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Vertical precedents in formal models of precedential constraint

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Abstract
The standard model of precedential constraint holds that a court is equally free to modify a precedent of its own and a precedent of a superior court—overruling aside, it does not differentiate *horizontal* and *vertical* precedents. This paper shows that no model can capture the U.S. doctrine of precedent without making that distinction. A precise model is then developed that does just that. This requires situating precedent cases in a formal representation of a hierarchical legal structure, and adjusting the constraint that a precedent imposes based on the relationship of the precedent court and the instant court. The paper closes with suggestions for further improvements of the model.

Keywords Legal reasoning · Factor-based reasoning · Default reasoning · Precedent

1 Introduction

How do common law precedents constrain later courts? According to what Horty (2015) calls the *standard model*, a precedent court constrains later courts by announcing a *rule*, which later courts are then required to respect.[1] But respecting a rule, on this view, does not require applying the rule to every dispute that falls within its scope. Rather, in certain circumstances, a later court can avoid an applicable precedent rule by *modifying* it so that it no longer applies. The standard model holds that a court is equally free to modify a precedent of its own and a precedent

[1] Though Horty (2015) was the first to formalize the standard model, he actually prefers the *reason model* that he develops in Horty (2011a), Horty (2011b), Horty and Bench-Capon (2012), Horty (2014), Horty (2015), Horty (2016), and Horty (2017). Intriguingly, Horty (2015, 12–13) argues that the reason model and the standard model are actually *equivalent*, in the sense that they permit precisely the same judicial behavior. He nevertheless prefers the reason model because he takes it to be more illuminating. For discussion, see Horty (2015, 13–23).

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of a superior court—like many formal models of common law reasoning, once the power to overrule is set aside, it does not differentiate *horizontal* and *vertical* precedents. This, I will argue, is a mistake, at least if the standard model is intended to explicate the doctrine of precedent in the United States.

The paper proceeds as follows. The next section briefly introduces the standard model, as well as an alternative called the strict rule model. Section 3 reviews Horty's formalization of the standard model. Section 4 argues on the basis of case law that, in the U.S., different constraints are imposed by horizontal and vertical precedents. In particular, while a horizontal precedent imposes a constraint akin, more or less, to that imposed by the standard model, a vertical precedent imposes a more severe constraint, akin to that imposed by the strict rule model. Accordingly, no formal model of precedential constraint can capture the U.S. doctrine of precedent unless it differentiates horizontal and vertical precedents. Section 5 discusses an historical example that suggests that a predictive model that recognizes the strictness of vertical precedents will outperform one that does not. Section 6 briefly considers a possible concern about my description of the role of precedent in the U.S. The main task of the rest of the paper is to develop a formal model of precedential constraint, the *hierarchical model*, that incorporates something like the U.S. doctrine of vertical precedent. Section 7 prepares for this task by formalizing the strict rule model. Section 8 then uses elements from the standard model and the strict rule model to build the hierarchical model. Sections 9 and 10 consider how the hierarchical model might be improved in future research. Section 11 briefly concludes.

2 The standard model

The doctrine of precedent requires courts to decide disputes in a way that respects what courts have done in the past. While this description ought to be uncontroversial, it leaves a lot open. A particular model of the doctrine of precedent, then, will have something to say about what sort of respect is required, and about what exactly is supposed to be respected. A plausible answer to this last question appeals to the notion of a *rule*.[2]

(R) For any judicial decision, there is a rule of the case. Future courts must respect this rule in deciding the cases that come before them.[3]

[2] Many theorists subscribe to a rule-based model of one form or another. See, for example, Raz (2009, chp. 10), Eisenberg (2007), Alexander and Sherwin (2007), Alexander and Sherwin (2001). Compare the "reason models" of Horty (2017), Rigoni (2017), Horty (2015, §4), Horty (2014, §4), Horty and Bench-Capon (2012), and Lamond (2005), and the "result models" of Horty (2017), Horty (2004), Levi (1949), and Paton (1946, 159).

[3] The implicit universal quantifier in the second sentence of (**R**) ranges over future courts *in the same legal system*. For now, I take the notion of a legal system for granted. For discussion, see Raz (2009, chp. 5), Raz (1970). Later, in Sect. 8, I will define a formal notion closely related to the informal notion of a legal system.

Thus, (**R**) specifies that what future courts must respect is the *rule of the case* established by a precedent court. Presumably the rule of a given case has something to do with the associated judicial opinion. But what exactly is the relationship between the two? And, given the opinion, how does one go about identifying the rule? In fact, there is a large literature devoted to this question.[4] But I will follow Horty in simply setting it aside, by adopting the simplifying assumption that the rule of the case is always clear.

A rule-based model now has to say what respecting a precedent rule requires. To a first approximation, a precedent rule is going to be something of the form

If *A*, *B*, *C*, then decide for *s*,

where *A*, *B*, and *C* are properties of a legal dispute, and *s* is either the plaintiff or the defendant.[5] The antecedent of the rule establishes its *scope*, so that the rule *applies* to a dispute just in case the dispute is characterized by *A*, *B*, and *C*. If a dispute falls outside the scope of the rule, then the rule simply does not apply, and rule-based models all agree that it is only when a court must decide a dispute to which a precedent rule applies that the court is constrained by the requirement to respect the rule.

So how is a court constrained by an *applicable* precedent rule? The most straightforward answer is perhaps the following.

(S) A court faced with an applicable precedent rule that it is unwilling or unable to overrule must decide as the rule directs.

Call the model of precedent that accepts both (**R**) and (**S**) the *strict rule model*.[6] In a legal system governed by the strict rule model, a court faced with a dispute and an applicable precedent rule that it is unwilling or unable to overrule is required to apply the rule and decide accordingly, no matter how strong the court's conviction that to do so would be unjust, or harmful, or inimical to the purposes motivating the adoption of the precedent rule in the first place.

The standard model avoids this result, by allowing courts to deflect unwelcome precedents without overruling them. This involves rejecting (**S**) in favor of (**D**).

(D) All courts have the power to distinguish applicable precedents.

[4] See, for example, Garner et al. (2016, 44–96), Schauer (2009, 44–57, 180–184), Duxbury (2008, 58–92), Abramowicz and Stearns (2005), Cross and Harris (1991, chp. 2), Simpson (1959), Simpson (1958), Montrose (1957a), Montrose (1957b), Simpson (1957), Goodhart (1930b).

[5] See Llewellyn (2011, 38, 67, 103) for discussion of this conception of legal rules. I actually think that precedent rules should not generally be understood as directed to *courts*, as opposed to ordinary people in the relevant jurisdiction. See Hart (1994, 35–42). But I will set this concern aside for purposes of this essay.

[6] The strict rule model is defended as the *normatively* best doctrine of precedent, the one that *ought* to be adopted by the courts, in Alexander and Sherwin (2008, 42–50), Alexander and Sherwin (2007), Alexander and Sherwin (2001, 145–156), and Alexander (1989). When Alexander (1989, 53–54) turns to the descriptive question of the *actual* U.S. doctrine of precedent, however, he suggests that while vertical precedents are governed by the strict rule model, horizontal precedents are governed by something weaker. This is the position that I will be defending in Sect. 4.

It is important to notice that (**D**) grants the power to distinguish *applicable* precedents. For there is another sense of the term 'distinguish', a *weaker* sense, on which one distinguishes a precedent simply by showing that the associated rule does not apply to the present dispute. In this sense of the term, it is nonsensical to suggest that a court can distinguish an applicable precedent, and nobody denies that all courts have the power to distinguish. This is a perfectly fine sense of the term, but it is not the sense in which I will use it here.[7] As I will use the term, *only* applicable precedents can be distinguished, and the claim that all courts have the power to distinguish is controversial.[8] Distinguishing, in this sense, involves *modifying* an applicable rule without overruling it, so that, in its modified form, it no longer applies to the instant case.[9] This is the power that (**D**) grants to *all* courts, including those inferior to the precedent court in the judicial hierarchy.

Precedents, however, are supposed to constrain courts. Given its commitment to (**D**), is the standard model not incapable of accounting for this truism? How can a court be constrained by a rule that it is free to modify?[10] The standard model attempts to capture the constraint of precedent, while allowing greater flexibility than the strict rule model, by limiting the power to distinguish in certain ways. According to the standard model, courts are not free to modify unwelcome precedent rules in any way they please. Rather, the power to distinguish is subject to the following two conditions.[11]

(C1) The modification must consist only in restricting the scope of the original rule.

(C2) The modified rule must support the outcome of all precedent cases in which the original rule was applied.

Condition (**C1**) limits courts to adding conditions to the antecedent of the precedent rule until it no longer applies to the instant case. Thus, a court can't simply replace a precedent rule with any rule at all—dropping certain conditions from the antecedent of the original, say, while adding others—so long as the modified rule is narrower. The antecedent of the modified rule must contain all of the conditions in the antecedent of the original, and at least one in addition. And condition (**C2**) does not merely require that the modified rule be consistent with the outcome of the original precedent case (and subsequent cases applying the rule). The modified rule must *justify* that outcome, in the sense that it applies to the underlying dispute, and its

[7] In fact, this sense is far more common in American courts than the one I adopt in this paper. I adopt the non-standard sense nevertheless in order to make contact with Horty (2015) and related work.

[8] So the term 'applicable' in (**D**) is redundant. I retain it as a reminder of the sense of 'distinguish' in play.

[9] For this use of the term, see, for example, Garner et al. (2016, 97), Raz (2009, 185), Duxbury (2008, 115), Hart (1994, 135). Compare the use of 'narrowing' in Re (2014), and the use of 'stealth overruling' in Friedman (2010).

[10] Cf. Dworkin (1977, 37): "If courts had discretion to change established rules, then these rules would of course not be binding upon them…"

[11] These conditions were first articulated in Raz (2009, 186). Compare Raz (1990, 140).

consequent enjoins the court to decide for the party that actually won. The standard model, then, is characterized by (**R**), (**D**), (**C1**), and (**C2**).

3 Formalization

This section begins by developing the formal representational framework that Horty uses to represent precedent cases and the common law of a given area more generally. This framework is then put to use in formalizing the standard model. Much of this section follows quite closely the presentation in Horty (2015).

We suppose that a legal dispute can be represented as a set of *factors*, where a factor is a property whose instantiation by a dispute favors either the plaintiff or the defendant.[12] Suppose, for instance, that a particular company D causes a harm to a person P that could have been prevented had D installed a technology T. If P sues D for negligence, then, in the United States, it counts in favor of a decision for P if companies in the relevant industry standardly invest in T.[13] In negligence cases of this sort, then, whether the safety technology is standard in the relevant industry can be represented as a factor, whose instantiation (indicating that the technology *is* standard) favors the plaintiff. Whether the defendant caused the injury to the plaintiff in the first place could be represented by another factor. And so on. An adequate formalization of common law reasoning would probably allow for the development of factor hierarchies, in which the instantiation of a higher-level factor depends on the instantiation of lower-level factors.[14] Similarly, it would probably represent cases not simply in terms of factors but in terms of *dimensions* as well, where a dimension is not a property that a dispute either has or lacks but an ordered set of legally significant values, representing, very roughly, the *degree* to which a dispute instantiates a certain property.[15] But, for the purposes of this paper, we follow Horty (2015) in representing disputes solely in terms of factors, and in representing cases as if they were decided in a single step, from the factors present to a decision for one party or the other.

Notice that even the representation of a dispute in terms of factors requires legal expertise: car crashes and oil spills and all the other dust-ups that end up in court do not come pre-tagged with useful legal concepts. Nevertheless, our formalizations will *start* with the representation of a dispute in terms of legal factors, which means that a step in the court's legal analysis has already occurred when we arrive on the scene. This is significant, because many of the techniques that courts use to avoid unwelcome precedents are applied to antecedent representations of the relevant

[12] Factors were introduced in Ashley (1990). They have since been adopted in a great deal of work on modeling legal reasoning. See, for example, Rigoni (2015), Horty and Bench-Capon (2012), Wyner et al (2011), Horty (2004), Prakken and Sartor (1998).

[13] See American Law Institute (2005, §13), American Law Institute (1965, §295A).

[14] See, for example, Aleven (1997, 20–24), Aleven and Ashley (1996). See also Al-Abdulkarim et al (2016), Grossi and Jones (2013), Lindahl and Odelstad (2013, §7), Roth and Verheij (2004), Ashley and Brüninghaus (2003), Brüninghaus and Ashley (2003), Bench-Capon (1999, 40–41), Branting (1994).

[15] On dimensions, see, for example, Bench-Capon (2017), Horty (2017), Rigoni (2017), Al-Abdulkarim et al (2015), Bench-Capon (1999), Ashley (1990).

events in order to *reach* a convenient characterization of the dispute in terms of legal factors.[16] Our models must simply ignore techniques of this kind.

With those preliminaries out of the way, we can now follow Horty (2015, 5–6) in building up a formal representation of a precedent case. We begin by postulating a set $F^\pi = \{f_1^\pi \ldots f_n^\pi\}$ of legal factors favoring the plaintiff, and a set $F^\delta = \{f_1^\delta \ldots f_n^\delta\}$ of legal factors favoring the defendant. Since we assume that any legal factor favors either the plaintiff or the defendant (but not both), $F = F^\pi \cup F^\delta$ is the entire set of legal factors. A *fact situation* (or, a *dispute*) is a set of factors $X \subseteq F$. Where X is a fact situation and s a side, we let X^s be the set of factors in X favoring s, so that $X^s = X \cap F^s$.

Horty (2015, 5–6) now uses factors to define *reasons*, and *reasons* to define *rules*. A *reason for a side s* is a set of factors $R^s \subseteq F^s$ favoring s, and a reason is a reason for one side or the other. So, for example, $\{f_1^\pi, f_2^\pi\}$ is a reason for the plaintiff, while $\{f_1^\pi, f_2^\pi, f_1^\delta\}$ is not a reason at all, because it contains factors for both sides. Reasons are read conjunctively, so that the reason $\{f_1^\pi, f_2^\pi\}$, for example, is understood as the proposition that the dispute is characterized by both f_1^π and f_2^π.

We can now define rules in terms of reasons. Where R^s is a reason for s and $R_1^{\bar{s}} \ldots R_n^{\bar{s}}$ $(0 \leq n)$ are reasons for the opposite side, a rule for s has the form

$$R^s \wedge \neg R_1^{\bar{s}} \wedge \cdots \wedge \neg R_n^{\bar{s}} \rightarrow s.$$

We understand this rule to say that, when R^s holds, and none of the reasons $R_1^{\bar{s}} \ldots R_n^{\bar{s}}$ hold, then the court should decide for s. A rule is simply a rule for one side or the other. In a rule for s, I will call any conjunct in the premise of the rule of the form $\neg R_i^{\bar{s}}$ an *exception* to the rule. While this does not entirely fit with ordinary usage, I trust that it is not so far off as to be jarring, and it will be useful to have a term for such conjuncts.

It will also be convenient to assume functions *Premise*, *Premise*^{s}, and *Conclusion* that, when given as argument a rule r for s, return as value its premise, that part of its premise that constitutes a reason for s, and its conclusion, respectively, as in the following illustrations:

$$\begin{aligned} r &= R^s \wedge \neg R_1^{\bar{s}} \wedge \cdots \wedge \neg R_n^{\bar{s}} \rightarrow s \\ \mathit{Premise}(r) &= R^s \wedge \neg R_1^{\bar{s}} \wedge \cdots \wedge \neg R_n^{\bar{s}} \\ \mathit{Premise}^s(r) &= R^s \\ \mathit{Conclusion}(r) &= s \end{aligned}$$

Before we define the notion of a precedent case, we need one more definition on the table. Where X is a fact situation and R a reason, we define $X \vDash R$ (read "R holds in X," or "X satisfies R"), and extend this definition to $X \vDash \boldsymbol{\Phi}$ for complex wffs $\boldsymbol{\Phi}$, by the following stipulations:

$$\begin{array}{lll} X \vDash R & \text{iff} & R \subseteq X^\pi \text{ or } R \subseteq X^\delta \\ X \vDash \neg\phi & \text{iff} & X \nvDash \phi \\ X \vDash \phi \wedge \psi & \text{iff} & X \vDash \phi \text{ and } X \vDash \psi \end{array}$$

[16] See Llewellyn (2008, 80), Llewellyn (1960, 84–91).

We say that a rule *r applies* to a fact situation X, then, just in case $X \vDash \mathit{Premise}(r)$.

We can now define our formal notion of a precedent case. A precedent case is a triple $c = \langle X, r, s \rangle$ such that X is a fact situation, r is a rule, and $s \in \{\pi, \delta\}$, all such that (1) $X \vDash \mathit{Premise}(r)$, and (2) $\mathit{Conclusion}(r) = s$. In a case $\langle X, r, s \rangle$, we say the court was presented with a dispute X and decided for s on the basis of r. Where $s = \pi$, the decision was for the plaintiff. Where $s = \delta$, it was for the defendant. Conditions (1) and (2) can be considered *coherence* constraints on the notion of a case. They say that a court must decide each dispute on the basis of an applicable rule, and the court must decide for the side favored by the rule it applies. Finally, we represent the common law in some area using the notion of a *case base*, which we define as a finite, non-empty set of precedent cases.

We can now use this framework to formalize the standard model. Following Horty (2015), we will ignore the possibility of overruling for the purposes of the formal model. As a result, when the model says that, given a certain background case base, a court is *required* to decide a dispute for s, say, we understand this to mean that, in those circumstances, the court is required to decide for s if it is unwilling or unable to overrule relevant precedents. Thus, the standard model applies to both vertical and horizontal precedents. It simply operates on the assumption that the only difference between the constraint that precedents place on higher and lower courts is that a higher court can sometimes overrule a precedent that a lower court can't. Since the model ignores the possibility of overruling, it treats all courts as on a par.[17] With that clarification out of the way, we turn now to the positive task of building the formal model.

Suppose that we have a background case base $\Gamma_1 = \{c_1\}$, where c_1 is as follows.

$$
\begin{aligned}
c_1 &= \langle X_1, r_1, s_1 \rangle \\
X_1 &= \{f_1^{\pi}, f_1^{\delta}\} \\
r_1 &= \{f_1^{\pi}\} \rightarrow \pi \\
s_1 &= \pi
\end{aligned}
$$

And suppose that, in the context of Γ_1, a court is faced with the dispute $X_2 = \{f_1^{\pi}, f_2^{\delta}\}$. Plainly, r_1 *applies* in X_2. The strict rule model, then, would require the court to decide for the plaintiff on the basis of r_1. The standard model, by contrast, allows the court to decide for the defendant, should it find the presence of the new factor f_2^{δ} sufficiently compelling.

Thus, on the standard model, the court might permissibly decide for the defendant on the basis of the rule $r_2 = \{f_2^{\delta}\} \rightarrow \delta$, producing the case c_2.

[17] Alternatives to the standard model are often built on similar assumptions. See, for example, Horty and Bench-Capon (2012, 199), Horty (2011b, 2, 3), Lamond (2005), Horty (2004, 20). Rigoni (2015, 134) may be an exception, however. He suggests that his reason model—rather than applying to both horizontal and vertical precedents, while setting aside the possibility of overruling—applies exclusively to vertical precedents. Yet Rigoni (2015, 157) seems to discuss the application of his model to cases in which the Supreme Court distinguishes its own past decisions. So it's ultimately unclear where the model is supposed to apply.

$$c_2 = \langle X_2, r_2, s_2 \rangle$$
$$X_2 = \{f_1^{\pi}, f_2^{\delta}\}$$
$$r_2 = \{f_2^{\delta}\} \rightarrow \delta$$
$$s_2 = \delta$$

This makes two changes to the common law in this area. First, c_2 is added to the case base. Second, c_1 is modified. For the c_2 court has *distinguished* c_1, and this means that it has modified r_1 so that it no longer applies in c_2. The modification consists in adding an *exception* to r_1 for cases in which the factor f_2^{δ} is present. So modified, r_1 becomes $r_1' = \{f_1^{\pi}\} \wedge \neg\{f_2^{\delta}\} \rightarrow \pi$. And notice that this modification is consistent with (**C1**) and (**C2**), for it merely restricts the scope of r_1 by adding a condition to its premise, and the modified rule r_1' supports the outcome of c_1, since r_1' *applies* to X_1 and $Conclusion(r_1') = s_1$. This modification is represented by replacing c_1 in the case base with $c_1' = \langle X_1, r_1', s_1 \rangle$. Going forward, then, the common law in this area is $\Gamma_2 = \{c_1', c_2\}$.

