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A Birth-Death Toy Model for a Measure of Consciousness

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The ancient Ouroboros symbolism (one who eats oneself) is here integrated into a simple birth-death clustering process that needed nothing but itself for a transition from indistinguishable phases to a sort of higher level "conscious" phases. Birth and death coefficients are formulated in terms of odd and even exponentials used to represent a suitable form for conscious states via the internal transfer of information. This toy model may ideally quantify conscious states having inner causes via an Ouroboros index $0 < \Upsilon_{\alpha,\Omega} < 1$. The value $\Upsilon_{\alpha,\Omega} = 0$ lends to infinite loops and the limiting values $\Upsilon_{\alpha,\Omega} \rightarrow 1$ disclose transformation into awareness. Relationships with physics theories of consciousness and the use of the Oroborous index to discern about conscious-like states in Artificial Intelligence systems are discussed. Consciousness and freedom of will may go side-by-side in the model when $\Upsilon_{\alpha,\Omega}$ is extended to be a complex number modulus.

Keywords: Birth-death processes; Consciousness; Artificial Intelligence; Ouroboros spaces.

1. Introduction

The living experience of consciousness and comprehension is personal. The nature of this private inner sentient journey has been postulated to be governed by the laws of quantum systems, rather than from a classical description of matter made of atoms –the basic constituents of the universe (Penrose [1996, 2016]; Faggin [2023]; D'Ariano & Faggin [2022]). Atoms could mean particles, quantum fields or quanta of information (Qbits). The outer aspect of reality feeds instant meaning to our internal consciousness to express itself. Thus, we stand on both sides of this co-evolution among, say, an inner aspect entangling qualia states (perception of sensations), comprehension and free-will, and the external observable universe. Under these hypothesis, consciousness is a fundamental property of nature, and the *unity* among the inner and outer realities forms the basis for new theories of consciousness. The complexity of our world reflects the desire of consciousness to know itself (Faggin [2023]; D'Ariano & Faggin [2022]).

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The continuous sequence of existence –physical and biological events that occurs in our living time in an irreversible succession from the past to the present, makes us to think that a proper analytical theory of consciousness should be dynamical. In fact, the co-evolution of inner being and outer inputs becomes richer as we age. The universe cannot return to the exact state in which it was at an earlier time step while we gain experiences from it. In this context, the *unity* between inner and outer realities at all times, which one cannot do without the other, has been symbolized by the ancient iconography Ouroboros (Greek word translated as "tail-devourer") which depicts a snake or serpent swallowing its own tail. This union of *"capo-coda"* opposites, seem to form a perfect whole (unity), same as the yin and yang in Chinese philosophy (Mahdihassan [1989]). The Ouroboros has been a potential source of inspiration associated with mythology, religion, transmutation belief, philosophy, pop culture, natural phenomenon, etc. Spanning different cultures and millennia the Ouroboros, has attached rich meanings, legends and representations of nature such as cosmic harmony, immortality, eternity… (PCD [2018]). Motivated by these associations, in this paper the Ouroboros is integrated into a simple discrete, mathematical birth-death process to represent conscious-like states driven by an ad-hoc Ouroboros index $\Upsilon_{\alpha,\Omega}$. We refer to this approach as a "Birth-Death Toy Model for a measure of Consciousness" (BDTMC).

A toy model based on birth-death processes may help us to abstract the dynamic of conscious paths. The coexistence of birth-death rates at all times are here specified functions of natural exponentials. These functions spread the encoded information between transient states without loss of generality. The other Ouroboros model proposed in Ref.Thomsen [2011], describes an algorithmic architecture for cognitive agents to sketch themes of interest in Artificial Intelligence (AI) and consciousness. Similarly, this model also has a cyclic procedure at its core and is based on self-referential recursive processes with alternating phases of data acquisition and evaluation.

