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NONLOCALITY IN THE EXPANDING INFINITE WELL

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According to D. Bohm's interpretation of quantum mechanics, a particle always has a well-defined spatial trajectory. A change in boundary conditions can nonlocally change that trajectory. In this note we point out a striking instance of this phenomenon that is easy to understand qualitatively.

Key words: Bohm interpretation of quantum mechanics, quantum potential, expanding infinite well, nonlocality.

In a recent paper, R. E. Kastner [5] uses D. Bohm's [1] interpretation of nonrelativistic quantum mechanics to give an account of a simple example of quantum nonlocality (for recent work on Bohm's theory, see [2, 3, 4]). The example is the change of a single particle wavefuncion when it is forced to go through a tube. We would like to emphasize, though Kastner does not, that according to Bohm's theory, a particle always has a well-defined spatial trajectory, and a change in boundary conditions can therefore nonlocally change that trajectory. In this note we wish to point out a different and striking instance of this phenomenon that is easy to understand qualitatively. It is the familiar example of the expanding one-dimensional infinite

[c] Sadly, Professor Weingard died on September 14, 1996.

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well.

According to Bohm's theory, particles have definite positions and velocities at all times. The motion of these particles is determined by the usual Schrodinger equation and the Bohmian equation for velocity. If we write the wavefunction in the form, $\psi = R\exp[-iS]$, then the Bohmian velocity is given by

$$v(x) = \frac{\hbar}{m}\nabla_x S(x). \tag{1}$$

Interestingly, as we can see from (1), wavefunctions whose phase S is position independent correspond to particles at rest. Stationary state wavefunctions with real spatial parts thus have zero velocity.

Consider a single particle trapped in a one-dimensional box along the x-direction. To represent the box of length L, let the potentials become infinitely high at $x = 0$ and $x = L$. The solutions are $\psi_n(x) = A\sin(n\pi x/L)$. Because the energy eigenfunctions ψ_n are real, a single particle in such a state is at rest in the box.

Now, if the box suddenly expands at $t = 0$ to length L', the states ψ_n no longer remain stationary. They start evolving according to

$$\psi(x,t) = \sum_n c_n\psi'_n(x)\exp[-iE'_n t], \tag{2}$$

where the $\psi'_n(x)$ are the energy eigenfunctions of the expanded box, with energy eigenvalues E'_n. For the specific case of $L \to 2L$, then up to an arbitrary phase, we have the well-known results

$$c_2 = \frac{1}{\sqrt{2}}, \quad c_n = \begin{cases} 0, & \text{for even } n > 2, \\ \frac{\sqrt{32}}{4\pi n^2}, & \text{for odd } n > 0. \end{cases} \tag{3}$$

Writing $\exp[-iE'_n t] = \cos(E'_n t) - i\sin(E'_n t)$, we see that the phase of $\psi'_n(x)$ is no longer constant, but has the form

$$S = \tan^{-1}\left[\frac{-\sum_n c_n\psi'_n(x)\sin E'_n t}{\sum_n c_n\psi'_n(x)\cos E'_n t}\right]. \tag{4}$$

Since $\nabla_x S(x)$ is no longer constant, the particle will no longer be at rest for $t > 0$.

We wish to emphasize only one point: the expansion of the sides of the box nonlocally causes the particle to start moving! This is nonlocal in the sense that there are no classical potentials present in the box. Note that L can be arbitrarily large, and so the change can occur arbitrarily far from the particle. The effect does not require the well to double in size, but just to grow. In other words, an arbitrarily quick change can produce an effect arbitrarily far away, instantaneously.

But this effect is not an instance of the familiar quantum nonlocality involving entangled particle pairs, as in the EPR state. Rather it is more like the nonlocality found in Newtonian gravitational theory, wherein the gravitational force is locally determined as the gradient of the gravitational potential, although this potential is related by Poisson's equation to the global mass density. Because of this relation, changes in the matter density can nonlocally disturb the local gravitational field despite the fact that this field is locally determined by the potential. In the case of interest the quantum force responsible for moving the particle is also locally determined by a field, the ψ-field. But like the gravitational field potential, the ψ-field also permits infinitely fast disturbances. A change in the quantum field instantaneously affects the motion of all the particles.

Finally, Kastner, like Bohm, regards effects like this as being mediated by the "'quantum potential" $Q = -\hbar^2\nabla^2 R/2mR$. This thought follows from the fact that $mdv/dt = -\nabla(V + Q)$ where V is the classical potential. This is to interpret Bohm's equation of motion on the model of Newtonian mechanics. As is well-known, substitution of $\psi = R\exp[-iS]$ into the Schrödinger equation implies a generalized *Hamilton-Jacobi equation*

$$\frac{dS}{dt} + \frac{(\nabla S)^2}{2m} + V + Q = 0. \tag{5}$$

Nonetheless, there may be good reasons for not thinking of Bohm's theory in this way, not the least of which is that the momentum (or velocity) of a particle is not a dynamical variable in Bohm's theory. This is because the motion of the particle is determined by a particle's position and its wavefunction. Instead of thinking in terms of the quantum potential, it is also possible to think simply of the particle's motion being determined directly by its wavefunction via the first-order equation (1) (this point of view has been emphasized by [3]). The particle starting to move despite the lack of forces on it is then understood as a direct result of instantaneous influences in the ψ-field and not as the product of the operation of a new potential.

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