

Existential Instantiation, Arbitrary Reference and Supposition

Wylie Breckenridge

Existential instantiation is a rule of inference that allows us infer, from the proposition that there are some p things, the proposition that a is a p thing. What role does 'a' play here? According to one account, recently defended by Breckenridge and Magidor, we use 'a' to refer to a p thing. I argue that this cannot be right. I propose an alternative account, according to which we use 'a' to refer to a *supposedly* p thing.

1. Existential instantiation

Suppose that p and q are properties. Here is a valid argument:

1. There are some p things
 2. All p things are q things
- Therefore,
- C. There are some q things

If its validity is not immediately apparent then we can make it more apparent by adding some intermediate steps, as follows:

1. There are some p things (Premise)
 2. All p things are q things (Premise)
 3. a is a p thing (From 1, by existential instantiation)
 4. a is a q thing (From 2 and 3, by universal instantiation)
- C. There are some q things (From 4, by existential generalisation)

Line 3 is an application of *existential instantiation*. When presenting this argument we might more naturally say, "Let a be p thing", but it is standardly presented in writing as "a is a p thing".

2. The role of 'a' and the p thing account

What role does 'a' play in line 3, and then again in line 4? Although existential instantiation is discussed in standard textbooks on first-order logic, typically not much is said about the role of 'a', other than that it 'stands for an arbitrary p thing' (or something like that). Proving that the rules involving the introduction and elimination of terms such as 'a' are sound and complete tells us that by following these rules we can prove exactly what we should be able to prove, but it does not tell us much about what 'a' means. This turns out to be complicated. Various accounts have been offered. According to some accounts we use 'a' as a meaningless symbol. According to others we use it as a variable. According to Kit Fine (1985) we use it to refer to the arbitrary p thing – a special kind of thing that is not an ordinary p thing. (For details and further references see B&M 2012.)

Breckenridge and Magidor (2012) have recently argued against each of these accounts. They defend an alternative referential account, according to which we use 'a' to refer

to a p thing (an ordinary p thing, not Fine's arbitrary p thing). On this view line 3 actually serves a dual role – we use it to fix the reference of 'a' to a p thing, and also to express the proposition that a is a p thing. By writing "a is a p thing" we emphasize the second of these two roles; we could instead emphasize the first role by writing "Let a be a p thing".

Defending this view of 'a' is not the main aim of B&M's paper – their main aim is to defend what they call 'arbitrary reference', the view that we can refer to things arbitrarily. Nevertheless, their main argument for arbitrary reference depends on this view of the role of 'a'. They argue: In line 3 we get 'a' to refer to a p thing; we can only get 'a' to refer to a p thing if we can arbitrarily refer; therefore, we can arbitrarily refer.

3. Problems for the p thing account

I am persuaded by B&M's arguments against meaningless symbol accounts of 'a', against variable accounts of 'a', and against Fine's referential account of 'a', and for the rest of this paper I will assume that these accounts are wrong and that some other referential account is right. But I'm not persuaded by the referential account defended by B&M. In fact, I think it must be wrong.

B&M do not say such much about what kind of p thing we get 'a' to refer to in line 3, and it is difficult to see what kind of p thing it could be. It cannot be an *actual* p thing, because for some properties p there are no actual p things. Nor can it be a *possible* p thing, because for some properties p there are no possible p things (it might also be impossible to refer to merely possible p things, at least for some properties p). So they can't claim that we get 'a' to refer to an actual or possible p thing.

It won't do for them to say that these properties are special – that for these properties either a different account is needed or that existential instantiation does not work. The original argument is valid, and the extended argument helps to show that it is valid, no matter which properties p and q are; in particular, no matter which property p is. So they need an account of 'a' that works for all properties p.

There are also problems for the idea that 'a' refers to a p thing even for some properties p that have actual instances.

First, let p be the property of being male. This has actual instances. But if q is a property that is inconsistent with p, such as being not male, then there are no possible circumstances in which the premises are both true. So, if 'a' is supposed to refer to some p thing in some circumstance in which the premises are both true then 'a' cannot refer to anything (because there are no such circumstances).

