Conditional Heresies

Abstract

The principles of Conditional Excluded Middle (CEM) and Simplification of Disjunctive Antecedents (SDA) have received substantial attention in isolation. Both principles are plausible generalizations about natural language conditionals. There is however little or no discussion of their interaction. This paper aims to remedy this gap and explore the significance of having both principles constrain the logic of the conditional. Our negative finding is that, together with elementary logical assumptions, CEM and SDA yield a variety of implausible consequences. Despite these incompatibility results, we open up a narrow space to satisfy both. We show that, by simultaneously appealing to the alternative-introducing analysis of disjunction and to the theory of homogeneity presuppositions, we can satisfy both. Furthermore, the theory that validates both principles resembles a recent semantics that is defended by Santorio on independent grounds. The cost of this approach is that it must give up the transitivity of entailment: we suggest that this is a feature, not a bug, and connect it with with recent developments of intransitive notions of entailment.

1 Introduction

David Lewis's logic for the counterfactual conditional (Lewis 1973) famously invalidates two plausible-sounding principles: simplification of disjunctive antecedents (SDA),[1] and conditional excluded middle (CEM).[2] Simplification states that conditionals with disjunctive antecedents entail conditionals whose antecedents are the disjuncts taken in isolation. For instance, given SDA, (1) entails (2).

(1) If Hiro or Ezra had come, we would have solved the puzzle.

(2) If Hiro had come, we would have solved the puzzle and if Ezra had come, we would have solved the puzzle.

[1] See Fine 1975; Nute 1975; Lewis 1977; and Nute 1980*a*.

[2] See Stalnaker 1981; von Fintel 1997; DeRose 1999; Williams 2010; and Klinedinst 2011.

Conditional Excluded Middle is the claim that the (boolean) disjunction of $A > C$ and $A > \neg C$ must be a logical truth. Against the background of classical logic, a distinctive consequence of CEM is that the negation of $A > C$ entails $A > \neg C$. For instance, (3) entails (4)

(3) It is not the case that if Hiro had come we would have solved the puzzle.

(4) If Hiro had come, we would not have solved the puzzle.

Much attention has been devoted to these heretical principles in isolation, but virtually no work has considered their interaction. Since there are strong reasons to accept both principles, it is urgent to investigate how they might be made to fit.

The pessimistic finding at the center of this paper is that the heresies do not mix easily. We present a battery of impossibility results showing that no traditional theory of conditionals or disjunction can allow them to coexist. The general shape of these results is that given a variety of minimal assumptions about conditionals, disjunction, and logical consequence, the combination of our two heresies requires that the conditional either be the material conditional or share some undesirable property with it.

Despite these negative findings, we argue that the project of combining CEM and SDA is not hopeless. To validate both principles, we synthesize two different tools that can independently be used to validate each principle individually. Speaking abstractly, these tools are operations on possible meanings of the conditional: given a possible meaning m and some inferential pattern P, the operation produces a new meaning m' which is related to m in some way and validates P (whether or not m did).

In the case of SDA, we turn to the idea that disjunctions involve alternatives (for example, Alonso-Ovalle 2006), introducing a general mechanism that turns any candidate meaning for the conditional into a derived conditional that is guaranteed to validate SDA. Unfortunately, this tool alone does not yield CEM in full generality—even if we start out with a conditional connective that validates CEM. In particular, CEM won't be guaranteed for conditionals with disjunctive antecedents. In the case of CEM, we turn to the growing semantic tradition that invokes homogeneity presuppositions (von Fintel 1997). Building on the idea of homogeneity, we identify a tool that forces the validity of CEM, no matter what conditional we begin with. The drawback of this tool is that some appealing generalizations of SDA fail. In particular, simplification fails for *might* conditionals like "if it rains or snows, you might need boots".

We show that these problems can be solved by using *both* tools in sequence. This, then, is the shape of our final proposal: start with any conditional meaning; inject alternative sensitivity, thus securing SDA; finally, inject homogeneity presuppositions, thus securing CEM. It turns out that the resulting conditional need not be trivial or equivalent to the material conditional.

in fact, this procedure can yield a conditional connective that was recently defended, on different grounds, by Santorio 2017.

The resulting theory leads, however, to one last heresy: the entailment relation must fail to be transitive. In particular, while CEM is valid in the resulting theory, other principles are invalid that are logical consequences of CEM. We conclude by discussing this feature of our view and connecting it with other work on intransitive entailment.

Our negative results and our positive discussion should be of particular significance to those theorists who are committed to frameworks that are founded on endorsing one of the heresies. For example, validating SDA is a founding assumption of the truth-maker semantics of Fine (2012). Given our results, friends of truth-maker semantics who also want to endorse CEM need broader revisions than one might otherwise anticipate. The same is true of those frameworks on which conditionals denote selection functions (Stalnaker, 1968, 1973, 1981), which in turn are founded on the idea that CEM is valid.

2 The Case for the Heresies

In this section, we restate and defend our two principles. Letting '$\vdash$' denote the consequence relation, simplification of disjunctive antecedents can be formulated as the following entailment:[3]

SDA. $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C} \vdash (\mathsf{A} > \mathsf{C}) \;\&\; (\mathsf{B} > \mathsf{C})$

The main argument for SDA seems to consist entirely in the observation that instances like the one from (1) to (2) sound extremely compelling (see Fine 1975, p.453-454).

Of course, a conclusive case for the validity of SDA does require some kind of defense beyond "it sounds pretty plausible". For instance, it is well known (Fine 1975; Ellis *et al.* 1977) that SDA has troublesome downstream consequences: in presence of a principle of substitution for logical equivalents, SDA entails antecedent strengthening ($\mathsf{A} > \mathsf{C} \vdash \mathsf{A}^+ > \mathsf{C}$, where $\mathsf{A}^+ \vdash \mathsf{A}$). We will investigate these consequences and their significance below, but the point for now is that the preliminary motivation for SDA tends to be clear and strong judgments about the validity of its instances.

Conditional excluded middle can be formulated abstractly as a validity claim:

CEM. $\vdash (\mathsf{A} > \mathsf{B}) \vee (\mathsf{A} > \neg \mathsf{B})$

[3] In formulating, SDA, we use *or*, as opposed to '$\vee$' because much of the evidence for SDA relies on the intuitive plausibility of its natural language instances and we do not assume at the outset that natural language disjunction is boolean.

Unlike SDA, CEM is not typically justified by direct intuitions of validity about its instances. Instead, defenders of CEM propose that various linguistic phenomena fall into their proper place once we posit the validity of this schema. For instance, one might think that (3), which is of the form $\neg(hiro > puzzle)$, intuitively entails (4), which is of the form $hiro > \neg puzzle$. Given CEM, this inference turns into an application of disjunctive syllogism.

More generally, conditionals involving *will* and *would* consequents fail to enter into scope relations that would be expected if CEM failed. (The playbook for this sort of argument is laid out in the seminal discussion of Stalnaker 1981, p.137-139). A recent version of this argument relies on data involving attitude verbs that lexicalize negation (see Cariani and Santorio, forthcoming, for a version of this argument involving *will*). In the case of *would*, the argument centers around the observation that (5) and (6) sound equivalent:

(5) I doubt that if you had slept in, you would have passed.

(6) I believe that if you had slept in, you would have failed.

The equivalence is easily explained if CEM is valid (and assuming that failing equals not passing). The speaker doubts $sleep > pass$; if there was a way for this conditional to be false other than by $sleep > fail$ being true, it should be possible to accept (5) without accepting (6). By contrast, it is hard, if not impossible, to explain without it.

This argument streamlines an older argument for CEM involving the interaction between conditionals and quantifiers.[4] Consider:

(7) No student will succeed if he goofs off.

(8) Every student will fail if he goofs off.

(7) and (8) are intuitively equivalent. They appear to involve quantifiers taking scope over conditionals. Given CEM and this scope assumption, they are predicted equivalent. Take an arbitrary student, and suppose it is false of him that he will succeed if he goofs off. By CEM it follows that he will fail if he goofs off. This batch of data involving *will*-conditionals looks equally compelling when considering counterfactual conditionals (Klinedinst, 2011).

(9) No student would have succeeded if he had goofed off.

(10) Every student would have failed if he had goofed off.

On reflection, we take the interaction of conditionals and quantifiers to also favor CEM, for both indicative and counterfactual conditionals.

[4]See Higginbotham 1986; von Fintel and Iatridou 2002; Leslie 2009; and Klinedinst 2011 for discussion.

