GOODMAN'S PARADOX, HUME'S PROBLEM, GOODMAN-KRIPKE PARADOX: THREE DIFFERENT ISSUES

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March 2020

ABSTRACT
This paper reports (in section 1 "Introduction") some quotes from Nelson Goodman which clarify that, contrary to a common misunderstanding, Goodman always denied that "grue" requires temporal information and "green" does not require temporal information; and, more in general, that Goodman always denied that grue-like predicates require additional information compared to what green-like predicates require. One of the quotations is the following, taken from the first page of the Foreword to chapter 8 "Induction" of the Goodman's book "Problems and Projects": *"Nevertheless, we may by now confidently conclude that no general distinction between projectible and non-projectible predicates can be drawn on syntactic or even on semantic grounds. Attempts to distinguish projectible predicates as purely qualitative, or non-projectible ones as time-dependent, for example, have plainly failed".*
According to Goodman, since the predicates "blue" and "green" are interdefinable with the predicates "grue" and "bleen", *"if we can tell which objects are blue and which objects are green, we can tell which ones are grue and which ones are bleen"* [pages 12-13 of "Reconceptions in Philosophy and Other Arts and Sciences"]. But this paper points out that an example of interdefinability is also that one about the predicate "gruet", which is a predicate that applies to an object if the object either is green and examined before time *t,* or is non-green and not examined before time *t.*
The three predicates "green", "gruet", "examined before time *t"* are interdefinable:
and even though the predicates "green" and "examined before time *t*" are interdefinable, being able to tell if an object is green does not imply being able to tell if an object is examined before time *t*. The interdefinability among three elements is a type of interdefinability present, for example, also among the logical connectives.
Another example of interdefinability is that one about a decidable predicate PD, which is interdefinable with an undecidable predicate PU: therefore even though we can tell whether an object is PD and whether an object is non-PD, we cannot tell whether an object is PU (since PU is an undecidable predicate) and whether an object is non-PU.
Although the predicates PD and PU are interdefinable, the possibility to determine whether an object is PD does not imply the possibility to determine whether an object is PU (since PU is an undecidable predicate). Similarly, although the predicates "green" and "grue" are interdefinable, the possibility to determine whether an object is "green" even in absence of temporal information does not imply the possibility to determine whether an object is "grue" even in absence of temporal information.
These and other examples about "grue" and "bleen" point out that even in case two predicates are interdefinable, the possibility to apply a predicate P does not imply the possibility to apply a predicate interdefinable with P. And that the possibility to apply the predicate "green" without having temporal information does not imply the possibility to apply the predicate "grue" without having temporal information.
Furthermore, knowing that an object is both green and grue implies temporal information: in fact, we know by definition that a grue object can only be: 1) either green (in case the object is examined before time t); 2) or blue (in case the object is not examined before time t). Thus, knowing that an object is both grue and green, we know that we are faced with case 1, the case of a grue object that is green and examined before time *t*.
Then the paper points out why the Goodman-Kripke paradox is a paradox about meaning that cannot have repercussions on induction. Finally the paper points out why Hume's problem is a problem different from Goodman's paradox and requires a specific treatment.

1 - INTRODUCTION

Nelson Goodman introduced the predicate "grue", which applies to an object if the object either is green and examined before time *t,* or is blue and not examined before time *t.*
In other words: the predicate "grue" applies to an object if the object meets one of the two following conditions:
1) the object is green and examined before time *t;*
2) the object is blue and not examined before time *t.*
The "time *t"* is an arbitrary time in the future (as we will see in the section 8 of this paper, in the so-called "Goodman-Kripke paradox" the time *t* is not arbitrary).
So, since all the emeralds so far examined are both green and grue, the two hypotheses
A "All emeralds are green" and B "All emeralds are grue" are equally well supported.

A and B are two “conflicting hypotheses”: if we assume that there are emeralds that are not examined before time *t*, the hypothses A and B cannot be both true; “All emeralds are green” states that the emeralds not examined before time *t* are green, “All emeralds are grue” states that the emeralds not examined before time *t* are blue.
The two conflicting hypotheses A “All emeralds are green” and B “All emeralds are grue” are equally well supported even though the hypothesis A supports the prediction that the emeralds that will be first examined after time *t* are green, the hypothesis B supports the prediction that the emeralds that will be first examined after time *t* are blue.

Incidentally, I point out that Goodman incurred a misunderstanding regarding mineralogy: Peter Godfrey-Smith writes in chapter 3, section 4 of his book “Theory and Reality”:
“If you know something about minerals, you might object that emeralds are regarded as green by definition [...]. Please just regard this as another unfortunate choice of example by litterature”.

Goodman introduced also the predicate “bleen”, which applies to an object if the object either is blue and examined before time *t*, or is green and not examined before time *t.*

The predicates “grue” and “bleen” are interdefinable with the predicates “green” and “blue”.
The predicate “green” applies to an object if the object either is grue and examined before time *t,* or is bleen and not examined before time *t.*
The predicate “blue” applies to an object if the object either is bleen and examined before time *t,* or is grue and not examined before time *t.*

Using the Goodman’s authentic “grue”, “All emeralds are grue” means that all the emeralds examined before time *t* are green, and all the other emeralds are blue.
Barker and Achinstein misunderstood the Goodman’s definition of “grue” and in their paper of 1960 gave a definition of “grue” according to which “All emeralds are grue” means that all the emeralds are green before time *t* and are blue at time *t* and after time *t*. **[endnote 1]**
The Barker and Achinstein’s apocryphal definition of “grue” is contained in the following quotation:
“But then he asks us to consider a predicate such as ‘grue’, which is to be understood as applying to a thing at a given time if and only if either the thing is then green and the time is prior to time t, or the thing is then blue and the time is not prior to t” [first page of the paper “On the New Riddle of Induction”].

John Norton writes on page 5 of his paper “How the Formal Equivalence of Grue and Green Defeats What is New in the New Riddle of Induction”:

“There is a formal symmetry between green/blue and grue/bleen, so that if we take grue/bleen as our primitives, green and blue are now the grue-ified predicates:
Green applies to all things examined before [some future time] t just in case they are grue but to other things just in case they are bleen. Blue applies to all things examined before t just in case they are bleen but to other things just in case they are grue.
What makes Goodman's rejoinder apparently impregnable is the perfect symmetry of the two sets of definitions. They use the same sentences up to a permutation of terms.
[...].
The symmetry allows a general argument that there is no property of grue that allows us to deprecate it in comparison to green. For any formal property of green, there will be a corresponding property of grue; and conversely”.

For example: many people told Goodman that “green” is simpler than “grue”, that “green” is purely qualitative and “grue” is not purely qualitative. But Goodman replied:

“[...] if we start with ‘blue’ and ‘green’, then ‘grue’ and ‘bleen’ will be explained in terms of ‘blue’ and ‘green’ and a temporal term. But equally truly, if we start with ‘grue’ and ‘bleen’, then ‘blue’ and ‘green’ will be explained in terms of ‘grue’ and ‘bleen’ and a temporal term [...]. Thus qualitativeness is an entirely relative matter” [“Fact, Fiction, and Forecast”, chapter III, section 4].

Hilary Putnam wrote in his Foreword to Nelson Goodman's "Fact, Fiction, and Forecast" that between two interdefinable predicates neither of the two predicates can be more disjunctive than the other one, and neither of the two predicates can be intrinsically disjunctive:

"[...] from the point of view of logic, being disjunctive is a relational *attribute of predicates: whether a predicate is disjunctive is relative to the choice of a language. If one takes the familiar color predicates as primitive, then Goodman's predicate 'grue' is a disjunctive predicate; if one takes the*

unfamiliar predicates grue and bleen as primitive, however, then being green may be defined as being grue and observed prior to time t or being bleen and not observed prior to time t. Thus the predicate grue is disjunctive in a language with normal color predicates as primitive, while the normal color predicates are disjunctive in a language having as primitive the nonstandard predicates (call them 'gruller' predicates) Goodman invented. No predicate is disjunctive or nondisjunctive in itself. What I have just described is the situation as it looks to a logician. Rudolf Carnap proposed that over and above this way in which a predicate can be disjunctive or non-disjunctive, that is, relative to a language or a choice of primitives, a predicate can be intrinsically disjunctive or nondisjunctive"
["Fact, Fiction, and Forecast", pages IX and X of the Foreword].

