Arguments, Suppositions, and Conditionals *

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Abstract Arguments and conditionals are powerful means natural languages provide us to reason about possibilities and to reach conclusions from premises. These two kinds of constructions exhibit several affinities—e.g., they both come in different varieties depending on the mood; they share some of the same connectives (i.e., 'then'); they also allow for similar patterns of modal subordination. In the light of these affinities, it is not surprising that prominent theories of conditionals—old and new suppositionalisms and dynamic theories of conditionals—as well as certain reductive theories of arguments tend to semantically assimilate conditionals and arguments. In this paper, I shall marshall some linguistic evidence as well as some theoretical considerations for thinking that, despite these similarities, arguments and conditionals should be given a different semantics.

Keywords: argument, conditional, Lewis Carroll, reductionism, suppositionalism, dynamic theories

1 Introduction

It is a familiar point that we do not just exchange information in language—we also give orders, ask questions, express preferences, etc. Language is a versatile tool. It is less frequently noted that, among the things we can do with language, there is *to give arguments*, *to argue*, and *to reason in language*, so that our reasoning and arguments can only be as good as the language that we use to reason and to argue allows for.

Natural languages offer us different tools to argue and to reason in them. One such tool—conditionals—has been widely studied by both philosophers and linguistics; the other—arguments—much less so. My contention is that we can learn a lot about these two different tools that language gives us to reason by comparing them and by contrasting them. My primary goal is to raise some difficulties for any semantic theory that assimilates arguments and conditionals: these theories stand in the way of understanding the different

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¹ See Pavese 2017, Pavese 2022b, Kocurek & Pavese 2022 for some recent work on the semantics of arguments.

resources that natural languages give us to argue and to reason in them. The second part of the article makes some progress towards modeling both constructions so to account both for their commonalities as well as for their differences.

2 Arguments and Conditionals

2.1 Arguments and Semantic Theory

Arguments are ubiquitous in ordinary discourse. They appear, from time to time, in political discourse:

- (1) Our cruel and unrelenting enemy leaves us only the choice of brave resistance or abject submission. We have, therefore, to resolve to conquer or die. (George Washington, 1776 General Orders, 2 Jul.)
- (2) All free men, wherever they may live, are citizens of Berlin, and therefore, as a free man, I take pride in the words, 'Ich bin ein Berliner.' (J.F. Kennedy, 1963 Speech at West Berlin City Hall)
- (3) I cannot deliver the mandate on which I was elected by the Conservative Party. Therefore, I am resigning as leader of the Conservative Party. (Truss in her short resignation speech.)

Arguments also appear in literary discourse (as in (4)) and in religious discourse (as in (5)):

- (4) The impossible could not have happened. Therefore, the impossible must be possible despite the appearances. (Agatha Christie, Murder on the Orient Express)
- (5) But seek first the kingdom of God and His righteousness, and all these things will be added unto you. Therefore, do not worry about tomorrow, for tomorrow will worry about itself. Today has enough trouble of its own. (Matthew 3:64)

These are rather simple arguments. More complex arguments can be found in both philosophical and scientific discourse. Here is Hume's argument that causation is mere custom:

(6) When I see a billiard ball moving towards another, my mind is immediately carried by habit to the usual effect, and anticipates my sight conceiving the second ball in motion. There is nothing in these objects, abstractly considered, and independent of experience, which leads me to form any such conclusion; ... It is not, therefore, reason which is the guide of life, but custom. (Hume 1896)

For an example of an argument in scientific discourse, consider Einstein's argument for the

existence of the continuum, which piles up several suppositions:

(7) Suppose we have a set of 'elements' of some sort. Suppose that these elements possess one or more fundamental identifying characteristics, analogous to the coordinates of a point... Suppose we find that no two elements of the set possess identically the same set of defining values. Suppose finally that the elements of the set are such that, no matter what numerical values we define by these an actual element of the set, that corresponds to this particular collection of values. Our elements then share with the real number system the property of sharing no holes, of constituting a continuous possession in every dimension which we possess. We then have a continuum. (Bird 1921: 148-9).

While people differ as to how good they are at *giving arguments*, a rich literature in psychology (e.g., Johnson-Laird 1983; Rips 1994) suggests that competent speakers are good at *interpreting arguments when given to them in their language*. This includes complex arguments too, such as arguments by conditional proof or arguments by cases. For example, we all can understand and interpret rather complex arguments such as (8):

(8) Whoever committed the crime left by the window. Anyone who had left by the window would have mud on his shoes. Suppose the butler committed the crime. Then he left by the window. In that case, he has mud on his shoes. So if the butler committed the crime, he has mud on his shoes. (Conditional proof)

If arguments are discourses which competent speakers can interpret, then semanticists should be concerned with arguments just as much as they have been concerned with conditionals and other linguistic constructions that are similarly interpretable—i.e., the interpretation of arguments definitely falls within the purview of semantic theory.

2.2 The Conventional Form of Arguments

Though arguments come in a variety of forms, we can isolate a conventional form for stereotypical arguments (fig. 1). Arguments have antecedents and consequents. Antecedents that can be a premise (as in (9a)), a list of premises (as in (9b)), or themselves full arguments (as in (10)). Consequents can be sentences (or *conclusions*) (as in (9a–f)) or arguments themselves (as in (12)). Arguments typically feature an argument connective (e.g., 'therefore', 'then', 'so', 'thus', 'hence', 'ergo').

Arguments with a premises/conclusion structure such as ' P_1 , ... P_n . Therefore, C' are *simple arguments*. A premise can be categorical (as in (9a–d)) or suppositional (as in (9e–f)); a simple argument is categorical if it has categorical premises, else it is suppositional; arguments' consequents can be declarative (as in (9a), (9e)), interrogative (as in (9c), (9f)),

imperative (as in (9d)), or even argumentative (as in(12)):

- (9) a. It is raining. Therefore, streets are wet.
 - b. It is raining. It is windy. Therefore, better to stay home.
 - c. It is raining. Will the street be, therefore, wet?
 - d. It is raining. Therefore, take the umbrella!
 - e. Suppose it is raining. Then, streets are wet.
 - f. Suppose it is raining. Then, will the street be wet?

