Logic Journal of IGPL Advance Access published January 12, 2011

Quantum-like non-separability of concept combinations, emergent associates and abduction

P. D. BRUZA, K. KITTO, B. RAMM and L. SITBON, *Faculty of Science and Technology, Queensland University of Technology, Brisbane, Australia.*
e-mail: p.bruza@qut.edu.au; kirsty.kitto@qut.edu.au; brentynramm@gmail.com; laurianne.sitbon@qut.edu.au

D. SONG, *School of Computing, Robert Gordon University, Aberdeen, Scotland.*
e-mail: d.song@rgu.ac.uk

S. BLOMBERG, *School of Biological Sciences, University of Queensland, Australia.*
e-mail: s.blomberg1@uq.edu.au

Abstract

Consider the concept combination 'pet human'. In word association experiments, human subjects produce the associate 'slave' in relation to this combination. The striking aspect of this associate is that it is not produced as an associate of 'pet', or 'human' in isolation. In other words, the associate 'slave' seems to be emergent. Such emergent associations sometimes have a creative character and cognitive science is largely silent about how we produce them. Departing from a dimensional model of human conceptual space, this article will explore concept combinations, and will argue that emergent associations are a result of abductive reasoning within conceptual space, that is, *below* the symbolic level of cognition. A tensor-based approach is used to model concept combinations allowing such combinations to be formalized as interacting quantum systems. Free association norm data is used to motivate the underlying basis of the conceptual space. It is shown by analogy how some concept combinations may behave like quantum-entangled (non-separable) particles. Two methods of analysis were presented for empirically validating the presence of non-separable concept combinations in human cognition. One method is based on quantum theory and another based on comparing a joint (true theoretic) probability distribution with another distribution based on a separability assumption using a chi-square goodness-of-fit test. Although these methods were inconclusive in relation to an empirical study of bi-ambiguous concept combinations, avenues for further refinement of these methods are identified.

Keywords: conceptual combination, emergence, quantum theory, abductive reasoning.

1 Introduction

Consider the concept combination 'pet human'. In word association experiments, human subjects quite readily ascribe the associate 'slave' to this combination when it is not an associate produced in relation to 'pet', or 'human' when either is individually presented as a cue in a free association experiment. So it seems the associate 'slave' is emergent in the sense it arises as a product of the concept combination and cannot be recovered from its constituent concepts. Such emergent associations sometimes have a creative character and the literature in cognitive science is largely silent about how we produce them. Concept combination, emergence and abduction are huge and

doi:10.1093/jigpal/jzq049

challenging research topics in themselves. Therefore, the aims of this article are quite modest. We aim to provide some background on the dominant models of concept combination as currently presented in the field of cognitive science and place them within the context of emergence. A speculative connection will then be established with human abductive reasoning. The gist of this speculation is some concept combinations are non-separable in a way similar to quantum entangled particles. The bulk of the article will provide an account of how concept combinations may be modelled as interacting quantum systems. Although the link to quantum theory may raise eyebrows, there has been growing speculation in the literature that concept combinations may behave like quantum entangled particles [1, 3, 5, 9]. This article can therefore be seen as a contribution to the emerging field of quantum cognition [7]. Quantum theory provides the hypothesis that concept combinations are non-separable systems, meaning a full account of conceptual combination cannot be provided by reductive modelling. By this we mean that the concept combination cannot be fully understood in terms of the constituent words in the combination. Colloquially speaking, a concept combination is *not* simply a 'sum of its parts'.

Concept combination is still largely an open question within cognitive science. Broadly speaking there are two schools of thought.

Schema-based theories of conceptual combination (e.g. [17, 21]) propose that the head noun is a schema-structure made up of various dimensions (e.g. colour, size, shape etc). These dimensions form slots that can be altered or filled by the modifier noun. Murphy's [17] specialization model extends the schema-model to account for noun–noun combinations by including slots in the header that represent relations, and secondary elaboration process. Thus the concept 'dog', as well as containing slots such as colour, size and shape, may also include relational dimensions such as functions, behaviours and habitat [17]. In the concept combination 'beach bicycle', 'beach' would fill a relational slot such as 'for riding on'. Elaboration processes would then infer based upon the properties of sand and tires, that typical tires would sink in sand, and thus need to be thicker than usual, with deep tread, etc. According to Wisniewski [22] this elaborative process involves the construction of a plausible scenario, including the role that each constituent plays in the scenario. Thus a 'bicycle' plays the role of *transport* and 'beach' the role of *surface* while the scenario may be 'recreation', 'racing', etc.

