On the meaning of ‘therefore’

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What does ‘therefore’ contribute to the meaning of a sentence such as (1)?

(1) He is an Englishman and he is, therefore, brave. (*therefore*-sentence)

Grice (1975) famously took the case of ‘therefore’ to be one of the clearest examples of a word associated with a conventional implicature:

If I say (smugly), “He is an Englishman; he is, therefore, brave,” I have certainly committed myself, by virtue of the meaning of my words, to its being the case that his being brave is a consequence of (follows from) his being an Englishman. But while I have said that he is an Englishman and said that he is brave . . . I do not want to say that my utterance of this sentence would be, strictly speaking, false should the consequence in question fail to hold. So some implicatures are conventional . . . (Grice 1975: 44–45)

Let us call the putative implication by a ‘therefore’-sentence (e.g. ‘He is an Englishman and he is, therefore, brave’) that a certain entailment holds (e.g. that his being brave follows from his being an Englishman) its target content. According to the conventional implicature analysis (henceforth CIA), the target content is conventionally associated with a ‘therefore’-sentence but neither it is part of that sentence’s core content – of what is said by it or of its ‘at issue’ content – nor is its truth required for the truth/falsity of the whole ‘therefore’-sentence (Bach 1999; Potts 2007).

Since Grice’s proposal, the implicature analysis of ‘therefore’ has been quite prevalent both in linguistics and in philosophy (Davis 2014; Neta 2013; Potts 2007).¹ Yet it faces some difficulties. The first worry is that, as also observed by Bach (1999: 330), the truth of a ‘therefore’-sentence might actually require its target content to be true. For example, consider the following weird-sounding sentences:

(2) a. ??Andrea has red hair and is, therefore, female.

b. ??13 is a number and is, therefore, odd.

CIA predicts that these ‘therefore’-sentences are true if their conjuncts are. But out of context, it is hard to hear either of these sentences as true. Moreover, in many ordinary contexts, they do not sound true at all: for example, (2-a) does not sound true in any context where the red-haired people under discussion come in both genders, even if Andrea is both red-haired and female.

red-haired and female. Nor does (2-b) sound true when both odd and even numbers are being discussed.

The proponent of CIA may insist that (2-a) and (2-b) are true but infelicitous because of the falsity of the relevant conventional implicatures. However, this proposal does not straightforwardly generalize from uses of ‘therefore’ as parenthetical (like the previous (1)) to uses of ‘therefore’ in discourses such as (3):

(3) Humans are animals; George is a human. Therefore, George is an animal. (*therefore*-argument)

In (3), ‘therefore’ appears to turn a list of sentences into an argument. And even more clearly than with the correctness of ‘therefore’-sentences (i.e. their truth), the correctness of an argument such as (3) (i.e. its soundness) does require its premisses to entail its conclusion.

In the light of these difficulties with CIA, one might be tempted to build the target content into the core (or at issue) content of ‘therefore’-sentences and ‘therefore’-arguments. On this analysis, a ‘therefore’-sentence of the form ‘$u$ and, therefore, $v$’ (and a ‘therefore’-argument of the form ‘$u$. Therefore, $v$’) would have the same content as a sentence of the form ‘$u$ and $v$ follows from $u$’. However, consider:

(4) a. Mark is an Englishman and from that it follows that he is brave.
    b. That is false.
(5) a. Mark is an Englishman and he is, therefore, brave.
    b. ??That is false.

As hinted by Grice (1975) in the passage quoted above, there seems to be a contrast in acceptability between directly challenging (4-a) with ‘That is false’ on the ground that the entailment from Mark’s being an Englishman to Mark’s being brave does not hold, and directly challenging (5-a) with ‘That is false’ on the same ground. The current proposal cannot straightforwardly account for this contrast.