Complex arguments are arguments that have other arguments as parts. For example, (10) has an argument as its antecedent; (11) is made of two suppositional arguments; (12) has an argument as its consequent:

- (10) Suppose there is a largest prime number p. Then p! + 1 is larger than p. But p! + 1 is prime, contradiction. Therefore, there is no largest prime number. (Complex & Categorical)
- (11) Suppose she is from Turin. Then she is Italian. Suppose instead she is from Madrid. Then she is Spanish. (Complex & Suppositional)
- (12) It is raining. Therefore, suppose you forget the umbrella. You will get wet. (Complex with Argumentative Conclusion)

2.3 Affinities between arguments and conditionals

Argumentative discourse bears several affinities with conditional discourse, which has been much more widely and extensively studied by both philosophers and linguists. Conditionals and arguments are used almost interchangeably to reach conclusions from premises and to talk about possibilities and both come in different varieties depending on their mood—whether indicative or subjunctive:

- (13) a. Suppose Oswald didn't kill Kennedy. Then someone else did. (Indicative Mood)
 - b. If Oswald did not kill Kennedy, someone else did. (Indicative Mood)
- (14) a. Suppose Oswald hadn't killed Kennedy. Then, someone else would have. (Subjunctive Mood)
 - b. If Oswald hadn't killed Kennedy, someone else would have. (Subjunctive Mood)

They also exhibit similar patterns of modal subordination—persistence beyond their syntactic scope (as in (15a-b)) as well as reversibility (as in (16a-b)):

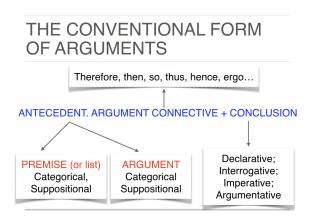


Figure 1 The Conventional Form of Arguments.

- (15) a. If a wolf comes in, we will use a gun. If we manage to shoot, we will be safe. If we bury the body, nobody will find out.
 - b. Suppose a wolf comes in. We will use a gun. Suppose we manage to shoot. We will be safe. Suppose we bury the body. Nobody will find out.
- (16) a. If it is raining, the park will be wet. If it is not, then the park will be dry.
 - b. Suppose it is raining. The park will be wet. Suppose it is not. The park will be dry.

Finally, arguments and conditionals allow for similar kinds of conclusions (interrogative, imperative, as well as declarative):

- (17) a. Suppose Oswald did not kill Kennedy. Then, someone else did. (Declarative)
 - b. If Oswald did not kill Kennedy, then someone else did. (Declarative)
- (18) a. Suppose Oswald did not kill Kennedy. Then, who did? (Interrogative)
 - b. If Oswald did not kill Kennedy, then who did? (Interrogative)
- (19) a. Suppose they chase you. Then run! (Imperative)
 - b. If they chase you, run! (Imperative)

2.4 Theories assimilating conditionals and arguments

So conditional discourse and argumentative discourse share many affinities. No wonder many prominent theories assimilate them. Consider, to start, suppositionalism about conditionals. It is a view of conditionals that has a long pedigree (e.g., Ramsey & Mellor

1929; Adams 1966; Mackie 1973; Barnett 2006; Edgington 1986; Edgington 1995). A clear statement of the view is due to Edgington (2020):

Hence, it appears, if Suppositionalism is right, conditionals shouldn't be construed as having truth conditions at all. A conditional judgment involves two propositions, which play different roles. One is the content of a supposition. The other is the content of a judgment made under that supposition. They do not combine to yield a single proposition which is judged to be likely to be true just when the second is judged likely to be true on the supposition of the first.

On this form of suppositionalism, a conditional—just like a suppositional argument—does not have truth conditions—i.e., there is no conditional proposition that is expressed by a conditional. Since suppositional arguments also do not have truth conditions, it is theoretically fitting to equate the conditional with the corresponding suppositional argument:

Suppositionalism If P then $Q \equiv Suppose P$. Then Q

Suppositionalism has been traditionally a speech act theory—a theory of the speech act of asserting a conditional—rather than a semantic claim. Recently, however, suppositionalism has been defended as a semantic claim. For example, Carter (2021) defends suppositionalism as a dynamic theory of conditionals, according to which "if-clauses are instructions to suppose" (p. 1066), and "if-clauses are sentence-level suppositions and suppositions are discourse-level if-clauses" (p. 1082). The difference between conditionals and suppositional arguments is supposed to be just that, in conditionals, the instruction to suppose is embedded as an antecedent, whereas in arguments it is a self-standing sentence. I will refer to antecedents of conditionals as to 'if-clauses' and to clauses such as 'Suppose...' as to 'supposes.' This form of suppositionalism explicitly assimilates if-clauses and supposes—indeed it is motivated by an alleged identity in meaning between them.

Neo-suppositionalism is rather explicit in equating conditionals and suppositional arguments. Other theories, such as dynamic theories of conditionals, arguably share the same commitment (Veltman 2012; Gillies 2010; Starr 2014a; Starr 2014b; Willer 2014). These views take conditionals *to perform a test*, which can be decomposed into two parts: (i) The *if-clause* expresses an instruction to temporarily update the context in such a way to see whether the context so updated supports the consequent; (ii) The consequent tests for such a support. If the context so updated passes the test, the result of the overall update is usually taken to be the original context.

So, on these views, *if-clauses* are *effectively* instructions to suppose. After all, an instruction to temporarily update the context in order to see if it supports the consequent

is, in effect, an instruction to suppose. Thus, dynamic theories of conditionals, just like suppositionalism, also draw on the analogy between conditionals and arguments and tend to assimilate the two. In this sense, they too are versions of suppositionalism.

