Logic and Pragmatism. Selected Essays by Giovanni Vailati

C. Arrighi, P. Cantù, M. De Zan, and P. Suppes

January 22, 2010

CENTER FOR THE STUDY
OF LANGUAGE
AND INFORMATION

Contents

Preface

This book is the result of a joint cooperation of several scholars that are interested in Vailati's works for different reasons, but who have in common the belief in the opportunity of making a wide selection of his texts available for the first time in the English language, thus enabling a revival of Vailati's studies.

Vailati's work became known quite early, given that his complete writings were published soon after his death in 1911.[1] However, in the first half of the 20$^{\text{th}}$ century, neoidealism in Italy came into a strong hegemonic position; this philosophical movement rejected the dialogue between science and philosophy and refused to consider pragmatism as a philosophical movement, and Vailati's figure, who had always pleaded against the polarization into 'two cultures', was soon forgotten. It was not until the fall of Fascism and the end of the Second World War that Vailati's writings came to be appreciated again in Italy. A list of references on Vailati that is included in the Selected Bibliography at the end of this volume shows an interest in several aspects of his work, ranging from economics to philosophy of knowledge, from logic to ethics and philosophy of religion, from history of science to psychology.

In 1959 Vailati's rich correspondence, his manuscript annotations and his library were acquired by the Department of Philosophy of the University of Milan. This allowed for the publication of a new, extended edition of Vailati's writings by Mario Quaranta in 1987,[2] and the edition in 1971 by Giovanni Lanaro of Vailati's correspondence including letters to Pareto, Mach, Welby, Fogazzaro, Vacca, Brentano, Papini, Prezzolini, and from Enriques, Croce, Calderoni, Amato Pojero, and

[1] M. Calderoni, U. Ricci, Giovanni Vacca, eds. *Scritti di G. Vailati, 1863-1909*. Leipzig-Firenze: Barth, 1911.

[2] M. Quaranta, ed. *Scritti. Giovanni Vailati*, Bologna: Forni, 1987.

Salvemini.[3] Although Lanaro's edition already contains a vast selection of Vailati's letters, further correspondence with Amato Pojero, Croce, and Brentano has been published at a later date,[4] and some unpublished correspondence with Russell, Pikler, Wohlwill, Duhem, and Schiaparelli has been recently edited on the Yearbook of the *Vailati Study Center*[5] in Crema—Vailati's hometown,— a center which promotes researches on Vailati and the publication of materials from the Archive.

This year, being the centenary of Vailati's death, a revival of interest in his works has taken place, especially in Italy, where two International Conferences (Milan, October 7–8; Bologna October 11–13) will analyze several aspects of Vailati's philosophy and epistemology. A great deal still remains to be said and this volume, which arose from a conjoint project of Mauro De Zan, the director of *Vailati's Center* in Crema, and Patrick Suppes, Lucie Stern Professor of Philosophy Emeritus in Stanford, is intended not only as a means to celebrate the centenary of Vailati's death, but also as a tool for future research.

The reasons for the choice of the essays contained in this volume are only partly evident from the title. On the one hand the chosen writings serve to illustrate Vailati's original form of pragmatism, which does not date only from Vailati's collaboration with the Italian Pragmatist group formed by Papini and Prezzolini around the journal *Leonardo*, but is also connected to Vailati's participation to the Italian school of Mathematical Logic. Paola Cantù and Mauro De Zan show in their presentation of Vailati's life and works the relevance of his interest for the logical problem of definition but also for the historical study of scientific ideas, and of his critical analysis of the conceptions of Hume, Berkeley, Brentano and of Positivist scientists. Vailati's contribution to pragmatism, as shown in the essay by Maria Caamano and Patrick Suppes, includes the development of a criterion to establish empirical meaningfulness based on predictive elements, and in particular on conditional expectations, underlining the important role of deliberate actions as the grounds for objectivity—both in language and science.

On the other hand the chosen writings will also show Vailati's in-

[3] G. Lanaro, ed. *Giovanni Vailati. Epistolario, 1891-1909*. Torino: Einaudi, 1971.

[4] See A. Brancaforte. *Giovanni Vailati e Amato Pojero. Epistolario: 1898-1908*. Milano: Angeli 1993; C. Rizza, ed. *Benedetto Croce-Giovanni Vailati. Carteggio (1899-1905)*. Acireale-Roma: Bonanno, 2006; and R. M. Chisholm and M. Corrado, eds. "The Brentano-Vailati Correspondence" *Topoi*, 1:3–30, 1982.

[5] See M. De Zan, "I carteggi europei di Vailati," in *Annuario del Centro Studi Giovanni Vailati*, 2004, pages 19–52, R. Pettoello, "Il Carteggio Pikler-Vailati (1892-1908)," in *Annuario del Centro Studi Giovanni Vailati*, 2005-06, pages 83–106, and M. De Zan, "Il Carteggio Vailati-Schiaparelli (1897-1900)," ibid., pages 107–118.

terest for science and the humanities, and his early criticism of the polarization of knowledge into "two cultures," based on the idea of certain similarities between their respective languages and argumentation schemes. The variety of Vailati's interests was probably related to his freedom from Academic duties, but was also a result of his defense of the unity of knowledge, and his belief in the fruitfulness of speculative analogies between different fields, including the newborn scientific disciplines of psychology, sociology and economics.

Translator's Notes

C. Arrighi

This translation project started while I was working as a research assistant for Patrick Suppes in Stanford, when he asked me to retrieve some material about the pragmatist views of Giovanni Vailati. While working on an article about Bruno de Finetti, he noticed that de Finetti, in *Theory of Probability* (1974), mentions Vailati's writings on pragmatism as a work that he "particularly admires." Given the scarce number of Vailati's writings available in English, I undertook the task of translating his two main articles on pragmatism (chap. 18 and 19 of this volume). Suppes really appreciated them, especially for the prominence of the role played by predictions and conditional expectations in Vailati's pragmatist approach to epistemology. In his recent article about de Finetti, Suppes writes: "It is unfortunate that the work of the Italian pragmatists is not more available in English."[6]

From here came the idea of extending this limited initial project, and the evolution of this idea into this volume has been possible thanks to a grant provided by the *Centro Studi Giovanni Vailati* in Crema and the scholarly collaboration of Mauro De Zan and Paola Cantù, respectively the president and one of the members of the *Centro Studi*.

I translated most of Vailati's articles in this volume, with the assistance of Wendy Hall for extensive corrections of the drafts and the advice of Patrick Suppes regarding technical terminology.[7] The excep-

[6] "Some philosophical reflections on de Finetti's thought," in Galavotti 2009, *Bruno de Finetti, Radical Probabilist*, London: College Publications, p. 21.

[7] Several excerpts translated into English by Robert Innis have also proved helpful. See Innis, Robert E. 2002. *Pragmatism and the Forms of Sense: Language,*

Logic and Pragmatism. Selected Essays by Giovanni Vailati.
C. Arrighi, P. Cantù, M. De Zan, and P. Suppes.

tions are the articles "Pragmatism and Mathematical Logic," "A Study of Platonic Terminology," "The Attack on Distinctions," and "On Material Representations of Deductive Processes" (chap. 12, 13, 14, and 15)—these articles were translated and published by contemporaries of Vailati (as indicated at the beginning of each article in this volume), and we are just reprinting them for the reader's convenience.

In Vailati's work we can see an abundance of quotations, many of which are reported in their original languages, including ancient Greek and Latin. Given that the ability to read several foreign languages is not common, we have opted, in general, for quoting the same passages from an English edition, as indicated in a footnote. The original passages in French, German, Italian, Latin and Greek have generally been omitted with some minor exceptions:

- well known passages as the one from Moliére's *The imaginary Invalid*;
- quotations and terms that are relevant for the understanding of Vailati's thesis, as some excerpts from Leibniz, and many words in ancient Greek (especially chap. 7 and 13);
- verses from poems, as in the case of Michelangelo, Dante, Schiller and Berni.

If no English translation is mentioned, then I have translated the passage myself from an Italian edition, with the help of Paola Cantù for some passages in Latin, French, German and ancient Greek. This is also the case for some passages (mainly from Aristotle) that Vailati most likely quoted by memory, and therefore do not correspond exactly to the original text.

At the beginning of every essay by Vailati a footnote indicates where the article was originally published. However, the articles used for the translation are the ones found in the collection of Vailati's writings published after his death (1911). The only exception is "The Difficulties Involved in a Rational Classification of the Sciences," (chap. 4) which Vailati originally wrote in French, and was recently published in Italian by Mario Quaranta in a selection of Vailati's philosophical writings (*Giovanni Vailati. Gli strumenti della ragione*, Il Poligrafo, 2003). This Italian version is the one used for the translation.

The translator's or editors' notes and additions, even when integrating Vailati's own footnotes, are indicated in squared brackets. Regarding the references, the editors have decided to leave in the articles those provided by Vailati himself, to keep the integrity of the original text,

Perception, Technics. University Park, PA: Penn State Press.

though we also have added a list of references at the end of each article, including:

- the references already mentioned in full by Vailati in the text;
- complete references to works mentioned by Vailati, even if we are not sure of the edition he had access to—sometimes we have chosen an edition known to be part of Vailati's personal library, of which we have record;
- references to volumes used to retrieve an English version of a passage quoted by Vailati. In general, this information is included in the bibliography entry of the original work. For example: "France, Anatole. 1885. *Le livre de mon ami*. Paris: Lévy. Engl. transl. by J. Lewis May and B. Miall. *My friend's book*. In *The Works of Anatole France*, vol. 5. London New York: Lane 1908."

To Paola Cantù goes the credit for retrieving many sources of quotations that were not clearly indicated neither in the original text nor in the Italian editions of Vailati's writings, and for organizing and formatting the extensive list of references. Mauro De Zan has collected the additional bibliography found at the end of this volume, to encourage further studies.

One last note on Vailati's style of writing. As Patrick Suppes pointed out in one of our exchanges, "the sometimes impenetrable style of thinking and writing in German philosophy of the 19$^{\text{th}}$ century has often been commented on. This tendency was present in more philosophers of the last half of the 19$^{\text{th}}$ century than we care to mention, not only those writing in German but also in Italian, French, and English. Vailati was affected by this style of writing, even if much less so in his thinking." As a translator, I necessarily had to make certain decisions about terminology and syntactic structures. I have followed Vailati's writing quite closely, because one of the intentions of the editors was to convey Vailati's style; but at the same time I have tried to ensure that the English would flow as well as possible, given the intrinsic differences between the two languages—sometimes I have opted for a paraphrase of an entire convoluted paragraph, not just because a close translation would have "sounded" wrong, but because it would have introduced ambiguities in the meaning. Notwithstanding these limitations and some inevitable mistakes on my part, I hope that this translation will be of some use.

Acknowledgments

The editors would like to thank the University of Milan, the Philosophy Department and the library staff for facilitating Paola Cantù and Mauro De Zan in their search for manuscripts and texts, the Vailati Center in Crema for offering Claudia Arrighi a grant that allowed her to translate Vailati's texts into English, Mr. Paolo Casini and Mrs. Anna Casini Paszkowski for the caricature of Giovanni Vailati (drawn by Armando Spadini) that is reproduced in the cover. We are grateful to Dikran Karagueuzian of CSLI Publications for believing in this project and in particular for his endless patience. A special thanks goes to Wendy Hall for painstakingly correcting the original drafts of translated articles, to Andrew Haigh for revising the English in the first introductory essay, and to Sarah Terman, Daphne Humes, and Maya Juarez for their proofreading. We would also like to warmly thank all colleagues, friends, and family members who helped us with comments and encouragement.

Part I

Introductory Essays

Life and Works of Giovanni Vailati

P. Cantù and M. De Zan

Giovanni Vailati (1863-1909)

Giovanni Vailati was born in Crema in April 1863. Crema is a small, medieval town, situated forty kilometres south-east of Milan, in Lombardy, but which for many years was under the dominion of the Venetian Republic. Vailati's parents belonged to the city's aristocracy, and so followed a typical Venetian lifestyle—spending the winter in a palace in the town center, where they frequented the theatre and the nobles' club ("ridotto dei nobili")—and spending the summer in their villa in Offanengo, where they remained to run the farm estate until the first autumnal mists began to rise up from the many ditches that irrigate the fertile country of Crema (Greenfield, 1934).

It was common for the children of the Lombard and Venetian aristocracy to be sent to renowned religious colleges belonging to secular orders, and so it was that Vailati first attended the Barnabite College (boarding school) in Monza and then the Barnabite Gymnasium in Lodi, where he studied classical humanities and sciences until 1880. Vailati later abandoned religion during his time at university, but maintained friendly relations with his old teachers, to whom he sent copies of his works.

Education and Academic activity in Turin (1880-1899)

In autumn 1880 Vailati moved to Turin, and for two years attended the courses of the Mathematics Faculty. He then passed to the Real School for Engineering, where he got a degree in Civil Engineering in

Logic and Pragmatism. Selected Essays by Giovanni Vailati.
C. Arrighi, P. Cantù, M. De Zan, and P. Suppes.

1885. After that, he did not start working as an engineer but continued his mathematical studies, finally getting a degree in mathematics in 1888.[8] Vailati was interested in continuing his life in Turin, where he got a position in 1892 as the assistant to Giuseppe Peano for his course in Infinitesimal Calculus.[9]

Turin, which had been the capital of the newborn Kingdom of Italy from 1861 to 1865, was becoming a modern industrial town, deeply influenced by a positivist and pragmatic mentality: the development of scientific and experimental laboratories had a great impact on the cultural scene. Among the scientists who were active in Turin at the time were sociologists such as Cesare Lombroso; physiologists such as Jacob Moleschott and Angelo Mosso; natural scientists such as Michele Lessona; psychologists such as Enrico Morselli, and economists such as Salvatore Cognetti de' Martiis. But in the same years Turin was also well-known for its lively cafés, where Nietzsche spent the last intellectually active period of his life (Verrecchia, 1978).

From 1880 onwards, Vailati took the habit of writing down annotations and comments on the books he had read in small notebooks (*quaderni*), some of which (about 130) are now conserved in the Vailati Archive of the Philosophical Institute of the University of Milan. Vailati's interests included ethics, epistemology, economics, statistics, linguistics, and psychology, but were mainly devoted to evolutionist positivism and experimental psychology. He read defenders of social Darwinism, including John Fiske, John Lubbock, and especially Herbert Spencer's *First principles* (1862). But he also read recent results in experimental psychology, including Ludwig Büchner, Théodule Ribot, Francis Galton, Henry Mandsley, Prosper Despine and Cesare Lombroso, since he was driven by an interest in the complex relations between hereditary characters, mental pathologies, special skills, criminal behavior and the subconscious. He shared Lombroso's idea that the investigation of people suffering from mental illnesses could be useful to the inquiry of the human mind.

His deep involvement in anthropology and psychology was accompanied by a growing interest in philosophy, in particular in authors who had developed a non-reductionist description of mankind. Vailati was known among his fellows as the 'philosopher,' spending most of his time in Turin's National Library, reading classics of philosophy. He devoted particular attention to the *Dialogues et fragments philosophiques* by

[8]Unfortunately many documents of the University Archive in Turin were lost, so the title, the content, and the name of the supervisor of Vailati's master's thesis are unknown.