The structure mapping theory of conceptual combination holds that similarity judgements and analogical reasoning depends upon a process of aligning shared dimensions, and finding alignable differences between the concepts [11, 12, 16]. Wisniewski [21] proposed that there were three types of conceptual combination:

1. Relation linking in which constituents are linked via a relation (e.g. 'ladder box' can be interpreted as a box *for* storing ladders).
2. Property mapping in which one or more properties of the modifier are mapped onto the header, but not the entire representation (e.g. a 'zebra crossing' as a painted stripes on the road).
3. Hybridization in which the object is either a conjunction of the two constituents or shares multiple properties of each of them (e.g. a 'clock radio' for example has the dual-function of 'clock' and 'radio').

Wisniewski [21] proposed that property-mapping and hybridization are based upon aligning the dimensions of concepts, and finding alignable differences. Results showed that the degree of similarity affected the type of interpretation. Highly similar combinations tended to be interpreted via property-mapping, e.g. 'whiskey beer' as 'whiskey flavoured beer'. Highly dissimilar combinations were more often interpreted via relation-linking, such as 'whiskey newspaper' as a newspaper used to conceal a whiskey bottle. Wisniewski thus concluded that a comparison process is an important step in interpreting conceptual combinations. Wilkenfeld and Ward [20] tested the hypothesis that the similarity of individual concepts influences the production of emergent properties for the combination. Similarity is also an indication of alignability of concepts, in that similar concepts (e.g. ‘guitar’ and ‘harp’) share many of the same dimensions (e.g. both have strings, and can be played) and dissimilar concepts (e.g. ‘helicopter’ and ‘blanket’) do not share many dimensions. Wilkenfeld and Ward predicted that low similarity concepts would require more reasoning processes to resolve conflicts that should result in more emergent properties.

Our hypothesis is that abductive reasoning is involved. Evidence for this can be found when one considers that such reasoning is ‘ignorance preserving’, which has been convincingly argued as being a hallmark of abduction [8]. For example, consider ‘helicopter blanket’. Abductive reasoning provides the scenario or interpretation we settle on, but we are ultimately ignorant whether it is referring to a tarpaulin for draping over a helicopter, or a boy’s blanket with a helicopter motif. Furthermore, we are interested to empirically investigate the question whether the non-separability of concept combinations is related to phenomenon of emergent associations and abduction. In short, we posit a link between the non-separability of concept combinations, emergent associates and abductive reasoning employed by humans to interpret such concept combinations.

2 Concept combination in conceptual space

We are strongly aligned with the view that concept combination takes place at the conceptual level of cognition. Gärdenfors puts forward a three level model of cognition in which how information is represented varies greatly across the different levels [10]. Within the lowest level, information is pre- or sub-conceptual and is carried by a connectionist representation. Within the uppermost level’ information is represented symbolically. It is the intermediate, conceptual level (or ‘conceptual space’), which is of particular relevance to this account. Here properties and concepts have a geometric representation in a dimensional space. For example, the property of ‘redness’ is represented as a convex region in a tri-dimensional space determined by the dimensions hue, chromaticity and brightness. The conceptual level is rich in associations, both explicit and implicit. We hold the view that such associations play an important role in how concepts are combined.

Let us first consider a single word ‘bat’. One way to probe how this word is associated with other words is via the classical free association experiments. Subjects are cued with the word ‘bat’ and asked to produce the first word that comes to mind. Over numbers of subjects, probabilities can be calculated that a certain associate is produced. Table 1 depicts such data gleaned from the University of South Florida word association norms [18]. For example, 25% of the subjects produced ‘ball’ is relation to the cue ‘bat’. The associate ‘ball’ derives from the sport sense of ‘bat’. Other associates in the table are clearly related to the animal sense of ‘bat’. When considered this way, ‘bat’ can be modelled in two-dimensional vector space, the basis vectors of which correspond to the two possible senses. This is illustrated in Figure 1. The basis vectors $|0\rangle$ and $|1\rangle$ correspond to the senses ‘sport’ and ‘animal’ respectively. The word ‘bat’ is modelled as $|\text{bat}\rangle = a_0|0\rangle + a_1|1\rangle$ which expresses the intuition that ‘bat’ is a linear combination of senses where a_0 and a_1 are scalars representing the likelihood of the given sense being produced in a free association experiment. These scalars relate to probabilities via Pythagoras’ theorem: $a_0^2 + a_1^2 = 1$. The preceding formalization of basic vector notions may be unfamiliar because the notation used is the same as that employed in quantum theory. Modelling ‘bat’ in the above way is very similar to modelling ‘bat’ as a quantum particle, like a photon, where $|0\rangle$ and $|1\rangle$ would correspond to polarization ‘down’ or ‘up’. In quantum theory, the linear combination is deemed to represent a superposition of basis states. The act of measuring the