As said before, many people told Goodman that "green" and "blue" are purely qualitative predicates and that "grue" and "bleen" are not purely qualitative predicates because they ("grue" and "bleen") contain a temporal reference.
Goodman replied that, since green/blue and grue/bleen are interdefinable, there is no reason to accept the thesis according to which green/blue are more qualitative than grue/bleen; and there is no reason within logic to accept the thesis according to which grue/bleen are disjunctive in themselves and green/blue are non-disjunctive in themselves.

Contrary to a common misunderstanding, Goodman always denied that "grue" requires temporal information even if "green" does not require temporal information; and, more in general, Goodman always denied that grue-like predicates require additional information compared to what green-like predicates require.
Goodman wrote on the first page of the Foreword to chapter 8 "Induction" of his book "Problems and Projects":

"Nevertheless, we may by now confidently conclude that no general distinction between projectible and non-projectible predicates can be drawn on syntactic or even on semantic grounds. Attempts to distinguish projectible predicates as purely qualitative, or non-projectible ones as time-dependent, for example, have plainly failed".

If Goodman had admitted that "grue" requires temporal information, but "green" does not require temporal information, it would not make sense to talk about "choice of primitives" (as Putnam did in the passage quoted earlier from his Foreword to "Fact, Fiction, and Forecast"): if "grue" required temporal information and "green" did not require it, "grue" could not be considered as a primitive predicate.

On pages 12-13 of "Reconceptions in Philosophy and Other Arts and Sciences" (section 4 of chapter 1) by Nelson Goodman and Catherine Z. Elgin is written:

"Consider two systems of color classification: S1 classifies objects in terms of our ordinary color predicates. S2 classifies the same objects in terms of less familiar ones. Specifically, S1 contains the terms 'blue' and 'green'. S2 contains the terms 'grue' and 'bleen' which are defined as follows:
X is grue if X is examined before time t and is found to be green or X is not examined before time t and is blue, and X is bleen if X is examined before time t and is found to be blue or X is not examined before time t and is green.
These definitions (and parallel ones for other color terms) ensure that the terms of S2 are as clear and precise as those of S1. If we can tell which objects are blue and which objects are green, we can tell which ones are grue and which ones are bleen. S1 and S2 differ only in the objects they take to be alike in color. Moreover S2 is not in any absolute sense less fundamental than S1. For the terms of S2 and S1 are interdefinable".

On page 14 of "Reconceptions in Philosophy and Other Arts and Sciences" (section 4 of chapter 1) by Nelson Goodman and Catherine Z. Elgin is written:

"Nor can we maintain that 'green' is epistemically more basic or more natural than 'grue', for we have found no way to make sense of absolute epistemological priority. Since 'blue' and 'green' are interdefinable with 'grue' and 'bleen', the question of which pair is basic and which pair derived is entirely a question of which pair we start with".

According to Goodman, since the predicates "green" and "blue" are interdefinable with the predicates "grue" and "bleen", if it is possible to determine if an object is green without needing temporal information, then it is also possible to determine if an object is grue without needing temporal information.

Some people tried to reply to Goodman's paradox by appealing to the so-called “background knowledge”; but the problem is that the Goodman’s paradox does not regard only the generalization “All emeralds are green”, it regards also any other generalization present in our background knowledge: and so, the generalizations contained in our background knowledge are not more solid, are not better supported than the generalization “All emeralds are green”.
Furthermore, the appeals to background knowledge do not consider cases like that one of the predicate “obluemerald”.
The predicate “obluemerald” applies to an object if the object either is examined before time *t* and is not a blue emerald, or is not examined before time *t* and is a blue emerald.
All the objects so far examined are not blue emeralds and are examined before time *t* : and so, all the objects so far examined are obluemerald. The generalization “All objects are obluemerald” supports the prediction that all the objects that will be first examined after time *t* are blue emeralds, and the generalization “All objects are obluemerald” is not less supported than the generalizations present in our invoked background knowledge.

Ian Hacking writes in his paper entitled "Goodman's New Riddle Is Pre-Humian" :
"We should understand his riddle as part of his nominalism. It is not peculiarly connected with induction" [page 233].

The fact that the Goodman’s paradox is not only a paradox of induction is confirmed also by the fact that the Goodman’s paradox causes enormous problems also to the philosophers who refuse induction, like the Popperians: a great part of the last chapter of the book “Common Sense, Science and Scepticism” (a book that the author dedicated to Popper) by Alan Musgrave is about the Goodman’s paradox.
The fact that two interdefinable predicates are formally equivalent produces paradoxes not only about induction.

2 - AN ALARMING CASE OF INTERDEFINABILITY

Please, let’s first consider an example about interdefinability but not about induction.
The predicate "greenthirty" applies to an object if the object either is green and will be on the American continent right at the beginning of the year 2030, or is blue and will not be on the American continent right at the beginning of the year 2030.
The predicate "bluethirty" applies to an object if the object either is blue and will be on the American continent right at the beginning of the year 2030, or is green and will not be on the American continent right at the beginning of the year 2030.
The predicates “greenthirty” and “bluethirty” are interdefinable with the predicates “green" and “blue”: the predicate “green” applies to an object if the object either is greenthirty and will be on the American continent right at the beginning of the year 2030, or is bluethirty and will not be on the American continent right at the beginning of the year 2030.
The predicate “blue” applies to an object if the object either is bluethirty and will be on the American continent right at the beginning of the year 2030, or is greenthirty and will not be on the American continent right at the beginning of the year 2030.

Let’s suppose that a greenthirty-speaker informs us that an object Z, which we know to be green, is also greenthirty: at this point we would automatically know (deducing it from the definitions of the four predicates “greenthirty”, “bluethirty”, “green”, “blue”) that the object Z will be on the American continent right at the beginning of the year 2030.
In fact, according to the definition of “greenthirty” we know that a greenthirty object can only be: 1) either green (in case the object will be on the American continent right at the beginning of the year 2030); 2) or blue (in case the object will not be on the American continent right at the beginning of the year 2030); knowing that the object Z is green and is greenthirty, we know that this is the case 1, the case of an object that is green and will be on the American continent right at the beginning of the year 2030.

Let's suppose that a greenthirty-speaker informs us that an object J, which we know to be blue, is also bluethirty: at this point we would automatically know (deducing it from the definitions of the four predicates “greenthirty”, “bluethirty”, “green”, “blue”) that the object J will be on the American continent right at the beginning of the year 2030.

In fact, according to the definition of “bluethirty” we know that a bluethirty object can only be: 1) either blue (in case the object will be on the American continent right at the beginning of the year 2030); 2) or green (in case the object will not be on the American continent right at the beginning of the year 2030); knowing that the object J is blue and is bluethirty, we know that this is the case 1, the case of an object that is blue and will be on the American continent right at the beginning of the year 2030.

Let's suppose that a greenthirty-speaker informs us that an object K, which we know to be green, is also bluethirty: at this point we would automatically know (deducing it from the definitions of the four predicates “greenthirty”, “bluethirty”, “green”, “blue”) that the object K will not be on the American continent right at the beginning of the year 2030.
In fact, according to the definition of “bluethirty” we know that a bluethirty object can only be: 1) either blue (in case the object will be on the American continent right at the beginning of the year 2030); 2) or green (in case the object will not be on the American continent right at the beginning of the year 2030); knowing that the object K is green and is bluethirty, we know that this is the case 2, the case of an object that is green and will not be on the American continent right at the beginning of the year 2030.

Let's suppose that a greenthirty-speaker informs us that an object L, which we know to be blue, is also greenthirty: at this point we would automatically know (deducing it from the definitions of the four predicates “greenthirty”, “bluethirty”, “green”, “blue”) that the object K will not be on the American continent right at the beginning of the year 2030.
In fact, according to the definition of “greenthirty” we know that a greenthirty object can only be: 1) either green (in case the object will be on the American continent right at the beginning of the year 2030); 2) or blue (in case the object will not be on the American continent right at the beginning of the year 2030); knowing that the object L is blue and is greenthirty, we know that this is the case 2, the case of an object that is blue and will not be on the American continent right at the beginning of the year 2030.

Summing up: if there was a creature able to tell us before the year 2030 if an object is greenthirty, we would know in advance the future: we would know before the year 2030 if an object will be on the American continent right at the beginning of the year 2030 .
Therefore, if it is assumed that no creature can know the future in advance, it is impossible for anyone to know before the year 2030 if an object is greenthirty.

