Identity Criteria: An Epistemic Path to Conceptual Grounding

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July 23, 2018

This is the penultimate draft of the the paper online first by *Synthese*. The final version is here: https://doi.org/10.1007/s11229-018-1878-5.

Abstract

Are identity criteria grounding principles? A *prima facie* answer to this question is positive. Specifically, *two-level identity criteria* can be taken as principles related to issues of identity among objects of a given kind compared with objects of a more basic kind. Moreover, they are grounding *metaphysical* principles of some objects with regard to others. In the first part of the paper we criticise this *prima facie* natural reading of identity criteria. This result does not mean that identity criteria could not be taken as *grounding principles.* In the second part, we propose some basic steps towards a *conceptual reading* of grounding. Such a way of understanding it goes along with an *epistemic reading* of identity criteria.

1 What are identity criteria?

The credit for introducing *identity criteria* is usually attributed to Frege. In his *Foundations of Arithmetic*, he introduces this idea in a context where he wonders how we can grasp or formulate the concept of *number* (see [14], sec. 62). This is the standard Fregean quotation:

> If we are to use the symbol a to signify an object, we must have a criterion for deciding in all cases whether b is the same as a, even if it is not always in our power to apply this criterion. (see [14], sec. 62)

Two famous examples of identity criteria provided by Frege are as follows:

- *Directions*: If a and b are lines, then the direction of line a is identical to the direction of line b if and only if a is parallel to b.
- *Hume's principle*: For any concepts F and G, the number of F-things is equal to the number of G-things if and only a one-to-one correspondence exists between F-things and G-things.

Even if it is not completely clear whether Frege thought of identity as related only to abstract entities, his considerations about identity criteria seem to be adaptable to both concrete and abstract objects.[1] He suggests that an identity criterion has the function of providing a general way of answering the following question, with a and b objects in a given domain:

[1] From a Fregean perspective, an example of an identity criterion for concrete entities is the Davidsonian criterion for events in [8]. For an application of identity criteria as tools to demarcate concrete and abstract objects, see [15].

Fregean Question: How can we know whether a is identical to b?

In the philosophical literature, the Fregean question has been reformulated in the following ways:

- *Ontological Question* (OQ): If a and b are Ks, what is it for the object a to be identical to b?
- *Epistemic Question* (EQ): If a and b are Ks, how can we know that a is the same as b?
- *Semantic Question* (SQ): If a and b are Ks, when do 'a' and 'b' refer to the same object?

These three questions are related, respectively, to the epistemic, ontological and semantic functions of identity criteria. To answer (EQ), we refer to conditions associated with a procedure for deciding identity questions concerning objects of some kind K. To answer (OQ), we refer to properties that objects of the same kind must share to be identical. Finally, an answer to (SQ) concerns sameness and difference in referring to simple or complex names.

Both Frege's examples and later philosophical formulations seem to assume that the *Fregean question* is to be restricted to particular kinds of objects.

Using identity criteria when answering (OQ) is standard. For example, when we talk of identity criteria in relation to a *principium individuationis* (see [22] and [21]), the word 'identity criterion' is used in a metaphysical/ontological sense–it is an answer to (OQ). In answering (OQ), we think of conditions that are meant to provide an ontological analysis of the identity between objects of some kind K. However, in the final clause of the above quotation Frege has been observed to point to an ontological reading without blocking off an epistemic one. This is, for example, Williamson's opinion in ([39], pp.148-149).[2]

Following Frege considering what the logical form of identity criteria looks like, even though different ways of conceiving the form have been proposed, is worthwhile. A general, formal way to represent identity criteria is as follows:

$$\forall x \forall y((K(x) \land K(y)) \to (x = y \leftrightarrow R(x, y))) \tag{IC}$$

R represents the identity condition; R holds between a pair of Ks x and y, iff x and y are identical. Given that $x = y$ is an equivalence relation, the right side of the biconditional R must be an equivalence relation. (IC) is also formulated in the following way (without a reference to K):

$$\forall x \forall y(x' = y' \leftrightarrow R(x, y)), \tag{IC*}$$

where "x'" and "y'" are terms representing entities of the kind K suitably connected with x and y. Frege's criterion of identity for directions is an example of (IC*):

$$\forall x \forall y((o(x) = o(y) \leftrightarrow P(x, y))) \tag{O}$$

where x and y range over lines, o stands for 'the direction of' and P means 'is parallel to'. As an example the direction of line a is identical to the direction of line b if and only if a is parallel to b. In (O), the identity sign is flanked by terms constructed with a functional letter, and the right-hand side of the

[2]We will return on this point later.

biconditional introduces a relation among entities (lines) different from the entities for which the criterion is formulated (directions).

For Williamson (IC*) is the logical form of a *two-level IC* (see [39], pp. 145-146). On the contrary, the *axiom of extensionality* for sets,

$$\forall x \forall y(x = y \leftrightarrow \forall z(z \in x \leftrightarrow z \in y)) \tag{A}$$

is an example of a *one-level identity criterion.* In (A) the identity sign is flanked by terms for sets, and the right-hand side states a relation extensionally equivalent to the identity between sets.

In the case of *two-level identity criteria*, the conditions of identity concern objects that are *not* of the same kind of objects for which the IC is provided. On the contrary, in the case of *one-level* identity criteria the conditions of identity concern objects that *are* the same kind of objects for which the identity criterion is provided.