TABLE 1. Free association probabilities for the word'bat'

Associate	Probability
ball	0.25
cave	0.25
vampire	0.07
fly	0.06
night	0.06
baseball	0.05
blind	0.04
…	…

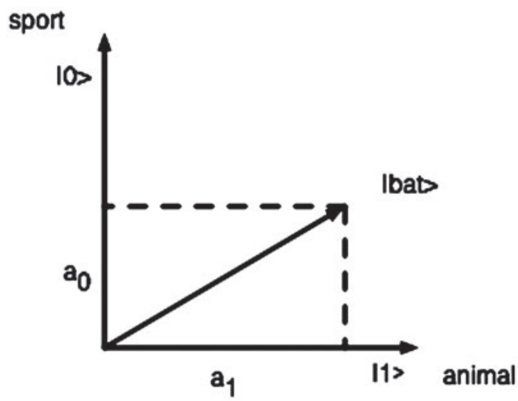

FIG. 1. The word 'bat' represented in a two-dimensional space.

particle's polarization collapses the superposition onto one of the basis states e.g. 'up', in much the same way presenting the cure 'bat' collapses the ambiguous word 'bat' onto one of the basis states corresponding to a sense of the word. In both cases, a disambiguation of the state takes place. Several authors have independently modelled words in human memory in this quantum-like way [2, 4, 19]. In addition, this style of modelling aligns very well with Gärdenfors' geometric account of the conceptual level of cognition, whereby concepts have a dimensional structure, although it should be noted that the formalization just presented is not as expressive as Gärdenfors' model of conceptual space.

2.1 Modelling concept combinations as interacting quantum systems

Motivated by Gärdenfors' dimensional structure of conceptual space, we adopt an approach whereby concept combination is modelled by use of the tensor product. This approach is inspired by the matrix model of memory [14]. The appealing intuition here is that the words being combined can be considered as 'interacting', and as we shall see, this opens the door to the intriguing question of whether such interactions can be modelled in a quantum-like way.

By way of illustration, the matrix model of memory would represent the interaction between two words as a dyadic (outer) product of the respective vectors representing the individual words. The result is a matrix with rank one. The significance of the matrix having rank one is simply that the matrix can be represented as a product of two vectors, that is the product of the two 'parts'

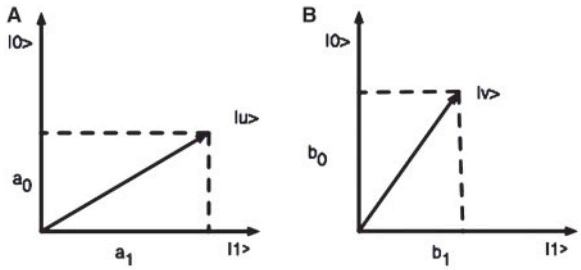

FIG. 2. Two quantum systems: A and B.

combining to produce a representation corresponding to the combination of the two parts. In other words, the interaction is decomposable because it can expressed by such a product. We shall see later that some interactions are 'non-separable' because they cannot be expressed as a product of vectors corresponding to the states of constituent words in the concept combination.

There is a growing body of literature describing quantum-like models of the conceptual level of cognition, e.g. [3–6, 9]. Indeed, quantum entanglement has been proposed as a useful way in which to model concept combinations, especially for the case of emergent properties.

In quantum theory interacting quantum systems are formalized via a tensor product of the individual systems. By way of illustration, Figure 2 depicts two systems A and B. System A is in state $|u\rangle = a_0|0\rangle + a_1|1\rangle$ and system B is in state $|v\rangle = b_0|0\rangle + b_1|1\rangle$. The state of the interacting systems is the tensor product of the two states:

$$|u\rangle \otimes |v\rangle = a_0 b_0|00\rangle + a_0 b_1|01\rangle + a_1 b_0|10\rangle + a_1 b_1|11\rangle. \tag{1}$$

Observe how state of the combined system is represented in a vector space with basis vectors $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. These basis vectors, or 'basis states' represent all possible outcomes, for example the basis state $|11\rangle$ may represent the photon in system A and the photon in system B both have a polarization 'up'. In addition the scalars are related to probabilities as follows: $(a_0b_0)^2 + (a_0b_1)^2 + (a_1b_0)^2 + (a_1b_1)^2 = 1$. The state represented in equation 1 is a superposed state. The probability $(a_0b_0)^2$, for example, corresponds to the probability the state will collapse on the basis state $|00\rangle$ (both photons in the system have polarization 'down').