[9]For a deeper analysis of Vailati's university years, see De Zan 2009.

Ernest Renan (1876), and, probably through the reading of the former, to the writings of Arthur Schopenhauer, whose aphorisms are often accurately copied in the notes. But he also developed a deep interest in William K. Clifford's writings about the philosophy of science, and in the theory of knowledge—especially the works of Locke, Hume, and John Stuart Mill. Reading Mill's *Examination on the Hamilton's Philosophy* (1865) and the *System of Logic* (1870) at the same time as the *First Principles* of Spencer (1862), Vailati argued against any philosophical attempt at hypostatizing scientific concepts, or at grounding knowledge on *a priori* intuition, thus adhering to Mill's criticism of Hamilton's theory of knowledge in order to develop a criticism of Spencer's metaphysics (De Zan, 2009).

This criticism is related to an anti-Kantian feature of Vailati's thought that is one of the main divergences from Peirce's form of pragmatism. While Peirce often considers Kant's theses in a positive way, tracing the origin of the expression 'pragmatism' to Kant himself,[10] Vailati always criticized any revival of Kant's theory of knowledge, repeatedly opposing Kant to Locke, whose *Essay Concerning Human Understanding* he had read carefully already in 1882. In particular, he praised Locke's criticism of the misuse of language in scientific and philosophical disputes, an issue that motivated his own pragmatism.[11]

Having finished his studies at the university Vailati went back, rather reluctantly it seems, to Crema, where he secured some administrative work for the local municipality, allowing him plenty of time to devote himself to theatre, modern literature, and philosophy. In 1890 he became a member of the *London Society for Psychical Research*, whose aim was—in Vailati's own words—"to promote by any means unprejudiced, and careful investigations, the knowledge of the causes of error in observations, and of the precautions to take against memory's illusions and the tendency of language to put in the description of facts something that was not actually observed."[12]

Vailati's interest for psychic phenomena, though never accompanied by specific investigations into this field, is evident from the number of writings he devoted to the topic from 1896 to 1900. In 1896 he participated in the *International Conference in Psychology* in Munich. In some articles published in Lombroso's "Archivio di Psichiatria" and

[10] See Peirce 1905.

[11] See the Inaugural Lecture to the Course in History of Mechanics that Vailati gave at the University of Turin in 1898 (this volume, chapter 2).

[12] See Vailati's letter to Giulio Cesare Ferrari—the future traslator of William James's *Principles of Psychology* into Italian—of 1896, September 5, in Vailati 1971, pp. 59–60.

in "Rivista di Studi Psichici," Vailati criticized Spiritism, but he believed in the possibility of telepathy. He was mainly concerned with the necessity of developing a rigorous method of investigation of psychic phenomena, avoiding both the dogmatic prejudices of Positivists, and the ingenuous approach of many scholars in psychical research. To the latter he recommended the study of the history of science in order to avoid the hypostatization of concepts, an error that in the course of history was repeatedly made, for example in mechanics and in physical sciences.

The collaboration with Peano and the journal "Rivista di Matematica"

Vailati's collaboration with Peano's "Rivista di Matematica" started in 1891, and in 1892 he began his academic activity at the Mathematics Faculty of the University of Turin, where he obtained a position as Assistant Lecturer in Infinitesimal Calculus, the course taught by Giuseppe Peano.

In the first two volumes of the *Rivista* (1891-1892) Vailati published three articles on mathematical logic and an article on the foundations of geometry: "Un teorema di logica matematica" [A theorem of mathematical logic] (1891b), "Le proprietà fondamentali delle operazioni della logica deduttiva" [The fundamental properties of the operations of Deductive Logic], "Dipendenza fra le proprietà delle relazioni" [Dependence between the properties of relations], and "Sui principi fondamentali della geometria della retta" [On the fundamental principles of the geometry of the straight line].[13] In the third and fourth volumes (1893-1894) he published some reviews of books in mathematical logic by Nagy (1892) and Burali-Forti (1894), and a presentation of the courses of the Mathematics Faculty of the University of Texas, where Halsted, Macfarlane, and Taylor directed the School of Pure Mathematics, the School of Physics, and the School of Applied Mathematics respectively.[14] In the fifth volume (1895) he wrote two more articles on geometry: "Sulle relazioni di posizione tra punti d'una linea chiusa" [On the relations of position between points of a closed line], and "Sulle proprietà caratteristiche delle varietà a una dimensione" [On the characteristic properties of one-dimensional varieties].[15] In the sixth volume of the journal (1896-99) he published his historical contributions to the *Formulario*,[16] the common project of the Peano school, and some re-

[13] See Vailati 1891a, 1892a, and 1892b.
[14] See Vailati 1893, 1894a, and1894b.
[15] See Vailati 1895b and 1895a.
[16] The *Formulario* had various editions between 1895 and 1908. See in particular

views of Hontheim (1895), Perez (1895), and Couturat (1901).[17] There were no more articles from Vailati in the two following volumes of the journal, published between 1900 and 1906, apart from some historical notes added to the *Formulario* (1902a).

After 1896 Vailati would be closer to Vito Volterra, professor at the University of Turin, even if he still had contacts with Peano's group, and particularly with Alessandro Padoa.[18]

Vailati's logical and mathematical writings concern mainly the analysis of relations and operations, both in algebra and in geometry. The essay on the fundamental properties of the operations of deductive logic focuses on the search for a minimal number of "combinatorial properties of an operation or a system of operations that have to be assumed as primitive in order to prove all other combinatorial properties that such an operation or system of operations actually possesses" (Vailati, 1891a, p. 2). This foundational approach is maintained in the mentioned geometrical writings. The interest for the theory of proportions, which goes back to a talk presented in Livorno (1902b), is evident in the historical article "Sulla teoria delle proporzioni" [On the theory of proportions] (1924), strongly influenced by Zeuthen 1896.

Vailati's contribution to the *Formulario* probably concerned mainly the paragraphs on functions and relations of the logical section and some historical notes, as Vailati 1902a. In the 1893 volume of the *Rivista* he added a note to the logical section explaining that it contains a revised presentation of the formulas already contained in the previous edition, and an entirely new paragraph devoted to functions and relations, because "from a certain point of view they belong to logic rather than to mathematics" (Peano and Vailati, 1893, p. 1).[19]

Peano 1901a.

[17] See Vailati 1898d, 1896c, and 1901c.

[18] See the letters from Padoa to Vailati (about thirty letters and postcards written from March 1896 to May 1908) kept in the Vailati Archive at the University of Milan. Most letters inform us about the friendly relations between Vailati and Padoa: a letter from 1898 shows that Padoa recommended Vailati as temporary teacher in Pinerolo during the time he spent in Brussels giving a series of lectures on mathematical logic (Padoa, 1898); several letters from 1900 show that Padoa and Vailati organized together their stay in Paris for the 1900 International Conferences. Some logico-mathematical issues are also addressed, especially in some letters—first published in Cantù (2007)—that Padoa sent to Vailati in 1905 concerning the relation of equality, and in particular the possibility or impossibility of deriving the reflexive property from substitutivity. Note that the derivability of reflexivity from other defining properties of equivalence had been discussed in Vailati 1891a and, with reference to the former text, also in De Amicis 1892.

[19] The only reference added to this paragraph of the logical section is Dedekind 1888.

Vailati also wrote an article on Peano's mathematical logic that was published in the "Revue de Métaphisique et de Morale" (1899b).

Vailati's Inaugural Lectures in History of Mechanics

In 1895 Vailati became Assistant Lecturer in Projective Geometry, a course given by Full Professor Luigi Berzolari, and in the following academic years he was demoted to Voluntary Assistant in Mathematics, a position as Lecturer in the Undergraduate Course in Mathematics that was practically unpaid, though it allowed him to freely choose the topic of his lectures. Vailati decided—for the first time in an Italian Faculty of Sciences—to give lectures in History of Mechanics, following a suggestion from Vito Volterra, who held him in high esteem, and encouraged him to publish his works on the history of science (De Zan, 2003). Between 1897 and 1898 Vailati published three essays in "Atti dell'Accademia delle Scienze di Torino" on the center of gravity in Archimedes' statics, on the development of virtual labor from Aristotle to Heron, and on the influence of Benedetti's observations concerning the fall of heavy bodies on Galileo. A fourth essay on Galileo's laws of motion was never finished.[20]

Vailati's inaugural lectures, which he published at his own expense and circulated in Italy and abroad thanks to Volterra's help, were deeply influenced by Ernst Mach, with whom he was in correspondence from 1896 to 1907 (Vailati, 1971, pp. 113-134). Vailati, who reviewed Mach's writings[21] and also promoted an Italian edition of *Die Mechanik* (1883),[22] was interested not only in the historical but also in the philosophical conceptions of Mach: he held Mach's phenomenism in great esteem and saw a similarity between Mach's epistemology and Pragmatism.

The three inaugural lectures already show the dominant interests of Vailati: history and philosophy of science (especially mechanics, economics and psychology), mathematical logic, argumentation theory, philosophy of language, and pragmatism.

On the Importance of the History of Science (1896)

In the first inaugural lecture, held in 1896 (December, 4) on the importance of researches in History of Science[23] Vailati claimed that an

[20] See Vailati 1897a, Vailati 1897b, and Vailati 1898c.

[21] See Vailati 1896a, Vailati 1896b, Vailati 1901b, and Vailati 1905d.

[22] Vailati's intention is reported by Papini in a letter to Prezzolini dated October 21, 1902. Vailati finally wrote the Introduction to the translation into Italian by D. Gambioli (Mach, 1909).

[23] *Sull'importanza delle ricerche relative alla storia delle scienze. Prolusione a un corso sulla storia della meccanica, letta il 4 dicembre 1896 nell'Università di*

analysis of the nature of history of science is necessary, for the latter has become a science in itself that is taught as an autonomous discipline in several universities. Vailati's conception of science here is not yet fully original—Vailati's remarks are still deeply influenced by positivism (Spencer and the positivist psychology in particular), Darwinism, and Mach's epistemology—nor fully coherent, for there is, as we will see, an unresolved tension between continuism and discontinuism.

Nonetheless some original issues are already present: 1) the idea of the development of science as a progressive accumulation of knowledge is tempered by the acknowledgment of the positive function of errors; 2) beyond the Machian application of the metaphor of the economy of nature, a new metaphor of the economy of economics is applied in the analysis of mathematical notations and formalism; 3) in the essay on the Italian mathematician Giovanni Benedetti, Vailati described Galileo's discovery of the laws of motion as a 'scientific revolution.'

1) Vailati argues that what varies in science is not the content but the standard of acceptance of proofs and arguments, which increased in the passage from ancient to contemporary works. As an example he mentions mathematics, and argues that the comparison between Euclid's treatise and modern formulations of mathematics shows that the differences in the content are mainly formal or of minor importance. There is an evident continuism in Vailati's conception of science, whose progress is compared to "a series of explorations of an unknown land, where every exploration corrects or refines the results of previous explorations and makes it easier, for those in the future, to achieve the common goal" (1897c, this volume, p. 6). The history of science—he argues—is "a series of successes, each of which has overtaken and eclipsed the one before it, just as the one before that had done" (ibid.). Darwin's evolutionism, and the analogy between the evolution of species and the evolution of knowledge by means of a fight for survival and the adaptation to the environment, are used to "give a new, and more concrete meaning" to Leibniz's aphorism *the present is child of the past but is also parent to the future* or to Pascal's remark that the succession of human generations in the course of the years has to be considered as the life of a single man *who lives forever and learns continuously.* (this volume, p. 7). This positivist approach is mitigated by the belief that the importance that "various competing trains of thought on a certain field of research are going to acquire or lose at a given time" cannot be formulated by "a scholar, who does not care about anything other than the current state of the science he is studying," just as a geometer

Torino (Vailati, 1897c).

cannot "determine the path of a curve from only one point or one linear element." (ibid.)

The positivist conception of the development of sciences as a progressive enterprise is also tempered by Vailati's attention to the positive role of errors in the history of science:

> An erroneous assertion, an inconclusive argument from a scientist of the past can be worthy of consideration as much as a discovery or an ingenious intuition, if they are equally useful in shedding light on the causes that have accelerated or delayed the progress of human knowledge, or in revealing how our intellectual faculties operate. Every error shows us an obstacle to avoid, while not every discovery shows us a path to follow (this volume, p. 5).

2) Darwin's evolutionism is mediated by Mach's comparison of the economy of nature with the economy of thought that is typical of the scientific progress. In *The Origin of Species* (1859) Darwin quotes the construction of hexagonal cells by bees as an example of natural selection. As Vailati recalls, the Alexandrine mathematician Pappus in his work on isoperimetry had already mentioned the fact that bees seem to know that the hexagon can hold more honey than a triangle or a square for the same expenditure of material used in constructing the different figures (Pappus, *Collections*, Book 5). As Darwin defines the cells constructed by bees to contain larvae as made with the greatest possible economy of labor and wax, Vailati judges theories that are produced by human thought to contain observational data as made "with a decreasing use of concepts and direct recourse to experience or intuition, with a decreasing need of distinctions and special considerations that vary from case to case, in other words with the maximum possible economy of that most precious of worldly materials, man's thought" (this volume, p. 19). Vailati's reference to Darwin does not imply a strict biological reductionism, but shows his belief in a continuity in the evolution of biological and cultural phenomena.

Another metaphor, this time from economics, is applied in the analysis of mathematical notations and formalism. In modern industries that are based on fixed capital, the replacement of production facilities, though necessary to increase productivity, might not result in an immediate increase in output, for the expenses might require some time to be amortized. Analogously the introduction of a new notation in mathematics, though more efficient in the production of knowledge, might be resisted, or considered as not sufficiently fruitful because it requires time and costs in order to be learned (this volume, p. 15). The balance of gain and costs is the reason why a method might not be abandoned, even if one knows that it is outdated or that it has been

surpassed by a new one. A pragmatist element is here clearly delineated: scientific concepts are "instruments which only have value depending on their usefulness in achieving the purpose so defined" (this volume, p. 16). But for Vailati a different method does not introduce novelties of content, but is only a way to produce the same result in a more efficient way.

3) The pragmatic analysis of gain and costs according to the aims of science introduces a discontinuity element in Vailati's epistemology. The history of mechanics, and more generally the history of sciences, is filled with "intellectual confrontations" between opposing ideas in the mind of scientists, where the ideas that can better satisfy the needs and the aims of science at a given historical moment ultimately prevail (this volume, p. 18). Even if the "intellectual confrontations" between different ideas might remind us of Kuhn's thesis on the development of science through successive paradigm changes by means of what he called scientific revolutions (1962), Vailati's intellectual battles are not radical discontinuities, but rather progressive changes in human mind that mirror the natural process of adaptation of living beings to the environment.

Discontinuity is more evident in the characterization of Galileo's theory of dynamics given in "The speculations of Giovanni Benedetti on the movement of heavy bodies" (Vailati, 1898c), where Benedetti is given the merit of having made Galileo's "big, scientific revolution" possible. Vailati introduced—long before Koyré (1939)—the term 'revolution' to designate the relevant change introduced by Galileo's laws of motion.