Moreover, if the target content were part of the core truth conditions of ‘therefore’-sentences, it would have to be entailed by those sentences. However, the target content can still be inferred from ‘therefore’-sentences when they occur in wider linguistic contexts, such as in the antecedents of conditionals (6-a), in questions (6-b) (see also Neta 2013: 394–395) as well as under negation (6-c):

(6) a. If Mark is an Englishman and he is, therefore, brave, he will act as such. (antecedent of a conditional)
    b. Is Mark an Englishman and, therefore, brave? (question)
    c. It is not the case that Mark is an Englishman and he is, therefore, brave. (negation)
Because it can ‘project’ out of these embeddings, the target content of (5-a) behaves more like a presupposition than an entailment. After all, it is usually considered to be the mark that a sentence such as (7-a) presupposes, rather than entails, the truth of (7-b) that (7-a) still licenses an inference to (7-b) when embedded under negation, as in (7-c) (Beaver 2001; Karttunen 1973; Langendoen and Savin 1971):

(7) a. It is the knave that stole the tarts.
    b. Somebody stole the tarts.
    c. It is not the knave that stole the tarts.

An immediate advantage of classifying ‘therefore’ as a presupposition trigger is that a semantic conception of presuppositions (Karttunen 1973, 1974) can predict that sentences such as (2-a) and (2-b) do not need to be true. For on this view, the truth of a sentence requires the truth of its presuppositions.

Furthermore, the presupposition analysis (henceforth PA) provides a natural framework within which to predict and to account for the remarkable context-sensitivity of ‘therefore’-sentences. For example, consider:

(8) Mark is a progressive and he is, therefore, from the North.

Mark’s progressive political inclination by itself does not entail his geographical origins. And yet (8) can be both literally true and felicitous in a context, for example, where a sociological experiment is conducted with a group of subjects that includes Mark and where all the progressive subjects of the experiment turn out to come from the North.

PA can predict this pattern of context-sensitivity for it is commonly accepted that presuppositions are to be ‘satisfied’ relative to their so-called local contexts (Heim 1983; Karttunen 1974; Rothschild 2011; Schlenker 2009, 2010). Simplifying a bit, a local context can be thought of as the current set of assumptions in a conversation augmented with the contribution of the linguistic environment closest to the presupposition trigger. For instance, in the case of a conjunction $\varphi \& \psi$ like (8), the local context for its second conjunct $\psi$ is the current set of assumptions $c$ augmented (+) with $\varphi$. For example, a standard explanation for why (9) is infelicitous in any context where nobody has been assumed to go to school is that the presupposition triggered by ‘too’ is not satisfied by the local context for (9)’s second conjunct:

(9) ??Mary went to the bank and Mark went to school too.

Now, in the case of (8), the local context for ‘He is, therefore, from the North’ is the global set of assumptions $c$ augmented with the sentence ‘Mark

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2 Some theories of local context have been formulated within an explicitly dynamic framework (Heim 1983; Karttunen 1974; Rothschild 2011); others within an explicitly non-dynamic framework (Schlenker 2009, 2010). Those differences will not matter here.
is a progressive’. Let us model \( c \) as the set of possible worlds compatible with the set of assumptions described in this scenario and let \(<\text{Mark is a progressive}>\) be the set of possible worlds where Mark is a progressive. Finally, let \( c + \text{‘Mark is a progressive’} \) be the intersection of the possible worlds in \( c \) with those where Mark is a progressive (\( = c \cap <\text{Mark is a progressive}>\)). In every possible world in this intersection, it will be true that Mark is from the North. In this sense, the local context for ‘Mark is, therefore, from the North’ ‘entails’ that Mark is from the North. Hence, the hypothesis that ‘therefore’ is a presupposition trigger, together with the related notion of local context, can naturally explain why (8) can be felicitous and true in the context described even though Mark’s being a progressive by itself does not entail his being from the North.