Suppositionalisms of different brands equate conditionals and suppositional arguments—on these views, conditionals are suppositional arguments in disguise. This is not the only possible way of assimilating conditionals and suppositional arguments.² Another sort of view assimilating conditionals and argument is one according to which arguments are conditionals in disguise. On this view, arguments are equivalent to certain discourses involving conditionals. Some call it the *Stoic Thesis*, since they attribute it to the Stoics.³ For our purposes, it will not matter if the attribution is correct—it is certainly a view out there in the literature. For example, Brasoveanu (2007: 279) observes that categorical arguments can be reduced to conditional discourses—e.g., (20a) is intuitively equivalent to (20b):

- (20) a. A man saw a women. Therefore, he noticed her.
 - b. A man saw a women. If he saw her, he must have noticed her.

Going forward, I will dub the REDUCTIVISM ABOUT ARGUMENTS—REDUCTIVISM for short—the semantic view that reduces suppositional arguments to conditionals (**Claim 1**) and that reduces categorical arguments to a sequence of conditionals together with the categorical statements of their antecedent (**Claim 2**):

Reductivism about Arguments

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Claim 1 Suppose P. Then Q = If P then Q;
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Claim 2 P. Therefore, Q = P. If P then Q.

Since any argument is made out of suppositional arguments or/and categorical arguments, this thesis boils down to the claim that *any* argument is reducible to a discourse made out of conditionals and/or categorical statements.

² Williamson (2020) also establishes a close connection between conditionals and suppositional arguments, though only as a heuristics to probe conditionals, rather than as a semantic equivalence. See also Nolan 2023 for discussion.

³ See, e.g., Iacona 2023. However, against what Iacona (2023) alleges, what is known as a 'Stoic Thesis' should not be attributed to the Stoics. Sextus Empiricus does say that the Stoics drew a connection between true conditionals and valid arguments. But as Bobzien (2019: 261, fn 57) notes, "the Stoic conditional cited in the main text ('A Stoic sequent is valid when the conditional that consists of the conjunction of the antecedent assumptions as antecedent and the succedent as consequent is sound/true') is not a biconditional. For example, in the Stoic view (p&p)→p is true, but p, p ⊢ p is not a valid sequent." I am very thankful to Lucas Champollion for drawing my attention to this important detail.

My goal in the next two sections is to marshall some linguistic evidence against suppositionalism about conditionals (§3) and against reductivism about arguments (§4). As we will see, these constructions differ in their distributions in discourse in a way that suggests that they must make different semantic contributions to discourse.

3 Against Suppositionalism

In this section I will marshal three different sets of data that highlight some crucial differences between conditionals and suppositional arguments.

3.1 Distribution in modus ponens arguments

It is a familiar point that it is felicitous to use an *if-clause* after a might-statement that has the *if-clause*'s content as prejacent ((21a)). The same holds for *supposes*, as in (21b):

- (21) a. Lucio might be Italian. ✓ If he is Italian, then he must be European. Therefore, Lucio is European.
 - b. Lucio might be Italian. ✓ Suppose he is Italian. Then he must be European. Therefore, Lucio is European.

However, an interesting difference between *if-clauses* and *supposes* is that, while *if-clauses* are allowed after a *categorical* statement of their content ((22a)), *supposes* are not (as in (22b)):

- (22) a. Lucio is Italian. ✓ If he is Italian, then he must be European. Therefore, Lucio must be European.
 - b. Lucio is Italian. # Suppose he is Italian. Then he must be European. Therefore, Lucio must be European.

The first discourse (22a) is a common way to argue by *modus ponens*. People routinely use discourses of this sort. By contrast, when arguing by modus ponens, we typically do not use *suppose* after the categorical statement of its content, as evidenced by the unacceptability of (22b).

This brings us to the first observation:

Observation 1 *If-clauses* and *supposes* distribute differently after categorical statements of their content. Consequently, they tend to distribute differently in arguments by *modus ponens*.

The second observation is that conditionals and suppositional arguments distribute differ-

ently in arguments by conditional proof.

3.2 Distributions in arguments by conditional proof

To set up the case, consider a circumstance in which Elliott and John have fought and their fight has divided their friends into two non-overlapping teams. Mary is organizing a party, and in order to prepare the list of invitees, she wants to make sure she knows exactly who is angry at whom. Her memory is only so good, so she writes down table 1, where it is recorded who will come to the party if either Elliott or John come. Just before the party, Mary is asked who will attend if John comes to the party. Having to answer that question, Mary looks at table 1 and on this basis reasons as follows:

(23) Suppose John comes. Then Georgi will attend. Therefore, if John comes, Georgi will attend. [Conditional proof, summary uses]

This is a typical and simple argument by conditional proof. We routinely use arguments by conditional proofs and, as we have seen at the outset, we have no trouble understanding them. If conditionals and suppositional arguments made the same contributions to discourse, we would expect them to be intersubstitutable with no loss in arguments by conditional proof. But now suppose we replace the conditional in the consequent of (23) with the corresponding suppositional argument. We get:

- Suppose John attends the party. Then Georgi will attend. Therefore, ?? suppose John attends the party. Then Georgi will attend.
- (24) is not ungrammatical *per se* nor is it ill-formed. I already argued that arguments can have other arguments as conclusions (cf. §2.2, (12)). So, the problem with (24) cannot be that an argument appears as its consequent. (24) *just* sounds very redundant. It is not a coincidence that natural deduction systems do not classify this as an argument of any kind. It is just a matter of repeating the *same* argument twice.

Now, suppose, instead, we replace the antecedent of (23) with the corresponding conditional. Now we get:

(25) If John attends the party, Georgi will attend. Therefore, if John attends the party, Georgi will attend. (Trivial Argument)

This is the trivial argument—not conditional proof anymore. The trivial argument differs from an argument by conditional proof in that it is *trivial*. Another way of stating this point is to compare the respective argument forms. Consider Simple Conditional Proof, The Trivial Argument and The Double Supposing Argument. They all differ in their degree of

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Table 1 Who is friend to whom

triviality:

Simple Conditional Proof Suppose P. Then, Q. Therefore, if P then Q;

The Trivial Argument If P, then Q. Therefore, if P then Q;

The Double Supposing Argument Suppose P. Then Q. Therefore, suppose P. Then Q.