Benedetti was the point of departure of a scientific revolution because he had a "clear awareness" not only of the insufficiency of the Aristotelian theories in the explanation of new phenomena generated by the introduction of firearms, but also of the "direction into which one should proceed [...] in order to forge better theories that deserve to be substituted to the former."[24] In the inaugural lecture on the deductive method as a research tool (1898a) Vailati insists on the role of the invention of firearms, which contributed to the discovery of the fundamental laws of motion, because it "made new facts available to the observers where the two main determining circumstances of the trajectory of a thrown body were energically surpassing the perturbating influences of the others" (this volume, p. 58). Discontinuism is here connected not to a methodological but to a technological change that allows new kinds of experiments, and thus new kinds of observations:

[24]Cf. Vailati 1898c in Vailati 1911, p. 161.

"The sieges and the wars, that afflicted our country in the century that separate the birth of Leonardo from that of Galileo, acted in this respect as real laboratories for experimental mechanics" (this volume, p. 58). The originality of this historical insight of Vailati was appreciated by Mach himself, who thanked him for his "valuable" critical remarks in the preface to *The Science of Mechanics* and gave him the credit for having first understood the connection between the construction of firearms and the development of mechanics.[25]

On the deductive method (1897)

The second inaugural lecture on the deductive method as an instrument of research[26] is entirely devoted to an analysis of Gaileo's method, based on the combination of induction and deduction (*sensate esperienze e certe dimostrazioni*). Vailati argues that modern science is not mainly inductive, but rather based on a new conception and a systematic application of the deductive method.

The reason why induction is not as relevant as deduction in the rise of modern science has nothing to do with a presumed higher grade of certainty of the assertions obtained by deduction; what makes deduction essential is its capacity of being not only a demonstrative means, as it was in Aristotle, but also an heuristic tool, that guides the research of the experimental scientist, as was the case in Galileo's works. Deduction is thus considered as a means to explain and anticipate experience rather than as a means to prove results.[27]

Experimental scientists made use of deduction in those cases in which "the propositions taken as a starting point were considered more in need of proof than the resulting ones, cases where, therefore, the resultant propositions were those which had to pass on, to the initial conjectures, the grade of certainty that they were directly acquiring from a comparison with facts and experimental verifications" (1898a, this volume, p. 31). Deduction does not transfer the certainty of the premises to the conclusions, but rather uses the certainty of the conclusions that can be observed experimentally to ground the plausibility of the conjectures assumed as hypotheses.

In this lecture the historical discontinuity between Aristotle and Galileo is seen as a change that concerns not only the notational ap-

[25] "As Vailati remarks, the rapid spread of firearms in the fourteenth century gave a distinct impulse to the study of the motion of projectiles, and indirectly to that of mechanics generally" (Mach, 1883, p. 526).

[26] *Il Metodo Deduttivo come Strumento di Ricerca. Prolusione ad un corso libero di Storia della Meccanica, 1897-1898* (Vailati, 1898a).

[27] For an analysis of the pragmatic themes involved in Vailati's conception of deduction as a means of knowing see Innis 2002.

paratus of science, but its theoretical approach. There is a fundamental rupture between Aristotle and modern science, that is not due to new tools (as in the previous metaphor from economics), but rather to a different theoretical use of well known tools: it is the heuristic use of deduction made by Galileo that allows him to make the best out of empirical experiments. This change in Vailati's perspective and its originality with respect to the continuist views of other historians of science of the time was probably the reason for the success of this essay, which was translated into Polish by Samuel Dickstein and into French in "Revue de Métaphysique et de Morale" (Vailati, 1898b).

On the role of language in the history of science and culture (1898)

The third inaugural lecture concerns the role of questions of words in the history of science and culture.[28] Vailati claims that the history of science offers many examples of questions of words that are not—contrary to the positivist dictum—useless and unnecessary with respect to matters of fact. An historical analysis of questions of words—either when they gave rise to controversies or when they failed to be noticed—shows that they can be relevant for the development of science, at least inasmuch as they can promote but also hinder its progress. Such an analysis can thus reveal the mechanism of language in the transmission of knowledge, and teach us to detect sophisms, learning to avoid them or to defend ourselves from their consequences. The belief in the fruitfulness and inevitability of errors, together with the refusal of a "clear distinction, established once and for all, between questions which can be the subject of scientific research and others to which such a privilege does not belong" (Vailati, 1899a, this volume, p. 65) are the cornerstones of Vailati's criticism of positivism. But the possibility of tracing such a distinction is not entirely abandoned, and the analysis of questions of words is taken to be most useful in distinguishing between solvable and unsolvable matters, and understanding "which and how many of such [unsolvable] questions get this characteristic from some fundamental flaw in our way of formulating them, or from the fact of being merely fictitious questions, i.e. that there is no corresponding determined sense that could be assigned to the compound of words used to state them." (ibid.).

Vailati remarked that there are cases in which the ordinary language is unable to distinguish the use of an expression as a definition (as a

[28] *Alcune osservazioni sulle Questioni di Parole nella Storia della Scienza e della Cultura. Prolusione ad un corso libero di Storia della Meccanica, 1898-98* (Vailati, 1899a).

proposition that determines the meaning of a name) or as an assertion (on some real objects). He believed this lack of distinction to be a common source of ambiguities and mistakes in psychology and social sciences, though rarely dangerous in the technical sciences, for if an assertion is acknowledged as true, it can also be taken as a definition. The lack of a clear distinction between definition and assertion is—according to Vailati's analysis—at the base of the misunderstanding of Berkeley's and Hume's remarks, who aimed at modifying the definition of the expressions 'substance,' 'existence,' and 'cause' rather than negating that there might be substances, causes, or things that exist (this volume, p. 75 ff.). Changing the definition of a name might change the objects that fall under the name in its new meaning, because what falls under a concept depends on its definition, as what exists depends on the definition of existence.

Questions of words are analyzed as a source of epistemological illusions that might have negative or positive effects on the development of science. In one case they generate "extremely pessimistic and discouraging theories on the limits that scientific research should impose on itself" (this volume, p. 77). In the other case they favor the progress of knowledge.

As an example of the first case, Vailati considers the notion of primitive propositions and notions. He argues that the distinction between explained and unexplained matters is often taken to coincide with the distinction between knowable and unknowable things, while it rather refers to our capacity or incapacity "to deduce our cognitions from one another and to order them in such a way so that part of them are consequences of those remaining" (ibid.). Just as admitting unexplained propositions does not imply an enlargement of what we do not or cannot know, similarly the acknowledgement of undefined notions does not imply that their meaning should be mysterious: on the contrary it is usually so well-known that we cannot find more familiar concepts. Two classes of phenomena can be explained one by means of the other: the fact that we usually use a class of phenomena to explain another class is just a consequence of our viewpoint, of our being more familiar with certain facts than with others, it is a matter of psychology. There is no place for philosophy as a superior judge of matters of foundations in Vailati's perspective, for if the deduction of classes of phenomena from other classes of phenomena can be symmetric, then there cannot be a unique list of plausible principles and general hypotheses for each science: the choice is rather conventional. If philosophy can play a role in matters of foundations, it cannot play the role of a "Supreme Court of Cassation," but rather the role of a "clearing house" that rules at a

meta-level the transactions between different sciences (this volume, p. 79).

As an example of illusions that might have a positive function, Vailati mentions the tendency to believe that one can associate an object with any name, as in the case of definitions by abstraction and in the case of the introduction of ideal ('fictional') elements.

> So, if two objects are in a certain relationship, and such relationship has properties similar to those of the equivalence or similarity, the supposition that they should actually resemble each other in some aspects may lead, and in fact has led in many cases, to the discovery of new properties in the objects in question, and to the realization of which are, among those properties, those whose common possession correlates with, or determine, the existence of their relationship. (this volume, p. 83).

Even when the analogy could not be precisely expressed by a relation of equality, "talking and arguing as if it had in fact been achieved, has often suggested important generalizations, which, notwithstanding their merely verbal and formal character, have provided the occasion and incentive for substantial scientific progress" (this volume, p. 83). Vailati considers several examples, including the introduction of the point at infinity in geometry, irrational numbers in arithmetic, the notion of latent heat in physics.

Defending the heuristic positive role of analogies, Vailati does not condemn the presence of metaphors in mathematics, refusing to take part in the crusade against intuition and impure proofs in mathematics. Vailati acknowledged that scientific technical language, like ordinary language, was actually full of metaphors, and believed they should at least be made explicit, for "even if after long use [they] have stopped reminding us of the image suggested in origin, [they] have not lost their ability to induce us to attribute to the facts that such expressions describe all the properties of the image to which they refer" (this volume, p. 87). The metaphor of 'dead analogies' is further developed in the paper "On material representations of deductive processes" (1905a, this volume, chapter 15), where Vailati shows the presence of several metaphors in the language that is used to describe logical operations and the influence that they have on the way we conceive them (see below, p. xliii).

Vailati's intermediate position between a dogmatic refusal of analogy and a skeptical attitude to logical investigations directed at the elimination of implicit intuitions from mathematics could be interpreted as a stand in the controversy between the Italian school of algebraic geometry, which arrived at many new results without giving rigorous proofs,

and the logical school of Peano. Segre (1891) argued that the first way to arrive at a mathematical truth does not usually consist in a satisfactory proof, because the scientific truth is placed at the top of a high mountain that is at first reached by difficult paths between dangerous peaks (safer ways can be traced only afterwards). Peano answered that mathematics is based on logical deduction, and any activity that is not deductive should be considered "poetry, but surely not mathematics" (1891, p. 67). Vailati's position is certainly nearer to Segre than to Peano:

> In contrast with Molière's character who was surprised when he realized he had always spoken in prose without knowing it, we should be surprised to have always spoken in poetry without noticing it. This is not harmful to us, in the same way as it is not harmful to the mathematician, who investigates the properties of functions, to use sentences that refer to, or are taken from, their geometric representation, and in the same way as it is not harmful for the geometer to talk about spaces with n dimensions, or about points in common to lines which do not meet each other (Vailati, 1899a, this volume, p. 87).

The inaugural lecture ends with an appreciation of theories of reasoning and argumentation, which, together with mathematics and linguistics, constitute a means of emancipation from the unconscious slavery of thought from words. An implicit reference to William James occurs again when, after having criticized the Positivists' contempt towards the "imperfect" scientific theories of the past, Vailati mentions the contributions of American experimental psychology and—"last not least"—of mathematical logic to the renewal of interest in history of sciences (this volume, p. 91).

Outside the Academy (1899-1903)

In October 1899 Vailati accepted a position as mathematics teacher in a high school in Syracuse, Sicily, definitively renouncing his academic career. In the following years Vailati would mainly devote himself to issues connected to philosophy of science: from the status of psychology to its connections with logic, philosophy of language and with the issue of determinism in science.

In Sicily Vailati met one of his previous correspondents: Giuseppe Amato Pojero, a member of the Mathematical Circle of Palermo, and founder of the *Società per gli studi filosofici*.[29] Thanks to Amato Pojero, Vailati got to know Franz Brentano, who began a rich correspondence

[29] Antonio Brancaforte has published their correspondence in Vailati and Amato Pojero 1993.

with Vailati (Chisholm and Corrado, 1982) and invited him to Austria in the summer of 1902.

In 1900 Vailati moved from Syracuse to a high school in Bari, Apulia, and then in 1902 to Como, Lombardy, not too far from his native town Crema. During his activity as a school teacher, Vailati diversified his researches, describing himself as a mole burrowing a lawn in different directions and running from one to the other gallery advancing a few decimetres in each direction:[30] in these years he mainly wrote reviews and took part in international conferences. In the summer holidays he traveled across Europe, meeting philosophers and scholars with whom he was in correspondence, and who praised him not only for his scientific merits but also for his humanity and whole-heartedness. So wrote Mario Calderoni after their first meeting at the Psychology Conference in Münich in 1896: "I saw him universally celebrated and requested from all intervening scholars; in the streets, in the pubs, in gatherings and meetings he was always in the middle of a group which he fascinated with his simple, whole-hearted, and nonetheless interesting, informative conversation" (Calderoni, 1924).

During these years Vailati continued his 'work' as reviewer that would lead him to write a huge number of reviews. Vailati believed in the philosophical relevance of reviews and had a preference for this 'literary genre':

> My preference for this literary genre that, though not separately considered in rhetoric textbooks, is a true, freestanding genre like madrigal, sonnet, or satire, does not depend only on the fact that I do not have time to write longer works, but from the belief that it is a very useful and too often neglected genre, that is wrongly let at the mercy of those who write only to praise (or blame) the works they are reviewing, rather than to separate what is good from the rest (Rizza, 2006, pp. 74–75).

Among the few essays of this period there are three texts that are strictly related to Vailati's paper presented in 1904 at the Heidelberg International Conference of Mathematicians: Vailati 1903a on Saccheri's *Demonstrative logic*, which he had the merit of rediscovering, an article on Aristotle's theory of definition (1903b, this volume, chapter 7), and Vailati 1902a—a historical contribution to Peano's *Formulario* on the distinctions between real and nominal definitions, and between definitions of classes and individuals and definitions of functions and relations.[31]

[30] Letter of Vailati to Papini, June 1, 1908 (Vailati, 1971, p. 463 ff.).

[31] See below, page xl, and page xlvii. See also Lolli 1985.

International Conferences

During the years he spent as teacher, Vailati also attended and presented papers at several International Conferences on different topics, including Psychology, Philosophy, Mathematics and History.

In Paris Vailati took part, together with Padoa and the other members of the Peano school, in the International Philosophy Conference, where Russell was impressed with the clarity of language and reasoning of the Italian group:

> The Congress was a turning point in my intellectual life, because I there met Peano. I already knew him by name and had seen some of his work, but had not taken the trouble to master his notation. In discussions at the Congress I observed that he was always more precise than anyone else, and that he invariably got the better of any argument upon which he embarked. As the days went by, I decided that this must be owing to his mathematical logic. I therefore got him to give me all his works, and as soon as the Congress was over I retired to Fernhurst to study quietly every word written by him and his disciples. It became clear to me that his notation afforded an instrument of logical analysis such as I had been seeking for years, and that by studying him I was acquiring a new and powerful technique for the work that I had long wanted to do (Russell, 1998, p. 147).

