c,R_{Alice}(w_3)}$. Billy does *not* assign this conditional probability 1 at w_3, for it is false at w_3: this conditional

The reason we avoid triviality is that the Shifty Link does not entail the Global Probability 1 Thesis; instead, it entails the Local Probability 1 Thesis:

> **The Local Probability 1 Thesis.** For any subject s, context c and world w: if $P_{s,w}(\llbracket\varphi\rrbracket^{c,R_s(w)}) > 0$, then $P_{s,w}(\llbracket\psi\rrbracket^{c,R_s(w)}|\llbracket\varphi\rrbracket^{c,R_s(w)}) = 1$ iff $P_{s,w}(\llbracket\varphi > \psi\rrbracket^{c,R_s(w)}) = 1$

The Local Probability 1 Thesis is weaker than the Global Probability 1 Thesis. It doesn't say that just *anyone* must assign probability 1 to $\llbracket\varphi > \psi\rrbracket^{c,R_s(w)}$ just in case they assign probability 1 to $\llbracket\psi\rrbracket^{c,R_s(w)}$ conditional on $\llbracket\varphi\rrbracket^{c,R_s(w)}$. It says that s must assign probability 1 to $\llbracket\varphi > \psi\rrbracket^{c,R_s(w)}$ just in case s assigns probability 1 to $\llbracket\psi\rrbracket^{c,R_s(w)}$ conditional on $\llbracket\varphi\rrbracket^{c,R_s(w)}$. The equation holds only when the evidence determining the probability function and the evidence determining the interpretation of the conditional are identical.

This localization blocks the triviality result from earlier. Assume again that $P_{s',w}$ is the probability function that results from conditionalizing $P_{s,w}$ on $\llbracket\psi\rrbracket^c$ and that $\llbracket\varphi\rrbracket^c$ and $\llbracket\psi\rrbracket^c$ are compatible relative to $P_{s,w}$; that is, that s assigns positive probability to $\llbracket\varphi\rrbracket^c$ conditional on $\llbracket\psi\rrbracket^c$.[30] The Global Probability 1 Thesis entails that s' has probability 1 in $\llbracket\varphi > \psi\rrbracket^{c,R_s(w)}$; that is, they assign probability 1 to what $\varphi > \psi$ expresses in s's context. Since $P_{s',w}$ is $P_{s,w}$ conditionalized on $\llbracket\psi\rrbracket^c$, this would allow us to conclude that s assigns probability 1 to their *own* conditional $\llbracket\varphi > \psi\rrbracket^{c,R_s(w)}$ conditional on $\llbracket\psi\rrbracket^c$; which, in turn, would mean that s was certain of $\llbracket\neg\psi\rrbracket^c$ conditional on $\llbracket\neg(\varphi > \psi)\rrbracket^{c,R_s(w)}$. But the Local Probability 1 Thesis does not have this consequence. That's because it does not require s' to have probability 1 in s's conditional, $\llbracket\varphi > \psi\rrbracket^{c,R_s(w)}$, in virtue of having probability 1 in $\llbracket\psi\rrbracket^c$ conditional on $\llbracket\varphi\rrbracket^c$; it only requires s' to have probability 1 in her *own* conditional, $\llbracket\varphi > \psi\rrbracket^{c,R_{s'}(w)}$, when she has probability 1 in $\llbracket\psi\rrbracket^{c,R_{s'}(w)}$ conditional on $\llbracket\varphi\rrbracket^{c,R_{s'}(w)}$. This means that s can have non-extreme credence in $\llbracket\neg\psi\rrbracket^c$ conditional on $\llbracket\neg(\varphi > \psi)\rrbracket^{c,R_s(w)}$.

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is about what *Alice* is sure of; and at w_3 Alice leaves open a world where Matt is in Paris, not London. So we have

(42) $P_{Billy,w_3}(\llbracket\text{If Matt isn't in Los Angeles, he's in London}\rrbracket^{c,\kappa_c}) \neq 1$

But in all worlds compatible with what Billy is sure of and where Matt isn't in Los Angeles, he's in Paris; so the following is true in c:

(43) Billy is sure that if Matt isn't in Los Angeles, he's in Paris.

[30] Note that, in the shifty framework, $\llbracket\varphi\rrbracket^c$ is $\llbracket\varphi\rrbracket^{c,\kappa_c}$, where κ_c is the set of worlds compatible with what the speaker in c is sure of.