There is no reason why this structure cannot be applied by means of analogy to modelling words. Recall that the word'bat' can be represented as a superposition between two senses 'sport' ($|0\rangle$) and 'animal' ($|1\rangle$) as depicted in Figure 1. The same can be said for the word 'boxer'. Table 2 details its free association data and it reveals that the associates express both an animal and sporting sense. As both words can be represented as a superposition between the basis states corresponding to the senses 'sport' and 'animal', this opens the possibility to represent the concept combination 'boxer bat' using the interacting quantum model described above.

Referring back to the state of the combined system as detailed by Equation 1. When the basis of the combined system is fixed, the tensor representation can equivalently represented as a matrix of amplitudes as shown in Figure 3. The matrix of amplitudes allows a connection to be made with the matrix model mentioned previously. Consider once again the concept combination 'boxer bat'. Assume 'boxer' is represented by the state $|u\rangle = a_0|0\rangle + a_1|1\rangle$ (system A) and 'bat' is represented by the state $|v\rangle = b_0|0\rangle + b_1|1\rangle$ (System B). The matrix model would represent 'boxer bat' as the outer product of the two vectors resulting in the matrix of amplitudes depicted in Table 3. This matrix has

TABLE 2. Free association probabilities for the word 'boxer'

Associate	Probability
fighter	0.14
gloves	0.14
fight	0.09
dog	0.08
shorts	0.07
punch	0.05
Tyson	0.05
…	…

TABLE 3. Matrix of amplitudes corresponding to the interacting systems in Figure 2

	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 0\rangle$	a_0b_0	a_0b_1
$\lvert 1\rangle$	a_1b_0	a_1b_1

TABLE 4. Matrix of amplitudes corresponding to a non-separable state of the combined system depicted in Figure 2 in a case where the two systems should be considered non-separable

	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 0\rangle$	0	x
$\lvert 1\rangle$	y	0

rank one, meaning there is a vector in system A and a vector in system B, which when combined gives the matrix in question. In other words, the state of 'boxer bat' is decomposable—it can be established as a product of the state 'boxer' with the state 'bat'. We will term such a decomposable state of the interacting systems as being *separable*.

Assume the state of the combined system evolves to the state depicted in Table 4. The state represented in Table 4 can be written in linear form as $x|01\rangle + y|10\rangle$ where $x^2 + y^2 = 1$. In terms of the example 'boxer bat' this state represents the situation where the human subject holds the possibility of two interpretations. With Figure 1 as reference, the basis state $|01\rangle$ represents 'boxer' collapsing onto the sport sense, and 'bat' collapsing onto the animal sense. This would correspond to the interpretation 'a small furry black animal with boxing gloves on'. The the basis state $|10\rangle$, on the other hand, represents 'boxer' collapsing onto the animal sense (a breed of dog), and 'bat' collapsing onto the sport sense. In this case the interpretation could be a 'toy bat a boxer dog plays with'. Ultimately the human subject will settle on an interpretation. In terms of the model, this corresponds to the superposed state represented in Figure 4 collapsing onto one of the basis states: $|01\rangle$ or $|10\rangle$. The link to abductive reasoning could be the following. The interacting systems provide possible interpretations via the basis states. Assuming that x^2 is significantly larger than y^2 then the interpretation corresponding to basis state $|10\rangle$ is used to interpret the concept combination because it is more plausible.

This state represented in Table 4 is *non-separable* because the matrix is not of rank one. In other words, there does not exist $|u\rangle \in A, |v\rangle \in B$ such that the outer product of $|u\rangle$ and $|v\rangle$ gives rise to the state in Table 4. This style of describing non-separable states has been used to characterize bipartite non-separable pure states in quantum mechanics (See Theorem 1 in [13]).

This section presented a simple quantum-like model of concept combinations. In particular, the non-separability of concept combinations has been formalized. The important question remains as to whether such non-separable combinations actually manifest in cognition.