Let us take stock. PA can account for the observed difference between ‘therefore’-sentences and ‘follow’-sentences because, just like CIA, it takes the target content of ‘therefore’-sentences not to be part of their core content. But PA fares better than CIA in other respects.\(^3\) It predicts that sentences such as (2-a) and (2-b) do not need to be true even if both of their conjuncts are; it provides a natural framework within which to understand both the projective behaviour and the context-sensitivity of ‘therefore’-sentences; finally, as will be clearer from the analysis to come, PA can offer a desirable unified treatment of the meaning of ‘therefore’ in both ‘therefore’-sentences and ‘therefore’-arguments, for it puts into the correctness conditions of both constructions that their target content be true.\(^4\)

To simplify the discussion a bit, I will take ‘therefore’-sentences and ‘therefore’-arguments to have the same syntax – that of the horizontal line in Argument Schema, taking a list of premisses and a conclusion into an argument:

1. **Argument schema**

\[
\varphi_1, \ldots, \varphi_n \quad \psi
\]

I uniformly refer to ‘therefore’-sentences and ‘therefore’-arguments as to “‘therefore’-discourses” and will indicate them both as follows:

(10) \( \Phi; \text{therefore}, \psi \).

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\(^3\) This is not to say that there are not ways of construing CIA on which it turns out to be compatible with PA. See, for example, Karttunen (1979) for a treatment of presuppositions as conventional implicatures.

\(^4\) As von Fintel (2004) teaches us, intuitions about sentences’s truth values alone are hardly conclusive evidence for the presence or absence of a presupposition. Accordingly, my emphasis here is on the projective behaviour of ‘therefore’-sentences and on the naturalness with which the machinery of local contexts, standardly employed in the treatment of presuppositions, can help explain the context-sensitivity of ‘therefore’-sentences.
Φ is the ordered set of premisses \( \varphi_1, \ldots, \varphi_n \) and \( \psi \) is the conclusion. As before, a context is the set of possible worlds where every assumption shared in a conversation holds true. A context \( c \) entails a sentence \( \varphi (= c \models \varphi) \) just in case \( \varphi \) is true at every world in \( c \). A context \( c \) augmented with a sentence \( \varphi (= c + \varphi) \) is the intersection of \( c \) with the set of worlds where \( \varphi \) is true \((= c \cap <\varphi>)\). A sentence \( \varphi \) is true at \( c \) just in case \( c \cap <\varphi> = c \). Finally, a context \( c \) augmented with an ordered set \( \Phi \) of premisses \((= c + \Phi)\) is the successive intersection of \( c \) with the sets of worlds where each of the premisses in \( \Phi \) is true \((= c \cap <\varphi_1> \cap \ldots \cap <\varphi_n>)\).\(^5\)

So much for the set-up. In order to reach an analysis for ‘therefore’, I propose we first look at the following dynamic entry for epistemic ‘must’:\(^6\)

**Dynamic Must**

\[
\text{Dynamic Must } c[\text{must } \varphi] = \begin{cases} c & \text{if } c \models \varphi \\ \emptyset & \text{if } c \not\models \varphi \end{cases}
\]

**Dynamic Must** assigns ‘must-\( \varphi \)’ a function which, given a context \( c \), returns \( c \) itself, if \( c \) entails \( \varphi \). Else, it returns the absurd context – i.e. the empty set. Dynamic ‘must’ is said to be a *test*, for the sentences where it occurs do not alter the context by eliminating possible worlds from it. Rather, they ‘test’ whether the context satisfies a certain constraint – that of entailing the sentence which ‘must’ embeds.

My proposal is that the semantic entry for ‘therefore’ resembles **Dynamic Must** in the kind of constraint that ‘therefore’ imposes on context. In particular, the presupposition associated with ‘therefore’ imposes that the context augmented with the premisses entail the conclusion. This said, there are some crucial differences between ‘therefore’ and ‘must’ that an analysis should capture.

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5 At logical form, we may need to appeal to an indexing mechanism to disambiguate certain logical forms:

(11) \( \varphi_{1,y}, \ldots, \varphi_{n,y}; \text{therefore}_y, \psi \).