Second, let p be the property of being a never-referred-to thing (see Haze (Forthcoming)). This has actual instances. But 'a' cannot refer to any of them, because that amounts to 'a' referring to a never-referred-to thing, which it cannot do, because if it did then that thing would not be a never-referred-to thing.

Again, it won't do for B&M to say that these properties are special. The original argument is valid, and the extended argument helps to show that it is valid, no matter which properties *p* and *q* are. They need an account that works for all properties *p* and *q*.

So I don't see how a referential account of 'a' can successfully maintain that we get 'a' to refer to a *p* thing.

4. A proposal: the supposedly *p* thing account

I have a suggestion, and presenting this suggestion is the main purpose of this paper. A referential account of 'a' cannot claim that we use 'a' to refer to a *p* thing (as we have just seen). But it might be able to claim that we use 'a' to refer to a *supposedly p* thing (that is, a thing that supposedly has *p*). Moreover, this might still give B&M an argument for arbitrary reference, as follows: In line 3 we get 'a' to refer to a supposedly *p* thing; we can only get 'a' to refer to a supposedly *p* thing if we can arbitrarily refer; therefore, we can arbitrarily refer. And it might allow them to avoid the problems that I have presented above.

My task now is to explain all of this.

Suppose that we are developing a referential account of the role of 'a' in the extended argument – that is, an account according to which we use 'a' to refer to something. Let's think through how the account should go.

One thing that we'd like the account to do is explain how the extended argument helps us to see that the original argument is valid. How might our account do this?

We might try saying this: by adding lines 3 and 4 to the argument we break the inference down into some smaller steps, each of which is more clearly valid than the step from 1 and 2 to *C*. (i.e. lines 3 and 4 are intermediate consequences.) The idea would be this: *C* follows from 1 and 2, because *C* follows from 4, and 4 follows from 2 and 3, and 3 follows from 1. This is a common way of adding lines to an argument to make its validity more apparent.

But we can't say this. In particular, we can't say that 3 follows from 1 (although we might be able to say that 4 follows from 2 and 3). According to the account that we are developing, 'a' refers to something. For 3 to follow from 1 we need the following to be the case: it is not possible for 1 to be true while 3 is false; that is, it is not possible for there to be some *p* things without a being a *p* thing. So we need 'a' to refer to something *x* such that: necessarily, if there are any *p* things then *x* is a *p* thing. For some properties *p* there is such a thing. The property of being Bill Gates is one such property (necessarily, if there are any things that have this property then Bill Gates has this property). So is the property of being a number (necessarily, if there are any things that have this property then 2 has this property), and the property of being even (necessarily, if there are any things that have this property then 2 has this property).

But, and this is the problem, for some properties p there is no such thing. The property of being human is one such property. There is no x such that: necessarily, if there are any humans then x is human. (i.e. every x is such that: possibly, there are some humans but x is not human.) We want an account that works for all properties p , so it cannot be part of our account that 3 follows from 1.

Another common way of adding lines to an argument to make its validity more apparent is to make a supposition. Perhaps we can say that in line 3 we are making a supposition? No, we cannot say this either. If 3 is a supposition then it has to be discharged before the argument is complete. There are two places that it might be discharged: either after line 4, or after C.

Suppose that it is discharged after line 4, by adding line 5 as follows:

1. There are some p things (Premise)
 2. All p things are q things (Premise)
 3. a is a p thing (Supposition)
 4. a is a q thing (From 2 and 3, by universal instantiation)
 5. If a is a p thing then a is a q thing (From 3 – 4 by conditional proof)
- C. There are some q things (From ?)

This cannot be the extended argument. For from which of the previous lines does C follow? We have lines 1, 2 and 5 available (3 and 4 are within the scope of the supposition). It is no help to say that it follows from lines 1 and 2 because that's what we are trying to show. It does not follow from lines 1 and 5 (it is possible for 1 and 5 to be true without C being true)(remember that according to the account that we are developing, ' a ' is not a variable – it refers to something – and 5 is not a universal quantification). It does not follow from lines 2 and 5 (it is possible for 2 and 5 to be true without C being true). And it does not follow from line 5 (it is possible for 5 to be true without C being true).

