No New Solutions to the Logical Problem of the Trinity

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Abstract

Analytic theologians have proposed numerous "solutions" to the Logical Problem of the Trinity (LPT), mostly versions of Social Trinitarianism (ST) and Relative Identity Trinitarianism (RI). Both types of solution are controversial, but many hold out hope that further "Trinitarian theorizing" may yield some as yet unimagined, and somehow importantly different, solution to the LPT. I first give a precise definition of the LPT and of what would count as a solution to it. I then show how, though there are infinitely many possible solutions, all solutions can be grouped together into a finite, exhaustive taxonomy, based precisely on those features which make them either controversial, heretical, or inconsistent. The taxonomy reveals why ST and RI have been the major proposed solutions, and also proves that there can be no *importantly* different, new solutions to the LPT.

1 What is the Logical Problem of the Trinity?

1.1 Introduction

Consider the following set S of natural language sentences:

(S1) The Father is God

(S2) The Son is God

(S3) The Holy Spirit is God

(S4) The Father is not the Son

(S5) The Father is not the Holy Spirit

(S6) The Son is not the Holy Spirit

(S7) There is exactly one God[1]

Call the set of propositions that the sentences of S express P, and call each of the propositions each S-n expresses P-n.

Of course, P is not the entirety of the doctrine of the Trinity.[2] But it is an important subset of it, or is at least logically entailed by it. Intuitively, what is called "the logical problem of the Trinity" (LPT) is just the question how, or whether, P is consistent.

On the one hand, at a certain level the anti-Trinitarian only wants to prove that the doctrine of the Trinity is *false*. And the anti-Trinitarian wins on that point if the doctrine of the Trinity simply includes or entails false metaphysical or theological claims. Thus, theological and metaphysical arguments are by no means irrelevant here. But, on the other hand, in the context of the LPT, the anti-Trinitarian typically portrays his argument as a "knock-down" – a matter of logic – not just another of the many uncertain arguments found in metaphysics, in philosophy generally, or in biblical hermeneutics. Thus, it would be a major embarrassment to the anti-Trinitarian if the purely formal argument against the doctrine of the Trinity were to fail. In that case, it would not, after all, be a "knock-down." It would be less like a refutation of the claim "it's both raining and not raining" – as the anti-Trinitarian wants to portray it – and more like a debate between, say, endurantists and perdurantists in the metaphysics of time.

And of course, it would be more than an embarrassment to the Trinitarian if the doctrine really could be shown to be formally inconsistent.

1.2 Outline

Perhaps because of a bad analogy to Plantinga's "defense" against the Logical Problem of Evil,[3] there has been an attitude to the effect that, even if all of the major

[1] [8] seems to be the first to have formulated it this way in the current debate, and most follow him.

[2] For example, the proposition that "the Father begets the Son" is plausibly an essential part of the doctrine of the Trinity. But of course, adding additional propositions can only yield an inconsistent set from a consistent one, not vice-versa. So adding additional propositions could only help the anti-Trinitarian, not the Trinitarian, so the Trinitarian has no reason to complain about focusing on just this set. On the other hand, anti-Trinitarians have not relied on additional propositions in their formulations of the LPT, and if they really *needed* more propositions to be added to make the set inconsistent, then it is unclear that the resulting problem would deserve the name "the Logical Problem of the Trinity." Thus, most in this debate would likely agree that if P could be shown to be consistent, we could say the LPT had been solved, even if there might be other possible arguments to raise against the doctrine.

[3] I discuss this bad analogy in [3].

proposed defenses fail for various reasons, there are infinitely many possibilities out there for a "defense" of the doctrine of the Trinity. In the words of one analytic theologian, the "business of trinitarian theorizing" is merely "unfinished."[4] At the same time, many authors will speak of Social Trinitarianism and Relative Identity Trinitarianism as "the major" or "most common" proposed solutions to the LPT. This comes close to acknowledging that these are really the only two viable solutions, but without quite committing to whether there couldn't be others.

In the interests both of making sharper the distinction between the arguments about the plausibility of Trinitarian *metaphysics*, and the (allegedly) purely *formal* inconsistency of the doctrine (the LPT), of clarifying what exactly the options for both sides of the debate are, and of hopefully pushing the discussion forward in light of both of those projects, in what follows I will do the following in turn. First, in section 2, I give a more precise definition both of what the problem is and of what exactly would constitute a "solution" to it. In section 3, I explain how various proposed solutions to the LPT implicitly attribute different logical forms to P. And finally in section 4, I show how, despite the fact that there are infinitely many logical forms one could attribute to P, we can create an exhaustive taxonomy of all possible logical forms attributable to P based precisely on the logical features of the proposed answers to the LPT that cause them to be either inconsistent, heretical or controversial. Although the result does not map onto the usual dichotomy between Social Trinitarianism (ST) and Relative Identity Trintiarianism (RI) precisely, the taxonomy allows one to see why these two approaches might appear to be the only viable ones, as well as the ways in which a possible solution might subtly differ from proposals given so far.

For anti-Trinitarians, the taxonomy will show that, if just a handful of objections could all be pressed simultaneously, the doctrine of the Trinity would be decisively defeated. For the Trinitarian, the taxonomy will reveal where one really ought to focus one's efforts if one wants to defend the doctrine.

2 What Is the Problem? What Would Be a "Solution"?

2.1 A Precise Statement of the Problem

If the LPT is supposed to be a "knock-down," if it is supposed to show the doctrine of the Trinity, or at least P, to be formally inconsistenct, the question is, how would one show, using the tools of modern logic, whether or not a set of *propositions* is formally consistent or inconsistent? The modern logician's methods of determining

[4] [26], p. 165. At least, this was his claim before finally giving up on the doctrine of the Trinity altogether.

consistency and inconsistency only apply to *sentences* or *formulae* within the artificial languages they construct. So in order to make any use of the tools developed by the logician, a set of propositions Π must be given some regimentation Φ in some formal language L such that the logical forms of the formulae in Φ accurately represent the logical forms of the propositions in Π. Within this artificial language, questions of consistency can be determined (if at all) with mathematical precision. Thus, if a regimentation, Φ in L, of Π can be found such that all parties to the debate can agree that:

1. the formal language L is suitably expressive that there are possible formulae of L that could capture the logical forms (or at least all of the relevant aspects of the logical forms) of the propositions in Π (for short "L is a formally adequate language for Π"), and

2. the logical forms of the formulae of Φ in L do reflect the logical forms (or at least the relevant aspects of the logical forms) of the propositions in Π (for short "Φ is a formally adequate regimentation of Π")

then the question of the formal consistency of the propositions in Π can be decided on the basis of the formal consistency of the formulae Φ in L.

So, in any debate over the formal consistency of a set of propositions Π, the real work, and matter for debate, lies not in proving the formal consistency or inconsistency of any set of formulae, but in finding a suitable artificial language L and a suitable regimentation Φ, such that it can be shown that:

1. L is a formally adequate language for Π, and

2. Φ in L is a formally adequate regimentation for Π.

With one important exception, most philosophers in this debate agree that standard versions of predicate logic with (Leibnizian, classical, absolute, non-relative) identity ("PLI" for short) would be formally adequate for P. What will be called "pure" Relative Identity Trinitarianism (Pure RI) seems to be the only camp in the debate that demands the use of an importantly different formal language in which to address the issue.[5] Since rejecting PLI is controversial in itself, we will adopt a certain mild "prejudice" toward PLI. Specifically, as long as a view *can* be given a

[5]Strictly speaking, it may be going too far even to say that Pure RI requires a different formal language. Peter van Inwagen has pointed out to me that every formula that is valid in standard PLI remains valid in his Relative Identity Logic. It is simply that a proponent of Pure RI refuses to *make use of* the standard identity predicate as a way of correctly formalizing any natural language statements. However, I will ignore this complication in what follows as I don't think it affects my

formally adequate regimentation in PLI, we will regiment it in PLI. In other words, if it is *possible* to represent a certain logical form *via* formulae of PLI, we will use formulae of PLI as the means by which we will represent that logical form.[6]

There are two senses in which we might say that a set of formulae Φ in a formal language L is inconsistent. It may be that, given the rules of inference that are valid in L, Φ entails a contradiction, entails its own negation, or for whatever reason, the inference rules for that language say that the conjunction of the members of Φ must be false. If this is so, Φ is "*syntactically inconsistent* in L." If there is no such valid proof in L, Φ is "*syntactically consistent*" in L. This is the strict meaning of "consistency," and pertains, obviously, to syntax.

On the other hand, it may be that there is no interpretation I of the non-logical constants of L such that all of the members of Φ are true in L on I (i.e. there is no "model" for Φ in L). If this is so, Φ is not "*satisfiable*" with respect to the class of possible interpretations of the non-logical constants of L. If there *is* such an interpretation (a model) for Φ in L, Φ *is* satisfiable in L. This is not strictly consistency. It pertains not to syntax but to semantics. But it is just as important a consideration, and in a formal language with the features of *soundness* and *completeness*, the syntactic feature of consistency and the semantic feature of satisfiability go hand in hand.

Since our concerns encompass both syntax and semantics, it would seem that our ordinary use of the word "inconsistent" should cover both strict, syntactic inconsistency, and the semantic notion of unsatisfiability. Thus, I will say that a set Φ of formulae of L is "inconsistent in L" if and only if it is *either* syntactically inconsistent in L *or* merely unsatisfiable in L. Likewise, I will say that Φ is "consistent" in L if and only if it is *both* syntactically consistent in L *and* satisfiable in L.

Now in any formal language L worth studying, any language with the property of "*soundness*," if Φ is syntactically inconsistent in L, then it will be unsatisfiable in L as well. So, although these are not the only ways to do so, a usually good strategy for showing the inconsistency of Φ is to *give a proof* of the negation of the conjunction of the members of Φ (because the syntactic feature of inconsistency will show the semantic feature of unsatisfiability as well), and a usually good strategy for

ultimate conclusion. Ultimately, I will conclude that one way to solve the LPT is to adopt an analysis of counting statements that does not work by way of classical identity, and that conclusion remains, even in the face of this complication. It may be that van Inwagen's proposal could be placed into the category of "Impure" RI, rather than "Pure" RI. But the general taxonomy I will construct can proceed at an abstract level, regardless of how precisely to classify van Inwagen himself, or his proposal.

[6]It's important to emphasize here that nothing in our proof hangs on this "prejudice," since, even if there are other languages that are formally adequate for P, as long as PLI is *one* formally adequate language, then we *may* choose to work entirely within that language if we so choose.

showing "formal consistency" is to *give a model* for all of the members of Φ (because the semantic feature of satisfiability will show the syntactic feature of consistency as well).

So, why does the anti-Trinitarian think that P is inconsistent?

Suppose we take "Father," "Son," and "Holy Spirit" univocally as names for individuals wherever they appear in S. Suppose we also take "God" in S1 through S3 univocally as the name of an individual. Suppose we take "is" univocally as the "is" of (classical) identity in S1 through S6. And suppose we analyze the counting statement expressed by S7 in a standard way, and understand "is God" as it occurs there in the same way we did in our interpretation of S1 through S3. The logical form of the claims expressed by S on this interpretation of it can be represented in PLI as:

$\Phi_{\text{LPT-1}}$:

($1_{\text{LPT-1}}$) $f=g$
($2_{\text{LPT-1}}$) $s=g$
($3_{\text{LPT-1}}$) $h=g$
($4_{\text{LPT-1}}$) $f\neq s$
($5_{\text{LPT-1}}$) $f\neq h$
($6_{\text{LPT-1}}$) $s\neq h$
($7_{\text{LPT-1}}$) $(\exists x)(\forall y)(x=g \;\&\; (y=g \rightarrow y=x))$

$\Phi_{\text{LPT-1}}$ is inconsistent in PLI.[7] (($7_{\text{LPT-1}}$) is not strictly necessary to derive a contradiction here; I include it only for completeness' sake.)

