The Indefensibility of the Scientific Concept of Probability

Alec Braynen

September 2024

Abstract

Whereas many philosophers accept the validity of 'probability' and confine themselves to interpreting it, this paper challenges its conceptual coherence by critically examining its use in the empirical world. While measure theory provides a rigorous mathematical framework for manipulating probability functions, we argue that applying precise probability measures to empirically uncertain outcomes introduces a fundamental contradiction. Probability measures claim to quantify uncertainty while simultaneously implying a degree of understanding about events that we do not fully possess. This inconsistency undermines the idea that probability offers objective or reliable insights into reality. Moreover, we argue that it is impossible to assign a correct probability in the empirical world—neither deduction nor induction provides a justifiable basis for doing so. Therefore, despite probability appearing to work as a practical tool for managing uncertainty, its theoretical foundation collapses under scrutiny in empirical applications.

One cannot participate in society for long without encountering the concept of probability. We use it to forecast the weather, guide financial decisions, assess risks in medicine, and most coherently, play gambling games. Gambling games, and **problems of fairness** concerning them, were actually central to the development of the foundations of probability theory[7].

In everyday conversation, we seem to use the concept of probability to communicate that some event could possibly happen. More precisely, we seem to utilize the notion of probability to express that the physical world W may encompass a particular set of facts S at some instant t:

$$S \subseteq W(t)$$

In this usage, where the answer to a question about a possible event is 'probably,' we speak rightly and communicate sensibly. In other words, we convey two things: (1) that we believe the event is likely to occur, and (2) that we do not know the exact likelihood of its occurrence. Here, the use of 'probably' is relegated to a statement about belief rather than a knowledge of precise likelihood of the possible event.

On the other hand, in various scientific disciplines, 'probably' is used in a formal and precise manner. In these fields, probability is treated as a rigorous mathematical tool to model uncertainty, make predictions, and infer relationships from data. Researchers and practitioners assign exact probabilities to outcomes, quantifying uncertainty in a way that allows them to make claims about likely results, often backed by empirical evidence and statistical models.

However, in these formal contexts where we use precise probability measures in regard to the empirical world, we place ourselves in an epistemic position that is fundamentally flawed. By assigning mathematically precise probabilities to events in a set of uncertain outcomes in the empirical world, we ridiculously claim to have knowledge about what we lack knowledge of—we quantify our uncertainty in a way that suggests we understand what we do not fully comprehend. In other words, we claim to know with precision what is probable or improbable, despite the fact that such assertions require an understanding of the system or event in question that we do not possess. The very act of utilizing probability measures in the first place reveals this lack of knowledge, as we use them precisely because we do not fully understand the system we are attempting to describe or predict.

This paradox of 'unknowing knowledge,' occurs because the concept of probability has been mangled from its correct context or usage. The foundation of probability theory was built by Pascal and Fermat in response to problems concerning gambling games[7]. The key insight here is that when probability is applied to gambling games, the possibility space is **arbitrated** by the game rules. In other words, unexpected events are irrelevant to the game, therefore relative probabilities in the possibility space are all that matter.

For example, suppose we have a gambling game where two players gain a point if a fair coin lands on either heads or tails. Applying probability to this scenario allows us to enumerate a possibility space $S = \{H, T\}$, where H represents heads and T represents tails; since the measured outcomes are arbitrated by the game, one can **rightly** apply the principle of indifference[5, 6] to propose that the probabilities are:

$$P(H) = P(T) = 50\% = \frac{1}{2}.$$

This is because the game only considers the relative probabilities between the events, where heads and tails are the only possible outcomes. Therefore, within the constraints of the game, the total probability distribution over the sample space S is complete:

$$P(S) = P(H) + P(T) = \frac{1}{2} + \frac{1}{2} = 1.$$

In contrast, if we attempt to make a factual or empirical statement about coin flipping in the empirical world, we can no longer say that the possibility space is $S = \{H, T\}$ because we can no longer arbitrate the possibility space. Relative probabilities do not apply here, so the possibility space must, at the very least, be expanded to $S = \{H, T, E\}$, where E represents the coin landing on its edge. In other words, now that the logical category underpinning our statement is *'ways the coin can land,'* one cannot justifiably exclude E as an outcome. The probability distribution now becomes incomplete:

