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Time's Arrow and Irreversibility in Time-Asymmetric Quantum Mechanics

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The aim of this paper is to analyze time-asymmetric quantum mechanics with respect to the problems of irreversibility and of time's arrow. We begin with arguing that both problems are conceptually different. Then, we show that, contrary to a common opinion, the theory's ability to describe irreversible quantum processes is not a consequence of the semigroup evolution laws expressing the non-time-reversal invariance of the theory. Finally, we argue that time-asymmetric quantum mechanics, either in Prigogine's version or in Bohm's version, does not solve the problem of the arrow of time because it does not supply a substantial and theoretically founded criterion for distinguishing between the two directions of time.

1. Introduction

The problems of irreversibility and of time's arrow were born with the discussions of the founding fathers of statistical mechanics about the mechanical meaning of the second law of thermodynamics. Since those days, much ink has been spilled on these subjects. Nevertheless, the debates have continued up to the present without leading to an overall agreement. In the second half of the century, chiefly since the 1960s, the works on quantum irreversibility have contributed to these discussions with the proposal of the so-called 'time-asymmetric quantum mechanics' (TAQM).

We shall subsume under the label 'time-asymmetric quantum mechanics school' (TAQM school) the members and the works of two groups led by Arno Bohm at Austin and Ilya Prigogine at Brussels. Since the very beginning of his scientific life, Prigogine was interested in time's arrow. For him, intrinsic irreversibility would establish an objective difference between past and future; then, he directed his efforts to introduce

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irreversibility in fundamental physics. Bohm, in turn, focused his interest on quantum scattering, where decaying processes are in need of an adequate description. For this reason, his main aim was to obtain a formalism capable of modelling those irreversible phenomena. The scientific contact between the two groups extended their concerns and formal resources: Bohm's group incorporated the interest in the quantum arrow of time, and Prigogine's group adopted the rigged Hilbert space formalism previously used in scattering processes. As a result of the intellectual evolution of both groups, at present it can be said that their main technical efforts have been directed to the formulation of a quantum mechanics capable of accounting for irreversible quantum phenomena. Besides this general aim, the two groups agree in the use of rigged Hilbert spaces for addressing the issue of irreversibility in quantum mechanics; according to their view, this formalism turns standard quantum mechanics into a 'time-asymmetric' theory where irreversible quantum descriptions can be obtained. Both groups also claim, although on a different basis, to have supplied a conceptually adequate theoretical account of the arrow of time in quantum mechanics.

The aim of this paper is to analyze the main claims of the TAQM school about the problems of irreversibility and of time's arrow. We shall begin with a precise elucidation of the central concepts on which those claims are based, in particular, time-reversal invariance, reversibility and time's arrow. This task will allow us to argue for the difference between both problems. On this conceptual basis, we shall show that, contrary to a common opinion, the ability of TAQM to describe irreversible quantum processes is not a consequence of the semigroup evolution laws of the theory. With respect to time's arrow in quantum mechanics, we shall argue that the proposal of the TAQM school, either in Prigogine's version or in Bohm's version, does not offer a conceptually adequate answer to the problem: it does not supply a substantial and theoretically grounded criterion for distinguishing between the two formal structures, one the temporal mirror image of the other, arising from the original time-reversal invariant quantum theory.

2. Disentangling Concepts

When the problems of irreversibility and of time's arrow are addressed, the main obstacle to be faced is conceptual confusion: the two problems are usually identified, as if irreversibility were the clue for understanding the origin and the nature of the arrow of time.

The identification or, at least, the close link between irreversibility and time's arrow is continuously present in the works of the TAQM school. For instance, Antoniou and Prigogine (1993, 443) conceive irreversibility as the fundamental difference between the two directions of time, and even 'as the emergence of a privileged direction of time'. After introducing the quantum mechanical arrow of time, Bohm claims that 'irreversibility … is the asymmetry of the time evolution based on this arrow' (Bohm and Harsh man 1998, 185); therefore, 'the exponential decay of resonances is just one manifestation of a quantum mechanical arrow of time' (Bohm et al. 1995, 2595). The close relationship between the two concepts is also emphasized by the authors who analyze the TAQM school's work: in one of his several papers devoted to study such a work, Bishop points out that 'intrinsic irreversibility is of prime interest to Bohm and his collaborators, as well as to Prigogine's Brussels–Austin group, because these types of irreversible processes are related to arrows of time' (Bishop 2004a, 1678).

In several previous papers. (Castagnino, Lara and Lombardi 2003a, 2003b Castagnino, Lombardi and Lara 2003; Castagnino and Lombardi 2004, 2005a, 2005b) we have addressed the problems of irreversibility and of time's arrow from a philosophically grounded point of view: on the basis of an elucidation of the main concepts involved in the debates, we have argued that these two problems are conceptually different and that their identification is the source of many confusions. In this section, we shall summarize the position developed in those previous works; this review will provide us with the conceptual basis for the analysis of the TAQM school's claims on the subject.

2.1. Time-Reversal Invariance and Reversibility

The two central concepts involved in the discussions about the problem of irreversibility are time-reversal invariance and reversibility.

> **Definition 1:** A dynamical equation (law) is *time-reversal invariant* if it is invariant under the application of the time-reversal operator **T**.

The operator **T** performs the transformation $t \to -t$ and reverses certain magnitudes which depend on the particular theory considered. Nevertheless, the central idea is that **T** must reverse all the dynamical variables whose definitions in function of t are non-invariant under the transformation $t \to -t$. For instance, in classical particle mechanics, the action of **T** reverses the momenta but not the positions of the particles: $\mathbf{Tp} = -\mathbf{p}$ and $\mathbf{Tq} = \mathbf{q}$. In electromagnetism, **T** leaves the electric fields unchanged and reverses the velocities of the charges and the magnetic fields, since such fields change their direction in accordance with the velocities of the charges: $\mathbf{Tv} = -\mathbf{v}$, $\mathbf{TB} = -\mathbf{B}$ and $\mathbf{TE} = \mathbf{E}$ (for details, cf. Earman 2002).

On the other hand, the concept of irreversibility has received many definitions in the philosophical literature. In general, it is said that a process is reversible if the temporal succession of the states $e_1, e_2, \ldots, e_n$ can occur in the opposite order, and irreversible otherwise. Of course, in this characterization much depends on how the 'can occur' is interpreted. When the opposite succession is precluded by the fact that certain initial conditions never occur, irreversibility is considered as a *de facto* property. In turn, irreversibility is nomological when the opposite succession is excluded by the dynamical law that rules the process.

With this broad characterization, there are many ways in which irreversibility is manifested in nature. However, in TAQM, the interest is directed towards the dynamical description of *intrinsic* irreversibility, that is, towards the irreversible behavior generated by the dynamics of a *closed* quantum system. On the other hand, the irreversible processes studied by this theory are decaying processes, such as the decay of excited states of molecules and nuclei, the weak decay of elementary particles or certain resonances such as those of the neutral Kaon system. In these cases, the time evolution tends to a final equilibrium state from which the system cannot escape: the irreversibility of the process is due precisely to the fact that the evolution leaving the equilibrium state is not possible. Since the aim of the TAQM is to find the adequate dynamical equations to describe this kind of irreversible behavior, in this context the concept of irreversibility can be elucidated in terms of the notion of *attractor* with no loss of generality.