On the other hand, suppose we instead take "is God" in S1 through S3 univocally but take "God" to be a predicate nominative ("a god") and "is" to be the "is" of predication. Suppose we again analyze the counting statement expressed by S7 in the standard way, and understand "is God" as it occurs there in the same way we did in our interpretation of S1 through S3. And suppose we otherwise leave our regimentation unchanged. The logical form of the claims expressed by S on this interpretation of it can be represented in PLI as:

$\Phi_{\text{LPT-2}}$:

($1_{\text{LPT-2}}$) Gf
($2_{\text{LPT-2}}$) Gs
($3_{\text{LPT-2}}$) Gh
($4_{\text{LPT-2}}$) $f\neq s$
($5_{\text{LPT-2}}$) $f\neq h$

[7] Proof is trivial.

($6_{\text{LPT-2}}$) $s \neq h$
($7_{\text{LPT-2}}$) $(\exists x)(\forall y)(Gx \ \& \ (Gy \rightarrow y{=}x))$

$\Phi_{\text{LPT-2}}$ is also inconsistent in PLI.[8]

Since both of the logical forms we have in mind here *can* be represented in PLI, we will use PLI. So, a more precise way to put the anti-Trinitarian argument would be as follows:

1. PLI is a formally adequate language for P, and
2. at least one of $\Phi_{\text{LPT-1}}$ in PLI or $\Phi_{\text{LPT-2}}$ in PLI is a formally adequate regimentation of P, and
3. both $\Phi_{\text{LPT-1}}$ and $\Phi_{\text{LPT-2}}$ are (syntactically) inconsistent in PLI.
4. So, by definition of "formally adequate language" and "formally adequate regimentation," P is formally inconsistent.

But, if this is the "problem"... what exactly would count as a *solution*?

2.2 What Would Be a Solution?

If the anti-Trinitarian argument above is right, then P is formally inconsistent, and that is the "answer" to the LPT. There is no solution, but rather there is, as we might say, a "non-solution." So the Trinitarian must maintain that *neither* regimentation, in PLI, is formally adequate for P (or else that PLI itself is not formally adequate for P).

Let us define a "*proposed* answer" to the LPT as a set that includes:

1. Exactly one formal language L in which to regiment P, and
2. Exactly one set Φ of formulae of L with which to regiment P,[9] and
3. A proof of the formal consistency or inconsistency of Φ in L.

Let us say that a "*formally adequate* answer" to the LPT is a proposed answer to the logical problem of the Trinity such that:

[8] Proof is trivial.

[9] We will relax this requirement in an obvious and non-problematic way in the case of a couple of dilemmas, where two different possible regimentations are offered, and the claim made is only that at least one of them is formally adequate. Specifically, the anti-Trinitarian regimentations LPT_1 and LPT_2, and the Naïve Modalist regimentations NM_1 and NM_2.

1. L is a formally adequate language for P, and

2. Φ in L is a formally adequate regimentation of P

3. (and the proof of formal consistency or inconsistency of Φ in L is correct.)

A "proposed *solution*" to the LPT is a proposed answer to the logical problem of the Trinity that proves that Φ is formally consistent in L.

A "proposed *non-solution*" to the LPT is a proposed answer to the logical problem of the Trinity that proves that Φ is formally inconsistent in L.

A "formally adequate *solution*" to the LPT is a formally adequate answer that is a proposed solution (i.e., a formally adequate answer to the LPT that proves that Φ is formally consistent in L).

A "formally adequate *non-solution*" to the LPT is a formally adequate answer that is a proposed non-solution (i.e., a formally adequate answer to the LPT that proves that Φ is formally inconsistent in L).

Thus, the anti-Trinitarian argument above can be seen as a constructive dilemma. One of two proposed non-solutions to the LPT (call those LPT_1 and LPT_2) is formally adequate. Thus, there is some formally adequate non-solution to the LPT.[10] So, by definition of "formally adequate language" and "formally adequate regimentation," P is formally inconsistent.

Thus, to defend P, the Trinitarian must argue at least that it might (for all we know) be the case that neither LPT_1 nor LPT_2 is a formally adequate answer to the LPT, or even that PLI itself is not a formally adequate language for P, that is, either:

1. PLI is not a formally adequate language for P, or

2. Neither Φ_{LPT-1} in PLI nor Φ_{LPT-2} in PLI is a formally adequate regimentation of P.

   (Or both.)

In principle, this very weak response ("for all we know," either the language is inadequate or the regimentations are) would be a sufficient defence of P. But most philosophers in the literature have wanted to do more. They have wanted to argue that it can be *shown* that P really is consistent (not just that it's not unreasonable

[10]It should be obvious that if there is one formally adequate solution, all formally adequate answers are solutions, and that if there is one formally adequate non-solution, all formally adequate answers are non-solutions, since a set of propositions is either consistent or not.

for us to *believe* that it is).[11] But it might seem hard to see how one would argue that PLI is not formally adequate for P, except by arguing that some other language L *is* formally adequate for P, and that L is importantly different from PLI in some relevant way. Likewise, assuming that PLI is formally adequate for P, it might seem hard to see how one would argue that neither Φ_{LPT-1} in PLI nor Φ_{LPT-2} in PLI is a formally adequate regimentation for P, except by arguing that some other regimentation Φ in PLI *is* a formally adequate regimentation for P, and that Φ's being a formally adequate regimentation of P in PLI is somehow incompatible both with Φ_{LPT-1} in PLI being a formally adequate regimentation of P and with Φ_{LPT-2} in PLI being a formally adequate regimentation of P (as will be the case if the alternative proposed answer has an importantly different logical form, which of course must be the case if it is a proposed solution instead of a non-solution). And so, the majority of the literature has centered around the search for alternative proposed answers to the LPT to supplant LPT_1 and LPT_2.

But if one wants to replace LPT_1 and LPT_2... *what alternative answers could one propose*?

3 Proposed Solutions

Thinking on this issue goes back to the earliest centuries of Christianity. And not just any way of understanding the "three-ness" and "one-ness" of God has been received as within the bounds of orthodoxy. Certain views, though consistent, were rejected as heretical during the course of the Trinitarian controversies of roughly the late 3rd through early 5th centuries AD. I will refer to these as the "Classical Trinitarian Heresies" (CTHs). CTH's may have interpretted S in consistent ways, but they do so only by being at odds with the orthodox doctrine of the Trinity. Thus, they in some sense count as proposed answers to the LPT, even "solutions," but ones that are not available to the Trinitarian. So, for completeness' sake, we will discuss the CTH's in some detail. However, contemporary views may be easier to understand and easier to regiment in a standard way. Also, some of that discussion will help to shed light on the CTHs. So, we will begin with contemporary proposed solutions to the LPT.

Our purpose at the moment is to collect various proposed solutions to the LPT. And what concerns us most at the moment is the matter of formal consistency.

[11] There is another approach, labelled "mysterianism" by Tuggy [27], which does take precisely the approach of avoiding the issue of consistency, but arguing for the epistemic acceptability of accepting a set of propositions that appears to be inconsistent. Addressing that approach is beyond the scope of the current paper, however, which has an eye only towards those who would offer a "defense" of the doctrine.

So we will not try to give exhaustively detailed discussions of any of these views, but only so much as to give us a clear enough idea of its *logical form* that we can represent it in a formal language and determine its consistency or inconsistency.

3.1 Contemporary Proposed Solutions

3.1.1 Social Trinitarianism (ST)

Probably the easiest contemporary proposal to understand is Social Trinitarianism (ST). Paradigmatic versions of ST hold that Father, Son and Holy Spirit are straightforwardly numerically distinct persons, each of whom is fully divine. Instances of the phrase "is God" in reference to the persons individually are read as *predications* ("is divine" or "is (a) god") rather than as *identifications* to an individual called "God." But ST (attempts to) escape(s) tritheism by claiming it is the Trinity *as a whole* – the collective or "community" or "society" they compose – to which the term "God" is properly applied when we speak of "the one God" (whether we treat this as a *name* for the collective, or as a predicate that is not, at least not precisely, univocal with "is (a) god" when applied to the persons).

Both proponents and critics of ST tend to focus on its taking the divine "persons" to be fully "*persons*" in our modern, post-Cartesian sense – fully aware centers of consciousness, reason, will, etc. This is thought to be its distinctive feature. But from the point of view merely of logical form, the issue is irrelevant. The "persons" could be *beans* as far as the LPT is concerned. But if there are three of them, and each is a bean, yet there is only one bean, LPT_2 would provide a formally adequate regimentation of the view, and the doctrine would be inconsistent.

So what features of ST *are* relevant to our concerns?

First, it is clear that Social Trinitarians insist on making a very strong, real distinction between the persons. But classical non-identity ($\neq$) is the weakest (real) distinction one can make.

It's clear then that Social Trinitarians will agree with LPT_1 and LPT_2 on their regimentation of P4 through P6. (Indeed, Social Trinitarians often want to go even *further* in distinguishing the persons, but they must *at least* admit the non-identity of the persons.) And in so doing they will (they may as well) take PLI to be a formally adequate language for P. In keeping with this emphasis on the distinctness of the persons, it is also clear that Social Trinitarians will want to treat "is god" in P1 through P3 not as identity claims to some individual, but as predications. Thus, Social Trinitarians will deny the formal adequacy of $1_{\text{LPT-1}}$ through $3_{\text{LPT-1}}$ in PLI.

It is also clear that Social Trinitarians make no distinctions *between* the persons as to their divinity. That is, each person is divine in exactly the same sense as either

of the other two persons. So, there will be no equivocation here. And while Social Trinitarians will deny the formal adequacy of $1_{\text{LPT-1}}$ through $3_{\text{LPT-1}}$ in PLI, they will admit to the formal adequacy of $1_{\text{LPT-2}}$ through $3_{\text{LPT-2}}$ in PLI (or something relevantly similar, in a sense that will become clear later.)

ST, then, so far *agrees* with LPT_2 on the logical form of P. If ST is to count as a *solution*, then, it must regiment P7 differently. But there is no indication in ST literature that Social Trinitarians have any problem with standard logical regimentations of counting statements in general or with the classical identity relation (=) in particular. (Indeed, one of the motivations for adopting ST is precisely to avoid having to give up on analyzing counting statements with classical identity. See the discussion of Relative Identity Trinitarianism below for more.) The only way, then, that ST could possibly avoid contradiction would be to equivocate on "is god," not among its applications to the persons themselves, individually (in S1 through S3) but between its application there on the one hand and in S7 on the other. But it is this purely *formal* feature that lies at the heart of a major criticism of ST. Brian Leftow writes:

> But even if Trinity monotheism avoids talk of degrees of deity, it faces a problem. Either the Trinity is a fourth case of the divine nature, in addition to the persons, or it is not. If it is, we have too many cases of deity for orthodoxy. If it is not, and yet is divine, there are two ways to be divine – by being a case of deity, and by being a Trinity of such cases. If there is more than one way to be divine, Trinity monotheism becomes Plantingian Arianism. But if there is in fact only one way to be divine, then there are two alternatives. One is that only the Trinity is God, and God is composed of non-divine persons. The other is that the sum of all divine persons is somehow not divine. To accept this last claim would be to give up Trinity monotheism altogether.
>
> I do not see an acceptable alternative here. So I think Trinity monotheism is not a promising strategy for ST.[12]

Leftow here uses "Trinity monotheism" for what he takes to be just one *version* of ST. But as we've seen, if all versions of ST admit the non-identity of the persons, and if all versions of ST treat "is god" as univocal across S1 and S3, and if no versions of ST take issue with standard logical regimentations of counting statements, then all versions of ST will have to confront the problem Leftow raises. (At least, they will have to confront the purely *formal* problem Leftow's argument relies on.) Namely,

[12] [14, p. 221].

first, that ST must equivocate on "is god" in S1 through S3 on the one hand and "is God" in S7 on the other hand (otherwise we end up with *four* gods instead of *one*). But then it follows that either (1) there is more than one "way" of being divine or being "a god" (a position Leftow calls "Plantingian Arianism," see below, p. 18, for more on Arianism), or else (2) the persons are not legitimately divine or "god," or else (3) the "one God" (i.e., according to ST, the Trinity as a whole) is not legitimately divine or "(a) god."

So, although, again, both proponents and critics tend to characterize ST in terms of its taking the divine "persons" to be distinct centers of consciousness and so forth, it is more useful for our purposes to characterize it in terms of the formal feature Leftow's criticism relies on. For even if "x is god" means *that x is a bean*, we can run essentially the same argument to the effect that one will have to equivocate on "is god." If the persons satisfy *any* predicate, and they are all non-identical, yet there is only one thing that satisfies that predicate, then LPT_2 is formally adequate, regardless of what the predicate in quesiton is, or what it means.