The circular Ouroboros, loosely depicted in Fig.1 with modern technologies, has also been interpreted as a symbol of self-reflection, represent the idea of seeking truth and wisdom within oneself and as a symbol for the pursuit of balance and harmony in spiritual life (Stanton [2023]). It came first in a 14th-century BC Egyptian religious text found in the tomb of King Tutankhamen (PCD [2018]). The deity Ouroboros also appears to illustrate creation through destruction in a passage about the origin of the sun god Ra through a union with the death god Osiris. Archaeologists discovered a dragon eating its tail engraved on a 7,000 years old jar in eastern China. The Ouroboros figure has also been a source of inspiration in science. It has played a role in the discovery of the linked carbon ring structure of a benzene molecule in 1872, and as an early symbol for mercury in 1625 –see Sterling [2013]; Stanton [2023] for excellent compendiums. The mathematical symbol for infinity ∞, and the two electron spin states, may be seen as a variant of the Ouroboros with the snake looped once before eating itself.

In nature, snakes can really swallow their own tails for a variety of plausible

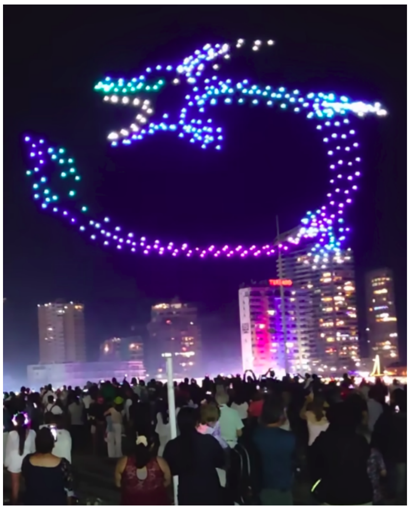

Fig. 1. Synchronized drone show at Cavancha Beach, Iquique (Chile), forming representative figures including a Ouroboros-like dragon –New year 2024.

reasons (see video [2024]). When a snake is unable to cool itself, hyper-metabolism coupled with disorienting effects, can result in a confused (unconscious) state. Since snakes are predators, they may be mistaking their own tail for prey –specially when in captivity. Stress-inducing factors such as living in a small tank, excessive light and noise are a manifestation of psychological disorder which can cause this occurrence. Other possibilities include illness or simply aging. However, there is not a definitive answer as to why this unawareness behavior of grabbing the end of their own tail it occurs. It is thankfully an extremely uncommon phenomenon which intrigues Herpetologists to this day.

2. Awareness Index

In 1D birth-death systems for studies of population growth (e.g., bacterial, customers queuing, etc (see Craw [2018], Canessa [2007])) the expected birth $\alpha(t) \geq 0$ and death $\Omega(t) \geq 0$ rates can be any specified functions of the time (or age) t. "Births" and "deaths" processes occur according to a simple relation for the normalized state probabilities p_j. Transient clusters or aggregates reflect the amount of cooperative information carried by the integer states $j = 0, 1, 2 \cdots N$ of a large system of size N. For two analytical birth-death processes these relations are

$$\hat{\alpha}_j(t)\; \tilde{p}_j(t) = \hat{\Omega}_{j+1}(t)\; \tilde{p}_{j+1}(t) \quad ; \quad \alpha_j(t)\; p_j(t) = \Omega_{j+1}(t)\; p_{j+1}(t) \quad . \tag{2.1}$$

Births and deaths processes ascend or descend the system state by one. Births rates start at state $j = 0$ and deaths at $j = 1$.

For times $t > 0$, each of these recursive relations yield the state distributions

$$\tilde{p}_j(t) = \frac{\hat{\alpha}_0(t)\,\hat{\alpha}_1(t)\,\cdots\,\hat{\alpha}_{j-1}(t)}{\hat{\Omega}_1(t)\,\hat{\Omega}_2(t)\,\cdots\,\hat{\Omega}_j(t)}\,\tilde{p}_0(t) \quad ; \quad p_j(t) = \frac{\alpha_0(t)\,\alpha_1(t)\,\cdots\,\alpha_{j-1}(t)}{\Omega_1(t)\,\Omega_2(t)\,\cdots\,\Omega_j(t)}\,p_0(t) \quad . \tag{2.2}$$

These processes can be determined subject to specification of the so-called absorbing states or initial conditions p_0 and the product of birth-death ratios at all previous states.

