A GENUINELY INTRODUCTORY INTRODUCTION TO C. S. PEIRCE'S DIAGRAMMATIC LOGIC

by

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3 disclaimers

sign-action → enables → regimented thinking → enables → practical engagement

	studied by semiotics	studied by logic	studied by pragmatism
Peirce:	Area of Specialization	AOS	AOS
Disclaimer # 1 Me:	AOS	AO Competence	AOS

moreover ↓

Disclaimer # 2

Not a native speaker in EGs (raised on standard symbolic logic) ⟼ I will avoid translating (can be done)

moreover ↓

Disclaimer # 3

There are currently no accessible textbooks on EGs (only a few experts)

↳ in this presentation, I have left out citations + asides

↓

pedagogical

The status of logic

① Voluntary (not what God is thinking)

First rule of logic is: "[...] In order to learn you must desire to learn and in so desiring not be satisfied with what you already incline to think"

(EP 2.48) →

② Habitual, not mechanical

Rooted in semiosis, where the most complex of the ten signs is a tendency, given some suitably arranged symbols, to produce a further symbol (the conclusion)

③ Validity of entailments is grasped by observation

④ Fallible (a human device)

We can err, but others can detect flaws

↓

So, the inquiry is self-corrective

↓

So, opinions converge on valid patterns

⑤ Normative

Logic is a normative science (alongside ethics and esthetics) that studies how one should think

(art of self-control)

Peirce's 2 goals with Existential Graphs

1) Place inferences before vision

"I do not think I ever *reflect* in words: I employ visual diagrams, firstly, because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose." (MS 619: 8, dated 1909)

2) Dissect reasoning into as many steps as possible

"aim was to make every demonstration as long as it possibly could be made without being circuitous" (Synthese 2015, p. 888)

Logic intended for rational agents, not machines

Why? Becomes clearer when status of logic ↘

Where to start?
The Sheet of Assertion

- Begin in real life: experience is complex (e.g.: Tricolor)
- Simple quality (Firstness), arrived at by prescission
- Sum total of what one could assert ("universe of discourse")
- Bare presence of SA is undeniable, but SA is undifferentiated.

↓

Because prescission artificially left out rest of the world, we can carefully re-introduce symbols for claims we wish to make.

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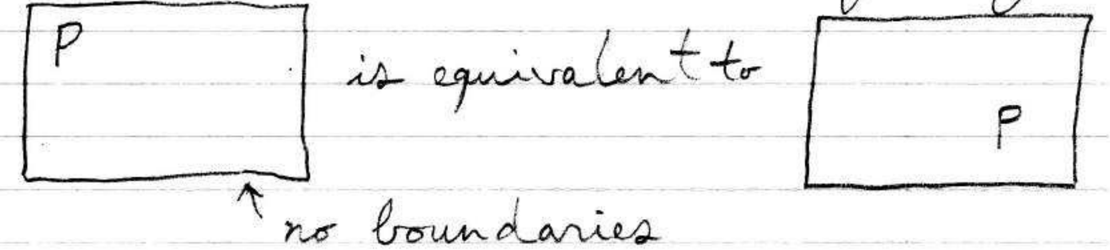
Code can be anything, but must stay constant

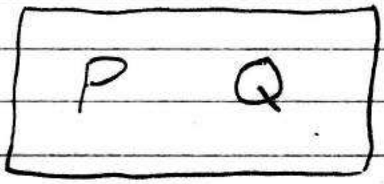
e.g.: P can mean "Marc loves Wendy"

- Spatial location does not matter (up to a point)



Juxtaposition
as the natural sign of conjunction

- Two claims juxtaposed (by any "distance") are conjoined.
- No extra sign for "and" between claims
- So,

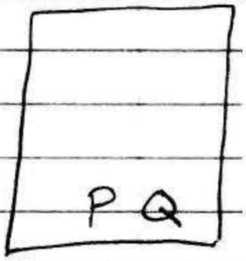


is the diagramm. represent. of "P and Q" (P•Q)

- Order and placement does not matter

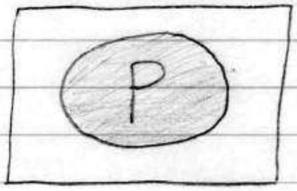
e.g.

is equivalent to



Cut

- If everything on the SA is affirmed, then anything not on the SA is denied.
- SA has no boundaries, so we can cut out a space (often shaded, for contrast).
- Thus,