Suppose that it is discharged it after C, by adding C' as follows:

1. There are some p things (Premise)
 2. All p things are q things (Premise)
 3. a is a p thing (Supposition)
 4. a is a q thing (From 2 and 3, by universal instantiation)
- C. There are some q things (From 4, by existential generalisation)
- C'. If a is a p thing then there are some q things (From 3 – C by conditional proof)

This cannot be the extended argument either, because it does not have the right conclusion. Perhaps there is an implicit further conclusion, C'': There are some q things. (This is just a repetition of C, but this time it is outside the scope of the supposition.) But from which of the previous lines does C'' follow? We have lines 1, 2 and C' available (3, 4 and C are within the scope of the supposition). It is no help to say that it follows from lines 1 and 2 because that's what we are trying to show. It does

not follow from lines 1 and C', it does not follow from lines 2 and C', and it does not follow from line C'.

So we have this: if our account is that we use 'a' to refer to something then to explain why adding lines 3 and 4 is helpful we cannot say that line 3 follows from previous lines, but nor can we say that it is a supposition.

So what can we say? I think we can say this: the extended argument helps us to see that the original argument is valid by helping us to see that C is true on the supposition that 1 and 2 are true. And this, in turn, gives our account a possible referent for 'a' – a supposedly p thing.

This needs some explaining.

We start by supposing that the premises are both true.

This is certainly *one* way that we can start to show that the original argument is valid. Showing that the original argument is valid is equivalent to showing that the following proposition is necessarily true: if there are some p things and all p things are q things then there are some q things. One way to do this is to start by supposing that the antecedent is true and then use conditional proof. (This is not the only way – we could start by supposing that the proposition is false and then use proof by contradiction.) Now, supposing that the antecedent is true amounts to supposing that the premises of the original argument are both true. So we can see that the original argument is valid by first supposing that the premises are both true. What I'm proposing here is that this is what we do when we use the extended argument to see that the original argument is valid.

So we start by supposing that the premises are both true. In particular, we suppose that there are some p things.

Next, line 3. According to the referential account that we are trying to develop, in this line we get 'a' to refer to something. For the reasons that I explained earlier, we can't say that we get 'a' to refer to a p thing, either actual or possible. But we might be able to say this: we get 'a' to refer to a *supposedly* p thing. The idea is this: by supposing that there are some p things we make it the case that there are some supposedly p things (that is, some things that supposedly have p), no matter which property p is (and no matter whether there are any actual or possible p things); then in line 3 we get 'a' to refer to one of these supposedly p things.

The crucial part of this proposal is the following claim: by supposing that there are some p things we make it the case that there are some supposedly p things. We need not claim that these supposedly p things are always *created* by the supposition. We can allow that we can suppose that there are some p things by thinking about some actual things and supposing that they have p – in this case we wouldn't be creating these things (there already are these things), we would just be making it the case that they supposedly have p. We can allow this, but we shouldn't require it – it could be

that to suppose that there are some p things we cannot think of any actual things and suppose them to have p. This is what we should say: if we suppose that there are some p things then there are some things that supposedly have p (which we may or may not create by making the supposition). If we do create these things in making the supposition then their existence depends on the supposition, and lasts just as long as the supposition lasts.

This claim, that if we suppose that there are some p things then there are some supposedly p things, is obviously a controversial claim. But it is in good company – there is a precedent for this kind of claim in theories of fiction. According to some theories of fiction, if an author creates a work of fiction, and it is part of the fiction that there are some p things, then the author thereby makes it the case that there are some fictionally p things (that is, some things that fictionally have p)(see for example Kripke (2013), Searle (1975), van Inwagen (1977)). These things need not be merely fictional things – they could be real things which feature in the fiction. Or they could be merely fictional things, things that are created when the fiction is created and last as long as the fiction lasts. Either way, in creating the work of fiction the author makes it the case that there are some fictionally p things. What I am suggesting is much the same thing, just applied to the making of suppositions rather than the authoring of fictions. Making a supposition is, after all, very much like telling a story. In fact, we can show that the original argument above is valid by telling a story instead: “Once upon a time there were some p things, and all p things were q things. Let’s call one of these p things ‘a’. So a was a p thing. And, since all p things were q things, a was also a q thing. So there were some q things.”

