Rationalizing Uncertainty
I: The Classical Probabilist

Alec Braynen

October 1, 2024

Abstract

This dialogue builds on a previous critique that highlights a potential paradox in the empirical application of probability theory. In the earlier work, it was suggested that while probability is mathematically sound, conceptual challenges arise when it is used to model real-world uncertainty. In this dialogue, Laplace, a proponent of classical probability, and Incredulus, a philosophical skeptic, engage in a thoughtful examination of these issues. Their inquiry explores the interplay between logic, probability, and empirical observation, raising questions about how past experiences shape our understanding of possible outcomes and the limits this may impose on probabilistic reasoning.

Dramatis Personae

- **Laplace**: A defender of traditional probability theory, confident in the ability of probability to quantify uncertainty in both theoretical and empirical contexts. Laplace represents the classical probabilistic viewpoint.
- **Incredulus:**: A philosopher deeply skeptical of the coherence of probability when applied to the empirical world. Incredulus embodies the spirit of incredulity toward long-held assumptions about probability, inspired by the recent identification of a methodological paradox.

Dialogue

Scene 1: Outside the College of Mathematics and Statistics

The conversation unfolds in a quiet resting area just outside the College of Mathematics and Statistics. Students pass by on their way to class, but Incredulus is engrossed in repeatedly flipping his silver coin. Laplace approaches, noticing his preoccupation, and pauses before speaking.

Laplace: Incredulus, what are you doing here? And why are you flipping that coin over and over? Shouldn't you be heading to our Statistics class?

Incredulus: (Looks up, flipping the coin again) Statistics class? Yes, Laplace, I plan to go. But I thought, to be a good student, I should first try to grasp this notion of probability, which, if you recall, we were told is the foundation of Statistics.

Laplace: Ah, always eager to get to the root of things, I see! Well, you're in luck, my friend as I'm quite the master of probability, therefore I can help you with whatever is puzzling you about it.

Incredulus: Really, Laplace? I didn't know! I did notice that you hardly ever have any questions for the professor, while I'm constantly holding up the lesson because I can never seem to understand what she says. But I never imagined that your silence was because you already mastered the foundations of statistics! In that case, I am very lucky indeed.

Laplace: Yes, you are. So tell me, what exactly is puzzling you about probability?

Incredulus: Well Laplace, I am trying to figure out, when, if ever it is appropriate to use probability in scientific work.

Laplace: Appropriate? What do you mean, Incredulus? Probability is absolutely essential in science! How else would we manage uncertainty while attempting to discover facts about the world? Without probability, we'd be left to guesswork. It provides a structured way to account for variability and unknowns in our observations and experiments.

Probability allows us to take imperfect evidence and still draw reasonable conclusions. For instance, when we don't know the exact outcome of a process, probability lets us measure the likelihood of various outcomes based on what we do know. It helps us bridge the gap between ignorance and knowledge.

Imagine conducting an experiment where the data isn't entirely conclusive or the measurements vary—without probability, how would we quantify the reliability of our results? Probability is the language we use to deal with chance, uncertainty, and incomplete information. It's what allows us to make predictions, build models, and understand systems that we can't directly control or predict with certainty.

In fields like physics, biology, and even medicine, probability is indispensable. When scientists analyze the likelihood of an event happening—be it the behavior of particles or the risk of a disease—what they're really doing is applying probabilistic reasoning. It's not just appropriate for science, Incredulus, it is science's most valuable tool for grappling with the inherent uncertainty of the world.

Incredulus: I know my doubt may sound, well, incredulous, but I just can't help wondering about this Probability. I mean, we use it when we're unsure of the outcome, right? If we're unsure of the outcome, how can we be so confident that the probability we assign is actually telling us something scientific?

Laplace: Incredulus, probability is a mathematical concept. It's designed to handle uncertainty in a structured way. Why would you doubt this?

Incredulus: Well Laplace, it seems to me that when we actually use Probability, we're admitting that we don't know exactly what's going to happen. And if we don't know exactly what's going to happen, how can we know if the numbers we assign are accurate? Isn't it like trying to measure something when we don't even know exactly what we're measuring?

Laplace: No, no, no, Incredulus, you misunderstand. It seems to me that you're conflating not knowing what is going to happen with not knowing what can happen. When we use probability, we know what can happen, we just don't know exactly which outcome will occur. But that's the beauty of it—probability gives us a way to quantify the uncertainty while still acknowledging the possible outcomes.

Incredulus: But Laplace, how do we know all of what can happen? Are we not using probability to figure this out in the first place? And if we're relying on probability to tell us what can happen, how can we ever be sure we know all the possibilities?