An attractor is the subset of the phase space towards which a set of evolutions tends for $t \to \pm\infty$. We can extend this definition by considering a generalized concept of attractor as a subset of the set of the possible states of a system towards which a set of evolutions tends for $t \to \pm\infty$; this concept can be applied not only to phase spaces but also to any kind of sets of states. Examples of generalized attractors are the attractors of classical dynamical systems (fixed point, limit cycle, fractal, etc) and any classical or quantum equilibrium state. With this characterization, the concept of reversibility can be defined as:

> **Definition 2:** A solution (evolution) $e(t)$ of a dynamical equation is *reversible* if it has no generalized attractors, for any representation of $e(t)$

When the time-dependent state $e(t)$ can be represented as an n-uple of dynamical variables in phase space, $e(t) = (v_1(t), \ldots, v_n(t))$, reversibility requires that, for any dynamical variable $v_i(t)$, the limit $\lim_{t\to\pm\infty} v_i(t)$ does not exist. In this case, it can be said that the evolution $e(t)$ is reversible if it has no attractors in phase space.

Independently of the details of these definitions (for further discussions, cf. Albert 2000, Arntzenius 2004), it is quite clear that the concepts of time-reversal invariance and irreversibility are different to the extent that they apply to different mathematical (physical) entities:[1] whereas time-reversal invariance is a property of dynamical equations and, *a fortiori*, of the sets of its solutions, reversibility is a property of a single solution of a dynamical equation. Furthermore, both properties are not even correlated; in fact, they can be combined with each other in the four possible cases (for examples of the four cases, cf. Castagnino, Lara and Lombardi 2003a).

When the concepts of time-reversal invariance and reversibility have been carefully distinguished, *the problem of irreversibility* can be clearly stated: *how to explain irreversible evolutions in terms of time-reversal invariant laws.* Let us note that this characterization of the problem includes the usual definition in terms of thermodynamic concepts, since entropy increase is a feature of irreversible thermodynamic evolutions that should be explained in terms of the time-reversal invariant laws of mechanics. However, our approach is more general than the usual one since it also includes other interesting cases of irreversibility, like those studied in TAQM.

This characterization of the problem of irreversibility shows that, in principle, there is no conceptual obstacle to its solution: nothing prevents a time-reversal invariant equation from having irreversible solutions. Of course, although the conceptual answer is simple, a great deal of theoretical work is needed to obtain irreversible evolutions from an underlying time-reversal invariant dynamics. But the point to stress here is that the question about the arrow of time does not need to be invoked for addressing the problem of irreversibility. In fact, when we talk about entropy-increasing processes, we are presupposing an entropy increase *towards the future*; or when we consider a process going from non-equilibrium to equilibrium, we implicitly locate equilibrium *in the future*. In general, any evolution that tends to an attractor is conceived as approaching it towards the future: the distinction between past and future is *usually presupposed* in the traditional treatments of the problem of irreversibility. This is not a shortcoming of those treatments, since their aim is to explain irreversibility and not to seek a physical distinction between the two directions of time. However, such a distinction becomes the central point when the arrow of time is the question at issue.

2.2. The problem of the Arrow of Time

The problem of the arrow of time owes its origin to the intuitive asymmetry between past and future. We experience the time order of the world as 'directed': if two events are not simultaneous, one of them is earlier than the other. Moreover, we view our access to past and future quite differently: we remember the past and predict the future. The ultimate metaphysical nature of time has been one of the traditional interests of philosophy since its birth. There seems to be something essentially evasive in our experience of time and its 'flow' from past to future through the present. Here, we shall not discuss the questions related with the time-asymmetry of our experience of time. As in the case of irreversibility, we shall address The Problem of the Arrow of Time within the limits of physics. In this context, the problem arises when we seek a *physical correlate* of the intuitive asymmetry between past and future: do physical theories pick out a preferred direction of time?

The main difficulty to be encountered in answering this question depends on our anthropocentric perspective: the difference between past and future is so deeply rooted in our language and our thoughts that it is very difficult to shake off these temporally asymmetric assumptions. In fact, traditional discussions around the problem of the arrow of time are usually subsumed under the label 'the problem of the direction of time', as if we could find an exclusively physical criterion for singling out *the privileged* direction of time, identified with what we call 'the future'. However, there is nothing in physical evolution laws that distinguishes, in a non-arbitrary way, between past and future as we conceive them in our ordinary language. It might be objected that theoretical physics implicitly assumes this distinction with the use of temporally asymmetric expressions, like 'future light cone', 'initial conditions', 'increasing time', and so on. However, this is not the case, and the reason relies on the distinction between *conventional* and *substantial*.

> **Definition 3:** Two objects are *formally identical* when there is a permutation that interchanges those two objects and nothing else in the system to which they belong, and preserves all the system's structural properties and relations.

Examples of formally identical objects are the two semicones of a light cone and the two spin senses.

> **Definition 4:** We shall say that we establish a *conventional* difference between two objects when we call two formally identical objects with two different names.

This is the case when we assign different signs to the two spin senses, or different names to the two light semicones.

> **Definition 5:** We shall say that the difference between two objects is *substantial* when we assign different names to objects that are not formally identical (Penrose 1979, Sachs 1987). In this case, although the particular names are conventional, the difference is substantial.

In the case of fundamental physics, the labels 'past' and 'future' are used in a conventional way. Therefore, the problem of the arrow of time cannot be posed in terms of singling out the future direction of time: it becomes the problem of finding a *substantial difference between the two temporal directions*. Therefore, we cannot project our independent intuitions about past and future or our technological abilities for solving the problem without begging the question. When we want to address the problem of the arrow of time from a perspective purged of our temporal intuitions, we must avoid the conclusions derived from subtly presupposing time-asymmetric notions. As Huw Price (1996) claims, it is necessary to stand at a point outside of time and to adopt the '*view from nowhen*': this atemporal standpoint prevents us from using temporally asymmetric expressions in a non-conventional way.[2]

But then, what does 'the arrow of time' mean when we accept this constraint? Of course, the traditional expression coined by Eddington has only a metaphorical sense. We recognize the difference between the head and the tail of an arrow on the basis of its geometrical properties; therefore, we can substantially distinguish between both directions, head-to-tail and tail-to-head, independently of our particular perspective. Analogously, *the problem of the arrow of time* should be conceived in terms of *the possibility of establishing a substantial distinction between the two directions of time exclusively by means of arguments based on theoretical physics.*

2.3. Time-Symmetric Twins

When the difference between the problems of irreversibility and of time's arrow has been accepted, a new question arises: why is time-reversal invariance an obstacle to solve the problem of the arrow of time?