Let us postulate basic expressions for the transmission-of-flow via (odd and even) increasing and decaying exponential events among two coexistent processes as follows

$$\hat{\alpha}_j(t) = \alpha_j(t)\,exp\left\{-\,\frac{\Upsilon^{2j+1}_{\alpha,\Omega}}{2j+1}\right\} \quad , \quad \hat{\Omega}_j(t) = \Omega_j(t)\,exp\left\{+\,\frac{\Upsilon^{2j}_{\alpha,\Omega}}{2j}\right\} \quad . \tag{2.3}$$

Here we introduce the Ouroboros index $0 < \Upsilon_{\alpha,\Omega} < 1$ which is interpreted as the transformation, or switch, from mindless to more mindful processes along the j-states.

The equilibrium state probabilities given by Eq. (2.2) then become

$$\tilde{p}_j(t) = \frac{\alpha_0(t)\,exp\left\{-\,\Upsilon_{\alpha,\Omega}\right\}\;\alpha_1(t)\,exp\left\{-\,\frac{\Upsilon^3_{\alpha,\Omega}}{3}\right\}\,\cdots\,\alpha_{j-1}(t)\,exp\left\{-\,\frac{\Upsilon^{2j-1}_{\alpha,\Omega}}{2j-1}\right\}}{\Omega_1(t)\,exp\left\{+\,\frac{\Upsilon^2_{\alpha,\Omega}}{2}\right\}\;\Omega_2(t)\,exp\left\{+\,\frac{\Upsilon^4_{\alpha,\Omega}}{4}\right\}\,\cdots\,\Omega_j(t)\,exp\left\{+\,\frac{\Upsilon^{2j}_{\alpha,\Omega}}{2j}\right\}}\,\tilde{p}_0(t) \quad , \tag{2.4}$$

or

$$\tilde{p}_j(t) = \frac{\alpha_0(t)\,\alpha_1(t)\,\cdots\,\alpha_{j-1}(t)}{\Omega_1(t)\,\Omega_2(t)\,\cdots\,\Omega_j(t)}\,exp\left\{-\,\Upsilon_{\alpha,\Omega} - \frac{\Upsilon^2_{\alpha,\Omega}}{2} - \cdots - \frac{\Upsilon^{2j-1}_{\alpha,\Omega}}{2j-1} - \frac{\Upsilon^{2j}_{\alpha,\Omega}}{2j}\right\}\,\tilde{p}_0(t) \quad . \tag{2.5}$$

Using expansion of the natural logarithm: $ln(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \cdots$ for values of $|x| < 1$, the above discrete expression can then further simplified to

$$\frac{\tilde{p}_j(t)}{\tilde{p}_0} \approx \frac{p_j(t)}{p_0}\,exp\left\{ln(1 - \Upsilon_{\alpha,\Omega})\right\} = \frac{p_j(t)}{p_0}\,\left(1 - \Upsilon_{\alpha,\Omega}\right) \quad , \tag{2.6}$$

which, in turn, it reduces to a simple linear relation for the absorbing states of our birth-death processes

$$p_0 \approx (1 - \Upsilon_{\alpha,\Omega})\,\tilde{p}_0 \quad . \tag{2.7}$$

This BDTMC approximation is derived from the normalization condition $\sum_{j=0}^{N\to\infty} p_j(t) \equiv 1$. To some extend this result may concern the idea behind self-replicating Ouroboros spaces in mathematics: $f = f(f(\cdots f(f)\cdots))$ (Soto *et al.* [2011]). This result may also relate conjectural awareness phases of consciousness as we shall discuss next.