But now suppose that a court, in the context of Γ_2, is faced with the dispute $X_3 = \{f_1^{\pi}, f_2^{\pi}, f_1^{\delta}\}$ and wishes to decide for the defendant. *This* the standard model rules out. For there is no modification that can be made to r_1', consistent with **(C1)** and **(C2)**, such that it no longer applies to X_3 but continues to justify the result in c_1'. These sorts of considerations lead Horty (2015, 8–9) to formalize the standard model using the notion of the *refinement* of a case base, which he defines as follows.

Definition 1 (*refinement*) Where Γ is a case base, its refinement Γ^{+} is the set that results from the following procedure. For each case $c = \langle X, r, s \rangle$ belonging to Γ:

(1) Let $\Gamma_c = \{c^* \in \Gamma : c^* = \langle X^*, r^*, \bar{s} \rangle \wedge X^* \vDash Premise^s(r)\}$.
(2) $\forall c^* \in \Gamma_c$, let $d_{\langle c, c^* \rangle} = \neg Premise^{\bar{s}}(r^*)$.
(3) Let

$$D_c = \bigwedge_{c^* \in \Gamma} d_{\langle c, c^* \rangle}.$$

(4) Replace the case $c = \langle X, r, s \rangle$ from Γ with $c' = \langle X, r', s \rangle$, where

$$r' = Premise^s(r) \wedge D_c \rightarrow s.$$

The process of refinement involves finding, for each case c in a case base, all those cases in which the rule of c applies but the court nevertheless reached a contrary result. In each of those cases, the court must have *distinguished* c. Accordingly, the rule of c is modified by adding a corresponding exception. Once this is done for every case in the case base, the cases—now with their modified rules—are collected together in a new set, and the refinement of the original case base is complete.

Now notice that the refinement of $\Gamma_1 \cup \{c_2\}$ produces exactly what was achieved above by developing Γ_1 with c_2 in accordance with the standard model. That is,

$(\Gamma_1 \cup \{c_2\})^+ = \Gamma_2$. Horty (2015, 9) suggests that this holds in general. But, now, imagine that the court faced with X_3 in the context of Γ_2 went ahead and decided for the defendant on the basis of $r_3 = \{f_1^\delta\} \rightarrow \delta$, producing c_3.

$$c_3 = \langle X_3, r_3, s_3 \rangle$$
$$X_3 = \{f_1^\pi, f_2^\pi, f_1^\delta\}$$
$$r_3 = \{f_1^\delta\} \rightarrow \delta$$
$$s_3 = \delta$$

To avoid c_1, the c_3 court must have *distinguished* it, by adding an exception to r_1 so that it becomes $r_1^* = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \wedge \neg\{f_1^\delta\} \rightarrow \pi$. But, since r_1^* does not *apply* to X_1, r_1^* does not support the outcome in the original case, in violation of **(C2)**. Thus, as I suggested earlier, this decision violates the standard model. What, then, would the refinement of $\Gamma_2 \cup \{c_3\}$ produce? It would produce a set that includes $c_1^* = \langle X_1, r_1^*, \pi \rangle$. But since $X_1 \not\models \mathit{Premise}(r_1^*)$, c_1^* is not a precedent case. And this means that $(\Gamma_2 \cup \{c_3\})^+$ is not a case base.

This leads Horty to give the following formalization of the standard model.

Definition 2 (*precedential constraint: the standard model*) Let Γ be a case base and X a new fact situation before a court. Then the court must decide X on the basis of a rule r leading to an outcome s such that $(\Gamma \cup \langle X, r, s \rangle)^+$ is a case base.[18]

There is a lot to be said for the standard model. But for present purposes, its most important characteristic is that it is *flat*. As we've seen, the model operates on the background assumption that the only difference between the constraint that precedents place on higher and lower courts is that a higher court can sometimes overrule a precedent that a lower court can't. But since the possibility of overruling is set aside for purposes of the model, any given precedent constrains any given court in precisely the same way. In the standard model, in short, all courts are on a par. I will now argue that this prevents the model from capturing the U.S. doctrine of precedent.

4 The doctrine of vertical precedent

Most judicial systems are organized in a *hierarchy*, giving superior courts authority over inferior courts. In the United States, for example, the federal courts are organized into three levels, ascending from (1) *district courts*,[19] to (2) *courts of appeals* (or *circuit courts*), and, finally, to (3) the *Supreme Court*. Each circuit court

[18] Note that this definition assumes that the case base Γ is consistent to begin with, in the sense that Γ^+ is a case base.

[19] Strictly speaking, the lowest level also includes the Court of International Trade and the Court of Federal Claims, which have nationwide jurisdiction to hear different sorts of specialized claims. We ignore these and other complications for ease of exposition.

exercises appellate review over a number of district courts, and the Supreme Court exercises appellate review over every other court in the system. In general, the judicial systems of U.S. states are similarly organized,[20] with trial courts on the bottom, appellate courts in the middle, and a supreme court on top. When a court situated in a hierarchy must decide a dispute, it may be constrained by (1) a past decision of that very court, or (2) a past decision of a *superior court*.[21] The former are called *horizontal precedents*; the latter, *vertical precedents*.

In this section, I defend the following claim.

(DVP) The U.S. doctrine of horizontal precedent and the U.S. doctrine of vertical precedent are significantly different, even once the possibility of overruling is set aside. In particular, while a U.S. court faced with an applicable *horizontal* precedent that it is unwilling or unable to overrule is often permitted to either apply the precedent or distinguish it, a court faced with an applicable *vertical* precedent is required to apply the rule and reach the outcome it prescribes.

Now, overruling aside, the standard model does not differentiate horizontal and vertical precedents. Rather, as we've seen, it holds that a court has the same power to distinguish in either case. It follows that the standard model can't capture the U.S. doctrine of precedent without serious revision.[22]

[20] Indeed, this judicial structure is found in many western democracies. See Taruffo (1997, 437).

[21] A more general taxonomy might also allow for the possibility that a court is constrained by (3) a past decision of an *inferior* court, or (4) a past decision of a court to which it is neither superior nor inferior. (A court falling under (4) is not necessarily a *coordinate* court, a court at the same level in the judicial hierarchy. The Second Circuit Court of Appeals, for example, is not superior, inferior, or coordinate to the United States District Court for the Eastern District of Arkansas.) But U.S. courts are not generally constrained by such decisions.

[22] So long as an adequate formal model of the U.S. doctrine would be worth having, it doesn't much matter whether past work on the standard model and its competitors was aimed at developing such a model. My concern is not ultimately to take issue with any particular claims that have been made on behalf of these models. Rather, it is simply to take up the standard model, and to ask, regardless of what may have motivated its development in the first instance, how well it captures the U.S. doctrine of precedent. Incidentally, though, I do think that the standard model and its competitors have often been taken to capture the U.S. doctrine of precedent (even if they have been employed for different purposes as well). While Raz (2009, 181) first presented the standard model (informally) as a descriptive model of the doctrine of precedent not in the U.S. but in England, subsequent discussions of the standard model and its competitors have taken the models to apply more generally. Horty (2011b, 1), for example, introduces the standard model, as well as his preferred reason model, as a specification of the doctrine of precedent along *roughly* Razian lines. Gone, though, is the explicit clarification that what is at issue is the doctrine of precedent *in England*. Instead, we find only generic mention of "the common law." The suggestion—fairly explicit in Horty (2014, 337) but more often left in the background—is that substantially the same doctrine of precedent binds judges in the United Kingdom, the United States, Canada, India, Israel, and other common law jurisdictions. If these models are supposed to capture the doctrine of precedent in all of these jurisdictions, then, *a fortiori*, they are supposed to capture the doctrine in the U.S., which is all that was wanted. I should say, though, that the suggestion that substantially the same doctrine is at work in all these jurisdictions strikes me as implausible (although this will of course depend on just how much work is being done by "substantially"). Many scholars have noted differences between the doctrines of precedent in different common law jurisdictions—for example, MacCormick and Summers (1997),

Before I defend **(DVP)**, however, we need to get clear on what it means. Let's say that a *doctrine of precedent* is a set of norms directing judges to treat precedents in certain ways. Similarly, let's say that a doctrine of *horizontal* precedent is a set of norms concerning the judicial treatment of horizontal precedents, and that a doctrine of *vertical* precedent is a set of norms concerning the judicial treatment of horizontal precedents. There are infinitely many possible doctrines of precedent, then, for there are infinitely many sets of this kind. Moreover, a doctrine of precedent, on this construal, is an abstract object intrinsically unrelated to any particular legal system.[23] So what makes a doctrine of precedent *the* doctrine of precedent in a particular legal system? Let's stipulate that a doctrine is *the* doctrine of a system just in case it contains the norms concerning the treatment of precedents that the judges of that system, in official statements, (i) claim that they are required to follow; (ii) criticize other judges for failing to follow; and (iii) appeal to in order to justify their official decisions.[24] The doctrine of precedent in a given legal system, in this stipulated sense, need not

Footnote 22 (continued)

Cross (1991, 19ff.), Goodhart (1930a). But we don't actually need to compare the doctrines of different common-law jurisdictions to see that there is no perfectly general doctrine of precedent baked into the very notion of common law reasoning. It's enough to notice that the doctrine in any given jurisdiction changes over time. The U.K. doctrine famously changed in 1966 when the House of Lords gave itself the power to overrule its own precedents, a power it had lacked since at least 1898. But the U.S. doctrine has changed too. For example, as discussed in Bradford (1990), it used to be an open question whether a federal circuit court is required to follow an applicable Supreme Court precedent that the court is reasonably sure the current Court would overrule if given the chance. Then, after many years of confusion in the lower courts, the Supreme Court finally decided, in *Rodriguez de Quijas v. Shearson/American Express*, 490 U.S. 477 (1989), that inferior courts must follow applicable Supreme Court precedents until the Court actually overrules them. Now this is a settled component of the U.S. doctrine of precedent. Plainly, then, there is no single doctrine of precedent to be found in every common law system at every time. There are similarities, to be sure, and as we develop our formal models, it may well make sense to stave off parochialism for a while by starting with features common to many jurisdictions. Fairly soon, however, assuming that our aim is to describe the doctrine of precedent in *some* jurisdiction, progress will require that we attend to the doctrine of some *particular* jurisdiction. This paper, of course, focuses on the U.S. In spite of the real differences between the doctrines of precedent in different common law jurisdictions, however, I suspect that, even once the possibility of overruling is set aside, the distinction between vertical and horizontal precedent will need to be incorporated into formal models for many of them, although the precise doctrine for each sort of precedent will need to be different in different models. I make no argument for this claim here, but see, for example, Eng (1997, 204) (concerning Norway), Bankowski et al. (1997, 325–326, 328, 349) (concerning the United Kingdom).

[23] Compare Lewis (1975) and Lewis (1969) on "languages and language" (or "possible languages and actual languages").

[24] To be clear, this is a stipulation, not a conceptual or linguistic claim. If it *were* a conceptual or linguistic claim, then one might reasonably object that it asks too little. For it is at least plausible that for a doctrine of precedent to be *the* doctrine of a legal system—in the ordinary, non-stipulative sense—we must have not only (i)–(iii), but also that the judges of the system (iv) generally act in accordance with the doctrine, and (v) generally think that they *ought* to act in accordance with the doctrine. Conditions (i)–(v) are closely related to the conditions that Hart (1994, 55–61) claims are necessary and sufficient for the existence of a *social rule*. Their plausibility in this context should hardly be surprising, of course, since Hart's rule of recognition is a social rule, and Hart (1994, 101) thought that the doctrine of precedent—part of it, anyway—is included in the rule of recognition in most modern legal systems. Alternatively, one might plausibly say that (iv) and (v) alone are necessary and sufficient for a doctrine to be *the* doctrine of a given system. (Indeed, Hart is sometimes read in this way with respect to social rules generally.) On this account, the fact that judges in a particular legal system, in official statements, claim that they are required to follow a certain doctrine of precedent, criticize other judges for failing to follow

perfectly conform to the pattern of decisions. Some judges in some systems may even routinely violate the prevailing doctrine of precedent. In doing so they will be violating the norms that they themselves insist they are duty-bound to follow. But such violations are possible. Accordingly, **(DVP)** cannot easily be supported or undermined by empirical evidence concerning patterns of decision in the U.S. courts. The key evidence concerns instead the second-order critical remarks of judges.

I begin my defense by considering what the Supreme Court has said about horizontal and vertical precedents. When it comes to horizontal precedents, the Court tells us that stare decisis is a "principle of policy" rather than a "mechanical formula of adherence to the latest decision."[25] Adherence to horizontal precedent "is the preferred course because it promotes the evenhanded, predictable, and consistent development of legal principles, fosters reliance on judicial decisions, and contributes to the actual and perceived integrity of the judicial process."[26] But it is not an "inexorable command,"[27] and the Court is free to depart from one of its holdings when it finds that there is a sufficiently good reason to do so.[28]

Footnote 24 (continued)
that doctrine, and so on, would merely be relevant *evidence* that they (v) generally think that they ought to comply with the doctrine, i.e., that they *accept* the doctrine. In any case, I do not intend to engage in these sorts of debates here. While I do think that judges in the U.S. generally accept and comply with the doctrine of precedent that **(DVP)** describes, I will be directly concerned, in this section, only with whether they claim that they are required to comply with it, criticize violations of it, and appeal to it in order to justify their official decisions. (In the next section, I will briefly address whether judges in the U.S. generally follow the U.S. doctrine of precedent. But the issue is not a central concern of this essay.)

[25] *Helvering v. Hallock*, 309 U.S. 106, 119 (1940).

[26] *Janus v. Am. Fed'n of State, Cty., & Mun. Employees, Council 31*, 138 S. Ct. 2448, 2478 (2018).

[27] *Payne v. Tennessee*, 501 U.S. 808, 828 (1991). See also *Hohn v. United States*, 524 U.S. 236, 251 (1998); *Seminole Tribe of Fla. v. Fla.*, 517 U.S. 44, 63 (1996); *Planned Parenthood of SE Pennsylvania v. Casey*, 505 U.S. 833, 854–855 (1992).

[28] Generally, the Court says that it must follow a horizontal precedent unless there is a "special justification" for departing from it. See, for example, *Dickerson v. United States*, 530 U.S. 428, 443 (2000). See also *Kimble v. Marvel Entm't, LLC*, 135 S. Ct. 2401, 2409 (2015); *Halliburton Co. v. Erica P. John Fund, Inc.*, 134 S. Ct. 2398, 2407 (2014); *Michigan v. Bay Mills Indian Cmty.*, 134 S. Ct. 2024, 2036 (2014). What constitutes a special justification? For starters, there is a "positive" reason to depart from precedent if the case was wrongly decided (and perhaps badly reasoned), its continued application would result in harm or injustice, or the lower courts have struggled to apply the associated rule or standard. See *Janus*, 138 S. Ct. at 2479; *Johnson v. United States*, 135 S. Ct. 2551, 2562 (2015); *Citizens United v. Federal Election Comm'n*, 558 U.S. 310, 363 (2010); *Montejo v. Louisiana*, 556 U.S. 778, 785 (2009); *United States v. South-Eastern Underwriters Ass'n*, 322 U.S. 533, 580 (1944) (Stone C.J. dissenting). Whether there is a "special justification" in a given case may depend, in part, on just how great a harm or injustice the continued application of the precedent would produce, or just how unworkable the rule has proven to be. But it will usually depend also on the presence or absence of countervailing reasons to adhere to precedent. For instance, if people have arranged their affairs in reliance on a precedent, this is a good reason not to depart from it. Accordingly, a "special justification" is more likely to exist if reliance interests are absent. See, for example, *Leegin Creative Leather Prod., Inc. v. PSKS, Inc.,* 551 U.S. 877, 925 (2007); *Hohn*, 524 U.S. at 251; *Payne*, 501 U.S. at 828. A handful of additional considerations are considered relevant, but the general idea is simply that the Court must "make certain that more harm will not be done in rejecting than in retaining a rule of even dubious validity." *South-Eastern Underwriters Ass'n*, 322 U.S. at 580 (Stone C.J. dissenting). See also *Citizens United*, 558 U.S. at 378 (Roberts C.J. concurring) ("When considering whether to reexamine a prior erroneous holding, we must balance the importance of having constitutional questions *decided* against the importance of having them *decided right*.").

When the Court decides to depart from an applicable precedent, it sometimes overrules the precedent outright. In *Lawrence v. Texas*,[29] for example, the Court was asked to determine the constitutionality of a Texas statute criminalizing gay sex.[30] Though it was convinced that the statute was unconstitutional, before striking it down the Court had to reckon with its decision to uphold a Georgia sodomy ban in *Bowers v. Hardwick*.[31] The Court might have tried to distinguish *Bowers*, perhaps on the grounds that where the Georgia statute, by its terms, outlawed *all* sodomy, heterosexual and homosexual, the Texas statute aimed solely at homosexual sodomy, arguably raising distinct equal protection concerns.[32] Instead, the Court overruled *Bowers* outright, and in no uncertain terms:

> *Bowers* was not correct when it was decided, and it is not correct today. It ought not to remain binding precedent. *Bowers v. Hardwick* should be and now is overruled.[33]

Lawrence thus deprived *Bowers* of all legal authority. As a result, *Bowers* is simply no part of the law.

Other times the Court will tread more lightly, though, departing from an applicable precedent not by overruling it but by *distinguishing* it. Consider, for example, the case of *New York v. Quarles*.[34] In that case, the police chased an armed man through a supermarket before finally apprehending him. By the time he was in handcuffs, the gun was gone. The police asked him where he had stashed it, and he told them. Only after retrieving the gun did the police read the man his rights. The question was whether his statement directing the police to the gun was admissible at trial. A majority of the justices were inclined to think that it was. The problem was that *Miranda v. Arizona*[35] held that a defendant's response to custodial interrogation is inadmissible if he has not been read his rights.[36] The majority in *Quarles* acknowledged the applicability of the *Miranda* rule—when he was asked about the gun, the suspect was plainly in police custody, and he had not been read his rights.[37] But, rather than apply *Miranda* or overrule it, the Court decided to *modify* it, by creating

[29] 539 U.S. 558 (2003).

[30] *Id.* at 562–563. For additional examples of the Supreme Court overruling its own precedents, see *Janus v. Am. Fed'n of State, Cty., & Mun. Employees, Council 31*, 138 S. Ct. 2448, 2486 (2018) (overruling *Abood v. Detroit Bd. of Ed.,* 431 U.S. 209 (1977)); *S. Dakota v. Wayfair, Inc.,* 138 S. Ct. 2080, 2099 (2018) (overruling *Quill Corp. v. North Dakota*, 504 U.S. 298 (1992) and *National Bellas Hess, Inc. v. Department of Revenue of Ill.,* 386 U.S. 753 (1967)); *Johnson v. United States*, 135 S. Ct. 2551, 2563 (2015) (overruling *Sykes v. United States*, 131 S. Ct. 2267 (2011) and *James v. United States*, 550 U.S. 192 (2007)); *Obergefell v. Hodges*, 135 S. Ct. 2584, 2605 (2015) (overruling *Baker v. Nelson*, 409 U.S. 810 (1972)); *Adarand Constructors, Inc. v. Pena*, 515 U.S. 200, 227 (1995) (overruling *Metro Broadcasting, Inc. v. FCC*, 497 U.S. 547 (1990)).

[31] 478 U.S. 186 (1986).

[32] See *Lawrence*, 539 U.S. at 579ff. (O'Connor J. concurring in the judgment).

[33] *Id.* at 578.

[34] 467 U.S. 649 (1984).

[35] 384 U.S. 436 (1966).

[36] *Id.* at 444.

[37] *Quarles*, 467 U.S. at 655. See also *id.*, at 664–665 (O'Connor J. concurring in the judgment in part and dissenting in part) (noting the majority's concession that *Miranda* applies).

an exception for interrogation related to immediate public safety concerns.[38] So modified, the rule of *Miranda*, in relevant part, became roughly the following:

> A defendant's response to custodial interrogation is inadmissible if he has not been read his rights and the interrogation is not necessary to secure the immediate safety of the police or the public.