Laplace: What do you mean, Incredulus? Surely we start with the possibilities first, and then probability comes in to measure their likelihood.

Incredulus: Is this how it works Laplace? Do we not use probability first, for example in Statistics, to determine the possibilities too?

Laplace: No, Incredulus! We don't rely on probability to figure out the possibilities—we use logic or reasoning to define them first. Take that coin you're flipping, for instance. Based on its design, we know it can only land on heads or tails, and that gives us two possibilities. Probability comes in after that to measure how likely each one is.

Incredulus: I see Laplace. So I am in error to think that we use probabilities to determine the possibilities, and instead we use logic and reason to determine them?

Laplace: Yes, Incredulus. It would be ridiculous to use probability to determine the possibilities, like you said, we would never be certain we had all the outcomes!

Incredulus: I see, Laplace! So we use logic and reason, like you did just now, to figure out the possibilities. But now that we have those possibilities, how do we know what probability to assign to each?

Laplace: Well, Incredulus, now that we have the possibilities, we can assign probabilities using logic and, in this case, symmetry. In the case of the coin, we assume each side is equally likely to appear because there's no reason to believe one outcome is more likely than the other. This is known as the principle of indifference—when we have no additional information, we assume each possibility has an equal chance. So for your coin, we assign a probability of 50% to heads and 50% to tails.

Incredulus: I see, Laplace. So, we used logic and the coin's symmetry to assume the outcomes are equally likely since we don't have any reason to think otherwise?

Laplace: Exactly, Incredulus. In classical probability, we often start by assuming that all possible outcomes are equally probable unless we have specific information to suggest otherwise. This is similar to how we approach many situations in science—when detailed knowledge is lacking, we rely on logic, reasonable assumptions, and often past evidence to guide us. It gives us a rational and informed starting point.

Incredulus: I see, Laplace! So if we had more information, like knowing the coin was weighted, that would change how we assign the probabilities?

Laplace: Precisely! If we knew, for instance, that the coin was weighted, or if past experiments showed a tendency for one side to appear more often, we'd adjust the probabilities accordingly. Probability is a flexible tool that adapts as we gain more knowledge. But in the absence of such information, we rely on logic and reason to guide us.

Incredulus: I think I'm starting to understand this tricky probability, but I still wonder—how were you able to determine so quickly and easily that the coin could only land on one of two sides?

Laplace: Well, Incredulus, how about you flip the coin a few times and see what happens?

Incredulus: (Flips the coin a few times) Alright... heads... heads... tails... heads...tails... it seems to have landed on one of both sides, just like you said Laplace.

Laplace: Exactly, Incredulus. Each time you flip the coin, it lands on one of the two sides, just as we expected. The physical structure of the coin—two flat faces—naturally limits the possibilities to heads or tails.

Incredulus: But Laplace, isn't this exactly what you said would be ridiculous, are we not using probabilities to determine possibilities?

Laplace: No, Incredulus! We didn't use probability to determine the possibilities; we used logic and reasoning. The outcomes—heads or tails—are determined by the physical properties of the coin, not by probability. I just asked you to flip it as a sort of proof, not to discover the possibilities.

Incredulus: But Laplace, I still don't understand. If you had me flip the coin and the outcomes are occurred by probability, how is that not using probability to determine the possibilities?

Laplace: I see why you're confused, Incredulus, but there's a distinction here. Logic and reasoning tell us what the possibilities are—based on the physical structure of the coin, we know it must land on one of two sides: heads or tails. Probability doesn't determine those possibilities; it simply measures how likely each one is. When I asked you to flip the coin, it wasn't to discover the possibilities, but to show that it always lands on one of those two sides. We already knew that heads and tails were the only possible outcomes.

Incredulus: I see, Laplace. Probability, helps us measure how likely each possibility is, but it's through logic and reasoning that we determine what the possibilities are. Thank you, Laplace! You really are a master of this tricky subject—and a great teacher too!

Laplace: I'm glad it's starting to make sense, Incredulus.

Incredulus: It is, Laplace. But I do have another question, how exactly did you use logic and reasoning to determine these possibilities?

Laplace: What do you mean, Incredulus?

Incredulus: What I mean, Laplace, is how did you determine that these were the only two possibilities? Isn't it possible that the coin flipped a certain number of times in the air before it landed? Or that it took a specific amount of time to land? What if I flipped it too hard and a hungry bird swooped in to eat it mid-air?