So, if ST is to offer a solution, it must reject the formal adequacy of $7_{\text{LPT-2}}$ and replace it with an equivocation on "is god," which we can represent formally by using "G_1" for one sense of "is god," and "G_2" for another sense. (The precise semantic content can be filled in however a particular proponent of ST likes. The important fact from a formal point of view is simply that there are *two* senses, whatever they might be.) Thus, we can pin down a formal regimentation for ST and give an ST proposed solution to the LPT as follows:

1. PLI is a formally adequate language for P.

2. Φ_{ST} in PLI is a formally adequate regimentation of P:

   Φ_{ST}:

   (1_{ST}) $G_1 f$

   (2_{ST}) $G_1 s$

   (3_{ST}) $G_1 h$

   (4_{ST}) $f \neq s$

   (5_{ST}) $f \neq h$

   (6_{ST}) $s \neq h$

   (7_{ST}) $(\exists x)(\forall y)(G_2 x \ \& \ (G_2 y \rightarrow x{=}y))$[13]

[13]It might be objected that this treats "is god" in P7 as another predication, whereas Social Trinitarians might claims it should be treated as a name in P7, thus: (7_{ST}) $(\exists x)(\forall y)(x{=}g \ \& \ (y{=}g \rightarrow$

3. Φ_{ST} is formally consistent in PLI.[14]

3.1.2 ("Pure") Relative Identity Trinitarianism (Pure RI)

The major strand of Relative Identity Trinitarianism (RI) in contemporary philosophy of religion, called "pure" RI by Mike Rea,[15] began with Peter Geach's discussions of relative identity, and his application of it to the doctrine of the Trinity.[16] A. P. Martinich also endorsed an RI view a few decades ago,[17] as did James Cain.[18] But probably the clearest, fullest and most influential statements of the view are van Inwagen's.[19] In his earlier statement of the view, van Inwagen does not answer the question whether classical identity exists or not.[20] But in his later statement, he explicitly rejects the existence of classical identity.[21]

Pure RI may be, in some sense, the most difficult proposed solution to the LPT to wrap one's head around, given that it rejects the existence of classical identity altogether, and given how intuitive classical identity seems to most of us. But in another sense (happily, the sense that will matter for us), it is among the easiest. This is especially so as it appears in van Inwagen's work, which, also happily, is what we might call the canonical version of Pure RI.

First, Pure RI explicitly rejects the very existence, or intelligibility, of classical identity, and so explicitly rejects PLI as a formally adequate language for P (PLI being "predicate logic *with identity*"). Van Inwagen has given his own preferred formal language for this purpose, Relative Identity Logic, which he shortens to "RI-logic,"[22] and which I will shorten even further to "RIL." So, Pure RI does not

x=y)). However, that is still an equivocation, and so, when we give a more general characterization of a "Family" of views into which ST will fall, such a version of ST will be included in our "Family" anyway. See p. 30 ff. below.

[14]It should be obvious that there is a model for Φ_{ST}, and the proof is left as an exercise for the reader.

[15]The distinction begins in [19] p. 433 and *passim*.

[16]See [11, pp. 43–48 and 69–70]; [10] and [9], both reprinted in [12]; and his chapter, [13].

[17][15] and [16].

[18][7].

[19][28], and [29] in [21, pp. 61–75].

[20]In [28], p. 241, van Inwagen considers three arguments concerning classical identity and its relation to relative identity, and says "I regard these arguments as inconclusive. In the sequel, therefore, I shall assume neither that classical identity exists nor that it does not exist." Thus, strictly speaking, in this paper, van Inwagen counted as an adherent of "impure" Relative Identity theory, to be discussed below.

[21]In [29, p 70], he says, "I deny that there is one all-encompassing relation of identity... there is no relation that is both universally reflexive and forces indiscerniblility."

[22][28, p. 231].

accept PLI as a formally adequate language for P, but claims that RIL is a formally adequate language for P.

Second, Pure RI replaces the classical (non-)identity predicate "$\neq$" in the regimentations of P1 through P6 with various relative (non-)identity predicates, the two relevant for our purposes being: "is the same being as" in its equivalents of P1 through P3, and "is (not) the same person as" in its equivalent of P4 through P6.[23] It can then use the "is the same being as" predicate in its equivalent of P7 without generating inconsistency. Although van Inwagen uses the English "is the same being as" and "is (not) the same person as," I will shorten these to "$=_{\mathrm{B}}$" and "$\neq_{\mathrm{P}}$," respectively.

One might think we could now state a Pure RI proposed solution to the LPT as:

1. RIL is a formally adequate language for P.
2. $\Phi_{\text{Pure-RI}}{}^*$ in RIL is a formally adequate regimentation of P:

   $\Phi_{\text{Pure-RI}}{}^*$:

   ($1_{\text{Pure-RI}}{}^*$) $\mathrm{f} =_{\mathrm{B}} \mathrm{g}$

   ($2_{\text{Pure-RI}}{}^*$) $\mathrm{s} =_{\mathrm{B}} \mathrm{g}$

   ($3_{\text{Pure-RI}}{}^*$) $\mathrm{h} =_{\mathrm{B}} \mathrm{g}$

   ($4_{\text{Pure-RI}}{}^*$) $\mathrm{f} \neq_{\mathrm{P}} \mathrm{s}$

   ($5_{\text{Pure-RI}}{}^*$) $\mathrm{f} \neq_{\mathrm{P}} \mathrm{h}$

   ($6_{\text{Pure-RI}}{}^*$) $\mathrm{s} \neq_{\mathrm{P}} \mathrm{h}$

   ($7_{\text{Pure-RI}}{}^*$) $(\exists \mathrm{x})(\forall \mathrm{y})\ (\ \mathrm{x} =_{\mathrm{B}} \mathrm{g}\ \&\ (\ (\mathrm{y} =_{\mathrm{B}} \mathrm{g}) \rightarrow (\mathrm{y} =_{\mathrm{B}} \mathrm{x})\ )\ )$
3. $\Phi_{\text{Pure-RI}}{}^*$ is formally consistent in RIL.

However, this would not be accurate. At least, not without some qualifications about the uses of "f," "s," and "h" in RIL. As van Inwagen points out,

> The philosopher who eschews classical, absolute identity must also eschew singular terms, for the idea of a singular term is – at least in currently orthodox semantical theory – inseparably bound to the classical semantical notion of reference or denotation; and this notion, in its turn, is inseparably bound to the idea of classical identity. It is a part of the orthodox semantical concept of reference that reference is a many-one relation. And it is a part of the idea of a many-one relation – or of a

[23] It will become clear why I say its "equivalents" shortly.

> one-one relation, for that matter – that if x bears such a relation to y and bears it to z, then y and z are absolutely identical.[24]

To cut a long story short, RIL must replace singular reference with *relative* singular reference, and this boils down to certain kinds of general or quantified statements employing relative identity relations. Thus, a Pure RI proposed solution to the LPT would instead come to something like this:

1. RIL is a formally adequate language for P.
2. $\Phi_{\text{Pure-RI}}$ in RIL is a formally adequate regimentation of P:

   $\Phi_{\text{Pure-RI}}$:

   $(\exists x)\ (\exists y)\ (\exists z)$

   $(Gx\ \&\ Gy\ \&\ Gz\ \&$ (cf. P1 to P3)

   $x \neq_P y\ \&\ x \neq_P z\ \&\ y \neq_P z\ \&$ (cf. P4 to P6)

   $(\forall v)\ (\forall w)\ ((Gv\ \&\ Gw) \rightarrow (v =_B w)))$ (cf. P7)
3. $\Phi_{\text{Pure-RI}}$ is formally consistent in RIL.[25]

$\Phi_{\text{Pure-RI}}$ is just one long formula. I have split it onto different lines for ease of reading. Obviously the first line is just the initial three quantifiers, which we must use in the place of singular terms. With that in place, the second line corresponds in a way to P1 through P3. The third line corresponds in a way to P4 through P6. And the fourth line corresponds in a way to P7. Thus, the different parts of $\Phi_{\text{Pure-RI}}$ are in some sense the "equivalents" of different parts of P. (One can usefully compare $\Phi_{\text{Pure-RI}}$ to other proposed answers by taking the conjunction of their regimentations of P1 through P7 in order, and then "Ramsifying" away the names of the persons.)

3.1.3 "Impure" Relative Identity Trinitarianism (Impure RI)

The final contemporary proposal we will look at has been defended by Mike Rea and Jeff Brower. In the Rea-Brower account of the Trinity, the persons stand in a "constitution" relation to one another, and the word "God" is systematically ambiguous between the persons.[26] The constitution relation does not entail classical identity,

[24] [28, p. 244].

[25] It is easy enough to see that this will be consistent, but for more, one can see [28, pp. 249–250].

[26] The view is explicated, defended, and developed in more detail over the course of a number of articles. See [19], [5], [6], [20], and [22]. See also Rea and Michael Murray's discussion of the Trinity in [17].

but the account does not deny the existence or intelligibility of classical identity as on the "pure" RI view. It simply holds that *our ordinary counting practices* rely not on classical identity, but on various relative identity relations. The constitution relation either is, or at least entails, a species of relative identity between the persons, such that we should count them as three persons but one god. (For the time being, we will follow Rea's terminology in calling this "impure Relative Identity" (Impure RI) as distinguished from "pure" RI. We will see later, p. 35 ff., why there may be a more useful term to cover both of these views.)

Since Impure RI accepts classical identity, it can (it may as well) accept PLI as a formally adequate language in which to regiment P. Furthermore, it can regiment P4 through P6 as classical non-identity claims just as in LPT_1 and LPT_2. However, like Pure RI, it rejects classical identity as the relation by which we *count*, and instead analyzes counting statements as operating by way of relative identity relations. So, it will regiment P7 differently.

To claim that we count by *classical identity* is to claim that we count one or two (...or n) Fs when there are one or two (...or n) terms (t_1, t_2, ... t_n) of which "F" is true and the appropriate claims of classical non-identity involving those terms ($\mathrm{t}_1 \neq \mathrm{t}_2$, ...) are all true, and any other term t_{n+1} of which "F" is true is such that at least some claim of classical identity involving t_{n+1} and one of the previous terms is true (thus, $\mathrm{t}_{n+1} = \mathrm{t}_1$ or $\mathrm{t}_{n+1} = \mathrm{t}_2$, or ... $\mathrm{t}_{n+1} = \mathrm{t}_n$).

To claim that we count by *relative identity* is to claim that we count one or two (...or n) Fs when there are one or two (...or n) terms (t_1, t_2, ... t_n) of which "F" is true and the corresponding claims of *relative* non-identity involving those terms and that predicate ($\mathrm{t}_1 \neq_{\mathrm{F}} \mathrm{t}_2$, ...) are all true, and any other term t_{n+1} of which "F" is true is such that at least some claim of relative identity involving t_{n+1} and one of the previous terms and the appropriate predicate is true (thus, $\mathrm{t}_{n+1} =_{\mathrm{F}} \mathrm{t}_1$ or $\mathrm{t}_{n+1} =_{\mathrm{F}} \mathrm{t}_2$, or ... $\mathrm{t}_{n+1} =_{\mathrm{F}} \mathrm{t}_n$).

Thus, Impure RI's regimentation of P7 will look much like Pure RI's in a way, but stated in PLI instead of RIL. But how does Impure RI analyze P1 through P3?