At the International Conference of Philosophy Giuseppe Peano presented a paper on definitions (1901b). Padoa presented his best known contribution to logic and the foundation of mathematics (1901), arguing that a logically perfect deductive theory should have a system of irreducible axioms and a system of primitive irreducible symbols, and used the so-called Padoa's definability criterion to verify the latter condition. Vailati did not give a talk on mathematical logic, having abandoned its study some years before, but on the classification of sciences, and especially on an analysis of Comte's classification (1901a).[32]

Like Peano, Padoa, and Volterra, Vailati also attended the International Conference of Mathematicians. Padoa presented two papers: a study on the principles of geometry that was later translated into Italian and into Spanish in the journals "Periodico di matematica" and "El Progreso Mathemático" respectively (1902a), and a paper on the definition of the field of natural numbers by means of two operations:

[32] See below, page xxxvii. It is possible that Vailati had originally planned to discuss this topic in a course of mechanics, given that the structure and the content of this paper are quite similar to those of the three inaugural lectures on the history of mechanics. In this same period Vailati discussed problems connected to the classification of science also in some reviews, especially in Vailati 1899d, Vailati 1900, and Vailati 1902c.

the successor of an integer and the symmetric of an integer (1902b). Volterra presented a paper on the role of Betti, Brioschi and Casorati in the foundation of the Italian school of algebrists (1902b), and a second paper on the equation of Poisson and its transformations (1902a).

The classification of sciences (Paris 1900)

In "Des difficultés qui s'opposent á une classification rationelle des sciences" [The difficulties involved in a rational classification of the sciences] (1901a) Vailati connected the classification of the different sciences to the differentiation of labor, showing once more his deep interest in sociology and economics. After raising two typically pragmatic objections —most classifications are based on a vague and mistaken notion of what it is to make a classification, and overlook "the practical motivations that have determined the division of intellectual labor"—Vailati discussed the conceptual distinctions introduced by Durand de Gros in *Aperçu de taxinomie générale*[33] and put forward three objections to Comte's classification, which are grounded in historical counterexamples and aim at a rehabilitation of the role of two social sciences: sociology, whose method can be not only inductive but also deductive, and psychology, which deserves to be considered as a science. Apart from showing the need for a conceptual clarification of the notion of classification, that should be taken to be descriptive—and thus historically based—rather than normative, Vailati's remarks were aimed at showing the fruitfulness of an analysis that takes into account the aims of the special sciences and limits itself to the relations between one science and its neighboring disciplines, renouncing the unattainable claim of giving a simultaneous classification of all sciences.

The historical awareness of the complexity of the ways in which sciences had developed prevented him from believing that any ideal scheme might adequately represent the many-sided relations between different sciences. Nonetheless he defended the usefulness of partial, flexible classifications, which might still serve various aims—to compile a bibliographical catalogue, to organize a didactic activity, or to establish an interdisciplinary, historical research project.

> Between the research of a perfect and ideal grouping of the various sciences according to a uniform and necessarily unilateral criterion, and the passive adhesion to the traditional divisions between the areas of research of the different sciences, divisions for which most of the time the historical causes of their origins have long disappeared, there is a large area open for useful and important attempts. If they are unable to order and unify according to new principles the variety of

[33] See Durand de Gros 1899 and Vailati 1900.

> human knowledge, it does not mean that they will be less effective for the progress of science and to better the economy of the efforts that tend to make it grow (Vailati, 1901a, this volume, p. 106).

The classification of mental states (Paris, 1900)

The analysis of Brentano's states of consciousness was the topic of a talk Vailati gave in 1900 at the III International Conference of Psychology in Paris. In "Sulla portata logica della classificazione dei fatti mentali proposta dal prof. Franz Brentano" [On the logical import of the classification of mental facts proposed by Franz Brentano] (1901e) Vailati remarked that Brentano's tripartion of mental facts in representations, expectations and volitions has a logical meaning, for it corresponds to the distinction between definitions or analytic propositions, factual propositions and judgments of value. As in the third inaugural lecture, Vailati argued that the lack of a clear distinction between these different kinds of propositions in the ordinary language has induced epistemological mistakes, as that of believing that one can derive propositions of one category from propositions of another category, for example geometrical propositions from geometrical definitions, or normative propositions from definitions and factual propositions. Against the naïve scientism of some contemporary Positivists, Vailati believed that the inability of science to predict wants or desires was not a weakness of science, but rather a consequence of the fact that one cannot derive predictions on judgments of value from factual propositions alone. "To blame science, or scientists, for their inability in this regard is only slightly less absurd than it is to blame the talent of a painter for the fact that the light of a lamp he has painted is not able to brighten the dark room where the painting is hanging." (this volume, p. 112). Without entering into a detailed discussion of Brentano's classification, Vailati used it to defend science from its opponents, arguing that the distinction "helps us to understand that we are wrong to expect something from science which, due to its own nature, it cannot give us" (this volume, p. 112).

On the concepts of cause and effect in historical sciences (Roma, 1903)

In April 1903 Vailati took part, in Rome, in the International Conference of Historical Sciences, where he gave two talks, one in history of science—"La dimostrazione del principio delle leva data da Archimede" [Archimedes's proof of the principle of the lever] (1904b)[34]—and one in philosophy of science—"Sull'applicabilità dei concetti di causa e di effetto nelle scienze storiche" [On the applicability of the concepts of

[34]This text is widely discussed in Palmieri 2008.

cause and effect in historical sciences] (1903c, this volume, chapter 6.).

Vailati took part in a debate about the nature of history in which Italian Positivists and neoidealist philosophers were deeply involved. Vailati argued that history should be considered as a scientific discipline, for physical laws—like historical laws—are not necessary, because they are general but subject to exceptions, and hypothetically valid. First, he claimed that, since the conditions that have to be verified for a physical law to be valid cannot be exhaustively formulated, physical laws—like historical laws —are general, but subject to exceptions, and only hypothetically valid. Secondly, he argued that most scientific laws are obtained by induction and not by deduction, and that even in the case of mathematical physics, which is mainly deductive (see this volume, chapter 2), only the connection between premises and conclusions is necessary, and not the laws themselves.

> Therefore the truth of a law is compatible, in every special case, both when the facts mentioned in it happen or not, because all it affirms is not that such and such a fact *happens* or *does not happen*, but only which are the facts that join it *when it happens*, or with which it would be joined *in the case it should happen* (this volume, p. 116).

Vailati's refusal of the determinism of human will is associated with his refusal of some interpretations of the materialist conception of history. He criticized those views according to which social phenomena are only ideological reflections, and economic conditions the causes of all social transformations.[35] Between economics and society there is not a cause-effect relation, but rather a relationship of mutual dependence. It is the practical restriction to a certain aim and to a certain point of view that induces the researcher to consider certain factors as causes of a given social phenomenon:

> [the researcher] allows himself, more or less consciously, to be induced to limit his attention and to consider as causes only those, among the conditions of a given fact, whose modification he believes would be necessary or useful to provide if we were to generate or prevent the fact in question or others of a similar nature, or to modify them in the way he desires. This kind of partiality should not be considered illegitimate, or confused with that which consists in allowing our passions and our interests to influence the evaluation of the proof of facts and theories (this volume, p. 118).

The notion of mutual dependence between cause and effect, and more generally, the concept of correlative properties was further developed in

[35] Vailati's rejection of the common interpretation of the materialist conception of history has to be distinguished from his judgment on Marx's philosophy (this volume, p. 117).

"La caccia alle Antitesi" [The attack on distinctions]—first published in 1905 in the journal "Leonardo" and translated into English in 1907 (this volume, chapter 14).

On the meaning of the difference between axioms and postulates in Greek geometry, Heidelberg 1904

In 1904 Vailati participated in the 3rd International Conference of Mathematicians in Heidelberg, where he presented a paper in the section on History of Mathematics, in which Cantor, Dickstein, Zeuthen, Tannery, Loria, and many others also participated. Vailati remarked that if axioms are distinguished from postulates on the basis of a higher degree of evidence or intuition, then the traditional distinction between axioms and postulates becomes useless in geometry, for the latter gives "less and less importance to 'intuition' and to the criterion of 'evidence' in choosing fundamental propositions" (1904a, this volume, p. 145).

Quoting Proclus' *Commentary on the first Book of Euclid's Elements* Vailati outlined three possible ways to distinguish between postulates and axioms. The expression 'postulates' might be reserved: 1) for propositions that include some assertion of existence (e.g. the postulate of Archimedes); 2) for propositions that belong to a single science and are not common to several sciences (e.g. the geometric proposition "All right angles are equal"); 3) for propositions that are used as *implicit* definitions of relations or operations, or of classes of entities that satisfy such relations and operations (e.g. "Two quantities equal to a third are equal to each other").[36]

Presenting postulates as propositions that include some assertion of existence, Vailati introduced a distinction between the Principle of Archimedes, which is considered as a postulate, and Dedekind's Continuity, which is considered as a definition, i.e. as one of those propositions which, "even if they are presented, as this one, in the form of assertions of existence, are really used only to extend and generalize the meaning of some locution, making it applicable to a broader field than the one it was previously reserved for" (1904a, this volume, p. 149).

The distinction between direct and indirect definitions (applied to the so-called 'implicit' definitions) is expressed in Vailati's essay "La teoria Aristotelica della definizione" [The Aristotelian theory of definition] (1903b, this volume, p. 7) and in a letter that he sent to Frege on March 17, 1904:

[36] At this Conference Hilbert defined equality by means of reflexivity and substitutivity. Vailati presumably attended Hilbert's talk (Hilbert, 1905), which might be relevant to the analysis of the correspondence with Padoa in 1905. See above footnote 18 and Cantù 2007.

> But I believe that if only Mr Hilbert could make up his mind to renounce his opinion that the axioms represent the 'fundamental facts of intuition' (an opinion whose source is perhaps to be sought in his irrational devotion to Kantian philosophical jargon, which is still deplorably popular among writers on the philosophy of science), all the rest of his exposition could be given an irreproachable form. When he says that a given group of axioms defines a 'relation' (for example 'between') or a class of objects (for example 'points'), he does not sufficiently distinguish the different nature of the definition in the two cases: 1) To define 'a *relation*' by means of axioms is to enunciate a *condition* (or functional equation) or several conditions which are supposedly satisfied by the relation being defined. 2) To define a *class of objects* by means of axioms is to characterize it as constituted by objects *between which* one can establish the 'relations' that satisfy the conditions enunciated in the axioms. In the first case we have, so to speak, a *direct* definition, and in the second an *indirect* definition (that is, a definition of *a class of objects* by means of other definitions, that is, by means of the definitions of the relations that can supposedly be established between them. (Frege, 1980, pp. 173–74)

Pragmatism, logic and language

In 1904 Vailati was asked to edit the works of Evangelista Torricelli on behalf of the Accademia dei Lincei, and he moved to Florence, where he taught at the Technical Institute "G. Galilei." Here he often met Giovanni Papini and Giuseppe Prezzolini, the founders of the militant journal "Leonardo." Vailati had already been in correspondence with Giovanni Papini since 1902, after having read *On the psychological theory of prevision* (1902), and he remained in contact with him until shortly before his death (the latter postcard is dated January 1909).[37]

The Pragmatism Club in Florence (1904-1905)

During the time he spent in Florence, Vailati, fascinated by the strong personality of Papini and by the opportunity to criticize the cultural backwardness of the Italian universities, started to collaborate on the journal, where he published reviews of recent works by Pierre Duhem, Ernst Mach, Henry Poincaré, Ernst Schröder, Charles S. Peirce, Louis Couturat, Federigo Enriques, and William James, as well as several essays on science, language, and philosophy that are included in this volume (see chapters 8,10–14).

[37] Vailati's letters to Giovanni Papini have been published by G. Lanaro in Vailati 1971, pp. 323–473.

The debate on pragmatism in the journal "Leonardo"

At about this time the editorial group of the journal "Leonardo" started a lively debate on the nature of pragmatism, having the aim of transforming the journal into the official organ of Italian Pragmatism. In November 1904 Mario Calderoni, a disciple of Vailati, published an essay entitled "Le varietà del Pragmatismo" [The varieties of pragmatism], in which he opposed Peirce's writings, considered as the original nucleus of pragmatism, based on the 'rule,' and James's conception, which he considered as a wider and derivative variant (Calderoni, 1904). Prezzolini, whose adhesion to pragmatism was quite short, replied in "Il mio prammatismo" [My pragmatism] (1905) that there are no profound distinctions between the two conceptions. In the same year, three more articles were published: Calderoni 1905, Schiller 1905, and Papini 1905, which was signed "The Florence Pragmatism Club."[38] Calderoni criticized Prezzolini, and Papini tried to mediate between the different conceptions of pragmatism that were opposing the collaborators to the journal (Casini, 2002).

The common ground was an instrumentalist conception of theories and beliefs and a related pluralist conception of pragmatism as a collection of methods, like the corridor of a hotel with many doors leading to innumerable rooms, a metaphor by Papini that James appreciated:

> As the young Italian pragmatist Papini has well said, it lies in the midst of our theories, like a corridor in a hotel. Innumerable chambers open out of it. In one you may find a man writing an atheistic volume; in the next some one on his knees praying for faith and strength; in a third a chemist investigating a body's properties. In a fourth a system of idealistic metaphysics is being excogitated; in a fifth the impossibility of metaphysics is being shown. But they all own the corridor, and all must pass through it if they want a practicable way of getting into or out of their respective rooms (James, 1908, Lecture 2).

Knowledge and Will

In 1905 Calderoni, Papini and Vailati published in "Leonardo" three talks on the relations between beliefs and will that they had presented at the 5th International Conference of Psychology in Rome: Calderoni's "Intorno a una definizione della volontà" [On a definition of will] is described in the editorial as considering will as an effect of beliefs; Papini's "Influenza della volontà sulla conoscenza" [Influence of will on knowledge] as considering will as the cause of beliefs; and Vailati's "Distinzione tra conoscere e volere" [Distinction between knowledge and

[38] The name of the group was written exactly as it appears here, in English.

will] as considering will as the cause of erroneous beliefs.[39] Calderoni, following Vailati, defined voluntary actions as those actions whose probable outcomes can be reasonably foreseen, arguing that one is juridically and morally responsible for all acts that imply a conscious expectation. Papini on the contrary wrote about the primacy of action and creativity on will. Vailati recalled here, as he had already discussed in 1901e (this volume, chapter 5), Brentano's distinction between expectations and volitions, arguing that beliefs and volitions have often been confused because of the linguistic similarities of the propositions that express them, and because of the meaning of the word 'cause' in ordinary language.

At the International Conference in Rome the group of Italian pragmatists met William James, whose talk "On the Notion of Consciousness" was published in an Italian translation in the July-August issue of "Leonardo." James enthusiastically described the meeting in a letter to his wife: "I have been having this afternoon a very good and rather intimate talk with the little band of 'pragmatists,' Papini, Vailati, Calderoni, Amendola, ..." (James, 1969, p. 227) and in a letter to George Santayana dated March 1905:

> "What I really write to you for is to tell you to send (if not sent already) your *Life of Reason* to the *Revue de Philosophie,* [...] and to the editor of "Leonardo" (the great little Florentine philosophical journal) [...]. The most interesting, and in fact genuinely edifying, part of my trip has been meeting this little cénacle, who have taken my own writings, entre autres, au grand sérieux, but who are carrying on their philosophical mission in anything but a technically serious way, inasmuch as "Leonardo" (of which I have hitherto only known a few odd numbers) is devoted to good and lively literary form." (James, 1969, p. 228)

Maybe because Vailati and Calderoni were quite far from the theories he had developed in his latest writings, James became more fond of Papini, with whom he started a regular correspondence and whom he considered to be the most relevant exponent of the Italian pragmatism (James, 1906).