3 Testing for non-separable concept combinations in cognition

3.1 Experimental rationale

The purpose of this experiment was to test which senses are interpreted for two ambiguous words in bi-ambiguous concept combinations (e.g. ‘boxer bat’). We tested this hypothesis by attempting to shift one word sense away from its dominant meaning in the hope that an non-separable compound would result in a noticeable effect upon the second word. Thus, we primed either the dominant or subordinate sense of word one or word two in the concept combination, while the sense of the other word was left free to vary. For example, in a experiment, the priming word ‘vampire’ is first shown to the subject. This primes the animal sense of ‘bat’ (its dominant sense). The concept combination ‘boxer bat’ is then shown to the subject, who is asked to provide an interpretation. For example, ‘a furry black animal with boxing gloves on’. In addition they are explicitly asked to declare the senses they attributed to each word, for example, animal sense to ‘bat’, sport sense to ‘boxer’. The goal of this manipulation was to test whether the word pairs had become non-separable, in particular by showing that when a word is forced into a specific sense (e.g. ‘boxer’ to the animal sense) this results in changes to the probability that the other word will be interpreted in its dominant sense (e.g. ‘bat’ into the animal sense). The use of primes also increased the chances that participants would use an even spread of the different senses in interpreting the compounds rather than just relying upon the dominant senses. We compared the primed compounds with a baseline group which received neutral primes. It was expected that this group would show which senses tend to be selected when no priming is used. The concept combinations chosen for the empirical study all had a novel character as it was felt this would promote cognitive processing (e.g. abductive reasoning) to furnish a possible interpretation, rather than just retrieving an interpretation from memory as would probably be the case with familiar concept combinations.

3.2 Methodology

Participants completed a web-based task in which they provided an interpretation for twelve bi-ambiguous compounds, e.g. ‘boxer bat’. (See Table 5). The compounds were only seen once by each participant. Participants were assigned to one of ten groups based on the order in which they logged into the experiment. For eight groups the compound was preceded by a priming word (e.g. ‘vampire’) and for two groups (baseline) the compound was preceded by a neutral prime (e.g. 36). For the priming groups, participants classified the priming word as ‘natural’ or ‘non-natural’. The goal of the classification task was to activate the prime in memory. The baseline groups classified whether a number was odd or even. This classification task was neutral, and while not expected to interfere with the interpretation of word senses, was chosen to balance the amount of cognitive processing across subjects.

The priming groups received one of four potential primes for each compound: Prime 1: Word 1 dominant sense (e.g. ‘fighter’), Prime 2: Word 1 subordinate sense (e.g. ‘dog’), Prime 3: Word 2 dominant sense (e.g. ‘vampire’), Prime 4: Word 2 subordinate sense (e.g. ‘ball’). The purpose of the primes was to set the dominant or subordinate sense of word one or word two. Participants received different primes according to their group, with the constraint that over the twelve compounds they all received three each of the four potential priming words. After interpreting the compound participants were asked to clarify which sense they chose for each word. For ‘bat’ subjects: (A) An animal (B) A piece of sporting equipment, or (C) Other (which they were asked to specify). In order

TABLE 5. The bi-ambiguous compounds, along with their dominant and subdominant prime cues

	Word 1		Word 2	
Compound	Prime 1 (dom)	Prime 2 (sub)	Prime 3 (dom)	Prime 4 (sub)
boxer bat	fighter	dog	vampire	ball
bank log	money	river	cabin	journal
star charge	moon	movie	account	volt
apple suit	banana	computer	vest	slander
stock tick	shares	cow	flea	mark
fan post	ceiling	football	lamp	web
ring pen	diamond	oval	ink	pig
seal pack	walrus	envelop	suitcase	leader
spring plant	summer	coil	seed	factory
racket pitch	tennis	noise	tone	throw
toast gag	jam	speech	choke	joke
poker strike	cards	fire	lightning	union

TABLE 6. boxer bat modelled as interacting quantum system ($n = 10$)

	$\lvert D\rangle$	$\lvert S\rangle$
$\lvert D\rangle$	0.3	0.3
$\lvert S\rangle$	0.1	0.3

to furnish a direct comparison to the quantum-like model described above, only the dominant (A) and subordinate sense (B) were retained in the analysis reported below.

3.3 Analyzing the non-separability of bi-ambiguous concept combinations

Two means for analyzing non-separability were employed, the one motivated from quantum theory described above and another based on 'classical' probability theory.

3.3.1 Quantum-like non-separability of bi-ambiguous combinations

For each bi-ambiguous concept combination, the data for those subjects in the baseline group can be pooled. For example, the data for 'boxer bat' is depicted in Table 6. The probability that both 'boxer' and 'bat' will collapse on their dominant senses (basis state $\lvert DD\rangle$) is 0.3. The rank of the resultant matrix is then determined. The concept combination is deemed to be non-separable if the rank of the matrix is greater than one.

3.3.2 A 'classical' probabilistic model of bi-ambiguous combinations

A concept combination of two words is modelled by two random variables A and B, where A corresponds to the first word in the combination and B corresponds to the second word.