For example in (12-a), ‘therefore’ can either take only ‘John is socially liberal’ as in (12-b), or both ‘John is economically conservative’ and ‘John is socially liberal’, as in (12-c):

(12) a. If John is economically conservative and socially liberal, and, therefore, from the North, he’ll vote for Clinton.

b. If John is economically conservative and socially liberal, and therefore, from the North, he’ll vote for Clinton.

c. If John is economically conservative, and socially liberal, and therefore, from the North, he’ll vote for Clinton.

(Thanks to Philippe Schlenker for discussion). I will omit this complication in the text.

The first is that, in contrast with ‘therefore’, the constraint that ‘must-ϕ’ imposes on context – i.e. that the context entail ϕ – does not project out of embeddings. To illustrate, the following sentences are not unacceptable if it is not known that John is in Holland:

(13) a. If John must be in Holland, he cannot be the thief. (antecedent of a conditional)
    b. It is not the case that John must be in Holland. (negation)

Hence, it would be a mistake to classify ‘must’ as a presupposition trigger. This suggests that the mechanism by which ‘must’ and ‘therefore’ test the context must be different in the two cases. In accordance with a rather standard dynamic treatment of presuppositions (Beaver 2001), we can capture this difference in the semantics by assigning ‘therefore’-discourses a function from contexts to contexts that is simply undefined (rather than returning the empty set) when the presupposition is not satisfied:

\[
\text{Dynamic Therefore } c[\Phi; \text{ therefore, } \psi] = \begin{cases} 
  c & \text{if } c + \Phi \models \psi \\
  \text{undefined} & \text{if } c + \Phi \not\models \psi
\end{cases}
\]

The difference between these two ‘fail’ values – undefinedness versus the empty set – is important. A semantic entry that returns the empty set receives a non-fail value – that of a tautology – under negation. But in order to account for the projection of the presupposition from a ‘therefore’-sentence to its negation, the negation of a ‘therefore’-sentence must also receive a fail value if the ‘therefore’-sentence does. Choosing ‘undefined’, rather than the empty set, gives the desired result here – i.e. that the negation of a ‘therefore’-sentence will also be undefined. Because of this, Dynamic Therefore enables us to account for the projection of the presupposition from a ‘therefore’-sentence to its negation.

As it stands, however, Dynamic Therefore only affords modelling one dimension under which an argument such as (3) may fail – i.e. its invalidity. But an argument can fail in another respect too – by being unsound though valid – and it would be desirable to capture this further dimension of failure. Moreover, instead of truth- (or soundness-) conditions, Dynamic Therefore assigns a ‘therefore’-discourse a dynamic semantic value – a function from contexts to contexts. But some may find it desirable to be able to assign truth conditions to ‘therefore’-sentences such as (1). Hence, here is what I propose:

\[
\text{First Pass } = \begin{cases} 
  \text{If } c + \Phi \models \psi, \\
  \{ \Phi; \text{ therefore, } \psi^{-}\text{ is true/sound in } c \} & \text{if } c + \Phi + \psi = c; \\
  \{ \Phi; \text{ therefore, } \psi^{-}\text{ is false/unsound in } c \} & \text{if } c + \Phi + \psi \neq c; \\
  \text{Else, } \{ \Phi; \text{ therefore, } \psi^{-}\text{ is undefined in } c. \}
\end{cases}
\]
According to First Pass, the discourse ‘Φ; therefore, ψ’ is only defined in c if adding the premisses in Φ to c results in a context that entails ψ. A ‘therefore’-argument is sound in c if, in addition, premisses in Φ are all true in c (and unsound otherwise); and a ‘therefore’-sentence is true in c if, in addition, the premisses in Φ are all true in c (and false otherwise).