VIII. CONCLUSION

The Qualitative Thesis is a plausible thesis about the indicative conditional: one direction is secured by Modus Ponens; the other by the Direct Argument and Stalnaker's Thesis. We gave two arguments that Conditional Locality is necessary to fully vindicate the Qualitative Thesis. First we argued that the weakest plausible precisification of the Qualitative Thesis has problematic epistemological consequences in standard frameworks: it is incompatible with the margin for error principle, a plausible principle about the nature of rational sureness. Second, we argued for a specific precisification of the Qualitative Thesis, the Strong Qualitative Thesis, but showed that it trivialises in standard frameworks. We proposed the Local, Shifty theory of conditionals, where the interpretation of a conditional is sensitive to its local context, and we assumed that attitude operators shift that local context. We showed that the resulting framework resolves both issues, allowing it to fully vindicate the Qualitative Thesis.

APPENDIX A. THE QUALITATIVE THESIS IN STANDARD FRAMEWORKS

A.1. The Variably Strict Framework. Our language $\mathcal{L}$ is the smallest set of sentences generated by the following grammar:

- $\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi > \psi \mid S\varphi$

A *variably strict frame* $\mathcal{F}$ is a tuple $\langle W, R, f\rangle$. W is a non-empty set of worlds. R is a binary relation on W representing doxastic accessibility: wRw' means that w' is compatible with what the subject is sure of in w. f, the selection function, is a function from a world and a set of worlds to a set of worlds and is used to interpret the conditional operator: $f(\mathbf{A}, w)$ is the set of selected $\mathbf{A}$-worlds at w. We say that a *normal* variably strict frame is any variably strict frame $\langle W, R, f\rangle$ such that f obeys the following constraints.

Success. $f(w, \mathbf{A}) \subseteq \mathbf{A}$

Minimality. If $w \in \mathbf{A}$, then $w \in f(w, \mathbf{A})$

Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $f(w, \mathbf{A}) \neq \emptyset$

We interpret the language with a model $\mathcal{M} = \langle \mathcal{F}, V\rangle$. $\mathcal{F}$ is a variably strict frame and V a function from propositional variables to sets of worlds. We recursively define truth at point in W:

$[\![p]\!]^w = 1$ iff $w \in V(p)$

$[\![\neg\varphi]\!]^w = 1$ iff $[\![\varphi]\!]^w = 0$

$$[\![\varphi \wedge \psi]\!]^w = 1 \text{ iff } [\![\varphi]\!]^w = [\![\psi]\!]^w = 1$$

$$[\![\varphi > \psi]\!]^w = 1 \text{ iff } f(w, [\![\varphi]\!]) \subseteq [\![\psi]\!]$$

$$[\![S\varphi]\!]^w = 1 \text{ iff } \forall w' \in R(w) : [\![\varphi]\!]^{w'} = 1$$

where $[\![\varphi]\!] = \{w : [\![\varphi]\!]^w = 1\}$.

Recall from section 4 our object language version of the The Qualitative Thesis:

QT $\neg S \neg \varphi \supset (S(\varphi > \psi) \equiv S(\varphi \supset \psi))$

And recall:

> **Indicative Constraint**. If $R(w) \cap \mathbf{A} \neq \emptyset$, then if $w' \in R(w)$, then $f(w', \mathbf{A}) \subseteq R(w)$.

Fact 1. QT is valid on a normal frame $\mathcal{F}$ iff $\mathcal{F}$ meets the Indicative Constraint.

Proof. $\Leftarrow$: We split QT into the following two principles and show that both must be valid on $\mathcal{F}$, if it meets the Indicative Constraint:

QT$_\Rightarrow$ $\neg S \neg \varphi \supset (S(\varphi > \psi) \supset S(\varphi \supset \psi))$

QT$_\Leftarrow$ $\neg S \neg \varphi \supset (S(\varphi \supset \psi) \supset S(\varphi > \psi))$

First we show QT$_\Rightarrow$ cannot fail on a normal frame $\mathcal{F}$. Suppose for contradiction it did. Then, for some w, $[\![\neg S \neg \varphi]\!]^w = [\![S(\varphi > \psi)]\!]^w = 1$ but $[\![S(\varphi \supset \psi)]\!]^w = 0$. So, for some $w' \in R(w) : [\![\varphi]\!]^{w'} = 1$ but $[\![\psi]\!]^{w'} = 0$. But, by Minimality, $w' \in f([\![\varphi]\!], w')$. So $[\![\varphi > \psi]\!]^{w'} = 0$ and $[\![S(\varphi > \psi)]\!]^w = 0$ after all; contradiction. So QT$_\Rightarrow$ holds on any normal frame; and in particular it holds on any normal frame that meets the Indicative Constraint.