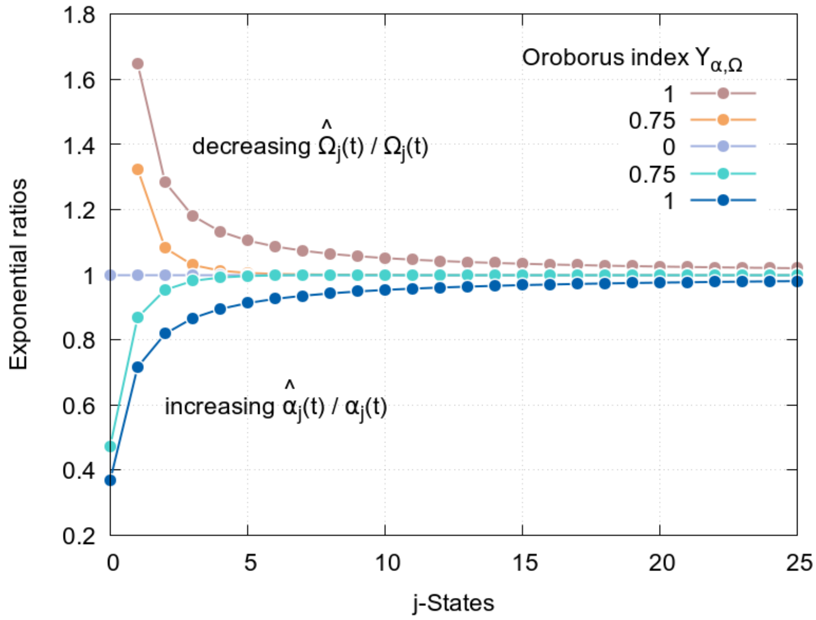

Fig. 2. Ratio of birth and death coefficients in terms of exponentials at each state j from Eq. (2.3) for different values of the Ouroboros index including limiting cases 0 and 1. These define a measure for the cooperative internal information through birth-death clustering processes.

3. Discussion

Recognizing signs of conscious experiences is important for the development of mathematical symbols of consciousness. Some degree of embedded consciousness may be mapped by focusing on the functional aspects of consciousness or by using the scientific method to explore strengths between active brain neurons and their synapses (Lahav & Neemeh [2022]). Neuroimaging experimental validations can only be statistical and limited to arbitrary thresholds. This experimental identification of brain activity from the connections between neurons may not necessary capture the subjective, soul experience of consciousness (Kent [2018]). If consciousness occurs from processes within brain micro-tubules of the cytoskeleton –as suggested by the ORC-OR theory of R. Penrose, 2020 Physics Nobel Prize winner (Penrose [1996, 2016]), quantum physical excitations have to take place at large scales. There is still room for new hypothesis behind consciousness and its relation to the physical domain (Riffini [2017]; Kleiner [2020]; Canessa [2020]).

A comprehensive approach of the consciousness has been recently formulated within relativistic theory as in Lahav & Neemeh [2022]. Idealized cognitive systems, in two different frames of reference, observe the same laws of nature in force and the same phenomenon of consciousness in their different frames. This formalism opens up interesting new possibilities for research. On the other hand, speculative approximations to the hard problem of consciousness by toy models have provided algorithms to illustrate consciousness in simple terms. For example, a simple toy model has been proposed that allow conscious perceptions of objects to be classical (without large quantum uncertainties) or highly quantum (having large variances within a single perception) (Page [2022]). Another toy model proposes a kind of brain system whose free energy contains an order parameter associated to the measure of a certain "amount of consciousness" (Silva [2021]). Let us discuss next how our alternative birth-death toy model also suggests a metrics to ideally quantify conscious states.