Already in 1912, Ehrenfest and Ehrenfest (1959) noted that when entropy is defined in statistical terms, if the entropy of a closed system increases towards the future, such an increase is matched by a similar increase in the past of the system. This old discussion can be generalized to any kind of evolution arising from time-reversal invariant laws: if e_t is a solution of a time-reversal invariant law L, then $\mathbf{T}e_t$ is also a solution of L. We have called these two solutions '*time-symmetric twins*' (cf. Castagnino, Lara and Lombardi 2003a; Castagnino and Lombardi 2004): they are twins because, without presupposing a privileged direction of time, they are only conventionally different; they are time-symmetric because one is the temporal mirror image of the other. The traditional example of time-symmetric twins is given by electromagnetism, where dynamical equations always have advanced and retarded solutions, respectively related with incoming and outgoing states in scattering as described by Lax-Phillips's (1979) theory. With this terminology, we can say that a time-reversal invariant theory always produce time-symmetric twins: the obstacle to solve the problem of the arrow of time relies on the fact that, in the context of the theory, the twins are only conventionally different.

The traditional arguments for discarding one of the twins and retaining the other in general invoke time-asymmetric notions which are not justified in the context of the theory. For instance, the retarded nature of radiation is usually explained by means of *de facto* arguments referred to initial conditions: advanced solutions correspond to converging waves that require a miraculous cooperative emitting behavior of distant regions of space at the temporal origin of the process. A different but related argument is put forward by those who appeal to the impossibility (or high difficulty) of preparing time-reversed states in laboratory experiments such as experiments of scattering. It seems quite clear that this kind of argument, not based on theoretical considerations, is not legitimate in the discussions about time's arrow:[3] they presuppose the arrow by introducing the difference between the two directions of time from the very beginning. In other words, they violate the 'nowhen' requirement of adopting an atemporal perspective purged of temporal intuitions like those related with the asymmetry between past and future or between initial and final conditions. Therefore, from an atemporal standpoint, the challenge consists in supplying a non-conventional theoretical criterion for choosing one of the time-symmetric twins as the physically meaningful one: such a criterion will establish a substantial difference between the two members of the pair and, *a fortiori*, between the two directions of time.

3. Rigged Hilbert Space Formalism in Time-Asymmetric Quantum Mechanics

Since the proposal of the TAQM school is based in the use of rigged Hilbert spaces (RHSs), in this section we shall review the features of this formalism that will be relevant to our further discussions.

A RHS (Gel'fand and Vilenkin 1964) is a triplet of spaces:

$$\Phi \subset \mathcal{H} \subset \Phi^{\times}, \tag{1}$$

where: (i) $\mathcal{H}$ is an infinite-dimensional separable Hilbert space, (ii) Φ is a topological vector space, dense in $\mathcal{H}$,[4] and (iii) $\Phi^{\times}$ is the antidual space of Φ, and its elements are continuous and antilinear functionals $F : \Phi \to \mathbb{C}$, whose action on $\phi \in \Phi$ is usually expressed as $\langle \phi | F \rangle$ (Dirac's notation). Under general assumptions, any operator A on $\mathcal{H}$ can be extended into the antidual $\Phi^{\times}$ as $A^{\times}$ by the *duality formula*:

$$\langle A^{\dagger}\phi \,|\, F \rangle = \langle \phi \,|\, A^{\times} F \rangle \qquad \forall \phi \in \Phi,\ \forall F \in \Phi^{\times}, \tag{2}$$

where $A^{\dagger}$ is the adjoint of A, and $A^{\times}$ is a linear and continuous operator on $\Phi^{\times}$.[5]

Different realizations of RHSs have been used in physics for distinct purposes. For instance, the Schwartz space S has been adopted to give a rigorous mathematical foundation to Dirac's formalism. In TAQM, Bohm and Gadella (1989) introduced the following structure:

$$S \cap \mathcal{H}^2_{\pm}\big|_{\mathbb{R}^+} \subset L^2(\mathbb{R}^+) \subset \left(S \cap \mathcal{H}^2_{\pm}\big|_{\mathbb{R}^+}\right)^{\times}, \tag{3}$$

where $\mathcal{H}^2_+$ $(\mathcal{H}^2_-)$ is the space of the Hardy functions on the upper (lower) half-plane, S is the Schwartz space, and $\big|_{\mathbb{R}^+}$ indicates the restriction of the functions of $S \cap \mathcal{H}^2_\pm$ to $\mathbb{R}^+$. The vector states $\phi^\pm \in \Phi_\pm$ correspond to wave functions whose energy representation, $\phi^\pm(\omega) = \langle\omega|\phi^\pm\rangle$, belong to the space $S \cap \mathcal{H}^2_\pm\big|_{\mathbb{R}^+}$

As a consequence of the use of Hardy functions in this realization, the original group of evolution operators of standard quantum mechanics, U_t $(t \in \mathbb{R})$, is split into two semigroups $U_t^+ = e^{-iH_+t}$ and $U_t^- = e^{-iH_-t}$, where the semigroup generators $H_\pm$ are the restrictions of the self-adjoint operator H to the subspaces $\Phi_\pm$. Then, the time evolutions in the antiduals spaces $\mathbf{\Phi}^\times_\pm$ are given by the duality formula (2):

$$\langle U_{-t}^+\phi^+ | F^+\rangle = \langle\phi^+ | U_t^{+\times}F^+\rangle \quad \forall\phi^+ \in \mathbf{\Phi}_+, \forall F^+ \in \mathbf{\Phi}_+^\times, \forall t \geq 0 \tag{4}$$

$$\langle U_{-t}^-\phi^- | F^-\rangle = \langle\phi^- | U_t^{-\times}F^-\rangle \quad \forall\phi^- \in \mathbf{\Phi}_-, \forall F^- \in \mathbf{\Phi}_-^\times, \forall t \leq 0, \tag{5}$$

where $U_t^{+\times} = e^{-iH_+^\times t}$ and $U_t^{-\times} = e^{-iH_-^\times t}$ are operators defined on $\mathbf{\Phi}_+^\times$ and $\mathbf{\Phi}_-^\times$ respectively, and $H_\pm^\times$ are the extensions of the self-adjoint operator H to the subspaces $\mathbf{\Phi}_\pm^\times$ (Bohm and Gadella 1989, Bohm and Scurek 2000, Bohm, Loewe and van de Ven 2003).

In addition to the states $\phi^\pm \in \Phi_\pm$ with smooth wave functions $\phi^\pm(\omega)$, this realization of the RHS formalism introduces new generalized vectors, that is, functionals on the spaces $\Phi_\pm$. Loosely speaking, in a RHS, the smaller the space Φ is, the bigger the space $\Phi^\times$ is. In this particular realization, the spaces $\Phi_\pm$ are restricted enough to allow their antiduals $\mathbf{\Phi}_\pm^\times$ to contain not only Dirac kets, but also more general kets. In fact, besides eigenkets with real eigenvalues, the spaces $\mathbf{\Phi}_\pm^\times$ may also contain eigenvectors of the Hamiltonian having complex eigenvalues. For instance, there may exist a '*decaying Gamow vector*' $\mathbf{\Psi}^D \in \mathbf{\Phi}_+^\times$ and a '*growing Gamow vector*' $\mathbf{\Psi}^G \in \mathbf{\Phi}_-^\times$, such that they are eigenvectors of $H_+^\times$ and $H_-^\times$ with complex eigenvalues $z_R = \omega_R - i\frac{\Gamma}{2}$ and $z_R^* = w_R + i\frac{\Gamma}{2}$, respectively, with $\Gamma > 0$ (Bohm and Gadella 1989, Bohm, Loewe and van de Ven 2003):

$$H_+^\times\mathbf{\Psi}^D = z_R\mathbf{\Psi}^D = (w_R - i\frac{\Gamma}{2})\mathbf{\Psi}^D \tag{6}$$

$$H_-^\times\mathbf{\Psi}^G = z_R^*\mathbf{\Psi}^G = (w_R + i\frac{\Gamma}{2})\mathbf{\Psi}^G. \tag{7}$$