The variable A ranges over $\{a_1, a_2\}$ corresponding to its two underlying senses, whereby a_1 is used to refer to the dominant sense of first word in the combination and a_2 refers to its subordinate sense. Similarly B ranges over $\{b_1, b_2\}$. This convention helps explain the model to follow but is not necessary for its probabilistic development.

TABLE 7. boxer bat: base joint distribution ($n = 10$)

	b_1	b_2
a_1	0.3	0.3
a_2	0.1	0.3

TABLE 8. boxer bat:$\lambda_1 =$ fighter, $n = 10$

	b_1	b_2
a_1	0.2	0.8
a_2	0	0

TABLE 9. boxer bat:$\lambda_2 =$ dog, $n = 11$

	b_1	b_2
a_1	0.2	0.8
a_2	0	0

Primes are designed to span four mutually exclusive cases. By way of illustration, the primes used for 'boxer bat' are {fighter, dog, vampire, ball}. The primes are modelled as a random variable λ ranging over $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$. In a probabilistic setting, separability is formalized by assuming the joint probability is factorizable:

$$\Pr(A, B|\lambda) = \Pr(A|\lambda)\Pr(B|\lambda) \tag{2}$$

Using Bayes' rule, this can be rewritten as:

$$\Pr(A, B, \lambda) = \Pr(A|\lambda)\Pr(B|\lambda)\Pr(\lambda) \tag{3}$$

Assuming the law of total probability:

$$\Pr(A, B) = \sum_{1 \leq i \leq 4} \Pr(A|\lambda_i)\Pr(B|\lambda_i)\Pr(\lambda_i) \tag{4}$$

The final equation opens the door to test the separability assumption. From the baseline group, the joint probability function is given empirically:

$$\Pr(A, B) \quad = \quad \{\Pr(a_1, b_1), \Pr(a_1, b_2), \Pr(a_2, b_1), \Pr(a_1, b_2)\}$$

whereby $\Pr(a_i, b_j)$ is shorthand notation for $\Pr(A = a_i, B = b_j)$. For example, the probability $\Pr(a_1, b_2)$ denotes that a given human subject interprets the combination whereby the first word is attributed with the dominant sense ($A = a_1$) and the second word is attributed its subordinate sense ($B = b_2$).

In the case of 'boxer bat' the baseline joint distribution is depicted in Table 7. Each priming condition expresses a distribution $\Pr(A, B|\lambda_i), 1 \leq i \leq 4$. For 'boxer bat' the four distributions were empirically determined (Tables 8–11): These data allow the joint probability distribution $\Pr(A, B)$ to be computed assuming separability (4) as well as assuming uniform prior probabilities of the primes. The question then is whether the base joint distribution (see Table 7) and that computed under the assumption of separability (see Table 12) are really different. To address this question, a chi-square goodness-of-fit test at the 95% confidence level.

TABLE 10. Boxer bat:$\lambda_3 = \text{vampire}, n = 7$

	b_1	b_2
a_1	0.71	0
a_2	0.29	0

TABLE 11. Boxer bat:$\lambda_4 = \text{ball}, n = 6$

	b_1	b_2
a_1	0.5	0.5
a_2	0	0

TABLE 12. Joint distribution of 'boxer bat' assuming separability

	b_1	b_2
a_1	0.42	0.41
a_2	0.16	0

TABLE 13. Analysis of separability, n denotes the number of subjects used to compute the baseline joint distribution, and n_p the number of subjects used in priming experiments to compute the joint distribution assuming separability

Compound	Rank	Chi-square
boxer bat	2	$p = 0.00014, (n = 10, n_p = 34)$
bank log	2	$p = 0.14, (n = 10, n_p = 51)$
star charge	2	$p = 0.08, (n = 5, n_p = 29)$
apple suit	2	$p = 0.42, (n = 10, n_p = 45)$
fan post	2	$p = 0.42, (n = 10, n_p = 34)$
seal pack	2	$p = 0.47, (n = 10, n_p = 48)$
spring plant	1	$p = 0.0073, (n = 11, n_p = 47)$
racket pitch	2	$p = 0.07, (n = 10, n_p = 45)$
toast gag	2	$p = 1.0, (n = 10, n_p = 52)$
poker strike	2	$p = 0.54, (n = 11, n_p = 37)$

3.4 Results

The analysis methods would be working perfectly if the rank criterium would deem non-separability and there would be a corresponding significant difference between the baseline joint distribution and the factorized joint distribution (Equation 4). Unfortunately Table 13 does not exhibit this characteristic. The table shows that all concept combinations except for 'spring plant' were deemed non-separable according to the rank criterium. If the separability assumption was not holding in the probabilistic model, the table of results should report a significant difference between the baseline joint distribution and the joint distribution computed assuming separability. Only in the case of 'boxer bat' and 'spring plant' were there significant differences reported between the respective distributions. The latter raises a contradiction as the rank analysis deems 'spring plant' to be separable and the probabilistic analysis deems it to be non-separable.