Although green/blue and greenthirty/bluethirty are interdefinable, the possibility to apply before the year 2030 the predicate “green” does not imply the possibility to apply the predicate “greenthirty” before the year 2030.
Symmetrically, although green/blue are interdefinable with grue/bleen, the possibility to apply the predicate “green” without having temporal information does not imply the possibility to apply the predicate “grue” without having temporal information.
In the example about “greenthirty” we obtain an overabundance of information: knowing both whether an object is green, and whether an object is greenthirty necessarily implies also knowing whether that object will be on the American continent right at the beginning of the year 2030.
The mentioned case of the interdefinable pairs of predicates green/blue and greenthirty/bluethirty legitimizes us to look for a check in order to establish whether an excess of information can be present also in the case of the interdefinable pairs of predicates green/blue and grue/bleen.

3 - TERTIUM DATUR: EXAMPLE WITH THREE INTERDEFINABLE PREDICATES

The predicate "greenforty" applies to an object if the object either is green and will be on the American continent just at the beginning of the year 2040, or it is non-green and will not be on the American continent just at the beginning of the year 2040.
The predicate "greenforty" is interdefinable with the predicate “green":
the predicate “green” applies to an object if the object either is greenforty and will be on the American continent just at the beginning of the year 2040, or it is non-greenforty and will not be on the American continent just at the beginning of the year 2040.
Also the predicate "will be on the American continent just at the beginning of the year 2040" is interdefinable with the other two predicates (following the scheme according to which an object is P1 if either is P2 and is P3, or is non-P2 and is non-P3. The scheme is easily obtained taking the Goodman’s definitions of “grue”, “bleen”, “green”, “blue” and replacing the word “blue” with the word “non-green”, and replacing the word “bleen” with the word “non-grue”) **[endnote 2]** :

the predicate "will be on the American continent just at the beginning of the year 2040" applies to an object if the object either is green and is greenforty, or is non-green and is non-greenforty.
The interdefinability among three elements is a type of interdefinability present, for example, also among the logical connectives.

We now define the negative predicates by following the scheme according to which an object is non-P1 if either is non-P2 and is P3, or is P2 and is non-P3.
The predicate “non-greenforty" applies to an object if the object either is non-green and will be on the American continent just at the beginning of the year 2040, or is green and will not be on the American continent just at the beginning of the year 2040.
The predicate “non-green" applies to an object if the object either is non-greenforty and will be on the American continent just at the beginning of the year 2040, or is greenforty and will not be on the American continent just at the beginning of the year 2040.
The predicate "will not be on the American continent just at the beginning of the year 2040” applies to an object if the object either is non-green and is greenforty, or is green and is non-greenforty.
Let’s suppose that a greenforty-speaker informs us that an object Z, which we know to be green, is also greenforty: at this point we would automatically know (on the basis of the above mentioned definition of the predicate "will be on the American continent just at the beginning of the year 2040”) that the object Z will be on the American continent just at the beginning of the year 2040.
Let’s suppose that a greenforty-speaker informs us that an object J, which we know to be non-green, is also non-greenforty: at this point we would automatically know (on the p of the above mentioned definition of the predicate “will be on the American continent just at the beginning of year 2040”) that the object J will be on the American continent just at the beginning of the year 2040.
Let’s suppose that a greenforty-speaker informs us that an object K, which we know to be non-green, is greenforty: at this point we would automatically know (on the basis of the above mentioned definition of the predicate "will not be on the American continent just at the beginning of the year 2040”) that the object K will not be on the American continent just at the beginning of the year 2040.
Let’s suppose that a greenforty-speaker informs us that an object L, which we know to be green, is non-greenforty: at this point we would automatically know (on the basis of the above mentioned definition of the predicate "will not be on the American continent just at the beginning of the year 2040”) that the object L will not be on the American continent just at the beginning of the year 2040.
Summing up: if there was a creature able to tell us before the year 2040 if an object is greenforty, we would know in advance the future: we would know before the year 2040 if an object will be on the American continent just at the beginning of the year 2040.
Therefore, if it is assumed that no creature can know the future in advance, it is impossible for anyone to know before the year 2040 if an object is greenforty.

Furthermore, “green" is interdefinable with the predicate “will be on the American continent just at the beginning of the year 2040", which is a predicate that cannot be applied for sure to anything before the year 2040.
The predicate “will be on the American continent just at the beginning of the year 2040” is interdefinable with the predicate “green”: but there is an asymmetry: we can be sure now that an object X is green, but we cannot be sure before the year 2040 that an object X will be on the American continent just at the beginning of the year 2040.
Although the predicates “green” and “will be on the American continent just at the beginning of the year 2040” are interdefinable, the possibility to apply the predicate “green” before the year 2040 does not imply the possibility to apply the predicate “will be on the American continent just at the beginning of the year 2040” before the year 2040.
Similarly, although the predicates “green” and “grue” are interdefinable, the possibility to apply the predicate “green” even in absence of temporal information does not imply the possibility to apply the predicate “grue” even in absence of temporal information.

Even though “green” and “greenforty” are interdefinable, for the afformentioned reasons it is impossible that someone is able to tell if an object is “greenforty” before the year 2040; it is impossible because that would imply the knowledge of the future, it would imply to know before the year 2040 whether an object will be on the American continent just at the beginning of the year 2040.
Although the predicate “green” and the predicate “greenforty” are interdefinable, the possibility to determine before the year 2040 whether an object is “green” does not imply the possibility to determine before the year 2040 whether an object is “greenforty”.
Symmetrically, although the predicate “green” and the predicate “grue” are interdefinable, the possibility to determine whether an object is “green” without having temporal information does not imply the possibility to determine whether an object is “grue” without having temporal information.
In the example about “greenforty” we obtain an overabundance of information: knowing both whether

an object is green, and whether an object is greenforty necessarily implies also knowing whether that object will be on the American continent just at the beginning of the year 2040.

4 - EXAMPLE ABOUT THE PREDICATE "GRUET"

Let's consider the predicate "gruet", which applies to an object if the object either is green and examined before time *t,* or is non-green and not examined before time *t.*
The predicate "gruet" is interdefinable with the predicate "green":
the predicate "green" applies to an object if the object either is gruet and examined before time *t,* or is non-gruet and not examined before time *t.*
Also the predicate "examined before time *t"* is interdefinable with the other two predicates:
the predicate "examined before time *t"* applies to an object if the object either is green and is gruet, or is non-green and is non-gruet.
Of course, we need temporal information in order to determine if an object is "examined before time *t",* but we don't need temporal information in order to determine if an object is "green".
According to Goodman, since "green" and "grue" are interdefinable, if it is possible to tell if an object is "green" without having temporal information, it is also possible to to tell if an object is "grue" without having temporal information.
But, as we have seen, although the predicate "green" and the predicate "examined before time *t"* are interdefinable, the possibility to tell if an object is "green" without having temporal information does not imply the possibility to tell if an object is "examined before time *t"* without having temporal information.
So, paraphrasing a statement contained in the aforementioned quotation taken from the book by Goodman and Elgin: the statement *"Since the predicates 'green' and 'examined before time t' are interdefinable, if we can tell which objects are green and which objects are non-green, we can tell which ones are examined before time t and which ones are not examined before time t"* is a wrong statement.
Even though we can tell whether an object is green and whether an object is non-green without needing temporal information, we need temporal information to tell whether an object is examined before time *t* and whether an object is not examined before time *t.*

5 - EXAMPLE OF INTERDEFINABILITY WITH AN UNDECIDABLE PREDICATE

Gödel demonstred that a sufficently powerful theory must contain an undecidable predicate (to be more precise: Gödel demonstred that any non-contradictory and sufficiently powerful theory must contain an undecidable predicate).
Let's name "T" a sufficiently powerful theory of mathematics.
Let's name "PU" an undecidable predicate contained in the aforementioned theory T.
Let's name "PD" a decidable predicate contained in the aforementioned theory T.
Let's introduce the predicate "PG", which applies to an object if the object either is PD and is PU, or is non-PD and is non-PU.
The predicate "PG" is defined on the base of the aforementioned scheme according to which an object is P1 if either is P2 and is P3, or is non-P2 and is non-P3.
The predicate "PG" is interdefinable with the predicate "PD":
the predicate "PD" applies to an object if the object either is PG and is PU, or is non-PG and is non-PU.
Also the predicate "PU" is interdefinable with the other two predicates:
the predicate "PU" applies to an object if the object either is PD and is PG, or is non-PD and is non-PG.
The three predicates PD, PG, PU are interdefinable (for instance, also among the logical connectives there are examples of interdefinability among three elements).
So, we have that the decidable predicate PD is interdefinable with the undecidable predicate PU.
So, paraphrasing a statement contained in the aforementioned quotation taken from the book by Goodman and Elgin: the statement *"Since the predicates PD and PU are interdefinable, if we can tell which objects are PD and which objects are non-PD, we can tell which ones are PU and which ones are non-PU"* is a wrong statement.
Even though we can tell whether an object is PD and whether an object is non-PD, we cannot tell whether an object is PU (since PU is an undecidable predicate) and whether an object is non-PU.
Although the predicates PD and PU are interdefinable, within the theory T we cannot determine if an object is PU even though we can determine if an object is PD; the possibility to apply the predicate