Williamson points out the following:

> The idea of a two-level criterion of identity has an obvious advantage. No formula could be more basic (in any relevant sense) than '$x = y$', but some might be more basic than '$ox = oy$', by removing the symbol 'o' and inserting something more basic than it. ([39], p. 147)

In such cases, one can also speak of a *reductivist* conception of identity criteria because the identity among the objects of a certain kind depends on the relations among more basic objects. Following Quine's example, rivers are individuated in terms of the momentary stages of the river ([27], p. 66). Reductivist identity criteria are open to some objections. One of the most important ones is as follows. If identity criteria have to provide an ontological analysis of identity, we have to admit objects for which there are no reductive criteria of identity and from which we move on in order to provide identity criteria for less-basic objects. Otherwise we run into a(n infinite) regress. Consider this example of identity criterion:

(M=) Material objects are identical if and only if they occupy the same place at all times.

One could ask for a criterion of identity for the notion of place, and this criterion, again, has to be given in terms of entities different from material objects and places, even if what these different entities could possibly be is unclear. Continuing to apply the same kind of demand produces a(n infinite) regress.

How can this regress be avoided in the Fregean framework of (IC*)? A simple answer is a sort of biting the bullet–introducing an *ontological* fundamental level of entities that are not reducible to others. The identity of these entities is primitive and can ground the identity of entities belonging to other levels of reality. Following this way of avoiding the regress, a natural reading of identity criteria is as *ontological grounding principles.* Is such a reading of identity criteria proper? In the following, we will show the difficulties with this *prima facie* plausible idea.

2 Grounding identity

Briefly, at least one *prima facie* reason exists for reading identity criteria as grounding metaphysical principles.[3] In general, to say that at a certain time t, x and y are distinct particulars or the same particular items seems to imply that there is something *in virtue of which* x and y are distinct particulars/are the same particular, i.e. a fact that grounds the distinctness of the two particulars at play. Consequently, to avoid the regress described in the last section, we have to presuppose some basic objects / facts for which there are no criteria of identity and from which we move on in order to give identity criteria for less-basic objects.

At least two general kinds of problems exist in understanding identity criteria as grounding principles. The first kind has to do with the *discrimination* of the facts involved in identity criteria, whereas the second kind concerns the *logical structure* of an *identity fact*. Let us take them into consideration in this order.

First, let us establish some terminologies. We use square brackets to denote facts: if A is a sentence, $[A]$ is the fact that A. By angle brackets $< \dots >$ we denote propositions; so, $< A >$ is the proposition that A. To indicate the grounding relation, we will use the symbol $\rhd$.

Then, the first problem is that the facts occurring in the identity criteria

[3] Grounding is one of the most discussed notions in contemporary philosophy. Roughly, grounding is a type of non-causal, primitive relation (or operation) such that the grounded entities, usually facts, are somehow explained, determined or constituted by the grounding entities. The *grounding revolution* ([33], 91) contributed to clarifying the meaning of locutions such as "in virtue of" and "because". The literature on grounding is massive. Some basic and introductory papers on it are: [12], [13], [7], [6], [31], [37], and [3]. An enlightening introduction to grounding and related notions is Chapter 5 of [35].

must be distinct. That is, generally speaking, if

$$[A] \rhd [B]$$

then the fact that A must be different from the fact that B; if not, we lose the anti-reflexivity of the grounding relation. An analogous point is considered in ([32], pp. 123-124). According to Rosen, a good guide for finding the grounding relations among facts is following the reduction relations among propositions. He sums up this strategy in the so-called *Grounding-Reduction Link*:

If $\langle p \rangle$ is true and $\langle p \rangle \Leftarrow \langle q \rangle$, then $[q] \rhd [p]$

In words, if the proposition that p is true and *p's being the case consists in q's being the case,* then the fact that q grounds the fact that p.

Now, construing IC as reductions *à la* la Rosen is perfectly plausible; in this case, we would have the following:

If $\langle d(a) = d(b) \rangle$ is true and $\langle d(a) = d(b) \rangle \Leftarrow \langle Par(a, b) \rangle$, then $[Par(a, b)] \rhd [d(a) = d(b)]$.

In words, if it is true that the direction of a is identical to the direction of b, and if their identity consists in being parallel a and b, then the fact that the direction of a is identical to the direction of b is grounded on the fact that a and b are parallel. However, things are not so easy:

> [...] The [Grounding-Reduction] Link presents us with a real puzzle. After all, if our definition of square is correct, then surely the fact that $ABCD$ is a square and the fact that $ABCD$ is an equilateral rectangle are not different facts; they are one and the same. But then the grounding-Reduction Link must be mistaken, since every

> instance of it will amount to a violation of irreflexivity. ([32], p. 124)

Something similar happens with directions; let us reflect on what it means that it is a fact that $Par(a, b)$. Very likely, this is a (geometrical) scenario in which at least two items are in a certain spatial relation. But in this specific scenario, the direction of a *is* the direction of b; in other words, it seems that there is not a further grounded fact based in the identity of the directions.

Rosen's solution is as follows:

> We can resist this [critique] by insisting that the operation of replacing a worldly item in a fact with its real definition never yields the same fact again. It yields a new fact that 'unpacks' or 'analyzes' the original. ([32], p. 124)

What Rosen means here by the notion of 'unpacking' a fact is not perfectly clear; the example he provides is the following:

> Suppose for the sake of argument that to be the number two just is to be the successor of 1. [In our notation: $\forall x\ (\langle x = 2\rangle \Leftarrow \langle x = s(1)\rangle)$.] One might accept this while rejecting the exotic view that the number 2 somehow contains the number 1 as a part or constituent. Simply from the fact that 1 figures in the *definition* of 2, it does not follow that 1 is a part of 2. But now propositions (and facts) are individuated by their constituents. [...] The former contains 2 as a constituent, but need not contain the successor function or the number 1; the latter contains *successor* and the number 1, but need not contain the number 2. ([32], p. 125)

Briefly put, Rosen's idea seems to be to identify facts and propositions through their constituents, as individuals, functions, attributes and so on. Therefore, according to him, the fact that $3 = 2 + 1$ is different from the fact that $3 = s(1) + 1$. Once this has been assumed, the nexuses of conceptual reduction are reliable guides to the genuine grounding relations.