The Gamow vectors are related with resonances, which are usually described by means of the scattering operator S in the energy representation, $S(\omega)$:[6] the analytical continuation of $S(\omega)$ in the upper and the lower half-planes of the complex energy plane possesses at least a pair of complex conjugate poles z_R, and z_R^*, which turn out to be the complex eigenvalues of the Hamiltonian (Gadella 1997).[7] The imaginary part of these eigenvalues is precisely what leads to exponentially growing and decaying evolutions. In fact, since the Gamow vectors belong to the antidual spaces $\mathbf{\Phi}_\pm^\times$, their time evolution has to be computed by means of the duality formula (2):

$$\langle e^{iH_+t}\phi^+ \mid \Psi^D \rangle = \langle \phi^+ \mid e^{-iH_+^{\times}t}\Psi^D \rangle \quad \forall \phi^+ \in \Phi_+, \forall t \geq 0 \tag{8}$$

$$\langle e^{iH_-t}\phi^- \mid \Psi^G \rangle = \langle \phi^- \mid e^{-iH_-^{\times}t}\Psi^G \rangle \quad \forall \phi^- \in \Phi_-, \forall t \leq 0. \tag{9}$$

Therefore, for $\forall t \geq 0$:

$$\langle \phi^+ \mid e^{-iH_+^{\times}t}\Psi^D \rangle = \langle \phi^+ \mid \Psi^D \rangle e^{-i(w_R - i\frac{\Gamma}{2})t} = \langle \phi^+ \mid \Psi^D \rangle \, e^{-iw_Rt} e^{-\frac{\Gamma}{2}t}. \tag{10}$$

This expression represents an exponentially decaying process with lifetime $\tau = \frac{2}{\Gamma}$. This means that Ψ^D describes an irreversible evolution that tends to 0 for $t \to \infty$ Analogously, Ψ^G describes an irreversible evolution that tends to 0 for $t \to -\infty$ (for a recent review, cf. Civitarese and Gadella 2004).

4. Irreversibility in Time-Asymmetric Quantum Mechanics

As is well known, in standard quantum mechanics the time evolution of a state vector belonging to the Hilbert space $\mathcal{H}$ is given by a unitary evolution governed by the Schrödinger equation. These evolutions are *always reversible*: they have no limit for $t \to \pm\infty$ because the unitary operator U_t does not change the angle of separation (the inner product) or the distance (the square modulus of the difference) between vectors representing two different states. However, there are several quantum phenomena, experimentally obtained in laboratory, that clearly manifest irreversible decaying processes. Quantum mechanics in RHS is proposed as the formal framework for describing this kind of quantum irreversibility.

In its many works, the TAQM school seems to suggest that the fact that evolutions are described by means of semigroups rather than groups is what permits irreversibility to be modeled in a natural way. For instance, according to Antoniou and Prigogine, semigroups are the formal elements that describe the intrinsic irreversibility of large Poincaré systems where the number of degrees of freedom tends to infinity and 'continuous sets of resonances' arise; in particular, the split of U_t into two semigroups, $U_t^+ (t \geq 0)$ and $U_t^- (t \leq 0)$, 'is the essence of intrinsically irreversible representations of dynamics' (Antoniou and Prigogine 1993, 459). For Bohm, the time-asymmetry of the new theory consists in its 'lack of symmetry with respect to time-reversal transformation' (Bohm et al. 1997, p 499) ('non-time-reversal invariance' in our terms) which, in turn, is the consequence of the difference between the two semigroups of evolution operators defined for $t \geq 0$ and $t \leq 0$. But he also considers that the time-reversal invariance of the quantum theory in Hilbert space 'is particularly detrimental for the description of decay processes and resonance scattering, which are intrinsically irreversible processes' (Bohm, Loewe and van de Ven 2003, 556); on the contrary, his theory leads to 'the incorporation of time-asymmetry in the quantum mechanical time evolution, of which the irreversibility of the (undisturbed and unobserved) decay of a resonance is a special case' (Bohm, Gadella and Mithaiwala 2003, 117). Bohm is even more clear about this relationship between semigroups and irreversibility when he asserts that, in the new formulation of quantum mechanics, 'the semigroup arrow is interpreted as microphysical irreversibility' (Bohm et al. 1995, 2593) and that 'the semigroup, $e^{-iH_+^{\times}t}$, $t \geq 0$, expresses intrinsic irreversibility on the microphysical level' (Bohm and Harshman 1998, 233; for a similar claim, cf. 189). These claims show that the TAQM school seems to establish a close link between the non time-reversal invariance of the theory, expressed by semigroup evolution laws, and the irreversibility of the processes described by it, as if the irreversible character of the particular evolutions were the consequence of the fact that they are described by semigroups. In fact, the relevance of the use of RHSs has been interpreted in this sense. For instance, Bishop says that 'one of the important features of the RHS is that evolution operators are often elements of semigroups rather than groups, so that irreversible behavior can be modeled naturally'. (Bishop 2004b, 17), and that 'compared to the standard HS framework, the RHS framework provides a significant advantage in the description of irreversible processes in that semigroup evolutions arise naturally in the latter' (Bishop 2004a, 1685), since 'semigroups of operators are the appropriate operators for the evolution of intrinsically irreversible processes' (Bishop 2004a, 1679). The same idea reappears when the author discusses, in particular, the works of Prigogine's and Bohm's groups: 'the intrinsic irreversibility of LPS [large Poincaré systems] *must* be described by semigroups' (Bishop 2004b, 18; our italics); 'these semigroups fall out of the analysis quite naturally in the RHS framework providing a rigorous description of irreversible behavior in a scattering experiment' (Bishop 2004a, 1680). In this section, we shall argue that, when the RHS's realization used by the TAQM school is analyzed from a mathematical viewpoint, the supposed link between the semigroup evolution laws ant the irreversibility of the processes described by the theory is not as close as these claims seem to suggest. In order to develop our argument, we have to begin with recalling the definition of Hardy functions.

A function $f(x)$ is a *Hardy function on the upper (lower) half-plane* Im $z > 0$ (Im $z < 0$) of the complex plane, $f(x) \in \mathcal{H}_+^2 (f(x) \in \mathcal{H}_-^2)$, iff:

(i) $f(x)$ is a complex function of real variable, $f : \mathbb{R} \to \mathbb{C}$

(ii) $f(x)$ represents the boundary values of an analytic function $f(z)$ on the upper (lower) half plane Im $z > 0$ (Im $z < 0$) of the complex plane. This means that, for any $y_0 > 0$, $y_0 \in \mathbb{R}$, the complex function $f(z) = f(x + iy_0)$ ($f(z) = f(x - iy_0)$) is analytic in the upper (lower) half-plane. In this case, it is said that $f(z)$ is the *analytical continuation* of the function $f(x)$ in the upper (lower) half-plane.