Laplace: Really, Incredulus? A bird swooping in to eat the coin mid-flip? You have quite the imagination sir! Are you trying to ask how I determined what to measure?

Incredulus: Yes, Laplace. Isn't it possible that the coin could flip, one, or ten, or one hundred times with each flip? And isn't it possible it may stay in the air for three seconds, or five seconds, or ten seconds, with each flip? And what's to stop a bird from swooping past and eating it mid-air Laplace? Is that really only possible in my imagination?

Laplace: Well, Incredulus, I suppose that it is possible, but do you not see that in any of those cases, the coin will still land?

Incredulus: Very true, Laplace! In any of those cases, the coin would land. But would it have been invalid to use probability to measure those other scenarios?

Laplace: No, Incredulus, it wouldn't have been invalid. However, it is more logical to measure what must happen in any particular case. Instead of getting caught up in endless possibilities, we rely on logic to guide what we will measure, covering all cases by what must ultimately occur.

Incredulus: I understand, Laplace. And what must ultimately occur, across all those cases, is that the coin will land?

Laplace: Exactly, Incredulus. While we can use probability to measure the likelihood of the number of flips before the coin lands, or the likelihood of the amount of time the coin will stay in the air, or even the likelihood of it absurdly getting eaten by a bird—in any of those cases, the coin lands, or settles. Therefore, it is most logical to focus on what is certain in all cases:

that the coin will land.

Incredulus: I see, Laplace, and in addition to being most logical, the coin landing also seems most easy to measure...but Laplace, I still have a doubt about all of this.

Laplace: What is it, Incredulus?

Incredulus: You will think me ridiculous when you hear it, Laplace, but my doubt, is this: if in all cases under consideration, the coin must land, and using this general case—which we determined from all considered cases that 'the coin will land,' we have determined the specific possible 'ways that the coin may land', are we sure we are not overlooking a possible outcome? I mean, within this general case of 'the coin will land', from which we determined it may land on one flat side or the other, is it not possible that it may land on its edge, which would be neither of the sides, but still a way that the coin can land?

Laplace: Well, Incredulus, I suppose it is theoretically possible for the coin to land on its edge... but that would be quite unlikely! Have you ever seen such a thing?

Incredulus: But Laplace, whether I have ever seen such a thing or not, if it is a possible way for the coin to land, should we not consider it as a relevant possibility for our probabilities, if as you said, we used logic to determine these possibilities? Did not logic and reasoning lead us to this general case of 'coin must land,' from which we then determined the possible ways it could land?

Laplace: Well, yes...

Incredulus: So, if it's a logical possibility, we should consider it. That would mean there are three possible outcomes, right? Heads, tails, and the coin landing on its edge. All of those are possible 'ways the coin can land.'

Laplace: Yes, those would be the three possibilities, though the edge is so unlikely that—

Incredulus: But didn't you say, Laplace, that we don't use probabilities to determine possibilities?

Laplace: Yes, I did.

Incredulus: And is not your stating it's unlikely, a use of probability, in regard to these possibilities, Laplace? I thought you said we use logic and reasoning to determine possibilities and NOT probability?

Laplace: Yes, Incredulus... The edge is a possibility.

Incredulus: Good. I like that I am understanding. So, if the edge is a possibility, we should now use logic and symmetry to determine the likelihood: roughly 33% for heads, 33% for tails, and 33% for the edge, following the principle of indifference?

Laplace: Yes, following the principle of indifference.

Incredulus: Very good! So we should expect to see the edge occur roughly one out of three times according to logic and reason yes?

Laplace: If this usage of the principle holds, yes.

Incredulus: Let's flip the coin, then.
Heads...Tails...Heads...Heads...Tails....Heads...Tails...there is no edge yet Laplace.

Laplace: No, it's as it was before: heads or tails.

Incredulus: But the edge was a logical possibility, and you said the possibilities are determined by logic and not probabilities, because using probabilities to determine possibilities would be ridiculous. Shouldn't it have appeared by now according to our logic and the principle of indifference?

Laplace: Yes, the principle of indifference does state that if we have no reason to favor one outcome over the other, we should give both an equal likelihood, but here, we have a reason to favor the two sides of the coin over the edge, Incredulus.

IC: Really? What is the reason Laplace?

Laplace: The reason, Incredulus, is that while logic helps us determine what is theoretically possible, it doesn't account for the practical realities of the physical world. Factors like the shape of the coin, gravity, how it's tossed, and the surface it lands on, make landing on its edge extremely unlikely in practice.