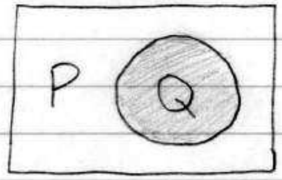
is the diagrammatic representation of "not-P"

$\sim P$

note that cut serves as tilda and parenthesis of what it encloses

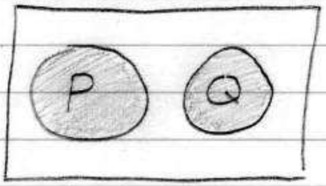
- Cuts occupy space, so they can be juxtaposed

e.g.:



P and not-Q

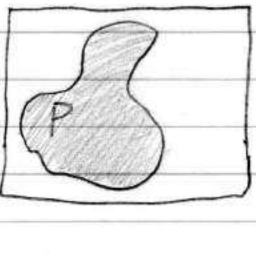
$P \cdot \sim Q$



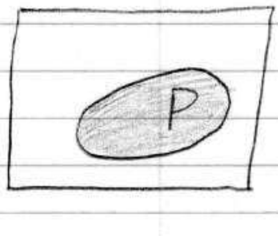
not-P and not-Q

$\sim P \cdot \sim Q$

- Shape of the cut does not matter

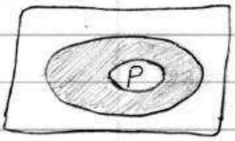


equivalent to



- A cut can be cut, restoring the force to assertion instead of negation

So, here, P is asserted ($\sim\sim P$)



Why should any of this be binding?
Contrapiction as our compass

- Everything in lived experience confirms principle of non-contradiction

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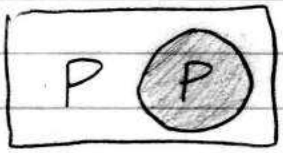
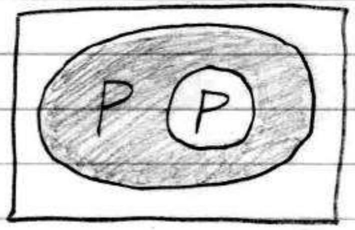
this will be our compass is evaluating arguments as good or bad

- 

is necessarily false

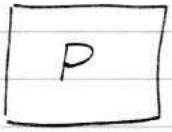
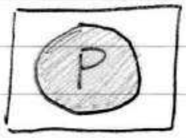
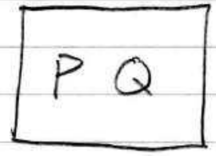
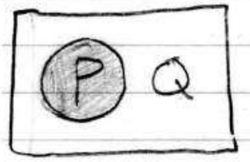
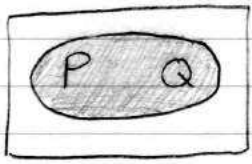
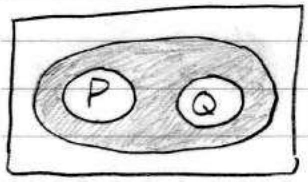
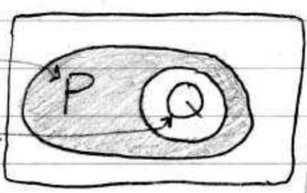
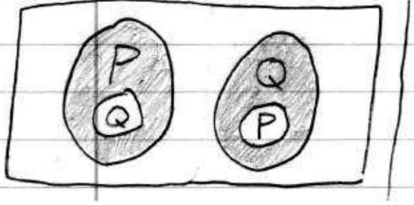
- 

is necessarily true

- Contradictions:
  Nature: Verbal / discursive (symbol)
  Why bad: Violates personal consistency
  ∴ hallmark of irrationality

- Contrapictions
  Nature: Qualitative / pictorial (icon)
  Why bad: Is impossible to implement
  ∴ hallmark of irrationality