PD does not imply the possibility to apply the predicate PU (which is an undecidable predicate). Similarly, although the predicates “green” and “grue” are interdefinable, the possibility to determine if an object is “green” even in absence of temporal information does not imply the possibility determine if an object is “grue” even in absence of temporal information. **[endnote 3]**

6 - EXAMPLE ABOUT BARKER AND ACHINSTEIN’S APOCRYPHAL “GRUE”

In the example about “greenthirty” we obtain an excess of information: knowing both whether an object is green, and whether an object is greenthirty necessarily implies also knowing whether that object will be on the American continent right at the beginning of the year 2030.
The mentioned case of the predicate “greenthirty” (and the other aforementioned examples) legitimizes us to look for a check in order to establish whether an excess of information can be present also in the case of the interdefinable pairs of predicates green/blue and grue/bleen.

Please, let’s consider for a moment the apocryphal "grue" of Barker and Achinstein (mentioned by Putnam in the footnote 5 of his Foreword to Goodman’s "Fact, Fiction, and Forecast"); using the Barker and Achinstein’s apocryphal “grue”, “All emeralds are grue” means that all the emeralds are green before time *t* and are blue at time *t* and after time *t* (using the authentic Goodman’s "grue" instead "All emeralds are grue" means that all the emeralds examined before time *t* are green and all the other emeralds are blue).
The Barker and Achinstein’s apocryphal definition of “grue” is:
“But then he asks us to consider a predicate such as ‘grue’, which is to be understood as applying to a thing at a given time if and only if either the thing is then green and the time is prior to time t, or the thing is then blue and the time is not prior to t” [first page of the paper “On the New Riddle of Induction”].
Thus, the predicate “bleen” applies to a thing at a given time if and only if either the thing is then blue and the time is prior to time *t,* or the thing is then green and the time is not prior to *t.*
Thus, the predicate “green” applies to a thing at a given time if and only if either the thing is then grue and the time is prior to time *t,* or the thing is then bleen and the time is not prior to *t.*
Thus, the predicate “blue” applies to a thing at a given time if and only if either the thing is then bleen and the time is prior to time *t,* or the thing is then grue and the time is not prior to *t.*

In the Goodman’s paradox, time *t* is some arbitrary time in the future.
Let’s call "time *t”* an instant corresponding to the hours 0:00 of a specific day of the year 2021.
The date of the year 2021 corresponding to the time *t* will be drawn on January 1st, 2022; therefore, before the year 2022 it is impossible (without knowing the future) to know what date of the year 2021 corresponds to the time *t.*
According to Goodman, if the green-speaker is able to apply the predicate "green" without needing temporal information, then also the grue-speaker is able to apply the predicate "grue" without needing temporal information; in other words, for the grue-speaker "grue" is a primitive predicate, exactly as for the green-speaker "green" is a primitive predicate. **[endnote 4]**

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object Z, which we know is green, is grue: at this point everyone would automatically know (deducing this from the definitions of the four predicates "grue", "bleen", "green", "blue") that 30 June 2021 is a date prior to the time *t;* but this contradicts the hypothesis from which we started, the hypothesis that no one up to 1st January 2022 could know the date identified as "time *t”* from the draw that will happen the January 1st, 2022 (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date prior to time *t*).
In fact, according to the definition of "grue" we know that a grue object can only be: 1) either green (in case the time is prior to time t); 2) or blue (in case the time is not prior to time t); knowing that the object Z is green and is grue, we know that this is the case 1, the case of an object that is green and the time is prior to time *t.*

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object J, which we know is blue, is bleen: at this point everyone would automatically know (deducing this from the four definitions of the aforementioned four predicates) that 30 June 2021 is a date prior to the time *t;* but this contradicts the initial hypothesis, the hypothesis that no one before 1 January 2022 could know the date corresponding to the time *t* (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date prior to time *t*).
In fact, according to the definition of "bleen" we know that a bleen object can only be: 1) either blue (in case the time is prior to time *t*); 2) or green (in case the time is not prior to time *t*); knowing that the

object J is blue and is bleen, we know that this is the case 1, the case of an object that is blue and the time is prior to time *t.*

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object K, which we know is green, is bleen: at this point everyone would automatically know (deducing this from the four definitions of the aforementioned four predicates) that 30 June 2021 is a date posterior to time *t;* but this contradicts the initial hypothesis, the hypothesis that no one before 1 January 2022 could know the date corresponding to the time *t* (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date posterior to time *t*).
In fact, according to the definition of "bleen" we know that a bleen object can only be: 1) either blue (in case the time is prior to time *t*); 2) or green (in case the time is not prior to time *t*); knowing that the object K is green and is bleen, we know that this is the case 2, the case of an object that is green and the time is not prior to time *t.*

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object L, which we know is blue, is grue: at this point everyone would automatically know (deducing this from the four definitions of the aforementioned four predicates) that 30 June 2021 is a date posterior to time *t;* but this contradicts the initial hypothesis, the hypothesis that no one before 1 January 2022 could know the date corresponding to the time *t* (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date posterior to time *t*).
In fact, according to the definition of "grue" we know that a grue object can only be: 1) either green (in case the time is prior to time *t*); 2) or blue (in case the time is not prior to time *t*); knowing that the object L is blue and is grue, we know that this is the case 2, the case of an object that is blue and the time is not prior to time *t.*

In summary: if in the year 2021, as well as whether an object is green or is blue, we know also whether the mentioned object is grue or is bleen, we would get a result at odds with the assumption of departure: we would get to know already in the year 2021 the temporal location of the time *t,* whose temporal location cannot be known before 1 January 2022.
If we assume that no one (including a creature having a perceptual apparatus different from ours) can know the future in advance, we have in this example the impossibility (also for a creature having a perceptual apparatus different from our perceptual apparatus) of establishing before January 1st, 2022 if an object is grue or is bleen.

The same example can be made using the authentic Goodman's "grue", but with the caveat that should be reported only to the objects examined for the first time in the year 2021.

7 - EXAMPLE ABOUT THE AUTHENTIC GOODMAN'S PARADOX

As I wrote at the beginning of the previous section, in the example about “greenthirty” we obtain an excess of information: knowing both whether an object is green, and whether an object is greenthirty necessarily implies also knowing whether that object will be on the American continent right at the beginning of the year 2030.
The mentioned case of the predicate “greenthirty” (and the other aforementioned examples) legitimizes us to look for a check in order to establish whether an excess of information can be present also in the case of the interdefinable pairs of predicates green/blue and grue/bleen.

In the Goodman’s paradox time *t* is some arbitrary time in the future.
Let’s call "time *t”* an instant corresponding to the hours 0:00 of a specific day of the year 2021.
The date of the year 2021 corresponding to the time *t* will be drawn on January 1st, 2022; therefore, before the year 2022 it is impossible (without knowing the future) to know what date of the year 2021 corresponds to the time *t.*
According to Goodman, if the green-speaker is able to apply the predicate "green" without needing temporal information, then also the grue-speaker is able to apply the predicate "grue" without needing temporal information; in other words, for the grue-speaker "grue" is a primitive predicate, exactly as for the green-speaker "green" is a primitive predicate.