On material representations of deductive processes

An interesting example of Vailati's interest in an analysis of non-deductive parts of the scientific discourse is offered by his writings on the role of metaphors in science. The role of metaphors in science had already been analyzed in the third inaugural lecture, where Vailati had defended the necessity of making such metaphors explicit.

[39] See the June-August issue of "Leonardo" 1905, especially p. 125.

Vailati himself made use of metaphors in his own writings, taking them not only from biological or natural sciences but also, if not especially, from economics, as was the case in the first inaugural lecture, where he compared the introduction of a new notation to the replacement of production facilities in an industry, or in the third inaugural lecture, where he presented philosophy as a clearing house.

In "I tropi della logica" [The tropes of logic]—an article published in "Leonardo" in 1905 and translated into English in the "Journal of Philosophy, Psychology and Scientific Methods" in 1908—Vailati analyzed "the employment of physical metaphors as means of representing mental facts" (1905a, this volume, p. 197) in order to draw "some indications of the means of regulating the play of mental activities" (this volume, p. 198).

In this article, which was quoted in Dewey (1909, chap. 7), Vailati considered the metaphors related to deductive processes, classifying them in three groups:

1. metaphors of supporting or upholding;
2. metaphors of ascending or descending;
3. metaphors of containing or including.

Different metaphors are connected to different conceptions of deduction that Vailati had outlined in the second inaugural lecture. If one considers an assertion as supporting another or as being drawn from another, one considers deduction as a means of proof. If one considers conclusions as being already contained in premises, deduction is considered as a process that cannot produce anything new, while if premises are the simple elements that compose conclusions, deduction is conceived as an analysis.

The most recent definition of mathematics

In accordance with the polemic spirit of the journal, in "La più recente definizione della matematica" [The most recent definition of mathematics] (1904c) Vailati considers a provocative definition attributed to Bertrand Russell: "mathematics is a science where we never need to know if what is said is true, nor do we need to know what we are talking about" (this volume, p. 137). Vailati explains the paradoxical content of the definition by means of two relevant changes in the mathematical discipline: the hypothetic-deductive approach introduced by modern axiomatics, which he describes in Poincaré's words, and the formal approach associated with Peano's mathematical logic and with Peirce's theory of relations.

Vailati argues that mathematics is not mainly interested in truth be-

cause it investigates different systems of axioms independently of their conformity to "real" facts. His explanation of the first part of Russell's definition is entirely based on Poincaré (1902): on the one hand different systems might all be compatible with experimental observations (e.g. non-Euclidean geometries) and their mathematical interest does not consist in their being true, but in their being more convenient; on the other hand assuming a false hypothesis might be fruitful, if it artificially simplifies the facts to which it is supposed to refer. As Poincaré had argued both in the analysis of the differences between the properties of the geometrical space and the perceptual spaces, and between the features of the physical and the mathematical continuum, the mathematical models do not correspond to reality as such, but they are artificial constructions used to describe physical facts.

Turning to the second part of Russell's quotation, Vailati argues that the power of a language is proportional to the number of words that are devoid of meaning if taken in isolation. Relying on Max Müller's assertion that language begins where interjections end (Müller, 1891, vol. 1, p. 507), i.e. where the meaning of words is not predetermined but depends on the way they occur in a sentence, Vailati argues that the force of mathematical language depends on the amount of symbols that denote relations, operations, functions.[40] Calling functional symbols meaningless Vailati means that mathematics is interested in the study of uninterpreted relations and operations. Peano's mathematical logic and Peirce's theory of relations are mentioned as the two recent developments that

> emancipate mathematical deductions from any appeal to facts or intuitions with reference to the meaning of the operations, or relations, under consideration. These are defined by the mere simple enunciation of a certain number of fundamental properties that, being able to be common to relations or operations with the most disparate and heterogeneous meanings, are compatible with the most various interpretations of the symbols appearing in their enunciation. (1904c, this volume, p. 141)

Pragmatism and mathematical logic

In the short essay "Pragmatism and Mathematical Logic" (1906b) published in "Leonardo," Vailati enumerates six pragmatic features of logical theories:

1. the value of each assertion is intimately connected to its use in the deduction of consequences: a postulate is a proposition that

[40] The same idea is at the base of *The grammar of algebra* (1908b, see this volume, chapter 16).

has the same epistemological status as any other proposition, but it is chosen as a postulate because of its usefulness for the specific aims of the theory;

2. assertions are distinguished on the basis of their relation to facts: possible or essential;
3. the recourse to historical analysis of science and language shows the compatibility of some modern theories with ancient theories;
4. the development of the theory of definition—based on the distinction between definitions by abstraction, by operations, by postulates, on the interest for the notion of definability, and on the consideration of implicit and local definitions—shows the importance of contextual considerations;
5. the construction of particular models to prove the consistency of a set of postulates shows the "mysterious ally" between abstract theories and "particular facts";
6. the introduction of the symbolic notation illustrates a tendency to the simplification of theories, which is a condition for their instrumental application.

Apart from the discussion of which of the above mentioned issues should be considered as truly pragmatic in nature, it is interesting to discuss the stance of Vailati on the one hand with regard to the conceptions of Peano and of the members of his school, and on the other hand with regard to the role of history, language, and philosophy in logic.

Notwithstanding relevant differences between the members of the school, which was far from homogeneous, the works of Peano and his disciples were mainly oriented to the search for convenient systems of postulates and of definitions for each mathematical theory (Pieri's axioms for geometry, Peano's axioms for arithmetic and for logic, and so on), rather than to a unified theory. Besides, they were based on the idea, clearly formulated by Vailati, that different interpretations of the same symbols could and should be admitted in order to study the independence of the primitive notions and propositions of a given theory (1904c, this volume, p. 141).

Concerning the role of language in mathematics, Vailati distinguished the mathematical project of expressing the whole of mathematics in a formal language, a project that he repeatedly praised but in which he did not participate, from the philosophical analysis of science, which includes the uncovering of metaphors, the study of the argumentative schemes, the logical analysis of ordinary language.

In "A Study of Platonic Terminology" (1906c) Vailati also mentioned the relevance of linguistic analysis for the work of the historian, who

should be aware of the "modifications of technical philosophical language which aim at expressing *new* ideas or *new* distinctions" (Vailati, 1906c, this volume, p. 174).

Vailati's pragmatic and logical interest in definitions

Definition emerges as a logical problem already in Vailati's inquiries on relations and operations. Vailati shares Peano's ideas that definitions are merely conventional: there is not a single definition but several possible definitions of the same concepts, and the choice between them depends on the point of view of the researcher. Vailati's arguments reveal a good knowledge not only of Peano's position, but also of Burali-Forti's and Padoa's results on definitions.[41]

If from a logical point of view different definitions might be equivalent at least in the general sense that they allow, given certain conditions, to derive the same propositions, different definitions might have radically divergent consequences. Different definitions might be heuristically not equivalent, for certain definitions might hinder the progress of science, while others favor it. So, while in the Peano school the analysis of different definitions was mainly a logical and foundational issue—determine which concepts are independent from other concepts or which definitions allow us to reduce the number of primitives—in Vailati's eyes it soon became an historical and epistemological issue: how can different definitions influence the efficiency of a theory or its foreseeing capacity? Which definition is more appropriate to the aims of a science at a given period of time?

Vailati's interest in the history of ideas is thus strictly connected to the analysis of different historically given definitions of the same words, showing how new ways of defining a word might generate new concepts. This was a viewpoint which Vailati acquired gradually over time, probably due to the development of an historical sensibility that drove him progressively away from his earlier positivist approach. In particular, while in the inagurual lecture Vailati still believed that mathematical logic was a difficult but more efficient way to achieve the same results, just as the change of machinery is a costly way to rationalize production, in his last works he became more and more interested in the positive and negative consequences of the adoption of one definition rather than another. Besides, he considered how definitions were obtained, because this could explain the origin of the concept defined, and also its meaning, given that the meaning of a concept is related to its context of use and to the way that it has been introduced. Vailati

[41] See in particular Burali-Forti 1894, Padoa 1901, and Padoa 1905-06.

was thus interested in the *Wirkungsgeschichte* of definitions, even if rather in an instrumentalist rather than hermeneutic way.

The interest in definitions, and in their logico-mathematical origin, might also explain why Vailati's historical essays were mainly devoted to the Socratic and the Aristotelian theory of definition, or to the Euclidean difference between axioms, postulates, and definitions, to the changes determined by the deductive method in the history of modern mechanics, and to the study of classifications of sciences.

The interest in definitions is also dominant in Vailati's approach to Brentano's psychology: he immediately transposes the distinction between different psychological facts on the logico-epistemological domain, interpreting it as a difference between assertions and definitions. So, even the 'pragmatic' distinction between knowledge and will, and its consequences on the analysis of the task of science that should foresee the former but not the latter, is again a matter of the correct definition of the concepts involved and of the role of scientific thought.

Some pragmatic themes that emerge from this collection of Vailati's essays are related to the distinction between questions of words and matters of facts, and especially to the pragmatic allowance of different definitions—depending on their degree of appropriateness to specific aims. The importance of definitions does not consist mainly in their being points of departure of the deduction, as if an agreement on the meaning of words were only a prerequisite of an argumentative practice. It rather consists in their making explicit the conceptual elements that play a significant role in the deduction. For this reason, Vailati's version of pragmatism has a logico-semantic vein that might allow an interesting comparison with the recent approach of Robert Brandom.

But we would like to recall here instead that Moritz Pasch was one of the first authors who insisted on the importance of analyzing scientific propositions in order to free deductions from an implicit or unconscious recourse to intuition. There is an echo of Pasch's epistemology in Vailaiti's approach, even if Vailati mentions him only once as the first "to analyze the fundamental propositions of geometry of position and to determine which notions and entities concerned by it (point, right line, plane, segment, angle, triangle, tetrahedron, and so on) can be considered as primitive, indefinable and such that all other notions and entities can be defined by means of them."[42] Pasch's geometrical results had influenced Peano,[43] but his epistemology influenced other Italian mathematicians, such as Veronese, who recognized the importance of

[42]See Vailati 1899b, repr. in Vailati 1911, pp. 239–40.
[43]See for example Gandon 2006.

tracing back the abstract mathematical and geometrical notions to their empirical analogues, including the results in a list of empirical observations that, though clearly distinguished from the definitions, might shed some light on the reasons why certain definitions are preferred (Cantù, 1999). Veronese had discussed the empirical origin of mathematics in correspondence with Vailati, as the latter had asked for his opinion on the opportunity of including empirical and practical mathematics in the programs for younger pupils (Cantù, 2000).

Definitions are a common ground to Vailati's and Peano's researches. Though never declaring any specific interest in philosophy, Peano shared with Vailati a pluralist, antidogmatic, and antifoundationalist conception of definitions. Defending the plurality of views that emerged in his own school, Peano argued that definitions by abstraction, by a nominal definition, by means of a relation, and by means of an operator are 'equally logic and equally rigorous': the best definition is nothing else but the definition that each teacher prefers (Peano, 1915, p. 409). The emergence of this 'pragmatic' theme shows that Vailati's claim that there were points in common between Peano's mathematical logic and pragmatism was grounded. But it does not show that Vailati's own version of pragmatism might be regarded as the explicit philosophical formulation of Peano's implicit epistemology, nor that the logical issues considered in "Pragmatism and mathematical logic" should faithfully correspond to Peano's conception of logic.

Nonetheless, apart from Vailati's rhetoric, especially in the articles published in "Leonardo," on the simplification of theories by elimination of concepts and words, as if the progress of science should depend more on the elimination of superfluous machinery rather than on the introduction of new, refined conceptual tools, Peano's and Vailati's perspectives on language share a common ground. Peano's project of *latino sine flexione* was also a simplification of the language that should determine

> a minimal number of words, prefixes and suffixes, that are necessary to express any idea, that is to build the *latino minimo*. This method is an application of the Mathematical Logic, which allows to decompose, by means of a series of equalities, a set of mathematical ideas into primitive and derivative ideas (Peano, 1903-04, pp. 279–80).

Educational projects and pedagogical theory (1905-1909)

The edition of Torricelli's works could not be realized, but Vailati asked to remain in Florence in order to continue his studies in the National

Library. Instead he was given another appointment from the Public Ministry of Education: he was nominated member of the Royal Commission in charge of the Educational Reform for Secondary Schools. He thus moved to Rome and started a series of trips around Europe to analyze and compare different scholastic systems.

The Italian Commission for the reformation of school

The Italian school system was based at the time on a rigid distinction between humanities and sciences after the first 5 common years: humanities were taught in the 'Ginnasio' (5 years) and 'Liceo' (3 years) that gave access to higher education; sciences were mainly taught in technical schools, which gave access only to some scientific universities. The proposal of the Ministerial Commission was driven by the idea that there should be a unique system lasting 8 years and three curricula in the 'Liceo' (5 years): classical, scientific and linguistic. Vailati agreed on the first part of the proposal but not on the second: he instead suggested a unique kind of liceo, where both humanities and sciences would be taught and where each would be given equal weight. Vailati did not favor the introduction of philosophy in the curricula of the secondary schools; he rather believed in the opportunity of introducing a Philosophy course in every university faculty, but his idea was never realized, and the Italian Academy became more and more divided into two opposed and non-communicating branches: humanities and sciences.

Apart from the General Reform of Secondary School, which in the end was never approved, Vailati was in charge of preparing new programs for the Mathematical Courses. He thus began a correspondence with many contemporary Italian mathematicians, in order to get their critical remarks on the project. Like the Italian geometer Veronese, who had devoted several studies to the problem of teaching mathematics to young pupils, Vailati was favorable to an empirical and practical approach, based on intuition and on concrete experiences. Vailati presented his ideas on the teaching of mathematics in many reviews, but also in "L'insegnamento della matematica nel primo triennio della scuola secondaria superiore" [The teaching of mathematics in the first three years of high school] (1907b).

This article suggested an operative or experimental method to teach geometry to younger students. He refused to call such a method 'intuitive'—as was done in the same years by Veronese—because what distinguishes this method from the 'logical' or 'rational' teaching of geometry (i.e. from the Euclidean style) does not depend on the fact that students restrict themselves to the learning of intuitive truths, but rather on the fact that they learn non-intuitive truths by exper-

iments (Cantù, 2000). The experimental discovery of truths does not exclude the introduction of proofs; on the contrary it is the best way to let the student develop "the desire and the need to understand 'how' and 'why' certain properties subsist, and to induce him to find interest in the learning of (or search for) deductive relations between properties and reasoning that lead him to acknowledge them as being consequences of one another" (Vailati, 1907b, p. 305). These remarks against the rational learning of geometrical proofs at an early age was motivated by a criticism of the mnemonic approach to the learning of definitions and theorems, based on the treatment of students as "recipients to be filled" rather than persons, "fields to be sown, plants to be grown, fires to be lighted." Students, he argued in a review of C. Laisant, should be asked to understand and not just to learn (Vailati, 1899c).