Yet another argument for CEM is based on the interaction between *if* and *only*.[5] CEM can help explain why *only if* conditionals imply their *if...then* counterparts. Consider the following conditionals:

(11) The flag flies only if the Queen is home.

(12) If the flag flies, then the Queen is home.

(13) The flag flies if the Queen isn't home.

(11) entails (12). In von Fintel 1997 this entailment is derived compositionally, on the assumption that *only* in (11) takes wide scope with respect to the conditional. *Only* then negates the alternatives to the conditional *the flag flies if the Queen is home*, which are assumed to include (13). Given some background assumptions, CEM and the negation of (13) imply (12).[6]

This short catalogue does not exhaust the motivation for CEM,[7] but it provides sufficient motivation to explore the relationship between CEM and other plausible principles like SDA.

Before moving on, it is worth highlighting that these arguments for CEM require that the relevant conditional connective not be the material conditional. The material conditional, of course, does validate CEM, but it does so in a way that is incompatible with the explanatory benefits we just noted. For instance, the quantifiers argument relied essentially on the 'Weak Boethius' Thesis'(WBT)—the claim that $A > C$ and $A > \neg C$ cannot both be true (when A is consistent).[8]

WBT. $A > C; A > \neg C \models \bot$

Without this assumption, it could be that some student will fail if he goofs off, but will also succeed if he goofs off. But the material conditional invalidates this principle, allowing both conditionals to be true when A is false. In this case, the negative conditional (10) would be stronger than the positive conditional (9), since the positive conditional can be true simply because the relevant individuals did not actually goof off. Similar points can be established for our other arguments.[9]

[5] See Barker 1993 and von Fintel 1997 for discussion.

[6] For some experimental evidence that support the inferential judgments involved in this argument, see Cariani and Rips (ms.).

[7] For other arguments and further discussion, see Cross 2009, Goodman (ms.), Williams (2010).

[8] For discussion, see Pizzi and Williamson 2005.

[9] Both arguments are incompatible with the material conditional because the negation of a material conditional tends to be quite strong, entailing the antecedent and the negation of the consequent. But the attitudes argument requires negating the conditional "if you had slept in, you would have passed" and the argument involving *only if* requires negating (13). One specific problematic consequence of this would be that, according to the material conditional analysis, *only if* conditionals would imply the truth of their consequent and the falsity of the antecedent. (11) clearly does not have this implication.

This last point is relevant to the interpretation of our negative results. For example, we will shortly show that SDA and CEM jointly imply collapse to the material conditional. One might reason as follows: CEM gives you something good (it explains the data in this section) and something bad (together with SDA it yields implausible consequences); the way to have both CEM and SDA is to go in for the material conditional; so if CEM's goodies outweigh the bad consequences, we have an argument for the material conditional. However, this interpretation would be incorrect, because the Weak Boethius Thesis (which the material conditional violates) is essentially involved in delivering the goodies. Without WBT all of the positive arguments in favor of CEM are undercut.

3 Incompatibility Results

Having introduced our favorite conditional heresies, we explain why it is difficult to jointly accept them. In §3.1, we show that, given modest logical assumptions, CEM and SDA collapse to the material conditional. The next result (in §3.2) dispenses with most of the logical assumptions and yields the conclusion that CEM and SDA collectively entail that the conditional is, in a certain respect, trivial. While we derive these results syntactically, we note in §3.3 that there are related collapse results that exploit semantic reasoning only.

In keeping with a distinction we have drawn in the previous section, we appeal to two notions of disjunction: (i) natural language *or* (which we used in stating SDA) and (ii) boolean disjunction, symbolized '$\vee$' (which we used in stating CEM). It will strengthen our argument to refrain from assuming that these connectives have the same meaning. Our results require classical assumptions about the logic of '$\vee$' but very few assumptions about the meaning of *or*.

3.1 Collapse

CEM and SDA together imply collapse to the material conditional, given relatively modest assumptions about the logic. We assume standard sequent rules for classical connectives as well as the standard structural rules governing classical logic.[10] Among the structural rules, the transitivity of entailment–which is itself a consequence of the Cut rule–will play a very important role in our discussion.

Transitivity. if $\mathsf{A} \vdash \mathsf{B}$ and $\mathsf{B} \vdash \mathsf{C}$, then $\mathsf{A} \vdash \mathsf{C}$

Cut. if $X \vdash \mathsf{A}$ and $Y, \mathsf{A} \vdash \mathsf{B}$, then $X, Y \vdash \mathsf{B}$

(notation: X and Y denote sets of sentences.) Several of our proofs rely on disjunction rules, so it is worth stating them explicitly.

[10] For contemporary sources on the sort system we presuppose, see Buss 1998, Troelstra and Schwichtenberg 2000; Restall 2000; Bimbó 2014; Negri and von Plato 2001.

Cases. if X, A $\vdash$ B and Y, B $\vdash$ C, then X, Y, (A $\vee$ B) $\vdash$ C

$\vee$-Intro. if X, A $\vdash$ B, then X, A $\vdash$ B $\vee$ C

To these, add specific assumptions about conditionals (three dedicated sequent axioms and one new rule):

Modus Ponens. A, A > C $\vdash$ C

Reflexivity. $\vdash$ A > A

Agglomeration. A > B, A > C $\vdash$ A > (B & C)

Upper Monotonicity. if X, B $\vdash$ C, then X, A > B $\vdash$ A > C

While these assumptions are not entirely uncontroversial, they are generally accepted in the literature.[11]

For ease of reference, we call this combination of assumptions *the classical package*. We can now state our result more precisely.

Fact 1 *Given the classical package,* CEM *and* SDA *imply that* A > C $\dashv\vdash$ ¬A $\vee$ *C*.

> We prove this equivalence in stages. First: SDA and CEM jointly imply the True Consequent paradox of material implication.
>
> True Consequent. C $\vdash$ A > C
>
> Next: this is enough to reach full collapse, in combination with the classical package.
>
> Comments on notation: Individual lines in the proofs below abbreviate multiple reasoning steps in the full sequent proof. At step 5 we note an implicit application of classical reasoning by citing something (LEM) that is not a rule in the actual proof system. Each line is annotated with a list of all the rules on which the suppressed piece of reasoning depends.
>
> First, we prove that True Consequent follows from SDA and CEM.
>
> 1. $\vdash$ ((A or ¬A) > C) $\vee$ ((A or ¬A) > ¬C) — CEM
> 2. $\vdash$ (A > C & ¬A > C) $\vee$ (A > ¬C & ¬A > ¬C) — 1, cases, SDA, cut
> 3. C, ¬(A > C) $\vdash$ (A > ¬C & ¬A > ¬C) — 2, cases, cut, weakening

[11] For instance, many of those who dispute the validity of modus ponens accept the validity of some special forms of the inference. In particular, conditionals that do not themselves contain further modals or conditionals are generally believed to validate modus ponens even by those who doubt the general validity of the inference (e.g. McGee (1985), Kolodny and MacFarlane (2010)). These restricted versions of modus ponens are sufficient to yield our results.

4. $(A \vee \neg A), (A > \neg C \;\&\; \neg A > \neg C) \vdash \neg C$ — cases, ponens
5. $C, \neg(A > C) \vdash \neg C$ — 3, 4, cut, LEM
6. $C \vdash A > C$ — 5, negation rules

We can then show that another paradox of material implication, False Antecedent, follows from True Consequent plus the classical package.

False Antecedent. $\neg A \vdash A > C$

1. $\neg A \vdash A > \neg A$ — TC
2. $\neg A \vdash A > \neg A \;\&\; A > A$ — 1, reflexivity, conjunction rule
3. $\neg A \vdash A > (\neg A \;\&\; A)$ — 2, agglomeration, cut
4. $\neg A \vdash A > C$ — 3, upper monotonicity, cut

Finally, we can use the classical package to show that TC and FA imply collapse.

1. $\neg A \vee C \vdash A > C$ — cases, FA, TC
2. $A > C, A \vdash C$ — ponens
3. $A > C \vdash \neg A \vee C$ — 2, material conditional intro

Previous work on SDA has shown that it sits in major tension with the substitution of logical equivalents (Fine 1975; Ellis *et al.* 1977). Interestingly, our own result makes no use of this principle. More generally, we make no assumptions about the semantic or logical properties of *or*, except that it supports SDA.

3.2 The Interconnectedness of All Things

Fact 1 requires a number of assumptions about the classicality of the conditional, disjunction, and logical consequence. Furthermore, the argument appeals to rather artificial instances of CEM—ones with antecedents of the form A or ¬A. If we are satisfied with a slightly weaker conclusion, a closely related result can be derived with far fewer assumptions and without appealing to conditionals with tautological antecedents.