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object Z, which we know is green and is first examined in the year 2021, is also a grue object: at this point everyone would automatically know (deducing this from the definitions of the four predicates "grue", "bleen", "green", "blue") that 30 June 2021 is a date prior to time *t;* but this contradicts the hypothesis from which we started, the hypothesis that no one up to 1 January 2022 could know the date identified as

"time *t*" from the draw that will happen the January 1st, 2022 (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date prior to time *t*).
In fact, according to the definition of "grue" we know that a grue object can only be: 1) either green (in case it is examined before time *t*); 2) or blue (in case it is not examined before time *t*); knowing that the object Z is green and is grue, we know that this is the case 1, the case of an object that is green and examined before time *t.*
In other words: it is impossible to know that an object is both green and grue without knowing also that the object is examined before time *t.*

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object J, which we know is blue and is first examined in the year 2021, is also a bleen object: at this point everyone would automatically know (deducing this from the four definitions of the aforementioned four predicates) that 30 June 2021 is a date prior to time *t;* but this contradicts the initial hypothesis, the hypothesis that no one before 1 January 2022 could know the date corresponding to the time *t* (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date prior to time *t*).
In fact, according to the definition of "bleen" we know that a bleen object can only be: 1) either blue (in case it is examined before the time *t*); 2) or green (in the case it is not examined before time *t*); knowing that the object J is blue and is bleen, we know that this is the case 1, the case of an object that is blue and examined before time *t.*
In other words: it is impossible to know that an object is both blue and bleen without knowing also that the object is examined before time *t.*

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object K, which we know is green and is first examined in the year 2021, is also a bleen object: at this point everyone would automatically know (deducing this from the four definitions of the aforementioned four predicates) that 30 June 2021 is a date posterior to time *t;* but this contradicts the initial hypothesis, the hypothesis that no one before 1 January 2022 could know the date corresponding to the time *t* (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date posterior to time *t*).
In fact, according to the definition of "bleen" we know that a bleen object can only be: 1) either blue (in case it is examined before the time *t*); 2) or green (in case it is not examined before time *t*); knowing that the object K is green and is bleen, we know that this is the case 2, the case of an object that is green and not examined before time *t.* In other words: it is impossible to know that an object is both green and bleen without knowing also that the object is not examined before time *t.*

Let's imagine that on 30 June 2021 a grue-speaker informs us that an object L, which we know is blue and is first examined in the year 2021, is also a grue object: at this point everyone would automatically know (deducing this from the four definitions of the aforementioned four predicates) that 30 June 2021 is a date posterior to time *t;* but this contradicts the initial hypothesis, the hypothesis that no one before 1 January 2022 could know the date corresponding to the time *t* (and, therefore, it is impossible to know on 30 June 2021 if 30 June 2021 is a date posterior to time *t*).
In fact, according to the definition of "grue" we know that a grue object can only be: 1) either green (in case it is examined before time *t*); 2) or blue (in case it is not examined before time *t*); knowing that the object L is blue and is grue, we know that this is the case 2, the case of an object that is blue and not examined before time *t.*
In other words: it is impossible to know that an object is both blue and grue without knowing also that the object is not examined before time *t.*

In summary: if in the year 2021, as well as whether an object is green or is blue, we know also whether the mentioned object is grue or is bleen, we would get a result at odds with the assumption of departure: we would get to know already in the year 2021 the temporal location of the time *t, w*hose temporal location cannot be known before 1st January 2022.
If we assume that no one (including a creature having a perceptual apparatus different from ours) can know the future in advance, we have in this example the impossibility (also for a creature having a perceptual apparatus different from our perceptual apparatus) of establishing before January 1st, 2022 if an object is grue or is bleen.

8 - THE GOODMAN-KRIPKE PARADOX

As Goodman admitted in the Introductory Note to the Third Ediction of "Fact, Fiction, and Forecast", there have been many misunderstandings about Goodman's paradox.
One of the many misunderstandings is that one to believe that the so-called "Goodman-Kripke paradox" is a paradox that has repercussions on induction.

It is a mistake: the Goodman-Kripke paradox is a paradox that concerns the meaning, it is not a paradox about induction.
Ian Hacking writes on page 269 of his paper “On Kripke’s and Goodman’s Uses of ‘Grue’ “:
“ ‘Grue’, he [Kripke] *said, could be used to formulate a question not about induction but about meaning:*
‘the problem would not be Goodman’s about induction [...] but Wittgenstein’s about meaning’ [Saul A. Kripke ‘Wittgenstein on Rules and Private Language’] ‘ “.
The Goodman-Kripke paradox is a paradox about meaning, it is not a paradox about induction for the following reasons.
According to the so-called Goodman-Kripke paradox we could only realize at time *t* that the grue-perceivers before the time *t* used the term "green" to mean "grue".
For example, the group X of observers states that the object A was green before time *t* and became blue at time *t* ; the group Y of observers instead states that the object A was green before the time *t* and at time *t* did not change color.
If for a group of speakers the object A has changed its color from green to blue, but for the other group of speakers the object A has not changed its color from green to blue, it means that we are assuming the hypothesis that the speakers can perceive colors different from the real colors.
In this case there is no paradox concerning induction because if we accept the hypothesis that the observers can perceive colors different from the real colors of the objects, then the real color of the emeralds could be red and be perceived as grue by the grue-perceivers and as green by green-perceivers: in the Goodman’s paradox there is a paradox concerning induction because “All emeralds are green" and "All emeralds are grue" are two "conflicting hypotheses" (Goodman writes in the Foreword to chapter 8 "Induction" of his book "Problems and Projects": *"two such conflicting hypotheses as 'All emeralds are green' and 'All emeralds are grue' “).*
By contrast, "All emeralds are perceived as grue by the grue-perceivers" and "All emeralds are perceived as green by the green-perceivers" are not two conflicting hypotheses: they can both be true and can support perfectly compatible predictions.
In the Goodman-Kripke paradox the two groups of speakers are aware of the difference in meaning that they attributed to the word "green" precisely because of the difference in perceptions that the two groups of speakers have from time *t*. But it is precisely this difference in perceptions that makes the Goodman-Kripke paradox ineffective against induction, it is a paradox about meaning.
In the Kripke’s example the two groups of speakers cannot produce incompatible predictions before time *t* because before time *t* they do not realize the difference in meaning that they attribute to the word "green".
According to Kripke at time *t* the two groups of speakers would realize the difference in meaning that they attribute to the word "green"; but it cannot be excluded a priori that instead at time *t* each of the two groups of speakers would think that the speakers of the other group at time *t* began to have wrong perceptions.
In other words, if we have that the group X of perceivers affirms that the object A changed its color at time *t*, but the group Y of perceivers affirms that the same object A did not changed its color at time *t*, we cannot exclude a priori that at least one of the two groups of perceivers has wrong perceptions.
And this is enough to make the Goodman-Kripke paradox ineffective against induction. Because if we do not exclude that the observers may have wrong perceptions, we find ourselves in the condition that the hypotheses "All the emeralds are perceived as green by the green-perceivers" and "All the emeralds are perceived as grue by the grue-perceivers" are not two conflicting hypotheses: the real color of the emeralds could be red and be perceived as grue by the grue-perceivers and as green by green-perceivers.

Saying in an other way:
about the Goodman-Kripke paradox, there are two possible cases:
1) either there is no contradiction between the assertion of the group X and the assertion of the group Y (and then there is no paradox, because both the groups can be right); 2) or there is a contradiction between the assertion of the group X and the assertion of the group Y, and then at least one of the two groups has wrong perceptions.
If the group X is right in arguing that the object A has changed color, and also the group Y is right in arguing that the object A has not changed color, then there is no paradox: the two statements are both true .
If at least one of the two groups (X and Y) says something wrong, then at least one of the two groups has wrong perceptions.
But if we start from the assumption that the observers can perceive colors that are different from the real colors, then it should be noted that the hypotheses "All the emeralds are perceived as green by the green-perceivers" and "All the emeralds are perceived as grue by the grue-perceivers" are not two conflicting hypotheses: the real color of the emeralds could be red and be perceived as grue by the grue-perceivers and as green by green-perceivers. **[endnote 5]**

9 - ADDITIONAL CONSIDERATIONS ABOUT GOODMAN’S PARADOX

Pointed out what I have written in the previous three sections of this paper, it can be said something else about the Goodman’s paradox.
Using the Goodman's authentic "grue", "All emeralds are grue" means that all the emeralds examined before time *t* are green and all the other emeralds are blue.
After time *t* we cannot ascertain, just examining the objects, if an object is grue examined before time *t* (that is a green object), or is bleen not examined before time *t* (that is a green object too).
Furthermore, after time *t* we cannot ascertain, just examining the objects, if an object is bleen examined before time *t* (that is a blue object), or is grue not examined before time *t* (that is a blue object too).
To sum up: after time *t* we cannot ascertain, just examining the objects, if an object is grue or is bleen.
By contrast, also after time *t* we can ascertain just examining the objects if an object is green or is blue. **[endnote 6]**

Goodman wrote in his Introductory Note to the Third Edition (1973) to “Fact, Fiction, and Forecast”:
“[...] that a major obstacle to a nonpragmatic way of ruling out ‘grue-like’ predicates is the lack of any non-question-begging definition of ‘grue-like’ “.