In a review of H. G. Wells's book *Mankind in the making* he did not spare his harsh criticism to the school system:

> Learned man, teachers, pedagogy scholars would refuse with horror the idea of taking part in three lectures a day, were it only for a week and only on topics they are deeply interested in. But they do not see how absurd it is, from an educational but also psychological and even hygienic point of view, to oblige students aged from 10 to 18 to remain riveted at least five hours a day for the whole year, as if there were no other means to attain the goals that should be reached in this way. For the result of this system of intensive culture—so similar to the barbaric nutrition system that is applied in the plain of Lombardy to obtain the exquisite goose liver—comes down to this, to cultivate in the students, especially in the most intelligent ones, such a repugnance for everything that is connected to school, or to what is taught in schools, that one might be glad that a large part of the school programs is not worth being known.[44]

In his review of Wells' book, Vailati argued that notionism was caused by scientific and technological backwardness: he polemically remarked that teaching was still organized as if Gutenberg's printing method "had not yet influenced our school system". In a technologically developed society the traditional lecture given by the teacher facing the students is just one among many different means to teach.[45] Vailati expressed a similar criticism of school didactics in the essay "Sull'arte di interrogare" [The art of asking questions] (1905e, this volume, chap. 10),[46] where he also encouraged teachers to modify the way

[44] See Vailati 1906a in Vailati 1911, p. 713.
[45] See Vailati 1906a in Vailati 1911, p. 713.
[46] This essay was influenced by James's *Talks to teachers on psychology: and to*

they ask questions to their students:

> The first formulation under which the question was asked represents, in my opinion rather characteristically, the type of question teachers should move towards as much as possible, either with the purpose of stimulating the student to reflect, or with the purpose of testing the condition of his knowledge. The best questions, for both purposes, are the ones that refer to the *prediction* of a specific fact, those where, after describing a given situation and a series of specific operations to the student, we ask what he would *expect* to find or to obtain if he were to perform them, or how he would *act* if he wanted to achieve a specific result given the circumstances (this volume, p. 156).

Pupils—he believed—should be left in a condition to learn autonomously, following their own interests and using all possible tools to acquire the cognitions they judge accessible and interesting. Notwithstanding the similarities, Vailati did not mention the laboratory school founded by Dewey in Chicago in 1896.[47] Recalling the criticism made in Papini and Prezzolini 1906, Vailati denounced the bad habit of publishing textbooks containing only pedantic lecture notes "compiled with dreadful uniformity according to the scholastic programs," rather than introductory books containing selected bibliographical references.[48]

Vailati's conception of school as a laboratory where the students participate in a dynamic process of learning does not correspond to the notion of a laboratory of scientific disciplines where the teacher or her assistants show to young students the 'canonical' experiments of modern science, but rather a place to realize the education of critical minds—a place enabling students to solve problems, learn autonomously by abstraction from sensible and concrete elements.[49] For this reason, Vailati insists on the importance of drawing in order to guide students to the learning of abstract geometry but also to the learning of manual work, intended not as an early specialization towards professional activity, but as an effective way to "stimulate and exercise [...] the various faculties of observation, discrimination, attention, and judgment, which are at stake in any kind of work" (Vailati, 1901d).

Vailati's last years (1907-1909)

In the years that preceded his death Vailati's interests ranged over various fields, from history of science to philosophy, from education (he

students on some of life's ideals (1899) delivered at Edinburgh in 1901-1902.

[47] Vailati mentioned Dewey's pragmatism only in two reviews of James's writings.

[48] See Vailati 1906a in Vailati 1911, p. 715.

[49] Cf. Vailati 1899c.Vailati's conception of the school as a laboratory has been emphasized in De Zan 1996. See also Minazzi 2000.

was still involved in the works of the Reform Commission) to religion.

In 1907 and 1908 he published several articles on the history of mechanics, for which he had developed a renewed interest after the publication of *The Origins of the Static* by Pierre Duhem (1905-1906). He also became a member of the scientific board of the Italian Society for the Progress of Sciences, directed by Volterra, who had transformed it from an aristocratic club into a democratic association based on interdisciplinary debate.[50]

A systematic volume on pragmatism

In this same period Vailati developed, together with Calderoni, the project of a systematic book on pragmatism, whose first two chapters "Le origini e l'idea fondamentale del Pragmatismo" [The origins and fundamental idea of pragmatism] (1909b) and "Il Pragmatismo e i vari modi di non dir niente" [Pragmatism and the various ways of saying nothing] (1909a) were published in the journal "Rivista di Psicologia Applicata" in 1909.[51]

Vailati and Calderoni 1909b acknowledged Peirce as the first to introduce the expression 'pragmatism' to denote a methodology that determines and clarifies the meaning of an assertion by indicating "which particular experiences, according to such an assertion, are going to take place, or would take place under specific given circumstances" (this volume, p. 234). After defending 'pragmatism' from the charges of 'utilitarianism' and 'relativism,' Vailati considered the relevance of this methodology to three cornerstones of his own philosophy: 1) the distinction between volitions and expectations; 2) the role of deductive inferences in the determination of the meaning of propositions; 3) the logical rather than psychological aim of the analysis of "predictive elements, that are always implicit in our assertions, even when absent from our present consciousness."

The distinction between volitions and expectations had already been introduced in the paper presented at the International Conference of Psychology in Paris on Brentano's classification of mental states (1901e), and in the article on knowledge published in "Leonardo" (1905c). The consideration of "deduction as an instrument of explanation and anticipation of experience" (this volume, p. 30) was discussed

[50] Volterra's inaugural talk at the first congress of the Italian Society for the Progress of Sciences, held in Parma on September 25th, 1907, has been published in an English translation in the appendix to Goodstein 2007, pages 261-269.

[51] See this volume, chapter 18 and chapter 19. For a deeper discussion of the 'pragmatic' elements contained in these two works, see the essay by M. Caamano and P. Suppes in this volume.

in the second inaugural lecture (1898a). The agent's relevance in the determination of the meaning of words and the importance of giving reasons as part of the meaning of words was hinted at in the third inaugural lecture (1899a). Recalling Sidgwick, Vailati remarked that "the advantage of research of this kind, on the sense of words, does not consist so much in the definitions that we find as in the operations that we have to do to find them, and that the fruit of such discussions is not in the conclusions reached, but in the reasons that we must discover and bring forward to justify them" (Vailati, 1899a, this volume, p. 70). An instrumentalist definition of meaning of an assertion as related to its deductive consequences was formulated in the essay on pragmatism and mathematical logic, where he attributed to Peano's mathematical logic and to Peirce's pragmatism "the tendency to regard the value, and even the meaning, of every assertion as being intimately related to the use which can be made, or which it may be desired to make, of it for the deduction and construction of particular consequences or groups of consequences" (1906b, this volume, p. 164).

Finally, the logical interest inherent in the analysis of language (tracing the distinction between assertions and definitions, or between volitions and expectations, classifying fallacies, developing a normative rather than descriptive theory of reasoning), and the opportunity of making unconscious aspects of language become explicit (as in the case of scientific metaphors) were already present in the third inaugural lecture Vailati (1899a), even if further developed in the essay on language as an obstacle to the elimination of illusory contrasts (1908a). Some of these aspects, which were related to the readings of Hume and Berkeley in the first writings, are here connected to Berkeley's *New Theory of Vision* (1732) and to Pikler's analysis of the judgments of existence of space and time in *The Psychology of the Belief in Objective Existence* (1890).

Vailati and Calderoni 1909a goes back to the analysis of the distinctions between assertions and definitions, relating it to the Lockean distinction between verbal (trifling) and real propositions, and with the Kantian distinction between analytic and synthetic judgments.

Definitions are thus once more at the center of Vailati's attention. An issue that was already discussed in the article on Saccheri (1903a), in the paper on the Aristotelian theory of definition (1903b) and in the historical notes to Peano's *Formulario* (1902a), concerns the problem of proving the compatibility of different properties that are assigned to the same object by a complex definition: a problem that can be solved by assuming or proving "the existence or constructability of figures or things that satisfy the conditions posed in the definitions" (1909a, this

volume, p. 253).

Vailati also developed a criticism of the use of abstract terms and of useless generalizations in philosophy, praising pragmatism for suggesting the substitution of abstract terms by concrete expressions as a therapy. The abuse of generalization takes place, according to Vailati, when generalization is considered not "as a means to certain logical or practical goals," but as a goal in itself. In "Dal Monismo al Pragmatismo" [From Monism to Pragmatism] (1907a), one finds several examples: the determination of too general classes of concepts of which one has nothing relevant to say, or the tendency to look for causes and explanations of certain facts beyond the point in which one is still capable of ascertaining the facts in questions, or the tendency to give and ask for definitions of what cannot be defined without recourse to other unexplained notions. Vailati's critical target is derived from Papini 1907, which is an attack against monists, and in particular against the philosophy of Enrico Morselli, a Professor of Psychiatry at Genoa University.

The beginning of this essay, as the conclusion of "Pragmatism and mathematical logic," reveals that some positivist elements that Vailati had absorbed in his youth were never really dismissed, notwithstanding the fact that positivism had soon become his main critical target. One of the advantages of pragmatism is described as the possibility of separating meaningful from meaningless propositions, arguing that it makes it easier "to distinguish those that can actually be proved or refuted from the ones that are not subject to any proper proof or refutation. This is because they refer to mind-states of which each individual subject is the only irrevocable judge, and also because these assertions are only apparent, and they are, in fact, sentences with *no meaning*" (1909a, this volume, p. 249).

The goal of the analysis of ordinary language and of the distinction between expressions containing volitions and expressions containing expectations is thus reduced to the mere elimination of trifling questions of words and false questions. Every expression that cannot be proved or refuted is considered as meaningless, and no analysis of its possible grounds is given. If no space is left for an objective evaluation of what falls outside the scientific domain, no rational argumentative practice or critical thinking would be applicable in many cases that Vailati himself had considered in his analyses of ordinary discourse. And the latter would have a very limited scope: banishing what is fallacious, rather than comparing the argumentative rules and schemas applied in differ-

ent contexts.[52]

Modernism

Vailati was not himself a believer, but he did not neglect the discussions that animated the catholic milieu of his time, and became interested in what is known as Roman Catholic Modernism. In 1893 he had attended the lectures given by Antonio Fogazzaro, the author of *Little Ancient World* and *Malombra*, on the compatibility of the Darwinian evolutionism with the Christian doctrine.[53] Fogazzaro belonged to a wider group of intellectuals—active between the end of the 19$^{\text{th}}$ century and the beginning of the 20$^{\text{th}}$ century in Italy, but also in France and in England,—who defended a rationalist interpretation of the Bible, secularism and the conciliation of faith and reason. This movement, that is known as 'modernism' after Pope Pius X introduced the name in the encyclical *Pascendi Dominici gregis*, was condemned by the Roman Catholic Church as the 'synthesis of all heresies': even if the compulsory oath against modernism that had been introduced for bishops and priests was not abolished until 1967, many Italian modernist priests and intellectuals continued to meet, as in the Roman circle organized by Ernesto Buonaiuti (Demofonti, 2003, p. 112) and frequented by Vailati himself.[54]

On the philosophy of language

Buonaiuti had described religion as an innovating movement that breaks up previous institutions, but finally becomes itself a dogmatic institution. In "Il linguaggio come ostacolo alla eliminazione di contrasti illusori" [Language as an obstacle to the elimination of illusory contrasts]—published on the Modernist journal "Rinnovamento" (1908a, this volume, chapter 17)—Vailati considers society as a network of obligations, responsibilities and commitments that individuals have not accepted nor are able to justify, but that impose themselves on everybody's life. Analogously, a language imposes on the speaker a number of classifications and distinctions that the speaker has not herself accepted, and cannot understand. This is—according to Vailati—common to physical and to social sciences, but the branch of philosophy that is devoted to the analysis of concepts should individuate unnecessary or unjustified expressions, as was done by the medieval theory

[52] For an analysis of Vailati's defense of mathematical logic and the ambivalence of his "practical" conception of logic, see Bozzi 2000, especially p. 106 f.

[53] Fogazzaro's last novels were banned by the Roman Catholic Church in 1905 because of his unorthodox ideas on Darwinism.

[54] Incidentally, Vailati was the only layman frequenting the circle.

of discourse that had classified fallacious questions—the so-called *exponibilia*.[55] Vailati defended not only the right but also the necessity for philosophy and science to oppose unjustified crystallizations of concepts, as happened when Galileo criticized the Aristotelian distinction between celestial and terrestrial phenomena. Independence from the language constraint does not consist in the arbitrariness of modifying the terminology used to designate given concepts, provided the meaning of the terms is preliminarily fixed by definition, but rather in the freedom of modifying conceptual distinctions and classifications that are "inappropriate or inadequate to the goals that one has in a given occasion." According to Vailati, modifying the concepts is much more important for philosophical and scientific discussions than modifying only the terms used to denote such concepts.[56]

The grammar of algebra

In 1908 Vailati attended the International Conference in Philosophy in Heidelberg and presented a paper "The grammar of algebra" at the Conference of the Italian Society for the Progress of Sciences (1908b).

In a recent inquiry on Vailati's linguistic analyses, Innis (2002) rightly claims that Vailati's approach to language is partly based on a "rhetoric of suspicion" and partly oriented to a descriptive and constructive task. If the previously mentioned article on language as an obstacle to the elimination of illusory contrasts was rather based on a negative conception of language, the essay on the grammar of algebra is based on a different approach, mediated by the comparison between ordinary language and the symbolic language of mathematics.

Vailati distinguished between phonetic and ideographic languages, between positional and non-positional writings, between nomenclatures and languages, and individuated the peculiarity of the algebraic language in being ideographic and positional. He then proceeded to a detailed analysis of the grammar and the syntax of algebraic language, comparing it to ordinary language. Relying on Müller's definition of language as composed of words that do not have a meaning if taken in isolation, Vailati analyzed the distinction between verbs that require to be completed by an object (and that he called 'generally transitive') and verbs that are strictly 'intransitive.'

Recalling Peirce's contributions to the theory of relations, Vailati compared transitive verbs with relative nouns and classified general transitive verbs according to the number of objects they need—a dis-

[55] Among Vailati's examples there is what modern theorists of argumentation call the "fallacy of the many questions."

[56] Vailati 1908a in Vailati, 1911, p. 899.

tinction that corresponds in algebra to the number of values of a function. After characterizing the algebraic language as containing only transitive relations—for it is taken to be based on equalities—Vailati discussed the differences between equality and identity, and the nature of definitions by abstraction, remarking that the choice of the kind of definition of a new symbol might depend on the nature of the concepts, but also on matters of convenience, clarity, and intelligibility.

Invoking the attention of philologists on symbolic languages, Vailati wrote another 'apologetic' essay: as in the case of the two essays on the definition of mathematics (this volume, chapter 8) and on the comparison between mathematical logic and pragmatism (this volume, chapter 12), Vailati mentioned other logicians, but discussed only the problems that were familiar to him from the debates with the logical school of Peano. For example, the interest for the debate on the use of one single sign to express different equivalence relations had been mentioned in the Frege-Peano discussion, and again in Burali-Forti and Marcolongo's analysis of vectorial notations, but also in Pieri's writings.[57] The interest for definitions by abstraction and for equality is also related to his friendship with Padoa, who had published an article on equivalence relations as conditions for the introduction of definitions by abstraction (1905-06).