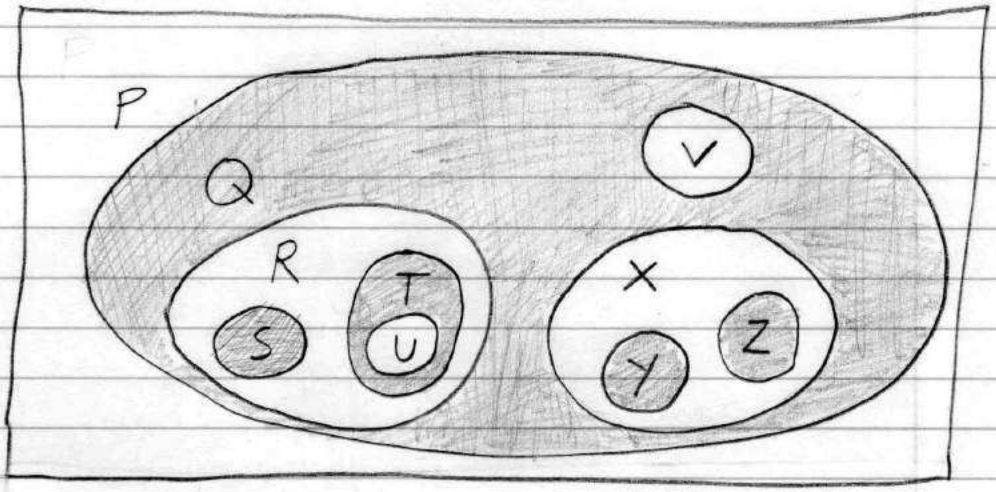
Some common compound propositions

	It is the case that P	P
	It is not case that P	~P
	It is the case that P and Q	P • Q
	It is not the case that P and it is the case that Q	~P • Q
	It is not the case that P and Q	~(P • Q)
	It is not the case that not-P and not-Q (alt: P and Q can't both be false)	P v Q
("scroll" outer and inner areas) 	It is not the case that P can hold yet Q does not hold	P → Q
	P implies Q and Q implies P	P ↔ Q

Tracking levels and nests

Peirce worked for geodetic survey

Take



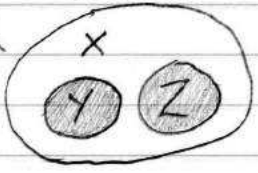
The diagrammatic manipulations that we will make as inference rules require that we track levels and nests.

For instance, Q is at an "odd" level (level 1)
S is at an "odd" level (level 3)

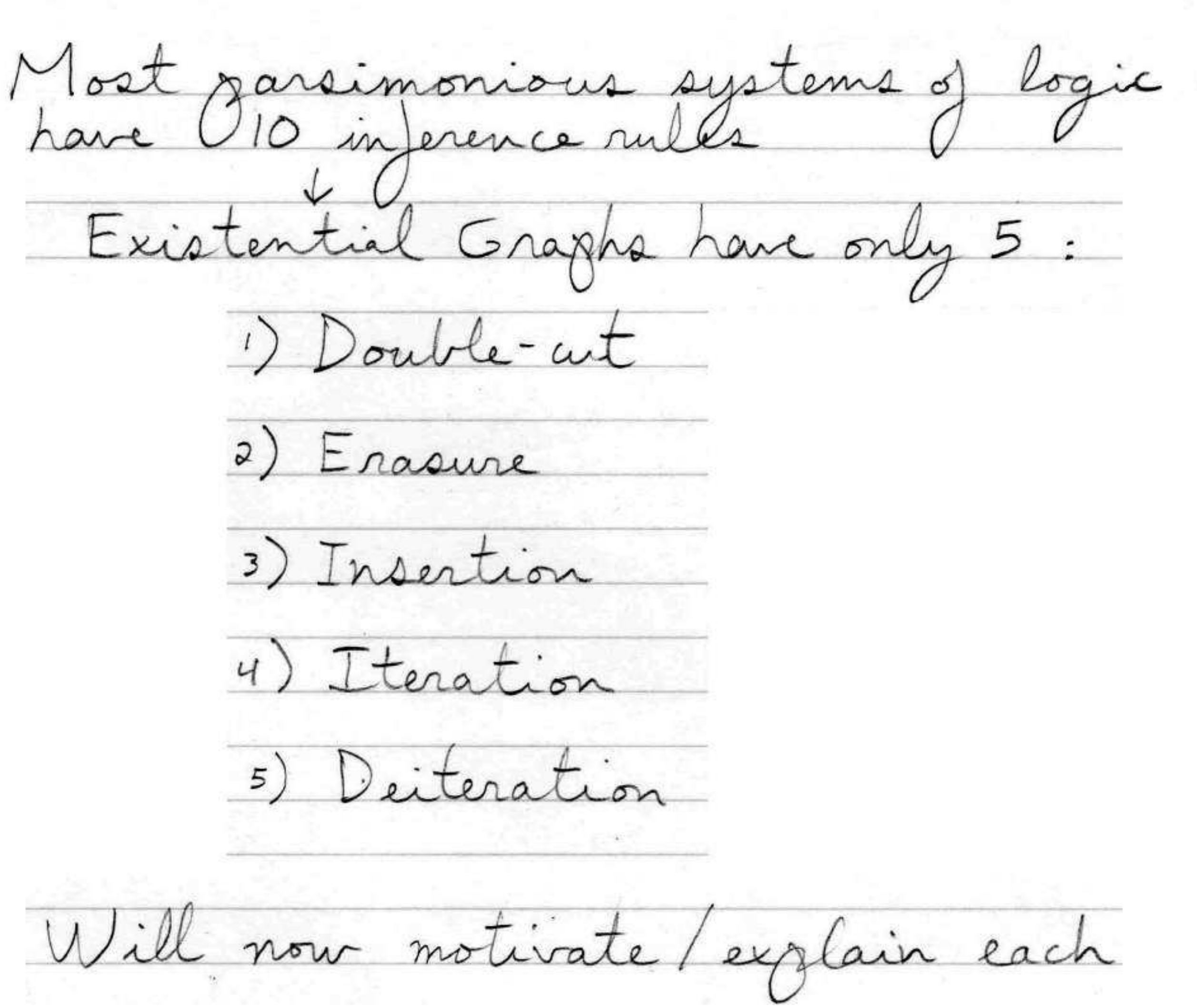
P is at an "even" level (level 0)
V is at an "even" level (level 2)