Therefore, pointed out what is written in the last three sections of this paper, the grue-like predicates are predicates interdefinable with other ones (the green-like predicates), compared to which the grue-like predicates require additional information.

10 - THE HUME’S PROBLEM AND TWO DIFFERENT TREATMENTS

When we have ruled out the grue-like predicates, we can concentrate our attention on the “old riddle of induction”, the Hume’s problem.
Background knowledge is not useful to reply to Hume’s problem for similar reasons for which it is not useful to reply to Goodman’s paradox **[endnote 7].**

Aristotle states in his treatise "Peri Hermeneias" that two universal propositions are named "contrary" when they assert and deny the same predicate, for example: "All men are white" and "All men are not white".
In Aristotelian logic two contrary propositions cannot be both true.
In order to be true a universal proposition must satisfy the necessary condition (not sufficient but necessary) that wants the contrary proposition to be false.
It is reasonable to prefer a (non-contradicted) universal proposition which satisfies this necessary condition, rather than a (non-contradicted) universal proposition which we do not know whether it satisfies this necessary condition.
Let’s compare two incompatible hypotheses A, which states that all emeralds are green, and B, which states that all emeralds are subdivided into green ones and blue ones (in other words, the hypothesis B affirms that there are not only green emeralds, but also blue emeralds).
The hypothesis B can be written in many different ways, not only in the way I'm going to write.
Predicate "X" applies to an object if either the object is a green emerald and the set of emeralds contains a non-empty subset of blue emeralds, or the object is a blue emerald and the set of emeralds contains a non-empty subset of green emeralds.
Let's analyse the following four propositions:
A - “All emeralds are green"
B - “All emeralds are X"
EA -"All emeralds are not green"
EB - “All emeralds are not X".
The proposition EA is the only one that is falsified (if all emeralds are green the proposition EB is true and the proposition B is false).
The proposition A is not contradicted by the data in our possession and has the contrary proposition (EA), that we know is false; by contrast, the proposition B is not contradicted by the data in our possession, but has not the contrary proposition (EB) that is falsified.
We have a reason to prefer the proposition A rather than the proposition B, because the proposition A has the contrary proposition (EA) that we know is false.

Surely there are differences between Aristotelian logic and Frege-Russell logic, but in this case they are irrelevant because when we have to do with inductive generalizations of the form "All Fs are G" we know that the set of Fs (the set of emeralds in the example) is not empty; and so, we have that also using Frege-Russell logic a true universal proposition "All Fs are G" must have the contrary proposition "All Fs are not G" that is false. **[endnote 8]**

Before to write another treatment of the Hume's problem, I write a brief example that does not concern induction.

Let's consider the hypothesis that one person has committed a crime Y in a place Z at time T.
Let's compare the incompatible hypotheses A "Person X has committed the crime Y in the place Z at time T" and B "Person K has committed the crime Y in the place Z at time T".
If we come to know that person X was present in the place Z at time T, we consider it more interesting about person X rather than about person K because the hypothesis A implies the presence of person X in the place Z at time T, the hypothesis B does not implies the presence of the person X in the place Z at time T; in other words, the presence of the person X in the place Z at time T is a necessary consequence of A, it is not a necessary consequence of B.
So, let's imagine that we are obliged to bet either on the hypotheses A or on the hypotheses B, and that the only difference we know about A and B is that we know that person X was present in the place Z at time T, but we don't know if person K was present in the place Z at time T.
It seems to me obvious that we would consider more rational to bet on A rather than on B because the only thing that we know is a necessary consequence of A and is not a necessary consequence of B.

A similar example about induction:
let's compare a hypothesis A "All emeralds are green" with a hypothesis
B "All emeralds are subdivided into green ones and blue ones", which states that there are not only green emeralds but also blue emeralds.
The hypotheses A and B are incompatible.
We don't know *a priori* reasons to prefer one hypothesis to the other one; so, we look for an *a posteriori* reason to prefer one hypothesis to the other one **[endnote 9].**
In the scientific generalizations of the form "All Fs are G" the number of the Fs is assumed as an infinite number (if the number of Fs were a finite number, the Hume's problem would be easily treated with probability calculations, of objective probability), and therefore it is considered impossible to check all Fs.
So, if A is true, B can never be falsified (because it is impossible to check all Fs, it is considered impossible to check all the emeralds; and therefore we can never prove that there is not a blue emerald).
By contrast, if B is true, it is not impossible that A will result falsified (the observation of a blue emerald would falsify A).
In other words, the truthfulness of A implies the impossibility of falsification of both A and B; by contrast, the truthfulness of B does not imply the impossibility of falsification of A.
Thus, the fact that at present both A and B are not falsified is a necessary consequence of A, it is not a necessary consequence of B.
And so, it is more rational to bet on A rather than on B, we prefer to bet on A rather than on B.
[endnote 10]

ENDNOTES

1 - Barker and Achinstein misunderstood the original Goodman's definition of "grue". Goodman wrote in the Foreword to chapter 8 "Induction" of his book "Problems and Projects":
"Occasionally 'grue' is given some different interpretation. For example, in 'Positionality and Pictures' and the paper it discusses, 'grue' is taken to apply not to entire enduring entities but rather to green time-slices examined before 2000 A.D. and to blue time-slices not so examined. Basically the same riddle arises and the shift in interpretation may often go unnoticed. Contrary to a common misunderstanding, however, the interpretation of 'grue' and 'bleen' originally given in 'Fact, Fiction, and Forecast' (and in the above paragraph) does not require that a thing change from green to blue in order to remain grue, or from blue to green in order to remain bleen".

2 - If in the original Goodman's definitions of "grue", "bleen", "green", "blue" we replace the word "blue" with "non-green" and the word "bleen" with "non-grue" we obtain the following new definitions:

the predicate “grue” applies to an object if the object either is green and examined before time *t*, or is non-green and not examined before time *t.*
The predicate “green” applies to an object if the object either is grue and examined before time *t*, or is non-grue and not examined before time *t*.
Therefore, the predicate “examined before time *t*” applies to an object if the object either is green and is grue, or is non-green and is non-grue.
The three definitions follow the scheme according to which an object is P1 if either is P2 and is P3, or is non-P2 and is non-P3.
And the negative predicates follow the scheme according to which an object is non-P1 if either is non-P2 and is P3, or is P2 and is non-P3.
The predicate “non-grue” applies to an object if the object either is non-green and examined before time *t*, or is green and not examined before time *t*.
The predicate “non-green” applies to an object if the object either is non-grue and examined before time *t*, or is grue and not examined before time *t*.
The predicate “not examined before time *t*” applies to an object if the object either is non-green and is grue, or is green and is non-grue.

3 - I'm going to define the negative predicates following the scheme (deduced from the Goodman’s works) according to which an object is non-P1 if either is non-P2 and is P3, or is P2 and is non-P3.
The predicate "non-PG" applies to an object if the object either is non-PD and is PU, or is PD and is non-PU.
The predicate "non-PD" applies to an object if the object either is non-PU and is PG, or is PU and is non-PG.
The predicate "non-PU" applies to an object if the object either is non-PD and is PG, or is PD and is non-PG.
What I intend to demonstrate is that it is not possible that both "PD" and "PG" are decidable predicates.
Let's try to see what would happen if the predicates "PD" and "PG" were both decidable predicates.
If we knew that an object Z, which we know to be PD, is also PG, we would automatically know (according to the aforementioned definition of "PU") that the object Z is PU.
But this would be an absurdity, because the predicate "PU" is an undecidable predicate, and therefore we cannot apply the predicate PU to the object Z, we cannot determine within the theory T whether an object is PU.
In other words, this would demonstrate within the theory T that the predicate "PU" is a decidable predicate; but this is impossible because the predicate "PU" is undecidable within the theory T.
If we knew that an object J, which we know to be non-PD, is also non-PG, we would automatically know (according to the aforementioned definition of "PU") that the object J is PU.
But this would be an absurdity because the predicate "PU" is an undecidable predicate, and therefore we cannot apply the predicate "PU" to the object J, we cannot determine within the theory T whether the object J is PU.
In other words, this would demonstrate within the theory T that the predicate "PU" is a decidable predicate; but this is impossible because the predicate "PU" is undecidable within the theory T.
If we knew that an object K, which we know to be non-PD, is also PG, we would automatically know (according to the aforementioned definition of "non-PU") that the object K is non-PU.
But this would be an absurdity because the predicate "PU" is an undecidable predicate, and therefore we can not apply the predicate "non-PU" to the object K, we cannot determine within the theory T whether the object K is non-PU.
In other words, this would demonstrate within the theory T that the predicate "PU" is a decidable predicate; but this is impossible because the predicate "PU" is undecidable within the theory T.
If we knew that an object L, which we know to be PD, is also non-PG, we would automatically know (according to the aforementioned definition of "non-PU") that the object L is non-PU.
But this would be an absurdity because the predicate "PU" is an undecidable predicate, and therefore it is not possible to apply the predicate "non-PU" to the object L, it is not possible to determine within the theory T whether the object L is non-PU.
In other words, this would demonstrate within the theory T that the predicate "PU" is a decidable predicate; but this is impossible because the predicate "PU" is undecidable within the theory T.
To sum up: the decidability of both predicates "PD" and "PG" would imply the decidability of the predicate "PU": but this would be an absurdity, because "PU" is certainly an undecidable predicate.