Over the course of several papers, the Rea-Brower view becomes fairly complex, involving the sharing by the persons of a trope-like divine nature that "plays the role of matter" for the persons, each of which is constituted by the divine nature plus its own hypostatic property (Fatherhood, Sonship, Spiritude). But the deeper importance of that theoretical machinery lies in its licensing of a relative identity claim involving the term "God" (here used as a name or other singular term again) and each of the names of the persons. We can symbolize this relative identity relation as "$=_{\mathrm{G}}$," which allows us to regiment the view more simply, and give an Impure RI proposed solution to the LPT as follows:

1. PLI is a formally adequate language for P.

2. $\Phi_{\text{Impure-RI}}$ in PLI is a formally adequate regimentation of P:

   $\Phi_{\text{Impure-RI}}$:

   ($1_{\text{Impure-RI}}$) $f =_G g$

   ($2_{\text{Impure-RI}}$) $s =_G g$

   ($3_{\text{Impure-RI}}$) $h =_G g$

   ($4_{\text{Impure-RI}}$) $f \neq s$

   ($5_{\text{Impure-RI}}$) $f \neq h$

   ($6_{\text{Impure-RI}}$) $s \neq h$

   ($7_{\text{Impure-RI}}$) $(\exists x)(\forall y)(x =_G g \ \& \ (y =_G g \rightarrow y =_G x))$

3. $\Phi_{\text{Impure-RI}}$ is formally consistent in PLI.[27]

Is it really OK to just ignore whatever more intricate logical structure might, given Rea and Brower's fuller account, be entailed by the "$=_G$" relation, such as a reference to the divine nature and the constitution relation? Yes. How so?

Whatever the "same god as" relation might entail, as long as "x is the same god as y":

1. is not in itself formally inconsistent, and

2. does not entail (classical) identity between x and some other term t_i,[28]

then ($1_{\text{Impure-RI}}$) through ($7_{\text{Impure-RI}}$) is still consistent.

On the other hand, if "x is the same god as y" *does* entail a (classical) identity between x and some other term t_i, then ($1_{\text{Impure-RI}}$) through ($6_{\text{Impure-RI}}$) will be inconsistent without even appealing to ($7_{\text{Impure-RI}}$). But not for any reasons interestingly related to Impure RI. It will be inconsistent for the same reasons ($1_{\text{LPT-1}}$) through ($6_{\text{LPT-1}}$) were.

More precisely, for any formula ϕ, where ${\phi^{t1,t2}}_{x,y}$ is the result of replacing every occurrence of the variables x and y in ϕ with the terms t_1 and t_2, respectively, if:

${\phi^{t1,t2}}_{x,y} \models t_1 = t_i$ for some $t_i \neq t_1$

[27] It should be obvious that there is a model for $\Phi_{\text{Impure-RI}}$, and the proof is left as an exercise for the reader.

[28] I include 1 merely to aid comprehension. Given 2, 1 is in fact redundant.

then

$\phi^{f,g}{}_{x,y}$ & $\phi^{s,g}{}_{x,y}$ & $\phi^{h,g}{}_{x,y}$ & $\mathrm{f} \neq \mathrm{s}$ & $\mathrm{f} \neq \mathrm{h}$ & $\mathrm{s} \neq \mathrm{h}$

is inconsistent anyway, but if:

$\phi^{t1,t2}{}_{x,y} \nvDash \mathrm{t}_1 = \mathrm{t}_i$ for any $\mathrm{t}_i \neq \mathrm{t}_1$

then

$\phi^{f,g}{}_{x,y}$ & $\phi^{s,g}{}_{x,y}$ & $\phi^{h,g}{}_{x,y}$ & $\mathrm{f} \neq \mathrm{s}$ & $\mathrm{f} \neq \mathrm{h}$ & $\mathrm{s} \neq \mathrm{h}$ &
$(\exists \mathrm{x})(\forall \mathrm{y})(\phi^{g}{}_{y}$ & $(\phi^{\mathrm{y,g}}{}_{\mathrm{x,y}} \rightarrow \phi^{\mathrm{y,x}}{}_{\mathrm{x,y}}))$

is consistent.

So, as long as "x is the same god as y" doesn't entail a classical identity claim between x and some other term, we are safe. And it doesn't seem that it would on the Rea-Brower account. The only other term that might be involved would be "the divine nature." But on the Rea-Brower account, the divine nature is definitely *not* classically identical to any of the persons. So, we needn't go into more detail on the precise logical structure, or further semantic content, of the "same god as" relation. The above will do to show the Rea-Brower account is at least formally consistent.

3.2 Classical Trinitarian Heresies

3.2.1 Arianism

Although not the first chronologically, the CTH of all CTHs was Arianism. It was Arianism that occasioned the First (and Second) Ecumenical Council(s) and the heated controversies of the 4th century. Historically, Arianism was not *motivated* by the search for a solution to the LPT. Nor was its rejection by the orthodox motivated by concerns about the LPT. Still, the logical problem of the Trinity did have a role in the debate, albeit a minor one. To the central question of whether the *Logos*, i.e., the "Angel of the Lord," i.e. Christ, was created or uncreated, the LPT was tacked on as an after-thought or "back-up" argument to other scriptural and metaphysical arguments.

Gregory Nazianzen in his *Fifth Theological Oration* (On the Holy Spirit) discusses an Arian argument:

> "If," they say, "there is God, and God, and God, how are there not three gods? Or how is that which is glorified not a poly-archy?"[29]

[29] [25], Fifth Theological Oration (31), *On the Holy Spirit*, section 13. Translation mine.

But why did those Arians not think the LPT was a problem for *them*? What was *their* proposed solution to the LPT?

For the first part of the answer, we must go back to Gregory's *Third Theological Oration (On the Son)*. Arians took the position that Father and Son have different natures (that they were not "consubstantial"). Second, they took the position that "is god" as applied to the Father predicates the divine nature. It follows directly from these two views that applying "is god" to the Son could only be done equivocally (regardless of concerns about the LPT).[30]

And this is a consequence they themselves acknowledged. We read in Gregory's *Third Theological Oration*:

> And when we advance this objection against them, "What do you mean to say then? That the Son is not properly God, just as a picture of an animal is not properly an animal?[31] And if not properly God, in what sense is He God at all?" They reply, "Why should not these terms be [both] ambiguous, and yet in both cases be used in a proper sense?"
>
> And they will give us such instances as the land-dog and the dogfish; where the word "dog" is ambiguous, and yet in both cases is properly used,[32] for there is such a species among the ambiguously named, or any other case in which the same appellative is used for two things of different nature.[33]

So there is step one in the Arian solution to the LPT: equivocate on "is god." Not between P7 on the one hand and P1 through P3 on the other, as in ST, but among P1 through P3 themselves.

[30] I should emphasize that this semantic claim, that "is god" predicates *the divine nature*, rather than a kind of *activity*, was, originally, a specificially *Arian* claim. It was not a part of the mainstream Christian tradition prior to that point, and was forcefully rejected by St. Gregory of Nyssa and others, while those church fathers who did not specifically reject it, at least refrained from affirming it. Only later was Augustine to be the first church father to actually accept this semantic claim that "is god" predicates divinity, rather than an activity, and it is not clear that his attempt to incorporate this originally Arian semantic claim into a Trinitarian theology was completely successful. See 4.4, p. 31 below for more.

[31] In Greek, ζῷον means either an animal or a painting.

[32] τὸν κύνα, τὸν χερσαῖον, καὶ τὸν θαλάττιον. In Greek, κύνη, "dog," refers to either a dog or a dog-fish, and neither is a metaphorical or secondary use of the term. A better example in English would be the word "bank," which is ambiguous for either a financial institution or the edge of a river, but neither is so only in a figurative sense. Both are perfectly proper and literal uses of the word "bank." Arians are saying that, just like the English "bank," the word "God" predicates two completely different natures, though neither is a metaphorical or improper sense of the word.

[33] [25] Third Theological Oration (29), *On the Son*, section 14. Translation from [24, p. 306].

Step two is that they paired this characteristically Arian equivocation on "is god" with a related view about counting statements involving ambiguous count nouns. Gregory continues a little later in the *Fifth Theological Oration*, speaking in the voice of his Arian opponents:

> "Things of one essence," you [=Arians] say, "are counted together," and by this "counted together," you mean that they are collected into one number. "But things which are not of one essence are not thus counted; so that *you* [=orthodox Trinitarians] cannot avoid speaking of three gods, according to this account, while *we* [=Arians] do not run any risk at all of it, inasmuch as we [=Arians] assert that they are not consubstantial."[34]

So the accusation made against Trinitarians by Arians is something like this. When we count by a count-noun F, for example "dog," that noun must express some essence or nature, in this case dog-hood. And the number of Fs will be the number of things instantiating this essence or nature. So, if there are three things that all instantiated dog-hood, then there are three dogs.

Applied to the Trinity, the Arian argues as follows. The orthodox Trinitarian holds precisely this sort of view with respect to the persons of the Trinity. That is, the orthodox Trinitarian holds that each of the persons instantiates god-hood (or "the Godhead," to use the old-fashioned word). So, given the Arian view of counting, the orthodox Trinitarian will have to say that there are three gods.

On the other hand, if we have a count-noun that is ambiguous between two essences or natures, then we have to precisify (whether explicitly, or tacitly, given a certain context), and only given that precisification can we answer the question how many Fs there are. For example, if "dog" is ambiguous between a kind of mammal and a kind of fish, and there is one land-dog and one dog-fish in the vicinity, and we ask "how many dogs are there?" the Arian will say that we have to precisify. In this context, there are two admissible precisifications. On one, the question comes to, "how many land-dogs are there?" and the answer is "one." On the other , the question comes to, "how many dog-fish are there?" and the answer is "one." So, on every admissible precisification in this context, the answer is "one." And on no admissible precisification in this context is the answer anything other than "one." So, it is right to answer "one" in a context like that.

Applied to the Trinity, the Arian argues that the three persons do *not* exemplify a single essence or nature, but three distinct natures. However, the count-noun "god" is ambiguous, and can predicate any of these three natures. So, in this context,

[34][25], Fifth Theologian Oration (31), *On the Holy Spirit*, section 17. Translation from [24, p. 323].

there are three admissible precisifications of the predicate "is god," which we can represent formally by using "G_1" for one sense of "is god," "G_2" for the second sense, and "G_3" for the third sense. (The precise semantic content of these predicates can be filled in however the Arian likes. The important fact from a formal point of view is simply that there are *three* senses, whatever they might be.) Then, on any admissible precisification of the question "how many gods are there?" in this context, the answer will be "one." (I.e., there is only one god in the sense of G_1, only one god in the sense of G_2, and only one god in the sense of G_3.) And on no admissible precisification of the question in this context is the answer anything other than "one." So, it is right (*for the Arian*) in a context like this to answer "one" to the question "how many gods are there" (likewise for, how many gods they believe in, worship, etc.)

So, we can state a proposed Arian solution to the LPT as follows:

1. PLI is a formally adequate language for P.
2. Φ_{AR} in PLI is a formally adequate regimentation for P:

   Φ_{AR}:

   (1_{AR}) G_1f

   (2_{AR}) G_2s

   (3_{AR}) G_3h

   (4_{AR}) $f \neq s$

   (5_{AR}) $f \neq h$

   (6_{AR}) $s \neq h$

   (7_{AR}) $(\exists x)(\forall y)(G_i x \ \& \ (G_i y \rightarrow x{=}y))$

   (for every admissible precisification of G_i in this context)
3. Φ_{AR} is formally consistent in PLI.[35]

3.2.2 (Naïve) Modalism

Modalism, also known as monarchianism, patripassianism or Sabellianism, was an early Trinitarian heresy, or family of heresies, that in some way denied the *distinctness* of the divine "persons" or "hypostases."

[35] Proof is left as an exercise for the reader.

We are in a more difficult position to determine precisely the content of Modalist doctrine, as compared to Arianism or orthodox Trinitarianism, due to lack of evidence. No complete modalist writings survive; what we have are fragments quoted by the church fathers and descriptions of their views by the church fathers. And the Fathers may not always have shared our concern for charitably interpreting one's opponents. Perhaps because of this, or perhaps for some other reasons, a certain interpretation of modalism has been quite popular. I have misgivings about the historical accuracy of that account, but since we will be able to do well enough with the standard account, I will not explore the issue, but will simply label the standard account of modalism "Naïve Modalism" (NM) and merely note that, in my opinion, there were probably at least some versions of modalism that were more sophisticated.

Now, what seems to me the less charitable interpretation (or perhaps a perfectly good interpretation of a much less plausible *version* of modalism) can be seen in passages such as this one from St. Basil:

> For they get tripped up [thinking] that the Father is the same as the Son, and that the Son is the same as the Father, and similarly also the Holy Spirit, so that there is one person, but three names.[36]

Similar statements can be found in other patristic descriptions of Sabellianism (as well as the related heresies of Praxaeus, Noetius, etc.)