Now suppose that QT$_\Leftarrow$ fails on $\mathcal{F}$. Then, for some w, $[\![\neg S \neg \varphi]\!]^w = [\![S(\varphi \supset \psi)]\!]^w = 1$ but $[\![S(\varphi > \psi)]\!]^w = 0$. So, for some $w' \in R(w)$, $[\![\varphi > \psi]\!]^{w'} = 0$. So there is some w'' such that $w'' \in f([\![\varphi]\!], w')$ and $w'' \notin [\![\psi]\!]$. So, by Success, $w'' \notin [\![\varphi \supset \psi]\!]$. But, since $[\![S(\varphi \supset \psi)]\!]^w$ it follows $R(w) \subseteq [\![\varphi \supset \psi]\!]$. So $w'' \notin R(w)$; the Indicative Constraint fails.

$\Rightarrow$: Suppose that the Indicative Constraint does not hold on $\mathcal{F}$. Then for some $\mathbf{A}$, there's some w and w' such that $R(w) \cap \mathbf{A} \neq \emptyset$, $w' \in R(w)$ but $f(\mathbf{A}, w') \not\subseteq R(w)$. So there's some $w'' \in f(\mathbf{A}, w)$ such that $w'' \notin R(w)$. But now we can build a model where QT fails. Let $V(p) = \mathbf{A}$ and $V(q) = \{w''\}$. We can see that for all $w' \in R(w)$ $[\![p \supset \neg q]\!]^{w'} = 1$, as $w'' \notin R(w)$. So $[\![S(p \supset q)]\!]^w = 1$. But $[\![p > q]\!]^{w'} = 0$, since $w'' \in f([\![p]\!], w')$. But $w' \in R(w)$, so $[\![S(p > q)]\!]^w = 0$. □

Now recall:

No Opposite Materials. For any two worlds w_1, w_2, if there's some w_3 such that $w_1 R w_3$ and $w_2 R w_3$, then, for any $\mathbf{A} \subseteq W$: if $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$, then there's no $\mathbf{C} \subseteq W$ such that $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$.

Fact 2. QT is valid on a normal frame $\mathcal{F}$ only if $\mathcal{F}$ satisfies No Opposite Materials.
Proof. Suppose for contradiction that on some normal frame $\mathcal{F}$ QT holds but No Opposite Materials does not. Then there are w_1 w_2, w_3 and $\mathbf{A}$ such that (i) $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$ but (ii) for some $\mathbf{C}$, $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$. Since QT is valid on $\mathcal{F}$, $\mathcal{F}$ obeys the Indicative Constraint. This means that $f(\mathbf{A}, w_3) \subseteq R(w_1)$ and $f(\mathbf{A}, w_3) \subseteq R(w_2)$. So $f(\mathbf{A}, w_3) \subseteq \mathbf{A} \supset \mathbf{C}$ and $f(\mathbf{A}, w_3) \subseteq \mathbf{A} \supset \neg\mathbf{C}$. Given Success, this means $f(\mathbf{A}, w_3) \subseteq \mathbf{C}$ and $f(\mathbf{A}, w_3) \subseteq \neg\mathbf{C}$. But this can only happen if $f(\mathbf{A}, w_3) = \emptyset$. But this is already ruled out by Non-Vacuity. Contradiction. □

A.2. The Qualitative Thesis in a Strict Framework. Our language $\mathcal{L}$ is as before. A *strict frame* $\mathcal{F}$ for $\mathcal{L}$ is a tuple $\langle W, R, h \rangle$. W is a non-empty set of worlds. R is a binary relation on W. h is a function from W to $\mathcal{P}(W)$. A *normal strict frame* obeys the following constraints on h:

Strict Minimality. $w \in h(w)$

Strict Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $h(w) \cap \mathbf{A} \neq \emptyset$.

The strict truth-conditions for the conditional are:

$$[\![\varphi > \psi]\!]^w = 1 \text{ iff } (h(w) \cap [\![\varphi]\!]) \subseteq [\![\psi]\!]$$

All of our other clauses remain the same as before.
Consider:

Strict Indicative Constraint. If $R(w) \cap \mathbf{A} \neq \emptyset$ then for all $w' \in R(w) : (h(w') \cap \mathbf{A}) \subseteq R(w)$