First Pass succeeds at modelling both dimensions of failure for ‘therefore’-discourses – invalidity or unsoundness for ‘therefore’-arguments and undefinedness or falsity for ‘therefore’-sentences – while providing a desirable unified analysis of ‘therefore’-arguments and ‘therefore’-sentences. Moreover, First Pass registers the difference between ‘must’ and ‘therefore’, while vindicating the idea that ‘therefore’ is a test in a broader sense – one that includes any expression whose semantic function is to impose certain constraints on context in order for the sentences where the expression occurs not to receive a ‘fail’ value in that context. While in the case of dynamic ‘must’, the relevant fail value is the empty set, in the case of ‘therefore’, the fail value is undefinedness.

There is, however, a further difference between ‘must’-sentences (and other expressions, such as ‘then’) and ‘therefore’-discourses which First Pass cannot predict. Consider:

(14) a. ??If Mark is an Englishman, therefore, he is brave.
   b. If Mark is an Englishman, he must be brave.

Whereas (14-b) is fine, (14-a) is slightly off. A similar contrast is also observable between (15-a) and (15-b):

(15) a. ?Suppose Mark is an Englishman. Therefore, he is brave.
   b. Suppose Mark is an Englishman. Then, he is brave.

How can we refine our semantic entry in order to capture this contrast?

When modelling the syntax of ‘therefore’-discourses, I have equated ‘therefore’ to the horizontal line in Argument Schema (see also Rumfitt 2015: 53 and Pavese 2016: 208). But the slight infelicity of (14-a) and (15-a) might indicate that ‘therefore’ is more similar to the square ‘œ’ that ends proofs than to the horizontal line:

Proof of Theorem . . . ∎

‘œ’ differs from the horizontal line in that it requires its premisses having been discharged and not being conditionally dependent on other premisses. Similarly, ‘therefore’ seems to require that its premisses be categorical.

Here is, then, the analysis. Recall that the local context for ‘Therefore, ψ’ in a ‘therefore’-discourse is a global context augmented with the ‘therefore’-discourse’s premisses. Now, premisses come with a mood and can be either categorical or hypothetical. For example, the premiss ‘He is an Englishman’ in (1) is categorical, while the premisses in (14-a)-(14-b) and in (15-a)-(15b) are
hypothetical. When the premisses \( \varphi_1, \ldots, \varphi_n \) are categorical, let \( c + \varphi_1, \ldots, \varphi_n \) be identical to \( c \uparrow \varphi_1, \ldots, \varphi_n \), where \( \uparrow \) indicates a ‘categorical update’ – an operation on the context \( c \) that amounts to intersecting \( c \) with the sets of worlds where the premisses are true \( (= c \cap <\varphi_1> \cap \ldots \cap <\varphi_n>) \). By contrast, when the premisses are of the hypothetical form ‘IF/SUPPOSE \( \varphi_1, \ldots, \varphi_n \)’ – such as ‘If he is an Englishman’ in (14-a) and ‘Suppose he is an Englishman’ in (15-b) – let \( c + \text{IF/SUPPOSE} \varphi_1, \ldots, \varphi_n \) be identical to \( c \downarrow \varphi_1, \ldots, \varphi_n \), where \( \downarrow \) indicates the hypothetical update. An hypothetical update is an operation on \( c \) that results in a different kind of local context – a hypothetical one – modelling a set of assumptions entertained and supposed but not necessarily accepted.7

Thus, we have that, while \( c + \varphi_1, \ldots, \varphi_n \) \( (= c \uparrow \varphi_1, \ldots, \varphi_n) \) is a categorical local context \( (C^+) \), \( c + \text{IF/SUPPOSE} \varphi_1, \ldots, \varphi_n \) \( (= c \downarrow \varphi_1, \ldots, \varphi_n) \) is a hypothetical local context \( (C^-) \). Let \( \Phi^* \) indicate the premisses together with their conditional or categorical mood (so that either \( \Phi^* = \Phi = \varphi_1, \ldots, \varphi_n \) or \( \Phi^* = \text{IF/SUPPOSE} \Phi = \text{IF/SUPPOSE} \varphi_1, \ldots, \varphi_n \)). Here is, then, the final entry for ‘therefore’-discourses:

\[
\text{Final} = \begin{cases} 
\text{If } c + \Phi^* \in C^+ \text{ and } c + \Phi^* \models \psi, \\
\text{\} \Phi^*; \text{ therefore, } \psi \text{ is true/sound in } c \text{ if } c + \Phi + \psi = c; \\
\text{or } \Phi^*; \text{ therefore, } \psi \text{ is false/unsound in } c \text{ if } c + \Phi + \psi \neq c; \\
\text{Else, } \Phi^*; \text{ therefore, } \psi \text{ is undefined in } c.
\end{cases}
\]