The formulation of the birth-death coefficients in terms of increasing and decaying exponentials at each single event for the ratio of births and deaths $\forall t$ as in Eq. (2.3), defines the cooperative internal information path between sequential clustering processes which is not produced by an arbitrary outsider observer. Such embedded information may relate analogous codings for the evolution of our consciousness. Different values of the Ouroboros index leads to the results shown in Fig.2. These test functions reflect simple processes relating $(\alpha(t),\Omega(t))$ cycles which tend to match if $\Upsilon_{\alpha,\Omega} \to 0$ and become distinct when $\Upsilon_{\alpha,\Omega} \to 1$, i.e., when ("conscious") variations associated to the Υ index are identified. Both exponential expressions tend to unity when $\Upsilon_{\alpha,\Omega} \to 0$ (and also for large states). According to Eq. (2.7), different realizations then become indistinguishable and independent with no computable correlations such that $p_0 \neq \tilde{p}_0$. The limiting values of the Ouroboros index relate infinite loops when $\Upsilon_{\alpha,\Omega} = 0$ and are interpreted as the awakening of awareness for $\Upsilon_{\alpha,\Omega} \to 1$.

As an illustrative example, in Fig.3 we display a simulated birth-death process for different values of Υ. The discrete simulation of birth-death processes at times $t_0 < t_1 < \cdots < t_n$ assume standard Markov chains and the use of our explicit odd and even exponential expressions for the probability distribution of the j-states (or population size for classical birth-death models). We account for both birth and death events by the so-called "first-reaction" method of the Gillespie algorithm starting from a given absorbing state p_0 (c.f., Eq. (2.7)). We first generate two random numbers and compare them to see which event happens first. After this event, the population size changes by (plus or minus) one depending on the event being birth or death. Then the total birth and death rates among the j-states grow accordingly multiplied by such states at the time points. We then repeat these steps and the simulation proceeds in a similar fashion one event at a time.

The simulated sample outcomes of Fig.3 reveals that for large time the event curves do not go extinct (say, run forever). Within our toy model, convergence of events becomes extremely unlikely when the Ouroboros index $\Upsilon_{\alpha,\Omega} \to 1$. This would mean that idealized consciousness along j-events grows up on time. These curves become statistically close to each other for $\Upsilon_{\alpha,\Omega} < 0.5$. Once the system is below these levels, our mindful states become fast merging due to significant stochastic fluctuations starting pretty early on time. Higher levels of consciousness