In order to see that Simple Conditional Proof is not as trivial as the Trivial Argument, consider that the validity of Simple Conditional Proof is not entirely out of the question. Indeed, denying the validity of Simple Conditional Proof is one of the most promising ways to overcome Curry's Paradox in its conditional form, as recently argued by Nolan (2016). By contrast, nobody has ever dreamt of invalidating the Trivial Argument. This is evidence that the Trivial Argument and Conditional Proof differ (at least) in their cognitive significance.

This discussion leads us to our second observation:

Observation 2 Conditionals and suppositional arguments distribute differently in arguments by conditional proof.

3.3 Coordination

The final datapoint elaborates on the phenomenon of conditionals coordination, recently discussed by Starr (2014b) and Khoo (2021). To set it up, consider as background a game of dice, in which only and all even numbers win. The die is tossed only once. In this context, it is equally bad to conjoin incompatible sentences under *if-clauses* and under *supposes*:

- (26) a. # If the die comes up 2 and the die comes up 4, Mark will win.
 - b. # Suppose the die comes up 2 and the die comes up 4. Then, Mark will win.

On the other hand, while it is fine to coordinate incompatible *if-clauses*, it remains unacceptable to coordinate incompatible *supposes*:

(27) a. \checkmark If the die comes up 2 and if the die comes up 4, Mark will win.

b. # Suppose the die comes up 2 and suppose the die comes up 4. Then, Mark will win.

One might wonder whether the explanation for this contrast might be syntactic—one might argue that perhaps the consequent is elided in (27a). That might explain why (27a) is acceptable. By contrast, the same kind of elision is not possible in (27b). On this explanation, that is so because (27b) is not a single sentence.

This syntactic explanation is not plausible, however, since, as noted by Starr (2014b), we find the same contrast with *supposing*. For example, (28) is as bad as (27b):

(28) # Supposing the die comes up 2 and supposing it comes up 4, Mark will win.

This observation is important to assess the plausibility of a syntactic explanation of the difference in coordination between *if-clauses* and *supposes*. Notice that *supposing* p, q is, like the conditional, a single sentence. So, if elision is possible to rescue the felicity of (27a), it should be possible to rescue the felicity of (28) too. But it is not. Thus, a syntactic explanation does not seem very plausible. This discussion leads us to the third observation:

Observation 3 Conjunctions of incompatible *if-clauses* are fine, whereas conjunctions of incompatible *supposes* are not.

3.4 Against a Syntactic Explanation

Conditionals and suppositional arguments do not have the same discourse distributions: they show a different distribution after categorical statements of their content; in arguments by conditional proof; and as far as coordination in antecedents goes.

To reiterate, the full set of data marshaled above is not easily explained on purely syntactic bases. The contrast between *if-clauses* and *supposes* after categorical statements of their contents (Observation 1) does not seem to be amenable to any syntactic explanation. There, the contrast does seem to be due to the different meanings of *if-clauses* and *supposes*. One might attempt to account for Observation 2 on purely syntactic bases. For example, a natural thought is that, e.g., arguments cannot occur as consequents whereas conditionals can. However, this explanation does not seem promising: as we have seen, the Double Supposing Argument—where an argument occurs in the conclusion—is not ungrammatical. It is *just* very redundant. Moreover, as we have seen at the outset (§2.2, (12)), arguments *can* generally occur as conclusions of arguments. Thus, one cannot simply explain away Observation 2 by invoking some syntactic constraints governing the distributions of suppositional arguments. Finally, we have already seen that Observation 3 cannot be

⁴ I am grateful to Dylan Bumford for pressing me on this point.

explained syntactically either. If the fact that they distribute differently in discourse cannot be explained syntactically, then there remains only another plausible explanation—that conditionals and suppositional arguments differ in their semantic contribution to discourse. We should infer that *if-clauses* and *supposes* do have different meanings.

3.5 'Supposing' and 'on the supposition that'

From the foregoing discussion, we should infer that suppositional arguments are not semantically equivalent to conditionals. But suppositionalists might object that conditionals are best assimilated to different kind of discourses—not to suppositional arguments but to supposing sentences such as 'Supposing p, q' or 'On the supposition that p, q':

Improved Suppositionalism 1 If p, $q \equiv Supposing p, q$;

Improved Suppositionalism 2 If p, $q \equiv On$ the supposition that p, q.

As we have seen in §3.3, supposing clauses do not coordinate incompatible contents. So, Observation 3 extends to invalidating these improved equivalences too. Here are, moreover, a few more independent reasons to doubt both of these equivalences. The gerund 'supposing' and the clause 'on the supposition that' feature aspectual complexities tied to the agentive verb that are spared for the conditional. These complexities show up in certain contexts. For example, consider Frege's contradictory Basic Law V and the following constructions:

- (29) a. ✓ Supposing that BL V is true, Frege got a contradiction.
 - b. ✓ Supposing that BL V is true, one gets a contradiction.
 - c. ✓ Supposing that BL V is true, a contradiction follows.
- (30) a. \checkmark On the supposition that BL V is true, Frege got a contradiction.
 - b. \checkmark On the supposition that BL V is true, one gets a contradiction.
 - c. ✓ On the supposition that BL V is true, a contradiction follows.
- (31) a. ## If BL V is true, Frege got a contradiction.
 - b. # If BL V is true, one gets a contradiction.
 - c. ?? If BL V is true, a contradiction follows.

(29a–c) are all coherent and meaningful; similarly for (30a–b). By contrast, (31a) and (31b) are infelicitous. (31c) sounds a bit better but also differs from (29c) and (30c) in an important respect: in a context in which we know about Basic Law V's contradictoriness, (29c) and (30c) are assertable but (31c) is not. This is some evidence that conditionals semantically differ not only from suppositional arguments but from supposing sentences as well.