The result is that combining CEM and SDA forces the conditional to validate an undesirable schema, which we call IAT for "the Interconnectedness of All Things".

IAT. $(A > C \;\&\; B > C) \vee (A > \neg C \;\&\; B > \neg C)$

The reason why validating IAT is undesirable is that it requires an extreme level of dependence among arbitrary distinct sentences. Consider an instance of the above in which A="Abe flies", B="Bea runs" and C="Cleo swims". Then it must be that either both *Abe flies > Cleo Swims* and *Bea runs > Cleo swims* are true or both *Abe flies > Cleo does not swim* and *Bea runs > Cleo does not swim* are. Among other things, this appears to entail that it is incoherent to reject both of the following:

(14) If Abe flies, then Cleo swims.

(15) If Bea runs, then Cleo does not swim.

It would be incorrect to say that nothing that is recognizably a conditional validates IAT. For one thing, the material conditional does.[12] Nonetheless, we comfortably assert that only unsatisfactory conditional connectives satisfy IAT.

With an eye to our later discussion, we provide an explicit statement and proof of the result.

Fact 2 *Given disjunction rules, cut,* CEM*, and* SDA*,* IAT *must be a logical truth.*

1.	$\vdash [(\text{A or B}) > \text{C}] \vee [(\text{A or B}) > \neg\text{C}]$	CEM
2.	$(\text{A or B}) > \text{C} \vdash (\text{A} > \text{C} \ \& \ \text{B} > \text{C})$	SDA
3.	$(\text{A} > \text{C} \ \& \ \text{B} > \text{C}) \vdash$ IAT	$\vee$-Intro
4.	$(\text{A or B}) > \text{C} \vdash$ IAT	2, 3, cut
5.	$(\text{A or B}) > \neg\text{C} \vdash (\text{A} > \neg\text{C} \ \& \ \text{B} > \neg\text{C})$	SDA
6.	$(\text{A} > \neg\text{C} \ \& \ \text{B} > \neg\text{C}) \vdash$ IAT	$\vee$-Intro
7.	$(\text{A or B}) > \neg\text{C} \vdash$ IAT	5, 6, cut
8.	$\vdash$ IAT	1, 4, 7, cases, cut

In addition to the intuitive reasons we gave against the validity of IAT, there is an important theoretical reason which helps put Fact 2 in perspective. Suppose we toss the Weak Boethius thesis into our cauldron of assumptions; then, IAT yields the further absurd consequence that $\text{A} > \text{C} \ \& \ \text{B} > \neg\text{C}$ is inconsistent

[12]This is to be expected given that it validates CEM and SDA. In fact the material conditional has an even stronger property. For any three sentences A, B, C at least three of the following four conditionals must be true: (i) $\text{A} > \text{C}$, (ii) $\text{B} > \text{C}$, (iii) $\text{A} > \neg\text{C}$, (iv) $\text{B} > \neg\text{C}$. Note that the material conditional is widely rejected as an analysis of the indicative conditional. One important class of references here is the seminal work in Edgington 1995 (but see also the survey Edgington, 2014). But rejection of the material analysis of the conditional is a tenet in the relevance logic tradition Anderson and Belnap (1975). What is even more important for our purposes is that we take our assumptions to be equally valid for counterfactuals as well, and virtually nobody believes that counterfactual conditionals are material conditionals.

(whenever A and B are consistent).[13] We submit that this would be an absurd consequence.[14]

3.3 Strict conditionals

As is to be expected, these proof-theoretic results correspond to related results at the semantic level. We approach the semantic landscape by considering how CEM interacts with semantic theses that are related to SDA. Let's begin with the idea that > is a strict conditional–that is, > universally quantifies over a fixed domain of worlds. More precisely, letting R be an accessibility relation over worlds, and letting R^w be the set of R-accessible worlds from w:

(S1) $[\![A > C]\!] = \{w \mid R^w \cap [\![A]\!] \subseteq [\![C]\!]\}$

Alonso-Ovalle 2006 notes that strict conditionals validate SDA.[15]

Unfortunately, this framework cannot accommodate both CEM and modus ponens. While this result follows already from the syntactic result of the previous sections, it can be further illuminated by thinking in purely semantic terms. In the strict framework, CEM and modus ponens correspond to constraints on the accessibility relation–respectively, uniqueness and reflexivity.

Uniqueness. $\forall w, v, v' : wRv \ \& \ wRv' \Longrightarrow v = v'$

Reflexivity. $\forall w : wRw$

The nature of the correspondence is the usual one from modal logic. That is to say: Uniqueness guarantees the validity of CEM and, moreover, any constraint on models that guarantees the validity of CEM must entail Uniqueness. Similarly, Reflexivity guarantees the validity of modus ponens and any constraint that suffices to guarantee the validity of modus ponens must entail uniqueness.

[13]Here is a sketch of the proof: this conjunction is not compatible with IAT under WBT because it entails that whenever A > C, we also must have ¬(A > ¬C)—ruling out the second disjunct of IAT— and whenever B > ¬C we must also have ¬(B > C)—ruling out the first disjunct of IAT.

[14]One response to our incompatibility results would be to bite the bullet, and allow that even counterfactuals are logically equivalent to material conditionals. We think the best way to pursue this strategy might be with some form of the dynamic conditional in Gillies 2007 and Gillies 2009. Indeed, Gillies 2009 proposes that the indicative conditional is logically equivalent to the material conditional, in order to validate Import-Export. But while the two conditionals are logically equivalent, they are not semantically equivalent; their meanings are more fine grained than logical equivalence, in a way that predicts different behavior of the two conditionals under higher operators, like negation. While we think this is an interesting strategy to pursue with respect to our own results, set it aside for later discussion. One concern about this strategy is that it requires not only the indicative, but also the subjunctive conditional to go in for the paradoxes of material implication.

[15]When > is a strict conditional, it is downward monotone—that is, A ⊢ B guarantees B > C | – A > C. In this setting, SDA follows from the validity of disjunction introduction.

The combination of these two constraints trivializes R, by entailing that every world only accesses itself.

Isolation. $\forall w, v : wRv \implies v = w$

Once again, > collapses onto the material conditional—its domain of quantification being limited to the singleton set of the world of evaluation.

Recent defenders of strict conditionals (von Fintel 2001; Gillies 2007) might shrug at this result, since they implement the view in the more complex framework of dynamic semantics. In the context of dynamic semantics, the correspondence results between axioms and constraints on models play out differently. However, reflection on the consequence relation $\models$ once again yields a collapse result even for such sophisticated views. No matter how sophisticated, strict analyses must accept, for some choice of $\Box$, the validity of:

Strictness. $A > C \dashv\models \Box(\neg A \vee C)$

This too collapses in the presence of basic assumptions. To see this, note that, given strictness, CEM corresponds to the validity of $\Box(\neg A \vee C) \vee \Box(\neg A \vee \neg C)$. Consider the instance $\Box(\neg\top \vee C) \vee \Box(\neg\top \vee \neg C)$. Given the identity rule for disjunction, $C \dashv\models (\neg\top \vee C)$. and because $\Box$ is closed under substitution of logical equivalents, $\models \Box C \vee \Box\neg C$. Suppose now that > validates modus ponens. In light of strictness, this corresponds to the principle that $\Box$ is strong, so that $\Box C \models C$. This last assumption allows us to infer the triviality of $\Box$. That is, it allows us to deduce $\Box C \dashv\models C$. In particular, $\Box(\neg A \vee C) \dashv\models (\neg A \vee C)$, which chained with strictness yields collapse.

We've now seen that CEM is quite difficult to reconcile with a strict conditional analysis, on both model theoretic and more abstract grounds. This is a surprising result. For von Fintel 1997's analysis of *only if* conditionals, one of the major arguments for the validity of CEM, requires that contraposition be valid. This in turn implies the validity of Antecedent Strengthening, and more generally implies that the conditional is strict. This suggests there is a serious tension within that analysis.

4 Heresies in isolation

The results in the previous section suggest that the space to validate both SDA and CEM is at best narrow. Before exploring it, we will constrain it further. Faced with these results, defenders of SDA might be tempted to simply reject CEM. After all, despite the evidence that supports it, the principle remains controversial, and Lewis (1973) presented several potential counterexamples to it.