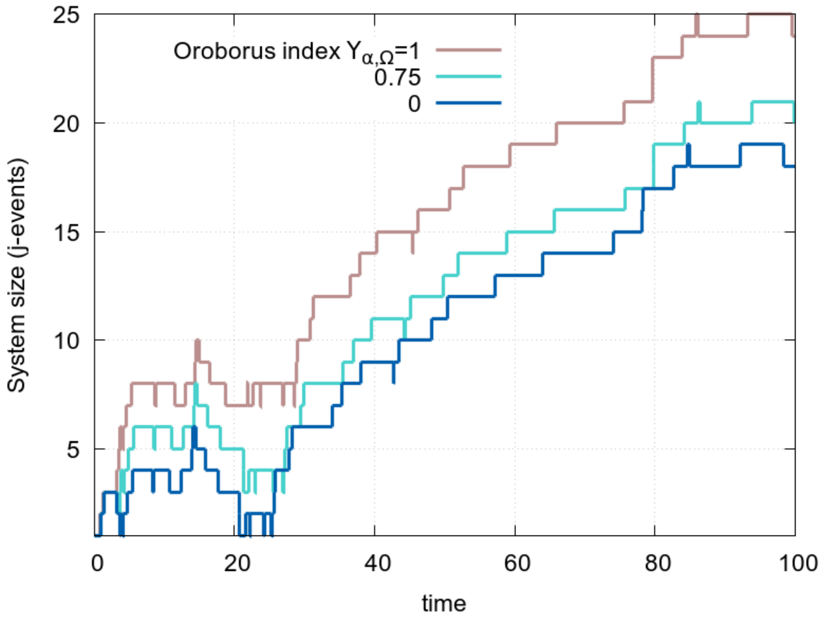

Fig. 3. Simulated sample outcomes of sequential birth-death processes in terms of exponentials for different values of the Ouroboros index $\Upsilon_{\alpha,\Omega}$. This birth-and-death toy model may define a metrics to ideally quantify conscious-like states. If an entity runs an algorithm on a loop, then it is defined as being on a mindless phase (i.e., it has a vanishing Oroborous index). When the entity departures from the Oroborous spaces $f \neq f(f(\cdots f(f)\cdots))$, the entity is say to be on some high level conscious state with the initial condition $p_0 \neq \tilde{p}_0$ of Eq. (2.7).

have been idealized in the limit $\Upsilon_{\alpha,\Omega} \to 1$. Processes become indistinguishable (absent of diversity) for $\Upsilon_{\alpha,\Omega} \to 0$. Besides these stochastics simulations, there exists quantitative investigations into birth-death processes, with birth or death rates given by an arbitrary function of the system size (Park [2023]). Temporality, measured with clocks, is another essential ingredient to discern on any probabilistic birth-death outcome that may account for conscious-likes states.

Let us uncover next some possible relationships between the proposed BDTMC and existing physics-based theories of consciousness, and also analyze the possibility of using the Oroborous index to recognize fictitious conscious-like states in AI systems.

3.1. *BDTMC and Physics theories of consciousness*

Without resorting to the transcendent or metaphysics, the ORC-OR theory on what consciousness could be, like the theory of D'Ariano & Faggin [2022], are first attempts to make hypotheses based on scientific knowledge of our physical reality. The Penrose-Hameroff microtubules conjecture (Penrose [2016]), and their associated quantum physical excitations, may give rise to increasingly refined and plausible points of view of conscious arousal, as our knowledge of the physical domain progresses. Consciousness and free will are properties that must have existed since the beginning of the Universe. For physicists, these do not emerge from a relation energy-matter which is without consciousness and without free will.

Human experiences are lived, and become comprehensible with the passing of time. The processes involved in conscious thinking are not so far entirely described by today's physics. An issue remaining open is the fact that the human experience of consciously understanding something is not computational. Consciousness by itself seems not to be a computational process. According to Faggin's framework, consciousness is a fundamental property of reality. Entanglement is inherent in quantum systems in a pure state which is conscious. It has the experience (qualia) of its own state, not computable. There must be "something (a field) outside the physics world" which seems to be still relevant.

The qualia aspects of human consciousness, e.g., appreciation of beauty, love, flavors, sensations... are not understood. We become aware of these, and recognize them, without having a proven comprehension. In this respect, within BDTMC, one may think of the Oroborous index as just disclosing a "degree" of awareness via the value of $\Upsilon_{\alpha,\Omega}$ for the temporal progression from mindless to more mindful phases due to some birth-death iterations driven by inner causes only. Thus, *birth-death processes may account for a measure of awarness in consciousness.* To this end, it is interesting to note that recent physics experiments has shown that a close system of three identical two-level atoms in free space prepared in particular entangled single-photon excited states can display "birth", "death," and a nonperiodic "revival" of spontaneous emissions in particular directions Mährlein [2020]. This observation of spontaneous decays at a distance may induce novel research in fundamental science as well as multiple applications using quantum technologies. Solvable examples of quantum versus classical birth and death processes have been recently discussed by Sasaki [2022]. All of these findings may be extended to include the present BDTMC by keeping in mind that quantum information cannot be copied.