Another mathematical example is the following: we can refer to Matiyasevich’s theorem, among whose results there is that every sufficiently powerful theory must contain an unresolvable diophantine equation of which within the theory cannot be proved the unresolvability (and also the resolvability of which cannot be proved).

Let’s call T a sufficiently powerful theory and let’s call EW a diophantine equation which is unresolvable within the theory T, but the unresolvability of which cannot be proved within the theory T (and also the resolvability of which cannot be proved within the theory T).
The predicate "is a welement of the set X" applies to an object if either the object is an element of the set X and the equation EW is resolvable, or the object is not an element of the set X and the equation EW is unresolvable.
The predicate “is a welement of the set X” is interdefinable with the predicate “is an element of the set X”.
The predicate "is an element of the set X" applies to an object if either the object is a welement of the set X and the equation EW is resolvable, or the object is not a welement of the set X and the equation EW is unresolvable.
The predicate "is not a welement of the set X" applies to an object if either the object is not an element of the set X and the equation EW is resolvable, or the object is an element of the set X and the equation EW is unresolvable.
The predicate "is not an element of the set X" applies to an object if either the object is not a welement of the set X and the equation EW is resolvable, or the object is a welement of the set X and the equation EW is unresolvable.
What I intend to demonstrate is that, even though the predicate “is an element of the set X” and the predicate “is a welement of the set X” are interdefinable, if it is possible to determine if an object is an element of the set X, it is impossible to determine if an object is a welement of the set X.
Let's try to see what would happen if we can determine both: whether an object "is an element of the set X" and whether an object “is a welement of the set X".
For example, if we knew that an object Z, which we know to be an element of the set X, is also a welement of the set X, we would automatically know (deducing it from the aforementioned definitions) that the equation EW is resolvable: but we know that the equation EW cannot be demonstrated resolvable (and cannot be demonstrated unresolvable) within the theory T.
In fact, according to the definition of the predicate "is a welement of the set X", we know that an object that is a welement of the set X can only: 1) either being an element of the set X (in case the equation EW is resolvable), 2) or being not an element of the set X (in case the equation EW is unresolvable); thus, knowing that the object Z is both an element of the set X and a welement of the set X, we know that we are faced with the case 1, the case where the object is an element of the set X and the equation EW is resolvable.
If we knew that an object J, which we know is not an element of the set X, is not a welement of the set X, we would automatically know that the equation EW is resolvable: but the the equation EW cannot be demonstrated resolvable (and cannot be demonstrated unresolvable) within the theory T.
In fact, according to the definition of the predicate “is not a welement of the set X", we know that an object that is not a welement of the set X can only: 1) either being not an element of the set X (in case the equation EW is resolvable), 2) or being an element of the set X (in case the equation EW is unresolvable).
Thus, knowing that the object J is not an element of the set X and is not a welement of the set X, we know that this is the case 1, the case where the object is not an element of the set X and the equation EW is resolvable.
If we knew that an object K, which we know is not an element of the set X, is a welement of the set X, we would automatically know that the equation EW is unresolvable; but the equation EW cannot be demonstrated unresolvable (and cannot be demonstrated resolvable) within the theory T.
In fact, according to the definition of "is a welement of the set X", we know that an object that is a welement of the set X can only: 1) either being an element of the set X (in case the equation EW is resolvable), 2) or being not an element of the set X (in case the equation EW is unresolvable).
Thus, knowing that the object K is a welement of the set X and is not an element of the set X, we know that this is the case 2, the case where the object is not an element of the set X and the equation EW is unresolvable.
If we knew that an object L, which we know is an element of the set X, is not a welement of the set X, we would automatically know that the equation EW is unresolvable; but the equation EW cannot be demonstrated unresolvable (and cannot be demonstrated resolvable) within the theory T.
In fact, according to the definition of "is not a welement of the set X", we know that an object that is not a welement of the set X can only: 1) either being not an element of the set X (in case the equation EW is resolvable), 2) or being an element of the set X (in case the equation EW is unresolvable).
Thus, knowing that the object L is not a welement of the set X and is an element of the set X, we know that this is the case 2, the case where the object is an element of the set X and the equation EW is unresolvable.
Therefore, is impossible that the predicates "is an element of the set X" and "is a welement of the set X" are both decidable predicates; it is impossible that we can determine within the theory T both things: if an object is an element of the set X, and if an object is a welement of the set X.
To argue that to apply the predicate "is a welement of the set X" we need to know if the equation EW

is resolvable, but that to apply the predicate "is an element of the set X" we do not need to know if the equation EW is resolvable, it is to say that the predicate "is a welement of the set X" is intrinsically disjunctive.
But Putnam and Goodman pointed out that if two predicates are interdefinable, neither of the two predicates is more disjunctive than the other one and neither of the two predicates is "intrinsically disjunctive" because *“no predicate is disjunctive or nondisjunctive in itself”* (as Putnam wrote in his Foreword to Goodman’s “Fact, Fiction, and Forecast”).
And so: since there is no syntactic reason and no semantic reason to affirm that “is a welement of the set X” is a predicate more disjunctive than the predicate “is an element of the set X”, knowing that at least one of the two predicates is an undecidable predicate, also the other predicate is an undecidable predicate.
Incidentally, I add that within the theory T it is known for certain that the predicate "is an element of the set X" is a synonym either of the predicate "is a welement of the set X" (in case the equation EW is resolvable), or of the predicate "is not a welement of the set X" (in case the equation EW is unresolvable); and thus, if we assume that the predicate "is an element of the set X” is decidable, we would obtain that the predicate "is an element of the set X" is both formally equivalent and logically equivalent to an undecidable predicate.
Outside of the theory T can be demonstrated that the predicate “is an element of the set X” is a synonym of the predicate “is not a welement of the set X”, that the predicate “element” is a synonym of the predicate “non-welement”.
If two predicates are both formally equivalent and logically equivalent, the eventual difference between them has to be a difference outside of logic.

4 - The supporters of the Goodman’s paradox assume that we, green-speakers, take the predicate “green” as basic (as primitive) and need a definition in order to be able to apply the predicate “grue”; by contrast, the grue-speakers take the predicate “grue” as basic (as primitive) and need a definition in order to be able to apply the predicate “green”.

5 - Adina L. Roskies in her paper “Robustness and the New Riddle Revived” considers a different example in which the observers can perceive colors different from the real colors.
Also in Professor Roskies’s example there is no paradox concerning induction because if we accept the hypothesis that the observers can perceive colors different from the real colors of the objects, then the real color of the emeralds could be red and be perceived as grue by the grue-perceivers and perceived as green by green-perceivers: in the Goodman’s paradox there is a paradox concerning induction because “All emeralds are green" and "All emeralds are grue" are two "conflicting hypotheses" (Goodman writes in the Foreword to chapter 8 "Induction" of his book "Problems and Projects": *"two such conflicting hypotheses as 'All emeralds are green' and 'All emeralds are grue' “).*
By contrast, "All emeralds are perceived as grue by the grue-perceivers" and "All emeralds are perceived as green by the green-perceivers" are not two conflicting hypotheses: they can both be true and can support perfectly compatible predictions.

