Span operators

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1. *Tensed plural quantifiers*

Presentists typically assent to a range of tensed statements, for instance, that there *were* dinosaurs, that there *was* a president named Lincoln, and that my future grandchildren *will be* on their way to school.[1] Past- and future-tensed claims are dealt with by introducing intensional tense operators, for instance, *it has been 12 years ago that*, *it was the case when I was born that*, and *it will be the case that* (Prior 1968). For example, 'there were dinosaurs', upon analysis, cashes out to:

(1) It was the case that (there are dinosaurs),

where *it was the case that* is a past tense operator. Quantifiers within the scope of past and future tense operators are not existentially committing. So, (1) does not commit us to the existence of dinosaurs.

But tensed plural quantifiers present a problem for presentism. Consider, for instance:

(2) There have been two kings named Charles.

As David Lewis (2004: 5) points out, the presentist cannot simply treat past- and future-tensed plural quantifiers as quantifiers within the scope of a slice tense operator. For (2) then comes out as:

(3) It has been that (there are two kings named Charles).

But since there have never been two kings named Charles at the same time, (3) is false. One alternative is to 'build a numerical quantifier out of two or more singular quantifiers' (2004: 5). (2) thus cashes out to: 'there has been a king named Charles, and there has been another king named Charles'. The presentist might attempt the following translation:

(4) It has been that (there is a king named Charles); and it has been that (there is another king named Charles).

But, says Lewis, this is not acceptable by presentist lights. For since the occurrence of 'another' is not within the scope of the first tense operator,

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[1] Presentism is the view that everything is present. For discussion, see e.g. Crisp 2003. Presentism is opposed to eternalism, which holds that wholly future and wholly past objects are just as real as presently existing objects. Presentism entails the A-theory of time. But the A-theory does not entail presentism. Eternalism plus the A-theory gives us the so-called *moving spotlight theory*. I shall here use 'eternalism' loosely to refer to non-presentism plus the B-theory.

it is unclear what it means. To deal with this sort of problem Lewis offers the following nested translation:

(5) It has been that (there is a king named Charles, and it has been that [there is another king named Charles]).

This is fine. But, says Lewis, the presentist runs into trouble if there have been or will be infinitely many Fs. For this would require 'a construction with tense operators nested ad infinitum' (2004: 7).

Quantifiers like 'there have been some kings' and 'there have been several kings' create further trouble. There have been some kings named George, it seems, but if presentism is true, it has never been that there are some kings named George. A translation of the claim 'there have been some kings named George' would require an infinitary construction, such as 'there has been one king named George, or there have been two or … or there have been infinitely many' (2004: 7). Lewis thinks that these unsuspected complexities require the presentist to 'abandon this brute-force method of translation, and seek some other way to handle tensed numerical or plural quantifiers' (2004: 7).

An appealing alternative to the brute-force translation method is to introduce span operators. Where the usual slice operators mean (as the eternalist would put it) 'at some past (or future) moment', a span operator means 'at some past (or future) interval' (2004: 12). Thus, where 'it HAS been that' is the past-tense span operator, Lewis suggests that we translate 'there have been two kings named Charles' as 'it HAS been that (there are two kings named Charles)'. By eternalist lights, this means that 'it is true of some interval in the past that there are two kings named Charles' (2004: 12). The presentist, of course, cannot accept this translation, but must presumably treat span operators as primitives.

According to Lewis, however, span operators are too ill-behaved to do any good. First, 'they create ambiguities even when prefixed to a sentence that is not itself ambiguous' (2004: 12). For example, 'it HAS been that (it is raining, and the sun is shining)' might mean: there is a past interval throughout some of which it is both rainy and sunny. Or: there is a past interval with a rainy subinterval and a sunny subinterval.

Second, truths sometimes result from prefixing span operators to contradictions. 'It HAS been that (it is raining and it is not raining)' is true, at least under one disambiguation.

In short, the span operators are too mischievous for anyone to claim primitive understanding of them. The eternalist, on the other hand, can easily use span operators, because 'she has another language available to remove ambiguities and to explain why sentences with embedded contradictions may nevertheless be true' (2004: 13).

2. Well-behaved span operators

However, I think that Lewis is too quick to deny the presentist the right to employ span operators. There is no reason why the presentist could not help herself to both primitive tensed slice operators, and primitive span operators. She would then have another device available to eliminate ambiguities and 'to explain why sentences with embedded contradictions may nevertheless be true'. The truth of 'it HAS been that (it is raining and it is not raining)', for example, can be explained as follows: 'it HAS been that (it is raining and it is not raining), but it was not the case that (it is raining and it is not raining)'. Or, as the eternalist would put it, there is some past interval with a rainy sub-interval and a non-rainy sub-interval, but there is no past time t such that at t it is raining and it is not raining.

Better yet: to say that span operators are primitive does not rule out imposing constraints on them. The presentist might impose a requirement on span operators to the effect that sentences containing them be weakly necessarily equivalent to (possibly infinitary) sentences containing slice operators but deny that slice operators provide the resources for an *analysis* of the span operators.[2] The totality of truths containing slice operators specifies, as Ted Sider puts it, 'a series of snapshots of the world at successive moments of time' (2001: 32). The totality of truths containing span operators should do no more than that.