3.2. *The Oroborous index and consciousness in AI systems*

Consciousness is a non-algorithmic phenomenon according to postulates by F. Faggin (Faggin [2023]; D'Ariano & Faggin [2022]). Consciousness is private, exists through self-awareness with one's own feelings and experience. This personal phenomenon posses challenges to the scientific community since proposed models theorize on an event which is unobservable from the outside. Experimental limitations to discern which theory of consciousness is the correct have open a great debate in the literature (Lenharo [2023]). So far, indexes of consciousness rely on the indirect prognosis of individuals interacting with the surroundings and by deriving protocols for the patterns obtained from sensory data (Casali *et al.* [2013]; Liuzzi *et al.* [2023]). It has been argued that consciousness may be related to the complexity of functional brain networks. However, it is far from clear how a group of neurons in a particular area of our brain may contribute to consciousness. We cannot rely on a definitive, complete understanding for what defines consciousness by measuring brain activity through signals. We experience different things across time and we are only conscious of the space around us. Perhaps the brain signals may be only part of these perceptions transformed into symbols. Although a simple birth-death toy model for a tentative measure of consciousness is not based on a trained deep-learning method or brain neural network approaches it may still offer some insights inspired by the human experience.

Consciousness has been mostly absent from discussions about AI systems (Lenharo [2024]). The new so-called indicator properties of consciousness elucidated in computational terms, could validate AI systems to develop recognizable human tasks (Butlin *et al.* [2023]). This does not imply AI models are actually being conscious. Bringing cognitive sciences and neuroimaging data into computational AI models have been presented as tempative approaches to get some insights into consciousness. In particular, Integrated information Theory (ITT) in relationship with AI and cognitive sciences may describe if an entity is in a conscious state or not (Guerrero [2023]). Let us see how BDTMC may be complementary to ITT.

There are correspondences between ITT and BDTMC formalisms. ITT is based on few statments. One of these statments is called intrinsic existence indicating that consciousness exists independently of external observers. We have represented this axiom through the Ouroboros circular symbolism. In ITT the underlying system contains a set of N elements A, B, C that interact with each other as illustrated in Fig.4. Such interactions change the state of elements by means of some mechanisms. Changes in BDTMC relate the different birth-death coefficients of Eq. (2.3) among two coexistent processes as also shown in Fig.4. ITT Elements can have two states (0 and 1) receiving inputs that can change their output state at the current moment on time t, this is equivalent to our birth and death progression but $\forall t$. A single state with probabilty $p_i(t)$ in the BDTMC influences the overall system. The inference of each component on the system is also quantified. Ouroboros indexes $\Upsilon_{\alpha,\Omega} \to 1$ represent higher levels of consciousness, and the limit $\Upsilon_{\alpha,\Omega} = 0$ concurrent processes become indistinguishable representing a complete annihilation of awareness.

To associate AI and ITT to conscious activity, the elements of a system are considered as neurons in the states “on” or “off”. Each neuron receives a 0 or 1 input from 3 nearest neighbours neurons (in 2D) depending on its state. If the sum of these inputs is larger that a given threshold, neurons then change their state and this state became the output. A similar process is adopted within the “first-reaction” method used in our 1D simulations shown in Fig.3. Random numbers are generated and compared within a given threshold to deduce if this is a birth or death event. Conditional probabilities in ITT constrain a future state and also a past state, and a minumum information partition gives the amount of interdependent information (Φ). The birth-death processes evolve over time under (the product of) some probabilities coefficients in terms of increasing and decaying exponentials