(iii) The following inequality holds:

$$\sup_{y_0>0} \int_{-\infty}^{\infty} |f(x \pm iy_0)|^2 \, dx \leq K \quad \text{with K} > 0 \tag{11}$$

where the sign + (−) corresponds to functions defined on the upper (lower) half plane, and the constant K depends on $f(z)$.[8]

A Hardy function on the upper (lower) half-plane is called 'smooth' if it is infinitely differentiable and fast decreasing. Therefore, the space of smooth Hardy functions on the upper (lower) half-plane is the intersection between the Hardy space $\mathcal{H}_+^2$ ($\mathcal{H}_-^2$) and the Schwartz space *S*. As a consequence of a theorem by Paley and Wiener (1934), the intersections $S \cap \mathcal{H}^2_\pm$ are dense in $\mathcal{H}^2_\pm$; then, if we endow $S \cap \mathcal{H}^2_\pm$ with the metric topology inherited from *S*, it can be shown that

$$S \cap \mathcal{H}^2_\pm \subset \mathcal{H}^2_\pm \subset (S \cap \mathcal{H}^2_\pm)^\times \tag{12}$$

are RHSs (cf. Bohm and Gadella 1989). In turn, as a consequence of a result of van Winter (1974), any $f_+(z)$ ($f_-(z)$) analytic in the upper (lower) half plane and fulfilling condition (iii) is uniquely determined by its boundary values on the positive real semi-axis. Therefore, instead of working with $S \cap \mathcal{H}^2_\pm$, we can work with the restriction of the functions of $S \cap \mathcal{H}^2_\pm$ to $\mathbb{R}^+$: $S \cap \mathcal{H}^2_\pm\big|_{\mathbb{R}^+}$. Since it can be proved that both $S \cap \mathcal{H}^2_\pm\big|_{\mathbb{R}^+}$ are dense in $L^2(\mathbb{R}^+)$ (Bohm and Gadella 1989), then

$$S \cap \mathcal{H}^2_\pm\big|_{\mathbb{R}^+} \subset L^2(\mathbb{R}^+) \subset \left(S \cap \mathcal{H}^2_\pm\big|_{\mathbb{R}^+}\right)^\times \tag{13}$$

are also RHSs: these are the particular realizations used in TAQM.

When time evolutions are governed by a unitary operator U_t, the time-reversal invariance of the evolution law implies that the evolution operators U_t ($t \in \mathbb{R}$) form a group, in particular, that there exists an operator U_{-t} such that $U_t U_{-t} = I$. By contrast, the time-evolutions in the TAQM's formalism are described by the operators $U_t^{+\times}$ and $U_t^{-\times}$, which are defined only for $t \geq 0$ and $t \leq 0$, respectively; this means that $U_t^{+\times}$ with $t < 0$ and $U_t^{-\times}$ with $t > 0$ do not exist and, therefore, the sets $\{U_t^{+\times} : 0 \leq t \in \mathbb{R}\}$ and $\{U_t^{-\times} : 0 \geq t \in \mathbb{R}\}$ of evolution operators form two semigroups. This is what breaks down the time-reversal invariance of the original theory: now, we have two semigroup evolution laws, each of which is non-time-reversal invariant. In turn, semigroups arise as a result of using Hardy functions in this particular realization of the RHS formalism. Such a mathematical result can be understood by considering the action of the evolution operator $U_{-t} = e^{iHt}$ on the vectors $\phi^\pm \in \Phi_\pm$, where U_{-t} is the adjoint (inverse) of U_t.[9] The operator U_{-t} is well defined on $\Phi_\pm$ as subspaces of the Hilbert space on which U_{-t} acts; however, its behavior on Φ_+ and Φ_- is very different for different values of *t*. In fact, it is desired that the action of e^{iHt} turn smooth Hardy functions on the upper half-plane into smooth Hardy functions on the upper half-plane:

$$\text{If } \phi^+(\omega) \in S \cap \mathcal{H}^2_+\big|_{\mathbb{R}^+}, \text{ then } \quad \varphi^+(\omega) = e^{iwt}\phi^+(\omega) \in S \cap \mathcal{H}^2_+\big|_{\mathbb{R}^+}. \tag{14}$$

However, this requirement is not fulfilled for all values of *t*, since the third property in the definition of the Hardy functions (Equation 11) does not hold for $t < 0$. Precisely, only for $t \geq 0$:

$$\sup_{y_0>0} \int_{-\infty}^{\infty} |\phi^+(\omega + iy_0) e^{i(\omega+iy_0)t}|^2 dx = \sup_{y_0>0} \int_{-\infty}^{\infty} |\phi^+(\omega + iy_0)|^2 e^{-2y_0 t} dx \leq K. \tag{15}$$

An analogous argument can be applied to functions $\phi^-(\omega) \in S \cap \mathcal{H}^2_-\big|_{\mathbb{R}^+}$: the evolution operator e^{iHt} turns smooth Hardy functions on the lower half-plane into smooth

Hardy functions on the lower half-plane only for $t \leq 0$ (cf. Bohm and Gadella 1989). This mathematical argument clearly shows that the impossibility of defining an evolution operator for $-\infty < t < \infty$ depends on the *third property* in the definition of the Hardy functions, which implies that, for any Hardy function $\phi^{\pm}(\omega)$ and for any $y_0 > 0$, the functions $\phi^{\pm}(\omega \pm iy_0)\, e^{i(\omega \pm iy_0)t}$ must be square-integrable, and all the integrals must be bounded by the same constant $K > 0$. This means that the non-time-reversal invariance of the theory is a consequence of working with a particular realization of the RHS formalism, based on Hardy functions.

On the other hand, in the TAQM's formalism, irreversibility is introduced by the fact that processes that exponentially decay (grow) as $e^{-\frac{\Gamma}{2}t}$ $(e^{\frac{\Gamma}{2}t})$ can be obtained: they have a well-defined limit for $t \to \infty$ $(t \to -\infty)$. And this, in turn, depends on the existence of the decaying (growing) Gamow vector Ψ^D (Ψ^G), which is an eigenvector of the Hamiltonian with complex eigenvalue $z_R = \omega_R - i\frac{\Gamma}{2}$ $(z_R^* = \omega_R + i\frac{\Gamma}{2})$. The possibility of defining Gamow vectors is a result of using functions that can be *analytically continued* in the lower and in the upper half-planes of the complex energy plane: each pair of Gamow vectors corresponds to the resonance determined by the pair of poles z_R and z_R^* of those analytical continuations. However, the property of having analytical continuation in the half-planes of the complex plane is weaker than the property of being a Hardy function, since it is only the *second property* in the definition of the Hardy functions. This means that the existence of Gamow vectors does not depend on the use of Hardy functions, that is, on the semigroup description of the evolution law, but on the use of functions having analytical continuations in the half-planes of the complex plane. Therefore, the theory's ability to describe irreversible evolutions is not a consequence of the fact that time evolutions are described by semigroups, that is, it does not depend on the non-time-reversal invariance of the theory. In fact, irreversible evolutions can also arise in a time-reversal invariant theory based on an adequate RHS. For instance, Gamow vectors can be obtained in a realization of the RHS formalism in terms of functions that have analytical continuations but are not Hardy functions (cf. Castagnino and Laura 1997, Castagnino et al. 2002). In this case, one may eventually define a pair of structures to describe resonances, one for positive and the other for negative values of time (cf. Castagnino et al. 2001). Here, resonances are also related with the poles of the corresponding function of complex variable; however, since the constraint imposed by the Hardy functions does not exist, the time evolutions are governed by group evolution laws, and as a consequence, the theory remains as time-reversal-invariant as standard quantum mechanics in separable Hilbert space.