If we today were to say that "Samuel Clemens" and "Mark Twain" are two names for the same person, then we would express that in PLI by making, say "s" in PLI have the same semantic value as "Samuel Clemens" in English, "m" in PLI have the same semantic value as "Mark Twain" in English, and asserting "s=m" in PLI. (At least, those of us who accept classical identity probably would.) So, if "Father," "Son," and "Holy Spirit" (in English, or their equivalents in Greek) are just three names for the same person, then, the persons of the Trinity are related in the way we would express using the "=" sign in PLI. So, if a Naïve Modalist accepts PLI (and he could), the NM view might be regimented as either of:

$\Phi_{\text{NM-1}}$:		$\Phi_{\text{NM-2}}$:	
($1_{\text{NM-1}}$)	f=g	($1_{\text{NM-2}}$)	Gf
($2_{\text{NM-1}}$)	s=g	($2_{\text{NM-2}}$)	Gs
($3_{\text{NM-1}}$)	h=g	($3_{\text{NM-2}}$)	Gh
($4_{\text{NM-1}}$)	f=s	($4_{\text{NM-2}}$)	f=s

[36] [23, pp. 308–310]. Translation mine.

($5_{\text{NM-1}}$) f=h	($5_{\text{NM-2}}$) f=h
($6_{\text{NM-1}}$) s=h	($6_{\text{NM-2}}$) s=h
($7_{\text{NM-1}}$) $(\exists x)(\forall y)(x{=}g \ \& \ (y{=}g \rightarrow y{=}x))$	($7_{\text{NM-2}}$) $(\exists x)(\forall y)(Gx \ \& \ (Gy \rightarrow y{=}x))$

And we can give a proposed NM solution to the LPT as:

1. PLI is a formally adequate language for P.
2. At least one of $\Phi_{\text{NM-1}}$ in PLI or $\Phi_{\text{NM-2}}$ in PLI is a formally adequate regimentation for P.
3. Both $\Phi_{\text{NM-1}}$ and $\Phi_{\text{NM-2}}$ are formally consistent in PLI.[37]

Similar considerations to those discussed in reference to Impure RI[38] show that it doesn't matter what further logical content might be packed into the Naïve Modalist understanding of "is god" in a regimentation of P1 through P3 as long as "x is god" doesn't entail $x \neq f$, $x \neq s$, or $x \neq h$. More precisely, if:

$$\phi^{t1}{}_x \models t_1{\neq}f \vee t_1{\neq}s \vee t_1{\neq}h$$

then

$$\phi^f{}_x \ \& \ \phi^s{}_x \ \& \ \phi^h{}_x$$

is inconsistent anyway. On the other hand, if:

$$\phi^{t1}{}_x \not\models t_1{\neq}f \vee t_1{\neq}s \vee t_1{\neq}h$$

then

$$\phi^f{}_x \ \& \ \phi^s{}_x \ \& \ \phi^h{}_x \ \& \ f{=}s \ \& \ f{=}h \ \& \ s{=}h \ \& \ (\exists x)(\forall y)(\phi x \ \& \ (\phi y \rightarrow y{=}x))$$

is consistent.

But although both of these regimentations are consistent (given the caveat in the preceding paragraph), neither is much in the way of a regimentation of P, because however P4 through P6 ought to be analyzed, *this isn't it*. NM avoids the inconsistency of LPT_1 and LPT_2, not by so much by offering legitimate alternative regimentations of P4 through P6, but by simply *denying* them. So NM is heretical by the lights of historical orthodoxy.

[37]It should be obvious that there is a model for $\Phi_{\text{NM-1}}$ as well as a model for $\Phi_{\text{NM-2}}$, and each proof is left as an exercise for the reader.

[38]Section 3.1.3, p. 15 ff., above.

3.3 The Big Question

This completes our discussion of representatives of the "major" answers to the LPT that have actually been proposed, both in ancient times and in our own. The question that faces us is whether these are the *only* ways one could *possibly* solve the LPT. And if not, *what other options could there be* for the Trinitarian? If there are no other options, *how could we know that*?

Some philosophers find fault with all of the on-offer solutions to the LPT, but hold out hope for new avenues in "Trinitarian theorizing." They hold that the "business of Trinitarian theorizing" is simply "unfinished," and that there may be fresh, new ways of creatively answering (and hopefully solving) the LPT. For example, Dale Tuggy in "The Unfinished Business of Trinitarian Theorizing," explores a few proposed Trinitarian theories, and finds fault with all of them. However, at least at the time of writing that paper, he still held out hope. "We Christian theologians and philosophers came up with the doctrine of the Trinity; perhaps with God's help we will come up with a better version of it."[39]

I think that sentiment is not atypical of many philosophers in the field. But could there really be any *importantly different* solution to the LPT? Something that is neither a form of RI nor of ST? Is there hope that further "Trinitarian theorizing" may someday pay off in a creative, new way of understanding the Trinity, heretofore undreamt of, and that avoids the anti-Trinitarian's criticisms in some previously unimagined way? Is "the business of Trinitarian theorizing" really "unfinished" in this sense?

In the next section, I will argue that this is not possible. I will show that, despite the fact that there are infinitely many other possible answers to the LPT, they can all be grouped together into a finite taxonomy of "Families" based on certain salient logical features. The ultimate result will be (1) a "Family" all of the members of which are logically inconsistent, (2) a "Family" all of the members of which would be either heretical or not usable by Trinitarians, and (3) and (4) two "Families" that would avoid those problems, and which closely, though not precisely, map onto ST and RI, but all of which will suffer from one or the other (or both) of the difficulties with those views we have already explored. I.e., they will either (3) equivocate on "is god" between P7 on the one hand and P1 through P3 on the other, or else (4) count in a non-standard way.

Thus, the Trinitarian who hopes that further "Trinitarian theorizing" might help is out of luck. Those who find fault with the on-offer solutions for the reasons we have discussed should simply close up the shop. Those who are willing to live with one or the other (or both) of those difficulties, are already in a position to claim

[39] [26, p. 179].

victory, at least with reference to the purely *logical* problem. In either case, no real work remains to be done on any purely *logical* problem for the doctrine of the Trinity.

4 Taxonomy of Possible Solutions

Method, Briefly

In this section, we will be grouping all of the infinitely many possible answers to the LPT together into a finite, and thus manageable, taxonomy. Here is how we will proceed. First, we will note certain key logical features of the already proposed answers to the LPT. Second, we will use these features to create a *jointly exhaustive* (though *not* mutually exclusive) taxonomy of sets, or "Families," of answers to the LPT.

Of course, there are infinitely many possible languages in which to regiment P, and within many of those languages, infinitely many sets of formulae with which to regiment P. But for the purposes of showing there to be a formally adequate solution to the LPT, it would be "overkill" to map out all of them.

For example, once we see how Pure RI avoids inconsistency by eschewing classical identity and positing alternative, relative identity relations in its place, it doesn't matter whether we go on to equivocate on "is god" among P1 through P3 or not. Once we see what minimal set of logical features of Pure RI allows it to avoid formal inconsistency, we can group together all proposed answers to the LPT that share those features into one set, or "Family," of answers to the LPT. Then we can go on to consider only other proposed answers that *do not* share those features.

We will proceed in 7 steps, plus three initial caveats.

Three Caveats

First, aside from the Pure RI-er, everyone involved in the debate seems to accept some version of PLI as a formally adequate language for P. Or in any case, they may as well. Therefore, we will continue with our "prejudice" towards PLI. Specifically, we will assume (or pretend) that: PLI is a formally adequate language for P if and only if there is such a thing as classical identity. And if we accept that PLI is a formally adequate language for P, PLI is what we will use to regiment P.[40]

[40] Again, nothing substantive hangs on this methodological choice. See footnote at 2.1, p. 4 above. Also, as I note below, even if one does find one of these three caveats problematic, we can always take answers to the LPT that exhibit one of the qualities discussed here and treat them as another "Family" in our taxonomy. See p. 26.

Second, almost nobody involved in the debate takes it to be legitimate to equivocate on the terms "the Father," "the Son," or "the Holy Spirit." Likewise, almost nobody takes them to be anything other than singular terms, *if* there are such things as singular terms.[41] So, we will also adopt the policy that, so long as we are working within a language in which there are such things as singular terms, we will insist on treating "the Father," "the Son," and "the Holy Spirit" as singular terms, and on regimenting them univocally wherever they appear.[42] (RIL, as we have seen, has its own way of analyzing what appear to be singular terms in natural languages that gets around the too-cozy relation between singular terms and classical identity.)

Third, while both Pure and Impure RI-ers count by a relation other than classical identity, neither they nor anybody else rejects the general *schema* with which logicians typically analyze counting statements. In other words, nobody denies that a formally adequate regimentation of "There is exactly one God" would have the *schema*:

(7_{SCHEMA}) $(\exists \mathrm{x})(\forall \mathrm{y})(\phi \mathrm{x}\ \&\ (\phi \mathrm{y} \rightarrow \mathrm{y}\ \mathbb{R}\ \mathrm{x}))$

(where $\mathbb{R}$ is a meta-linguistic variable to be filled in with a predicate standing for whatever relation we count by).

Further, it's hard to see what other schema one *could* count by. So, we will only consider answers to the logical problem of the Trinity where P7 is regimented as *some* instance of (7_{SCHEMA}), whether those instances give $\mathbb{R}$ the value of classical identity, some relative identity relation, or whatever.

It should be noted that, even if one were to disagree with *all three* of these provisos, it would by no means wreck the attempt to create a complete taxonomy of possible answers to the LPT. It would only mean that there would be, at most, an additional three Families of answers to the LPT – one Family of answers that does not treat "Father," "Son" and "Holy Spirit" as singular terms (despite accepting the formal adequacy of a language that includes singular terms) and/or equivocates on those terms, one Family that acknowledges classical identity but that for some reason

[41] Of course, there is an exception to every rule. See [1].

[42] In what follows, we shall always let those singular terms be, respectively, "f," "s," and "h," when we are using PLI. Strictly speaking, then, we are leaving out formulae that use other terms, other logical names, in PLI, such as "a," "b," "c," etc., to refer to the persons. To be more logically precise, we should instead use meta-linguistic variables such as "α," "β," and "γ" to range over all possible terms in the language, with the stipulation that $\alpha \neq \beta \neq \gamma$ (i.e., that the values of these meta-linguistic variables, the *terms* or "logical names," be distinct, *not* necessarily that their *bearers* be distinct, which would be the substance of P4 through P6 in all non-NM regimentations). But while this latter course is the more logically precise, it would introduce needless complexity in what will already be a complex taxonomy. So, we will simply choose always to use "f," "s," and "h," in PLI as the terms for the persons.

does not find PLI formally adequate, and one Family that regiments P7 according to some schema other than the usual one. I don't find any of those suggestions plausible enough to warrant attention, but even if I am wrong in ignoring these possibilities, we could still give a complete taxonomy of all possible answers to the LPT by simply grouping all answers to the LPT that have any of these three features into a "Bastard Step-Child Family." In what follows, appropriately enough, I will ignore the members of this family.

4.1 The $\mathbf{LPT_1}$ Family

As we did in Section 1, suppose that in P1 through P3 we take "is God" to be univocal, treat "God" as the name of an individual, and treat "is" as the "is" of (classical) identity. Then suppose we take "is not" in P4 through P6 as univocal claims of (classical) non-identity. In this case, there is such a thing as classical identity, so we take PLI to be a formally adequate language for P, and we use it. The result is, or at least entails, LPT_1, or something just like LPT_1 except for $7_{\text{LPT-1}}$.

But since $7_{\text{LPT-1}}$ is not necessary in order to derive a contradiction, we will group together any proposed answers to the LPT that share the problematic features of its regimentation of P1 through P6. What exactly are those problematic features?

It might seem that the most salient feature of LPT_1 is that it treats "God" as a logical name instead of a predicate. But of course, a contradiction would arise even if there were another name being used besides "God." And a contradiction would arise even if we treated P1 through P3 not as identity claims, but in a way that *entailed* a certain kind of identity claim.