The probabilistic analysis rests on two assumptions—the joint distribution corresponding to the constituent words is factorizable and also the prior probabilities on the priming words are uniform.

The latter assumption is contestable. Perhaps all that can be safely concluded is robust separability analysis that cannot be proceeded based on assuming a uniform distribution across the primes. Actual prior probabilities for the primes may be approximated using the statistical profile of the words appearing in language, or estimated from the University of South Florida word association norms based on the size of their respective associative networks. Having justifiable estimates of the priors will allow the focus to be solely on determining the validity of separability assumption (i.e. whether the joint distribution is factorizable, or not).

A further refinement of the above separability analysis is possible via the CHSH inequality of quantum theory [15], which provides an experimental test for distinguishing between local hidden variable theories. In the basic scenario, a source S emits two entangled photons, one travels left through a polarizer at region a, the other right through a different polarizer at b. The photons can reflect from each polarizer, or transmit through it, and the state describing the system becomes more complex again representing the different likelihoods of this occurring. Finally, two detectors in this system 'click', one on the left side, and one on the right. Coincidence is measured in this scenario, with $|11\rangle$ representing a situation where the two detectors requiring transmission through the polarizer click, and so on for the other states. Finally, the orientation of these polarizers can be changed, and this leads to a different proportion of photons being transmitted or reflected.

The results of this experiment are used to calculate expectation values for the four available combinations of two different polarizer settings, a, a', b, b':

$$E(i,,j) = \frac{N_{11} + N_{00} - N_{10} - N_{01}}{N_{11} + N_{00} + N_{10} + N_{01}} \text{ where } i \in \{a, a'\}, j \in \{b, b'\}. \tag{5}$$

If the two different sides of this experiment can be considered separately, then the expectation values for this experimental scenario will satisfy the CHSH inequality:

$$-2 \leq E(a,b) - E(a,b') + E(a',b) + E(a',b') \leq 2 \tag{6}$$

which provides us with a numerical test for the separability (or not) of a quantum system. If the system can be considered separable then the CHSH inequality will be satisfied. It is important to note that the motivating intuition behind equation (5) harbours the same form of separability assumption as used above in (4). It is assumed that the photon which passes through polarizer a does not interact with the one that passes through b and hence the joint probability distribution can be factorized [15].

In order to apply the CHSH inequality to concept combinations, the experimental protocol above needs to be refined to take into account the four polarizer settings by appropriate use of priming words For example, the compound 'boxer bat' would require the following polarizer settings:

1. $(a,b) = (\mathit{fighter}, \mathit{ball})$, primes = (sport,sport) senses
2. $(a,b') = (\mathit{fighter}, \mathit{vampire})$, primes = (sport,animal) senses
3. $(a',b) = (\mathit{dog}, \mathit{ball})$, primes = (animal,sport) senses
4. $(a',b') = (\mathit{dog}, \mathit{vampire})$, primes = (animal,animal) senses

where we have set the dominant meanings to the experimental markers, (a,b), etc. These choices are not maximal, as the biggest violation of the CHSH inequality with photons occurs for polarizer settings at $0°, 45°, 22.5°$ and $67.5°$. Unfortunately, we cannot yet identify the maximal 'angles' for violating the CHSH inequality in relation to the semantics of words. This is obviously an area for future work.

4 Summary and conclusions

This article draws a speculative connection between the non-separability of concept combinations, emergent associates and human abductive reasoning. Concept combinations are modelled as interacting quantum systems of superposed 'particles' which may become non-separable. The interaction is formalized by the tensor product of the interacting systems. The basis states of the tensor product correspond to possible hypotheses as to how the concept combination may be interpreted, and these interpretations are assumed to be superposed in human cognition. It is possible that human abductive reasoning selects the most likely in order to furnish an ultimate interpretive scenario for the concept combination in question.

Two methods of analysis were presented for empirically validating the presence of non-separable concept combinations. One method was based on quantum theory and another based on comparing a joint (true theoretic) probability distribution with another distribution based on a separability assumption using a chi-square goodness-of-fit test. Although these methods were inconclusive in relation to an empirical study of bi-ambiguous concept combinations, avenues of further refinement of these methods are identified. The door is definitely still open with respect to determining whether concept combinations are non-separable in human cognition.