To import Rosen's intuition to our case, we should maintain that in

$$\forall x \forall y((o(x) = o(y) \leftrightarrow P(x, y))) \tag{O}$$

the fact that certain a and b are parallel is different from the fact that the direction of a is identical to the direction of b. Previously, we argued that maintaining just one set of realities is plausible, and then that we have a violation of anti-reflexivity of grounding. However, the reply here is that the constituents of the facts at issue are different: in the former, we have lines, whereas in the latter, we have directions of lines.

Does this strategy work? Well, following Rosen's suggestion, it seems that we have to accept that $[3 = 2 + 1] \neq [3 = s(1) + 1]$, even if, arithmetically, $2 = s(1)$. So it should be a(n arithmetical) fact, that $2 = s(1)$. Now, understanding how this sub-fact must not enter into the constitution of both the facts $[3 = 2 + 1]$ and $[3 = s(1) + 1]$ is not easy; otherwise, if $2 = s(1)$ was a relevant fact, how could these facts be different? Put in other terms, how can we explain that these facts are different in virtue of the distinction of their constituents even though these constituents are identical?

Admittedly, [4] one could argue for Rosen's account by saying that even

[4] We want to thank an anonymous referee for pointing out this issue.

if $2 = s(1)$ is a fact, this fact does not enter as a constituent in those two other facts. The general idea is that, according to many grounding theorists, grounding is a hyper-intensional relation: necessarily equivalent facts can be discriminated with respect to their grounding relation. Let us concede that grounding is a hyper-intensional relation and let us follow this train of thought. Accordingly, the two facts $[3 = 2 + 1]$ and $[3 = s(1) + 1]$ are different in virtue of their constituents. But if we plug, so to speak, the fact that $2 = s(1)$ into the facts in examination, we would obtain two composed facts as follows: $[[3 = 2 + 1], [2 = s(1)]]$ and $[[3 = s(1) + 1], [2 = s(1)]]$. Now, are these two facts identical or different? If they are identical, they are so in virtue of their constituents. However, not *all* the constituents are identical and connected by the same relationship. Therefore, the reason of their identity has to be identified with what they have in common, that is, that $2 = s(1)$. But this is exactly the same reason we advanced to say that the original facts ($[3 = 2+1]$ and $[3 = s(1) + 1]$) are identical.[5]

2.1 Irreflexivity of grounding reduction

As we have seen from the previous section, usually, the grounding relation is considered *irreflexive*: it is one formal basic property of the relation (see, for instance, [7], [13]).[6] However, some scholars argue that irreflexivity is not trivial at all.[7] For our purpose, we think that Jenkins' analysis [18] (see also

[5]For a discussion on the ontology of mathematics in the light of the grounding approach, see [11]).

[6]We would like to thank an anonymous referee for emphasising this important point and, moreover, for suggesting some possible developments of the argument.

[7]For a very useful overview on this topic with a focus on the reasons for which we might find reflexive instances of dependence unacceptable, see [2].

[17]) is illuminating.

Imagine a situation in which we would like to save the following intuitions:

(a.) S's pain depends on the brain state B

(b.) S's pain does not depend on S's pain

(c.) S's pain and the brain state B are identical

Let us indicate by [B] the fact constituted by the neuronal patterns; by [P] the feeling of pain; by $\rhd$ the relation of dependence/grounding, and by $\sim$ the relation of identity between facts; we therefore have:

(a.*) $[B] \rhd [P]$

(b.*) $\neg([P] \rhd [P])$

(c.*) $[B] \sim [P]$

A first strategy Jenkins proposes is to isolate within facts at stake an *aspect*, or a *mode of presentation* (a variant of this is the recourse to Lewis' qua-objects, see [19]). Following this train of thought, [B] and [P] are the same fact that can be, so to speak, presented in two different ways. The intuitions (a.) – (c.) characterise the position according to which just one fact can be presented as pain (probably, from a 'first person' perspective) and as a neuronal pattern (probably, from a 'third person' perspective); moreover, a specific grounding relation exists between these two aspects: the phenomenal mode is grounded on the neuronal configuration.

How can the above-mentioned idea be applied to the case of identity criteria? The idea would be the following: the fact that a and b are parallel and the fact that the direction of a is identical to the direction of b are the same fact, even if they are presented in two different ways – a *parallel* mode and a *direction* mode. So it seems that the relation of grounding does not hold between two different facts (there is just a geometrical scenario) or between the *same* (in the strong sense) fact (Jenkins agrees that that saying that [A] depends on [A] is meaningless); on the contrary, the grounding relation holds between two aspects of the same (worldly) fact.

The proposal we aim to develop in the second part of this work is in line with Jenkins' suggestion: aspects (or modes of presentation) of the fact at issue can be intended as the *concepts* we use to understand reality. The same geometrical fact can be intended either as a 'fact' concerning directions or as a 'fact' concerning parallel lines; *it depends on the concepts we are using.* This fact is in accordance with the idea of *recarving the content* which the abstraction principles aim to.

Of course, some substantive questions need to be addressed. First of all, one could ask for a detailed formal characterisation of aspects (or modes); second, one should address the question about the mental or worldly nature of the items involved. In the following, we will provide an account with a clear epistemic orientation; however, nothing prevents us from modifying the intended construal and bending it towards a more Fregean view of concepts as objective ways to capture the fact in question.