For example, suppose I regiment "x is God" as a predication meaning "x is divine," but then analyze "x is divine" as meaning "x is identical to Lucifer." I will still have a contradiction, and for essentially the same reasons, logically speaking, as LPT_1. Indeed, if there is any term t_i such that $\mathrm{t}_i \neq \mathrm{x}$ and my analysis of "x is God" entails "$\mathrm{x} = \mathrm{t}_i$" I will end up with a contradiction. That is because "The Father is God" will now entail "The Father = t_i" and "The Son is God" will entail "The Son = t_i." And those together will entail "The Father = the Son," and that will contradict $4_{\text{LPT-1}}$, or anything that entails $4_{\text{LPT-1}}$. So, we can group together any answers to the LPT that:

(1) use PLI, and

(2) give some univocal regimentation ϕ to "is God" in P1 through P3, such that

(3) $\phi\alpha \models \alpha = \mathrm{t}_i$ for some term t_i such that $\mathrm{t}_i \neq \alpha$, and

(4) either regiment "is not" in P4 through P6 univocally as $\neq$, or for any other reason entails $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$

into the "LPT_1 Family."[43] Any member of the LPT_1 Family will be a non-solution to the LPT, since its analyses of P1-P3 versus P4-P6 will yield a contradiction.

So, from here on, we will only consider proposed answers to the LPT that do at least one (or more) of the following:

(1) use a language other than PLI (and so, given our "prejudice" in favor of PLI, must reject the existence of classical identity), *or*

(2) fail to give a univocal regimentation ϕ to "is god" in P1 through P3, *or*

(3) give a univocal regimentation ϕ to "is god" in P1 through P3 such that $\phi\alpha \nvDash \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$, *or*

(4) regiment "is not" in any of P4 through P6 in some way other than $\neq$, and do not for any other reason entail $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, or $6_{\text{LPT-1}}$.

4.2 The Non-PLI (Pure RI) Family

We've seen how Pure RI escapes inconsistency by rejecting classical identity, and with it PLI (option (1) immediately above). This means that, *perforce*, classical identity cannot be the relation by which we count gods in P7. This is *both* a feature that allows it to escape inconsistency *and* a feature that makes it controversial.

Since we are assuming (or pretending) that PLI is a formally adequate language for P if and only if there is such a thing as classical identity, we will group together all answers to the LPT that reject classical identity, and with it PLI, into the "Non-PLI Family" of answers – the family of answers all of which choose option (1) above. Since we have already seen at least one member of the Non-PLI Family that has a logically consistent regimentation of P (our Pure RI proposed solution), we know that the Non-PLI Family contains solutions to the LPT.[44]

[43] 1 is strictly speaking redundant, given 3 and our "prejudice" that, as long as there is such a thing as classical identity, PLI is a formally adequate language for P, and the language we will use to regiment P.

[44] It also contains non-solutions, but that will not matter for our concerns, since our taxonomy provides a kind of "process of elimination" proof. And if one rejects a particular proposed answer to the LPT as not formally adequate *because* that answer eschews classical identity and PLI, then one should reject *all* proposed answers to the LPT that eschew classical identity and PLI as not being formally adequate, and thus one should reject all proposed answers that fall within this Family.

So, from here on, we will only consider proposed answers to the LPT that accept the existence of classical identity and that (therefore, given our "prejudice" towards PLI) use PLI as the language in which to regiment P.

4.3 The Naïve Modalist Family (and Cousins)

We've seen how NM escapes inconsistency by analyzing P4 through P6 in such a way as to essentially *reject* them. Again, this is *both* a feature that allows it to escape inconsistency *and* a feature that makes it controversial (actually, in this case, heretical).

A related move would be to regiment P4 through P6 in a a non-committal way that simply *does not entail* any of the relevant classical identity claims, i.e. $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, or $6_{\text{LPT-1}}$, despite accepting that there is such a thing as classical identity, thus falling into option (4) above.

We've seen that orthodox Trinitarians intend to draw a strong, real distinction between the persons. And, assuming classical non-identity exists, it is the *weakest* real distinction that can be drawn. So, if the orthodox Trinitarian accepts the existence of classical (non-)identity, he himself will insist on regimenting P4 through P6 as classical non-identity claims ($4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$). And if the orthodox Trinitarian wanted to analyze P4 through P6 as drawing an *even stronger* distinction than classical non-identity, he would still at least *accept* $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$. Indeed, if his preferred analysis involved a stronger distinction, he would no doubt insist that, in some way or another, his preferred analysis at least *entails* $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$. Thus, we will group together all proposed answers to the LPT that (a) accept the existence of classical (non-)identity, but (b) do *not* entail all of $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$, into the "Naïve Modalist Family" ("NM Family") of answers.[45] Since we have already seen at least one member of the NM Family that has a logically consistent regimentation of P, we know that the NM Family contains solutions to the LPT.[46]

[45] A bit of logical housekeeping is in order. What about an answer to the LPT that, say, entails $5_{\text{LPT-1}}$ and $6_{\text{LPT-1}}$, but fails to entail $4_{\text{LPT-1}}$? Thus, the Holy Spirit would be distinct from the Father and from the Son, but the Father aned Son could be identical, a possibility St. Photios calls "a semi-Sabellian monstrosity" in his arguments against the *filioque*, [18, p. 73]. As we've defined the NM Family (any regimentation that does not entail $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, *and* $6_{\text{LPT-1}}$ – all three), it includes such "semi-Sabellian monstrosities." And this seems like a reasonable grouping. Clearly the orthodox Trinitarian wants to understand P4 through P6 univocally. Any kind of semi-Sabellian view is just about as bad, from the point of view of orthodoxy, as all-out Sabellianism.

Of course, this will mean some members of the NM Family will still be inconsistent, and for just the same reasons (at least some subset of the same reasons) as LPT_1 is. But that is fine. All we are claiming here is that *some* members of the NM Family are consistent – not that all of them are.

[46] Again, it also contains non-solutions, but that will not matter for our concerns, since the

Note that defining the NM Family this way means there will be certain "cousins" of Naïve Modalism included in the NM Family that will regiment, for example, "the Father is not the Son" simply as some "ho-hum" relation, "f R s," that neither commits us to the classical identity of the persons (characteristic of NM), *nor* the classical non-identity of the persons (characteristic of orthodox Trinitarianism). Is it right to include such non-committal answers in the NM Family?

I think so. Again, the *intent* of the orthodox Trinitarian in saying that "the Father is not the Son," is to draw a strong, real distinction between the two, and, at least within a framework that accepts classical non-identity in the first place, classical non-identity will be the weakest real distinction there is. Thus, any regimentation of "is not" that does not even *entail* classical non-identity (within a framework that admits the existence of classical non-identity) clearly subverts the intent of the claim. Or in any case, it clearly fails to say what the orthodox Trinitarian wants to be saying when he says "the Father is not the Son." On the other hand, regimentations of "is not" in P4 through P6 that are *not* classical non-identity statements but that *do entail* them will still be inconsistent with anything that $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$ are inconsistent with anyway (since they will entail $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$).

So, from here on, we will only consider proposed answers to the LPT that do not reject P4 through P6 and that either *do* regiment them univocally as classical non-identity claims between the persons, or else as some formula χ that in some other way at least *entails* those classical non-identity claims.

That means that at this point we can "lock in" our regimentation of P4 through P6 as:

($4_{\text{LPT-2-FAMILY}}$) χ_1 such that $\chi_1 \models \text{f} \neq \text{s}$
($5_{\text{LPT-2-FAMILY}}$) χ_2 such that $\chi_2 \models \text{f} \neq \text{h}$
($6_{\text{LPT-2-FAMILY}}$) χ_3 such that $\chi_3 \models \text{s} \neq \text{h}$

4.4 The Equivocation$_1$ Family

We've seen how Arianism escapes inconsistency by equivocating on "is god" among P1 through P3 (option (2) above). Again, this is *both* a feature that allows it to escape inconsistency *and* a feature that makes it controversial or problematic, though not in exactly the same way as NM.

In the case of NM, it is clear that the same formal feature that allows it to escape inconsistency makes it (unavoidably) heretical. That is to say, NM avoids inconsistency by admitting the strict identity of the persons, but there is no way

orthodox Trinitarian must reject *all* answers in this Family as heretical. Being a non-solution to the LPT is only *more* reason for the orthodox Trinitarian to reject a proposed answer to the LPT.

one could strictly identify the divine persons (in the sense of classical identity) and not fall into the heresy of Modalism. And strict identity (at least within PLI) is part of the purely *formal* apparatus of the language. Thus, here a purely formal, logical feature lands us in heresy, regardless of how we interpret the non-logical vocabulary.[47] Is the same the case with the characteristic equivocation that allows Arianism to escape inconsistency?

Trinitarians clearly want to say that the Father and Son share the same divine nature. However, if we are considering purely *formal* features, we cannot assume any particular semantic value for "god" or "divine." In particular, we cannot assume that "god" or "divine" must mean "a thing with the divine nature." Supposing it did, the equivocation here would certainly yield a heretical result. But, for one thing, a long line of Christian authors, from St. Justin Martyr up through St. Gregory of Nyssa, and beyond, deny that "god" means "thing with the divine nature."[48] We are thus in fact in a quite different situation with respect to the non-logical vocabulary "god" or "divine" that relates to Arianism, as opposed to the "is" and "is not" that relates to Modalism, since "is" counts as "logical" vocabulary, regardless of which sense of "is" it is, whereas "god" and "divine" are obviously part of the non-logical vocabulary, and thus take us into substantive questions of semantics, rather than purely formal questions. It is also the case that there is a sense in which many even of the pro-Nicenes would say that the Father alone is "the One God."[49] On the other hand, no orthodox Trinitarian would say that the persons are strictly identical. However, we can say that, in the sense in which the Father alone is "the One God," the Son and Spirit are simply *other than* the One God.[50] And in that case, the LPT simply does not arise in the first place. Thus, it is only in senses of the word "god" such that each person *does* count (univocally) as "(a) god" that the LPT even becomes an issue. And the fact that there may be *some* sense in which the Father alone is "the One God" does nothing to solve the LPT, so long as there is *any* sense

[47]Of course, strictly speaking it is not *purely* a matter of logical form that makes the propositions heretical! Rather, it is the fact that we are holding constant our uses of "Father" or "f," "Son" or "s," and "Holy Spirit" or "h." Given that bit of non-logical content *plus* the purely logical machinery of classical identity is what gives us heresy. But again, we are ignoring interpretations of P that treat these names in any other way.

[48]See [2, pp. 134–151]. Available at `www.beaubranson.com/research`.

[49]The Nicene Creed itself begins, "I believe in One God, the Father," and statements can be found in St. Athanasius and all three Cappadocian fathers to the effect that the One God is the Father. For a fuller explanation of this sort of view, see [4].

[50]E.g., Gregory Nazianzen, *Carmina Dogmatica* 1, *On the Father* says, "There is one God without source, without cause, uncircumscribed... the mighty Father of a mighty, Only-Begotten, and faithful Son... The Logos of God is *other than* the One God, but not other *in divinity*." (PG 37. Translation mine; emphasis mine.)

of the term "god" that applies to all of the persons univocally. And certainly any Trinitarian would say that there is *some* such sense.

Thus, what we can say is not that there is *no* sense in which one can equivocate on the predicate "is god" among P1 and P3 and remain within the bounds of orthodoxy, but that *there is some sense* in which the predicate "is god" applies univocally to the persons in P1 through P3 (at least, by the lights of orthodox Trinitarianism). And it is this sense (or these senses, if there are multiple such senses), which give rise to the LPT and which we therefore have in view when discussing the LPT. Any sense of the predicate "is god" which would apply only to the Father simply would not give rise to the LPT in the first place. It is only those senses of "is god" that should apply to all of the persons equally, if they apply to them at all, that we have in view here. Thus, while not all analyses of P that equivocate on the predicate "is god" among P1 and P3 are necessarily heretical, they are all *either* heretical *or* irrelevant to the LPT (since they would not be the sense(s) that give rise to the LPT in the first place).

So, we will group together all answers to the LPT that equivocate on the predicate "is god" among P1 through P3 into the "Equivocation$_1$ Family" of answers. Since we have already seen at least one member of the Equivocation$_1$ Family that has a logically consistent regimentation for P, we know that the Equivocation$_1$ Family contains solutions to the LPT.[51]

So, from here on, we will only consider proposed answers to the LPT that do *not* equivocate on "is god" among P1 through P3.