If convincing evidence were found that concept combinations are non-separable in human cognition, then this would undermine reductive models which understand concept combinations solely in terms of the constituent words in the combination. Such reductive models explicitly or tacitly adhere to Frege's principle of compositionality. We don't argue that this principle is false, but rather a better understanding is required of when it can be legitimately applied. It is our conjecture that it may not always apply in human conceptual space.

Funding

This project was supported in part by the Australian Research Council Discovery grants DP0773341 and DP1094974, and by the UKs Engineering and Physical Sciences Research Council, grant number: EP/F014708/2.

References

[1] D. Aerts, S. Aerts,J. Broeckaert, and L. Gabora. The violation of Bell inequalities in the macroworld. *Foundations of Physics* **30**, 1378–1414, 2000.

[2] D. Aerts and M. Czachor. Quantum aspects of semantic analysis and symbolic artificial intelligence. *Journal of Physics A-Mathematical and General*, **37**, L123–L132, 2004.

[3] D. Aerts and L. Gabora. A theory of concepts and their combinations II: a Hilbert space representation. *Kybernetes*, **34**, 176–205, 2005.

[4] P. Bruza and R. Cole. Quantum logic of semantic space: an exploratory investigation of context effects in practical reasoning. In *We Will Show Them: Essays in Honour of Dov Gabbay*, S. Artemov, H. Barringer, A. S. d'Avila Garcez, L. C. Lamb, and J. Woods, eds, Vol. 1, pp. 339–369. College Publications, 2005.

[5] P. Bruza, K. Kitto, D. Nelson, and C. McEvoy. Is there something quantum-like in the human mental lexicon? *Journal of Mathematical Psychology*, **53**, 362–377, 2009.

[6] P. Bruza, D. Widdows, and J. Woods. A quantum logic of down below. In *Handbook of Quantum Logic and Quantum Structures*. vol. 2. K. Engesser, D. Gabbay, and D. Lehmann, eds. pp. 625–660, 2009. arXiv:quant-ph/0612051.

[7] P. D. Bruza, J. Busemeyer, and L. Gabora. Special issue on quantum cognition. *Journal of Mathematical Psychology*, **53**, 2009.
[8] D. Gabbay and J. Woods. Advice on abductive logic. *Logic Journal of the IGPL*, **14**, 189–219, 2006.
[9] L. Gabora, E. Rosch, and D. Aerts. Toward an ecological theory of concepts. *Ecological Psychology*, **20**, 84–116, 2008.
[10] P. Gärdenfors. *Conceptual Spaces: The Geometry of Thought*. MIT Press, 2000.
[11] D. Gentner. The mechanisms of analogical learning. In *Similarity, Analogy and Thought*. S. Vosniadou and A. Ortony, eds, Cambridge University Press, pp. 199–241, 1989.
[12] D. Gentner and A. Markman. Structural alignment in comparison: No difference without similarity. *Pschological Science*, **5**, 152–158, 1994.
[13] S. Hu and Z. Yu. The necessary and sufficient conditions of separability for bipartite pure states in infinite dimensional hilbert spaces. [*quant-ph*] arXiv:0704.0969v1, 2007.
[14] M. Humphreys, R. Pike, J. Bain and G. Tehan. Global matching: A comparison of the siam, minerva ii, matrix and todam models. *Journal of Mathematical Psychology*, **33**, 36–67, 1989.
[15] F. Laloe. Do we really understand quantum mechanics? strange correlations, paradoxes and theorems. *American Journal of Physics*, **69**, 655–701, 2001.
[16] A. Markman and D. Gentner. Structural alignment during similarity comparisons. *Cognitive Psychology*, **23**, 431–467, 1993.
[17] G. Murphy. Comprehending complex concepts. *Cognitive Science*, **12**, 529–562, 1988.
[18] D. Nelson, C. McEvoy, and T. Schreiber. The University of South Florida, word association, rhyme and word fragment norms. *Behavior Research Methods, Instruments & Computers*, **36**, 408–420, 2004.
[19] D. Widdows. *Geometry and Meaning*. CSLI Publications, 2004.
[20] M. Wilkenfeld and T. Ward. Similarity and emergence in conceptual combination. *Journal of Memory and Language*, **45**, 21–38, 2001.
[21] E. J. Wisniewski. Construal and similarity in conceptual combination. *Journal of Memory and Language*, **35**, 435–453, 1996.
[22] E. J. Wisniewski. When concepts combine. *Psychonomic Bulletin and Review*, **42**, 167–183, 1997.