Fact 3. QT is valid on a normal strict frame $\mathcal{F}$ iff $\mathcal{F}$ meets the Strict Indicative Constraint.
Proof. $\Leftarrow$ Again we split QT into $\text{QT}_{\Leftarrow}$ and $\text{QT}_{\Rightarrow}$. First we show $\text{QT}_{\Rightarrow}$ cannot fail on a normal frame $\mathcal{F}$. Suppose for contradiction it did. Then, for some w, $[\![\neg S \neg\varphi]\!]^w = [\![S(\varphi > \psi)]\!]^w = 1$ but $[\![S(\varphi \supset \psi)]\!]^w = 0$. So, for some $w' \in R(w) : [\![\varphi]\!]^{w'} = 1$ but $[\![\psi]\!]^{w'} = 0$. But, by Strict Minimality, $w' \in h(w) \cap [\![\varphi]\!]$. So $[\![\varphi > \psi]\!]^{w'} = 0$ and $[\![S(\varphi > \psi)]\!]^w = 0$ after all, contradicting our initial supposition. So $\text{QT}_{\Rightarrow}$ holds on any

normal strict frame; and in particular it holds on any normal strict frame that meets the Strict Indicative Constraint.

Now suppose that $\mathrm{QT}_{\Leftarrow}$ fails on a normal strict frame $\mathcal{F}$. Then, for some w, $[\![\neg S \neg \varphi]\!]^w = [\![S(\varphi \supset \psi)]\!]^w = 1$ but $[\![S(\varphi > \psi)]\!]^w = 0$. So, for some $w' \in R(w)$, $[\![\varphi > \psi]\!]^{w'} = 0$. This means there is some $w'' \in h(w) \cap [\![\varphi]\!]$ such that $w'' \notin [\![\psi]\!]$. So $w'' \notin [\![\varphi \supset \psi]\!]$. But since $[\![S(\varphi \supset \psi)]\!]^w$ it follows $R(w) \subseteq [\![\varphi \supset \psi]\!]$. So $w'' \notin R(w)$; the Strict Indicative Constraint fails.

$\Rightarrow$: Suppose that the Strict Indicative Constraint does not hold on $\mathcal{F}$. Then for some $\mathbf{A}$, there's some w and w' such that $R(w) \cap \mathbf{A} \neq \emptyset$, $w' \in R(w)$ but $f(\mathbf{A}, w') \nsubseteq R(w)$. So there's some $w'' \in f(\mathbf{A}, w)$ such that $w'' \notin R(w)$. But now we can build a model where QT fails. Let $V(p) = \mathbf{A}$ and $V(q) = \{w''\}$. We can see that for all $w' \in R(w)$ $[\![p \supset \neg q]\!]^{w'} = 1$, as $w'' \notin R(w)$. So $[\![S(p \supset q)]\!]^w = 1$. But $[\![p > q]\!]^{w'} = 0$, since $w'' \in f([\![p]\!], w')$. But $w' \in R(w)$, so $[\![S(p > q)]\!]^w = 0$. □

Fact 4. A normal strict frame $\mathcal{F}$ validates QT only if No Opposite Materials holds on that frame.

Proof. Suppose, for contradiction, that on some normal strict frame $\mathcal{F}$ QT holds but No Opposite Materials does not. Then there are w_1 w_2, w_3 and $\mathbf{A}$ such that (i) $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$ but (ii) for some $\mathbf{C}$, $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$. Since QT is valid on $\mathcal{F}$, $\mathcal{F}$ obeys the Strict Indicative Constraint. This means that $h(w_3) \cap \mathbf{A} \subseteq R(w_1)$ and $h(w_3) \cap \mathbf{A} \subseteq R(w_2)$. So $(h(w_3) \cap \mathbf{A}) \subseteq \mathbf{A} \supset \mathbf{C}$ and $(h(w_3) \cap \mathbf{A}) \subseteq \mathbf{A} \supset \neg\mathbf{C}$ i.e. $(h(w_3) \cap \mathbf{A}) \subseteq \mathbf{C}$ and $(h(w_3) \cap \mathbf{A}) \subseteq \neg\mathbf{C}$. But this can only happen if $h(w_3) \cap \mathbf{A} = \emptyset$. But this is already ruled out by Strict Non-Vacuity. Contradiction. □

APPENDIX B. THE QUALITATIVE THESIS IN THE SHIFTY LOCAL FRAMEWORK

Our language $\mathcal{L}$ is as before. A *shifty frame* $\mathcal{F}$ for $\mathcal{L}$ is a tuple $\langle W, R, f_\kappa \rangle$. f_κ is a shifty selection function, a function from $\mathcal{P}(W)$ to a selection function. The other elements of the tuple are as before. A normal shifty frame obeys the following constraints on f_κ:

Success. $f_\kappa(w, \mathbf{A}) \subseteq \mathbf{A}$

Minimality. If $w \in \mathbf{A}$, then $w \in f_\kappa(w, \mathbf{A})$.