Since *Miranda* concerned station house questioning unrelated to any immediate threat to public safety, it fell outside the exception, and the modified rule continued to justify the holding of inadmissibility in that case.[39] But, the Court reasoned, since a handgun hidden somewhere in a supermarket poses a danger to the public, the facts of *Quarles* fell within the exception, and, so, the modified rule was inapplicable.[40] The Court thus cleared the way for a finding of admissibility. But, since it distinguished *Miranda* instead of overruling it, the rule of *Miranda*, in its modified form, remained good law.[41]

So the Supreme Court has some flexibility in dealing with its past decisions—it can avoid an applicable horizontal precedent rule either by overruling the precedent case or by distinguishing it. What, then, of vertical precedents? How must the Court's holdings be treated by lower courts? Plainly, a lower court lacks the authority to *overrule* a Supreme Court precedent. But can it *distinguish* such a precedent, modifying its holding until it no longer applies to a new dispute? The Court has made clear that the answer is *No*. An applicable Supreme Court precedent rule must be strictly applied by the lower courts "no matter how misguided the judges of those courts may think it to be."[42]

Consider, for example, *Thurston Motor Lines, Inc. v. Jordan K. Rand, Ltd*.[43] In many ways, this was the sort of routine contract dispute that generally winds up in state court—a carrier simply wanted to recover unpaid freight charges from a shipper. And, indeed, when the case reached the Ninth Circuit, the issue was whether the dispute "arose under" federal law, so that the district court had jurisdiction to hear it.[44] The Ninth Circuit found it "obvious" that the court lacked jurisdiction.[45] The problem, however, was that the Supreme Court had held in *Louisville & Nashville*

[38] *Id.* at 655–656. See also *id.*, at 679 (Marshall J. dissenting) (noting the majority's candor concerning its modification of *Miranda*).

[39] See *Miranda*, 384 U.S. at 491–492.

[40] *Quarles*, 467 U.S. at 657.

[41] The Court has modified *Miranda* in other cases as well. See, for example, *Berghuis v. Thompkins*, 130 S. Ct. 2250 (2010); *Oregon v. Elstad*, 470 U.S. 298 (1985); *Harris v. New York*, 401 U.S. 222 (1971). See also *Dickerson v. United States*, 530 U.S. 428, 441 (describing *Quarles* and *Harris* as *modifying* the rule of *Miranda*). Additional examples of the Court (arguably, at least) modifying an applicable horizontal precedent include *Arizona v. Gant*, 556 U.S. 332 (2009) (modifying *New York v. Belton*, 453 U.S. 454 (1981)); *Flast v. Cohen*, 392 U.S. 83 (1968) (modifying *Frothingham v. Mellon*, 262 U.S. 447 (1923)).

[42] *Hutto v. Davis*, 454 U.S. 370, 375 (1982). Otherwise, the Court says, "anarchy [would] prevail within the federal judicial system." *Id.*

[43] 460 U.S. 533 (1983) (per curiam).

[44] *Thurston Motor Lines, Inc. v. Jordan K. Rand, Ltd.*, 682 F.2d 811 (9th Cir. 1982).

[45] *Id.* at 812.

R.R. Co. v. Rice[46] that actions for the collection of unpaid freight charges "arise under" the Interstate Commerce Act (because that Act required carriers to collect the charges shown in their tariffs).[47] The Court of Appeals nevertheless tried to *distinguish* this Supreme Court decision.[48] In *Rice*, it pointed out, the carrier charged the shipper a fee that, while listed in the carrier's tariffs under the Act, fell outside the parties' original agreement. Thus, the Court of Appeals reasoned, the claim in *Rice* depended exclusively on the Act. In *Thurston Motor*, by contrast, since the disputed charges were addressed in the parties' contract, the Act seemed superfluous. So the Court of Appeals effectively proposed to modify the rule of *Rice*, so that it became:

> If an action is (i) for the collection of unpaid freight charges that are (ii) consistent with tariffs set in compliance with the Interstate Commerce Act but (iii) inconsistent with the parties' agreement, then the action arises under the Interstate Commerce Act.

This modified rule continued to justify the outcome of *Rice*, but since the charges at issue in *Thurston Motor* were consistent with the parties' original agreement, the modified rule no longer applied. Thus, the Ninth Circuit found itself free to hold that the federal courts lacked jurisdiction to hear the case.[49]

The Supreme Court reversed in a per curiam opinion that took a dim view of the lower court's handiwork.[50] *Rice*, the Court pointed out, had "squarely held" that federal courts have jurisdiction to hear actions for the collection of unpaid freight charges.[51] And, indeed, "[o]ther federal courts have had no difficulty in following" this unambiguous holding.[52] What, then, of the Ninth Circuit's attempt to distinguish this "most troublesome case"? It was, the Court declared, "wholly unconvincing."[53] There was simply "no support" for the lower court's "interpretation" in the Supreme Court's opinion in *Rice*, or in any other Supreme Court opinion, for that matter. To be sure, the Ninth Circuit was correct that the carrier in *Rice*, but not in *Thurston Motor*, charged the shipper a fee that was legal under the tariffs but outside the parties' agreement. But this, the Court insisted, was *irrelevant*: "the Court of Appeals has simply confused the factual contours of *Rice* for its unmistakable holding."[54] Thus, it was the *rule* of *Rice* that mattered, and if the rule applied to a dispute before a lower court, any remaining factual differences between that dispute

[46] 247 U.S. 201 (1918).
[47] *Id.* at 202.
[48] *Thurston Motor*, 682 F.2d at 812.
[49] *Id.* at 814.
[50] *Thurston Motor*, 460 U.S. at 533.
[51] *Id.* at 534.
[52] *Id.*
[53] *Id.* Why "wholly unconvincing" and not, say, *wholly inappropriate*? Because the Ninth Circuit framed its attempt to *modify* a vertical precedent as an attempt to *interpret* it. And it is hardly inappropriate for an inferior court to interpret a superior court opinion in order to identify its holding.
[54] *Id.* at 535.

and *Rice* were beside the point—the lower court was required to apply the rule and decide accordingly.

The Court has made this point repeatedly, insisting that lower courts must follow the Court's "explication of the governing rule of law."[55] Thus, a lower court judge is not allowed to distinguish a Supreme Court precedent based on her own view that it was wrongly decided in the first place or would produce injustice if applied to a new set of facts. In fact, she is not even allowed to distinguish a Supreme Court precedent based on her view that the *present Supreme Court* believes that it was wrongly decided and plans to overrule it as soon as it gets the chance. As the Court has said, "If a precedent of this Court has direct application in a case, yet appears to rest on reasons rejected in some other line of decisions," the lower court "should follow the case which directly controls."[56] In short, lower courts are required to strictly follow the rules handed down by the Supreme Court, no matter what they think of the rules themselves or the results they produce. If the rule applies, a lower court must decide accordingly.

So we find two doctrines of precedent articulated by the Supreme Court, one for horizontal precedents and another for vertical precedents. While horizontal precedents may generally be either overruled or distinguished, a lower court faced with an applicable vertical precedent must apply the rule and reach the outcome it prescribes. We find the same doctrine of vertical precedent articulated in the following passage from the Ninth Circuit Court of Appeals:

> [Vertical precedent] is very powerful medicine. A decision of the Supreme Court will control that corner of the law unless and until the Supreme Court itself overrules or modifies it. Judges of the inferior courts may voice their criticisms, but…if a [vertical] precedent is determined to be on point, it must be followed.[57]

In fact every federal court of appeals understands the doctrine of vertical precedent to require strict adherence to an applicable vertical precedent rule. The following statements are representative.[58]

[55] *County of Allegheny v. American Civil Liberties Union, Greater Pittsburgh Chapter,* 492 U.S. 573, 668 (1989) (Kennedy J. concurring and dissenting). See also *Bosse v. Oklahoma*, 137 S. Ct. 1, 2 (2016); *Ashcroft v. al-Kidd*, 563 U.S. 731, 742 (2011); *Seminole Tribe*, 517 U.S. at 63; *Rivers v. Roadway Exp., Inc.,* 511 U.S. 298, 312 (1994).

[56] *Rodriguez de Quijas*, 490 U.S. at 484. See also *Bosse v. Oklahoma*, 137 S. Ct. 1, 2, (2016); *Jaffree v. Bd. of Sch. Comm'rs of Mobile Cty.,* 459 U.S. 1314, 1316 (1983); *Hicks v. Miranda*, 422 U.S. 332, 344–345 (1975).

[57] *Hart v. Massanari*, 266 F.3d 1155, 1171–1172 (9th Cir. 2001).

[58] See also *United States v. Bd. of Cty. Commissioners of Cty. of Otero*, 843 F.3d 1208, 1214 (10th Cir. 2016); *United States v. Walker*, 351 F. App'x 16, 18 (6th Cir. 2009); *Omega S.A. v. Costco Wholesale Corp.,* 541 F.3d 982, 987 n. 5 (9th Cir. 2008); *United States v. Sampson*, 486 F.3d 13, 20 (1st Cir. 2007); *Cardenas v. Dretke*, 405 F.3d 244, 253 (5th Cir. 2005); *Johnson v. DeSoto Cty. Bd. of Commissioners*, 72 F.3d 1556 (11th Cir. 1996); *Butts v. City of New York Dep't of Hous. Pres. & Dev.,* 990 F.2d 1397, 1409 (2d Cir. 1993); *Vujosevic v. Rafferty*, 844 F.2d 1023, 1030 (3d Cir. 1988); *U.S. ex rel Shore v. O'Leary*, 833 F.2d 663, 667 (7th Cir. 1987).

- In a hierarchical system, decisions of a superior court are authoritative on inferior courts. Just as the courts of appeals must follow decisions of the Supreme Court whether or not we agree with them, so district judges must follow the decisions of this court whether or not they agree. *Reiser v. Residential Funding Corp.,* 380 F.3d 1027, 1029 (7th Cir. 2004) (Easterbrook J.).
- Vertical stare decisis is absolute and requires lower courts to follow applicable Supreme Court rulings in every case…Vertical stare decisis applies to Supreme Court precedent in two ways. First, the result in a given Supreme Court case binds all lower courts. Second, the reasoning of a Supreme Court case also binds lower courts. So once a rule, test, standard, or interpretation has been adopted by the Supreme Court, that same rule, test, standard, or interpretation must be used by lower courts in later cases. *U.S. v. Duvall*, 740 F.3d 604, 609–610 (D.C. Cir. 2013) (Kavanaugh J. concurring).
- [W]e are bound to apply the doctrine…the Supreme Court [has] articulated…and we must leave to the Supreme Court the decision of whether…to create an exception to, or otherwise limit, that rule. *Stop Reckless Econ. Instability Caused by Democrats ("Stop PAC") v. Fed. Election Comm'n*, 814 F.3d 221, 231 (4th Cir. 2016).
- If the Supreme Court's decision in a case is to be modified, overruled or disregarded, that will have to be done by the Supreme Court. *United States v. Pate*, 754 F.3d 550, 554 (8th Cir. 2014).
- As a circuit court…we are bound to follow a controlling Supreme Court precedent until it is explicitly overruled by that Court. *Nunez-Reyes v. Holder*, 646 F.3d 684, 692 (9th Cir. 2011) (internal quotation marks and citations omitted).

Thus, in *Reiser*, Judge Easterbrook emphasizes the importance, when determining the constraint a precedent decision places on a later court, of the hierarchical rank of both the precedent court and the later court. When a court is dealing with a vertical precedent, Easterbrook tells us, the precedent is "authoritative" and must be followed, whether or not the lower court agrees with the results.[59] Still, this passage leaves open just what exactly in a vertical precedent opinion a lower court is required to follow. Here, Judge Kavanaugh's statement in *Duvall* clarifies that it is indeed the rule or standard articulated in the superior court's holding that must be applied.[60] And *Stop PAC* and *Pate* assure us that this does not mean merely that a lower court lacks the power to *overrule* a vertical precedent outright. Lower courts

[59] The implicit contrast, of course, is with the situation in which a court is faced with a *horizontal* precedent that the court would rather not apply to a new set of facts. In *this* situation, the court is *not* (always) required to follow the precedent.

[60] In fact, many circuit courts have also suggested that they have a duty to follow the *dicta* of the Supreme Court. See, for example, *Utah Republican Party v. Cox*, 892 F.3d 1066, 1079 (10th Cir. 2018); *American Civil Liberties Union of Ky. v. McCreary Cnty.,* 607 F.3d 439, 447 (6th Cir. 2010) ("Lower courts are obligated to follow Supreme Court dicta, particularly where there is not substantial reason for disregarding it, such as age or subsequent statements undermining its rationale."); *Winslow v. F.E.R.C.,* 587 F.3d 1133, 1135 (D.C. Cir. 2009) (stating that "carefully considered language of the Supreme Court, even if technically dictum, generally must be treated as authoritative"); *McCoy v. Massachusetts Inst. of Tech.,* 950 F.3d 13, 19 (1st Cir. 1991) (stating that "federal appellate courts are bound by the Supreme Court's considered dicta almost as firmly as by the Court's outright holdings, particularly when…[they are] of recent vintage and not enfeebled by any subsequent statement").

also lack the power to *limit* or *modify* the rules embodied in vertical precedents.[61] *A fortiori*, they lack the power to distinguish them, in our strong sense of that term, for this is simply the power to modify them in certain ways under certain conditions. And this leaves us with the straightforward conclusion of *Nunez-Reyes* that whenever a vertical precedent applies, the court must decide accordingly.

We find the same understanding of the doctrine of vertical precedent articulated in the state courts.[62] Again, we'll limit ourselves to a handful of representative passages.[63]

- Michigan has a hierarchical judicial system, and trial courts are required to follow applicable rules, orders, and caselaw established by appellate courts… This structure is essential to the orderly, uniform, and equal administration of justice. A trial court is not free to disregard rules, orders, and caselaw with which it disagrees or to become a law unto itself. Although a trial court is not required to agree with appellate rules, orders, and caselaw, as with litigants and all other citizens seeking to comply with the law, the court is required in good faith to follow those rules, orders, and caselaw. *Pellegrino v. AMPCO Sys. Parking*, 486 Mich. 330, 352–353 (2010).
- The lower courts are bound by our decisions, and this Court alone is responsible for modifying that precedent. *Sell v. Gama*, 231 Ariz. 323, 330 (2013).
- As the court of appeals correctly noted, it was without authority to modify, withdraw, or otherwise change the holding in [a Wisconsin Supreme Court opinion] even if it wanted to. *Casper v. Am. Int'l S. Ins. Co.,* 336 Wis. 2d 267, 293 (2011).
- [E]ven if we were inclined, we lack the authority to alter, overrule or decline to follow the holding of cases the Supreme Judicial Court has decided. *Commonwealth v. Gallagher*, 91 Mass. App. Ct. 385, 399 (2017) (internal quotation marks and citations omitted).

[61] See also *Duvall*, 740 F.3d at 617 (Kavanaugh J. concurring) ("[T]he Supreme Court is free to reconsider or refine or tweak its own precedents…and it does so in appropriate cases. Lower courts, by contrast, are not free to reconsider or refine or tweak Supreme Court precedents.").

[62] And in the federal district courts as well. See, for example, *Poonjani v. Shanahan*, 319 F. Supp. 3d 644, 650 (S.D.N.Y. 2018); *McGary v. Crowley*, 266 F. Supp. 3d 254, 261 (D.D.C. 2017); *Exergen Corp. v. Kids-Med, Inc.,* 189 F. Supp. 3d 237, 243–244 (D. Mass. 2016); *In re Ashai*, 211 F. Supp. 3d 1215, 1221 (C.D. Cal. 2016); *Haith ex rel. Accretive Health, Inc. v. Bronfman*, 928 F. Supp. 2d 964, 971 (N.D. Ill. 2013) ("The reach of Supreme Court decisions are not limited to the particular facts and circumstances presented in the case being decided; lower courts must apply the reasoning of those decisions even to cases that are factually dissimilar."); *Panayoty v. Annucci*, 898 F. Supp. 2d 469, 479–480 (N.D.N.Y. 2012); *United States v. Rosenau*, 870 F. Supp. 2d 1109, 1115 (W.D. Wash. 2012); *Does 1–7 v. Round Rock Indep. Sch. Dist.,* 540 F. Supp. 2d 735, 749 (W.D. Tex. 2007); *In re Reveal*, 148 B.R. 288, 292 (Bankr. S.D. Ohio 1992).

[63] See also *Evans v. Rockdale Hosp., LLC*, 345 Ga. App. 511, 521 (2018); *Harris v. State*, 407 P.3d 348, 355 (Nev. App. 2017); *People v. Etherton*, 82 N.E.3d 693, 697 (Ill. 2017); *Stringer v. Stringer*, 544 S.W.3d 714, 723 (Tenn. Ct. App. 2017); *George v. Hercules Real Estate Servs., Inc.,* 339 Ga. App. 843, 854 (2016); *Rodriguez v. Nat'l City Bank*, 277 FRD 148, 154 (E.D. Pa. 2011); *State v. Hausmann*, 277 Neb. 819, 824 (2009); *Alaska Pub. Interest Research Grp. v. State*, 167 P.3d 27, 43–44 (Alaska 2007); *Sanford v. Clear Channel Broad, Inc.,* 14 Neb. App. 908, 915–916 (2006); *Com v. Millner*, 585 Pa. 237, 260 (2005); *Howell Lumber Co. v. City of Tuscaloosa*, 757 So.2d 1173 (Ala. Civ. App. 1997); *Stephenson v. Perlitz*, 524 S.W.2d 786, 788–789 (Tex. Civ. App. 1975).

– [I]t is well established that this court, as an intermediate appellate tribunal, is not at liberty to discard, modify, reconsider, reevaluate or overrule the precedent of our Supreme Court. *State v. Frazier*, 181 Conn. App. 1, 37 (2018) (internal quotation marks and citations omitted).

Notice, again, the lower courts' acknowledgement that they are not, under the doctrine of vertical precedent, free to *modify* or *limit* a vertical precedent. They are saying more than that they lack the power to *overrule*. They are saying that, when it comes to vertical precedent, they also lack the power to *distinguish*.

I conclude that **(DVP)** is correct.[64] In the United States, the doctrine of vertical precedent differs significantly from the doctrine of horizontal precedent. While the doctrine of horizontal precedent often allows a court to either overrule or distinguish a past decision of its own, the doctrine of vertical precedent requires a court faced with an applicable vertical precedent to apply the rule and reach the outcome it prescribes. As a result, any formal model of precedent, if it is to capture the U.S. doctrine, must treat horizontal precedents and vertical precedents differently, even once the possibility of overruling is set aside. If the standard model, in particular, is to capture the U.S. doctrine of precedent, it requires serious revision.[65]

[64] Although some may find it surprising, I do not think that **(DVP)** should be terribly controversial. In fact, it is so widely accepted among traditional legal scholars that it passes for a platitude in the law reviews. See, for example, Garner et al. (2016, 42, 101, 243), Kozel (2014, 203), Mead (2012, 791), Sloan (2009, 718), Abramowicz and Stearns (2005, 957), Lundmark (1998, 212), Summers (1997, 369–370, 374), Caminker (1994, 818), Dorf (1994, 2025), Alexander (1989, 53–54).