Summing up, in TAQM the semigroup description of the evolution law, which leads to the non-time-reversal invariance of the theory, is a consequence of the use of Hardy functions in the realization of the RHSs; the irreversibility of certain evolutions is obtained by means of Gamow vectors, which depend on working with functions that can be analytically continued in the two half-planes of the complex energy plane and, in particular, on the existence of poles in those continuations. But since the existence of Gamow vectors does not depend on the use of Hardy functions, the theory's ability to describe irreversible evolutions is not a consequence of its non-time-reversal invariance. Therefore, the links between semigroup evolution laws and irreversible evolutions are not as strong as usually suggested in the literature on TAQM.

5. Time's Arrow in Time-Asymmetric Quantum Mechanics

5.1. Prigogine's Version

As is well known, one of the motivations of Prigogine in his scientific work was to introduce in physics the objective difference between past and future that we perceive in our life. Whereas we structure our experience in terms of an asymmetric time 'directed' towards the future, the fundamental theories of physics do not distinguish between the two directions of time. According to Prigogine, the second law of thermodynamics would establish the desired difference between the two directions of time; the problem is that this non-time-reversal invariant law cannot be adequately explained in terms of the underlying time-reversal invariant laws of mechanics. Prigogine's aim was to throw a bridge from being to becoming by introducing the arrow of time in the fundamental laws of physics (Prigogine 1980). In this context his final purpose consisted in developing a theory where 'the existence of an "arrow of time" is taken to be a fundamental fact' (Misra and Prigogine 1983, 421). During a first period, from the 1960s to the mid-1980s, Prigogine's approach was based on the attempt to reconcile irreversible macroscopic dynamics with reversible microscopic dynamics in highly unstable systems by means of a similarity transformation mapping trajectory descriptions of unstable classical systems into a description in terms of probabilistic Markov processes. However, several technical difficulties of this first approach led him to adopt RHSs as the mathematical framework for describing irreversible phenomena (for details, cf. Bishop 2004b). For him, the fact that, in the appropriate realization of a RHS, evolution operators are elements of semigroups rather than groups, is the key element in the account for the arrow of time.

As we have seen, two RHSs arise when we use Hardy functions in the particular realization of the space Φ, which determines the properties of the antidual $\Phi^{\times}$. Thus, two antiduals can be defined, $\mathbf{\Phi}_{\pm}^{\times}$, which contain not only all the physically realizable states, but also generalized states like the Gamow vectors and the Dirac kets. Time evolution has a semigroup structure on the antiduals: the semigroups $U_t^{+\times}$ with $t \geq 0$ and $U_t^{-\times}$ with $t \leq 0$ govern the time evolutions on $\mathbf{\Phi}_{+}^{\times}$ and $\mathbf{\Phi}_{-}^{\times}$, respectively. We have also shown that the fact that $U_t^{+\times}$ and $U_t^{-\times}$ form semigroups is what breaks down the original time-reversal invariance of quantum mechanics in its standard version. On the basis of these previous results, now it is not difficult to see that those two RHSs with their corresponding evolution operators lead to time-symmetric twins, since they are two non-time-reversal invariant formalisms, one the temporal mirror image of the other. In fact, if $\mathbf{T}$ is the time-reversal operator in the Hilbert space $\mathcal{H}$, it can be readily shown that (Gadella and de la Madrid 1999):

$$\mathbf{T}\mathbf{\Phi}_{\pm} = \mathbf{\Phi}_{\mp}, \tag{16}$$

and if $\mathbf{T}^{\times}$ is the extension of $\mathbf{T}$ to the antiduals $\mathbf{\Phi}_{\pm}^{\times}$:

$$\mathbf{T}^{\times}\Phi_{\pm}^{\times} = \Phi_{\mp}^{\times}. \qquad (17)$$

If we want to distinguish between the two directions of time, $t \geq 0$ and $t \leq 0$, the challenge consists in supplying a non-conventional criterion, based on theoretical considerations, for choosing one of the twins of each pair as the physically meaningful one.

In his analysis of Friedrichs model for quantum scattering, Antoniou and Prigogine adopt a time-ordering rule to decide upon the direction of integration of the Hardy functions around the poles in the upper and lower complex half-planes: 'excitations are considered as past oriented, therefore they are extended from the lower to the upper half-plane. … de-excitations or mode–mode transitions are understood as future oriented and they are extended from the upper to the lower half-plane' (Antoniou and Prigogine 1993, 454–455). On this basis, the authors conceive both Gamow vectors, $\Psi^{D} \in \Phi_{+}^{\times}$ and $\Psi^{G} \in \Phi_{-}^{\times}$, as representing decaying processes, directed to the future and to the past, respectively, and evolving according their corresponding semigroup of evolution operators: 'The unitary group U_t when extended from the Hilbert space $\mathcal{H}$ to the space $\Phi_{+}^{\times} + \Phi_{-}^{\times}$ splits therefore into two semigroups, the forward semigroup $U_t^{+}, t > 0$, describing decay in the future, and the backward semigroup $U_t^{-}, t < 0$, describing decay in the past' (Antoniou and Prigogine 1993, 459). This means that $U_t^{+\times}$ carries states into the *forward direction of time* and then describes evolutions reaching equilibrium *in the future*; $U_t^{-\times}$ carries states into the *backward direction of time* and then describes evolutions reaching equilibrium *in the past*.

It is clear that the time-ordering rule does not yet answer the problem of the arrow of time to the extent that we still have a pair of time-symmetric twins only conventionally different: the theory by itself gives no basis for selecting one of the elements of the pair as the physically relevant one. Therefore, Antoniou and Prigogine adopt an observational criterion for retaining one of the semigroups and discarding the other: since no physical system has ever been observed evolving to equilibrium towards the past, the physically relevant semigroup of evolution operators is the semigroup corresponding to $U_t^{+\times}$, valid for $t \geq 0$; this is 'the semigroup corresponding to our observation' (Antoniou and Prigogine 1993, 459; cf. also Petrosky and Prigogine 1997).

Although this appeal to observational considerations is a legitimate move in the everyday work of physicists, it is not acceptable when the problem at issue is to supply a conceptually adequate account of the arrow of time, since the fact that our observations are time-directed was known from the very beginning. Furthermore, the question about the arrow of time arises precisely when we seek a physical correlate of that intuitive asymmetry between past and future. For this reason, the problem consists in accounting for the difference between the two directions of time *by means of theoretical arguments*.