- Odd levels are (or would be) shaded
- Even levels are (or would be) unshaded

A nested level is any level you can reach without going up (imagine a quarry dig site)

Thus, the sub-graph  is nested for Q but not for V

Introducing the 5 permissions



Most parsimonious systems of logic have 10 inference rules

↓

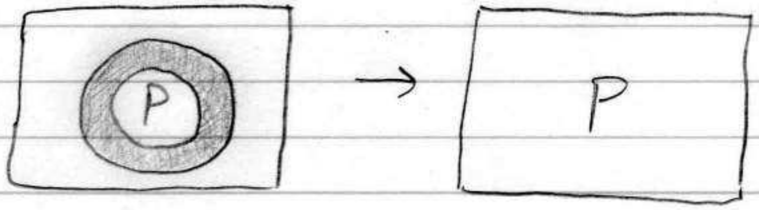
Existential Graphs have only 5 :

1) Double-cut
2) Erasure
3) Insertion
4) Iteration
5) Deiteration

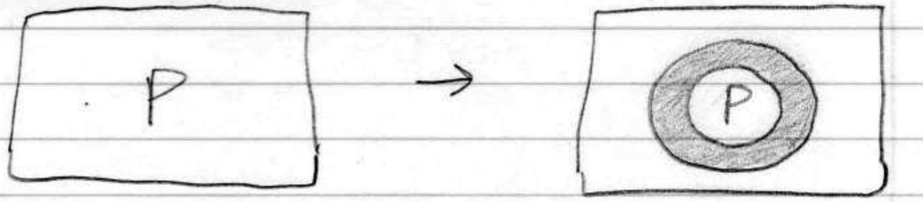
Will now motivate / explain each

Double-cut

- A double-denial is equivalent to an assertion, so
    - An (empty) double-cut can always be erased



    - An (empty) double-cut can always be added



Erasure

Rule: Any evenly enclosed (unshaded) graph may be erased

- One can go from $\boxed{P}$ to $\boxed{}$

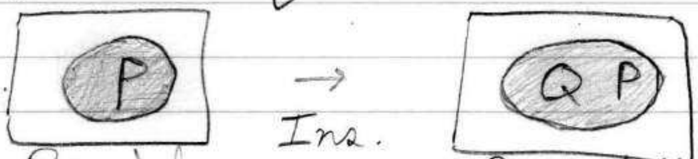
because one did not have to affirm P

Insertion

Rule: Any graph may be scribed on any oddly enclosed (shaded) area

Because a negated claim must start somehow

Insertion in a shaded area cannot make an already existing statement true



P is false

P is still false

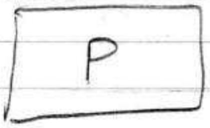
Therefore, the negation of that false area must be at least as true as before

(although $\sim P \not\equiv \sim (Q \cdot P)$)

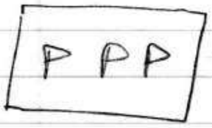
Iteration

Rule: Any graph that already exists
may be scribed again within
the same area
or in a nested area that already exists

Why?
In same area:
Because

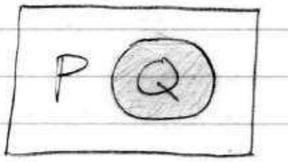


is equivalent to



(see type/token distinct.)