In this section, we want to highlight a few results to the effect that SDA *alone* requires us to tread carefully. We have already noted a classic result (Fine, 1975; Ellis *et al.*, 1977; Santorio, 2017) to the effect that SDA together with substitution of logical equivalents yields antecedent strengthening (which pattern is invalidated by many theories). We present two more results in a similar vein. The first establishes that a fully general statement of SDA—specifically one that involves modal consequents—is not compatible with the semantic assumption that conditionals are strict conditionals. The second shows that we can get collapse just by adding SDA to a more modest relative of conditional excluded middle—the principle that $\mathsf{A}\ \&\ \mathsf{C}$ entails $\mathsf{A} > \mathsf{C}$ (this result was first presented in Nute 1980*b*).

4.1 *Might* counterfactuals

Alonso-Ovalle 2006 observes that simplification of disjunctive antecedents also occurs with *might* counterfactuals, as in the inference from (16) to (17).

(16) If Hiro or Ezra had come, we might have solved the puzzle.

(17) If Hiro had come, we might have solved the puzzle.

Additionally, he shows that strict accounts of counterfactuals cannot validate this form of simplification, given a boolean semantics for disjunction.

It will be convenient to take *If A, might B* as idiomatic. Formally, we write this as $\mathsf{A} >_\diamond \mathsf{B}$. With this in hand we can state:

$\diamond$-SDA. $(\mathsf{A}\text{ or }\mathsf{B}) >_\diamond \mathsf{C} \vdash (A >_\diamond C)\ \&\ (B >_\diamond C)$

To explore whether $\diamond$-SDA is valid, we must provide $>_\diamond$ with a semantics. Towards this goal, suppose that *might*-counterfactuals existentially quantify over the very same domain that *would*-counterfactuals universally quantify over.

(S2) $[\![\mathsf{A} >_\diamond \mathsf{C}]\!] = \{w \mid R^w \cap [\![\mathsf{A}]\!] \cap [\![\mathsf{C}]\!] \neq \emptyset\}$

Suppose finally that the accessibility relation R is reflexive–which as we noted corresponds to the validity of modus ponens.[16]

Note that, because we do not derive $>_\diamond$ compositionally, $\diamond$-SDA is not simply a special case of SDA. Nonetheless, $\diamond$-SDA is very much in the spirit of SDA itself, and plausibly supported by many of the same intuitive considerations that support SDA.

We can prove a surprising result that constrains the range of acceptable meanings for disjunction. Under the assumptions we made about $>_\diamond$, the semantic value of disjunction cannot be a proposition, even if the disjuncts are propositional. (By proposition, we mean a set of worlds.)

[16]The result of this subsection can be proven on the basis of a weaker assumption than reflexivity: that for every world w, there is a world v such that vRw. However, the main path to a justification of this constraint is via reflexivity.

Fact 3 *Assume (S1), (S2), the reflexivity of R and the validity of both* SDA *and* $\Diamond$-SDA. *Then disjunction is not propositional.*

> *Proof:* Consider arbitrary propositions $[\![A]\!]$ (abbreviated: $\mathbf{A}$) and $[\![B]\!]$ (abbreviated: $\mathbf{B}$). We prove that there can be no function **or**, such that $\mathbf{or}(\mathbf{A},\mathbf{B})$ (which we write in infix notation as **A or B**) such that $\mathbf{A}$ **or** $\mathbf{B} = [\![A \text{ or } B]\!]$. We establish that (i) $\Diamond$-SDA requires $\mathbf{A}$ **or** $\mathbf{B} \subseteq \mathbf{A} \cap \mathbf{B}$; and (ii) SDA requires $\mathbf{A} \cup \mathbf{B} \subseteq \mathbf{A}$ **or** $\mathbf{B}$. These two requirements are inconsistent whenever $\mathbf{A} \neq \mathbf{B}$.
>
> *Ad (i):* let $\mathbf{C}$ be an arbitrary proposition. The validity of $\Diamond$-SDA requires that if $R^w \cap \mathbf{C}$ overlaps **A or B**, then it overlaps $\mathbf{A} \cap \mathbf{B}$. But now suppose $\mathbf{A}$ **or** $\mathbf{B} \not\subseteq \mathbf{A} \cap \mathbf{B}$. Then there is a v in the former set, but not the latter. By reflexivity $v \in R^v$. Now, consider the special case $\mathbf{C} = \{v\}$. Then we must have that $\mathbf{C}$ overlaps both $\mathbf{A}$ and $\mathbf{B}$, but because it's a singleton, it cannot do that without overlapping both, so $v \in \mathbf{A} \cap \mathbf{B}$ after all, which is contradictory.
>
> *Ad (ii)*: let $\mathbf{C}$ be an arbitrary proposition and w an arbitrary world. The validity of SDA requires that if $(R^w \cap (\mathbf{A}$ **or** $\mathbf{B})) \subseteq \mathbf{C}$, $(R^w \cap \mathbf{A}) \subseteq \mathbf{C}$ and $(R^w \cap \mathbf{B}) \subseteq \mathbf{C}$. Now, suppose for *reductio* that $\mathbf{A} \cup \mathbf{B} \not\subseteq \mathbf{A}$ **or** $\mathbf{B}$. Then there is a u that belongs to the former set but not the latter. By reflexivity $u \in R^u$. Now let $\mathbf{C} = (\mathbf{A}$ **or** $\mathbf{B})$. We claim that $u \in \mathbf{C}$ which would be contradictory, since u was chosen so that $u \notin (\mathbf{A}$ **or** $\mathbf{B})$. This must follow because $u \in \mathbf{A} \cup \mathbf{B}$, and hence either $u \in \mathbf{A}$ or $u \in \mathbf{B}$. In the first case $u \in (R^u \cap \mathbf{A}) \subseteq \mathbf{C}$; in the second, $u \in (R^u \cap \mathbf{B}) \subseteq \mathbf{C}$. Either way $u \in \mathbf{C}$.

In fact, this proof establishes something slightly stronger. There can be no sentences A and B with distinct meanings where SDA and $\Diamond$-SDA hold for them, paired with any consequent.

4.2 Strong Centering

In the previous section, we saw that even without CEM, SDA requires a non-standard theory of disjunction, at least when simplification also holds for *might* conditionals. In this section we consider another collapse result that does not depend on CEM. We show that SDA leads to collapse into the material conditional when combined with the commonly accepted Strong Centering principle that A & C implies A > C.

Strong Centering. A & C $\vdash$ A > C

CEM implies strong centering in the presence of modus ponens. But strong centering does not imply CEM. Indeed, Lewis 1973 rejects CEM but accepts Strong

Centering, in the form of a constraint on similarity that whenever A is true at w, w is the unique closest world to itself where A is true.

This collapse result requires additional assumptions (beyond what we used in Facts 1 and 1). Specifically, it requires some degree of classicality for *or*, the disjunction operator relevant to SDA. Even more specifically, we must assume the identity rule that $\top$ is equivalent to $\top$ *or* A.

Identity. $\top \dashv\vdash \top \text{ or } A$

Given identity, SDA and Strong Centering imply collapse to the material conditional.[17]

Fact 4 *Given the classical package,* SDA, *Strong Centering, and Identity:*

$$A > C \dashv\vdash \neg A \vee C$$

Proof. The crucial step in this proof is to show that our assumptions validate the True Consequent inference, that $C \vdash A > C$. From there, the proof proceeds the same as in the proof of Fact 1.

1. $C \vdash (\top \text{ or } A) \ \& \ C$ — identity, classical package
2. $(\top \text{ or } A) \ \& \ C \vdash (\top \text{ or } A) > C$ — Strong Centering
3. $(\top \text{ or } A > C \vdash A > C$ — SDA
4. $C \vdash A > C$ — 1-3, transitivity

Fact 4 is in one sense stronger than Fact 1, because it relies on a principle weaker than CEM. However, it does not make Fact 1 redundant since, unlike that earlier result, it appeals to a specific assumption about natural language *or*. Even more significantly, it does not make Fact 2 redundant, since that result did not depend on assuming modus ponens for $>$.

5 Alternatives

We have developed a variety of incompatibility results, showing that SDA and CEM are in considerable tension with one another. Given these results, the prospects for reconciling these principles might appear bleak. We now turn to developing strategies for dealing with this tension.

[17] The connection between SDA and Strong Centering was to our knowledge first presented, albeit in a somewhat different form, in Nute 1980*b*, p. 40.

5.1 The alternatives package

Our first attempt builds on earlier attempts to validate SDA using alternative semantics in the style of Alonso-Ovalle 2006. While that framework holds fixed the variably strict analysis in Stalnaker 1968 and Lewis 1973, we show that the crucial ideas from alternative semantics are independent of the underlying theory of conditionals. We provide a general mechanism for taking any truth conditional semantics for the conditional and producing an alternative semantics from it that validates SDA. The resulting theory restricts the validity of CEM to non-disjunctive antecedents, which is how it blocks the proofs of the incompatibility results.