Non-Vacuity. If $\kappa \cap \mathbf{A} \neq \emptyset$ then $f_\kappa(w, \mathbf{A}) \neq \emptyset$.

We recursively define truth at a world and a local context, i.e. a set of worlds in $\mathcal{W}$:

$$[\![p]\!]^{\kappa,w}= 1 \text{ iff } w \in V(p)$$

$$[\![\neg\varphi]\!]^{\kappa,w}= 1 \text{ iff } [\![\varphi]\!]^{\kappa,w}= 0$$

$$[\![\varphi \wedge \psi]\!]^{\kappa,w}= 1 \text{ iff } [\![\varphi]\!]^{\kappa,w}=[\![\psi]\!]^{\kappa,w}= 1$$

$$[\![\varphi > \psi]\!]^{\kappa,w}= 1 \text{ iff } f_{\kappa}(w,[\![\varphi]\!]^{\kappa}) \subseteq [\![\psi]\!]^{\kappa}$$

$$[\![S\varphi]\!]^{\kappa,w} = 1 \text{ iff } \forall w' \in R(w) : [\![\varphi]\!]^{R(w),w'} = 1$$

where $[\![\varphi]\!]^{\kappa} = \{w : [\![\varphi]\!]^{\kappa,w} = 1\}$.

Recall the following property of shifty frames from section 7:

> **Localized Indicative Constraint**. If $\mathbf{A} \cap \kappa \neq \emptyset$, then $\forall w' \in \kappa$: $f_{\kappa}(w', \mathbf{A}) \subseteq \kappa$

We prove the following fact stated in the text:

Fact 14. If a normal monotonic shifty frame $\mathcal{F}$ obeys the Local Indicative Constraint, then it validates QT.

Proof. Suppose the QT fails on a minimal monotonic shifty frame $\mathcal{F}$. Then for some κ and w, one of two cases obtains: i) $[\![\neg S\neg\varphi]\!]^{\kappa,w}= 1$, $[\![S(\varphi > \psi)]\!]^{\kappa,w}= 1$ and $[\![S(\varphi \supset \psi)]\!]^{\kappa,w}= 0$; or ii) $[\![\neg S\neg\varphi]\!]^{\kappa,w}= 1$, $[\![S(\varphi \supset \psi)]\!]^{\kappa,w}= 1$ and $[\![S(\varphi > \psi)]\!]^{\kappa,w}= 0$.

Case i) is ruled out by Minimality. For suppose i) obtains. Since $[\![S(\varphi > \psi)]\!]^{\kappa,w}= 1$, for all $w' \in R(w) : f_{R(w)}(w', [\![\varphi]\!]^{R(w)}) \subseteq [\![\psi]\!]^{R(w)}$. Since $[\![S(\varphi \supset \psi)]\!]^{\kappa,w}= 0$, there is some $w' \in R(w) : [\![\varphi]\!]^{R(w),w'} = 1$ and $[\![\psi]\!]^{R(w),w'} = 0$. But by Minimality, this $w' \in f(w', [\![\varphi]\!]^{R(w)})$. So $[\![\psi]\!]^{R(w),w'} = 1$ after all. Contradiction.

In case ii), the Local Indicative Constraint fails. Since $[\![\neg S\neg\varphi]\!]^{\kappa,w}= 1$, there is some $w' \in R(w)$ s.t. $[\![\varphi]\!]^{R(w),w'} = 1$; so the antecedent of the Local Indicative Constraint is satisfied when $\kappa = R(w)$ and $\mathbf{A} = [\![\varphi]\!]^{R(w)}$. Since $[\![S(\varphi \supset \psi)]\!]^{\kappa,w}= 1$, for all $w' \in R(w)$: either $[\![\varphi]\!]^{R(w),w'} = 0$ or $[\![\psi]\!]^{R(w),w'} = 1$. Since $[\![S(\varphi > \psi)]\!]^{\kappa,w}= 0$, there is some $w' \in R(w)$ such that $f_{R(w)}(w', [\![\varphi]\!]^{R(w)}) \not\subseteq [\![\psi]\!]^{R(w)}$. Since by Success $f_{R(w)}(w', [\![\varphi]\!]^{R(w)}) \subseteq [\![\varphi]\!]^{R(w)}$, it cannot be that $f_{R(w)}(w', [\![\varphi]\!]^{R(w)}) \subseteq R(w)$. So the Indicative Constraint fails. □

Note that the Localized Indicative Constraint is not *necessary* for validating QT: we only need the instances where $\kappa = R(w)$ for some w. But it seems to us that, from a semantic point of view, the more general principle is the more natural one.

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