Why?
In nested area:
Because



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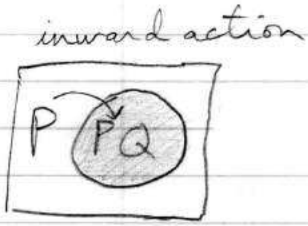
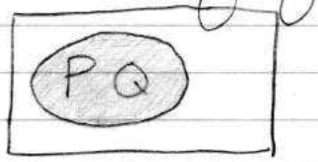


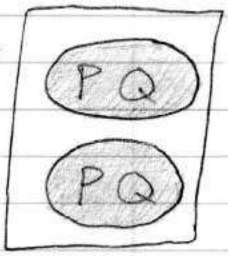
Truth-table proof of iteration equivalence

P	·	~	Q	≡	P	·	~	(P	·	Q)
T	F	F	T	T	T	F	F	T	T	T
T	T	T	F	T	T	T	T	T	F	F
F	F	F	T	T	F	F	T	F	F	T
F	F	T	F	T	F	F	T	F	F	F

- Can iterate sub-graphs, not just atomic propositions. Thus



is equivalent to



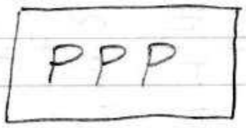
Deiteration

Rule: Any graph that is, or could be, the result of iteration, may be erased.

Why?

In same area:

Because



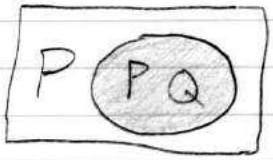
is equivalent to

(so long as one token survives, the type survives)

Why?

In a nested level:

Because



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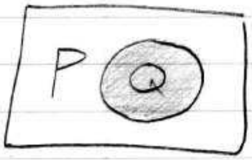
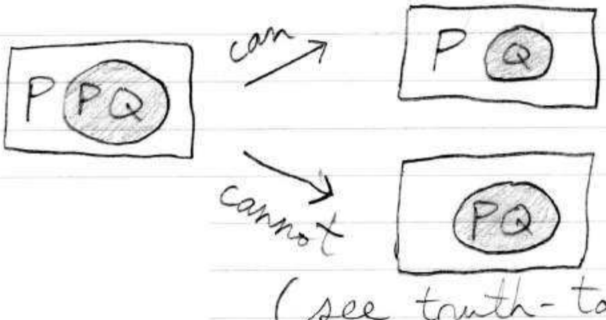


(see previous truth-table in opposite direction)

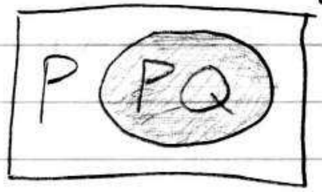
- Remember, deiteration is iteration backwards, so deiterations works only in nested levels



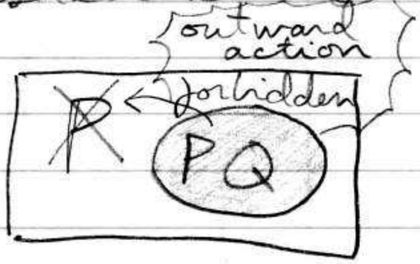
(see truth-table on next page...)

Why deiteration will not allow you to crawl out of a nested level

from



to



P	•	~	(P	•	Q)	$\not\equiv$	~	(P	•	Q)
T	F	F	T	T	T	T	F	T	T	T
T	T	T	T	F	F	T	T	T	F	F
F	F	T	F	F	T	F	T	F	F	T
F	F	T	F	F	F	F	T	F	F	F

Now for some hands-on practice...

Here is some step-by-practice with 5 basic arguments, each justified 2 ways:

5 arguments

- Modus Ponens: $P \rightarrow Q / P // Q$
- Modus Tollens: $P \rightarrow Q / \sim Q // \sim P$
- Disjunctive syllogism: $P \vee Q / \sim P // Q$
- Hypothetical syllogism: $P \rightarrow Q / Q \rightarrow R // P \rightarrow R$
- Constructive dilemma: $P \rightarrow R / Q \rightarrow S / P \vee Q // R \vee S$

2 ways

For the proofs, I will show how to transform the graphs of the premises into the graph of the conclusion, using only the 5 permissions.