Incredulus: But Laplace, aren't we then using observations of what happens most often, as the determinant of the possibilities to consider for probability? You said that we used logic and reason to determine the possibilities. And, in regard to the coin, logic told us the coin will land, and therefore, considering the possible ways it could land: heads, tails, or edge. But now you are saying that the reason we shouldn't consider the edge, nor apply the principle of indifference to these three possibilities, is based on past observations of what was more probable, rather than on the logical determination of all possible outcomes.

Laplace: But Incredulus, don't you see that the edge is so improbable that it's almost absurd to consider? Please apply some critical thinking—you're graduate student for goodness sake!

Incredulus: Are you sure you're not the one lacking in critical thinking Laplace? Because, there you go again, referencing probabilities to defend possibilities, when you already admitted earlier that using probabilities to determine possibilities would be ridiculous! Whether the edge is unlikely or not, as a possibility, it should be considered–is that not one of the requirements of probability? That at least one of the considered outcomes must occur? If the edge is a possible outcome, it must be considered, or else wouldn't our possibility space be incomplete, which would violate one of the assumptions required for probability theory to work?

Laplace: Yes, Incredulus, one of the requirements of probability theory is that of the set of possible outcomes, one of them **must** occur, which would require that if the edge is a distinct possibility it must be considered... Incredulus, I thought you said that you didn't understand probability?

Incredulus: No, Laplace, I said that I don't understand when it is appropriate to use probability scientifically.

Laplace: I see. I still think that you're overthinking all of this, Incredulus. The edge is so unlikely that it is impractical to consider it, once we ensure that our inquiry is practical, the principle of indifference then works well for us.

Incredulus: I see Laplace, so our inquiry should be of 'practical ways the coin can land,'?

Laplace: Yes, exactly, Incredulus.

Incredulus: And 'practical ways the coin can land' are the ways it lands that are most likely to occur in observation?

Laplace: Exactly, Incredulus!

Incredulus: And most likely means most probable, so another name for our category is 'most probable ways the coin can land,' or 'ways the coin can land with highest probabilities.' Is that right, Laplace?

Laplace: Yes, Incredulus.

Incredulus: Really, Laplace? Is this how a *master* of probability inquires? By simply ignoring the fact that the probability of one of the considered possibilities occurring must be one—a fundamental axiom of probability—and instead, assuming what's likely before even determining the full set of possibilities and their respective probabilities?

Laplace: I'm not ignoring the axiom, Incredulus! I'm just saying it's practical to focus on the most likely outcomes—heads or tails—since the edge is so rare–it may even be impossible!

Incredulus: Ah, I see! So, now we're calling it 'practical' to assume what's likely instead of considering the full set of possibilities and their probabilities? Tell me, Laplace, how can you even use probability if you don't consider the complete set of possible outcomes? Aren't we supposed to be working with possibilities where the total probability of all them added together sum to one?

Laplace: Yes, but focusing on the most likely outcomes doesn't mean we're entirely dismissing the other possibilities. It's simply a matter of efficiency.

Incredulus: Efficiency, you say? So we consider only the outcomes that seem most efficient to us and ignore everything else? Brilliantly efficient! Again, by doing that, you're ignoring the very foundation of probability theory. If we don't acknowledge *all* the possible ways the coin can land—including the edge—we have no right to even talk about probabilities or 'most probable ways the coin can land.' Without formally considering the edge, your event space isn't complete, and without that, Laplace, you can't even calculate the probabilities in the first place, because you no longer have one of the axioms required to use probability!

Laplace: But certainly such an improbable outcome need not be considered, Incredulus!

Incredulus: And that's precisely where you're mistaken, Laplace. You don't get to assign probabilities without considering the full set of possibilities. If the probability of at least one outcome occurring must be one, we can't just pick what seems likely and ignore the rest. We need to **know** the full possibility space for probability theory to even function. By focusing only on what's 'likely,' you've undermined this very 'probability' that you said you were a master of.

Laplace: But Incredulus, surely there is a place for practicality in all of this? Yes, the edge may be possible, I admit that, but in real-world scenarios, focusing on heads and tails—what's most likely to occur—makes sense, doesn't it? We're just prioritizing what we know actually happens, which seems far more useful than spending time on outcomes that rarely, if ever, occur.

Incredulus: Surely, there is a place for practicality, Laplace—but not at the cost of rigor. Probability, as you should know, does not care for convenience or practicality. You can't start by weighing what's 'likely' or 'efficient' when the first task was to formally account for every possible outcome. To talk about probabilities without first determining the full set of possibilities undermines the entire structure of the math you claimed to have mastered! Your precious practicality means nothing if you lack the complete possibility space to ensure the probabilities add to 1. So tell me, Laplace, what exactly is practical about ignoring a fundamental assumption or requirement of probability theory?