4 Against Reductionism

In this section, I would like to put forward an argument against reducing categorical arguments to conditional discourses—that is, against REDUCTIVISM. I am going to argue that, if REDUCTIVISM were true, then it would turn out to be impossible to argue to a categorical conclusion. Since we do routinely argue to categorical conclusions, I conclude that REDUCTIVISM cannot be true.

This conclusion is one of the lessons of a long standing paradox about inference and reasoning known as Carroll 1895's regress. The paradox features two characters, Achilles and the Tortoise. Achilles starts from two premises P and if P then Q. He wants to conclude Q, by modus ponens. The tortoise opposes that: well, Q follows only if P and if P then Q and if P and if P then Q then Q. At this point, Achilles retorts: now finally, I can conclude Q. The Tortoise is not happy yet: Q follows only if P and If P then Q, and if P and If P then Q, then Q, then Q.

A satisfactory solution to the regress should explain why it arises and how to stop it. A popular diagnosis is that the regress shows that we should not conflate arguments with conditionals (Russell 1903, Dummett 1981, Smiley 1995). Perhaps, the clearest statement of this diagnosis is due to Dummett (1981: 303), who remarked that Lewis Carroll's discovery was that an argument of the form (A) cannot be conflated with a conditional discourse (C):

A P. If P then Q. Therefore, Q;

C P. If P and if P then Q, then Q.

Pavese (2022a) argues that one respect in which conditional discourses such as (C) and categorical arguments such as (A) differ is that categorical connectives such as 'therefore' are presupposition triggers (cf. also Pavese 2017). One virtue of this diagnosis is that it helps explain the dynamics between the Tortoise and Achilles—why, e.g., the Tortoise is not willing to reach the categorical conclusion because they are not willing to grant that it follows from the premises. It also helps explain the inevitability of the regress and what it would take for the regress to be avoided.⁵

⁵ On some inferentialist theories of conditionals—see Khoo 2022 for a recent implementation of this kind of view—the paradox does not even arise, if both the Tortoise and Achilles properly understand the meaning of the conditional: since conditionals on these views encode inferential dispositions, the Tortoise cannot fail to be disposed to infer the conclusion if they understand what the conditional means. One problem with this kind of view is that it does seem that one can understand the meaning of conditionals without always being disposed to infer in the appropriate way (see Williamson 2003 for discussion of this point). Another problem is that these views risk obliterating the difference in meaning with conditionals and arguments.

Table 2

While this account is definitely part of the story, it cannot be *the* only difference between categorical argument connectives such as 'therefore' and conditional connectives such as 'then'. Another important difference between discourses such as (C) and discourses such as (A) is that consequents in (C) are not discharged from their antecedents. By contrast, reaching a categorical conclusion as in (A) requires discharging it from the premises. So, if we only had conditionals, and discourses made out of conditionals, effectively, we could never reach a categorical conclusion.

This discussion raises an important question. Prima facie, the mechanism of discharging the premises in natural languages does not seem to be the same as that of natural deduction systems. In natural deduction systems, the discharging of the premises is indicated by the scope of the vertical line. For example in table 2, the fact that the conclusion u is outside the scope of the vertical line indicates that the conclusion u is no longer under the scope of the premises.

However, this cannot be the mechanism by which premises are discharged in natural languages since there are no vertical lines in discourse. So the question arises how natural languages accomplish the discharging of the premises. This brings us to Observation 4:

Observation 4 Categorical arguments differ from conditional discourse in that their conclusions discharge the premises. But the mechanism of discharging the premises in natural language discourse is not accomplished through the same mechanisms that natural deductive systems employ.

Summarizing, here below are the four central observations. I am adding one more observation to the effect that conditionals and arguments share several affinities—those discussed in §2.3—which ought to be accounted for:

- **Observation 1** *If-clauses* and *supposes* distribute differently after categorical statements of their content;
- **Observation 2** Conditionals and suppositional arguments distribute differently in arguments by conditional proof;
- **Observation 3** *If-clauses* and *supposes* exhibit different patterns of subordination;
- **Observation 4** Categorical arguments differ from conditional discourses in that their conclusions discharge the premises. But the mechanism of discharging of the premises in natural language discourse is not accomplished through the same mechanisms that natural deductive systems employ;

Observation 5 Conditionals and arguments share several affinities.

In §5, I will introduce and motivate a framework that can account for Observation 4; in §6, I will (only) sketch how the framework might be supplemented to account for Observation 1, 2, 3, and 5 as well. The framework is partly based on work that I have done with my colleague Arc Kocurek in Kocurek & Pavese 2022. There, however, we ended up *assimilating* conditionals and arguments. As I have argued in the foregoing, I now think that that was a mistake. So I can only follow that framework up to a point. Moreover, Kocurek & Pavese (2022) was trying to model certain additional discourse phenomena, such as the use of parentheticals, which will not be my concern here. Thus, the framework presented below is in some respects more simplified.

5 How Natural Languages Discharge the Premises (Observation 4)

5.1 The anaphoricity of argument connectives

The main motivating evidence for the framework that I will introduce below is the anaphoricity of argument connectives. 'Therefore' is the paradigmatic example. As other anaphors, it cannot lack an explicitly articulated antecedent:

- (32) a. ?? Therefore/Hence/Thus, we should leave.
 - b. ?? Therefore/Hence/Thus, streets are wet.
 - c. ?? Therefore/Hence/Thus, either it is raining or it is not raining.

It can be ambiguous what the antecedent is. For example, (33) can have both a categorical and a suppositional reading. On the categorical reading, you should take the umbrella regardless of whether it is raining or not; on the suppositional reading, you should take the umbrella only assuming that it is not raining:

(33) Either it's raining or it's not. Suppose it's raining. Then you should take the umbrella. Suppose it is not raining. Then taking the umbrella will do no harm. Therefore, you should take an umbrella.