Span operators are *in a sense* like our ordinary quantifiers. If everything had a name, our quantified sentences would be materially equivalent to indefinitely long disjunctions or conjunctions of non-quantified sentences. For example, 'everything is self-identical' would be materially equivalent to an indefinitely long conjunction mentioning all things in the universe, 'Dick is identical to Dick, Harry is identical to Harry, … , and Sue is identical to Sue.'

Quantifiers come in handy for a number of reasons. First, not every object has a name. Second, quantified sentences are in many cases less unwieldy than complexes of subject-predicate sentences. So, even if every object had a name, quantifiers would be convenient devices for specifying complicated facts. Third, quantifiers play a communicative role that complexes of subject-predicate sentences cannot play. If I do not remember the name of the princess who suffered from an eating disorder and who recently died in a car accident, I can use a quantifier in its place (I just did).

Span operators come in handy for much the same reasons. First, sentences containing span operators are in many cases less unwieldy than sentences containing slice operators. 'It HAS been that (there are some

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[2] They are not logically equivalent to such sentences, for presentism is not a logically necessary thesis. Thanks here to an anonymous referee. I leave open the question of the force of the modal operator.

kings of England named George)' is manageable; the infinitary disjunction 'it has been that (there is one king named George), or it has been that (there are two kings named George), or … or it has been that (there are infinitely many)' is not. Second, span operators play a communicative role that slice operators cannot play. I can say 'it HAS been within the last five years that there are 200 students taught by me', even if I fail to remember how many students I taught when. Finally and most importantly, unlike 'it has been that (there are some kings of England)', 'it HAS been that (there are some kings of England)' seems to be a plausible candidate for the meaning of the *English* sentence 'it has been that there are some kings of England', which is to say, span operators seem to be among the prefixes of English.

There is, however, a further worry about span operators (even well-behaved ones): they are *hyperintensional* (Lewis 2004: 13).[3] The intension of a sentence created with a span operator is not a function of the intension of the embedded sentence.

This may seem disturbing. Standard modal operators, for example, are intensional, but they are not hyperintensional. The intension of 'Kerry might have been president' is a function of the intension of 'Kerry is president'.

However, the standard modal operators do not exhaust the realm of intensional operators to which the span operators could be likened. Story prefixes, such as *according to the Conan Doyle stories*, are hyperintensional: the intension of 'according to the Conan Doyle stories, Sherlock Holmes lives on Baker Street' is not a function of the intension of 'Sherlock Holmes lives on Baker Street'. The former expresses a (complete) proposition, the latter does not. The presentist who likens her span operators to story prefixes can thus wholeheartedly buy into their hyperintensionality.

The presentist in fact has good reason to liken her span operators to story prefixes: span operators create the (convenient) illusion that spans of time existed. But it is an illusion only, since span operators are devices for avoiding the 'unexpected complexities' generated by slice operators.

Ted Sider has provided a different reason for disposing of span operators. According to him, the presentist cannot coherently make use of them. Here is the argument:

> Presentists do not think that it merely happens to be the case now that only currently existing objects exist. They think that it is *always*

[3] Note that if eternalism is true, then span operators are not hyperintensional: 'Socrates is old, and Socrates is not old' is false on a 'slice reading', but true on a 'span reading'. So, the intension of 'it WAS the case both that Socrates is old, and Socrates is not old' is a function of the intension of the embedded sentence.

the case – indeed, that it *necessarily* is always the case – that only (then-)currently existing objects exist.… Presentists cannot admit, therefore, that there once were exceptions to presentism. But they would have to admit just this, if they accepted the span operators. For example, the sentence 'WAS $\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' comes out true, since its component sentence '$\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' is true of many spans of time in the past. And yet since there is no one instant at which Kant and Socrates exist, this component sentence '$\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' constitutes a violation of the presentist doctrine that there cannot exist non-present things – if two things never exist at the same instance then one or both must fail to exist at the present time. (2001: 27)

It is not entirely clear what the argument is. 'It is always the case that only present objects exist' is not straightforwardly incompatible with 'it WAS the case that $\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$'. *It is always the case that* is a slice operator, *it WAS the case that* a span operator.

Granted, 'it WAS the case that $\exists x \exists y(x = \text{Socrates} \ \& \ x = \text{Kant})$' entails '~(it WAS the case that only present objects exist)':

(6) ~It WAS the case that (only present objects exist).

Prima facie, it is awkward for the presentist to assert (6). However, it is not at all clear that (6) entails:

(7) ~It was the case that (only present objects exist).

Perhaps the argument is this. For the sentence 'it WAS the case that $\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' to be true, the embedded sentence $\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$ must be true of at least one span of time in the past. But in order for it to be true of a span of time in the past, it must be true of a past time. Since the presentist denies that $\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$ is true of a past time, presentism plus span operators is incoherent.