Being perhaps aware that to use observational arguments in this context amounts to begging the question, Antoniou and Prigogine appeal to a previous work by Prigogine and George (1983), where the second law of thermodynamics is interpreted as a selection principle on initial conditions, retaining only the conditions that lead to equilibrium in the future. In a similar sense, when referring to such a work, Antoniou and Prigogine (1993, 459) claim that the difference between the two directions of time emerges 'as a selection principle of the semigroup which reaches equilibrium in our future'. With this move, the authors seem to endow the observational criterion used to select the future directed semigroup and to disregard the past directed one with a certain nomological character. But, if this is the case, the new strategy is even more questionable than the previous one since it commits a *petitio principii*: now, we have a theoretical criterion for retaining one semigroup, but such a criterion is precisely the non-time-reversal invariant law whose microscopic foundation had to be explained.

5.2. Bohm's Version

Although the original aim of Bohm was focused on the description of the decaying phenomena produced in quantum scattering, the scientific contact with Prigogine's group leads him to incorporate the interest in the issue of the arrow of time. In fact, in several papers, he begins his technical presentation with a review of the different arrows of time treated in the literature, including not only the arrows arising in the different chapters of physics (the thermodynamic arrow TA, the radiation arrow RA, the cosmological arrow CA) but also the psychological arrow of time (cf. Bohm et al. 1995, introduction; Bohm and Harshman 1998, preface; cf. also Bohm et al. 1997, section 4). In the context of this broad discussion, Bohm restricts his interest to the quantum arrow of time (QAT), 'given by the time evolution semigroup generated by the Hamiltonian and used previously for the Gamow vectors' (Bohm et al. 1995, 2593).

As we have seen, Prigogine and Bohm agree in the use of RHSs to develop their theories. However, there is a conceptual difference between the two groups with respect to the interpretation of the formalism (for a comparison between both approaches, cf. Bishop 2003, 2004b). Whereas, in Prigogine's proposal, states and observables are represented by mathematical objects belonging to the same spaces, Bohm introduces a formal distinction between states and observables. In particular, Bohm replaces the representational postulate of standard quantum mechanics—according to which states are represented by the vectors of a separable Hilbert space and observables are represented by self-adjoint operators on that space—by a new postulate that distinguishes between the mathematical descriptions of states ψ and of observables φ:

$$\{\psi\} \equiv \Phi_- \subset \mathcal{H} \subset \Phi_-^\times \tag{18}$$

$$\{\varphi\} \equiv \Phi_+ \subset \mathcal{H} \subset \Phi_+^\times . \tag{19}$$

The new representational postulate asserts that the vectors $|\psi\rangle \in \Phi_-^\times$ represent the states of the system in the sense that a state is $\rho = |\psi\rangle\langle\psi|$, and the vectors $|\varphi\rangle \in \Phi_+^\times$ represent the observables of the system in the sense that an observable is $A = |\varphi\rangle\langle\varphi|$. If the time $t = 0$ is considered as the time at which preparation ends and detection begins, the energy distribution $\langle\omega|\psi\rangle$ produced by the accelerator must be zero for $t > 0$, and the energy distribution $\langle\omega|\varphi\rangle$ of the detected state must be zero for $t < 0$ (Bohm et al. 1997, Bohm, Gadella and Mithaiwala 2003):

$$|\psi\rangle \in \Phi_-^\times \qquad \langle\omega|\psi\rangle = 0 \quad \text{for } t > 0 \tag{20}$$

$$|\varphi\rangle \in \Phi_+^\times \qquad \langle\omega|\varphi\rangle = 0 \quad \text{for } t < 0. \tag{21}$$

The time evolution for a state $\rho = |\psi\rangle\langle\psi|$ (Schrödinger picture) is given by:

$$\rho(t) = e^{-iH_-t}|\psi(0)\rangle\langle\psi(0)|e^{iH_-t} = e^{-iH_-t}\rho(0)e^{iH_-t} \quad \text{for} \quad t \geq 0. \tag{22}$$

This equation makes sense for $t \geq 0$ since $e^{iH_-t}\psi \in \Phi_-$ for any $\psi \in \Phi_-$ if and only if $t \leq 0$. This means that, although the energy distribution $\langle\omega|\psi\rangle$ corresponding to the state $|\psi\rangle$ is zero for $t > 0$, states evolve forward in time. On the other hand, the time evolution for an observable $A = |\varphi\rangle\langle\varphi|$ (Heisenberg picture) is given by:

$$A(t) = e^{iH_+t}|\varphi(0)\rangle\langle\psi(0)|e^{-iH_+t} = e^{iH_+t}A(0)e^{-iH_+t} \quad \text{for} \quad t \geq 0. \tag{23}$$

This equation makes sense for $t \geq 0$, since $e^{iH_+t}\varphi \in \Phi_+$ for any $\varphi \in \Phi_+$ if and only if $t \geq 0$. Therefore, observables also evolve forward in time. In turn, the Gamow vectors $\Psi^G \in \Phi_-^\times$ and $\Psi^D \in \Phi_+^\times$ represent growing and decaying processes, respectively, both evolving towards the future. However, the time evolution corresponding to the growing Gamow vector Ψ^G can be defined only for $t \leq 0$, and the time evolution corresponding to the decaying Gamow vector Ψ^D can be defined only for $t \geq 0$.

As we can see, Bohm's approach breaks the symmetry between the time-symmetric twins—resulting from the two semigroups of evolution operators—by means of a new representational postulate which distinguishes between sates and observables. In turn, such a distinction is grounded on the so-called '*preparation-registration arrow of time*', expressed by the slogan '*no registration before preparation*' (Bohm et al. 1995).[10] The key intuition behind this proposal is that observable properties of a state cannot be measured until the state acting as a bearer of those properties has been prepared. For instance, in a scattering process, it makes no sense to measure the scattering angle until an incoming state is prepared by an accelerator. Sometimes, the adoption of this arrow seems to be based on merely empirical considerations: 'experimentally, these two entities can be distinguished and can be given operational definitions: A state W is prepared by a preparation apparatus while an observable A is registered by a registration apparatus' (Bohm and Wickramasekara 2002, 316). Nevertheless, in other cases, Bohm emphasizes that the preparation-registration arrow does not have to be understood as expressing an experimental fact: the arrow 'is also there without experimentalists performing any experiment' (Bohm, Gadella and Mithaiwala 2003, 138). In some papers, the preparation-registration arrow is presented in terms of causality: 'it is nothing but a general expression of causality' (Bohm et al. 1995, 2594; for a similar claim, cf. Bohm and Harshman 1998, 208). Of course, if the preparation-registration arrow is adopted by appealing to empirical arguments or to a pretheoretical notion of causality, from a conceptual viewpoint Bohm's account of the arrow of time has the same shortcomings as Prigogine's proposal: neither of these two positions supply a conceptually adequate response to the problem of the arrow of time in quantum mechanics.