Vailati's death and the project of the collected works

In the autumn 1908 Vailati planned to go back to his teaching post in Florence, but in December he contracted influenza, from which he failed to recover. Hoping for a milder winter, he moved to Rome, where he was hospitalized in March 1909, suffering from a rheumatic fever with cardiac complications.[58] The onset of tonsillitis led to his death on May 14, 1909. Right up until his death Vailati had hoped to recover,[59] devoting himself to reading: Marcus Aurelius's *Meditations* and Spinoza's *Ethics*, as well as novels by Stendhal, Rolland and Maupassant. The official funeral took place in Rome, attended by the Senate vice-president, Pietro Blaserna, the senator Vito Volterra and the deputy Andrea Torre; several articles and obituaries were published (De Zan, 2000a). Volterra asked Mario Calderoni, Umberto Ricci and Giovanni Vacca to edit a volume of Vailati's collected works (1911), whose costs were paid for by a public subscription. Among the subscribers there were not only Italian intellectuals, but also international scholars, such as Samuel

[57] See Frege 1980, Burali-Forti and Marcolongo 1909, and Marchisotto and Smith 2007.

[58] See Orazio Premoli's biography of Vailati in Vailati 1911.

[59] See the letter from Einaudi to Papini, March 6, 1909, in Kühn 1960.

Dickstein, Pierre Duhem, Knud Ferlov, Bèla Fogasari, Daniel and Élie Halévy, Johan Ludwig Heiberg, William James, Adolphe Landry, A. Lasson, Léon Xavier, Bertrand Russell, Victoria Welby, Gyula Pikler, who would later dedicate a book to Vailati,[60] and Vernon Lee (Violet Paget), who would dedicate a work on pragmatism to the memory of the friend Giovanni Vailati, "who better than anyone else, explained the incompatibility between 'willing to believe' and 'making one's ideas clear' (Lee, 1912)."

Conclusion

Vailati's interests were so varied that one can hardly present them all satisfactorily in a single essay: this was surely not the intention of our introduction. We have rather tried to offer a sampling of the variety of topics that he was interested in, while summarizing the main events of his life and introducing the reader to a figure who was not academic, but was strongly connected to the new research waves in philosophy and science and to the social ferment at the beginning of the 20^{th} century.

In the variety, if not serendipity of Vailati's interests, there are some important themes to be traced: one is surely Vailati's adhesion to pragmatism, which is discussed in the following essay by M. Caamano and P. Suppes. Another main theme can be detected in Vailati's interest for the definition of concepts. This 'pragmatist' theme is related to Peano's logical inquiries on the topic, but it is also deeply connected to several aspects of Vailati's own research, and especially to the interpretation of the common features of pragmatism and mathematical logic.

References

Berkeley, George. 1732. *An Essay Towards a New Theory of Vision*. London: J. Tonson.

Bozzi, Silvio. 2000. Vailati e la logica. In M. De Zan (2000b), pages 88–111.

Burali-Forti, Cesare. 1894. *Logica Matematica*. Milano: Hoepli.

Burali-Forti, Cesare and Roberto Marcolongo. 1909. Per l'unificazione delle notazioni vettoriali. In G. Castelnuovo, ed., *Atti del IV congresso internazionale dei matematici, Roma 6-11 Aprile 1908*. Roma: Tipografia dell'Accademia dei Lincei. Repr. Nendeln (Liechtenstein): Kraus, 1967.

Calderoni, Mario. 1904. Le varietà del pragmatismo. *Leonardo* 2(3).

Calderoni, Mario. 1905. Variazioni sul pragmatismo. *Leonardo* 3(2):15–21.

Calderoni, Mario. 1924. Giovanni Vailati, commemorazione letta a Crema il 20 giugno 1909. In O. Campa, ed., *Scritti*, vol. 2, pages 161–180. Firenze: La Voce.

[60] He wrote: "Giovanni Vailati in Freundschaft und Hochachtung zugeeignet" (Pikler, 1908).

Cantù, Paola. 1999. *Giuseppe Veronese e i fondamenti della geometria*. Milano: Unicopli.

Cantù, Paola. 2000. L'insegnamento della geometria nelle scuole medie inferiori. Una lettera inedita di Giuseppe Veronese a Giovanni Vailati. *Il Voltaire* 5:109–118.

Cantù, Paola. 2007. Il carteggio Padoa-Vailati. Un'introduzione alle lettere inviate da Chioggia. *Chioggia. Rivista di Studi e ricerche* 30:45–70.

Casini, Paolo. 2002. *Alle origini del Novecento. Leonardo, 1903-1907*. Bologna: Il Mulino.

Chisholm, Roderick M. and Michael Corrado. 1982. The Brentano-Vailati Correspondence. *Topoi* 1:3–30.

Couturat, Louis. 1901. *La Logique de Leibniz d'après des documents inédits*. Paris: Alcan.

Darwin, Charles. 1859. *The Origin of Species*. London: Murray.

De Amicis, Enrico. 1892. Dipendenza fra alcune proprietà notevoli delle relazioni fra enti di un medesimo sistema. *Rivista di Matematica* 2:113–127.

De Zan, Mauro. 1996. Attualità di Giovanni Vailati e superamento della classe come struttura base dell'organizzazione scolastica. In D. Generali and F. Minazzi, eds., *La scuola italiana. Tra delusione e utopia*, pages 183–193. Padova: Edizioni Sapere.

De Zan, Mauro. 2000a. Giovanni Vailati letto dai contemporanei. In M. De Zan (2000b), pages 37–49.

De Zan, Mauro, ed. 2000b. *I Mondi di Carta di Giovanni Vailati*. Milano: Angeli.

De Zan, Mauro. 2003. Sul carteggio tra Vito Volterra e Giovanni Vailati. *Annuario del Centro Studi Giovanni Vailati* 1:79–89.

De Zan, Mauro. 2009. *La formazione di Giovanni Vailati*. Lecce: Congedo Editore.

Dedekind, Richard. 1888. *Was sind und was sollen die Zahlen?*. Braunschweig: Vieweg.

Demofonti, Laura. 2003. *La riforma dell'Italia del primo Novecento*. Edizioni di Storia e Letteratura.

Dewey, John. 1909. *How We Think*. London: Heath.

Duhem, Pierre. 1905-1906. *Les origines de la Statique*. Paris: Hermann. 2 voll.

Duporcq, Ernest, ed. 1902. *Compte rendu du deuxième Congrès international des mathematiciens tenu à Paris du 6 au 12 août 1900*. Paris: Gauthier-Villars.

Durand de Gros, Pierre-Joseph. 1899. *Aperçus de taxinomie générale*. Paris: Alcan.

Frege, Gottlob. 1980. *Philosophical and Mathematical Correspondence*. Chicago: University of Chicago Press. Ed. by G. Gabriel et al.

Friedlein, Gottfried. 1873. *Procli Diadochi in primum Euclidis Elementorum librum commentarii*. Teubner. Repr. Hildesheim: Olms 1992.

Gandon, Sébastien. 2006. La réception des *Vorlesungen über neure Geometrie* de Pasch par Peano. *Revue d'Histoire des Mathématiques* 12(2):249–290.

Goodstein, Judith R. 2007. *The Volterra Chronicles. The Life and Time of an Extraordinary Mathematician (1860-1940)*. American Mathematical Society and London Mathematical Society.

Greenfield, Kent Roberts. 1934. *Economics and Liberalism in the Risorgimento; a Study of Nationalism in Lombardy, 1814-1848*. Baltimore: The Johns Hopkins Press.

Hilbert, David. 1905. Über die Grundlagen der Logik und der Arithmetik. In A. Kraser, ed., *Verhandlungen des III. Internationalen Mathematiker Kongresses in Heidelberg 1904*, pages 174–185. Leipzig: Teubner.

Hontheim, J. 1895. *Der logische Algorithmus in seinem Wesen, in seiner Anwendung, und in seiner philosophischen Bedeutung*. Berlin: Verlag von Felix L. Dames. Repr. in Vailati (1911), pages 149–153.

Hultsch, Fridericus. 1875-1878. *Pappi Alexandrini collectionis quae supersunt e libris manu scriptis edidit Latina interpretatione et commentariis*. Berlin: Weidmann.

Innis, Robert E. 2002. Paleo-Pragmatism's Linguistic Turn. Lessons from Giovanni Vailati. In *Pragmatism and the Forms of Sense: Language, Perception, Technics*, pages 99–130. University Park, PA: Penn State Press.

James, William. 1899. *Talks to Teachers on Psychology: and to Students on Some of Life's Ideals*. Longmans, 1901.

James, William. 1906. G. Papini and the Pragmatist Movement in Italy. *Journal of Philosophy, Psychology and Scientific Methods* 3:337–341.

James, William. 1908. *Pragmatism*. New York: Longmans & Green.

James, William. 1969. *The Letters of William James*. Boston: Athlantic Monthly Press.

Koyré, Alexandre. 1939. *Études galiléennes*. Paris: Hermann.

Kühn, Eva. 1960. *Vita con Giovanni Amendola*. Firenze: Parenti.

Kuhn, Thomas Samuel. 1962. *The Structure of Scientific Revolutions*. Chicago: The University of Chicago Press, 1970.

Lee, Vernon. 1912. *Vital Lies; Studies of Some Varieties of Recent Obscurantism*. London, New York: J. Lane.

Lolli, Gabriele. 1985. *Le ragioni fisiche e le dimostrazioni matematiche*. Bologna: Il Mulino.

Mach, Ernst. 1883. *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt*. Leipzig: Brockhaus (1889), 2nd edn. Engl. trans. by T.J. McCormack. *The Science of Mechanics. A Critical and Historical Account of Its Development*. Open Court Publishing Co.: Chicago, London 1919.

Mach, Ernst. 1909. *I principii della Meccanica esposti criticamente e storicamente nel loro sviluppo*. Roma-Milano: Albrighi Segati. Italian transl. by D. Gambioli.

Marchisotto, Elena A. and James T. Smith. 2007. *The Legacy of Mario Pieri in Geometry and Arithmetic*. Boston, Mass.: Birkhäuser.

Mill, John Stuart. 1865. *An Examination of Sir William Hamilton's Philosophy, and of the Principal Philosophical Questions Discussed in his Writings*. London: Longman, Roberts & Green, 1872.

Mill, John Stuart. 1870. *A System of Logic, Ratiocinative and Inductive; Being a Connected View of the Principles of Evidence and the Methods of Scientific Investigation*. New York: Harper & Brothers.

Minazzi, Fabio. 2000. Vailati e la scuola italiana. In M. De Zan (2000b), pages 223–251.

Müller, Max F. 1891. *The Science of Language: Founded on Lectures Delivered at the Royal Institution in 1861 and 1863*. London: Longmans, Green.

Nagy, Albino. 1892. *Principi di logica, esposti secondo le dottrine moderne*. Torino: Loescher.

Padoa, Alessandro. 1898. *Conférences sur la logique mathématique*. Bruxelles: Université nouvelle de Bruxelles.

Padoa, Alessandro. 1901. Essai d'un théorie algébrique des nombres entiers précédé d'une introduction logique à une théorie déductive quelconque. In *Bibliothèque du congrès international de philosophie. Paris 1900*, vol. 3, pages 309–365. Paris: Colin. Engl. transl. in J, van Heijenoort, ed. *From Frege to Gödel*, Cambridge, Mass.-London: Harvard Univ. Press. 1967, 119-123.

Padoa, Alessandro. 1902a. Un nouveau système des définitions pour la géométrie euclidienne. In Duporcq E., ed., 1902, pages 353–363.

Padoa, Alessandro. 1902b. Un nouveau système irréductible de postulats pour l'algèbre. In Duporcq E., ed., 1902, pages 249–256.

Padoa, Alessandro. 1905-06. Che cosa è una relazione? *Atti della Accademia Reale delle Scienze di Torino* 41:818–826.

Palmieri, Paolo. 2008. The Empirical Basis of Equilibrium: Mach, Vailati, and the Lever. *Studies in History and Philosophy of Science* 39:42–53.

Papini, Giovanni. 1902. La teoria psicologica della previsione. *Archivio per l'Antropologia e l'Etnologia* 32(2):351–375.

Papini, Giovanni. 1905. Pragmatismo messo in ordine. *Leonardo* 3(2):45–48.

Papini, Giovanni. 1907. Non bisogna essere monisti. In *Ricerche e Studi di Psichiatria, Nevrologia, Antropologia e Filosofia*. Milano: Vallardi. Repr. in *Sul pragmatismo (Saggi e Ricerche) 1903-1911*. Milano: Libreria Editrice Milanese. 1913, pages 83–102.

Papini, Giovanni and Giuseppe Prezzolini. 1906. *La coltura italiana*. Firenze: Lumachi.

Peano, Giuseppe. 1891. Osservazioni del Direttore (ad una lettera di C. Segre). *Rivista di Matematica* 1:66–69.

Peano, Giuseppe, ed. 1901a. *Formulaire de Mathématiques*. Turin: Bocca-Clausen.

Peano, Giuseppe. 1901b. Les définitions mathématiques. In *Bibliothèque du congrès international de philosophie. Paris 1900*, vol. 3. Logique et Histoire des Sciences, pages 279–288. Paris: Colin.

Peano, Giuseppe. 1903-04. Il latino, quale lingua ausiliare internazionale. *Atti della Reale Accademia delle Scienze di Torino* 39:273–283.

Peano, Giuseppe. 1915. Le definizioni per astrazione. *Bollettino della Mathesis* 7:106–120.

Peano, Giuseppe and Giovanni Vailati. 1893. Sulla raccolta di formule. *Rivista di Matematica* 3:1.

Peirce, C. S. 1905. What Pragmatism Is. *The Monist* 15(2):161–181.

Perez, E. 1895. El cultivo de la matemática y la forma deductiva de la inferencia. *Memorias de la Sociedad Científica Antonio Alzate* 8:315–363.

Pikler, Gyula. 1890. *The Psychology of the Belief in Objective Existence*. London: Williams and Norgate.

Pikler, Gyula. 1908. *Über Theodor Lipp's Versuch einer Teorie des Willens. Eine kritische Untersuchung, zugleich in Beitrag zu einer dynamischen Psychologie*. Leipzig: Barth.

Poincaré, Henri. 1902. *La science et l'hypothèse*. Paris: Flammarion. Engl. transl. *Science and hypothesis*. New York: The Science Press, 1905.

Prezzolini, Giuseppe. 1905. Il mio prammatismo. *Leonardo* 3(2):48.

Renan, Ernest. 1876. *Dialogues et fragments philosophiques*. Paris: Calmann-Lévy.

Rizza, Cinzia, ed. 2006. *Benedetto Croce – Giovanni Vailati Carteggio (1899-1905)*. Acireale-Roma: Bonanno.

Russell, Bertrand. 1998. *Autobiography*. London: Routledge.

Schiller, F.C.S. 1905. The Definitions of Pragmatism. *Leonardo* 3(2):44–45.