2.2 Identity facts

The second general worry about the metaphysical interpretation of grounding has to do with the same idea of an identity fact namely, the fact that, for instance, the direction of a is identical to the direction of b.[8] Take Leibniz's law:

$$a = b \leftrightarrow \forall F(Fa \leftrightarrow Fb) \tag{LL}$$

One could assume that it is an ontological explanation of identity. That is, identity facts are actually *indiscernibility facts*. An indiscernibility fact is the fact that a and b share all the properties. Applying this notion to identity criteria, we get the following:

$$[Par(a, b)] \rhd [\forall F(F(d(a)) \leftrightarrow F(d(b)))] \tag{ICG}$$

Its reading is as follows: the fact that a and b are parallel grounds the fact that their directions are indiscernible. Does all this work? We will proceed as follows: first, we will check if Rosen's general strategy exploiting the *grounding-reduction link* can be exported to the analysis of identity facts. Second, we will investigate whether the recourse to indiscernibility facts is really less problematic than that to identity facts.

That said, let us see how identity is supposed to work by using Rosen's grounding reduction link. Assuming that for a, to be identical to b is simply to share all the properties of b is plausible. The identity relation reduces to a universal equivalence. So speaking about directions, we have

$$\langle d(a) = d(b) \rangle \Leftarrow \langle \forall F(F(d(a)) \leftrightarrow F(d(b))) \rangle \tag{RLL}$$

[8]For similar reflections about the *fundamentality* of the identity facts see [24] and [34].

If this is true, we obtain the corresponding grounding relation:

$$[\forall F(F(d(a)) \leftrightarrow F(d(b)))] \rhd [d(a) = d(b)] \tag{GLL}$$

And this, too, is in the vein of Rosen's intuition. But now we have a problem; in fact, from (GLL) and from $[Par(a,b)] \rhd [d(a) = d(b)]$, it does not follow that

$$[Par(a,b)] \rhd [\forall F(F(d(a)) \leftrightarrow F(d(b)))], \tag{1}$$

which was the intended interpretation of IC for directions as a grounding statement. Our result could be obtained if we reversed the (GLL) principle,

$$[d(a) = d(b)] \rhd [\forall F(F(d(a)) \leftrightarrow F(d(b)))] \tag{GLL*}$$

But in order to get it, we also have to reverse the reduction link, i.e.:

$$\langle \forall F(F(d(a)) \leftrightarrow F(d(b))) \rangle \Leftarrow \langle d(a) = d(b) \rangle. \tag{RLL*}$$

But this is implausible because in that case, how the reduction is supposed to work is unclear. So we conclude that to provide a metaphysical explanation of an identity fact through Leibniz's Law, the path that Rosen suggested is not applicable.

3 Toward conceptual grounding

We have seen that in light of the encountered problems, construing identity criteria as ontological grounding principles is more difficult than it would appear *prima facie*. The greatest difficulties are in the metaphysical characterisation of the so-called *identity facts*, such as the fact that Plato is identical to Aristotle's teacher or that the direction of line *a* is identical to the direction of line *b*.

A more general and pervasive problem with the metaphysical characterisation of identity is as follows: from one hand, an alleged identity fact is described by a sentence in which *two* individual terms occur (identity is formally a dyadic relation). But from one other, the very fact that Plato is identical to Aristotle's teacher 'means' that just *one* individual exists, and the fact at play which should concern Plato is not clear. In addition, these facts can be considered either *primitive* or *derivative*. But, observe, if they are primitive they cannot be grounded and, by consequence, the ontological interpretation of IC is flawed. On the contrary, if the identity facts are not primitive, they have to be derivative. However, we have argued that the attempts to describe the metaphysical nature of identity through various theoretical devices are bankrupt. [9]

So we have a stalemate; our intuition pushes in the direction of construing the identity criteria as grounding statements; on the contrary, however, insurmountable obstacles to pursuing this project exist. In other words, if the notion of identity is, in a sense, so poor, how can we follow Frege's suggestion that identity criteria are *explicative*?[10]

Something went wrong. A suggestion to approach the question is by means of the 'cognate' notion of *explanation*; according to a monist point of view, implicitly adopted by some of the first proponents of grounding (see, for example,

[9]The same problem has been observed by Shumener in [34], 6.
[10]A new proposal on the above question is in [1].

[13]), *grounding* and *explanation* are one and the same relation: for arbitrary facts $[A]$ and $[B]$, whenever $[A]$ explains $[B]$, it is also the case that $[A]$ grounds $[B]$; thus, without the former, the later does not obtain. As argued, however, for identity criteria, we have explanation without (ontological) grounding.

Consider, again, what Frege says on the identity criterion for numbers. His suggestion is that it is explicative: it gives us an explanation of the sense of an identity sentence for numbers. Following the above suggestion, we adopt, at least partially, an *epistemic* reading of identity criteria. As said previously, this is in line with the above-mentioned quotation:

> If we are to use the symbol a to signify an object, we must have a criterion for deciding in all cases whether b is the same as a, even if it is not always in our power to apply this criterion. (see [14], sec. 62)

The idea, put in other terms, is that there could be an algorithm which can decide all the arithmetical problems of a certain kind, even if we are not always able to apply it. However, the algorithm is a way of knowing the answers to the arithmetical problems of the kind in question. It is not, at least not completely, that *in virtue of* which the answer is the correct one.

But, how can a (partly) epistemic reading of identity criteria be understood? We cannot think of them as a way of discovering whether any sentence of the form $a = b$, when a and b are Ks, is true or false. If so, the criterion would provide a way of discovering whether any sentence is true or false. But this is an absurd output. Here is the argument (it is in [20], 246). Take the identity statement: $a = (\iota x)\ (x = a \wedge S)$, where S is any sentence. It is logically true that:

S if and only if $a = (\iota x)(x = a \wedge S)$

(Case left-right: S is true. Then, $(a = a \wedge S)$ is true. Hence, there is an x such that $(x = a \wedge S)$ and a is the only object satisfying the condition: $(x = a \wedge S)$, i.e. $a = (\iota x)(x = a \wedge S)$. Case right-left: $a = (\iota x)(x = a \wedge S)$ is true. Then, a satisfies the condition: $(x = a \wedge S)$ and so S is true).[11] It follows that because such a criterion provides a way of discovering whether an identity sentence is true or false, it will also provide, in particular, a way of discovering whether an identity sentence of the form $a = (\iota x)(x = a \wedge S)$ is true or false, i.e. whether S is true or false.