$$P(S) = P(H) + P(T) + P(E) = ? + ? + ? \stackrel{?}{=} 1.$$

In this case, we hope that the probability distribution sums to 1, as we cannot determine the exact values for each outcome. And while it may be reasonable to assume that the possibilities in this example are exhaustive, in more complex scenarios, additional variables or unforeseen outcomes may arise that we have not accounted for, making it even harder to determine probabilities with any certainty.

To reiterate, in the game, it is acceptable to exclude the edge, because landing on the edge would simply lead to another flip for point calculation.[1] This is because the possibility space is **arbitrated** by the rules of the game. However, in empirical propositions about the world, the goal is to convey facts, and thus one cannot ignore the edge in the category of *'ways the coin can land'* underpinning the model.

Therefore, the assumptions that justified using probability in gambling games collapse when extended to the empirical world. Hume's problem of induction surfaces monstrously, making it impossible to justify assigning probabilities from past observations—probabilities are even less inductively reasonable than phenomena that can be modeled with definite concepts[3]. And neither deduction nor philosophy can provide repose, as empirical outcomes cannot be arbitrated by arbitrary rules like those in gambling games. In other words, the constrained, willed, rule-bound framework that made probability coherent in its original context is absent in empirical applications, leaving it without justification.[2]

To conclude, consider a simple coin-flip experiment conducted in an idealized and controlled environment, where we apply precise variations in force to measure potential outcomes. Our findings indicate that the coin is fair, prompting us to construct a probability model. However, a critical fallacy emerges: the empirical correctness of this probability model is only known because we eliminated all uncertainty from the experiment. This highlights the fundamental paradox in the empirical use of probability—it is a concept used to contend with uncertainty, yet its accuracy can only be known once uncertainty has been eliminated.

The question elucidated in this paper is: **Can probability theory be justifiably applied to empirical phenomena?**

[1] It is acceptable to exclude any non-point-deciding outcome.

[2] To put it in Wittgensteinian terms, 'probability' is being used in the wrong language game, as its application in the empirical world does not align with the context in which it has meaning[9]. From a Platonic perspective, there would be no 'Form' of probability, as it lacks an existence in the realm of perfect Ideas[8, 2]. In Aristotelian terms, probability is not immanent in the natural world, as it does not pertain to the essential qualities or causes of substances[1]. From a Kantian perspective, probability belongs to the realm of subjective judgment, relating to appearances and the limitations of human cognition, rather than any objective property of the noumenal world[4]. Even Pascal, who helped formalize probability, lamented that those who engage in games of chance live in an 'artificial world' detached from genuine understanding[7].

References

[1] Aristotle Aristotle and Aristotle. *Metaphysics*, volume 1. Harvard University Press Cambridge, MA, 1933.

[2] Edith Hamilton, Huntington Cairns, et al. *The collected dialogues of Plato*, volume 18. Princeton University Press, 1961.

[3] David Hume. *A treatise of human nature*. Oxford University Press, 2000.

[4] Immanuel Kant. Critique of pure reason. 1781. *Modern Classical Philosophers, Cambridge, MA: Houghton Mifflin*, pages 370–456, 1908.

[5] John Maynard Keynes. *A treatise on probability*. Courier Corporation, 2013.

[6] Pierre Simon Marquis de Laplace. *A philosophical essay on probabilities*. Wiley, 1902.

[7] Oystein Ore. Pascal and the invention of probability theory. *The American Mathematical Monthly*, 67(5):409–419, 1960.

[8] Plato Plato et al. *The republic*, volume 7. Wiley Online Library, 2008.

[9] Ludwig Wittgenstein. *Philosophical investigations*. John Wiley & Sons, 2009.