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Isolated Single Valued Neutrosophic Graphs

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Abstract: Many results have been obtained on isolated graphs and complete graphs. In this paper, a necessary and sufficient condition will be proved for a single valued neutrosophic graph to be an isolated single valued neutrosophic graph.

Keywords: Single valued neutrosophic graphs, complete single valued neutrosophic graphs, isolated single valued neutrosophic graphs.

1. Introduction

The notion of neutrosophic sets (NSs) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (*t*), an indeterminacy-membership function (*i*) and a falsity-membership function (*f*) independently, which are within the real standard or nonstandard unit interval]−0, 1+[. In order to conveniently use NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/* [38].

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science.

If one has uncertainty regarding either the set of vertices or edges, or both, the model becomes a fuzzy graph. The extension of fuzzy graph [2, 4, 25] theory have been developed by several researchers, e.g. vague graphs [27], considering the vertex sets and edge sets as vague sets; intuitionistic fuzzy graphs [3, 15, 26], considering the vertex sets and edge sets as intuitionistic fuzzy sets; interval valued fuzzy graphs [16, 17, 23, 24], considering the vertex sets and edge sets as interval valued fuzzy sets; interval valued intuitionistic fuzzy graphs [35], considering the vertex sets and edge sets as interval valued intuitionistic fuzzy sets; bipolar fuzzy graphs [18, 19, 21, 22], considering the vertex sets and edge sets as bipolar fuzzy sets; m-polar fuzzy graphs [20], considering the vertex sets and edge sets as m-polar fuzzy sets.

But, if the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and their extensions fail. For this purpose, Smarandache [5, 6, 7, 37] defined four main categories of neutrosophic graphs; two are based on literal indeterminacy (*I*), called: *I*-edge neutrosophic graph and *I*-vertex neutrosophic graph, deeply studied and gaining popularity among the researchers due to their applications via real world problems [1, 38]; the two others are based on (*t*, *i*, *f*) components, called: (*t*, *i*, *f*)-edge neutrosophic graph and (*t*, *i*, *f*)-vertex neutrosophic graph, concepts not developed at all by now.

Later on, Broumi et al. [29] introduced a third neutrosophic graph model, which allows the attachment of truth-membership (*t*), indeterminacy-membership (*i*) and falsity-membership degrees (*f*) both to vertices and edges, and investigated some of their properties. The third neutrosophic graph model is called the single valued neutrosophic graph (SVNG for short). The single valued neutrosophic graph is a generalization of fuzzy graph and intuitionistic fuzzy graph. Also, the same authors [28] introduced neighborhood degree of a vertex and closed neighborhood degree of a vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of a vertex in fuzzy graph and intuitionistic fuzzy graph. Recently, Broumi et al. [31, 33, 34] introduced the concept of interval valued neutrosophic graph as a generalization of fuzzy graph, intuitionistic fuzzy graph and single valued neutrosophic graph and discussed some of their properties with proof and examples.

The aim of this paper is to prove a necessary and sufficient condition for a single valued neutrosophic graph to be a single valued neutrosophic graph.

2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graphs, relevant to the present article. See [8, 9] for further details and background.

Definition 2.1 [8]

Let X be a space of points (objects) with generic elements in X denoted by x; then, the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X → $]^-0,1^+[$ define respectively a truth-membership function, an indeter-minacy-membership function and a falsity-membership function of the element x ∈ X to the set A with the condition:

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0,1^+[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [9] introduced the concept of SVNS, which is an instance of a NS, and can be used in real scientific and engineering applications.

Definition 2.2 [9]

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1]. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}. \quad (2)$$

Definition 2.3 [29]

A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G= (A, B), where:

1. The functions T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element v_i ∈ V, respectively, and:

$$0 \le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3,$$

for all v_i ∈ V.

2. The functions T_B: E ⊆ V x V →[0, 1], I_B:E ⊆ V x V →[0, 1] and F_B: E ⊆ V x V →[0, 1] are defined by $T_B(v_i, v_j) \le \min [T_A(v_i), T_A(v_j)]$, $I_B(v_i, v_j) \ge \max [I_A(v_i), I_A(v_j)]$ and $F_B(v_i, v_j) \ge \max [F_A(v_i), F_A(v_j)]$, denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge (v_i, v_j) ∈ E respectively, where:

$$0 \le T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \le 3,$$

for all (v_i, v_j) ∈ E (i, j = 1, 2, …, n)

We call *A* the single valued neutrosophic vertex set of *V*, and *B* the single valued neutrosophic edge set of *E*, respectively.

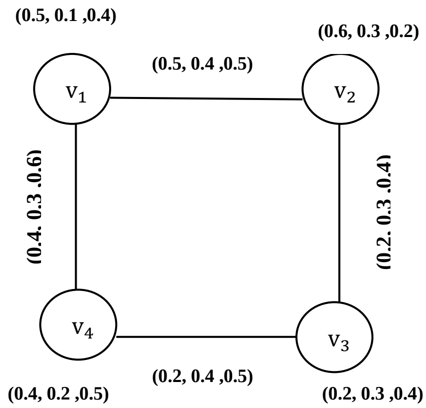

Figure 1: Single valued neutrosophic graph.