[65] While this paper focuses on the standard model, I also take the arguments in this section to show that if the *reason* model—elaborated in, for example, Horty (2017), Rigoni (2017), Horty (2015), Horty and Bench-Capon (2012), Horty (2011b), Lamond (2005)—is to capture the U.S. doctrine of precedent, then it too requires revision. An anonymous reviewer disagrees. In particular, the reviewer suggests that, for the reason model theorist, the prohibitions on modifying, tweaking, altering, and limiting vertical precedent—as found in *Stop PAC*, *Pate*, *Casper*, and elsewhere—do nothing to show that vertical precedents can't be distinguished, because distinguishing, on the reason model, does not involve modifying precedent rules. (Instead, it involves strengthening, in a particular way, the priority ordering on reasons implicit in the case base.) The reviewer's suggestion is, I think, mistaken. Before addressing the relevance to the reason model of the prohibitions on modifying vertical precedent rules, I will simply note that the conclusion that vertical precedents can't be distinguished under the U.S. doctrine is supported by many judicial statements that make no reference to rule modification, including, for example, the statements in *Rodriguez de Quijas*, *Duvall*, and *Hart* that if the holding of a vertical precedent applies, the lower court must follow the rule. Thus, even if the prohibitions on modifying vertical precedent rules were irrelevant to the reason model, the evidence in this section would still indicate that the reason model can't capture the U.S. doctrine of precedent without considerable revision. In fact, though, these prohibitions are as relevant to the reason model as they are to the standard model. To see this, note that, on the reason model, no court *ever* modifies a precedent rule, whether the precedent is horizontal or vertical. Yet courts in the U.S. certainly *say* that they can and do modify horizontal precedent rules. In *Dickerson*, for example, Chief Justice Rehnquist, writing for the Court, refers to the many "modifications" that the Court has made to the rule of *Miranda* and declares that "no constitutional rule is immutable." 530 U.S. at 441. The reason model, if it is to capture the U.S. doctrine of precedent, can't simply *ignore* all this talk of rule modification. There is evidently *something*, a certain way of treating precedents, that these courts are using the language of rule modification to talk about. And they are saying that it is *permissible* to treat horizontal precedents, but *impermissible* to treat vertical precedents, in that way. The reason model must account for this, presumably by translating talk of modifying rules into talk of strengthening priority orderings among reasons. Recall, in this connection, that Horty (2015) maintains that the reason model and the standard model are *equivalent*, in the sense that they permit precisely the same

5 Predicting case outcomes

In the rest of the paper, I will present a formal model of precedential constraint, the *hierarchical model*, that attempts to capture the U.S. doctrine of precedent. Thus, the hierarchical model will be a model of the doctrine of precedent that U.S. judges, in official statements, (i) claim that they are required to follow; (ii) criticize other judges for failing to follow; and (iii) appeal to in order to justify their official decisions. It is *not*, at least not in the first instance, a model of how U.S. judges in fact decide cases, and it does not generate any predictions about how cases will be decided. Of course, one could conceivably adapt some version of the hierarchical model to serve as one component of a model that *is* used to predict judicial decisions.[66] In this context, the determination that a court is required by the relevant doctrine of precedent to decide for *s*, say, would presumably be taken to raise the probability that the court will in fact decide for *s*. Whether such a project would be worthwhile depends on whether U.S. judges generally *comply* with the U.S. doctrine of precedent, including the strict doctrine of vertical precedent highlighted by **(DVP)**.

Now, I suspect that U.S. courts *do* generally comply with the strict doctrine of vertical precedent, so that the following claim is true.

(Comp) U.S. courts in fact treat horizontal and vertical precedents differently, even once the possibility of overruling is set aside. In particular, a court faced with an applicable precedent rule that it is unwilling or unable to overrule is more likely to decide as the rule directs if the precedent is a vertical precedent than if it is a horizontal precedent.

If this is right, then predictive models that treat horizontal and vertical precedents differently will generally outperform those that do not.[67] Of course, short of actually building and testing a predictive model that incorporates this assumption, a convincing argument for **(Comp)** would probably require an extensive discussion of the

Footnote 65 (continued)
judicial decisions. If this is right, then if these judicial statements suffice to show that distinguishing vertical precedents is prohibited on the standard model, they also suffice to show that it is prohibited on the reason model. Even if the two models are not equivalent, however, or the term 'distinguish' is used differently by reason model theorists, I submit that accounting for the data canvassed in this section will require significant changes to the reason model.

[66] For recent work in AI & Law aimed at predicting judicial decisions, see, for example, Chen and Eagel (2017), Conrad and Al-Kofahi (2017), Grabmair (2017), Ashley and Brüninghaus (2003), Brüninghaus and Ashley (2003).

[67] The comparison I intend is between (i) models that capture the distinction between horizontal and vertical precedent and (ii) models that don't capture that distinction *but still appeal to a doctrine of precedent*. It's possible, of course, that the best predictive models would make no appeal to doctrinal considerations at all, although this seems unlikely if the models are intended to predict decisions outside the Supreme Court.

empirical literature on judicial behavior.[68] Such an argument falls outside the scope of this paper, in which I am mainly concerned to establish **(DVP)** and to develop the hierarchical model. Nevertheless, in this section I will briefly attempt the modest task of making **(Comp)** plausible, by presenting an historical example of an inferior court faced with an applicable vertical precedent. Given what we know of the court, the facts, and the particular precedent at issue, we should expect that, if the option were really available, the court would distinguish the precedent. Instead, it applies the rule handed down from the superior court.

The example is a Seventh Circuit antitrust case called *Khan v. State Oil Co.*[69] Antitrust law is based on the assumption that market competition reduces the prices of consumer goods and promotes the efficient distribution of resources, thus providing broad social benefits.[70] Accordingly, the fundamental antitrust law in the U.S., the Sherman Act, prohibits contracts and conspiracies "in restraint of trade."[71] The *Khan* case concerned a practice called *resale price maintenance* (RPM), in which a producer places a floor or ceiling on the price at which sellers can offer a product. The question of the appropriate antitrust stance on RPM was, for a time, controversial. On the one hand, RPM is a form of price fixing, which, in the paradigm case of an agreement among competitors, is a clear violation of the Sherman Act. On the other, RPM sometimes actually *encourages* (non-price) competition, in the form of point-of-sale services. Despite this uncertainty, in *Dr. Miles Medical Co. v. John D. Park & Sons Co.,*[72] the Supreme Court declared RPM illegal *per se*. This means that once a court determines that an RPM agreement has been made, the inquiry is at an end, and the court is to hold that the agreement violates the Sherman Act. Subsequent cases reaffirmed this result again and again.[73] Indeed, while *Dr. Miles* had involved a scheme to fix a *minimum* resale price—the sort of RPM agreement that most nearly resembles the paradigm Sherman Act violation of a seller's cartel—the Court made clear in *Albrecht v. Herald Co.*[74] that the rule of *Dr. Miles* extended to agreements to fix *maximum* resale prices as well.[75]

[68] See, for example, Epstein et al (2013), Klein and Devins (2013), Kassow et al (2012), Westerland et al (2010), Hansford and Spriggs (2006), Cross (2005), Haire et al (2003), Segal and Spaeth (2002), Reddick and Benesh (2000), Brenner and Spaeth (1995), Songer et al (1994). Two useful surveys are Hansford (2017) and Klein (2017).

[69] 93 F.3d 1358 (7th Cir. 1996) (Posner C.J.).

[70] Hovenkamp (2008) is a good introduction.

[71] The task of developing standards to determine just what should count as a restraint on trade was left to the courts. As a result, despite the underlying statute, antitrust law in the U.S. is effectively common law.

[72] 220 U.S. 373 (1911).

[73] See *United States v. A. Schrader's Son*, 252 U.S. 85 (1920); *Frey & Son v. Cudahy Packing Co.,* 256 U.S. 208, 210 (1921); *Fed. Trade Comm'n v. Beech-Nut Packing Co.,* 257 U.S. 441, 452–453 (1922); *United States v. Socony-Vacuum Oil Co.,* 310 U.S. 150, 223 (1940); *United States v. Bausch & Lomb Optical Co.,* 321 U.S. 707, 721 (1944); *Kiefer-Stewart Co. v. Joseph E. Seagram & Sons*, 340 U.S. 211, 71 (1951); *United States v. McKesson & Robbins, Inc.,* 351 U.S. 305, 76 (1956); *United States v. Parke, Davis & Co.,* 362 U.S. 29 (1960); *Simpson v. Union Oil Co.,* 377 U.S. 13 (1964).

[74] 390 U.S. 145 (1968).

[75] *Id.* at 152–153.

From the early 1970s, Judge Posner was a vocal critic of the Court's antitrust decisions in general, and its rule against RPM in particular.[76] "For several generations," Posner proclaimed in one law review article, "the Supreme Court has been deciding [RPM] cases without any theory at all" of the economic interests involved. The rationale offered for the per se rule was "flimsy,"[77] the cases full of "nonsense" distinctions[78] that merely served to illustrate the justices' "inability to understand what was actually going on."[79] Posner went so far as to suggest that the only reason the Court was sticking with the per se rule was to discourage antitrust litigation so that the Court would not be bothered with it.[80] Otherwise, he suggested, the case law was simply inexplicable: "The [RPM] cases have no economic rationale, and no other rationale either."[81] In the absence of massive market power among the sellers, the proper approach, by Posner's lights, was not a per se *illegality* rule but a per se *legality* rule.[82]

What matters, for our purposes, is not whether Judge Posner was right but simply that he had clearly expressed a strong policy preference for permitting RPM agreements in the absence of market power. Given this preference, if the doctrine of vertical precedent were ineffectual, then we should expect Posner, when presented with an RPM agreement, to try to distinguish the Supreme Court's precedents in which the per se prohibition was applied. The *Khan* case presented just such an agreement. It involved a contract between State Oil, a gasoline distributor, and Khan, a gas station operator. The agreement effectively fixed the maximum price at which Khan could resell the gas he bought from State Oil.[83] After Khan lost his lease on the station, he got the idea that he could have been making more money all along if he had been able to raise his prices. So he sued State Oil for lost profits, arguing that the maximum price provision violated antitrust laws. When the dispute landed in the Seventh Circuit, State Oil asked the court to distinguish the precedents and hold the contract enforceable. But, in his opinion for the court, Posner refused[84]:

[76] See, for example, Posner (1979), Posner (1976), Posner (1975a), Posner (1975b), Posner (1970). So too was Judge Easterbrook, also of the Seventh Circuit. See, for example, Easterbrook (1984), Easterbrook (1982), Easterbrook (1981).

[77] (Posner 1975a, 290).

[78] Judge Posner was thinking of such cases as *United States v. Colgate & Co.,* 250 U.S. 300 (1919); *United States v. General Electric Co.,* 272 U.S. 476 (1926); and *Parke, Davis*, 362 U.S. at 29.

[79] Posner (1975b, 143).

[80] Posner (1975b, 141).

[81] Posner (1975a, 297).

[82] In the case of massive market power among the sellers, Posner suggested that the court ought to undertake the usual open-ended antitrust inquiry under the "rule of reason." Posner (1975a, 298–299).

[83] In fact, Khan was not contractually *prohibited* from charging a higher price, so that if he did so he would be in breach. Rather, the contract provided that, if Khan *did* raise the price, then State Oil was entitled to the difference between the new price and the old multiplied by the number of gallons sold at the new price. Thus, the contract made it worthless for Khan to raise the price. See *Khan*, 93 F.3d at 1360.

[84] Notably, Judge Ripple wrote a concurring opinion to register that he too opposed the Court's per se prohibition of maximum RPM agreements as a substantive matter. So, if the option to distinguish the Court's precedents were really available, Judge Posner would evidently have had the votes to do so. See *Khan*, 93 F.3d at 1367 (Ripple J. concurring).

> [T]he Supreme Court has thus far refused to reexamine the cases in which it has held that resale price fixing is illegal per se regardless of the competitive position of the price fixer or whether the price fixed is a floor or a ceiling. The key precedent… is [*Albrecht*,] …where the Court held…that the action of a newspaper publisher in fixing a ceiling at which its distributors could resell the newspaper to the public was illegal per se. State Oil seeks to distinguish *Albrecht* by pointing out that the initiative for the newspaper to take action against the plaintiff distributor had come from another distributor, giving the scheme a "horizontal" flavor. True, but this was not a factor on which the Court relied. It stated its holding broadly: maximum price fixing is illegal per se even if entirely "vertical," that is, even if the only parties in the picture are a single supplier and a single dealer, as in this case…It is not cricket to distinguish a precedent by pointing to a fact mentioned by the court in the previous opinion but clearly given no weight by it.[85]

Looking back on *Khan*, Posner (2001, 188–189) describes his opinion as "dutifully bowing to the authority" of vertical precedent. Even so, duty did not prevent him from including in the opinion a blistering critique of *Albrecht*, and a call for the Court to overrule it.[86] And sure enough, on appeal, the Court did just that.[87] In the course of overruling *Albrecht*, however, the Court made sure to note that the Seventh Circuit was correct to apply it. This, after all, was what the doctrine of vertical precedent required.[88]

Khan lends some plausibility to the claim that, in the U.S., not only is strict adherence to the rule announced in an earlier case of a superior court part of the professed *doctrine* of precedent, it is part of the *practice* as well.[89] I don't want to make too

[85] *Khan*, 93 F.3d at 1362.

[86] See, for example, *id.* at 1363 ("*Albrecht* was unsound when decided, and is inconsistent with later decisions by the Supreme Court. It should be overruled. Someday, we expect, it will be.").

[87] *State Oil v. Khan*, 522 U.S. 3 (1997).

[88] *Id.* at 20.

[89] An anonymous reviewer suggests that Judge Posner's *Khan* opinion actually *undermines* **(DVP)**. The objection can be paraphrased as follows.

Posner says it's not cricket to distinguish by pointing to a fact the precedent court mentioned but gave no weight. This implies that it *would* be "cricket" to distinguish by pointing to a novel reason, a relevant fact favoring the otherwise losing side. That's exactly what the reason model says is required for distinguishing. If vertical precedent were strict, why would it matter that the fact was mentioned in the older opinion? Shouldn't Posner just have to follow the rule?

This strikes me as confused. State Oil *was* claiming—and Posner agreed—that its dispute with Khan presented a novel reason, not present in *Albrecht*, to find the contract enforceable. The reason, present in *Khan* but absent in *Albrecht*, was that the dispute was entirely "vertical," in the sense that it involved a single supplier and a single dealer. The dispute in *Albrecht*, by contrast, was more "horizontal," in the sense that it involved multiple dealers. So Posner's suggestion that it's not cricket to distinguish a vertical precedent by pointing to a fact the precedent court mentioned but gave no weight does *not* imply that it's permissible to distinguish a vertical precedent by pointing to a novel reason. It implies that it's permissible to distinguish a vertical precedent—in the *thin sense* of 'distinguish'—by pointing to a factor that *was given weight* by the precedent court, a factor whose presence or absence the precedent court *relied on* by incorporating into its *holding*. The holding in *Albrecht*, Posner tells us, was: "Maximum price fixing is illegal per se." Had the Court relied on the "horizontal flavor" of the dispute in finding an antitrust violation, then it might have held: "Maximum price fixing schemes involving multiple dealers [in some speci-

much of this—a single example can hardly be conclusive, and I do not take myself to have provided any real argument for **(Comp)**. If I have argued for anything in this brief section, it is only for the proposition that **(Comp)** is worth investigating, at least for those interested in building models capable of predicting the decisions of U.S. courts. I leave the investigation, and the predictive models, for another day.

6 Discretion and creativity in the lower courts

Before turning to the development of the hierarchical model, I would like to briefly address a possible concern. I have just argued that lower courts in the U.S. are more constrained than the standard model acknowledges. Where the standard model would allow an inferior court to distinguish an applicable vertical precedent, I argued in Sect. 4 that, in fact, the U.S. doctrine of precedent requires strict adherence to applicable vertical precedents. And I suggested in Sect. 5, without argument, that courts in the U.S. generally comply with this requirement. Plainly, then, I have left inferior courts with less to do. And one might worry that I have left them with too little to do. Surely, one might think, adjudication in the lower courts—indeed, adjudication that is *permissible* under the doctrine of precedent—is not exhausted by "the tidy deductive pattern of discovering an applicable rule or principle of law[,] subsuming the facts under that rule, and inferring the outcome," without recourse to social, economic, or moral considerations?[90]

My response is to fully acknowledge that adjudication, even in the lower courts, involves discretion, creativity, and moral judgment. I also acknowledge that, at least plausibly, much of that discretion, creativity, and moral judgment is *permitted* by the U.S. doctrine of precedent. But I maintain that all of this is consistent with **(DVP)** and **(Comp)**. Let me explain.

Apart from the obvious point that lower courts have a great deal of discretion in disputes to which no vertical precedent applies, recall that, in the course of stating

Footnote 89 (continued)
fied way] are illegal per se." If the Court had taken this tack, then State Oil could rightly have pointed out that *Albrecht* did not apply in *Khan*, since *Khan* did not involve multiple dealers.

The objection seems to assume that distinguishing a precedent involves either pointing to some fact about the precedent dispute or pointing to some fact about the instant dispute, but not both. This is a mistake. A precedent can be distinguished (in either the thick sense or the thin sense) only if there is some factual difference between the precedent dispute and the instant dispute. And to show that two things are *different*, one must say something about each of them. A member of a law school admissions committee does not differentiate two candidates, after all, merely by pointing out that one aced the LSAT. Rather, she must somehow indicate that the other did *not*. Similarly, to differentiate its dispute with Khan from the dispute in *Albrecht*, State Oil needed to say something about its dispute with Khan, and it needed to say something about the dispute in *Albrecht*. What it said was that its dispute with Khan was entirely "vertical," while the dispute in *Albrecht* was more "horizontal." Why did State Oil think that the dispute in *Albrecht* was more horizontal? Because the Supreme Court said so in its *Albrecht* opinion. "If vertical precedent were really strict," the objection asks, "then why would it matter that the *Albrecht* Court mentioned that the case involved multiple dealers?" But the very point that Judge Posner is making is that it *doesn't* matter. What matters, he tells us, is the *holding* in *Albrecht*, the *rule of the case*: Maximum price fixing is illegal per se. Since the contract in *Khan* fixed maximum prices, that was the ballgame.

[90] Leiter (2007a, 88). See also Shapiro (2011, chp. 8).

and formalizing the standard model, we made the simplifying assumption that, for any judicial decision, the *holding*—the *rule of the case*—is always clear. This assumption concerns which statements associated with a judicial opinion establish valid law that future courts are required to respect. After all, courts do not make law with every sentence they write. An opinion will often include an extended recital of the facts (some thought to be legally relevant, others merely thought to provide a more interesting or coherent story); historical remarks about how the relevant law reached its current state; predictions about how it might evolve in the future; more or less informed musings on economics, psychology, and sociology; and much else besides. Among the rules of our legal system are rules that tell judges what to make of such a document. These rules provide a set of authoritative *methods* for judges to apply to the opinions of precedent courts in order to determine the associated holdings.[91] By assuming that the rule of the case is always clear, we assume that these methods instantiate a mathematical *function* on the set of all possible judicial opinions, so that the competent application of the methods to any opinion will always produce a single determinate rule of the case. The standard model then takes up the question of what the doctrine of precedent requires *once the precedent rules have been identified*. And because my arguments in the last section are specifically intended to bear on the adequacy of the standard model, they too ignore the problem of identifying the holdings of precedent cases and of dealing with precedent cases whose holdings are genuinely indeterminate.

And, of course, such cases are common, for our assumption that there is always a determinate rule of the case is unrealistic. For example, a judicial opinion may contain several apparent statements of the holding, each one slightly different from the rest.[92] Which version of the rule provides the basis for the court's decision? Competent lawyers will often disagree. The opinion may instead be written, as Leiter (2009, 1233) reminds us, in the classic "shotgun" fashion: "canvass all possible arguments in support of a position, repeat them for emphasis, and present them all without any regard for how they actually hang together as a coherent, principled position." Which premise of which of these arguments is the holding? Or are they all holdings? Separately or somehow combined? It may be unclear. The court may instead, after an extended recital of the facts, say, "We hold that, *in these circumstances*," the plaintiff wins (or whatever). Which circumstances form the antecedent of the holding? Surely not *all* of them—not the identities of the parties, for instance. Which ones, in particular, then? In these ways and others, the holding of a given case may be difficult to identify and possibly underdetermined, even when all the authoritative methods have done their work.[93]

And when the holding of a precedent case is genuinely indeterminate, it seems plausible that the doctrine of precedent grants even lower courts discretion to

[91] Llewellyn (2011, 41) refers to these as "trade techniques."

[92] This phenomenon is noted by Llewellyn (2008, 44–45).

[93] There is also the possibility that the authoritative methods themselves conflict, thus producing rather than removing indeterminacy, a possibility advanced in Llewellyn (2008, 68–69), Llewellyn (1950), Llewellyn (1940, 210). For discussion, see Leiter (2007b, 73–79), Leiter (2005, 64).

"interpret" the case in whatever way seems best, within the range of indeterminacy left open by the higher court. And the lower court may, if application of the precedent rule to the present dispute seems unwise, read the precedent more narrowly than it might have, so that the rule does not reach the present dispute. If the court takes this tack, then it has legitimately distinguished a vertical precedent, in *some* sense of that term. But in what sense? Recall that there are at least two senses of 'distinguish' that are relevant in this context, a thin sense and a thick sense. In the thin sense, to distinguish a precedent is simply to determine that the associated rule does not apply to a new dispute. In the thick sense, to distinguish a precedent is to *modify* the associated rule, so that it no longer applies. Here, the court seems to have done more than distinguish the precedent in the thin sense, since it has not merely *discovered* that the precedent rule does not apply to the present dispute (although the court will usually say that it has). Yet it does not seem to have distinguished the precedent in the thick sense either, since it has not necessarily *narrowed the scope* of the rule handed down by the higher court. Instead, it has *resolved an ambiguity* in that court's formulation of the rule, in a way that is sensitive to considerations of good policy. We might quibble about how best to describe what's going on in this sort of example. The point, however, is that the kind of lower court creativity that it illustrates, although plausibly permitted by the U.S. doctrine of precedent, falls outside of any formal model that, like the standard model, assumes that the rule of the case is always clear.