Segre, Corrado. 1891. Su alcuni indirizzi nelle investigazioni geometriche. Osservazioni dirette ai miei studenti. *Rivista di Matematica* 1:42–66.

Sidgwick, Henry. 1874. *The Methods of Ethics*. London: Macmillan, 1963.

Spencer, Herbert. 1862. *First Principles of a New System of Philosophy*. London: Williams and Norgate.

Vailati, Giovanni. 1891a. Le proprietà fondamentali delle operazioni della logica deduttiva studiate dal punto di vista d'una teoria generale delle operazioni. *Rivista di Matematica* 1. Repr. in Vailati (1911), pages 2–8.

Vailati, Giovanni. 1891b. Un teorema di logica matematica. *Rivista di Matematica* 1. Repr. in Vailati (1911), page 1.

Vailati, Giovanni. 1892a. Dipendenza fra le proprietà delle Relazioni. *Rivista di Matematica* 2. Repr. in Vailati (1911), pages 14–17.

Vailati, Giovanni. 1892b. Sui principi fondamentali della geometria della retta. *Rivista di Matematica* 2. Repr. in Vailati (1911), pages 9–13.

Vailati, Giovanni. 1893. Recensione a A. Nagy, Principi di logica esposti secondo le teorie moderne. *Rivista di Matematica* 3. Repr. in Vailati (1911), pages 18–19.

Vailati, Giovanni. 1894a. Recensione a C. Burali-Forti, Logica matematica (Milano, Hoepli, 1894). *Rivista di Matematica* 4. Repr. in Vailati (1911), pages 22–25.

Vailati, Giovanni. 1894b. Recensione a Catalogue of University of Texas for 1893-94. *Rivista di Matematica* 4. Repr. in Vailati (1911), pages 20–21.

Vailati, Giovanni. 1895a. Sulle proprietà caratteristiche delle varietà a una dimensione. *Rivista di Matematica* 5:183–185. Repr. in Vailati (1911), pages 30–32.

Vailati, Giovanni. 1895b. Sulle relazioni di posizione tra punti d'una linea chiusa. *Rivista di Matematica* 5. Repr. in Vailati (1911), pages 26–29.

Vailati, Giovanni. 1896a. Recensione a E. Mach, Populär-wissenschaftliche Vorlesungen. *Rivista di Studi Psichici* Repr. in Vailati (1911), pages 60–63.

Vailati, Giovanni. 1896b. Recensione a E. Mach, Populär-wissenschaftliche Vorlesungen. *Rivista sperimentale di Freniatria* 22(3). Repr. in Vailati (1911), pages 43–45.

Vailati, Giovanni. 1896c. Recensione a E. Perez, El cultivo de la Matematica y la forma deductiva de la inferencia. *Rivista di Matematica* 6. Repr. in Vailati (1911), pages 33–34.

Vailati, Giovanni. 1897a. Del concetto di Centro di Gravità nella Statica di Archimede. *Atti della Regia Accademia delle Scienze di Torino* 32. Repr. in Vailati (1911), pages 79–90.

Vailati, Giovanni. 1897b. Il principio dei Lavori Virtuali da Aristotele a Erone d'Alessandria. *Atti della Regia Accademia delle Scienze di Torino* 32. Repr. in Vailati (1911), pages 91–106.

Vailati, Giovanni. 1897c. *Sull'importanza delle Ricerche relative alla Storia delle Scienze. Prolusione a un corso sulla Storia della meccanica, letta il 4 dicembre 1896 nell'Università di Torino*. Torino: Roux e Frassati. Repr. in Vailati (1911), pages 64–78. Engl. transl. "On the Importance of Research Regarding the History of Science," this volume, chap. 1.

Vailati, Giovanni. 1898a. *Il Metodo Deduttivo come Strumento di Ricerca. Prolusione ad un corso libero di Storia della Meccanica, 1897-1898*. Torino: Roux-Frassati. Repr. in Vailati (1911), pages 118–148. Engl. transl. "The Deductive Method as an Instrument of Research," this volume, chap. 2.

Vailati, Giovanni. 1898b. La méthode déductive comme instrument de recherche. *Revue de Métaphysique et de Morale* 6:667–703.

Vailati, Giovanni. 1898c. Le speculazioni di Giovanni Benedetti sul moto dei gravi. *Atti della Regia Accademia delle Scienze di Torino* 33. Repr. in Vailati (1911), pages 161–178.

Vailati, Giovanni. 1898d. Recensione a Joseph Hontheim, Der Logische Algorithmus in seinem Wesen, in seiner Anwendung und in seiner philosophichen Bedeutung. *Rivista di Matematica* 6. Repr. in Vailati (1911), pages 149–153.

Vailati, Giovanni. 1899a. *Alcune osservazioni sulle Questioni di Parole nella Storia della Scienza e della Cultura. Prolusione ad un corso libero di Storia della Meccanica, 1898-98*. Torino: Bocca. Repr. in Vailati (1911), pages 203–228. Engl. transl. "Some Observations on the Questions of Words in the History of Science and Culture," this volume, chap. 3.

Vailati, Giovanni. 1899b. La logique mathématique et sa nouvelle phase de développement dans les écrits de M. J. Peano. *Revue de Métaphysique et de Morale* 7(1). Repr. in Vailati (1911), pages 229–242.

Vailati, Giovanni. 1899c. Recensione a C. Laisant. La Mathématique: philosophie, enseignement. Paris: Carré et Naud 1898. *Il Nuovo Risorgimento* 9(8). Repr. in Vailati (1911), pages 258–259.

Vailati, Giovanni. 1899d. Recensione a C. Trivero, Classificazione delle Scienze. Hoepli, Milano 1899. *Rivista italiana di Sociologia* 3(4). Repr. in Vailati (1911), pages 249–250.

Vailati, Giovanni. 1900. Recensione a J.P. Durand (De Gros), Aperçus de taxinomie générale, Alcan, Paris 1900. *Rivista di Scienze Biologiche* 1-2. Repr. in Vailati (1911), pages 298–299.

Vailati, Giovanni. 1901a. Des difficultés qui s'opposent à une classification rationelle des sciences. In *Bibliothèque du Congrès international de Philosophie. Paris 1900*, vol. 3. Logique et Histoire des Sciences. Paris: Colin. Repr. in Vailati (1911), pages 324–335. Italian transl. in Quaranta (2003), pages 179–202. Engl. transl. "The Difficulties Involved in a Rational Classification of Sciences," this volume, chap. 4.

Vailati, Giovanni. 1901b. Recensione a E. Mach. Analyse der Empfindungen und das Verhältnis des Physischen zum Psychischen. Jena: Fischer 1900. *Rivista di Biologia generale* 1-2. Repr. in Vailati (1911), pages 346–350.

Vailati, Giovanni. 1901c. Recensione a L. Couturat, La logique de Leibniz d'après des documents inédits, Alcan, Paris 1901. *Rivista di matematica* 7. Repr. in Vailati (1987), vol. 2, pages 193–204.

Vailati, Giovanni. 1901d. Recensione a M. Begey *Del lavoro manuale educativo*. Torino: Paravia 1900. *Rivista di Biologia generale* 1-2. Repr. in Vailati (1911), pages 343–345.

Vailati, Giovanni. 1901e. Sulla portata logica della classificazione dei fatti mentali proposta dal prof. Franz Brentano (Comunicazione presentata al III Congresso Internazionale di psicologia di Parigi, agosto 1900). *Rivista Filosofica* 2(1). Repr. in Vailati (1911), pages 336–340. Engl. transl. "On the Logical Import of the Classification of Mental Facts Proposed by Franz Brentano," this volume chap. 5.

Vailati, Giovanni. 1902a. Aggiunte alle Note Storiche del Formulario. *Rivista di Matematica* 8(3). Repr. in Vailati (1911), pages 449–453.

Vailati, Giovanni. 1902b. *Di un modo di riattaccare la teoria delle Proporzioni fra segmenti e quella dell'Equivalenza (II Congresso degli Insegnanti di Matematica delle Scuole secondarie, promosso dall'Associazione Mathesis, Livorno 17-22 agosto 1901)*. Livorno: Giusti. Repr. in Vailati (1911), pages 399–402.

Vailati, Giovanni. 1902c. Recensione a A. Naville, Nouvelle classification des sciences. Alcan, Paris 1901. *Rivista di Biologia generale* 3. Repr. in Vailati (1911), pages 429–439.

Vailati, Giovanni. 1903a. Di un'opera dimenticata del P. Gerolamo Saccheri ("Logica Demonstrativa" 1697). *Rivista Filosofica* 4. Repr. in Vailati (1911), pages 477–484.

Vailati, Giovanni. 1903b. La teoria aristotelica della definizione. *Rivista di Filosofia e scienze affini* 2(5). Repr. in Vailati (1911), pages 485–496. Engl. transl. "The Aristotelian Theory of Definition," this volume, chap. 7.

Vailati, Giovanni. 1903c. Sull'applicabilità dei concetti di causa e di effetto nelle scienze storiche. International Conference of Historical Sciences, Rome, April 1903. *Rivista Italiana di Sociologia* 7(3). Repr. in Vailati (1911), pages 459–464. Engl. transl. "On the Applicability of the Concepts of Cause and Effect in Historical Sciences," this volume, chap. 6.

Vailati, Giovanni. 1904a. Intorno al significato della differenza tra gli assiomi ed i postulati nella geometria greca. In *Verhandlungen des III Internationalen Mathematiker Kongresses in Heidelberg 8–13 aug. 1903*. Leipzig: Teubner, 1905. Repr. in Vailati (1911), pages 547–552. Engl. transl. "On the Meaning of the Difference Between Axioms and Postulates in Greek Geometry," this volume, chap. 9.

Vailati, Giovanni. 1904b. La dimostrazione del principio delle leva data da Archimede nel libro primo sull'equilibrio delle figure piane. In *Atti del Congresso Internazionale di Scienze Storiche, Roma, 1903*, vol. 12. Roma: Lincei. Repr. in Vailati (1911), pages 497–502.

Vailati, Giovanni. 1904c. La più recente definizione della matematica. *Leonardo* pages 7–10. Repr. in Vailati (1911), pages 528–533. Engl. transl. "The Most Recent Definition of Mathematics," this volume, chap. 8.

Vailati, Giovanni. 1905a. I tropi della logica. *Leonardo* 3:3–7. Repr. in Vailati (1911), pages 564–571. Engl. transl."On Material Representations of Deductive Processes," *Journal of Philosophy, Psychology and Scientific Methods*, 5(12):309–316, 12 June 1908. Repr. this volume, chap. 15.

Vailati, Giovanni. 1905b. La caccia alle antitesi. *Leonardo* 3:53–57. Engl. trans. "The Attack on Distinctions", *Journal of Philosophy, Psychology and Scientific Methods*, 4(26):701–709, December 19, 1907. Repr. this volume, chap. 14.

Vailati, Giovanni. 1905c. La distinzione tra conoscere e volere. *Leonardo* 3. Repr. in Vailati (1911), pages 626–629. Engl. transl. "The Distinction Between Knowledge and Will," this volume, chap. 11.

Vailati, Giovanni. 1905d. Recensione a E. Mach. Erkenntnis und Irrtum. Skizzen zur Psychologie der Forschung. Barth: Leipzig 1905. *Leonardo* 3:193–94. Repr. in Vailati (1911), pages 667–69.

Vailati, Giovanni. 1905e. Sull'arte di interrogare. *Rivista di Psicologia* 1(2). Repr. in Vailati (1911), pages 572–576. Engl. transl. "The Art of Asking Questions," this volume, chap. 10.

Vailati, Giovanni. 1906a. Idee pedagogiche di H.G. Wells. *Rivista di Psicologia applicata alla Pedagogia ed alla Psicopatologia* 2(3). Repr. in Vailati (1911), pages 713–717.

Vailati, Giovanni. 1906b. Il pragmatismo e la logica matematica. *Leonardo* 4(1):16–25. Repr. in Vailati (1911), pages 689–694. Engl. transl. by H. D. Austin "Pragmatism and Mathematical Logic," *Monist* 16:481–491, 1906. Repr. this volume, chap. 12.

Vailati, Giovanni. 1906c. A study of Platonic Terminology. *Mind* 15(60):473–485. Repr. this volume, chap. 13.

Vailati, Giovanni. 1907a. Dal monismo al pragmatismo. *Rivista di Psicologia applicata alla Pedagogia ed alla Psicopatologia* 3(4). Repr. in Vailati (1911), pages 787–790.

Vailati, Giovanni. 1907b. L'insegnamento della matematica nel primo triennio della scuola secondaria. *Bollettino di Matematica* 6(8-9). Repr. in Vailati (1911), pages 805–809.

Vailati, Giovanni. 1908a. Il linguaggio come ostacolo alla eliminazione di contrasti illusori. *Rinnovamento* 2(5-6). Repr. in Vailati (1911), pages 895–99. Engl. transl. "Language as an Obstacle to the Elimination of Illusory Contrasts," this volume, chap. 17.

Vailati, Giovanni. 1908b. La grammatica dell'algebra. *Rivista di Psicologia Applicata* 4. Repr. in Vailati (1911), pages 871–889. Engl. transl. "The Grammar of Algebra," this volume, chap. 16.

Vailati, Giovanni. 1911. *Scritti di G. Vailati, 1863-1909*. Leipzig-Firenze: Barth-Seeber. M. Calderoni, U. Ricci, and G. Vacca, eds.

Vailati, Giovanni. 1924. Sulla teoria delle proporzioni. In F. Enriques, ed., *Questioni riguardanti le matematiche elementari*, pages 143–191. Bologna: Zanichelli.

Vailati, Giovanni. 1971. *Epistolario, 1891-1909*. Torino: Einaudi. G. Lanaro, ed.

Vailati, Giovanni and G. Amato Pojero. 1993. *Epistolario: 1898-1908*. Milano: Angeli. A. Brancaforte, ed.

Vailati, Giovanni and Mario Calderoni. 1909a. Il Pragmatismo e i vari modi di non dir niente. *Rivista di Psicologia applicata* 5(9). Repr. in Vailati (1911), pages 933–41. Engl. transl. "Pragmatism and the Various Ways of Saying Nothing," this volume, chap. 19.

Vailati, Giovanni and Mario Calderoni. 1909b. Le origini e l'idea fondamentale del Pragmatismo. *Rivista di Psicologia applicata* 5(1). Repr. in Vailati (1911), pages 920–32. Engl. transl. "The Origins and Fundamental Idea of Pragmatism," this volume, chap. 18.

Verrecchia, Anacleto. 1978. *La catastrofe di Nietzsche a Torino*. Torino: Einaudi.

Volterra, Vito. 1902a. Betti, Brioschi, Casorati, trois analystes italiens et trois manières d'envisager les questions d'analyse. In Duporcq E., ed. 1902, pages 43–57.

Volterra, Vito. 1902b. Sur les équations aux dérivées partielles. In Duporcq E., ed., 1902, pages 377–378.

Zeuthen, Hieronymus Georg. 1896. *Geschichte der Mathematik im Altertum und Mittelalter*. Kopenhagen: A. F. Höst und Sön.