Because there is no recursive procedure which allows one to decide on any sentence, such a conclusion can be accepted only if the way of discovering whether any identity sentence is true or false is not a recursive one. However, imagining what such a non-recursive general method could be is difficult.

So there cannot be any general recursive procedure, and very likely any non-recursive procedure either, for deciding the truth value of any identity sentence in which terms of any form can occur.

However, identity criteria can be recipes for working out the problem of the recognition of an entity in a *weaker sense*. For example, one cannot exclude decision methods for particular classes of identity sentences. Neither can it be excluded that identity criteria may have an epistemic function which does not consist in providing an infallible decision method. In this sense, one can characterise them as *partially (or weakly) epistemic*: they can be fallible

[11] The formal proof is in [4], 223.

recipes, susceptible to different levels of precision. From this point of view, epistemic identity criteria could provide partial and not infallible identification procedures, depending on the *contexts of application* and on different *levels of precision*.

This characterisation seems to be in line with the idea that explanation has agent-relative features (see [36]). According to a pre-theoretical understanding of explanation as shaped by our uses of why-questions and because-answers, which why-questions and because-answers are appropriate and thus feature in successful explanation is, partly, a contextual matter.

Following the above suggestions and Jenkins' remarks on *aspects* of facts as *concepts*, it is arguable that in the case of directions and parallel lines, the *same geometrical fact* can be intended either as a 'fact' *concerning* directions or as a 'fact' concerning parallel lines; it depends on the *concepts* we are using. Moreover, it can be considered an explanation of the one in terms of the other: it is a simply way to understand our because-answers to why-questions, as appropriate.

Adopting monism (or *unity*, as it is sometimes called the monist view (see [36] and [23]) one can metaphysically deflate the notion of grounding in terms of *conceptual grounding*. This deflated notion has recently been championed by Dasgputa in ([9]). Our driving idea is that the mastery of the concept of *parallelism* can ground the application of the concept of *direction*.

An epistemic reading of identity criteria says that understanding, grasping and applying certain concepts depends on understanding, grasping and applying other concepts.

Summarising the above remarks, one can conclude the following:

(i) With a monist point of view on explanation and grounding taken, the grounding relations between concepts are, as explanation, intersubjective and agent-relative.

(ii) As for explanation, in the identity criteria case, we do not have any general recursive procedure for determining when a concept grounds another one, so the grounding relation is *weakly epistemic* or *contextually determined*.

(iii) Moreover, the explanation explicitly or tacitly presupposes a *level of precision*, so for grounding, a certain level of precision for concepts is presupposed.

(iv) The classical examples of identity criteria (*directions* and *numbers*) can be taken as *limit* cases of more common cases of epistemic grounding. We will discuss this point later.

In the next sections our project is the following: we want to provide an epistemic interpretation of identity criteria. However, our account must consider cases in which the general context and the background conditions are not *ideal*. We are to face situations in which, for instance, there are two (or more) plausible candidates for grounding a certain concept.

For this reason, we will provide the sketch of a model which allows the characterisation of the idea, according to which the concepts that are at a more fine-grained level are more plausible candidates for being the epistemic grounds of other concepts.

From the discussed examples (borrowed from Williamson, Horsten and others) it will be straightforward that our model is informative in cases in which concepts with empirical content occur (for instance, the indistinguishability of colors). The classical Fregean cases (directions and numbers) seem less apt to be characterised by our model. However, one can think that cases about *abstract objects* (in particular, geometrical and arithmetical objects) are paradigmatic situations in which the epistemic conditions are perfect; in such contexts, therefore, the relations of conceptual grounding will be univocal and perfectly determinate.

To develop our proposal, let us therefore start considering cases of non-ideal conditions; in the following, we will see situations in which there are relations considered as intuitively good candidates for being the condition R of an identity criterion, but they fail to be transitive. Consider some examples offered by Williamson [38]:

Example 1. *Let $x, y, z, ...$ range over colour samples and f be the function that maps colour samples to perceived colours. A plausible candidate for R might be the relation of indistinguishability. It is easy to verify, though, that such an R is not necessarily transitive, it might happen that x is indistinguishable from y and y from z, but x and z can be perceived different in colour.*

Example 2. *If $f(x)$ is a physical magnitude, to determine $f(x) = f(y)$ we measure x and y. If x and y differed marginally, the measurement operation could give the identity of the physical magnitudes as a result. If R were defined on the basis of the measurement operations, it would turn out to be not-transitive, as the sum of many little differences is not itself little.*

Example 1 and *example 2* show how some relations that are intuitively plausible candidates as identity conditions do not meet the logical constraints that identity criteria demand. However, instead of the refusal of this kind of plausible but inadequate identity criterion, the suggestion has been to *approximate* the relation R whenever it is not transitive. This means that, given a non-transitive R, we can obtain equivalence relations that approximate R by some operations. Some approaches have been suggested–two of them are by (Williamson [38], [39]), whereas a third approach is by (De Clercq and Horsten [10]).

4 Approximations of identity conditions

For Williamson, we can simply concentrate on the best approximation to a non-transitive relation giving up the requirement for the identity criterion to be both necessary and sufficient. Consider R a non-transitive relation that we take to be the best candidate for being R for some kind of objects $f(x)$s, such as directions.

Consider such an R a constant. Consider, then, variables on relations $R', R'', ...$ as possible approximations to R. Williamson proposes two ways to find an adequate equivalence relation to substitute a non-transitive R–an *approach from above* and an *approach from below*.