Similarly, the process of characterizing a dispute in terms of legal factors falls outside the standard model. But this too provides the occasion for some legitimate (and plenty of illegitimate) exercises of lower court discretion. Suppose that a vertical precedent rule is couched in terms of a "reasonable" level of care, an "excessive" speed, or an "undue" burden. In these circumstances, unlike those discussed above, a lower court may have no trouble at all identifying the precedent rule. Yet it may still be unclear whether a particular dispute falls within its scope. Likewise in pedestrian cases of vagueness, as in the famous discussion in Hart (1994, 127) of the rule prohibiting vehicles in the park. (Do bicycles count? How about airplanes? Skate boards?) Where there is genuine indeterminacy, again, the lower court may legitimately exercise some discretion. But, again, this sort of lower court creativity falls outside of a formal model that *begins* with characterizations of disputes in terms of legal factors.

It's possible that some theorists were led to recognize a general power for courts to distinguish (in the thick sense) even vertical precedents, as the standard model does, by a reasonable desire to capture some of the creativity and discretion that we find legitimately exercised in the lower courts. But this is not the way to go about it. For, as we have seen, where the vertical precedent rule is clear, and it clearly applies to a new dispute, the U.S. doctrine of precedent requires the court to decide as the rule directs. If we want to capture these phenomena in a formal model, we must do so by placing *inside the model* the processes of extracting and interpreting precedent rules and characterizing disputes in terms of legal factors. I'm frankly not sure whether we should be trying to capture these phenomena in a model of *precedential*

constraint, as opposed, say, to a general model of *legal argument* or *legal reasoning*.[94] In any case, I make no attempt to do so here.

7 The strict rule model

In the remainder of the paper, I will develop a formal model of precedential constraint, the *hierarchical model*, that accommodates something like the U.S. doctrine of vertical precedent. The hierarchical model will be a *mixed model*, in the sense that different standards will apply to horizontal and vertical precedents. In particular, the familiar requirement articulated by the standard model (or something quite close) will apply to horizontal precedents, while the requirement articulated by the strict rule model will apply to vertical precedents. It will be helpful then, before turning to this more complicated mixed model, to address the simpler task of formalizing the strict rule model.

Recall that the strict rule model is committed to **(R)** and **(S)**.

(R) For any judicial decision, there is a rule of the case. Future courts must respect this rule in deciding the cases that come before them.

(S) A court faced with an applicable precedent rule that it is unwilling or unable to overrule must decide as the rule directs.

Here, then, is a first shot at formalizing the strict rule model.

> Let Γ be a case base and X a new fact situation before a court. Then the court must base its decision on a rule r leading to an outcome s such that there is no case $c' = \langle X', r', \bar{s}\rangle$ in Γ such that $X \vDash \mathit{Premise}(r')$.

This seems to be on the right track.[95] It says that if a precedent rule for the plaintiff applies to a new dispute before a court, then the court can't decide for the defendant. But there is still more work to do. For suppose that $\Gamma_3 = \{c_4, c_5\}$, where c_4 and c_5 are as follows.

$$
\begin{array}{ll}
c_4 = \langle X_4, r_4, s_4\rangle & c_5 = \langle X_5, r_5, s_5\rangle \\
X_4 = \{f_1^{\pi}, f_2^{\delta}\} & X_5 = \{f_2^{\pi}, f_1^{\delta}\} \\
r_4 = \{f_1^{\pi}\} \rightarrow \pi & r_5 = \{f_1^{\delta}\} \rightarrow \delta \\
s_4 = \pi & s_5 = \delta
\end{array}
$$

And now suppose that a court is presented with the dispute $X_6 = \{f_1^{\pi}, f_1^{\delta}\}$. According to our provisional formalization, the court must decide for the plaintiff on the basis of r_4, *and* the court must decide for the defendant on the basis of r_5. Clearly the court

[94] See, for example, Prakken and Sartor (2013), Wyner et al (2011), Bench-Capon and Sartor (2003), Bench-Capon (1999), Ashley (1989).

[95] I'm assuming a background requirement that $\langle X, r, s\rangle$ constitute a case, in the sense that $X \vDash \mathit{Premise}(r)$ and $\mathit{Conclusion}(r) = s$.

can't do both. Evidently, there is a problem for the strict rule model here, due to the possibility of a case base in which each case complies with the model but the case base as a whole seems to impose inconsistent demands on future courts.

This problem does not arise for the standard model. In particular, the standard model permits a court faced with X_6 in the context of Γ_3 to decide for either the plaintiff or the defendant. Neither decision is required. More generally, let $\Gamma = \{c_i, c_j\}$ be a case base, with $c_i = \langle X_i, r_i, \pi \rangle$ and $c_j = \langle X_j, r_j, \delta \rangle$, and let X_k be a dispute. And suppose that the standard model requires a court faced with X_k in the context of Γ (i) to decide for the plaintiff *and* (ii) to decide for the defendant. Then, from (i), we have $X_i^\pi \subseteq X_k^\pi$ and $X_k^\delta \subseteq X_i^\delta$. And, from (ii), we have $X_j^\delta \subseteq X_k^\delta$ and $X_k^\pi \subseteq X_j^\pi$. So we have $X_i^\pi \subseteq X_k^\pi \subseteq X_j^\pi$, which gives us $X_i^\pi \subseteq X_j^\pi$. And we have $X_j^\delta \subseteq X_k^\delta \subseteq X_i^\delta$, which gives us $X_j^\delta \subseteq X_i^\delta$. It follows that whichever case in Γ was decided second was decided in violation of the standard model. Thus, while the standard model permits *defeasible* rules to conflict, a series of cases decided in accordance with the model never produces conflicting (strict) *requirements* on future courts.

How can we ensure that this same result holds for the strict rule model? *A priori*, there are any number of possibilities. We might say, for instance, that a court faced with two precedents that conflict in the context of X must follow the one whose underlying facts are most similar to X. Alternatively, we might say that the court is free to decide on the basis of *either* rule,[96] or that the court is free to decide on any basis at all. We might say that, in cases of conflicting precedent, the court must decide for the defendant (or for the plaintiff).[97] Each of these options, suitably specified, would save the strict rule model from the sort of paralysis that afflicts our provisional formalization. So would many other possible revisions. Clearly we need some grounds for choosing one option over the rest.

Since we are aiming to describe the doctrine of U.S. courts, however, the appropriate criterion is clear: What standards do the courts actually cite in cases of conflict? As it happens, the answer depends on the hierarchical rank of the relevant courts.[98] For example, a trial court faced with two conflicting precedent rules—one issued by its court of appeals and the other by the supreme court in its jurisdiction—is required to simply follow the rule issued by the supreme court.[99] Likewise, if an appellate court determines that a precedent rule of its own conflicts with a supreme court precedent, it is required to follow the supreme court precedent.[100]

[96] Cf. Bergholtz and Peczenik (1997, 311) on conflicting precedents in Swedish law.

[97] We might say that the court is required to invoke something like the reason model of Horty (2015) as a tie-breaker. (Notice, though, that this would not help the court faced with X_6 in the context of Γ_3. So we would need further tie-breakers beyond the reason model.)

[98] See Garner et al. (2016, 300–307).

[99] See, for example, *United States v. Snyder,* 5 F. Supp. 3d 1258, 1262 (D. Or. 2014) ("When a Ninth Circuit decision becomes clearly irreconcilable with the reasoning or theory of intervening higher authority, this court must follow the higher authority.") (internal quotation marks omitted).

[100] *United States v. Madden*, 733 F.3d 1314, 1319 (11th Cir. 2013) (stating that "our prior precedent is no longer binding once it has been substantially undermined or overruled by Supreme Court jurisprudence") (internal quotation marks and ellipses omitted); *U.S. v. Singletary*, 268 F.3d 196 (3d Cir. 2001) (stating that "our respect for the uniformity of decisions within this Court yields when a prior panel's holding conflicts with a holding of the Supreme Court").

But what if both rules were issued by the *same* court? In that case, although hierarchical rank still matters, the crucial question is *when the precedents were decided*. If, for example, both precedent rules were issued by the same supreme court, then an inferior court, whether a trial court or an appellate court, is generally required to follow whichever rule was issued *more recently*, on the assumption that the supreme court, in deciding the later case as it did, implicitly limited or overruled the earlier one.[101] By contrast, an intermediate appellate court faced with conflicting *horizontal* precedents is generally required to follow the *earlier* decision.[102]

This rule is followed almost universally by the federal courts of appeals. To make sense of it, we need a bit more detail about how these courts are structured. Each appellate court has a number of active judges that participate in its work. But the court does not round them all up for every appeal that gets filed. Instead, most appeals are decided by a three-judge panel. And in nearly every circuit, a three-judge panel *lacks the authority to overrule* the holding of an earlier panel.[103] Indeed, in many circuits, a later panel lacks even the authority to *distinguish*—in the thick sense of the term—the holding of a prior panel.[104] In these jurisdictions, if a panel's holding is to be distinguished or overruled, this can only be done in an *en banc* session of the court, in which all of the active judges participate.[105] So the assumption that is taken to justify following the *later* of two conflicting supreme court precedents—namely, that the court intended to limit or overrule the earlier case when it decided the later one—does not apply. In this context, then, when two rules (issued by panels) come into conflict, it is the later rule that must give way, because a later panel is prohibited from encroaching upon the scope of a rule laid down by an earlier panel.

Let's return, now, to the task of formalizing the strict rule model. We've seen two *types* of standards employed by U.S. courts to resolve conflicts between precedent rules. The first appeals to the hierarchical rank of the courts that issued the rules. Although we will eventually want to incorporate this sort of standard in our

[101] See, for example, *Luhman v. Beecher*, 144 Wis.2d 781, 787–788 (1988) ("If decisions of the supreme court are inconsistent, we follow the court's practice of relying on the most recent pronouncement."). To be clear, the inferior court must determine that both precedent rules *directly apply* to the present dispute. If an earlier supreme court precedent directly applies, and a later supreme court precedent, while casting doubt on the reasoning of the earlier case, does not directly apply (and does not explicitly overrule the earlier holding), then an inferior court is generally required to follow the earlier case. See, for example, *U.S. v. Leija-Sanchez*, 602 F.3d 797, 799 (7th Cir. 2010) (Easterbrook C.J.).

[102] *United States v. Madden*, 733 F.3d 1314, 1319 (11th Cir. 2013) ("When we have conflicting case law, we follow our oldest precedent."); *Mader v. U.S.*, 654 F.3d 794, 800 (8th Cir. 2011); *Rios v. City of Del Rio, Tex.,* 444 F.3d 417, 427 (5th Cir. 2006); *McMellon v. U.S.*, 387 F.3d 329, 333 (4th Cir. 2004).

[103] *Theile v. Michigan*, 891 F.3d 240, 245 (6th Cir. 2018); *White v. Chafin*, 862 F.3d 1065, 1067 (10th Cir. 2017); *Jackson v. Ault*, 452 F.3d 734, 736 (8th Cir. 2006) (stating that "one panel may not overrule an earlier decision by another").

[104] See, for example, *Rios*, 444 F.3d at 427 ("No panel is empowered to hold that a prior decision applies only on the limited set of facts set forth in that opinion, and a prior panel's explication of the rules of law governing its holdings may not generally be disregarded as dictum.") (internal quotation marks and citations omitted). For discussion, see Garner et al. (2016, 37ff.).

[105] Actually, some circuit courts have so many active judges that *en banc* cases may be heard by a proper subset. But we can ignore such details.

hierarchical model, we plainly do not want to invoke it here, since we are aiming for a formalization of the strict rule model that, like the standard model, ignores judicial hierarchy and the distinction between horizontal and vertical precedent. The second type of standard appeals to the *age* of the precedent rules. This is the sort of standard that we'll incorporate into our formalization of the strict rule model. But *which standard* of this type should we incorporate? Since the plan is for the strict rule model to ignore the possibility of overruling and distinguishing, the standard governing most federal appellate court panels seems appropriate. So we will formalize a requirement according to which, in the case of conflicting rules, the older rule controls.

To incorporate this requirement into our model, we first need to add a temporal dimension to our definition of a case base, to form something we'll call an *ordered case base*.[106] An ordered case base is a pair $\mathbf{B} = \langle \Gamma, \prec \rangle$ such that Γ is a case base and $\prec$ is a linear ordering on Γ. We can then say that, for cases $c, c' \in \Gamma$, c precedes c' iff $c \prec c'$. And now we can use this notion of an ordered case base to define a process akin to refinement, which we'll call *correction*.

Definition 3 (*correction*) Where $\mathbf{B} = \langle \Gamma, \prec \rangle$ is an ordered case base, its correction is $(\mathbf{B})\downarrow = \langle \Gamma^*, \prec^* \rangle$, where Γ^* and $\prec^*$ are defined as follows.

(1) For each $c \in \Gamma$, we let $c\star = \langle X, r', s \rangle$, where $c' = \langle X', r', s' \rangle$ is the oldest case in

$$App_c = \{c'' = \langle X'', r'', s'' \rangle \in \Gamma : X \vDash Premise(r'')\}.$$

Then we let $\Gamma^* = \{c\star : c \in \Gamma\}$.

(2) $\prec^*$ is a linear ordering on Γ^* such that, for any $c_i\star, c_j\star \in \Gamma^*$, $c_i\star \prec^* c_j\star$ iff $c_i \prec c_j$.

The process of correction takes each case $c = \langle X, r, s \rangle$ in an ordered case base $\mathbf{B}$ and finds the oldest case $c' = \langle X', r', s' \rangle$ in $\mathbf{B}$ to announce a rule that applies in c. (There must be some such case, for c itself belongs to $\mathbf{B}$, and, since c is a case, r applies in X.) The correction process then replaces r with r'. Why? Because, assuming c' was decided before c, this was the rule that, according to the strict rule model, the c court was *required* to apply.[107] If the c court complied with this requirement, then r' will simply be r, and the substitution will make no change. If, however, $Conclusion(r') = s' = \bar{s}$, then the c court violated the requirements imposed by the strict rule model. Accordingly, the substitution of r' for r will produce something that is not a case, in our technical sense of the term, because $Conclusion(r') \neq s$.

This suggests the following formalization of the strict rule model.

[106] For more work in AI & Law that takes up, in one way or another, the temporal context of legal decisions, see, for example, Al-Abdulkarim et al (2016), Grabmair (2016), Governatori et al (2005), and Berman and Hafner (1995).

[107] If c' was not decided before c, then $c' = c$ and obviously the substitution is harmless.

Definition 4 (*precedential constraint: the strict rule model*) Let $\mathbf{B} = \langle \Gamma, \prec \rangle$ be an ordered case base and X a new fact situation before a court. Then the court must decide for a side s on the basis of a rule r such that $(\mathbf{B}')\downarrow$ is an ordered case base, where $\mathbf{B}' = \langle \Gamma \cup \{\langle X, r, s\rangle\}, \prec' \rangle$, letting $\prec'$ extend $\prec$ so that $c \prec' \langle X, r, s\rangle$ for every $c \in \Gamma$.

To be clear, I do not think that there is anything much to recommend this model as a *general* description of the U.S. doctrine of precedent. I have developed it here to introduce, in a simpler setting, some ideas that we will be adapting in the context of the hierarchical model, to which we now turn.

8 Building the hierarchical model

Our goal is to develop the standard model and the strict rule model to define a hierarchical model that captures the distinction between horizontal and vertical precedents in the U.S. doctrine. To begin, we'll need to add some additional structure to our representational framework. In particular, we will need to situate our cases within something we will call a *legal structure*.

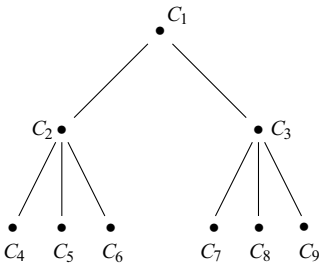
Definition 5 (*legal structure*) A legal structure is a pair $\mathbf{S} = \langle Courts, > \rangle$ such that

(1) *Courts* is a non-empty set $\{C_1, C_2, \ldots, C_n\}$ of courts;
(2) $>$ is a partial ordering on *Courts* that is transitive, irreflexive, and *treelike*, in the sense that, for any courts C_i, C_j, C_k in *Courts*, if $C_i > C_k$ and $C_j > C_k$, then $C_i > C_j$, $C_j > C_i$, or $C_i = C_j$; and
(3) *Courts* has a greatest element, i.e., there is some C^* in *Courts* such that $C^* > C$ for every C in $Courts - \{C^*\}$.

A legal structure $\mathbf{S} = \langle Courts, > \rangle$ represents the hierarchical organization of a legal system. Where $C_i > C_j$, we understand this to mean that C_i is superior to C_j in $\mathbf{S}$, in the sense that the holdings of C_i are *vertical precedents* for C_j, while the holdings of C_j are not precedents for C_i at all.

Notice that our definition of a legal structure does not require $>$ to be *connected*, so that for any distinct C_i, C_j in *Courts*, either $C_i > C_j$ or $C_j > C_i$. This makes sense. For consider Figure 1, where a downward edge from C_i to C_j indicates that $C_i > C_j$. In this legal structure, although C_3 is evidently an appellate court and C_4 only a trial court, C_3 is not *superior* to C_4 in the relevant sense, because $C_3 \not> C_4$. That is, C_3 does not create vertical precedents for C_4. Indeed, only C_2 and C_1 can create vertical precedents for C_4. This is precisely how the federal courts are organized in the U.S. A holding of the San Francisco-based Court of Appeals for the Ninth Circuit, for example, creates a binding vertical precedent for the Eastern District of California but not for the Northern District of Georgia, which is overseen by the Court of Appeals for the Eleventh Circuit. Accordingly, we should not rule out this sort of structure by definition.

Fig. 1 A possible legal structure



Now, in order to make use of these legal structures, we will need a new formal representation of a case, so that a case will indicate the court that decided it. So let's call the triples that we have been working with so far *triplet cases*. Then we can say that a case for a legal structure $\mathbf{S} = \langle Courts, > \rangle$ is a quadruple $c = \langle C, X, r, s \rangle$ such that $\langle X, r, s \rangle$ is a triplet case and $C \in Courts$. And let's assume functions *Court*, *Rule*, and *Outcome*, that, when given a case c as argument, return as value the court that decided c, the rule of c, and the winning side in c, respectively. Finally, a case base for **S** will be a set of cases for **S**, and an ordered case base for **S** will be a pair $\mathbf{B} = \langle \Gamma, \prec \rangle$ such that Γ is a case base for **S** and $\prec$ is a linear ordering on Γ.

With these resources, we can now straightforwardly define the notion of a vertical precedent and that of a horizontal precedent.

Definition 6 (*vertical precedent, horizontal precedent*) Let $\mathbf{B} = \langle \Gamma, \prec \rangle$ be an ordered case base for a legal structure $\mathbf{S} = \langle Courts, > \rangle$, and let c be a case decided against **B**.

- The set of vertical precedents for c in **B** is the set of all $c' \in \Gamma$ such that $Court(c') > Court(c)$.
- The set of horizontal precedents for c in **B** is the set of all $c' \in \Gamma$ such that $Court(c') = Court(c)$.

And let's assume functions *Horizontal* and *Vertical* that, when given as argument a triple consisting of a legal structure **S**, an ordered case base **B** for **S**, and a new case c decided against **B**, return as value the set of all horizontal precedents for c in **B** and the set of all vertical precedents for c in **B**, respectively. It will also be convenient to assume a couple more functions over these triples. First, a function *Inferior* that returns the set of all cases c' in **B** such that $Court(c) > Court(c')$. Second, a function *Disconnected* that returns the set of all cases c' in **B** such that $Court(c) \not> Court(c')$, $Court(c') \not> Court(c)$, and $Court(c) \neq Court(c')$. The first returns the set of all cases decided by courts that are inferior to the c court. The second returns the set of all cases decided by courts that are disconnected from the c court in the judicial hierarchy.