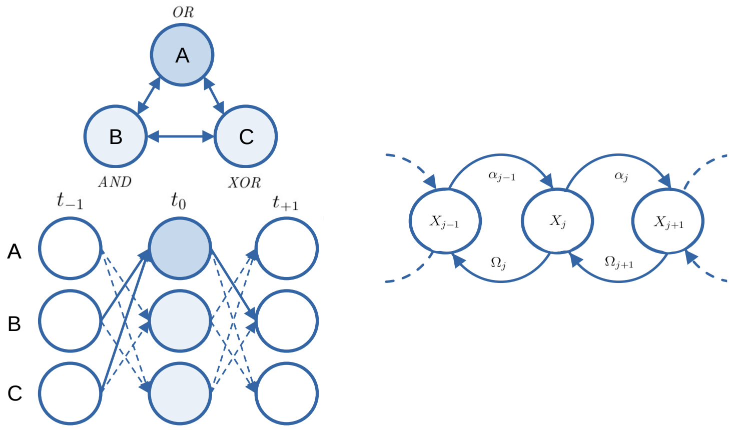

Fig. 4. *L*eft: Causal graph of different integrated mechanisms M={A,B,C} in a state at present time. *R*ight: Birth-death Markov chain. $\forall t$, the process can go from state j to $j+1$ or $j-1$ independently. In both approaches inner inputs create outcomes and such outcomes create inputs for succesive events.

defining internal information between sequential aggregation processes.

IIT has been criticized since its claims are not scientifically established or testable (Lenharo [2023]). However the theory could be still useful as a symbolic representation of conscious states in AI and the possible emergence of conscious-like states in machines. It may be also useful to delineating AI ethics guidelines for the behaviour or action performed by an AI entity independently to whether the AI system feels or understands anything (Chella [2023]). As a complement, within BDTMC, if a machine or entity keeps following or doing a pre-defined algorithm on a loop, then it is not a conscious entity (i.e., it has a vanishing Oroborous index). Whereas when the entity departures far from the Oroborous spaces $f \neq f(f(\cdots f(f)\cdots))$, this means that the entity is in a conscious state far from the absorbing, initial state (i.e., $p_0 \neq \tilde{p}_0$ as in Fig.3). Our hope with the BDTMC methodology is to recreate an alternative idealized scenario for consciousness as an educated guess.

4. Conclusion

A cosmic Ouroboros has been suggested by S. Glashow, 1979 Physics Nobel Prize winner, for a visual descriptor of the continuity of vastly different size scales between the quantum and cosmic worlds (Glashow & Bova [1988]). Cycled processes may represent a symbol of immortality, since the Ouroboros destroy himself, nurtures himself, and gives birth to himself. In Hinduism, the Ouroboros symbolizes the cycle of life (samsara) from birth to death to reincarnation and rebirth (Stanton [2023]). It also relates the concept of karma. Everything eventually circle around and come back. Yet in both Hinduism and Buddhism, the goal is to reach a state of total enlightenment and get off of this loop (the Ouroboros exit). In our highly idealized birth-death toy model inner inputs create outcomes and these outcomes, in turn, are used as inputs for succesive events. Higher levels of consciousness awake for Ouroboros indexes $\Upsilon_{\alpha,\Omega} \to 1$. In the limit $\Upsilon_{\alpha,\Omega} \to 0$ concurrent processes become indistinguishable (via infinite loops) representing the annihilation of awareness. Thus, birth-death processes as proposed could be a marker for awarness in consciousness.

Our model is not meant to describe actual human sensations –the feelings we experience in life. In real life, the emergence of conscious experiences is still a mystery. Our inner experience is not reproducible. We cannot transfer exactly what we sense inside ourselves. Feeling meanings references to ourselves. Innovative new experiments and analyses are much needed for the hard problem of consciousness (Bayne *et al.* [2023]). It is also a great challenge to understand free-will theoretically (D'Ariano & Faggin [2022]). One may extend the Oroubori index as being the modulus $\Upsilon_{\alpha,\Omega} = |z|^2 = \Re(z)^2 + \Im(z)^2$ of a complex number $(\Re(z), \Im(z))$ or complex wave $\psi = Ae^{i\theta}$. The real part could identify conscious states (private sense of self), and the imaginary part free-will (public unpredictable event). Within BDTMC, consciousness and freedom of will may then go side-by-side.

Our system simulations with exponential channels undergoing birth-death events are just a rough approach to reveal an indirect measure of inner conscious states, consistent with the current formalisms in which consciousness exists independently of external observers. BDTMC aims at stimulating further discussions along these lines, specially when reasoning on fictitious conscious-like states within AI entities.

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