Definition 2.4 [29]

A partial SVN-subgraph of SVN-graph G= (A, B) is a SVN-graph H = (V′, E′), such that:

- V′ ⊆ V,

where $T'_A(v_i) \le T_A(v_i)$, $I'_A(v_i) \ge I_A(v_i)$, $F'_A(v_i) \ge F_A(v_i)$, for all v_i ∈ V;

- E′ ⊆ E,

where $T'_B(v_i, v_j) \le T_B(v_i, v_j)$, $I'_{Bij} \ge I_B(v_i, v_j)$, $F'_B(v_i, v_j) \ge F_B(v_i, v_j)$, for all $(v_i\ v_j)$ ∈ E.

Definition 2.8 [29]

A single valued neutrosophic graph G = (A, B) of G* = (V, E) is called complete single valued neutrosophic graph, if:

$$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)],$$
$$I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)],$$
$$F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)],$$

for all $v_i, v_j \in V$.

Definition 2.9 [29]

The complement of a single valued neutrosophic graph G (A, B) on G^* is a single valued neutrosophic graph $\overline{G}$ on G^*, where:

1. $\overline{A}$ =A= (T_A, I_A, F_A);

2. $\overline{T_A}(v_i) = T_A(v_i)$, $\overline{I_A}(v_i) = I_A(v_i)$, $\overline{F_A}(v_i) = F_A(v_i)$,

for all $v_j \in V$.

3. $\overline{T_B}(v_i, v_j) = \min\left[T_A(v_i), T_A(v_j)\right] - T_B(v_i, v_j)$,

$\overline{I_B}(v_i, v_j) = \max\left[I_A(v_i), I_A(v_j)\right] - I_B(v_i, v_j)$

and

$\overline{F_B}(v_i, v_j) = \max\left[F_A(v_i), F_A(v_j)\right] - F_B(v_i, v_j)$,

for all $(v_i, v_j) \in E$.

3. Main Result

Theorem 3.1

A single valued neutrosophic graph G = (A, B) is an isolated single valued graph if and only if its complement is a complete single valued neutrosophic graph.

Proof

Let G : (A, B) be a single valued neutrosophic graph, $\bar{G}$ = $(A, \bar{B})$ be its complement, and G : (A, B) be an isolated single valued neutrosophic graph.

Then,

T_B(u, v) = 0,

I_B(u, v) = 0

and

F_B(u, v) = 0,

for all (u, v) ∈ V× V.

Since

$\overline{T_B}$(u, v) = min $(T_A(u), T_A(v)) - T_B$(u, v),

for all (u, v) ∈ V× V,

$\overline{T_B}$(u, v) = $\min(T_A(u), T_A(v))$

and

$\overline{I_B}$(u, v) = $\max(I_A(u), I_A(v)) - I_B$(u, v),

for all (u, v) ∈ V× V,

$\overline{I_B}$(u, v) = $\max(I_A(u), I_A(v))$

and

$\overline{F_B}$(u, v) = $\max(F_A(u), F_A(v)) - F_B$(u, v),

for all (u, v) ∈ V× V,

$\overline{F_B}$(u, v) = $\max(F_A(u), F_A(v))$,

hence $\bar{G}$ = $(A, \bar{B})$ is a complete single valued neutrosophic graph.

Conversely, let $\bar{G}$ = $(A, \bar{B})$ be a complete single valued neutrosophic graph

$\overline{T_B}$(u, v) = $\min\big(T_A(u), T_A(v)\big)$,

for all (u, v) ∈ V× V.

Since

$\overline{T_B}$(u, v) = $\min(T_A(u), T_A(v)) - \overline{T_B}$(u, v),

for all (u, v) ∈ V× V,

= $\overline{T_B}$(u, v) $-\overline{T_B}$(u, v),

for all (u, v) ∈ V× V,

= 0,

for all (u, v) ∈ V× V,

T_B(u, v) = 0,

for all (u, v) ∈ V× V.

$\overline{I_B}$(u, v) = $\max(I_A(u), I_A(v))$,

for all (u, v) ∈ V× V.

Since

$\overline{I_B}$(u, v) = $\max(I_A(u), I_A(v)) - \overline{I_B}$(u, v),

for all (u, v) ∈ V× V

=$\overline{I_B}$(u, v) $-\overline{I_B}$(u, v),

for all (u, v) ∈ V× V

= 0,

for all (u, v) ∈ V× V,

I_B(u, v) = 0,

for all (u, v) ∈ V× V.

Also,

$\overline{F_B}$(u, v) = $\max(F_A(u), F_A(v))$,

for all (u, v) ∈ V× V.

Since

$\overline{F_B}$(u, v) = $\max(F_A(u), F_A(v)) - \overline{F_B}$(u, v),

for all (u, v) ∈ V× V,

=$\overline{F_B}$(u, v) $-\overline{F_B}$(u, v),

for all (u, v) ∈ V× V

=0,

for all (u, v) ∈ V× V

F_B(u, v) = 0 for all (u, v) ∈ V× V,

hence G = (A, B) is an isolated single valued neutrosophic graph.

4. Conclusion

Many problems of practical interest can be represented by graphs. In general, graph theory has a wide range of applications in various fields. In this paper, we defined for the first time the notion of an isolated single valued neutrosophic graph. In future works, we plan to study the concept of an isolated interval valued neutrosophic graph.

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