According to \( \text{Final} \), the discourse ‘\( \Phi^*; \text{ therefore, } \psi \)’ is defined at a context \( c \) only if both \( c + \Phi^* \) entails \( \psi \) and it is a categorical context. By contrast, the lexical entry for ‘must’-sentences imposes no requirement on their local context that it be categorical. That explains the contrast in acceptability between (14-a) and (14-b). \( \text{Final} \) also has the resources to explain why, in contrast with (14-a), (16) is perfectly acceptable:

(16) If Mark is an Englishman and he is, therefore, brave, he will act as such.

In (16), ‘He is, therefore, brave’ is not in the scope of a conditional antecedent. So, its premiss is not ‘If Mark is an Englishman’ but, rather, the categorical ‘Mark is an Englishman’. Thus, in (16), the local context for ‘He is, therefore, brave’ is categorical, not hypothetical. Hence, whereas ‘therefore’s requirement that the \( \text{local} \) context be categorical is not met in the case of (14-a), it is met in the case of (16). (This example interestingly shows that a proposition (in this case, that Mark is an Englishman) can be

7 The distinction between categorical contexts and hypothetical contexts is independently motivated by a satisfactory treatment of indicative conditionals (Starr 2014; Willer 2010). On different kinds of updates, see Murray 2014.
accepted locally (by the local categorical context) without having to be accepted by the conversation (or by the non-local context).\textsuperscript{8)

In this essay, I argued for an analysis of ‘therefore’ as presupposition trigger against the more standard CIA originally put forward by Grice (1975).\textsuperscript{9} I proposed that we model the relevant presupposition as ‘testing’ the context in a way that is similar to how dynamic ‘must’ tests the context. But whereas PA is plausible for ‘therefore’, ‘must’ is not plausibly a presupposition trigger. Moreover, while ‘must’ can operate on both categorical contexts and hypothetical contexts, ‘therefore’ requires that its local contexts be categorical. On these bases, I suggested that tests come in different types, depending both on the mechanisms whereby they test the context (whether through a presupposition or through their core content, for example) and on the kind of contexts they can operate on.\textsuperscript{10}

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\textsuperscript{8} A more detailed discussion of this sort of interaction between local and categorical contexts must await further work.

\textsuperscript{9} On my analysis, ‘therefore’ discourses do not alter the context by eliminating the worlds where the premisses do not entail the conclusion. Rather, they test that the context satisfies certain constraints. In this sense, I am advocating a form of non-factualism about ‘therefore’ similar in spirit to Yalcin’s (2007) non-factualism about epistemic modals. I develop this idea and its consequences in more detail in Pavese 2016 and in other work in progress.

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References


Abstract
I argue for an analysis of ‘therefore’ as presupposition trigger against the more standard conventional implicature story originally put forward by Grice (1975). I propose that we model the relevant presupposition as ‘testing’ the context in a way that is similar to how, according to some dynamic treatments of epistemic ‘must’, ‘must’ tests the context. But whereas the presupposition analysis is plausible for ‘therefore’, ‘must’ is not plausibly a presupposition trigger. Moreover, whereas ‘must’ can naturally occur under a supposition, the same is not true for ‘therefore’. In the light of these differences, I suggest we distinguish between different sorts of tests on the basis of the mechanisms whereby these expressions test the context (whether through a presupposition or through their core content) and on the basis of whether they can operate only on categorical contexts or on both categorical and hypothetical contexts.