In alternative semantics, sentence meanings are not propositions, but instead sets of propositions (or 'alternatives'). The idea is that a disjunction A or C presents both of A and C as alternatives.

(S3) $[\![\mathsf{A \text{ or } B}]\!] = \{[\![\mathsf{A}]\!], [\![\mathsf{B}]\!]\}$

Disjunction contributes a set of propositions as its meaning. To validate SDA, we let the conditional operate on each alternative in this set.[18]

The main idea is to derive the meaning of the conditional from an underlying propositional conditional operator >—the 'proto-conditional'—which maps a pair of propositions to a new proposition. The proto-conditional regulates the behavior of the conditional ≫ when the antecedent is not an alternative. It also helps determine how ≫ behaves when its antecedent denotes a non-trivial sets of alternatives. We illustrate this for the case in which $[\![\mathsf{A}]\!]$ denotes a set of propositions.

(S4) $[\![\mathsf{A} \gg \mathsf{C}]\!] = \bigcap \left\{ [\![>]\!](\mathbf{B}, [\![\mathsf{C}]\!]) \mid \mathbf{B} \in [\![\mathsf{A}]\!] \right\}$

To simplify a bit more, suppose the set of propositions in $[\![\mathsf{A}]\!]$ is $\{\mathbf{B}_1, ..., \mathbf{B}_j\}$ denoted by the sentences $\mathsf{B}_1, ..., \mathsf{B}_j$. Then $\mathsf{A} \gg \mathsf{C}$ is true just in case each of the conditionals $(\mathsf{B}_1 > \mathsf{C}), ..., (\mathsf{B}_j > \mathsf{C})$ is true. In general, the alternative sensitive conditional is a generalized conjunction of a series of protoconditionals, distributed over the antecedent alternatives.[19] To recycle one of our early examples, the truth-conditions of *Hiro or Ezra* ≫ *puzzle* demand the truth of both: *Hiro* > *puzzle* and *Ezra* > *puzzle*.

Before showing how this framework can engage our collapse results, we must make some bookkeeping adjustments. Once we access the higher type

[18] We can think of several other approaches to SDA as offering different proposals about what exactly the alternatives for a disjunction are. For example, the state-based semantics from Fine 2012 and Briggs 2012 can be interpreted so that the meaning of a sentence is a set of propositions, in which case roughly it claims that the alternatives for a disjunction are the proposition expressed by each disjunct, as well as their intersection. These alternatives can then be processed further through different notions of truthmaking. Similarly, Santorio 2017 suggests a syntactic procedure for determining the alternatives for a sentence.

[19] For an implementation of the same idea in inquisitive semantics, with a similar purpose to the one we have here, see Ciardelli 2016 and Ciardelli *et al.* 2017.

of sets of propositions, we need a route connecting them back with propositional meanings. Without such a route, we would not be able to make sense of logical consequence. Furthermore, and relatedly, (S4) does not provide for non-disjunctive antecedents without such a bridge.

These problems are often solved by introducing a closure operator ! and type raiser ↑ to move from propositions to sets of propositions, and back (Kratzer and Shimoyama 2002). ! maps a set of propositions to its union, while ↑ takes a proposition to its singleton. However, introducing such operations into our semantics would considerably complicate our LFs and make it difficult to consider our collapse results directly.[20]

Instead, we opt for a different approach. First, we avoid the need for a type raising operation by defining our conditional operator polymorphically. ≫ can either take a proposition or a set of propositions as input. When it takes a proposition as input, it simply applies >; otherwise, it universally quantifies over alternatives.

(S5) $$[\![\mathsf{A} \gg \mathsf{C}]\!] = \begin{cases} [\![\mathsf{A} > \mathsf{C}]\!] & \text{if } [\![\mathsf{A}]\!] \subseteq W \\ \bigcap \left\{ [\![>]\!](\mathbf{A}, [\![\mathsf{C}]\!]) \mid \mathbf{A} \in [\![\mathsf{A}]\!] \right\} & \text{otherwise.} \end{cases}$$

Instead of an explicit type raising operator, we invoke an explicit existential closure operator !. And instead of obligatorily placing this operator in LF, we use it only to define entailment. Just like the conditional, we can define our closure operator polymorphically. When [[A]] is a proposition, ! has no effect on *A*. But when [[A]] is a set of propositions, ! takes the union of all of the *A* alternatives.

(S6) $$[\![!\mathsf{A}]\!] = \begin{cases} [\![\mathsf{A}]\!] & \text{if } [\![\mathsf{A}]\!] \subseteq W \\ \bigcup [\![\mathsf{A}]\!] & \text{otherwise.} \end{cases}$$

Say that !A is the closure of A. Then an argument is valid just in case the closure of the conclusion is true whenever the closure of all the premises are true.

(S7) $$\mathsf{A}_1, \ldots, \mathsf{A}_n \models \mathsf{C} \text{ iff } \bigcap_{i \in [1,n]} [\![!\mathsf{A}_i]\!] \subseteq [\![!\mathsf{C}]\!]$$

This proposal guarantees that disjunction behaves as classically as possible. Since entailment is only sensitive to the closed form of a sentence, the alternative sensitive disjunction *or* must satisfy both disjunction introduction and proof by cases. In addition, the proposal yields the DeMorgan equivalence that A or B is equivalent to ¬(¬A & ¬B).

A further consequence is that logical equivalence is less fine grained than equivalence of meaning. While A or B and ¬(¬A & ¬B) are co-entailing, they

[20] This is because we would have to reinterpret what exactly CEM, SDA, and other principles even say. For example, we might then formulate CEM as the validity of $![(\uparrow \mathsf{A} \gg \mathsf{C}) \vee (\uparrow \mathsf{A} \gg \neg \mathsf{C})]$.

do not have the same meaning. The disjunction, but not the negated conjunction, denotes a set of alternatives. For this reason, our logic for conditionals is hyperintensional in the sense that substituting logical equivalents in conditional antecedents does not guarantee equivalence of the resulting conditionals. Conditionals with disjunctive antecedents simplify, while conditionals with negated conjunctions in the antecedent do not.[21]

It is time turn to our collapse results. In this framework, regardless of what $>$ means,

$$[\![(\mathsf{A} \text{ or } \mathsf{B}) \gg \mathsf{C}]\!] = [\![\mathsf{A} > \mathsf{C}]\!] \cap [\![\mathsf{B} > \mathsf{C}]\!]$$

This evidently guarantees that SDA is valid.

Whether CEM is valid depends in part on the choice of proto-conditional $>$. Suppose for instance that, following Stalnaker (1968), we interpret $>$ in terms of a selection function f that, given a world w and proposition $\mathbf{A}$, returns the unique closest world to w where $\mathbf{A}$ holds.

$$[\![\mathsf{A} > \mathsf{C}]\!] = \{w \mid f(w, [\![\mathsf{A}]\!]) \in [\![\mathsf{C}]\!]\}$$

Then CEM is valid for $\gg$ when the antecedent is not disjunctive.[22]

More generally, the validity of CEM for non-disjunctive antecedents corresponds to the following condition restricted to non-disjunctive A:

$$W = [\![\mathsf{A} > \mathsf{C}]\!] \cup [\![\mathsf{A} > \neg\mathsf{C}]\!]$$

Because of this correspondence, even this restricted validation of CEM will fail if the proto-conditional does not itself validate CEM.

It is a simple corollary of our negative result that there is no non-trivial choice of proto-conditional that validates CEM for disjunctive antecedents. In a case where some alternatives guarantee C and some guarantee ¬C, CEM fails.

We summarize the two signature properties of the semantics above in a single statement.

Fact 5

1. *For any operator $>$, $(A \text{ or } B) \gg C \models (A \gg C)$ & $(A \gg C)$*
2. *For any operator $>$, if $>$ validates* CEM, *then $\gg$ validates* CEM *for any A not containing $\vee$.*

[21] This is why this proposal avoids the classic collapse result in Ellis *et al.* 1977, connecting simplification with antecedent strengthening.

[22] Our definition of entailment in terms of closure plays an important role in proving this. We have:

$$\models (\mathsf{A} \gg \mathsf{C}) \vee (\mathsf{A} \gg \neg\mathsf{C}) \text{ iff } [\![!\,[(\mathsf{A} \gg \mathsf{C}) \vee (\mathsf{A} \gg \neg\mathsf{C})]]\!] \subseteq W$$

But $[\![(\mathsf{A} \gg \mathsf{C}) \vee (\mathsf{A} \gg \neg\mathsf{C})]\!]$ is the set containing $[\![\mathsf{A} > \mathsf{C}]\!]$ and $[\![\mathsf{A} > \neg\mathsf{C}]\!]$, so its closure is the set of worlds where one of these conditionals holds. Since either C or ¬C is guaranteed to hold at $f(w, [\![\mathsf{A}]\!])$, this last is guaranteed.