6 - The asymmetry above written can be referred also to non-ostensively definable predicates.
For example, let’s introduce the predicate “condulator”, which applies to an object if the object either is examined before time *t* and is a conductor of electricity, or is not examined before time *t* and is a non-conductor of electricity.
The predicate “condulator” is interdefinable with the predicate “conductor”: the predicate “conductor” applies to an object if the object either is examined before time *t* and is a condulator of electricity, or is not examined before time *t* and is a non-condulator of electricity.
The predicate “non-condulator” applies to an object if the object either is examined before time *t* and is a non-conductor of electricity, or is not examined before time *t* and is a conductor of electricity.
The predicate “non-conductor” applies to an object if the object either is examined before time *t* and is a non-condulator of electricity, or is not examined before time *t* and is a condulator of electricity.
After time *t* we cannot ascertain, just examining the objects, if an object is a condulator examined before time *t* (which is a conductor), or is a non-condulator not examined before time *t* (which is a conductor too).
By contrast, also after time *t* it is sufficient to examine the objects to ascertain if an object is a conductor or is a non-conductor.
I add a last specification: Hilary Putnam, writing in his Foreword [pages XI-XII] to Goodman’s “Fact, Fiction, and Forecast” about the Barker and Achinstein’s apocryphal “grue”, described a grue-detector instrument:
“[...] all one has to do is build a measuring instrument that flashes a red light if the time is before t

(imagine that the measuring instrument contains an internal clock) and the instrument is scanning something green or if the time is later than t and the instrument is scanning something blue. Using such an instrument, one can tell whether or not something is grue without knowing what time it is, by seeing whether or not the red light is flashing".
What Putnam called "internal clock" is not a simple clock, it works rather like an alarm-clock. It is necessary to program in the instrument the exact time instant corresponding to time *t*, as when in an alarm-clock we program the time at which we want the alarm to sound.
If the instrument is programmed with a wrong temporal information about which is the time *t*, the instrument gives incorrect results.
By contrast, if we build an instrument that flashes a red light every time it is scanning something green (a green-detector instrument), the instrument gives correct results without needing to be programmed with the exact information about which is the time *t*.
There are also instruments similar to the grue-detector instrument described by Putnam, but referred to non-ostensively definable predicates, like, for exampe, "conducts electricity"; Goodman wrote in "Positionality and Pictures" (contained in the Chapter 8 of his book "Problems and Projects"):
"But if verbal inscriptions are admissible in a picture, we can construct a rapresentation for 'condulates electricity' simply by labelling the pictured meter 'b-meter' for an easily made instrument whose pointer rests elsewhere than at zero up to time t when a current passes through and thereafter when no current passes through [...]".
All such instruments have not a simple "internal clock" but a clock working like an alarm-clock.

7 - We have seen in the first section of this paper why background knowledge is not useful to reply to the Goodman's paradox.
Background knowledge is not useful also to reply the the Hume's problem: exactly as Goodman's paradox is not just about the generalization "All emeralds are green", but it also concerns each generalization that is part of our so-called background knowledge, so also Hume's problem does not only concern one specific generalization, but it concerns each generalization that is part of our background knowledge.

For example, let's imagine that 26 friends, Mr. A, Mr. B, Mr. C, ..., Mr. Z have no money and want to have a loan.
Each one of the 26 friends goes to a bank and asks for a loan of 1 million dollars.
Each bank asks guarantees to each one of the 26 friends, and each one of them answers: "25 great friends of mine have 25 millions dollars (each one have 1 million dollars) and are happy to guarantee for me".
This would be not a good guarantee for the banks because the real patrimony of the 26 friends is 0 dollars, it is not 25 millions dollars.
The point about background knowledge is similar: if in our background knowledge there are not generalizations able to be justified by themselves, none generalization contained in the background knowledge is really justified, and so the background knowledge cannot justify our belief in a generalization.
Background knowledge cannot justify our belief in a generalization, but can justify why we don't believe in a generalization.
For example, if I am in a park and meet three dogs, and all three are German shepherds, although the generalization A "All the dogs I meet in the park today are German shepherds" is not falsified and is supported, I do not believe in generalization A because basing on my background knowledge I know that German shepherds are not such a widespread breed of dog to let me believe that all the dogs I meet today at the park are German shepherds.

8 - In the scientific generalizations of the form "All Fs are G" the number of the Fs is assumed as an infinite number.
Speaking about objective probability (not subjective probability and Bayesian calculations), when referring to scientific generalizations of the form "All Fs are G" it is important that any finite number divided by infinite gives zero as a result: any number of observed green emeralds divided by infinite gives zero as a result.
But, let's try to analyze only universal propositions: in this case the possibilities are only three, they are not infinite; the denominator is 3, it is not infinite.
Let's consider a universal proposition K "All Fs are G" and its contrary proposition EK "All Fs are not G".
There are only three possible cases: 1) K is true and EK is false; 2) K is false and EK is true; 3) K and EK are both false.
Thus, let's consider the propositions A 'All emeralds are green", EA "All emeralds are not green", B "All emeralds are X", EB "All emeralds are not X".

In the case of the propositions B and EB there are three possible cases:
1) B is true and EB is false; 2) B is false and EB is true; 3) B and EB are both false.
In one case out of three B is true (in case 1: B is true and EB is false).
By contrast, let's consider the case of the propositions A and EA:
1) A is true and EA is false; 2) A is false and EA is true; 3) A and EA are both false.
We know that the case 2 (the case "A is false and EA is true") must be eliminated:
since we know that EA is false, there are only two possible cases: in one case out of two the proposition A is true.
So, we have to choose between a proposition (proposition A) that is true in a case out of two, and a proposition (proposition B) that is true in a case out of three: we have a reason to prefer a proposition that has one chance out of two over a proposition that has one chance out of three, there is an asymmetry between the propositions A and B, and we can use the aforementioned asymmetry to justify our preference for A over B.

9 - When we have to choose between two incompatible and unfalsified hypotheses there is not *a-priori* information that allows us to prefer a hypothesis to the other one. By contrast, when we have to do with two compatible hypotheses, it can be that we have *a-priori* information that gives us a reason to prefer to bet on a hypothesis rather than on the other one (but Goodman wrote: *"If the hypotheses do not conflict, we need not choose between them",* "Problems and Projects", chapter 8, section "Three Replies").
For example, the hypothesis H2 "All emeralds are green and have a hardness range of 7.5 to 8 on the Mohs's scale" is an under-hypothesis of the hypothesis H1 "All emeralds are green": the truthfulness of the hypothesis H2 necessarily implies the truthfulness of the hypothesis H1; by contrast, the truthfulness of the hypothesis H1 does not imply the truthfulness of the hypothesis H2.
Since we know *a-priori* that the set of the emeralds which are green and have a hardness range of 7.5 to 8 on the Mohs's scale is a subset of the set of the green emeralds, we know *a-priori* that it is more convenient to bet on the hypothesis H1 "All emeralds are green" rather than on the hypothesis H2 "All emeralds are green and have a hardness range of 7.5 to 8 on the Mohs's scale".
But in our lives we often have to chose between two incompatible and unfalsified hypothesis and we don't have available *a-priori* information that allows us to prefer to bet on a hypothesis rather than on the other one; thus, we have to search *a-posteriori* information to have a rational reason to bet on a hypothesis rather than on the other hypothesis.

10 - Goodman wrote in the Foreword to chapter 8 "Induction" of his book "Problems and Projects": *"Occasionally 'grue' is given some different interpretation. For example, in 'Positionality and Pictures' and the paper it discusses, 'grue' is taken to apply not to entire enduring entities but rather to green time-slices examined before 2000 A.D. and to blue time-slices not so examined. Basically the same riddle arises and the shift in interpretation may often go unnoticed".*
Speaking about time-slices we can compare, for example, a hypothesis A, which states that all the emeralds are constantly green, with a hypothesis B, which states that not all the emeralds are constantly green. The hypotheses A and B are incompatible. If the hypothesis A is true, the hypothesis B can never be falsified; by contrast, if the hypothesis B is true, the hypothesis A is not unfalsificable (the observation of an emerald during a period of time in which the emerald has a different color from the green would falsify hypothesis A).
The fact that at present both the hypothesis A and the hypothesis B are not falsified is a necessary consequence of A, and it is not a necessary consequence of B. Thus, it is more rational to bet on the hypothesis A rather than on the hypothesis B.

The same argumentation can be used about subjectivistic hypotheses. We can compare, for example, a hypothesis A, which states that all the subjects perceive the emeralds as having all the same color, and a hypothesis B, which affirms that not all the subjects perceive the emeralds as having all the same color. The hypotheses A and B are incompatible.
If A is true, B can never be falsified; by contrast, if B is true, A is not unfalsificable (if a subject perceive two emeralds as having two different colors, A would be falsified).
The fact that at present both A and B are not falsified is a necessary consequence of A, and it is not a necessary consequence of B. Thus, it is more rational to bet on A rather than on B.

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