Finally, here are plausible donkey sentences for 'therefore':

- (34) a. Whenever one believes a certain view, one has to believe that its consequences are therefore true.
 - b. If one derives a contradiction from a claim, one may infer that it is therefore false.

5.2 Towards a comprehensive framework

To start, let \therefore stands for argument connectives such as 'therefore' and 'then'. Let us introduce a supposition operator '+' to the language and a conditional \rightarrow . Our new syntax for sentences is as follows:

$$\phi ::= p \mid \neg \phi \mid (\phi \land \phi) \mid (\phi \lor \phi) \mid (\phi \to \psi) \mid \Box \phi \mid \Diamond \phi$$

$$\sigma ::= \phi \mid \therefore \phi \mid +\phi \mid \therefore +\phi.$$

In view of the anaphoricity of argument connectives, we want to think of argumentative discourse as establishing anaphoric relations between premises and the conclusions. To do that, we cannot simply think of argumentative discourse as a list of sentences, since these will not suffice to track the relevant anaphoric relations. So, we introduce the notion of a LABELED SENTENCE. A LABELED SENTENCE is a pair of the form $\langle n, \phi \rangle$, which we write as n: ϕ for short. The labels are supposed to track the anaphoric relations established in discourse.

- We write $\langle n_1, \dots, n_k \rangle$ in decimal form as $n_1.n_2....n_k$;
- We use "0" to stand for the empty tuple $\langle \rangle$ (the "categorical" label)

A DISCOURSE is therefore a sequence of labeled sentences. It is an interesting question what constraints, if any, to put on well-formed discourses. Kocurek & Pavese (2022: 434) suggest some plausible constraints. One important constraint that will be helpful in the following of my discussion is that suppositions cannot be "idle" — i.e., introduced without a consequent (or without a discourse whose first element contains its label as an initial segment). This rules out, e.g., discourses of the form $n: +\phi, n: +\psi$, where the supposition ϕ is introduced but not used. Thus, a sequence of suppositions must be interpreted as

introducing additional levels. To illustrate, sequences of suppositions like (35) sound marked since the second supposition is interpreted in the scope of the first (as in (35a)) rather than as a separate supposition (as in (35b)).

- (35) Suppose physicalism is true. ?? Suppose physicalism is false...
 - a. Suppose₁ physicalism is true. Suppose_{1,1} physicalism is false...
 - b. Suppose₁ physicalism is true. Suppose₂ physicalism is false...

Because the second supposition is interpreted within the scope of the first, as in (35a), and cannot be interpreted as in (35b), we have explained why (35) is infelicitous.

Kocurek & Pavese (2022: 438) give several examples of this syntax. Here is one example that they do not mention but it is worth being explicit about:

Example (nested suppositions)

Alessandro is either from Turin or from Madrid. Suppose₁, on the one hand, that he is from Turin. Then₁ either he did his PhD there or he did it in the US. Suppose_{1.1} he did his PhD in Turin. Then_{1.1}, he studied Umberto Eco's work. Suppose_{1.2} instead he did his PhD in the US. Then_{1.2} he studied linguistics. Therefore₁, he either did continental philosophy or philosophy of language. Now on the other hand, suppose₂ he is from Madrid. Then₂ he definitely did his PhD in the US. Therefore₂, he studied linguistics. Either way, therefore, he did either continental philosophy or philosophy of language.

0:
$$(t \lor m)$$
, 0: $+t$, 1: \therefore $(phd_t \lor phd_u)$, 1: $+phd_t$, 1.1: \therefore u , 1: $+phd_u$
1.2: \therefore l , 1: \therefore $(cp \lor pl)$, 0: $+m$, 2: \therefore phd_u , 2: \therefore l , 0: \therefore $(cp \lor pl)$

Now that we have clarified the syntax of argumentative discourse, it is time to think of how to interpret it. Since argumentative discourses are stretches of labeled sentences, their interpretation requires keeping track of the anaphoric relations that labeled sentences establish in discourse. While in dynamic semantics, contexts are usually modeled as information states, or as information states with some structure on it, this notion of context will not do for our theoretical purposes, since it will not allow us to track the suppositions that are made in argumentative discourse and the anaphoric relations that they establish in context. So Kocurek & Pavese (2022) propose to think of contexts not as single information states but rather as *labeled trees* of information states — i.e., a tree where each node is given its own label.⁶

⁶ Bumford & Charlow (forthcoming) also propose to think of contexts as trees. I leave it to further work to explore the parallels between frameworks.

The root of the tree represents the categorical information state. The other nodes of the tree represent suppositional information states. The "labels" keep track of which information states go with which labels in a discourse. More formally:

Definitions

- An INFORMATION STATE is a set $s \subseteq W$ of worlds.
- A CONTEXT is a partial function $c: \mathbb{N}^{<\omega} \to \mathscr{D}W$ from labels (i.e., sequences of numbers) to information states. We assume:
 - (1) $0 \in dom(c)$
 - (2) if $\langle n_1, \ldots, n_{k+1} \rangle \in \text{dom}(c)$, then $\langle n_1, \ldots, n_k \rangle \in \text{dom}(c)$.

The first constraint just says the categorical state (which is the root of the tree) is always defined. The second constraint says, in effect, that a suppositional state is defined only when its parent state is defined. This rules out the possibility of "disconnected" segments of a branch.

- Where *n* is a label, we write c_n as short for c(n).
- We call c_0 the CATEGORICAL STATE of c.
- We call c_n (where $n \neq 0$) a SUPPOSITIONAL STATE of c.

Updating the context c with n: ϕ (basically) amounts to updating c_n with ϕ . The only exception is updating with n: $+\phi$, which also requires adding a new information state above c_n that's updated with ϕ . $c \oplus_n \phi$ is the result of extending c with an additional suppositional state that is copied from c_n and then updated with ϕ .

For clarity, while sentences receive a Simple Dynamic Semantics, labeled sentences are interpreted as in General Dynamic Semantics.