One last remark: the alternatives approach does not require that conditionals with disjunctive antecedents always go in for simplification. It is compatible with what we have said to have the closure operator ! occur in the antecedent, generating the form $!(A \vee B) \gg C$. This would account for some localized failures of simplification, such as the classic example *If Spain had fought on the side of the Allies or on the side of the Nazis, it would have fought for the Allies.*[23]

5.2 Evaluation

The alternative semantics approach dodges our first two theorems because those results rely on applying CEM to a disjunctive antecedent, and then applying simplification. By blocking CEM for disjunctive antecedents, both proofs are blocked. The approach reflects a conservative response to our incompatibility results: it quarantines conditionals with disjunctive antecedents and allows CEM only for conditionals that cannot be manipulated *via* SDA.

This immediately raises the question whether the motivation for CEM requires its validity for disjunctive antecedents. If it does not, then we can rest content with the proposal of this section as a solution to our motivating concerns. Unfortunately, however, the arguments for CEM do not appear to discriminate against disjunctive antecedents. Let us run through those arguments again with the specific case of disjunctive antecedents in mind.

I. Scope relations. (18) and (19) sound equivalent in just the same way that (5) and (6) do.

(18) I doubt that if you had slept in or goofed off, you would have passed.

(19) I believe that if you had slept in or goofed off, you would have failed.

Similarly, we observe a duality effect with disjunctive antecedents under *no* and *every*. As before, (20) and (21) appear equivalent.

(20) No student would have succeeded if he had goofed off in class or partied the night before the exam.

(21) Every student would have failed if he had goofed off in class or partied the night before the exam.

By restricting CEM to non-disjunctive antecedents, the analysis renounces these predictions. Suppose that goofing off does imply failure, but that partying does not. In this case, the analysis predicts that the scope of (20) is false for any student, and so (20) is true. By contrast, (21) is false, since partying does not guarantee failure.

[23] See McKay and Inwagen 1977 and Alonso-Ovalle 2006 for discussion.

This gives rise to several questions. First, do (20) and (21) have any reading on which they are not equivalent? Such a reading is predicted to exist by the account above, by removing the closure operator. If no such reading is available, the account above would need supplementation with a theory of the distribution of existential closure operators. Perhaps such operators are for some reason obligatory when disjunction occurs under the scope of a quantifier. This allows disjunction to behave classically, leading to the equivalence of (20) and (21). Moreover, this is a tool that is independently needed.

Another natural question here is whether there is a reading of the quantified conditionals (20) and (21) on which they are consistent, and yet both go in for simplification. Indeed, Santorio 2017 suggests that at least negative quantified conditionals display exactly this effect:

(22) None of my friends would have fun at the party if Alice or Bob went.

(23) None of my friends would have fun at the party if Alice went.

(24) None of my friends would have fun at the party if Bob went.[24]

Such a combination of effects would be quite paradoxical, since it seems to require the validity of SDA and CEM even for disjunctive antecedents, and yet also require the validity of the Weak Boethius Thesis (WBT). Yet these principles seem jointly inconsistent, since the former two principles imply collapse to the material conditional, which is inconsistent with WBT. Concluding, we do think that the interaction between quantifiers and conditionals with disjunctive antecedents presents a problem for the current analysis.

II. 'Only if' conditionals. we saw that CEM helps derive the meaning of *only if* conditionals compositionally—i.e. on the basis of the interaction of *only* and conditionals. But, as above, it is implausible to restrict this phenomenon to conditionals with non-disjunctive antecedents.

(25) The flag flies only if the King or Queen is home.

(26) If the flag flies, then the King or Queen is home.

(27) The flag flies if the King or Queen isn't home.

Here, it is clear that (25) does imply (26), just as we saw earlier that (11) implied (12). This is a problem for the analysis above, which denies CEM for conditionals with disjunctive antecedents. For, again, a natural way to predict this entailment is through the idea that *only* negates alternatives, and that (27) is an alternative to the conditional in (25). But if CEM fails for disjunctive antecedents, then the negation of (27) will not imply the contraposition of (26), which is essential in von Fintel 1997's account.

[24] See Santorio 2017 10.

6 Homogeneity

In the previous section, we developed a tool for taking any theory of conditionals and enriching it with alternatives. While this theory provides an elegant treatment of SDA, it faces problems with CEM. The theory invalidates CEM for disjunctive antecedents.

In this section, we develop an approach with complementary features. Building on von Fintel 1997, we now develop a tool for taking any theory of conditionals and enforcing CEM. This new tool will have an advantage: any pattern that is valid relative to the underlying conditional remains valid when the conditional is enriched with presuppositions. So if we start with an SDA validating conditional, we can force it to validate CEM. To avoid our collapse results, the theory gives up the transitivity of entailment.

6.1 Homogeneity presuppositions

The theoretical device that will yield this result are homogeneity presuppositions. Homogeneity presuppositions have been invoked to explain certain otherwise problematic variants of excluded middle for plural definites.[25] It is worth indulging on plural definites because there is a parallel problem to one of our impossibility results providing us with a template for how we might go about addressing it.

Here is the problem: observe first that predications involving plural definites, like (28), plausibly license inferences to universal claims like (29).

(28) The cherries in my yard are ripe.

(29) All the cherries in my yard are ripe.

If some but not all cherries are ripe, one would not be in a position to assert (28). Furthermore, plural definites plausibly exclude the middle. That is, the following sounds like a logical truth:

(30) Either the cherries in my yard are ripe or they (=the cherries in my yard) are not ripe.

If someone were to utter (30), they would sound just about as informative as if they had made a tautological statement (although you might learn from it that they have cherries in their yard). The problem is that, starting with (30) and exploiting entailments like the one from (28) to (29) as well as standard validities for disjunction, we can reason our way to

(31) Either all the cherries in my yard are ripe or all the cherries in my yard are not ripe.

[25] See for example von Fintel 1997 and Kriz 2015.

That seems puzzling: did we just prove from logical truths and valid inferences that my yard cannot have some ripe cherries and some non-ripe ones? Of course, something must have gone wrong. The homogeneity view of plural definites explains what that is: first, plural definites carry a presupposition of homogeneity: *the F's are G's* presupposes that the *F*'s are either homogenously *G*'s or homogenously not *G*'s. If this presupposition is satisfied, their content is that all *F*'s are *G*'s. The sense in which (30) sounds tautological is that it cannot be false if its homogeneity presupposition is satisfied. Similarly, the sense in which (28) entails (29) is that if the presupposition of (28) is satisfied and (28) is true, (29) cannot fail to be true. But even if we can exploit these to deduce (31) we do not have license us to claim that (31) is valid: our justification for (30) and for the (28)-(29) entailment did not discharge the homogeneity presupposition.

6.2 Forcing CEM via homogeneity

A treatment of CEM using homogeneity presuppositions is found in von Fintel 1997. It allows that there may be more than one relevant world where the antecedent of a conditional is true. The key idea is that $\mathsf{A} > \mathsf{C}$ presupposes that either all of the relevant worlds where A is true are worlds where C is true, or they are all worlds where C is false. The A-worlds must be "homogeneous" with respect to the consequent.

We can generalize von Fintel 1997 by reformulating the theory without any appeal to quantification over worlds. Instead, we provide a general recipe for taking any conditional operator >, and enriching it with homogeneity presuppositions to create a new conditional, $\cdots\!>$.

(S8) $[\![\mathsf{A} \cdots\!> \mathsf{C}]\!](w)$ is defined only if $[\![\mathsf{A} > \mathsf{C}]\!](w) = 1$ or $[\![\mathsf{A} > \neg\mathsf{C}]\!](w) = 1$.
If defined, $[\![\mathsf{A} \cdots\!> \mathsf{C}]\!](w) = [\![\mathsf{A} > \mathsf{C}]\!](w)$.

To talk about SDA and CEM, we also need appropriate assumptions about $\neg$ and $\vee$. These connectives must allow homogeneity presuppositions to project in the right way.

(S9) $[\![\neg\mathsf{A}]\!](w)$ is defined only if $[\![\mathsf{A}]\!](w)$ is defined.
If defined, $[\![\neg\mathsf{A}]\!](w) = 1 - [\![\mathsf{A}]\!](w)$.