The approach from above seeks the smallest equivalence relation R^+ such that $R \subseteq R^+$. This means that some $f(x)$ and $f(y)$ that are not identical under R turn out to be identical under R^+, or, equivalently, R^+ is a super-relation of R. The IC of this form

$$\forall x \forall y (f(x) = f(y) \leftrightarrow R^+(x, y)) \tag{IC$^+$}$$

provides a sufficient, but not necessary, condition for the identity of $f(x)$s.

The approach from below seeks the largest equivalence relation R^- such that $R^- \subseteq R$. This means that R^- is a sub-relation of R because not all the ordered pairs in R are ordered pairs in R^-. R^- always exists on the assumption of the *axiom of choice*, but it is not unique. To decide which relation can be preferable over others, some constraints can be added. One of these is what Williamson calls the *minimality constraint*. According to such a constraint, the relation R^- to be preferred is the one with the minimum number of equivalence classes. The IC of this form,

$$\forall x \forall y (f(x) = f(y) \leftrightarrow R^-(x, y)) \tag{IC$^-$}$$

provides a necessary, but not sufficient, condition for the identity of $f(x)$s.

De Clercq and Horsten [10] suggest an approach to find approximating relations which is an alternative to that proposed by Williamson and is called the *overlapping approach*–the equivalence relation that is sought partially overlaps R, instead of being a sub- or a super-relation with respect to R.

Their proposal presupposes that R is not indeterminate: any two objects either stand in the relation R or they do not. The authors propose to define an equivalence relation $R^{\pm}$ that closely approximates R and achieves that task better than R^+ or R^- in the following way:

Example 3. *Given a function f, let the domain of objects for f be the following:*

$$\mathcal{D} = \{a, b, c, d, e\}$$

Assume there is a candidate relation R, reflexive and symmetric, for the identity condition for $f(x)$s. When R holds between two objects x and y, we denote this as $\overline{xy}$ (as De Clercq and Horsten do). Put otherwise, $\overline{xy}$ means $R(x, y)$ and $R(y, x)$. Let R on $\mathcal{D}$ be the following:

$$R = \{\overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd}, \overline{de}\}$$

R is not an equivalence relation. In fact, it fails to be transitive. For instance, R holds between a and d and between d and e, but it does not hold between a and e.

Consider now how R^+ looks in this case. It is unique, and it is the smallest equivalence relation that is a superset of R, i.e.

$$R = \{\overline{ab}, \overline{ac}, \overline{ad}, \overline{ae}, \overline{bc}, \overline{bd}, \overline{be}, \overline{cd}, \overline{ce}, \overline{de}\}.$$

On the contrary, R^- is not unique. For instance, one of the largest equivalence relations included in R is the following:

$$R^- = \{\overline{bc}, \overline{bd}, \overline{cd}\}.$$

To determine whether R^+ or R^- is the best approximation of R, first measure the *degree of unfaithfulness* (DOU) of R^+ and R^- with respect to R. Such a degree is the number of revisions you make to get R^+, R^- from R. A revision is any addition or removal of an ordered pair to or from R. In the example considered above, R^+ is obtained by adding four ordered pairs to R and R^- by removing three ordered pairs. The DOU of R^+ is 4 and the DOU R^- is 3.

Thus, R^- is closer to R than R^+ is; that is, R^- should be modified less than R^+ to obtain R.

Consider now the following equivalence relation:

$$R^{\pm} = \{\overline{ab}, \overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd}.\}.$$

With respect to R, $R^{\pm}$ adds one ordered pair and takes off another one. So, the DOU of $R^{\pm}$ is 2, i.e. less than both R^+ and R^-. Formally, the DOU is given by the symmetric difference $\triangle$:

$$(R^{\pm}, R) = |R \triangle R^{\pm}|. \qquad \text{(DOU)}$$

$R^{\pm}$ is an overlapping relation with respect to R, and it is a kind of hybrid relation between R^+ and R^- because it both adds and removes one ordered pair. An overlapping relation can be *closer* to R than the relations obtained with the approach from below and from above are.

5 Contexts and levels of specification. Two basic elements of the conceptual grounding

Consider *example 1* about phenomenal colours. As stated, this is an example in which indistinguishability transitivity fails.

In general, observe that, in non-problematic cases, that is, when we have to make judgments on very different colours, we report our observations by using coarse-grained predicates because we do not need to express shade differences.

It is when we have to deal with borderline cases of colours that we tend to be more precise in using colour predicates. More fine-grained colour predicates are used in colour science and technology, but in everyday life people do not use them; this is not just because there are limits to colours discriminability, but because of "something like the limit of useful naming of phenomenal hues for the purposes of communicating between people" ([16], p. 221). Put otherwise, the number of possible colour discriminations is much higher than the number of colour terms normally used.

The reason is the variability in discrimination between observers. Moreover, people observe colours under normal conditions such as changing light, contrast and shadows and not under standard conditions, and normal conditions make colour comparisons problematic. Finally, comparing a colour with a mental standard (such as the standard of *red* that one could have seen in the Munsell Chart) is more difficult than comparing a colour with another perceived at the same time.

So colour perception is influenced by many factors, and the use of colour predicates is somewhat sloppy. In [16] Hardin maintains that to answer a question such as 'What are the boundaries of red?' we must first

> specify, explicitly or tacitly, a context and a level of precision and [...] realize the margin of error or indeterminacy which that context and level carry with them. ([16], p. 230)

Can we apply the above-mentioned elements–contexts and levels of precision–to the indistinguishability condition?