Thus, we can now "lock in" at least the univocality of our regimentation of P1 through P3 as follows:

($1_{\text{LPT-2-FAMILY}}$) ϕf such that $\phi\alpha \nvDash \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$
($2_{\text{LPT-2-FAMILY}}$) ϕs such that $\phi\alpha \nvDash \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$
($3_{\text{LPT-2-FAMILY}}$) ϕh such that $\phi\alpha \nvDash \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$

Why will ϕ be such that $\phi\alpha \nvDash \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$? After Step 1 we decided to only consider answers to the LPT that do one of the following:

(1) use a language other than PLI (and so reject the existence of classical identity), *or*

(2) fail to give a univocal regimentation ϕ to "is god" in P1 through P3, *or*

[51] Again, it also contains non-solutions, but that will not matter for our concerns, since the orthodox Trinitarian must reject *all* answers in this Family as *either* heretical *or* as pertaining to an interpretation of "god" that is not relevant to the LPT. Being a non-solution to the LPT is only *more* reason for the orthodox Trinitarian to reject a proposed answer to the LPT.

(3) give a univocal regimentation ϕ to "is god" in P1 through P3 such that $\phi\alpha \nvDash \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$, *or*

(4) regiment "is not" in any of P4 through P6 in some way other than $\neq$, and do not for any other reason entail $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, or $6_{\text{LPT-1}}$.

After Step 2 we decided only to consider proposed answers to the LPT that do use PLI (so, option (1) is no longer open). After Step 3 we decided only to consider proposed answers to the LPT that *do* regiment "is not" in P4 through P6 as $\neq$ (or at least for some other reason entail $4_{\text{LPT-1}}$, $5_{\text{LPT-1}}$, and $6_{\text{LPT-1}}$) (so, option (4) is no longer open). And after Step 4, we decided to no longer consider answers to the LPT that equivocate on their regimentation of "is god" among P1 through P3 (so, option (2) is no longer open).

But since we are only considering answers to the LPT that choose at least *one* of the above four options, we can from now on only consider answers that take option (3), that is, that give a univocal regimentation ϕ to "is God" in P1 through P3, but such that $\phi\alpha \nvDash \alpha = \mathrm{t}_i$, for any term t_i such that $\mathrm{t}_i \neq \alpha$.

Since $\phi\alpha \nvDash \alpha = \mathrm{t}_i$, for any term t_i such that $\mathrm{t}_i \neq \alpha$, it will not contradict any of:

($4_{\text{LPT-2-FAMILY}}$) χ_1 such that $\chi_1 \models \mathrm{f} \neq \mathrm{s}$
($5_{\text{LPT-2-FAMILY}}$) χ_2 such that $\chi_2 \models \mathrm{f} \neq \mathrm{h}$
($6_{\text{LPT-2-FAMILY}}$) χ_3 such that $\chi_3 \models \mathrm{s} \neq \mathrm{h}$

simply on the basis of the non-identity claims. That is, whatever other logical form may be buried within χ_1, χ_2, and χ_3, could still generate a contradiction, but the non-identity claims themselves will not.

So, from here on out, we know we are dealing with families of answers to the LPT such that their regimentations of P1 through P6 will be consistent barring any problematic logical features that might be tucked away in the regimentation of "is not" beyond mere non-identity. Their regimentations of P1 through P6 will certainly be consistent if "is not" in P4 through P6 is simply analyzed univocally as classical non-identity.

So, our focus now will be on the regimentation of P7. It is here that we will see why ST and RI have seemed intuitively like the only options or the "major" options. Aside from the Non-PLI Family, the Families we have considered so far have all been either inconsistent, heretical, or irrelevant to the LPT. The remaining two Families will map onto ST and and Impure RI in a certain sense. We will later consolidate these into just two Families that roughly map onto ST and (Pure or Impure) RI.

4.5 The Equivocation$_2$ Family (Social Trinitarian)

We've seen how Social Trinitarianism escapes inconsistency by equivocating on "is god" between P7 on the one hand, and P1 through P3 on the other. Again, this is *both* a feature that allows it to escape inconsistency *and* a feature that makes it controversial.

So we will group together all such answers to the LPT into the "Equivocation$_2$ Family" of answers.[52]

Proposed solutions of this variety, therefore, will give regimentations of the form:

($1_{\text{LPT-2-FAMILY}}$) ϕf such that $\phi\alpha \not\models \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$
($2_{\text{LPT-2-FAMILY}}$) ϕs such that $\phi\alpha \not\models \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$
($3_{\text{LPT-2-FAMILY}}$) ϕh such that $\phi\alpha \not\models \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$
($4_{\text{LPT-2-FAMILY}}$) χ_1 such that $\chi_1 \models \mathrm{f} \neq \mathrm{s}$
($5_{\text{LPT-2-FAMILY}}$) χ_2 such that $\chi_2 \models \mathrm{f} \neq \mathrm{h}$
($6_{\text{LPT-2-FAMILY}}$) χ_3 such that $\chi_3 \models \mathrm{s} \neq \mathrm{h}$
($7_{\text{SCHEMA-}\psi}$) $(\exists \mathrm{x})\ (\forall \mathrm{y})\ (\psi \mathrm{x}\ \&\ (\psi \mathrm{y} \rightarrow \mathrm{y}\ \mathbb{R}\ \mathrm{x}))$

That is, the regimentation ϕ of "is god" for P1 through P3 will be different from the regimentation ψ of "is god" in P7. As we said earlier, not all such answers to the LPT will involve anything particularly "social." This is simply the salient *logical* feature of ST that allows it to escape contradiction.

Since we have already seen at least one member of the Equivocation$_2$ Family that has a logically consistent regimentation, we know that the Equivocation$_2$ Family contains solutions to LPT.[53]

So, from here on, we will only consider proposed answers to the LPT that do *not* equivocate on "is god" between P7 on the one hand, and P1 through P3 on the other.

[52]Note that, as we are defining the Equivocation$_2$ Family, it *is* necessary that a member of the Equivocation$_2$ Family family equivocate on "is god," but it is *not* necessary that it employ classical identity. A view that *both* equivocates in this way *and* employs a relation other than classical identity here would still fall into the Equivocation$_2$ Family as we are defining it. Of course, if one finds it more useful, one could have a separate "hybrid" family, the members of which would both equivocate on "is god" and count by a relation other than classical identity, then have a "pure" Equivocation$_2$ Family, the members of which equivocate on "is god" and *do* count by classical identity. For now, I will find it more convenient simply to group these all together into one Equivocation$_2$ Family, albeit a family, like Joseph's, that is "splittable" into the half-tribes of "Pure Equivocation$_2$ Family" and "Hybrid Equivocation$_2$ Family."

[53]Again, it also contains non-solutions, but that will not matter for our concerns, since anyone who objects to the characteristic equivocation involved here must reject *all* of the proposed answers in this Family. Being a non-solution to the LPT is only *more* reason for the orthodox Trinitarian to reject a proposed answer to the LPT.

But we have already "locked in" regimentations of P1 through P6. And we are no longer considering proposed answers to the LPT that equivocate on "is god" between P7 on the one hand, and P1 through P3 on the other. So, however we regiment "is god" in P1 through P3, it will have to be the same as our regimentation of "is god" in P7. And since we are assuming that counting works according to the usual schema (only the precise relation may be disputed), we can now "lock in" regimentations of all of P1 through P7 as:

($1_{\text{LPT-2-FAMILY}}$) ϕf such that $\phi\alpha \not\models \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$
($2_{\text{LPT-2-FAMILY}}$) ϕs such that $\phi\alpha \not\models \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$
($3_{\text{LPT-2-FAMILY}}$) ϕh such that $\phi\alpha \not\models \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$
($4_{\text{LPT-2-FAMILY}}$) χ_1 such that $\chi_1 \models \text{f} \neq \text{s}$
($5_{\text{LPT-2-FAMILY}}$) χ_2 such that $\chi_2 \models \text{f} \neq \text{h}$
($6_{\text{LPT-2-FAMILY}}$) χ_3 such that $\chi_3 \models \text{s} \neq \text{h}$
($7_{\text{SCHEMA-}\phi}$) $(\exists \text{x})(\forall \text{y})(\phi\text{x} \ \& \ (\phi\text{y} \rightarrow \text{y} \ \mathbb{R} \ \text{x}))$
(such that $\phi\alpha \not\models \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$)

leaving open only the question of precisely what relation "$\mathbb{R}$" will represent in ($7_{\text{SCHEMA-}\phi}$).

4.6 The Non-Classical-Identity-Counting Family

We've seen how Impure RI escapes inconsistency by claiming that our counting practices (at least sometimes) employ some relation(s) other than classical identity. Again, this is *both* a feature that allows it to escape inconsistency *and* a feature that makes it controversial.

We are assuming that the logical form of "is god" is not itself formally contradictory and that it does not entail a classical identity claim to some single individual.[54] Thus, as long as the relation we give for $\mathbb{R}$ in ($7_{\text{SCHEMA-}\phi}$) is not classical identity and as long as y $\mathbb{R}$ x does not *entail* y=x, no contradiction will be derivable.

So, we will group together all answers to the LPT that analyze counting statements *via* a relation other than classical identity, and that do not entail classical identity, into the "Non-Classical-Identity-Counting Family" ("NCIC Family") of answers.[55] Since we have already seen at least one member of the NCIC Family that

[54] Again, the assumption that the logical form of "is god" is not in itself contradictory is redundant, given the assumption that it doesn't entail a certain kind of identity claim. A contradiction entails anything.

[55] Note that this means that Pure RI will fall into *both* the Non-PLI Family *and* the NCIC Family. That is fine, since this is only intended to be a jointly exhaustive, not mutually exclusive, taxonomy of answers to the LPT. I will have more to say about this below under the heading "Consolidating

has a logically consistent regimentation of P, we know that the NCIC Family contains solutions to the LPT.[56]

So, from here on, we will only consider proposed answers to the LPT that *do* count by classical identity. But since we are assuming that counting works according to the usual schema, and since we are not equivocating on "is god" between P7 on the one hand, and P1 through P3 on the other, if we use classical identity as the relation to count by in P7, we can fill in the variable $\mathbb{R}$ in:

($7_{\text{SCHEMA-}\phi}$) $(\exists \text{x})(\forall \text{y})(\phi \text{x} \ \& \ (\phi \text{y} \rightarrow \text{y} \ \mathbb{R} \ \text{x}))$

with "=" and have:

($7_{\text{LPT-2-FAMILY}}$) $(\exists \text{x})(\forall \text{y})(\phi \text{x} \ \& \ (\phi \text{y} \rightarrow \text{y} = \text{x}))$

(And if we used any other relation R such that R *entails* classical identity, then our regimentation of P7, whatever it might be, would still at least *entail* ($7_{\text{LPT-2-FAMILY}}$).)

Thus, we are now out of formally consistent alternatives to LPT_1. We can now "lock in" our entire regimentation of P1 through P7 as:

4.7 The LPT_2 Family

($1_{\text{LPT-2-FAMILY}}$) ϕf such that $\phi\alpha \nvDash \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$
($2_{\text{LPT-2-FAMILY}}$) ϕs such that $\phi\alpha \nvDash \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$
($3_{\text{LPT-2-FAMILY}}$) ϕh such that $\phi\alpha \nvDash \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$
($4_{\text{LPT-2-FAMILY}}$) χ_1 such that $\chi_1 \models \text{f} \neq \text{s}$
($5_{\text{LPT-2-FAMILY}}$) χ_2 such that $\chi_2 \models \text{f} \neq \text{h}$
($6_{\text{LPT-2-FAMILY}}$) χ_3 such that $\chi_3 \models \text{s} \neq \text{h}$
($7_{\text{LPT-2-FAMILY-SCHEMA}}$) $(\exists \text{x})(\forall \text{y})(\phi \text{x} \ \& \ (\phi \text{y} \rightarrow \text{y} \ \mathbb{R} \ \text{x}))$
(such that $\phi\alpha \nvDash \alpha = \text{t}_i$ for any term t_i such that $\text{t}_i \neq \alpha$)
(such that $\text{y} \ \mathbb{R} \ \text{x} \models \text{y} = \text{x}$)

All proposed answers to the LPT that fall into the LPT_2 Family will be non-solutions.[57]

our Taxonomy of Proposed Answers," section 5, p. 37.