Bohm's proposal turns out to be more interesting from a philosophical point of view when the intuition behind the preparation-registration arrow is conceived as having an ontological content. This interpretation has been suggested by Bishop (1999): the existence of a state is ontologically prior to the existence of its properties; therefore, observable properties do not exist unless there exists a state which carries them. Of course, this interpretation makes Bohm's account of time's arrow at the quantum mechanical level subtler than the response given by Prigogine and his coworkers. However, it is worth noting that the ontological assumption of the priority of states over observables is based on the implicit adoption of an Aristotelian-style ontology of substances and properties, where the ontological priority of the substance is a consequence of the fact that it exists in and by itself, whereas properties require the substance to exist. But, of course, from a different metaphysical picture (for instance, in a bundle-theory metaphysics), the ontological priority of substances over properties cannot be defended. Nevertheless, even if we accept a traditional ontology of substances and properties, ontological priority implies nothing about temporal priority. If the asymmetry between past and future is not presupposed, we cannot say that the ontologically previous entity must also be temporally previous to the other: in fact, if we have not defined the relation of 'temporally previous' in advance, we cannot relate ontological priority with temporal priority. Even if we accept the ontological interpretation of the preparation-registration arrow, the assumption of a correlation between ontological priority and temporal priority is the result of the projection of our intuitions about past and future into ontology, violating the 'nowhen' requirement of adopting an atemporal perspective purged of time-asymmetric preconceptions.

This argument shows that the preparation-registration arrow says much more than what ontological priority imposes: it presupposes the arrow of time in advance by adding a particular temporal relationship to the ontological priority of states with respect to observables. In fact, without the time-asymmetric intuition introduced by the preparation-registration arrow, nothing prevents us from reversing the representational postulate by stating that $\Phi_- \subset \mathcal{H} \subset \Phi_-^\times$ is the RHS for the representation of observables, and $\Phi_+ \subset \mathcal{H} \subset \Phi_+^\times$ is the RHS for the representation of states. In this case, we would obtain the temporal mirror image of Bohm's theory, where the Gamow vectors $\Psi^G \in \Phi_+^\times$ and $\Psi^D \in \Phi_-^\times$ correspond to growing and decaying processes, respectively, both evolving towards the past. This new representational postulate restores the symmetry because now we have two postulates leading to two non-time-reversal invariant theories, one the temporal mirror image of the other: they also lead to time-symmetric twins, and the challenge consists, again, in supplying a theoretical and non-conventional criterion for retaining one of them and discarding the other. When analyzed in detail, Bohm's decision of selecting the future directed alternative is not merely based on an ontological assumption, but on presupposing the arrow of time from the very beginning on the basis of pretheoretical intuitions.

In conclusion, the TAQM school's proposal, either in Prigogine's or in Bohm's version, does not supply a conceptually acceptable answer to the problem of the arrow of time in quantum mechanics. We have argued elsewhere (Castagnino and Lombardi 2004, 2005a, 2005b) that this conclusion is not surprising to the extent that the problem of the arrow of time cannot be solved in local terms. Nevertheless, the detailed discussion of this point goes beyond the limits of the present paper.

6. Conclusions

In this paper, we have shown that, although in the case of unitary evolutions time-reversal invariance and reversibility seem to go hand in hand, both properties are different to the extent that they are related with distinct features of the formalism: whereas time-reversal invariance implies the *group* structure of the evolution operators, reversibility is a consequence of the *unitary* character of such operators. These considerations are applicable to time-asymmetric quantum mechanics, where the non-time-reversal invariance of the theory is due to the semigroup structure of the evolution laws which, in turn, is a consequence of the use of Hardy functions in the realization of the RHS. In turn, the irreversibility of the evolutions is due to the existence of Gamow vectors, which depend on the use, not of Hardy functions, but of functions having analytical continuations in the imaginary half-planes of the complex plane. Since irreversible evolutions given by Gamow vectors can be obtained in time-reversal invariant versions of the theory where evolutions are described by groups, irreversibility is clearly not a consequence of semigroup evolution laws as it is usually suggested.

With respect to the arrow of time, we have seen that the TAQM school does not solve the problem either in Prigogine's version or in Bohm's version. In fact, we have identified the time-symmetric twins in Prigogine's and Bohm's proposals, showing that they do not supply a non-conventional and theoretically grounded criterion for selecting one of the twins as the physically relevant one.

Of course, these criticisms do not affect the scientific value of the TAQM school's works: time-asymmetric quantum mechanics is a powerful theory for the description of intrinsic irreversibility. Here, our only purpose has been to supply a conceptual clarification of the school's proposal, where the discussions about irreversibility and time's arrow are closely entangled, with no clear elucidation of the concepts involved in the arguments. Our main aim is to contribute to the adequate interpretation of a theory that has been successfully applied in different areas of physics.

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Notes

[1] From here, we will not distinguish between mathematical entities (equations and solutions) and physical entities (laws and evolutions).

[2] The fact that we recognize the relevance of Price's claims about the need of adopting the 'nowhen' standpoint does not mean that we agree with his proposal for the solution of the problem of time's arrow (for a detailed discussion, cf. Castagnino, Lombardi and Lara 2003; Castagnino and Lombardi 2005a, 2005b).

[3] Let us note that we are saying that these arguments are not legitimate in the discussions about *the arrow of time*: we are not talking about the problem of irreversibility, where such arguments may be acceptable when *de facto* irreversibility is the question under discussion.

[4] The space Φ has its own topology, which is stronger than the topology that Φ possesses as a subspace of $\mathcal{H}$. The topology in Φ is not given by a norm but usually by a countable infinite family of norms.

[5] The assumptions are: (i) the domain $D(A^\dagger)$ of $A^\dagger$ includes the space Φ, (ii) for each $\phi \in \Phi$, $A^\dagger \phi \in \Phi$, and (iii) the $A^\dagger$ is continuous on Φ in the own topology of Φ (cf. Schäffer 1970). The duality formula also applies when A is self-adjoint; in this case, $A^\dagger = A$, and the duality formula becomes:

$$\langle A\phi \mid F\rangle = \langle \phi \mid A^\times F\rangle \qquad \forall \phi \in \Phi,\ \forall F \in \Phi^\times \cdot$$

[6] The possibility of complex generalized eigenvalues of self-adjoint operators was suggested by Gamow (1928) in the context of the decay of quantum systems. As poles of the resolvent, Gamow vectors were first introduced by Grossmann (1964), independently of RHSs. Later, they were unexpectedly obtained in the RHS formalism as generalized eigenvectors of self-adjoint operators with complex eigenvalues (Lindblad and Nagel 1970). The association between the poles of the S-matrix with the vectors in the RHS was established in the 1980s (Bohm 1981, Gadella 1983, 1984): these works showed that RHSs supply the formal representation to the decaying states and resonances heuristically constructed by Gamow.

[7] Second-and higher-order poles of the scattering operator S are treated in Bohm et al. (1997) and in Antoniou, Gadella and Pronko (1998).

[8] Therefore, any $f(z)$ analytic in the upper (lower) half-plane fulfilling condition (iii) uniquely determines its boundary values on the real line, given by a Hardy function on the upper (lower) half-plane. After the Titchmarsh theorem (Titchmarsh 1937), the reciprocal is also true.

[9] Recall that we want to obtain the evolution operator $U_t^\times$ for the vectors belonging to $\Phi_\pm^\times$ and, therefore we have to begin with the adjoint of U_t: $U_t^\dagger = U_{-t}$ (cf. the duality formula (4))

[10] Bohm recognizes that the origin of the idea of a preparation-registration arrow can be traced back to the works of Günther Ludwig (1983–1985).

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