Before the details of the analysis are discussed, it is remarkable to observe that the identity criteria for phenomenal colours are an example of identity criteria that have mostly an epistemic function: we do not know precisely whether two colours are identical. We only rely on our perception, which is fallible. So we express the identity criteria for colours in a logically inadequate way. From this perspective, identity criteria can be recipes for working out the problem of the recognition of an entity in a weaker sense. From this point of view, epistemic identity criteria could provide partial and not infallible identification procedures. Williamson and De Clercq and Horsten are going in this direction when they believe that there are logically adequate identity criteria and try to capture them by approximating our intuitively good, but logically inadequate, identity criteria.

Consider the following variations of the example of the identity criterion for perceived colours:

Example 4. *You see two monochromatic spots, A and B, and you do not detect any difference with respect to their colour. Following Williamson, you claim that they have the same colour because they are perceptually indistinguishable. Now, suppose you add two further monochromatic spots, C and D, such that they are perceptually distinguishable. However, A is indistinguishable from C and B from D. In such a scenario, you can revise your previous judgment and say that A and B are distinct.*

Example 5. *You see two colour samples A and B from a distant point of view such that you are not able to distinguish A-colour from B-colour. You say that A and B have the same colour. Now, you get closer to them and detect a difference between them. So, you revise your previous judgement and say that*

A and B are distinct.

Example 6. *You see two monochromatic spots again, A and B. You perceive them as equally, say, orange. Nevertheless, a friend of yours, who is a painter, tells you that she perceives them actually different: B is more yellowish than A. According to her colour perception, which is more refined than yours, there are more differences among colour samples than what you detect.*[12]

Example 4 shows how our perception of colours can be different, depending on the range of colours we see at the same moment. Better said, comparing a colour sample with one or more colour samples makes our judgements about colours differ. Thus, a relation R expressed by a criterion of identity can vary across contexts. Examples 5 and 6 concern levels of specification. In the following, we show how De Clercq and Horsten's framework can be improved if you consider the use of identity criteria in a *context* with a *level of specification* (Hardin in [16] talks of *precision*).

Consider a domain $\mathcal{D} = \{a, b, c, d, e\}$ and a context o, i.e. a subset of $\mathcal{D}$: $o = \{a, b\}$. Suppose $R = \{\overline{ab}\}$ in the context o. Consider now an enlarged context, o' containing a and b plus other elements, c and d: $o' = \{a, b, c, d\}$. In o' you may have the following R-pairs: $\overline{ac}$ and $\overline{ad}$ but not $\overline{ab}$. Example 4 can be specified using different contexts with an R varying across contexts.

Example 5 and 6 present a different issue than 4. Given the same context, R varies along different *levels of specification* of observation. When you are distant from the objects for which you have to make an identity statement, you are looking at them from a coarse point of view. Getting closer to the

[12]Examples 4-6 are in [5].

elements of the context, you reach a more fine-grained observational level and so you can make a different identity statement. The point of view of the painter can be seen, as well as a fine-grained observational level. In short, you can look at the elements of a context under different standards of precision, each of them corresponding to a level of observation. The finer is the level, the more differences between the individuals are detected. The different standards are introduced in a framework using a different R, which is more precise in the sense of better approximating R, having fixed a certain context.

In the next section we specify a way to understand *conceptual grounding* in terms of approximating identity criteria.

6 Sketch of a model for conceptual grounding

Let $\mathcal{L}$ be a formal standard language through which we can represent English expressions. The formulas of $\mathcal{L}$ are formally defined in the usual way.

Let $\mathcal{S} = \langle \mathcal{D}, R \rangle$ be a granular structure. Put otherwise, $\mathcal{S}$ is constituted by the domain $\mathcal{D}$, together with some (or a) context(s) o specified in $\mathcal{D}$ and a binary relation R (a two-arity predicate). A context is a subset of elements of $\mathcal{D}$. R gives rise to a set of ordered pairs in a certain specific context. R is the relation that makes identity statements about the elements of the domain, according to a certain identity criterion. As sketched before there are two changing elements:

- contexts o

- *levels of specification*

At first, consider *levels of specification* as in examples 5 and 6: R varies across what we call *levels of specificity.* One way to formalise the idea just mentioned is simply to fix a certain context o, obtaining different granular structures by different sets of ordered pairs generated by R.

Consider the following example. Let $o = \{a, b, c, d, e\}$ be our context o. The relation R in $\mathcal{S}_1$ holds between a and b, b and c and d and e. The situation is as follows:

- $o_1 = \{a, b, c, d, e\}$
- $R^{\mathcal{S}_1}_{o_1} = \{\overline{ab}, \overline{bc}, \overline{de}\}$

Consider now a more fine-grained structural level, $\mathcal{S}_2$. In this case the relation R–in our example, perceptual indiscriminability–changes along the grain size of the structure because of a different level of specification. The relation R in this case is the following: $R = \{\overline{bc}, \overline{cd}, \overline{de}\}$. The situation is then the following:

- $o_2 = \{a, b, c, d, e\}$
- $R^{\mathcal{S}_2}_{o_1} = \{\overline{bc}, \overline{cd}, \overline{de}\}$

Why is $\mathcal{S}_1$ more precise than $\mathcal{S}_2$? According to $\mathcal{S}_1$, we have $R = \{\overline{ab}, \overline{bc}, \overline{de}\}$. It is not transitive ($a$ is indistiguinshable from b and b from c, but a is not indistinguishable from c). The best overlapping approximation is as follows: $R^{\pm} = \{\overline{ab}, \overline{bc}, \overline{ac}\}$. The pair $\overline{ac}$ has been added, and the pair $\overline{de}$ has been removed. The DOU of $R^{\pm}$ is 2.

According to $\mathcal{S}_2$, we have the following: $R = \{\overline{ab}, \overline{bc}, \overline{cd}, \overline{de}, \overline{ce}\}$. In this case, R is not transitive either. But, observe that in this case the best overlapping approximation removes just the pair $\overline{bc}$: $R^{\pm} = \{\overline{cd}, \overline{de}, \overline{ce}\}$. The DOU of $R^{\pm}$ is 1. $\mathcal{S}_2$ is a more fine-grained structural level than $\mathcal{S}_1$.