This argument is unsound, however. It involves the following move:

(A) $$\frac{\text{It WAS the case that } \exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})}{\text{It was the case that } \exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})}$$

In the language of the eternalist, the premiss says that '$\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' is true of at least one span of time in the past. The conclusion says that '$\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' is true of at least one single instant in the past. If the latter means that there is a past time t such that Socrates and Kant both exist at t, then the conclusion is false. So, if the premiss is true, and the conclusion is interpreted this way, then (A) is obviously invalid.

The only option for the eternalist is to interpret the conclusion unrestrictedly. So interpreted, '$\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' means that the domain of our most unrestricted quantifiers contains Socrates and Kant. So, 'it was the case that $\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$' must mean that there is a past time t such that at t it holds that the domain of our most unrestricted quantifiers contains Socrates and Kant. This is true if eternalism is. For, if eternalism is true, then it is *always* the case that the domain of our most unrestricted quantifiers contains Socrates and Kant.

If the conclusion is so interpreted, then (A) is eternalistically valid, that is, it is valid if eternalism is necessarily true. But the argument was intended to show that presentism plus span tense operators is incoherent. Yet if presentism is true, then (A) is invalid: it WAS the case that $\exists x \exists y(x = \text{Socrates} \ \& \ y = \text{Kant})$, but it was never the case that the presentist's most inclusive domain of quantification contains Socrates and Kant. So, the argument fails to establish that *presentism* plus span tense operators is incoherent.

3. *Other issues*

Well-behaved span operators can do nothing semantically that slice operators cannot do. But they make our lives a lot easier. Besides aiding with the translations of tensed plural quantifiers, they make it easier to translate sentences containing adverbs of quantification. Consider, for instance:

(8) When I was a child I usually behaved well.

Eternalists take adverbs of quantification – such as 'usually', 'generally', 'always' – to quantify over cases, where cases are n-tuples of individuals, times or events (Lewis 1975). The cases quantified over in (8) are times at which I am a child. But if presentism is true, these times do not exist. So, (8) cannot be taken to quantify over them. Furthermore, I am unable to express what is asserted in (8) with nested slice operators. For I do not remember exactly when I behaved well as a child.

With span operators there is no problem. The presentist acknowledges exotic (composite) slice operators in addition to the more mundane ones, for instance, *it was the case 23 minutes ago that*, *it was the case when I turned on the stove that* and *it will be the case when most of my students have turned in their papers that* (Sider 2001: 25). The exotic slice operators are devices for avoiding quantification over past (or future) times that satisfy some condition (e.g. the condition of being an x such that my students have turned in their papers at x). In order to translate (8) we need an exotic span operator, namely: *it WAS the case most of the time when I was a child that*. With this prefix (8) cashes out to:

(9)   It WAS the case most of the time when I was a child that (I behave well).

The span operator *it WAS the case most of the time when I was a child that* is a device for avoiding quantification over times satisfying the property of being an x such that I am a child at x.

One further span operator benefit: events are often construed as entities with temporal parts (Quine 1960: 131). They are extended in time. But the presentist cannot literally hold that there are, were, or will be entities extended in time.

This gives trouble. For the presentist hardly wants to deny that there have been world wars, weddings, happy hours and APA meetings. For example, it seems that the following sentence could be true:

(10) The conference lasted three days.

But if 'the conference' picks out an object extended in time (if it picks out anything at all), then presentism predicts that (10) is false.

There are several ways of dealing with this problem. The presentist could, for example, conceive of events in much the same way that endurantists conceive of objects: conferences, happy hours and so on persist by being wholly present at successive times.[4]

Assuming that it makes sense to construe events as enduring entities, (10) can be expressed by specifying a series of snapshots of the form 'it was the case n units of time ago that the conference is taking place, and it was the case $n + 1$ units of time ago that the conference is taking place, and it was the case $n + 2$ units of time ago that the conference is taking place, …'. But this gets very tedious very quickly. Span operators do away with the tedium. With span operators (10) translates as:

(11) It WAS the case for three days that (the conference is taking place).

'It WAS the case for three days that' is a device for avoiding quantification over days and times, as in 'there are three days at each instant of which the conference is taking place'.

The same strategy can be used to translate:

(12) Bush Senior was in office for four years.

With slice operators 'Bush Senior was in office' translates effortlessly as 'it was the case that Bush Senior is in office'. But to capture the content of (12) we must specify each successive moment of time at which Bush

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[4] Other ways include treating events as exemplifications of properties of enduring objects (Kim 1969), states of affairs (Chisholm 1970), instantaneous changes of objects (Lombard 1986: ch. 7), or *sui generis* abstract entities.

was in office. With slice operators this is easier said than done. But with span operators it can be done with little ink wasted.

We have said very little about natural language. But it should be evident that that slice operators are often terrible candidates for the meaning of the English prefixes 'it has been that' and 'it will be that', witness 'it has been that there are three kings'. Span operators are perhaps not perfect in this role either, but they evidently fit the role better than slice operators.

In conclusion, span operators are neither obscure nor ill-behaved. They are convenient devices for avoiding quantification over entities that never co-existed.[5]

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