(S10) $[\![\mathsf{A} \vee \mathsf{B}]\!](w)$ is defined only if $[\![\mathsf{A}]\!](w)$ and $[\![\mathsf{B}]\!](w)$ are defined.
If defined, $[\![\mathsf{A} \vee \mathsf{B}]\!](w) = max([\![\mathsf{A}]\!](w), [\![\mathsf{B}]\!](w))$.

Here, we assume that a disjunction is defined only if each disjunct is defined. This assumption is slightly stronger than what we need in order to predict the validity of CEM. For example, we could also allow a more complex pattern of presupposition projection for disjunction, where the second disjunct treats the first disjunct as part of its local context, as in Heim 1992. But this won't be relevant in what follows, so we stick to the current formulation for simplicity.

Finally, to get predictions about our collapse results, we need a definition of consequence. The leading candidate for languages involving presuppositions is Strawson-validity (Strawson, 1952; von Fintel, 1997, 1999, 2001). According to this notion, an argument is valid just in case the conclusion is true whenever the conclusion is defined and the premises are true.

(S11) $\mathsf{A}_1;\ldots;\mathsf{A}_n \models \mathsf{C}$ iff $[\![\mathsf{C}]\!](w) = 1$ whenever:

- $[\![\mathsf{A}_1]\!](w);\ldots;[\![\mathsf{A}_n]\!](w)$ are defined.
- $[\![\mathsf{A}_1]\!](w) = 1$ and ... and $[\![\mathsf{A}_n]\!](w) = 1$.
- $[\![\mathsf{C}]\!](w)$ is defined.

Now let's turn to our collapse results. Here, the first important result is that CEM is valid regardless of the choice of proto-conditional. That is, for any operator $>$, $\cdots>$ satisfies CEM. Here, the key is that $\cdots>$ builds in a homogeneity presupposition, that either $\mathsf{A} > \mathsf{C}$ or $\mathsf{A} > \neg\mathsf{C}$ is true. Further, the validity of CEM requires that $(\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})$ is true whenever it is defined. Since disjunctions inherit the presuppositions of their first disjunct, we know that this whole disjunction is therefore defined only if it is true.

Our conditional $\cdots>$ is guaranteed to validate CEM. To deal with our collapse results, let's now consider SDA. Here, the key result is that any proto-conditional $>$ that validates SDA induces a new conditional $\cdots>$ that also validates SDA. Indeed, this is not unique to simplification. To see why, note first that any conditional $>$ differs from its homogenous counterpart $\cdots>$ only in their presuppositions. But this guarantees that any inference containing $>$ remains valid when $>$ is replaced with $\cdots>$. After all, the resulting formula differs from the original only by containing extra presuppositions. But given our definition of validity, this only makes it easier for the relevant inference to be valid. So in particular if we know that $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C}$ implies $(\mathsf{A} > \mathsf{C})\ \&\ (\mathsf{B} > \mathsf{C})$, we can infer that $(\mathsf{A} \text{ or } \mathsf{B}) \cdots> \mathsf{C}$ implies $(\mathsf{A} \cdots> \mathsf{C})\ \&\ (\mathsf{B} \cdots> \mathsf{C})$. This follows from the monotonicity of classical entailment. After all, an argument is Strawson valid just in case the result of strengthening the argument's premises with the presuppositions of the conclusion is classically valid.

Fact 6

1. *For any operator* $>$, $\models (A \cdots> C) \vee (A \cdots> \neg C)$
2. *For any operator* $>$, *if* $>$ *validates* SDA, *then* $\cdots>$ *validates* SDA.

We now have a completely general recipe for validating both SDA and CEM. But is it a recipe for triviality? That is, do we have that for any operator $>$ that validates SDA, $\cdots>$ collapses to the material conditional? The answer to both questions is "no".

There are many choices of protoconditional for which $\cdots>$ is not trivial. A first example is if we let $>$ be a generic strict conditional. To see how this theory avoids triviality, let us look at the semantic correlates of some of the entailments we used in the proof of our first collapse result. For simplicity, let's simply see how a generic strict conditional $>$ deals with the problem. The first step of the proof corresponds to this semantic fact: (32) is a logical truth.

(32) $[(\mathsf{A} \vee \neg\mathsf{A}) \cdots> \mathsf{C}] \vee [(\mathsf{A} \vee \neg\mathsf{A}) \cdots> \neg\mathsf{C}]$

Although (32) is true whenever defined, it is quite difficult for it to be defined. Given our account of $\vee$, the definedness of $(\mathsf{A} \vee \neg\mathsf{A}) > \mathsf{C}$ is equivalent to the requirement that either $R^w \subseteq [\![\mathsf{C}]\!]$ or $R^w \subseteq [\![\neg\mathsf{C}]\!]$. One of C and ¬C must be necessary at w for (32) to be defined.

Now, the reasoning connecting the first two steps of our proof also has a matching semantic fact: (33) entails (34).

(33) $[(\mathsf{A} \vee \neg\mathsf{A}) \cdots> \mathsf{C}] \vee [(\mathsf{A} \vee \neg\mathsf{A}) \cdots> \neg\mathsf{C}]$

(34) $[(\mathsf{A} \cdots> \mathsf{C} \ \& \ \neg\mathsf{A} \cdots> \mathsf{C}) \vee [(\mathsf{A} \cdots> \neg\mathsf{C} \ \& \ \neg\mathsf{A} \cdots> \neg\mathsf{C})]$

This holds because if (32) is defined, then the domain R^w uniformly consists of C-worlds or it uniformly consists of ¬C-worlds. Either way, (34) must be true.

Despite the validity of (32) and the entailment from (32) to (34), (34) is not itself valid. The definedness conditions of (34) are laxer than those of (32): for this reason (34) has a much better shot of being false. For instance (34) is false in a model that contains two worlds w and v with w verifying A and C and v verifying ¬A and ¬C.[26] But such a model does not impugn the validity of (32) under Strawson entailment, because its disjuncts are undefined.

In broad strokes, an instance of transitivity—in particular, one of the form $\models \mathsf{A}$, $\mathsf{A} \models \mathsf{B}$, therefore $\models \mathsf{B}$—fails for Strawson entailment (Smiley, 1967). This is possible because $\models \mathsf{A}$ only requires that A be true if defined; meanwhile, $\mathsf{A} \models \mathsf{B}$ also holds because the presuppositions of A are essentially involved in guaranteeing the truth of B. But $\models \mathsf{B}$ fails because here we are not allowed to assume that the presuppositions of A are satisfied. The same diagnosis applies to our second impossibility result. The first step of the proof claims the validity of $[(\mathsf{A} \text{ or } \mathsf{B}) > C] \vee [(\mathsf{A} \text{ or } \mathsf{B}) > \neg\mathsf{C}]$. The argument establishes that this claim entails IAT.[27] However, the validity of IAT does not follow.

6.3 Synthesis

Let us take stock of what we have argued: a generic strict conditional $>$ can validate both SDA and CEM, when enriched with homogeneity presuppositions.

[26] Here we assume & is definable in terms of ¬ and ∨, with the analogous definedness conditions.

[27] Despite involving two applications of transitivity, the argument up to step (7) can be replicated for Strawson entailment.

Here, however, we must take care. The resulting theory validates SDA, but invalidates $\Diamond$-SDA. That is, the analogue of simplification of disjunctive antecedents for *if ... might ...* fails. This is a problem because $\Diamond$-SDA sounds no less plausible than SDA itself.

We faced a symmetrical problem with $\gg$. There, simplification was unrestrictedly valid. But CEM was valid only for non-disjunctive antecedents. To fully validate simplification, we propose a synthesis of our two tools. In particular, we suggest that the English conditional recruits *both* alternatives and homogeneity presupposition. To signal this, we introduce the new connective $\cdots\gg$. Start with any conditional meaning $>$. Then apply the alternative sensitive enrichment from (S5), so as to get $\gg$. In light of Fact 1, the resulting semantics validates both SDA and $\Diamond$-SDA, but invalidates CEM for disjunctive antecedents. To force the unrestricted validity of CEM, enrich this conditional with homogeneity presuppositions according to the recipe in (S8), so as to get $\cdots\gg$.

More precisely, given an arbitrary proto-conditional $>$, we characterize $\cdots\gg$ by the clauses:

(S12a) If $[\![\mathsf{A}]\!] \subseteq W$, then $[\![\mathsf{A}\cdots\gg \mathsf{C}]\!](w)$ is defined only if $[\![\mathsf{A}>\mathsf{C}]\!](w) = 1$ or $[\![\mathsf{A} > \neg\mathsf{C}]\!](w) = 1$.
If defined, $[\![\mathsf{A}\cdots\gg \mathsf{C}]\!] = [\![\mathsf{A}\gg \mathsf{C}]\!] = [\![\mathsf{A}> \mathsf{C}]\!]$.