[56] Again, it also contains non-solutions, but that will not matter for our concerns, since anyone who rejects the view that counting works by way of some relation other than classical identity must reject *all* answers in this Family anyway. Being a non-solution to the LPT is only *more* reason for the orthodox Trinitarian to reject a proposed answer to the LPT.

[57] Proof is left as an exercise for the reader.

5 Consolidating our Taxonomy of Proposed Answers

Finally, we can usefully reduce the number of options by grouping together some of these families of answers in three steps, as follows.

(1) Anti-Trinitarians need not be picky about whether it is some member of the LPT_1 Family or of the LPT_2 Family that is formally adequate. If any member of either of these families is formally adequate for P, then P is inconsistent, and the anti-Trinitarians win. So we can combine these Familes into one and talk simply of the "LPT Family." This leaves only 6 families of answers to the LPT.

(2) Orthodox Trinitarians will want to reject *all* of the answers in the NM Family as heretical and all of the answers in the Equivocation_1 Family as either heretical (if "(a) god" is intended to mean a thing with the divine nature, or something else shared by the persons) or else as dealing with a sense of "god" that isn't what gives rise to the LPT in the first place. If any member of either of these Families is formally adequate for P (under an interpretation that is relevant to the LPT in the first place) then the orthodox conception of the Trinity is incorrect, and the heretics win. Thus, we can usefully group all of these answers together into one "CTH Family." (And since we are including the Equivocation_1 Family into the CTH Family, we will also now allow ourselves to refer to the Equivocation_2 Family simply as "the Equivocation Family.")

This leaves only 5 families of answers to the LPT.

Note that, while we define the LPT Family and the CTH Family as above, we do so with the caveat that while *all* members of the LPT Family are inconsistent, and *all* members of the CTH Family are either heretical or irrelevant, not all inconsistent regimentations of P (all non-solutions to the LPT) go into the LPT Family, and not all heretical views about the Trinity go into the CTH Family. (One can usefully think of the LPT Family, then, as the *Purely* Inconsistent Family, and the CTH Family as the *Purely* Heretical or Irrelevant Family. Regimentations of P found in other families of answers to the logical problem of the Trinity could still be inconsistent for other reasons, or be used to express heretical views about the Trinity once the content is filled in.)[58] The point is simply that the Trinitarian must reject *all* the members of the LPT Family and *all* the members of the CTH Family. Doing so is a necessary, but not a sufficient, condition for the Trinitarian to "win" the debate.

(3) Finally, by rejecting classical identity altogether, Pure RI perforce counts by a relation other than classical identity. But that is the characteristic feature of Impure RI that allows it to escape contradiction. But from the point of view of formal consistency, it is really irrelevant whether one then goes on to accept or

[58] "God is good" and "God is evil" have the same logical form. So clearly there is more to heresy and orthodoxy than simply logical form!

reject the existence of classical identity and the formal adequacy of PLI. That is, as long as a Pure RI answer agrees with an Impure RI answer in its regimentation of P7,[59] as involving a relation other than classical identity, and which *doesn't entail* classical identity (and Pure RI must agree with Impure RI about that), and as long as whatever formula ϕ it uses in its regimentations of P1 through P3 (or its equivalent of P1 through P3) is such that $\phi\alpha \nvDash \alpha = \mathrm{t}_i$ for any term t_i such that $\mathrm{t}_i \neq \alpha$ (and Pure RI *must* agree with Impure RI about that as well), then it is irrelevant whether we say that there is such a thing as classical identity or not. And it is irrelevant whether we regiment P4 through P6 as involving classical non-identity or not.[60]

Furthermore, since we grouped together all answers to the LPT that claim that counting works by some relation other than classical identity, and that does not entail classical identity, into the NCIC Family of answers, Pure RI is already included in it anyway.[61]

We can see that the appearance of Pure RI being importantly distinct from Impure RI (in a sense relevant simply to the question of formal consistency at least) is an illusion. Pure RI may have *rhetorical* (or other) advantages over Impure RI. But any advantages it may have are not *formal*.

The rejection of PLI is in itself controversial. And no proposed answers to the LPT that fall into the Non-PLI Family do not also fall into the NCIC Family. Thus, we can eliminate talk about the Non-PLI Family and simply speak about the NCIC Family.

That leaves only 4 families of answers to the LPT,[62] namely:

(1) the Equivocation Family,

(2) the NCIC Family,

(3) the CTH Family, and

(4) the LPT Family.[63]

[59] Or that part of its regimentation that is parallel to P7, see section 3.1.2, p. 15, above

[60] Or the Pure RI equivalent of P4 through P6.

[61] This is one example of why the categories of our taxonomy are jointly exhaustive, but not mutually exclusive.

[62] Aside from the Bastard Stepchild Family, which we are, appropriately enough, ignoring.

[63] Again, we are ignoring the Bastard Step-Child Family. But if one wants to take these sorts of regimentations seriously, one can simply add them in as a fifth family of answers to the LPT. The features that lead me to ignore them altogether would then simply count as more "controversial" features, since they are at least that.

All answers in the LPT Family (4) are non-solutions. All answers in the CTH Family (3) will be unusable by the orthodox Trinitarian. So, if the orthodox Trinitarian wants to give an analysis of P, that is, an interpretation of S, that is both (a) non-heretical and (b) offers a *solution* (rather than a non-solution) to the LPT, it must fall into either:

(1) the Equivocation Family, which equivocates on "is god" between P7 on the one hand, and P1 through P3 on the other hand, or

(2) the NCIC Family, which counts by a relation other than classical identity.[64]

So, as promised above (section 1.2, p. 3), although there are infinitely many logical forms one could attribute to P, we have created an exhaustive taxonomy of all possible logical forms attributable to P based precisely on the logical features of the major proposed answers to the LPT that cause them to be either inconsistent, heretical or controversial. Although the result does not map onto Social Trinitarianism and Relative Identity Trintiarianism precisely, the taxonomy allows one to see why these two approaches might appear to be the only viable ones, as well as the ways in which a possible solution might subtly differ from proposals given so far. (Specifically, there could be other members of the Equivocation Family in which the non-logical content doesn't necessarily have to do with "centers of consciousness," "divine societies," etc., and there could be other members of the NCIC Family that count by various other relations.)

6 Conclusion

Anyone who takes the "business of Trinitarian theorizing" to be "unfinished" in the sense that there may be new solutions to the purely *formal* difficulty with the doctrine of the Trinity is out of luck. Every answer to the LPT must fall into one (or more) of the categories we have discussed. Only two of these categories contain any solutions to the LPT that are non-heretical. These two categories do indeed roughly correspond to the usual divide between Social Trinitarianism and Relative Identity Trinitarianism, though there is room for additional proposals that may differ in the specific content they employ.

[64] It could fall into both, since, again, these categories are jointly exhaustive but *not* mutually exclusive. If one prefers a mutually exclusive taxonomy here, one could stipulate that the NCIC Family *not* equivocate on "is god," then split the Equivocation$_2$ Family into the "pure" and "hybrid" ST families, and relabel them as the "pure NCIC Family," "pure ST family" and "hybrid family," respectively.

However, anyone who rejects ST *on the basis of* its characteristic equivocation must reject *all* answers in the Equivocation Family. And anyone who rejects RI *on the basis of* its analysis of counting must reject *all* answers in the NCIC Family. The Trinitarian speculations of philosophers might help with the *metaphysics* of the Trinity, with establishing the Biblical basis for it, or with some rhetorical or other issue. But from a purely *formal* point of view, they will always be just another member of one of the Families of answers to the LPT we have defined here, and will necessarily share the controversial features that define those families.

References

[1] H. E. Baber. Trinity, filioque and semantic ascent. *Sophia*, 47(2):149 – 160, 2008.

[2] Beau Branson. *The Logical Problem of the Trinity.* Dissertation, University of Notre Dame, 2014.

[3] Beau Branson. Ahistoricity in analytic theology. *American Catholic Philosophical Quarterly*, 92(1):195–224, 2018.

[4] Beau Branson. The neglected doctrine of the monarchy of the father, and its implications for the analytic debate about the trinity. *Ephemerides Theologicae Lovanienses*, forthcoming.

[5] Jeffrey Brower and Michael Rea. Material constitution and the trinity. *Faith and Philosophy*, 22(1):57–76, 2005.

[6] Jeffrey Brower and Michael Rea. Understanding the trinity. *Logos: A Journal of Catholic Thought and Culture*, 8(1):145–157, 2005.

[7] James Cain. The doctrine of the trinity and the logic of relative identity. *Religious Studies*, 25(2):141–152, 1989.

[8] Richard Cartwright. On the logical problem of the trinity. *Philosophical Essays*, pages 187–200, 1987.

[9] Fred Feldman and Peter Geach. Geach and relative identity (with rejoinder and reply). *The Review of Metaphysics*, 22(3):547–561, 1969.

[10] Peter Thomas Geach. Identity. *The Review of Metaphysics*, 21(1):3 – 12, 1967.

[11] Peter Thomas Geach. *Reference and Generality: An Examination of Some Medieval and Modern Theories.* Cornell University Press, Ithaca, New York, 1968.

[12] Peter Thomas Geach. *Logic Matters.* University of California Press, Berkeley, 1972.

[13] Peter Thomas Geach. Ontological relativity and relative identity. In Milton Karl Munitz, editor, *Logic and Ontology.* New York University Press, New York, 1973.

[14] Brian Leftow. Anti social trinitarianism. In Stephen T. Davis, Daniel Kendall, and Gerald O'Collins, editors, *The Trinity : An Interdisciplinary Symposium on the Trinity*, pages 203–249. Oxford Univ Press, 1999.

[15] A. P. Martinich. Identity and trinity. *The Journal of Religion*, 58(2):169–181, 1978.

[16] A. P. Martinich. God, emperor and relative identity. *Franciscan Studies*, 39(1):180–191, 1979.

[17] Michael Murray and Michael Rea. Philosophy and christian theology. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. The Metaphysics Research Lab, Stanford, CA, 2012.

[18] Photius. *On the Mystagogy of the Holy Spirit*. Studion Publishers, 1983. Includes "Saint Photios and the Filioque" by Michael Azkoul and "The Life of Saint Photios the Great" by St. Justin Popovich, translated by Ronald Wertz.

[19] Michael Rea. Relative identity and the doctrine of the trinity. *Philosophia Christi*, 5(2):431 – 445, 2003.

[20] Michael Rea. Polytheism and christian belief. *Journal of Theological Studies*, 57:133–48, 2006.

[21] Michael Rea. *Oxford Readings in Philosophical Theology 1. 1.* Oxford Univ. Press, Oxford, 2009.

[22] Michael Rea. The trinity. In *The Oxford Handbook of Philosophical Theology*, pages 403–429. Oxford, Oxford University Press, 2009.

[23] St. Basil the Great. *Bibliothek der Symbole und Glaubensregeln der Alten Kirche*. E. Morgenstern, Breslau, 3 edition, 1897.

[24] St. Cyril of Jerusalem and St. Gregory Nazianzen. *A Select Library of Nicene and Post-Nicene Fathers of the Christian Church: S. Cyril of Jerusalem. S. Gregory Nazianzen.*, volume 7 of *A Select Library of Nicene and Post-Nicene Fathers of the Christian Church: Second Series*. Christian literature Company, Oxford; New York, 1894.

[25] St. Gregory Nazianzen. *Die Fünf Theologischen Reden; Text und Ubersetzung mit Einleitung und Kommentar.* Patmos-Verlag, Dusseldorf, 1963.

[26] Dale Tuggy. The unfinished business of trinitarian theorizing. *Religious Studies*, 39(2):165–183, 2003.

[27] Dale Tuggy. On positive mysterianism. *International Journal for Philosophy of Religion*, 69(3):205–226, 2011.

[28] Peter van Inwagen. And yet they are not three gods but one god. In Thomas Morris, editor, *Philosophy and the Christian Faith*, pages 241–278. University of Notre Dame Press, 1988.

[29] Peter van Inwagen. Three persons in one being: On attempts to show that the doctrine of the trinity is self-contradictory. In Melville Y. Stewart, editor, *The Holy Trinity*, pages 83–97. Springer, 2003.

Received 27 May 2019