With these definitions in place, we turn to the task of developing the hierarchical model. What do we want it to require? Well, we want it to require courts to respect horizontal precedents, where this involves treating them roughly as the standard model requires, and to require courts to respect vertical precedents, where this involves

treating them roughly as the strict rule model requires. So imagine that we have a legal structure $\mathbf{S}_1 = \langle Courts_1, > \rangle$, where $Courts_1 = \{C_S, C_A, C_T\}$, with $C_S > C_A > C_T$. And suppose we have an ordered case base $\mathbf{B}_4 = \langle \Gamma_4, \prec_4 \rangle$ for $\mathbf{S}_1$, where $\Gamma_4 = \{c_6\}$.

$$
\begin{aligned}
c_6 &= \langle C_S, X_6, r_6, s_6 \rangle \\
X_6 &= \{f_1^\pi, f_1^\delta\} \\
r_6 &= \{f_1^\pi\} \rightarrow \pi \\
s_6 &= \pi
\end{aligned}
$$

And suppose now that a court in $Courts_1$ is faced with a dispute $\{f_1^\pi, f_2^\delta\}$. Can the court decide for the defendant on the basis of the rule $\{f_2^\delta\} \rightarrow \delta$? We want this to depend on which court is rendering the decision. If the deciding court is again C_S, so that c_6 is a *horizontal* precedent, then the model should allow the court to distinguish it. If, however, the court is an *inferior* court—either C_A or C_T—then the court is faced with a *vertical* precedent, and the model should require a decision for the plaintiff. Thus, in the context of $\mathbf{B}_4$, we want to permit c_7 and prohibit, for example, c_8.

$$
\begin{aligned}
c_7 &= \langle C_S, X_7, r_7, s_7 \rangle & \quad c_8 &= \langle C_A, X_8, r_8, s_8 \rangle \\
X_7 &= \{f_1^\pi, f_2^\delta\} & \quad X_8 &= \{f_1^\pi, f_2^\delta\} \\
r_7 &= \{f_2^\delta\} \rightarrow \delta & \quad r_8 &= \{f_2^\delta\} \rightarrow \delta \\
s_7 &= \delta & \quad s_8 &= \delta
\end{aligned}
$$

Suppose that C_S does decide c_7 in the context of $\mathbf{B}_4$. Then the basic idea is that we can develop the case base in accordance with the hierarchical model by simply adding c_7 and *refining* the result (with just a couple of tweaks to the refinement process to deal with the new structure we've added to our representations of cases and case bases). And in this instance, that would work just fine. We would end up with something like $\mathbf{B}'_4 = \langle \Gamma'_4, \prec'_4 \rangle$, where $\Gamma'_4 = \{c'_6, c_7\}$, with c'_6 like c_6 but with r_6 replaced by $r'_6 = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \rightarrow \pi$. But there are also case bases for which this will not work.

Suppose, for example, that c_{10} is decided against the background case base $\mathbf{B}_5 = \langle \Gamma_5, \prec_5 \rangle$ for $\mathbf{S}_1$, where $\Gamma_5 = \{c_9\}$.

$$
\begin{aligned}
c_9 &= \langle C_S, X_9, r_9, s_9 \rangle & \quad c_{10} &= \langle C_S, X_{10}, r_{10}, s_{10} \rangle \\
X_9 &= \{f_1^\pi, f_1^\delta\} & \quad X_{10} &= \{f_3^\pi, f_1^\delta\} \\
r_9 &= \{f_1^\delta\} \wedge \neg\{f_1^\pi, f_2^\pi\} \rightarrow \delta & \quad r_{10} &= \{f_3^\pi\} \rightarrow \pi \\
s_9 &= \delta & \quad s_{10} &= \pi
\end{aligned}
$$

Corresponding to the cases c_9 and c_{10} are the triplet cases $\mathfrak{c}_9 = \langle X_9, r_9, s_9 \rangle$ and $\mathfrak{c}_{10} = \langle X_{10}, r_{10}, s_{10} \rangle$. And corresponding to Γ_5 is $\Gamma'_5 = \{\mathfrak{c}_9\}$. Consider, now, the refinement of $\Gamma'_5 \cup \{\mathfrak{c}_{10}\}$.

$$\begin{aligned}
\text{Step 1:} \qquad \Gamma_{\mathfrak{c}_9} &= \{\mathfrak{c}_{10}\} \\
\Gamma_{\mathfrak{c}_{10}} &= \varnothing \\
\text{Step 2:} \qquad d_{\langle \mathfrak{c}_9, \mathfrak{c}_{10}\rangle} &= \neg\{f_3^{\pi}\} \\
\text{Step 3:} \qquad D_{\mathfrak{c}_9} &= \neg\{f_3^{\pi}\} \\
\text{Step 4:} \qquad \mathfrak{c}_9^* &= \langle X_9, r_9^*, s_9\rangle, \\
\text{where} \quad r_9^* &= \{f_1^{\delta}\} \wedge \neg\{f_3^{\pi}\} \rightarrow \delta \\
(\Gamma_5' \cup \{\mathfrak{c}_{10}\})^+ &= \{\mathfrak{c}_9^*, \mathfrak{c}_{10}\}
\end{aligned}$$

Now, one would expect

$$r_9 = \{f_1^{\delta}\} \wedge \neg\{f_1^{\pi}, f_2^{\pi}\} \rightarrow \delta,$$

upon refinement, to become

$$\{f_1^{\delta}\} \wedge \neg\{f_1^{\pi}, f_2^{\pi}\} \wedge \neg\{f_3^{\pi}\} \rightarrow \delta.$$

But that's not what we find here. Instead, we find that it has become

$$\{f_1^{\delta}\} \wedge \neg\{f_3^{\pi}\} \rightarrow \delta.$$

The process of refinement has evidently *discarded* the exception $\neg\{f_1^{\pi}, f_2^{\pi}\}$.

Now, one might think that this is a serious problem for the standard model regardless of whether or not vertical precedents are strict. In fact, though, if we suppose that the standard model is otherwise correct, then it makes no difference whether the refinement process discards exceptions. To see this, we can quickly define a formal model that is just like the standard model except for being built on a case base development procedure that does not discard exceptions. Where Γ is a case base, let Γ^e be the set that results from a procedure just like refinement, except that, in the final step, the case $c = \langle X, r, s\rangle$ from Γ is replaced with $c' = \langle X, r', s\rangle$, where r' is *not*

$$Premise^s(r) \wedge D_c \rightarrow s$$

but rather

$$Premise(r) \wedge D_c \rightarrow s.$$

Now let's define precedential constraint on the SM^e model as follows. Where Γ is a case base and X a new fact situation confronting a court, then the court must decide X on the basis of a rule r leading to an outcome s such that $(\Gamma \cup \langle X, r, s\rangle)^e$ is a case base. Given these definitions, SM^e is exactly like the standard model except for being defined on the basis of a procedure that retains exceptions. But now it is straightforward to prove that the standard model and SM^e are equivalent, in the sense that, for any case base Γ and case c, a court that decides c in the context of Γ complies with

the standard model iff it complies with SM^e. This follows directly from the observation that, for any case base Γ, Γ^+ is a case base iff Γ^e is a case base.

The standard model aside, however, the hierarchical model plainly must not discard exceptions. For compare the cases c_9 and c'_9, again in the context of legal structure $\mathbf{S}_1$.

$$\begin{aligned} c_9 &= \langle C_S, X_9, r_9, s_9\rangle & c'_9 &= \langle C_S, X_9, r'_9, s_9\rangle \\ X_9 &= \{f_1^\pi, f_1^\delta\} & X_9 &= \{f_1^\pi, f_1^\delta\} \\ r_9 &= \{f_1^\delta\} \wedge \neg\{f_1^\pi, f_2^\pi\} \rightarrow \delta & r'_9 &= \{f_1^\delta\} \rightarrow \delta \\ s_9 &= \delta & s_9 &= \delta \end{aligned}$$

And imagine that, with either c_9 or c'_9 in the background, the lowest court in the system, C_T, decides the following case.

$$\begin{aligned} c_{11} &= \langle C_T, X_{11}, r_{11}, s_{11}\rangle \\ X_{11} &= \{f_1^\pi, f_2^\pi, f_3^\pi, f_1^\delta, f_2^\delta\} \\ r_{11} &= \{f_1^\pi, f_2^\pi, f_3^\pi\} \rightarrow \pi \\ s_{11} &= \pi \end{aligned}$$

If the background case base is $\{c_9\}$, then we want the hierarchical model to hold that the C_T court has behaved permissibly, for the rule r_9 simply doesn't apply to X_{11}. By contrast, if the background case base is $\{c'_9\}$, then r'_9 *does* apply, and the model should hold that C_T, as an inferior court, was required to decide accordingly. The upshot is that the process that we want to incorporate into the hierarchical model is not actually *refinement* but rather a variation that ensures that r_9 retains the $\neg\{f_1^\pi, f_2^\pi\}$ clause when the case base is developed.

We also make a second change to the process. Refinement, in addition to ignoring hierarchical structure, also ignores *temporal* structure, so that, in effect, an *earlier* case can distinguish a *later* case. We will instead define a process that is to be applied at each successive stage in the development of an ordered case base, capturing the changes made when a new decision distinguishes past horizontal precedents. This process, which we'll call *refinement*†, is defined as follows.

Definition 7 (*refinement*†) Let $\mathbf{B} = \langle \Gamma, \prec\rangle$ be an ordered case base for a legal structure $\mathbf{S} = \langle Courts, >\rangle$ and $c = \langle C, X, r, s\rangle$ a case decided against $\mathbf{B}$. Let H be the set $Horizontal(\mathbf{S}, \mathbf{B}, c)$. Then the refinement of H in light of c is the set $(H)\dagger_c$, which we define by the following procedure.

(1) Let $Con = \{c_i = \langle C_i, X_i, r_i, \bar{s}\rangle \in H : X \vDash Premise(r_i)\}$.
(2) $\forall c_i = \langle C_i, X_i, r_i, \bar{s}\rangle \in Con$:

- Let $D_{c_i} = \neg Premise^s(r)$.
- Let $c_i^\star = \langle C_i, X_i, r_i^\star, \bar{s}\rangle$, where $r_i^\star = Premise(r_i) \wedge D_{c_i} \rightarrow \bar{s}$.

(3) And let $Con^{\star} = \{c_i^{\star} : c_i \in Con\}$.
(4) Then $(H)\dagger_c = (H - Con) \cup Con^{\star} \cup \{c\}$.

The process of *refining*† H in light of a new case c decided for s involves finding every case c' in H decided for $\bar{s}$ such that the rule of c' applies in c. The c court, in deciding for s despite the applicability of the rule of c', must have *distinguished* c'. Accordingly, the rule of c' is modified by adding a corresponding exception. Once this is done for every applicable precedent decided for $\bar{s}$ in H, the cases—now with their modified rules—are collected together with c in a new set, and the *refinement*† of H in light of c is complete.

We avoid the problem of discarding exceptions by defining $r_i^{\star}$, in step (2) of the *refinement*† process, not as

$$Premise^{s}(r_i) \wedge D_{c_i} \rightarrow \bar{s}$$

but as

$$Premise(r_i) \wedge D_{c_i} \rightarrow \bar{s}.$$

And we avoid the problem of ignoring temporal structure, so that an earlier case can effectively distinguish a later case, by defining *refinement*† as a function over ordered pairs $\langle H, c\rangle$ consisting of a case base and a case, where the members of H are treated as past decisions, and c is treated as a new decision. As a result, the *refinement*† of H in light of c can't result in the modification of c but only in the modification of the members of H. This will be the process that we use to capture the doctrine of horizontal precedent in our hierarchical model.

Let's turn now to vertical precedents. The plan is for the hierarchical model to apply, in the case of vertical precedents, something like the standard we developed for the strict rule model. When we formalized that model, we saw that we needed to deal with the situation in which multiple rules apply to a dispute and direct the court to decide in different ways. Since that model was entirely flat, however, we could not resolve the conflict by appealing to the rank of the precedent courts. In the hierarchical model, this is no longer the case. So suppose, for example, that we have the background case base $\mathbf{B}_6 = \langle \Gamma_6, \prec_6\rangle$ for $\mathbf{S}_1$, where $\Gamma_6 = \{c_{12}, c_{13}\}$, with c_{12} and c_{13} as follows.

$$\begin{array}{ll} c_{12} = \langle C_S, X_{12}, r_{12}, s_{12}\rangle & c_{13} = \langle C_A, X_{13}, r_{13}, s_{13}\rangle \\ X_{12} = \{f_1^{\pi}, f_1^{\delta}\} & X_{13} = \{f_2^{\pi}, f_2^{\delta}\} \\ r_{12} = \{f_1^{\pi}\} \rightarrow \pi & r_{13} = \{f_2^{\delta}\} \rightarrow \delta \\ s_{12} = \pi & s_{13} = \delta \end{array}$$

And suppose that the C_T court is now faced with the dispute $X_{14} = \{f_1^{\pi}, f_2^{\delta}\}$. Both c_{12} and c_{13} represent vertical precedents for C_T, since $C_S > C_T$ and $C_A > C_T$. And both r_{12} and r_{13} *apply* to the dispute X_{14}. But r_{12} directs the court to decide for the plaintiff, while r_{13} directs the court to decide for the defendant. What, then, should the hierarchical model require the court to do? Taking its cue from the U.S. courts, the hierarchical model should require C_T to follow r_{12} because the court that issued r_{12} is

superior to the court that issued r_{13}—because, in our formalism, $C_S > C_A$. Thus, the model should require the court to decide as in c_{14}.

$$
\begin{aligned}
c_{14} &= \langle C_T, X_{14}, r_{14}, s_{14} \rangle \\
X_{14} &= \{f_1^{\pi}, f_2^{\delta}\} \\
r_{14} &= \{f_1^{\pi}\} \rightarrow \pi \\
s_{14} &= \pi
\end{aligned}
$$

What, though, if the conflicting rules were issued by the same superior court? In our formalization of the strict rule model, because we were ignoring the possibility of overruling and distinguishing, we adopted the standard applied by most three-judge panels in the federal courts of appeals to the decisions of prior panels, which privileges the *oldest* of the conflicting rules. In the hierarchical model, however, since we are allowing courts to distinguish earlier horizontal precedents, it is perhaps more appropriate to adopt the standard that inferior courts in the U.S. apply to conflicting rules issued by the supreme court in the jurisdiction, which privileges the *most recent* of the conflicting rules. This is admittedly somewhat arbitrary. A formal model of precedential constraint that fully captures the U.S. doctrine of precedent will need to invoke both standards in different circumstances. It is only in the interest of simplicity that we limit ourselves to a single standard here.

To capture these standards in our formal model, we need to define a variation on our notion of *correction*. In preparation for that definition, let's say that the *top case* in an ordered case base $\mathbf{B} = \langle \Gamma, \prec \rangle$ for a legal structure $\mathbf{S} = \langle Courts, > \rangle$, if it exists, is the case c in Γ such that (i) $Court(c) \geq Court(c')$ for every case c' in Γ, and (ii) $c' \prec c$ for every case c' other than c in Γ that was decided by $Court(c)$. In other words, the top case in an ordered case base, if it exists, is the most recent case decided by the highest court to have decided any case at all. With this, we are prepared to define our variation on correction—call it *correction*$^{\Downarrow}$—as follows.

Definition 8 (*correction*$^{\Downarrow}$) Let $\mathbf{B} = \langle \Gamma, \prec \rangle$ be an ordered case base for a legal structure $\mathbf{S} = \langle Courts, > \rangle$, and let $c = \langle C, X, r, s \rangle$ be a case decided in the context of $\mathbf{B}$. Let App_V be the set of all vertical precedents for c in $\mathbf{B}$ that are applicable in X. Let $c' = \langle C', X', r', s' \rangle$ be the top case in $App_V \cup \{c\}$. Then the correction of c by $\mathbf{B}$ is $(c)\Downarrow \mathbf{B} = \langle C, X, r', s \rangle$.

We know that there must be a top case in $App_V \cup \{c\}$, because every case in that set is either a vertical precedent for c or c itself. And, by the definition of a legal structure, $\mathbf{S}$ must be *treelike*, in the sense that, for any courts C_i, C_j, C_k in *Courts*, if $C_i > C_k$ and $C_j > C_k$, then $C_i > C_j$, $C_j > C_i$, or $C_i = C_j$.

Notice that, in addition to capturing the standards discussed above, correction$^{\Downarrow}$ differs from correction by accepting as argument not an ordered case base but, in effect, a triple consisting of a legal structure, an ordered case base, and a new case,

and that it yields, as value, not an ordered case base but a *case*.[108] The process of correction$^{\Downarrow}$, given a case c and an ordered case base $\mathbf{B}$ (for a legal structure), checks to see whether there are any vertical precedents for c in $\mathbf{B}$. If there *are*, then it takes the *top* vertical precedent for c in $\mathbf{B}$—that is, the most recent vertical precedent for c decided by the highest court to have decided any vertical precedent for c at all—and "corrects" c by replacing the rule of c with the rule of its top vertical precedent. This, of course, is the rule that the c court was required to apply in the first place. So, if the court behaved appropriately, this substitution will leave c just as it was. Similarly, if there are no vertical precedents for c in $\mathbf{B}$, then correction$^{\Downarrow}$ will leave c alone. If, however, there are vertical precedents for c in $\mathbf{B}$, and although the top vertical precedent for c in $\mathbf{B}$ directed the c court to decide for s, the court decided for $\bar{s}$, then correction$^{\Downarrow}$ will return a "case" whose rule does not support its outcome. And this is not a case at all, in our technical sense.

We are going to build the hierarchical model on the basis of a process called *consolidation* that incorporates both correction$^{\Downarrow}$ and refinement†. When given a new case to add to an ordered case base, correction$^{\Downarrow}$ effectively looks *up*, to see if there are any vertical precedents that bear on the new decision. Refinement†, on the other hand, looks to the *side*, to see if there are any horizontal precedents that need to be taken into account, and, if there are, to determine the effects of the new case on those past decisions. We may also, however, wish to look *down*. Not, to be sure, because the past decisions of inferior courts constrain the present decisions of superior courts but, rather, because a new superior court decision may have some effect on the state of the law in inferior courts. In particular, a new decision may *overrule* a case decided by an inferior court. The definition is straightforward.

Definition 9 (*overruled*) Let $\mathbf{B} = \langle \Gamma, \prec \rangle$ be an ordered case base for a legal structure $\mathbf{S} = \langle Courts, > \rangle$ and $c = \langle C, X, r, s \rangle$ a case decided against $\mathbf{B}$. Then for any $c' = \langle C', X', r', s' \rangle \in \Gamma$, c' is overruled by c iff:

(1) $C > C'$;
(2) $X' \vDash Premise(r)$; and
(3) $s' \neq Conclusion(r)$.

This says that a new case c decided for s overrules all earlier cases in which an inferior court decided for $\bar{s}$ a dispute to which the rule of c applies. Let's also assume a function *Overruled* that, when given a triple consisting of a legal structure $\mathbf{S}$, an ordered case base $\mathbf{B}$ for $\mathbf{S}$, and a new case c decided against $\mathbf{B}$, returns the set of all cases in $\mathbf{B}$ overruled by c. Our plan will be for the process of consolidation to simply eliminate cases that are overruled from its developing representation of the current state of the law.

We are finally ready to define the process of consolidation.

[108] Or, at any rate, a quadruple consisting of a court, a set of facts, a rule, and a side. Whenever a court has failed to follow binding precedent, this quadruple will fail to be a case in our technical sense.

Definition 10 (*consolidation*) Let $\mathbf{B} = \langle \Gamma, \prec \rangle$ be an ordered case base for a legal structure $\mathbf{S} = \langle Courts, > \rangle$. Then Γ is a set of n precedent cases for some finite $n > 0$. So let $\mathbf{c}_1 = \langle \mathbf{C}_1, \mathbf{X}_1, \mathbf{r}_1, \mathbf{s}_1 \rangle$ be the oldest case in Γ, $\mathbf{c}_2 = \langle \mathbf{C}_2, \mathbf{X}_2, \mathbf{r}_2, \mathbf{s}_2 \rangle$ the second oldest, and so on, up to $\mathbf{c}_n = \langle \mathbf{C}_n, \mathbf{X}_n, \mathbf{r}_n, \mathbf{s}_n \rangle$. Then the consolidation of $\mathbf{B}$ is the pair $\mathbf{B}! = \langle \Gamma^n, \prec^n \rangle$, where Γ^n and $\prec^n$ are defined recursively as follows.