Note that the account of ‘a’ that I am suggesting here is different from the p thing account defended by B&M. Both accounts are referential accounts (i.e. both claim that we use ‘a’ to refer to something), but whereas the B&M account says that we use it to refer to a p thing, this account says that we use it to refer to a *supposedly* p thing. Although it is different from their account, it might still give them an argument for arbitrary reference. For we might still need to appeal to arbitrary reference to account for how we can refer to a particular one of these supposedly p things in the cases in which there are more than one. If so, B&M can argue as follows: In line 3 we get ‘a’ to refer to a supposedly p thing; we can only get ‘a’ to refer to a supposedly p thing if we can arbitrarily refer; therefore, we can arbitrarily refer.

I don’t know whether the account that I am proposing is true (I’m worried about the crucial claim, that if we suppose that there are some p things then there are some supposedly p things). But if we want to maintain that in line 3 we get ‘a’ to refer to something then I don’t see any alternative.

Actually, there *is* an alternative, but it is not one that is open to a referential account of ‘a’, because it is not one on which we use ‘a’ to refer to something. The alternative is to say that in line 3 we merely *suppose* that ‘a’ refers to a p thing (rather than getting ‘a’ to actually refer to a supposedly p thing). On this account we do not get ‘a’ to actually refer – we merely suppose that it does. And this alternative does not give B&M an argument for arbitrary reference: since ‘a’ does not refer it does not have its

reference fixed, so there is no need to appeal to arbitrary reference to explain how it has its reference fixed. Note that on this account we don't even need to suppose that 'a' got its reference fixed arbitrarily – supposing that 'a' refers to one of many p things does not require us to suppose that it had its reference fixed arbitrarily to one of those things. So this account does not commit us in any way to an ability to arbitrarily refer to things. So it does not help B&M's case for arbitrary reference.

But I don't think that this account is right. Here is one argument (I'll give another one later, a better one). In line 3, rather than introduce 'a' we might instead just direct our thoughts to one of the p things, and argue as follows: "Consider one of these p things; it is a q thing; so there are some q things." When we do this we are not *supposing* that we are considering one of these p things – we actually *are* considering one of them. But by introducing 'a' in line 3 all that we are doing is giving ourselves a way of talking about the considering that we are doing. Since we actually are considering one of these supposedly p things, we are also actually referring to one of them with 'a', not merely supposing that we are.

So we start by supposing that the premises are both true; in particular, that there are some p things. This makes it the case that there are some things that supposedly have p. Then, in line 3, we get 'a' to refer to one of these things that supposedly have p (fixing the reference of 'a' arbitrarily to one of them, if there are more than one).

Having fixed the reference of 'a' to one of the supposedly p things we can then talk about this thing, and say what is true of it on the supposition that the premises are both true. On this supposition line 3 ("a is a p thing") is true, not because it follows from line 1 but because of the way that we have fixed the reference of 'a' (we have fixed its reference to a thing that supposedly has p). And, on this supposition, line 4 is also true, since 2 and 3 are true and 4 follows from 2 and 3. And C is true too, since it follows from 4. This establishes that C is true on the supposition that 1 and 2 are both true. And this is how lines 3 and 4 help us to see that the original argument is valid – they help us to see that C is true on the supposition that 1 and 2 are both true.

5. The problems avoided

We are now in a position to see how the account that I am proposing would work for all properties p and q, avoided the problems for the p thing account that I described above.

First, what if p is such that there are no p things? Answer: if we suppose that there are some p things then there are some things which supposedly have p, even if there are no actual p things, and these are available for 'a' to refer to.