For each context o we know which is/are the finest R i.e., the R which is/are the best approximation(s) of a criterion of identity that is *extensionally adequate* i.e., expressing an R. Consider, for example, o_1. In o_1 we have a certain number of structures:

- $R_{o_1}^{\mathcal{S}_1}$, $R_{o_1}^{\mathcal{S}_2}$, $R_{o_1}^{\mathcal{S}_3}$, $R_{o_1}^{\mathcal{S}_4}$

and we obtain a way to specify which structure is the most precise one, i.e. which structure is the *winner structure* (*WS*).

Definition 1. *Given a context o, a certain structure $\mathcal{S}_i$ is the winner structure (WS) iff R_i in $\mathcal{S}_i$ has a low DOU. R_i is the winner relation (WR). If R_i has the same DOU of another R_j in $\mathcal{S}_j$ and the DOU is the lowest one we have two equal merit WRs.*

The *WS* is the one that, given a certain context, best approximates the *criterion of identity* and *conceptually grounds* the others. If we have two equal merit *WR*s we obtain two *WS*s.

As specified before, we can have different levels of specification and different contexts. For each, we have individuated an *WR* or two (or more than two) equal merit *WR*s. Can we compare the different *WR*s of the contexts? A *WR* in o is extensionally characterised by a certain number of pairs. A way to compare *WR* in different contexts o_l is to say that

Definition 2. *Given two contexts o_l and o_k, the interpretation of R varies across o_l and o_k iff there is not an empty intersection $o^* = o_l \cap o_k \neq \emptyset$ such that $\exists x \in o^* \exists y \in o^* : (\overline{xy} \in R^{\mathcal{S}_i}_{o_l} \wedge \overline{xy} \notin R^{\mathcal{S}_i}_{o_k}) \vee (\overline{xy} \in R^{\mathcal{S}_i}_{o_k} \wedge \overline{xy} \notin R^{\mathcal{S}_i}_{o_l})$.*

Definition 2 is extendable to three or more contexts in the same way, so obtaining it given a certain number of contexts o, a certain R varies across the o. In this way, we obtain a certain number of contexts o_1, o_n shared by structures.

Given a certain number of contexts, you can order the *WR*s from the coarsest to the finest with respect to the number of intersecting items.

The *finest winner relation* (FWR) is the best approximation given a certain number of contexts and levels of specification. Even in the case of the FWR you can have two (or more) equal merit FWRs.

In other words, the *FWR(s)* is (are) the *best* concept(s) we can manage to understand the (identity of) notions we are interested in.

In summary, conceptual grounding works towards *two dimensions*:

i. The *WR* conceptually grounds, as stated, the concept in which we are interested (in a case discussed before, we could have a notion of parallelism, which is the *WR* for the notion of direction);

ii. At the same time, however, the *WR* conceptually grounds the other competitor relations, which are then more vague and imprecise (for instance, other imprecise notions of quasi-parallelism).

7 Conclusions

We have seen a *prima facie* reading of the identity criteria as grounding principles; the identity condition is what in virtue of objects of a certain kind are identical (or different). This construal is corroborated by the fact that the so-called two-level identity criteria ground, as we have seen, the identity of a certain kind of entity on the basis of other entities. This fact gives a layered structure, which is akin to the general view of grounding advocates.

If the identity criteria are (readable as) grounding principles, the question involves understanding which kind of grounding relation we are talking about. The first choice was the metaphysical grounding–the identity criteria make explicit the grounding relation among certain kinds of *facts*. We have seen, however, that this construal gives rise to some problems in particular, the facts occurring within the criteria must be distinct, and this has to do with their very inner structure. Moreover, what constitutes an identity fact is unclear.

Nevertheless, the encountered problems are not sufficient to abandon our starting intuition; reading identity criteria as principles of conceptual grounding is possible. But, what is the conceptual grounding? In which sense are two concepts intertwined by this relation? To provide an *explanation* of conceptual grounding, we look at another philosophical question on the identity criteria, i.e. the problem of their extensional adequacy.

Williamson, Leclerq and Horsten developed models to approximate the identity relations to transitive relations, thus getting closer to extensional adequacy. Our idea is, in a nutshell, to provide a sketch of a model by which, across various levels of specification and contexts, identifying a *WR* and a *WS* that is the best candidate to ground the identity we are searching for is possible. This model can give a sense of the relation of conceptual grounding and vindicate an epistemic reading of identity criteria. This fact obviously does not exclude the possibility of an ontological reading of identity criteria or worldly grounding relations.

8 Acknowledgments

Early versions of this paper have been read at the following conferences, workshops or seminars: *Metaphysical and Mathematical Explanation II – Explanation, Grounding, and Dependence* (Pavia, December 2017), *ECAP9* (Munich, August 2017), *IUC Conference in Metaphysics*, (Dubrovnik, June 2017), *From Foundations to Applications: Workshop in the Philosophy of Mathematics* (Milan, June 2017), *Ontoformat Seminar* (Milan, April 2017), *Grounding Grounding* (Alghero, October 2016). We are indebted to the participants in events for stimulating discussions. Thanks also to the referees of *Synthese* for their useful and detailed comments. Finally, many thanks for criticisms, suggestions and encouragement to: Lorenzo Azzano, Mark Balaguer, Otávio Bueno, Daniele Chiffi, Ilaria Canavotto, Alessandro Giordani, Øystein Linnebo, Vittorio Morato, Elisa Paganini, Francesca Poggiolesi, Marco Panza, Andrea Sereni, Tuomas Tahko, Paolo Valore, and Luca Zanetti.

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