Simple Dynamic Semantics (without the conditional) Where $s \subseteq W$ is an information state:

$$s[p] = \{w \in s \mid w(p) = 1\}$$

$$s[\neg \phi] = s - s[\phi]$$

$$s[\phi \land \psi] = s[\phi][\psi]$$

$$s[\phi \lor \psi] = s[\phi] \cup s[\psi]$$

$$s[\neg \phi] = \{w \in s \mid s[\phi] = s\}$$

$$s[\lozenge \phi] = \{w \in s \mid s[\phi] \neq \varnothing\}$$

$$s[:.\phi] = \begin{cases} s & \text{if } s[\phi] = s \\ \text{undefined} & \text{otherwise} \end{cases}$$

General Dynamic Semantics for Arguments Where ϕ does not contain \therefore or +:

$$c[n: \phi] = \begin{cases} c_n[\phi] & \text{if } c_n \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$c[n: \cdot \cdot \cdot \phi] = \begin{cases} c[n: \phi] & \text{if } c_n \text{ is defined and } c[n: \phi]_n = c_n \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$c[n: +\phi] = \begin{cases} c \oplus_n \phi & \text{if } c_n \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

For illustration, consider the case where n=0. If ϕ does not contain + or $\dot{}$, then updating c with n: ϕ is the result of updating c_0 , as well as any suppositional states that have been defined, with ϕ (or, more precisely, the information contained in $c_0[\phi]$). If $n \neq 0$, then the update effect is the same, except we only update information states above c_n . If ϕ is of the form $+\psi$, then updating c with n: $+\psi$ amounts to (i) checking whether c_n is defined, and (ii) adding a suppositional state above c_n that is the result of updating c_n with ψ .

5.3 Discharging the premises as a side effect of anaphoricity

This framework provides a nice way of modeling discharging of the premises. Premises are discharged when the conclusion is anaphorically linked to a node lower in the tree than the hypothetical node updated with the premises—which, in the basic case, will be the categorical node. According to it, categorical conclusions can be reached thanks to the anaphoricity of argument connectives.

Semantically, what discharging the premises does in an argument is to instruct to return to the categorical node of the context—or to the suppositional node that is just below in the tree to the node updated with the antecedents of the argument—and to test that that node supports the consequent of the argument. In this sense, discharging the premises is made possible by the fact that therefore-sentences can be linked to a node that is different from that updated with the antecedent of the argument. Thus, the framework models the discharging of premises in discourse as a side effect of the anaphoricity of argument connectives.

6 An Overly Brief Discussion of Observations 1-3, 5

6.1 Observation 2

Kocurek & Pavese (2022: 445) model the conditional as the following update:

Generalized Update for the Conditional

$$c[n:\phi\to\psi] = \begin{cases} c \oplus_n \phi & \text{if } c[n:\phi]_n \text{ is defined and } c[n:\phi][n:\psi]_n = c[n:\phi]_n \\ c \uparrow_n \varnothing & \text{otherwise} \end{cases}$$

In the framework we proposed there, *if-clauses* effectively work as *supposes*—they augment the context with one suppositional node updated with the antecedent. Consequents of conditionals then test the suppositional node, just like consequents of arguments do. So, this analysis effectively obliterates the differences between arguments and conditionals. Thus, in order to account for Observations 1, 2, 3, and 5, this aspect of Kocurek & Pavese 2022's framework has to be revised.

One possible way of revising it might be the following. Instead of the conditional adding a suppositional node to the context, we simply let the conditional to first update and then test the categorical nod. On this account *if-clauses* would introduce *sui generis* suppositions. The problem with this solution is that treating *if-clauses* as suppositions of sort stands in the way of an explanation of Observation 2. Recall (§5) that one constraint on argumentative discourses be that suppositions cannot be "idle"—i.e., introduced without a consequent. Now consider again:

- (37) a. # Suppose the dice comes up 2 and suppose the dice comes up 4. Then, Ben will win.
 - b. Suppose₁ the dice comes up 2 and suppose_{1.1} the dice comes up 4. Then Ben will win
 - c. Suppose₁ the dice comes up 2 and suppose₂ the dice comes up 4. Then Ben will win.

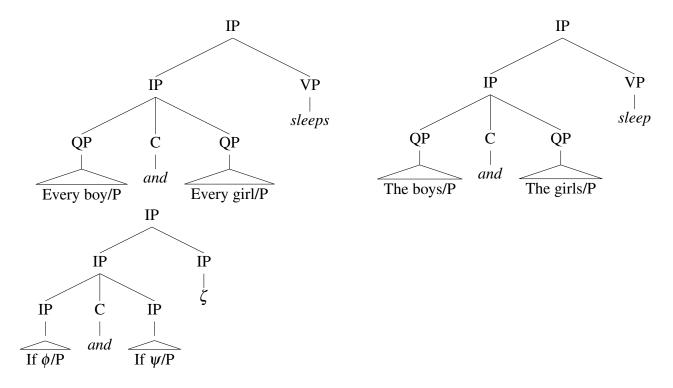
Because the second supposition in (37a) cannot be interpreted as in (37c), it has to be interpreted within the scope of the first, as in (37b). But in (37b) we are asked to suppose at once two incompatible events. So, we have explained why (37a) is infelicitous. If it is a general constraint of suppositional thinking that suppositions cannot be idle, we can explain why these discourses are off. But, if we allow that *if-clauses* are suppositions of some sort, then we foreclose the possibility of explaining why *if-clauses* can coordinate in the way they can.