Second, what if p is such that there are no possible p things? Answer: if we suppose that there are some p things then there are some things which supposedly have p, even if there are no possible p things, and these are available for 'a' to refer to.

Third, what if p and q are such that the premises are inconsistent? Answer: if we suppose that there are some p things then there are some things which supposedly have p , even if other assumptions are inconsistent with this assumption, and these things are available for 'a' to refer to. Compare the fictional case: there is a fictional character Sherlock Holmes, even if the stories about Sherlock Holmes are inconsistent. Note that under inconsistent suppositions everything is true, so the account predicts that if p and q are such that the premises are inconsistent then we can use the extended argument to derive any conclusion from these premises. But that is the right result.

Fourth, what if p is the property of being a never-referred-to thing? Answer: we can get 'a' to refer to something that supposedly is never-referred-to, because 'a' refers from *outside* the supposition, and the thing it refers to is never-referred-to only *inside* the supposition. 'a' refers, from outside the supposition, to something which, inside the supposition, is never-referred-to. There is no contradiction in that (just as there is no contradiction in our naming and referring to a character in a story who, in the story, has no name).

You might wonder whether the present account faces something like this last problem but with a different property – the property of being a never-supposed-of thing. For we might argue as follows:

1. There are some never-supposed-of things
 2. All never-supposed-of things are q things
- Therefore,
- C. There are some q things

According to the present account, we use 'a' to refer to a supposedly never-supposed-of thing. Isn't that impossible? No, and for the same reason. We can suppose, of something, that it is never-supposed-of, because the supposing occurs *outside* the supposition, and the thing of which we are supposing is never-supposed-of only *inside* the supposition. We suppose, from outside the supposition, something of something which, inside the supposition, is never-supposed-of. There is no contradiction in that.

6. One more argument

I can now give another argument against the idea that in line 3 we *suppose* that 'a' refers to a p thing. If this were right then we could use existential instantiation to show that the proposition that there are some never-referred-to things entails the proposition that grass is green (or any other proposition). For we could argue as follows:

1. There are some never-referred-to things (Premise)
 2. a is a never-referred-to thing (From 1, by existential instantiation)
- C. Grass is green (From 2)

In line 2 we suppose, on this account, that 'a' refers to a never-referred-to thing. But that is to suppose something inconsistent. So on this supposition everything is true. In particular, it is true that grass is green. So if this account were true then we could use existential instantiation to show that the proposition that there are some never-referred-to things entails the proposition that grass is green. But we cannot do that. So this account is not true.

7. Conclusion

I have argued that if, in the example of existential instantiation that we started with, we use 'a' to refer to something then it cannot be to a p thing, either actual or possible. I have suggested an alternative referential account – that that we use 'a' to refer to a *supposedly* p thing. I have explained how using 'a' in this way helps us to see that the original argument is valid, no matter which properties p and q are. And I have pointed out that this alternative account still gives B&M their hoped-for argument for arbitrary reference. But the alternative account relies on something controversial – that if we suppose that there are some p things there are some things that supposedly have p. So I'm not sure whether this is an account that we can ultimately maintain. But if we want a referential account of the role of 'a' then I don't see any alternative.

References

- Breckenridge, W. & Magidor, O. (2012), 'Arbitrary Reference', *Philosophical Studies* **158**, pp. 377-400.
- Fine, K. (1985), *Reasoning with Arbitrary Objects*, Oxford: Basil Blackwell.
- Haze, T. (2016), 'A Counterexample to the Breckenridge-Magidor Account of Instantial Reasoning', *Journal of Philosophical Research* **41**, pp 539-541.
- Kripke, S. (2013), *Reference and Existence*, Oxford University Press.
- Searle, J. (1975), 'The Logical Status of Fictional Discourse', *New Literary History* **6**, pp. 319-332.
- Van Inwagen, P. (1977), 'Creatures of Fiction', *American Philosophical Quarterly* **14**, pp. 299-308.