(1) $\mathbf{B}^1 = \langle \Gamma^1, \prec^1 \rangle$, where $\Gamma^1 = \{\mathbf{c}_1\}$ and $\prec^1 = \varnothing$.
(2) For all i such that $1 \leq i < n$, $\mathbf{B}^{i+1} = \langle \Gamma^{i+1}, \prec^{i+1} \rangle$, where

(a) $\Gamma^{i+1} = Vertical_{i+1} \cup (Inferior_{i+1} - Overruled_{i+1})$
$\cup\, (Horizontal_{i+1})\dagger_{(\mathbf{c}_{i+1})\Downarrow \mathbf{B}^i} \cup Disconnected_{i+1}$,
letting f_{i+1} abbreviate $f(\mathbf{S}, \mathbf{B}^i, (\mathbf{c}_{i+1})\Downarrow_{\mathbf{B}^i})$ for any function f; and
(b) $\prec^{i+1}$ is the linear ordering on Γ^{i+1} that meets the following conditions.

(i) For every $c, c' \in \Gamma^i \cap \Gamma^{i+1}$, $c \prec^{i+1} c'$ iff $c \prec^i c'$;
(ii) For every $c_m^\star, c_n^\star \in (Horizontal_{i+1})\dagger_{(\mathbf{c}_{i+1})\Downarrow \mathbf{B}^i} - (\Gamma^i \cup \{\mathbf{c}_{i+1}\})$, and every $c \in \Gamma^i \cap \Gamma^{i+1}$,
– $c_m^\star \prec^{i+1} c_n^\star$ iff $c_m \prec^i c_n$,
– $c_m^\star \prec^{i+1} c$ iff $c_m \prec^i c$,
– $c \prec^{i+1} c_m^\star$ iff $c \prec^i c_m$; and
(iii) For every $c \in \Gamma^{i+1} - \{\mathbf{c}_{i+1}\}$, $c \prec^{i+1} \mathbf{c}_{i+1}$.

Where $\mathbf{B}$ is an ordered case base, the consolidation of $\mathbf{B}$ represents the state of the law after the final case in $\mathbf{B}$ is decided. And the process of consolidation builds up to that representation, starting with a representation $\mathbf{B}^1 = \langle \Gamma^1, \prec^1 \rangle$ of the state of the law just after the earliest case $\mathbf{c}_1$ in $\mathbf{B}$ was decided. We might think of an even earlier representation in which the "case base" is simply the empty set. The point is just that adding $\mathbf{c}_1$ to that set is entirely straightforward, since there are no vertical precedents for the court to follow, no horizontal precedents for it to distinguish, and no decisions of inferior courts to overrule.[109] Next, the consolidation process uses $\mathbf{B}^1$ and the second earliest case $\mathbf{c}_2$ in $\mathbf{B}$ to create a representation $\mathbf{B}^2$ of the state of the law just after $\mathbf{c}_2$ was decided. Then it uses $\mathbf{B}^2$ and the third earliest case $\mathbf{c}_3$ to create $\mathbf{B}^3$, and so on.

As the definition specifies, we have $\mathbf{B}^{i+1} = \langle \Gamma^{i+1}, \prec^{i+1} \rangle$, where

$$\begin{aligned} \Gamma^{i+1} = & Vertical_{i+1} \cup (Inferior_{i+1} - Overruled_{i+1}) \\ & \cup\, (Horizontal_{i+1})\dagger_{(\mathbf{c}_{i+1})\Downarrow \mathbf{B}^i} \cup Disconnected_{i+1}, \end{aligned}$$

[109] There may, of course, be other sorts of constraints placed on the $\mathbf{c}_1$ court—by constitutional provisions or legislation, for example. The point is just that there are no *precedential* constraints placed on the $\mathbf{c}_1$ court.

using f_{i+1} to abbreviate $f(\mathbf{S}, \mathbf{B}^i, (\mathbf{c}_{i+1})\Downarrow \mathbf{B}^i)$ for any function f. Let's approach this definition of Γ^{i+1} one piece at a time. First, Γ^{i+1} includes $Vertical_{i+1}$, which is the set of all vertical precedents for $\mathbf{c}_{i+1}$ in Γ^i. It makes sense to include in Γ^{i+1} all of the vertical precedents for $\mathbf{c}_{i+1}$ in Γ^i unchanged, because $\mathbf{c}_{i+1}$ can't distinguish or overrule a precedent issued by a superior court. Likewise, $\mathbf{c}_{i+1}$ can't distinguish or overrule a precedent issued by a distinct court to which the $\mathbf{c}_{i+1}$ court is neither inferior nor superior. (This would be akin to a Ninth Circuit holding changing the law in the Eleventh Circuit.) Accordingly, we find that the cases in $Disconnected_{i+1}$ are included in Γ^{i+1} unchanged as well.

What about the cases in Γ^i decided by courts *inferior* to the $\mathbf{c}_{i+1}$ court? These are dealt with by the $Inferior_{i+1} - Overruled_{i+1}$ clause of the definition, and the basic idea is simply to retain in Γ^{i+1} those inferior court cases in Γ^i that were not overruled by $\mathbf{c}_{i+1}$. Notice, however, that in our equation defining Γ^{i+1}, we are using $Overruled_{i+1}$ *not* to abbreviate $Overruled(\mathbf{S}, \mathbf{B}^i, \mathbf{c}_{i+1})$, which is the set of all the cases in Γ^i overruled by $\mathbf{c}_{i+1}$, but to abbreviate $Overruled(\mathbf{S}, \mathbf{B}^i, (\mathbf{c}_{i+1})\Downarrow \mathbf{B}^i)$, which is the set of all the cases in Γ^i overruled by the $correction^{\Downarrow}$ of $\mathbf{c}_{i+1}$ by $\mathbf{B}^i$. This is solely in the interest of brevity. For $\mathbf{c}_{i+1}$ will overrule an inferior court case only if there are no applicable vertical precedents for $\mathbf{c}_{i+1}$ in Γ^i. And if there are no applicable vertical precedents for $\mathbf{c}_{i+1}$ in Γ^i, then the correction$^{\Downarrow}$ of $\mathbf{c}_{i+1}$ by $\mathbf{B}^i$ is just $\mathbf{c}_{i+1}$. If, on the other hand, there *are* applicable vertical precedents for $\mathbf{c}_{i+1}$ in Γ^i, then, while $(\mathbf{c}_{i+1})\Downarrow \mathbf{B}_i$ may or may not be $\mathbf{c}_{i+1}$, the set $Overruled(\mathbf{S}, \mathbf{B}^i, (\mathbf{c}_{i+1})\Downarrow \mathbf{B}_i)$ will in any case be empty. For any inferior case that the $\mathbf{c}_{i+1}$ court might have overruled by issuing the rule of its top vertical precedent will already have been overruled when that same rule was issued by the superior court in an earlier stage of the consolidation process.

What of horizontal precedents? Here we simply take the horizontal precedents for $\mathbf{c}_{i+1}$ in Γ^i and refine† them in light of $\mathbf{c}_{i+1}$.[110] This involves adding appropriate exceptions to any horizontal precedents that the $\mathbf{c}_{i+1}$ court distinguished, and then adding $\mathbf{c}_{i+1}$ to the collection. When, finally, we take the union of all these sets—the set of all vertical precedents, the set of all inferior cases that have not been overruled, the refinement† of the set of all horizontal precedents, and the set of all cases decided by courts that are unrelated in the judicial hierarchy—we arrive at Γ^{i+1}. As for $\prec^{i+1}$, it is essentially just like $\prec^i$, except that $\prec^{i+1}$ records that $\mathbf{c}_{i+1}$ was decided more recently than any other case in the case base.

To illustrate, suppose that we again have the background legal structure $\mathbf{S}_1 = \langle Courts_1, > \rangle$, where $Courts_1 = \{C_S, C_A, C_T\}$, with $C_S > C_A > C_T$. And suppose that we have an ordered case base $\mathbf{B}_7 = \langle \Gamma_7, \prec_7 \rangle$ for $\mathbf{S}_1$, where $\Gamma_7 = \{c_{15}, c_{16}\}$, and $c_{15} \prec_7 c_{16}$.

$$
\begin{array}{ll}
c_{15} = \langle C_A, X_{15}, r_{15}, s_{15} \rangle & c_{16} = \langle C_A, X_{16}, r_{16}, s_{16} \rangle \\
X_{15} = \{f_1^{\pi}, f_1^{\delta}\} & X_{16} = \{f_1^{\pi}, f_2^{\delta}\} \\
r_{15} = \{f_1^{\pi}\} \rightarrow \pi & r_{16} = \{f_2^{\delta}\} \rightarrow \delta \\
s_{15} = \pi & s_{16} = \delta
\end{array}
$$

[110] Again, the definition actually uses the $correction^{\Downarrow}$ of $\mathbf{c}_{i+1}$ to refine this set, but, as we've seen, if either (i) there were no applicable vertical precedents for $\mathbf{c}_{i+1}$ in Γ^i, or (ii) there *were* applicable vertical precedents for $\mathbf{c}_{i+1}$ in Γ^i, but the $\mathbf{c}_{i+1}$ court applied the top applicable precedent rule and decided accordingly, then the $correction^{\Downarrow}$ of $\mathbf{c}_{i+1}$ is simply $\mathbf{c}_{i+1}$.

What, then, does the consolidation of $\mathbf{B}_7$ produce? Well, $\mathbf{B}_7! = \langle \Gamma_7^2, \prec_7^2 \rangle$, where Γ_7^2 and $\prec_7^2$ are calculated as in the definition. To begin, $\mathbf{B}_7^1 = \langle \Gamma_7^1, \prec_7^1 \rangle$, with $\Gamma_7^1 = \{c_{15}\}$, since c_{15} is the oldest decision in Γ_7. (Of course, $\prec_7^1 = \varnothing$.) Now we use $\mathbf{B}_7^1$ and c_{16} to calculate $\mathbf{B}_7^2$. Since there are no vertical precedents for c_{16} in $\mathbf{B}_7^1$, no inferior court decisions for c_{16} in $\mathbf{B}_7^1$, and no cases in $\mathbf{B}_7^1$ decided by courts not connected to C_A in the judicial hierarchy of $\mathbf{S}_1$, $Vertical_2$, $Inferior_2$, and $Disconnected_2$ are all empty. So

$$\Gamma_7^2 = (Horizontal_2)\dagger_{(c_{16})\Downarrow \mathbf{B}_7^1}.$$

$Horizontal_2$ is just $\{c_{15}\}$, of course, and, since there are no vertical precedents for c_{16} in $\mathbf{B}_7^1$, $(c_{16})\Downarrow \mathbf{B}_7^1 = c_{16}$. So Γ_7^2 is just the refinement† of $\{c_{15}\}$ in light of c_{16}. So $\Gamma_7^2 = \{c'_{15}, c_{16}\}$, where $c'_{15} = \langle C_A, X_{15}, r'_{15}, \pi \rangle$, with $r'_{15} = \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \rightarrow \pi$. As for $\prec_7^2$, this is determined entirely by the final clause in the definition, which, applied to this example, states that $c \prec_7^2 c_{16}$ for every $c \in \Gamma_7^2 - \{c_{16}\}$. The only case in $\Gamma_7^2 - \{c_{16}\}$ is c'_{15}. Accordingly, we have $c'_{15} \prec_7^2 c_{16}$. In this scenario, the c_{16} court, in the face of an applicable horizontal precedent rule r_{15} issued in c_{15}, decided to *distinguish* c_{15}. Accordingly, the rule r_{15} was modified to include an exception so that it no longer applied in c_{16}. The case base was then updated to reflect this modification, as well as to include the new case c_{16}. This is all as it should be.

It is also nothing new. Since every case in the ordered case base $\mathbf{B}_7$ was decided by C_A, only horizontal precedents were involved. And while it is important that consolidation deal with horizontal precedents appropriately, the process was built to handle vertical precedents. So let's add some of those to the mix. Suppose that, in the context of $\mathbf{B}_7$, the following two cases are decided in turn.

$$\begin{array}{ll} c_{17} = \langle C_T, X_{17}, r_{17}, s_{17} \rangle & c_{18} = \langle C_S, X_{18}, r_{18}, s_{18} \rangle \\ X_{17} = \{f_2^\pi, f_3^\pi, f_1^\delta, f_2^\delta\} & X_{18} = \{f_3^\pi, f_4^\pi, f_2^\delta, f_3^\delta\} \\ r_{17} = \{f_2^\delta\} \rightarrow \delta & r_{18} = \{f_3^\pi\} \rightarrow \pi \\ s_{17} = \delta & s_{18} = \pi \end{array}$$

This gives us the ordered case base $\mathbf{B}_8 = \langle \Gamma_8, \prec_8 \rangle$ for $\mathbf{S}_1$, where Γ_8 contains c_{15}, c_{16}, c_{17}, and c_{18}, and $c_{15} \prec_8 c_{16} \prec_8 c_{17} \prec_8 c_{18}$. What, then, does the consolidation of $\mathbf{B}_8$ give us? Well, $\mathbf{B}_8! = \mathbf{B}_8^4 = \langle \Gamma_8^4, \prec_8^4 \rangle$, where Γ_8^4 and $\prec_8^4$ are calculated in the usual way. Plainly, $\mathbf{B}_8^2 = \mathbf{B}_7^2$. And since we have already calculated $\mathbf{B}_7^2$, we can start there. We now use $\mathbf{B}_8^2$ and c_{17} to calculate $\mathbf{B}_8^3$.

First, what are the vertical precedents for c_{17} in $\mathbf{B}_8^2$? Well, both c'_{15} and c_{16} were decided by C_A, and $C_A > C_T$. So both of these cases represent vertical precedents for c_{17}, and $Vertical_3 = \{c'_{15}, c_{16}\}$. Since there are no cases in $\mathbf{B}_8^2$ decided by courts that are *inferior* to C_T, the set $Inferior_3 - Overruled_3$ is empty. Likewise the set $Disconnected_3$. What about horizontal precedents for c_{17} in $\mathbf{B}_8^2$? Clearly, there are none. Does this mean that $(Horizontal_3)\dagger_{(c_{17})\Downarrow \mathbf{B}_8^2}$ is empty? No. It does mean that $Horizontal_3$ is empty. But we still must find the refinement† of the empty set in light of the correction$^\Downarrow$ of c_{17} by $\mathbf{B}_8^2$. The correction$^\Downarrow$ process tells us to replace r_{17} with the rule of the top vertical precedent for c_{17} in $\mathbf{B}_8^2$ whose rule applies in X_{17}. Since

the top vertical precedent is c_{16}, we replace r_{17} with r_{16}. These, of course, are the same, which suggests that the c_{17} court complied with the requirements of the hierarchical model. So $(c_{17})\Downarrow \mathbf{B}_8^2 = c_{17}$. And the refinement† of $Horizontal_3 = \varnothing$ in light of c_{17} is simply $\{c_{17}\}$. Thus, we have $\mathbf{B}_8^3 = \langle \Gamma_8^3, \prec_8^3\rangle$, where $\Gamma_8^3 = \{c'_{15}, c_{16}, c_{17}\}$, and $c'_{15} \prec_8^3 c_{16} \prec_8^3 c_{17}$.

Now, in the final step of the consolidation process, we use $\mathbf{B}_8^3$ and c_{18} to calculate $\mathbf{B}_8^4$. Since there are no vertical precedents for c_{18} in $\mathbf{B}_8^3$—after all, no court is superior to C_S—$Vertical_4$ is empty. Same with $Disconnected_4$. Since there are no horizontal precedents for c_{18} in $\mathbf{B}_8^3$, $Horizontal_4$ is also empty, and, as we've seen, this means that we have $(Horizontal_4)\dagger_{(c_{18})\Downarrow \mathbf{B}_8^3} = \{c_{18}\}$. Since every case in $\mathbf{B}_8^3$ was decided by a court inferior to C_S, we have $Inferior_4 = \{c'_{15}, c_{16}, c_{17}\}$. What about $Overruled_4$? Any case c in $Inferior_4$ is overruled by c_{18} just in case r_{18} applies in c and c was decided for the defendant. Since c_{17} fits that description, c_{17} is overruled. Since *only* c_{17} fits that description, we have $Overruled_4 = \{c_{17}\}$. Accordingly, $Inferior_4 - Overruled_4 = \{c'_{15}, c_{16}\}$. Thus, we have $\mathbf{B}_8^4 = \langle \Gamma_8^4, \prec_8^4\rangle$, where $\Gamma_8^4 = \{c'_{15}, c_{16}, c_{18}\}$, and $c'_{15} \prec_8^4 c_{16} \prec_8^4 c_{18}$. And this gives us the consolidation of $\mathbf{B}_8$.

Notice that, although c_{17} is intuitively inconsistent with c_{18}, this did not lead the consolidation process to produce something that is not a case base. This is because we want the consolidation process to give us something that is not a case base only when a court violates the doctrine of precedent imposed by the hierarchical model. And the c_{17} court did not do that. Suppose, however, that, in the context of $\mathbf{B}_8$, the following case is decided.

$$
\begin{aligned}
c_{19} &= \langle C_A, X_{19}, r_{19}, s_{19}\rangle \\
X_{19} &= \{f_2^\pi, f_3^\pi, f_1^\delta, f_2^\delta\} \\
r_{19} &= \{f_2^\delta\} \rightarrow \delta \\
s_{19} &= \delta
\end{aligned}
$$

This gives us the ordered case base $\mathbf{B}_9 = \langle \Gamma_9, \prec_9\rangle$ for $\mathbf{S}_1$, where Γ_9 contains c_{15}, c_{16}, c_{17}, c_{18}, and c_{19}, and $c_{15} \prec_9 c_{16} \prec_9 c_{17} \prec_9 c_{18} \prec_9 c_{19}$. Now, the c_{19} court, unlike the c_{17} court, *has* violated the doctrine of precedent imposed by the hierarchical model. For the case c_{18} imposes a binding vertical precedent rule $r_{18} = \{f_3^\pi\} \rightarrow \pi$ on the c_{19} court, and, although that rule applies in X_{19}, the c_{19} court refused to decide accordingly. In this case, then, we want the consolidation of $\mathbf{B}_9$ to produce something that is not an ordered case base. And this is precisely what it does. For the consolidation of $\mathbf{B}_9$ is $\mathbf{B}_9! = \mathbf{B}_9^5 = \langle \Gamma_9^5, \prec_9^5\rangle$, with Γ_9^5 and $\prec_9^5$ defined in the usual way. Clearly, $\mathbf{B}_9^4 = \mathbf{B}_8^4$. So we can simply use $\mathbf{B}_9^4$ and c_{19} to calculate $\mathbf{B}_9^5$. The new case c_{19} has two horizontal precedents in $\mathbf{B}_9^4$, namely c'_{15} and c_{16}. To find $(Horizontal_5)\dagger_{(c_{19})\Downarrow \mathbf{B}_9^4}$, we need to find the correction of c_{19} by $\mathbf{B}_9^4$. The top applicable vertical precedent for c_{19} in $\mathbf{B}_9^4$ is the only vertical precedent for c_{19} in $\mathbf{B}_9^4$, namely c_{18}. So we replace r_{19} with r_{18}, and $(c_{19})\Downarrow \mathbf{B}_9^4 = c'_{19} = \langle C_A, X_{19}, r_{18}, s_{19}\rangle$, where, recall, $r_{18} = \{f_3^\pi\} \rightarrow \pi$. Of course, c'_{19} is not a case, in our technical sense, since $Conclusion(r_{18}) \neq s_{19}$. And since $c'_{19} \in (Horizontal_5)\dagger_{(c_{19})\Downarrow \mathbf{B}_9^4}$, and $(Horizontal_5)\dagger_{(c_{19})\Downarrow \mathbf{B}_9^4} \subset \Gamma_9^5$, we have $c'_{19} \in \Gamma_9^5$.

So Γ_9^5 is not a case base, which means that $\mathbf{B}_9^5$ is not an ordered case base. Which is just what we wanted.

This all suggests that we can define the hierarchical model as follows.