Laplace: Well... Didn't I say earlier that it would be ridiculous to use probabilities to determine possibilities? This is exactly what I meant, Incredulus! You see, in the heat of the argument, I let myself get carried away with practicality! Of course, we must account for the full set of possibilities before we can even begin to talk about likelihoods.

Incredulus: Oh... of course, Laplace! You did say that didn't you? But then, why did you say that? Perhaps understanding that will help the both of us poor thinkers.

Laplace: Well, Incredulus. I had said it would be ridiculous because if we allow probabilities to determine possibilities, we would never be certain we've accounted for all possibilities. We'd be using probabilities to define the very framework we're supposed to measure. It's circular thinking—where we risk missing outcomes or misjudging their likelihoods because we're building everything on only what we've already observed, rather than considering the full range of possibilities. In doing so, we're not truly exploring the unknown; instead, we're limiting ourselves to the narrow scope of what we have seen.

Rather than discovering what is possible, we end up validating only what we think we know. When possibilities are determined by probabilities derived from observations that are themselves thought of probabilistically, we risk ignoring outcomes that fall outside our current experience—and worse, we may never even realize we're missing them. If something unexpected arises—something beyond our assumed framework of possibilities—we may not recognize it at all because our model was never designed to accommodate it. This creates a kind of intellectual blindness, where the very tool meant to help us confront uncertainty instead blinds us to new possibilities and reinforces our ignorance.

As I say all of this, I am beginning to understand your doubts about using probability empirically, Incredulus. Probability should help us manage uncertainty, but it can only do so when we are already certain of what can happen—a certainty I doubt we can ever have, if we are to, as we say, scientifically inquire about the world. If we limit probability to what we've observed, we risk confining our inquiry to the familiar, rather than pushing beyond it. In that regard, probability, tied as it must be, to past observations, seems ill-suited to guide us through the vast uncertainties of the empirical world.

Probability fundamentally depends on us knowing the exact set of possibilities to utilize it, and how can we ever determine that using probabilistic reasoning, the very tool that presupposes we know the possibilities? We cannot—which is why I insisted that its use be grounded in logic and reasoning to determine the possibilities first. Without this grounding, the probabilities we assign remain incomplete, and our results risk being built on unstable assumptions— we would believe we are managing uncertainty, but in reality, we are only managing conceit of knowledge—operating within a limited framework that cannot account for the unknown. And isn't that ridiculous, Incredulus? Relying on a tool to manage uncertainty, that falters precisely when we need it—when we want to inquire about what **is** possible?

Incredulus: I see, Laplace. Using probability, which presupposes a certainty of possibilities, in a context of fundamental uncertainty about what is possible, creates a contradiction. We attempt to quantify the unknown with a framework that cannot accommodate its own limitations.

Laplace: Yes, Incredulus. If we use it to determine possibility, we're only pretending to manage uncertainty. A comforting illusion, perhaps, but not one that will lead us to any real understanding.

Incredulus: I see, well you surely—oh shoot, Laplace... we're late for class!

Laplace: You're right. Let's get going, Incredulus!

Incredulus: Yes, let's! And let's ask the professor about what we talked about just now, perhaps she will find it interesting and worth discussing!

Laplace: Perhaps... Incredulus!

Bibliography

[1] Alec Braynen. The indefensibility of the scientific concept of probability. 2024.

[2] Rudolf Carnap. The two concepts of probability: The problem of probability. *Philosophy and phenomenological research*, 5(4):513–532, 1945.

[3] Rudolf Carnap. *Logical foundations of probability*, volume 2. Citeseer, 1962.

[4] David Hume. *A treatise of human nature*. Oxford University Press, 2000.

[5] John Maynard Keynes. *A treatise on probability*. Courier Corporation, 2013.

[6] Pierre Simon Marquis de Laplace. *A philosophical essay on probabilities*. Wiley, 1902.

[7] Oystein Ore. Pascal and the invention of probability theory. *The American Mathematical Monthly*, 67(5):409–419, 1960.

[8] Jochen Rau. On quantum vs. classical probability. *Annals of Physics*, 324(12):2622–2637, 2009.

[9] Mauricio Suárez. *Philosophy of probability and statistical modelling*. Cambridge University Press, 2020.

[10] Anubav Vasudevan. Chance, determinism and the classical theory of probability. *Studies in History and Philosophy of Science Part A*, 67:32–43, 2018.