(S12b) Otherwise, $[\![\mathsf{A}\cdots\gg \mathsf{C}]\!](w)$ is defined only if either $[\![>]\!](\mathbf{A})([\![\mathsf{C}]\!])(w) = 1$ for every $\mathbf{A} \in [\![\mathsf{A}]\!]$, or $[\![>]\!](\mathbf{A})([\![\mathsf{C}]\!])(w) = 0$ for every $\mathbf{A} \in [\![\mathsf{A}]\!]$.
If defined, $[\![\mathsf{A}\cdots\gg \mathsf{C}]\!] = [\![\mathsf{A}\gg \mathsf{C}]\!] = \bigcap\{[\![>]\!](\mathbf{A})([\![\mathsf{C}]\!]) \mid \mathbf{A} \in [\![\mathsf{A}]\!]\}$.

Not every result of applying this recipe to a proto-conditional is guaranteed to yield a non-collapsing conditional. For example, if we choose the material conditional as a proto-conditional, we get back a conditional that agrees with the material conditional whenever defined. On the other hand, no choice of proto-conditional can generate the identical definedness conditions and truth conditions of the material conditional.

Importantly, however, there are choices of proto-conditional for which the recipe does not yield a collapsing conditional. In particular, a natural option for the proto-conditional is the Lewisian variably strict conditional. The underlying Lewisian operator allows that there may be multiple worlds where the antecedent is true that are relevant to the evaluation of the consequent. Then the conditional that results from applying the procedure above is doubly homogenous. First, the conditional presupposes that the antecedent alternatives either all guarantee the consequent, or all guarantee the consequent's negation. Second, for each antecedent alternative, the conditional presupposes that either all of the relevant worlds where that alternative holds are worlds where the consequent is true, or they are all worlds where the consequent is false. Perhaps surprisingly, this theory more or less has already been developed and endorsed, for somewhat different reasons, in Santorio 2017. Another option would be to start with a strict proto-conditional, and apply both of our procedures. Furthermore, subjunctive and indicative conditionals might differ in exactly this respect: in which proto-conditional operator they are generated from.[28]

7 Intransitive Entailment?

It may seem that rejecting the transitivity of entailment is too high a price to pay. If entailment is understood as necessary truth-preservation (which of course is not how we characterized it), transitivity should be a basic property. How much do we give up by moving to an intransitive notion of entailment? And is it worth it?

The first thing to notice is that violations of transitivity in our framework are highly localized. Recall our formulation of Transitivity and Cut:

Transitivity. if $\mathsf{A} \models \mathsf{B}$ and $\mathsf{B} \models \mathsf{C}$, then $\mathsf{A} \models \mathsf{C}$

Cut. if $X \models \mathsf{B}$ and $Y, \mathsf{B} \models \mathsf{C}$, then $X, Y \models \mathsf{C}$

Generally speaking: Transitivity (and Cut) can only fail, in the context of Strawson entailment, if B plays an essential role in satisfying the presuppositions of C.

As a consequence, various restricted forms of Transitivity and Cut are unaffected. Specifically, there are no violations of Transitivity and Cut when C is presupposition-free. A bit more generally, there are no violations of Cut when Y alone guarantees that all the presuppositions of C are satisfied (the case in which C is presupposition-free is a limit case of this condition).

[28]Our proposal results from applying the two tools in a specific order: the *alternatives-first* strategy we follow, denoted by $\cdot\cdot\!\gg$, takes a protoconditional and transforms it into an alternative sensitive conditional. Then it adds on a layer of homogeneity presuppositions, requiring that either every antecedent alternative make the protoconditional true, or every such alternative make it false. . But, of course, there is another option: the *homogeneity-first* strategy, represented by $\succ\cdot\cdot>$, takes a protoconditional and first adds homogeneity presuppositions. It then adds in a layer of alternative sensitivity, where $\mathsf{A} \succ\cdot\cdot> \mathsf{C}$ says that for every antecedent alternative, either every relevant scenario where that alternative holds makes the consequent true, or every such scenario makes the consequent false.

It turns out that this homogeneity first strategy makes different predictions than the alternatives first strategy. In particular, it leads to a conditional with strictly weaker presuppositions than our own, but with the same truth conditions whenever defined. For consider some arbitrary disjunction A $\mathtt{or}$ B (with A and B non-disjunctive) and suppose we have $\mathsf{A} > \mathsf{C}$ and $\mathsf{B} > \neg\mathsf{C}$ for some proto-conditional satisfying the Weak Boethius Thesis. Then our preferred conditional above makes $(\mathsf{A}\ \mathtt{or}\ \mathsf{B})\cdot\cdot\!\gg \mathsf{C}$ undefined. By contrast, in the homogeneity first approach the relevant disjunctive conditional $((\mathsf{A}\ \mathtt{or}\ \mathsf{B}) \succ\cdot\cdot> \mathsf{C})$ is defined. After all, our layer of alternative sensitivity simply says that every alternative in the antecedent leads to true when plugged into a protoconditional. If each input to this operation is defined, the whole thing should be as well. For this reason, the homogeneity first approach gives up CEM on the disjunctive fragment of the language. It thus offers no advance with respect to our incompatibility results, compared with an ordinary alternative sensitive treatment of SDA.

Depending on what we think motivates Transitivity (and Cut), these restricted forms might be all we need.[29] If we thought that these generalizations are supported by the intuitive plausibility of their instances, we should not be troubled by the retreat to the restricted forms of the rules. The cases in which transitivity fails are not only relatively localized, but they are also not obviously cases in which transitivity is supported.

But perhaps the justification of these structural rules is not empirical or intuitive, but purely conceptual. For example, Dummett 1975 (p. 306) famously objects that giving up transitivity "undermine[s] the whole notion of proof and, indeed, to violate the concept of argument itself". Such theorists would likely find our rejection of transitivity to border on the contradictory (or, worse, to just be contradictory).

While we are not convinced, it is well beyond the scope of this paper to engage whether the very idea of entailment relation rules out intransitive relations. Even so, it is possible to reconcile our results with the Dummettian stance. Instead of claiming that *entailment* is intransitive, we might adopt a two-level explanatory scheme on which the apparent validity of certain inferences is accounted for in terms of a secondary, pragmatic relation. The playbook for this reconciliation is set by Stalnaker's discussion of *reasonable inference*. Stalnaker famously characterizes reasonable inference as follows:

> an inference from a sequence of assertions or suppositions (the premises) to an assertion or hypothetical assertion (the conclusion) is reasonable just in case, in every context in which the premises could appropriately be asserted or supposed, it is impossible for anyone to accept the premisses without committing himself to the conclusion.[30]

There is a tight connection between this pragmatic notion and Strawson entailment. Without necessarily trying to reduce them to a common notion, we might notice a few important similarities.

First, reasonable inference is also intransitive. For example, a signature application of reasonable inference is to the direct argument, from A or C to the indicative conditional $\neg A \rightarrow C$. In Stalnaker 1975, this argument is invalid, but is still a reasonable inference. Likewise, because any classically valid inference is a reasonable inference, the inference from A to A or C is also a reasonable inference. Nonetheless, these two inferences cannot be chained together:

[29]There is much prior work on restrictions of Cut or Transitivity, although generally not in the context of Strawson entailment. See Smiley 1958, Tennant 1992, Tennant 1994, Ripley 2013, Ripley 2015, Cobreros *et al.* 2012, Cobreros *et al.* 2015. Ripley's program provides a particularly interesting point of connection here, even if it is not based around the notion of Strawson entailment. The connection is that one of its key selling point is the ability to retain a constellation of plausible principles and theses in the face of paradoxes. (General issues surrounding the correct formulation of transitivity are taken up in Ripley forthcoming.)

[30]Stalnaker 1975

the inference from A to $\neg A \rightarrow C$ is unreasonable, because the intermediate step A or C can be unassertable in some scenarios in which A is true and assertable.

Second, Strawson entailment can be viewed as involving a specialized notion of "appropriate assertibility". In Stalnaker's characterization this concept is entirely general, but we may view Strawson entailment as restricting focus to failures of assertibility due to presupposition failure.

In sum, what is central to our proposal is that, whatever the aggregate of notions that is involved in explaining the acceptability of inferences, it must involve at least one notion that is entailment-like (in the sense of being amenable to rigorous formalization, e.g., in terms of a proof system) and intransitive. There is certainly no purely conceptual argument against this.

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