Definition 11 (*precedential constraint: the hierarchical model*) Let $\mathbf{B} = \langle \Gamma, \prec \rangle$ be an ordered case base for a legal structure $\mathbf{S} = \langle Courts, > \rangle$, and let X be a new fact situation confronting a court $C \in Courts$. Then C must decide for a side s on the basis of a rule r such that $\mathbf{B}'!$ is a case base, where $\mathbf{B}' = \langle \Gamma \cup \langle C, X, r, s \rangle, \prec' \rangle$, where $\prec'$ extends $\prec$ so that $c \prec' \langle C, X, r, s \rangle$ for every $c \in \Gamma$.

This completes our development of the hierarchical model. While it does not perfectly capture the U.S. doctrine of precedent, it improves on the standard model by incorporating distinct constraints for horizontal and vertical precedents.

9 A complication

Suppose that we again have the background legal structure $\mathbf{S}_1 = \langle Courts_1, > \rangle$, where $Courts_1 = \{C_S, C_A, C_T\}$, with $C_S > C_A > C_T$. And suppose that we have an ordered case base $\mathbf{B}_{10} = \langle \Gamma_{10}, \prec_{10} \rangle$ for $\mathbf{S}_1$, where $\Gamma_{10} = \{c_{20}, c_{21}, c_{22}\}$, and $c_{20} \prec_{10} c_{21} \prec_{10} c_{22}$.

$$
\begin{array}{ll}
c_{20} = \langle C_A, X_{20}, r_{20}, s_{20} \rangle & c_{21} = \langle C_A, X_{21}, r_{21}, s_{21} \rangle \\
X_{20} = \{f_1^\pi, f_1^\delta\} & X_{21} = \{f_1^\pi, f_2^\pi, f_1^\delta, f_2^\delta, f_3^\delta\} \\
r_{20} = \{f_1^\pi\} \rightarrow \pi & r_{21} = \{f_1^\delta, f_2^\delta, f_3^\delta\} \rightarrow \delta \\
s_{20} = \pi & s_{21} = \delta \\
c_{22} = \langle C_S, X_{22}, r_{22}, s_{22} \rangle & \\
X_{22} = \{f_2^\pi, f_4^\delta, f_5^\delta, f_7^\delta\} & \\
r_{22} = \{f_2^\pi\} \rightarrow \pi & \\
s_{22} = \pi &
\end{array}
$$

We see that the c_{21} court, when faced with the applicable horizontal precedent c_{20}, decided to distinguish that case. Accordingly, the law in this jurisdiction just after c_{21} can be represented by the ordered case base $\mathbf{B}_{10}^2 = \langle \Gamma_{10}^2, \prec_{10}^2 \rangle$, where $\Gamma_{10}^2 = \{c'_{20}, c_{21}\}$, with c'_{20} just like c_{20} except that $r'_{20} = \{f_1^\pi\} \wedge \neg\{f_1^\delta, f_2^\delta, f_3^\delta\} \rightarrow \pi$ has been substituted for r_{20}. Thus, by distinguishing c_{20}, the c_{21} court added an exception to r_{20} so that it no longer applied to X_{21}. So far, so good.

But now consider what happens after c_{22}. Since r_{22} applies in c_{21}, but c_{21} was decided for the plaintiff, c_{22} overrules c_{21}. Thus the consolidation of $\mathbf{B}_{10}$ gives us $\mathbf{B}_{10}! = \langle \Gamma_{10}^3, \prec_{10}^3 \rangle$, where $\Gamma_{10}^3 = \{c'_{20}, c_{22}\}$, and $c'_{20} \prec_{10}^3 c_{22}$. We see that c_{21} is not in Γ_{10}^3, which is what we want. But notice, now, that c_{20} is not in Γ_{10}^3 either—instead, c'_{20} remains. This is probably *not* what we want. In general, once a case c is overruled, we

probably want to remove from our representation of good law in the jurisdiction not only c but any exceptions that c added to earlier precedent rules as well.

How can this be achieved? The most straightforward way, it seems to me, would involve complicating our representations of *rules*. So far, a rule has been something of the form

$$R^s \wedge \neg R_1^{\bar{s}} \wedge \cdots \wedge \neg R_n^{\bar{s}} \rightarrow s.$$

The problem that this sort of representation raises in the context of an ordered case base like $\mathbf{B}_{10}$ is that it provides no information about the *source* of an exception $R_i^{\bar{s}}$. And without that information, the consolidation process can't know whether to remove the exception $R_i^{\bar{s}}$ from the rule when any given case is overruled. The thing to do, then, is to simply add that information to our representation of the rule. We can capture our usual representation of a rule with a triple

$$\langle R^s, \{R_1^{\bar{s}}, \ldots, R_n^{\bar{s}}\}, s\rangle.$$

Our new representation will change each $R_i^{\bar{s}}$ into a *pair*, where the first member is the exception $R_i^{\bar{s}}$ and the second is the case responsible for adding that exception to the rule:

$$\langle R^s, \{\langle R_1^{\bar{s}}, c_i\rangle, \ldots, \langle R_n^{\bar{s}}, c_j\rangle\}, s\rangle.$$

If we were to take this route, we could of course continue, in many circumstances, to represent rules using the form

$$R^s \wedge \neg R_1^{\bar{s}} \wedge \cdots \wedge \neg R_n^{\bar{s}} \rightarrow s.$$

This would simply be a form of shorthand.

Notice that this strategy would probably require complicating our representation of a *case* as well. For suppose that a case c distinguishes another case c^*, adding an exception $R_i^{\bar{s}}$ to the rule r^*. In our new formalism, we would represent this portion of the revised rule with the pair $\langle R_i^{\bar{s}}, c\rangle$. But suppose now that c gets distinguished a couple of times, so that c becomes c', and c' becomes c'', before c'' is finally overruled. How is our improved version of consolidation to recognize that this should prompt the removal of the exception $R_i^{\bar{s}}$? After all, the case responsible for adding the exception to r^* was not c'' but c. It seems, then, that we would probably want to add to our representation of a case a canonical name for the case that persists through various changes to the associated rule. The consolidation process could then use this canonical name to link c'' to $R_i^{\bar{s}}$.

More elegant solutions may be possible. But these changes do succeed in reducing the problem presented by an ordered case base like $\mathbf{B}_{10}$ to a bit of bookkeeping. For reasons of space, I leave their implementation for another occasion.

10 Some remaining issues

In this section, I simply note a handful of ways in which the hierarchical model fails to reflect the U.S. doctrine of precedent. (There are no doubt others.) These present opportunities for the model to be improved in future work. Of course, as we continue to

complicate the model, we will need to trade off accuracy against other values, and some complications might not be worth the trouble. I don't address those tradeoffs here.

(1) To this point, we have been assuming that although the standard model misconstrues the U.S. doctrine of vertical precedent, it gets the U.S. doctrine of *horizontal* precedent basically right. Accordingly, the hierarchical model adopts the doctrine of horizontal precedent imposed by the standard model with only relatively modest revisions. But is this assumption justified? In fact, we have already been given good reason to think that it is not.

Consider the cases c_{23} and c_{24}, decided, again, in the context of $\mathbf{S}_1$.

$$\begin{aligned} c_{23} &= \langle C_S, X_{23}, r_{23}, s_{23}\rangle & c_{24} &= \langle C_S, X_{24}, r_{24}, s_{24}\rangle \\ X_{23} &= \{f_1^\pi, f_1^\delta\} & X_{24} &= \{f_1^\pi, f_1^\delta\} \\ r_{23} &= \{f_1^\pi\} \rightarrow \pi & r_{24} &= \{f_1^\pi\} \wedge \neg\{f_2^\delta\} \rightarrow \pi \\ s_{23} &= \pi & s_{24} &= \pi \end{aligned}$$

Now, it might seem that c_{24} leaves future courts with greater discretion than does c_{23}. For, on the face of it, a court saddled with the precedent rule "If A, then φ" is *more constrained* than a court faced with the precedent rule "If A but not B, then φ." The reason, of course, is simply that the first rule has *broader scope*. It applies in every situation in which the second rule applies, and in some more situations besides. So imagine that we select either c_{23} or c_{24} to place in the background case base. Intuitively, the key set of facts for which it might matter which one we choose is $X_{25} = \{f_1^\pi, f_2^\delta\}$. If the background ordered case base is $\mathbf{B}_{11} = \langle \Gamma_{11}, \prec_{11}\rangle$, with $\Gamma_{11} = \{c_{24}\}$, then X_{25} seems to present a case of first impression. But if it is $\mathbf{B}_{12} = \langle \Gamma_{12}, \prec_{12}\rangle$, with $\Gamma_{12} = \{c_{23}\}$, then the c_{23} court seems already to have answered the question posed by X_{25}. Of course, this is just the same intuition again, that the one rule is more permissive than the other.

Following the pronouncements of U.S. courts, this is precisely how the hierarchical model treats these rules when the court faced with X_{25} is *inferior* to C_S. Thus, while the hierarchical model permits the C_T court to decide for the defendant on the basis of the rule $r_{25} = \{f_2^\delta\} \rightarrow \delta$ in the context of $\mathbf{B}_{11}$, it prohibits the court from doing so in the context of $\mathbf{B}_{12}$.

$$\begin{aligned} c_{25} &= \langle C_T, X_{25}, r_{25}, s_{25}\rangle \\ X_{25} &= \{f_1^\pi, f_2^\delta\} \\ r_{25} &= \{f_2^\delta\} \rightarrow \delta \\ s_{25} &= \delta \end{aligned}$$

But suppose that the court faced with X_{25} is again C_S. In that case, according to the hierarchical model, even the intuitively stronger rule r_{23} represents no constraint at all. The court can decide for the plaintiff or for the defendant. If the court wishes to decide for the defendant, then any rule of the form

$$\{f_2^\delta\} \wedge \neg R_1^\pi \wedge \cdots \wedge \neg R_n^\pi \rightarrow \delta,$$

with $0 \leq n$, is a permissible basis for decision, so long as $\{f_1^\pi\} \neq R_i^\pi$ $(i = 0, 1, \ldots, n)$. Thus, although we had thought c_{24} more permissive than c_{23}, we see now that, when the same court C_S is faced with the key fact pattern X_{25}, c_{23} permits anything at all. So, according to the hierarchical model, there is no difference between c_{23} and c_{24} *qua* horizontal precedents.

Suppose, now, that C_S is the Supreme Court of the United States. In that case, the hierarchical model is certainly correct to hold that the Court, when faced with X_{25} in the context of $\mathbf{B}_{11}$ is *permitted* to decide for the defendant. Consider, though, whether the doctrine of precedent *discourages* the Court from deciding X_{25} for the defendant in the context of $\mathbf{B}_{11}$.[111] Because if it does discourage the Court from deciding for the defendant in the context of $\mathbf{B}_{11}$—but *not* in the context of $\mathbf{B}_{12}$—then there is a difference between c_{23} and c_{24}, even *qua* horizontal precedents. And this suggests that the hierarchical model is missing something about the U.S. doctrine of horizontal precedent.

In fact, the U.S. doctrine of precedent *does* discourage the Court from deciding X_{25} for the defendant in the context of $\mathbf{B}_{11}$. To see this, note first that it is not merely the combination of facts and outcome in a past case that carries the force of precedent. Rather, the doctrine of horizontal precedent urges the Court to follow the *rule* established by a horizontal precedent:

> As a general rule, the principle of *stare decisis* directs us to adhere not only to the holdings of our prior cases, but also to their explications of the governing rules of law.[112]

And the Court insists that it is required to follow an applicable horizontal precedent rule unless there is some *special justification* for departing from it.[113] Whatever exactly constitutes a special justification, the Court has been clear that it is not enough for the precedent to have been wrongly decided, or for the application of

[111] We might put this differently. We might say that the Court is at least *sometimes* permitted to decide for the defendant in this context. The question, then, would be whether the Court is *always* permitted to decide for the defendant in this context. We could then continue to speak in terms of the doctrine of precedent permitting and prohibiting decisions, rather than switching to talk of the doctrine "discouraging" decisions and "placing a thumb on the scale." I'm speaking here in terms of discouragement simply because it seems more natural. Frankly, though, I'm not sure that it's the best way to go from an analytical perspective.

[112] *Seminole Tribe*, 517 U.S. at 67 (internal quotation marks and citations omitted). Accord *Cty of Allegheny v. Am. Civil Liberties Union Greater Pittsburgh Chapter*, 492 U.S. 573, 668 (1989). See also *United States v. Virginia*, 518 U.S. 515, 596 (1996) ("The Supreme Court of the United States does not sit to announce "unique" dispositions. Its principal function is to establish precedent—that is, to set forth principles of law that every court in America must follow. As we said only this Term, we expect both ourselves and lower courts to adhere to the rationale upon which the Court based the results of its earlier decisions.").

[113] *Dickerson*, 530 U.S. at 443. See also *Citizens United v. Fed. Election Comm'n*, 558 U.S. 310, 362 (2010) ("Our precedent is to be respected unless the most convincing of reasons demonstrates that adherence to it puts us on a course that is sure error."). To be sure, most of the Court's explicit discussion of the "special justification" requirement has taken place in the context of decisions whether to *overrule* a precedent. But see, for example, *Quarles*, 467 U.S. at 660 (O'Connor J. concurring in the judgment in part and dissenting in part), for the suggestion that a special justification is required even to distinguish (in the thick sense) a horizontal precedent.

the rule in the present case to produce an undesirable result.[114] All of this suggests that the U.S. doctrine of precedent does indeed discourage the Supreme Court, faced with X_{25} in the context of $\mathbf{B}_{11}$, from distinguishing c_{23} and deciding for the defendant. To be clear, the Court is not *prohibited* from distinguishing c_{23}, any more than it is prohibited from overruling it. But the doctrine of precedent does place a thumb on the scale.[115] Its failure to capture this aspect of the doctrine represents a shortcoming of the hierarchical model—one whose solution, unfortunately, is far from obvious.[116]

(2) Notice that we have been discussing the doctrine of horizontal precedent in the Supreme Court specifically. Why not simply the U.S. doctrine of horizontal precedent? Because, alas, there is no single (non-disjunctive) U.S. doctrine of horizontal precedent. Consider the federal courts of appeals. While a circuit court sitting en banc is generally subject to the same doctrine of horizontal precedent as the Supreme Court, this is not typically the case for three-judge panels (which decide the vast majority of cases in the circuit courts). In most circuits, the doctrine of horizontal precedent for three-judge panels is more demanding.[117] In a recent comprehensive treatment of the U.S. law of precedent, Garner et al (2016, 37) describe the situation thus:

> Within each circuit, the rules for three-judge panels are generally strict. Traditionally speaking, three-judge panels are absolutely bound by prior decisions of the en banc court. They are also strictly bound by the decisions of prior panels…[with] one obvious and commonsense caveat: a panel may depart from or overrule an earlier panel's decision when it has been repudiated or undermined by later controlling authority, such as a statute, an intervening Supreme Court decision, or [an] en banc decision.

This standard is evidently distinct from that employed by the Supreme Court to deal with its own past decisions. But even this standard is not uniform among the federal

[114] See *Planned Parenthood of SE Pa. v. Casey*, 505 U.S. 833, 864 (1992).

[115] It's hard to say what impact, if any, this additional constraint built into the doctrine of horizontal precedent has on the actual *practice* of the Court. Certainly there are many cases in which the Court appears to have departed from a horizontal precedent based on nothing more than the bare policy preferences of the majority. And indeed the evidence suggests that precedent in general influences the decisions of Supreme Court justices far less than it influences the decisions of lower court judges. See, for example, Epstein et al. (2013, chp. 3), Segal (2008, 23), Segal and Spaeth (2002, 298), Segal and Spaeth (1996), Brenner and Spaeth (1995, 109). (Of course, there are a variety of reasons to predict this result. For discussion, see Klein (2017, 238–239), Posner (2008, chp. 10).) But even staunch attitudinalists acknowledge that the doctrine of precedent has *some* influence on the decisions of (some) Supreme Court justices. See Segal and Spaeth (2002, 295, 302–03), Segal and Spaeth (1996, 977). Our question, however, concerns the causal effects of a particular *component* of the doctrine of horizontal precedent, and this question has not, to my knowledge, been studied empirically.

[116] Consider, in this regard, the suggestion in Strauss (2010, 40) that common law reasoning is about "attitudes, not algorithms." Note that similar difficulties will arise once the hierarchical model is expanded, as it presumably ought to be, to account for the overruling of horizontal precedents.

[117] The Seventh Circuit, however, allows one panel to depart from the holding of another whenever there are "compelling reasons" to do so. *United States v. Reyes-Hernandez*, 624 F.3d 405, 413–414 (7th Cir. 2010). And this sounds quite like the doctrine of horizontal precedent in the Supreme Court.

circuits. The First Circuit, for instance, allows one panel to "overrule another in those relatively rare instances in which authority that postdates the original decision, although not directly controlling, nevertheless offers a sound reason for believing that the former panel, in light of fresh developments, would change its collective mind."[118] And there is variation among state courts as well. Indiana appellate courts, for example, reject *any* form of horizontal stare decisis.[119] All of this suggests that, as our formal models of common law reasoning become increasingly sophisticated, we may eventually want to allow for distinct doctrines of horizontal precedent for different courts in a legal structure.

(3) The hierarchical model gives every court the power to create precedents that future courts are required to respect. Even a court at the bottom of the judicial hierarchy may, on the hierarchical model, create horizontal precedents that must be respected by that same court in later cases. In fact, however, as the Supreme Court has stated, "[a] decision of a federal district court judge is not binding precedent in either a different judicial district, the same judicial district, or even upon the same [district] judge in a different case."[120] Thus, in future work, we may want to develop the hierarchical model to allow for courts that are constrained by vertical precedents but unable to create precedents of their own.

(4) The hierarchical model does not allow one court to be "superior" to another in certain respects but "inferior" (or unrelated) to it in others. But this prevents the model from capturing the relationship between federal and state courts in the U.S. For while a Supreme Court holding on a matter of federal constitutional law, for example, establishes a binding vertical precedent for state courts, when the Court must resolve a question of *state* law, it is bound by the precedents of the highest court in the relevant state.[121] Eventually, we may want our models to be able to capture this sort of complex relationship among courts.

11 Conclusion

I have argued that the U.S. doctrine of precedent treats horizontal precedents and vertical precedents differently, even once the possibility of overruling is set aside. In particular, while courts are often permitted to distinguish applicable horizontal precedents, courts are *never* permitted to distinguish applicable vertical precedents.

[118] *United States v. Rodriguez-Pacheco*, 475 F.3d 434, 442 (1st Cir. 2007) (quotation marks and internal citations omitted).

[119] *Martinez v. State*, 82 N.E.3d 261, 266 (Ind. Ct. App. 2017) (stating that "this court does not recognize horizontal stare decisis"); *In re F.S.,* 53 N.E.3d 582, 596 (Ind. Ct. App. 2016) ("[W]e do not recognize horizontal stare decisis in Indiana."); *Smith v. State*, 21 N.E.3d 121, 126 (Ind. Ct. App. 2014).

[120] *Camreta v. Greene*, 563 U.S. 692, 709 n.7 (2011) (internal citations omitted). See also *Smentek v. Dart*, 683 F.3d 373, 377 (7th Cir. 2012) (Posner J.) (stating that "a district court decision does not have precedential effect"); *Threadgill v. Armstrong World Indus., Inc.,* 928 F.2d 1366, 1371 (3d Cir. 1991) (stating that "there is no such thing as 'the law of the district'").

[121] *Erie R.R. v. Tompkins*, 304 U.S. 64, 78 (1938). See also *Commissioner v. Estate of Bosch*, 387 U.S. 456, 465 (1967); *Williams v. Kaiser*, 323 U.S. 471, 473 (1945).

Rather, faced with an applicable vertical precedent rule, an inferior court is required to decide as the rule directs. Since the standard model does not differentiate horizontal and vertical precedents, it fails to capture this doctrine. It does a decent job, though, with one part of that doctrine, namely the doctrine of *horizontal* precedent. For the doctrine of *vertical* precedent, however, I have argued that the strict rule model gets it right. Accordingly, I have developed the hierarchical model of precedential constraint, which requires courts to treat horizontal precedents roughly as the standard model requires and to treat vertical precedents roughly as the strict rule model requires. This involved supplementing our representation of a case base with a formal representation of a hierarchical legal structure, as well as complicating our representation of an individual precedent case to indicate the deciding court. While the hierarchical model does not perfectly capture the U.S. doctrine of precedent, it does improve on the standard model and on a number of other formal models in the literature.

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