An alternative, independently motivated, explanation is to equate *if-clauses* to other phrases that are susceptible of similar patterns of coordination. A long tradition in semantics

takes *if-clauses* to be quantifiers over possible worlds, or restrictors of hidden quantifiers (Kratzer 2012). Yet another prominent analysis takes *if-clauses* to be plural descriptions (e.g., Schlenker 2004). Coordinating *if-clauses* is not surprising if *if-clauses* are quantifiers or plural descriptions, since quantifiers phrases and plural descriptions coordinate too:

- (38) a. \checkmark Every boy and every girl sleeps.
 - b. \checkmark The boys and the girls sleep.
 - c. \checkmark If the dice comes up 4 and if it comes up 2, Ben will win.
 - d. # Suppose the dice comes up 4 and suppose it comes up 2. Then Ben will win.

Like other quantifier phrases or plural descriptions (Stanley & Gendler Szabó 2000), *if-clauses* have a hidden variable *P* that restricts the scope of the quantification depending on what is salient in the context.



6.2 Observation 1

Thinking of *if-clauses* as quantifiers or plural descriptions of sort also makes progress towards an explanation of Observation 1. Recall that this observation was that in modus ponens contexts, adding an *if-clause* after a categorical statement of its content is fine—not

so with *suppose*. This contrast is easy to understand if *supposes* are instructions to open a suppositional information state in order to discuss a possibility introduced by it. When a proposition is already part of common ground—by having been added to it through a categorical statement—there is no point in instructing to open a new suppositional node updated with that very same proposition—in that case, in order to discuss that possibility, one can steadily remain at the categorical level. This explaination accounts for why *supposes* are infelicitous after categorical statements of their content. By contrast, *if-clauses* do not instruct to open a new suppositional information state to discuss the possibilities that are supported by the current proposal, they quantify, or describe, possibilities that are supported by the current information state. Since *if-clauses* do not instruct to open a new suppositional information state, it is no mystery why *if-clauses* are allowed after a categorical statement of their content.

6.3 Observation 2

In the current framework, the difference in cognitive significance between the Trivial Argument, Simple Conditional Proof, and the Double Supposing Argument can be appreciated. Arguments exemplifying each of these forms effectively amount to very different updates on context. The Trivial Argument adds a conditional proposition to the context and then it tests whether the result of so updating the context supports the conclusion. The Double Supposing Argument instructs to first open a suppositional node updated with the premise P, and to check whether the node supports the consequent Q; then it instructs to open a second suppositional node updated with P and to check again whether this suppositional node supports the conclusion Q. So, effectively arguments of this form instruct to open a suppositional node with the same premise twice and to test each node so opened. Hence, the redundancy of the overall update. Conditional proof instructs to open a suppositional node, to update it with P and to check whether the conclusion O follows; then it instructs to check whether the conditional proposition that if P then Q is supported by the categorical context. Hence, an argument of this form is neither trivial as the Trivial Argument—since the test can fail depending on what kind of conditional proposition conditionals express—nor redundant like the Double Supposing Argument—since it instructs to only open a suppositional node one time. For these reasons, this approach has the resources to account for the difference in cognitive significance of these three argument forms.

6.4 Observation 5

Here, I will not be able to lay out a comprehensive framework that accounts for all the commonalities between conditionals and arguments. I will just note that, on the current framework, patterns of modal subordinations allowed by conditionals and arguments will

have to be the result of different kind of mechanisms. Consider again patterns of modal subordination and of modal insubordination in §2.3. Conditionals create pattern of modal subordination through their anaphoric *if-clauses*. As we have seen in §6.1, treating *if-clauses* as quantifiers makes it natural to think of them as involving an anaphoric element that can restrict the scope of the quantifier (Stanley & Gendler Szabó 2000). So, we might account for persistance beyond syntactic scope in (15a) as resulting from the anaphoric element's referring to the possibility introduced by the former conditional and thereby restricting the scope of the quantifier. For example, in (15a), the second conditional's *if-clause* will only quantify on possibilities in which the wolf have come in and we have to use a gun. When the pattern of modal subordination is reversed—as in (15a)—the *if-clause* anaphorically refers to a different set of possibilities—those compatible with the information state prior to the update with the first conditional in the sequence. Arguments instead create patterns of modal subordination by creating derived suppositional information states that are updated with their antecedents (Kocurek & Pavese 2022; 442-7). In general, an argumentative discourse is modally subordinated to another if the former has a label that was introduced by the latter. Which labeled sentences are modally subordinated to which labeled sentences is represented by the labels: either the subordinated argument have the same label or they have an incremental label. For example the modal subordination in (15b) is modeled as follows:

$$n: +A,^{+}: \therefore B, n^{+}: +C, (n^{+}).1: \therefore D, (n^{+}).1: +E, (n^{+}).1.1: \therefore F$$

$$n^{+}.1.1 \boxed{s[A][C][E]}$$

$$if s[A][C][E][F] = s[A][C][E]$$

$$n^{+}.1 \boxed{s[A][C]}$$

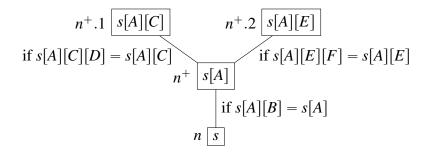
$$if s[A][C][D] = s[A][C]$$

$$n^{+} \boxed{s[A]}$$

$$if s[A][B] = s[A]$$

And the modal insubordination in (16b) is modeled as follows:

$$n: +A, n^+: \therefore B, n^+: +C, (n^+).1: \therefore D, n^+: +E, (n^+).2: \therefore F$$



7 Conclusions and open ends

Arguments discourse and conditionals are powerful resources to reason and to argue in language. Despite their affinities, these linguistic constructions are substantially different tools. Without arguments, we could not reach categorical conclusions. And without conditionals, we could not argue by conditional proof. Thus, we should resist the temptation to assimilate them. Failure to do so forecloses our ability to explain how we reason in language in the way we do.

I have outlined a framework that makes some progress in modeling argumentative discourse and some of the ways argumentative discourse differs from conditional discourse. A lot remains to do—e.g., a comprehensive account would have to model subjunctive argumentative discourse, as well as to model conditionals with non-declarative conclusions (such as imperatives and questions) with a propositionalist semantics for conditionals. I have to leave all of this to future work.

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