For the indirect derivations, I will show how obtain a contradiction once the conclusion is assumed false

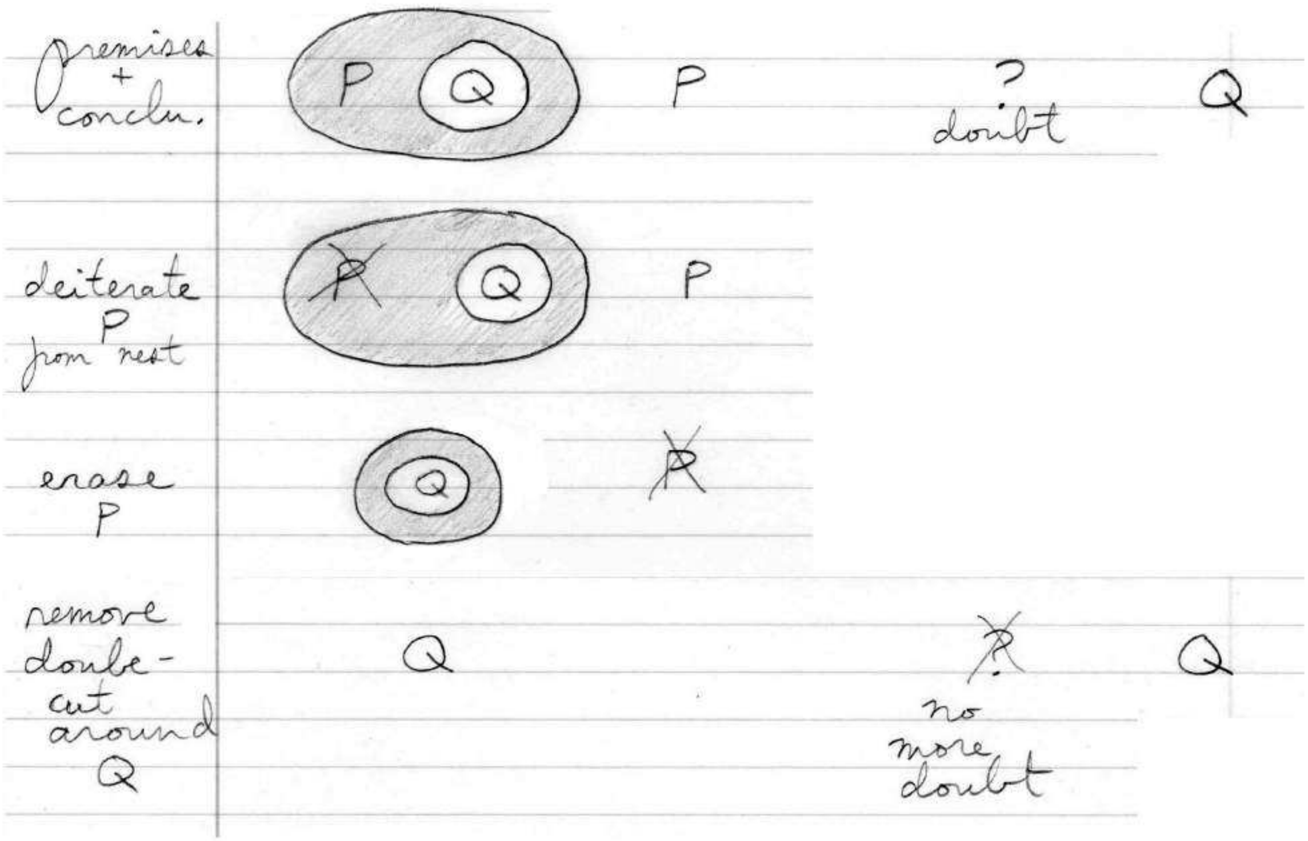
Note that I will rewrite the graphs at each step, even if this is cumbersome.

Consider an argument like the following:

"If I go to the restaurant (P), then I will miss the game (Q). Now, I am going to the restaurant (P). Therefore, I will miss the game (Q)."

Does the conclusion (after the "therefore") really follow? To test this with Existential Graphs, we construct a diagram of the premises, and then we see if the permissions allow us to obtain a diagram of the conclusion.

Derivation of Modus Ponens

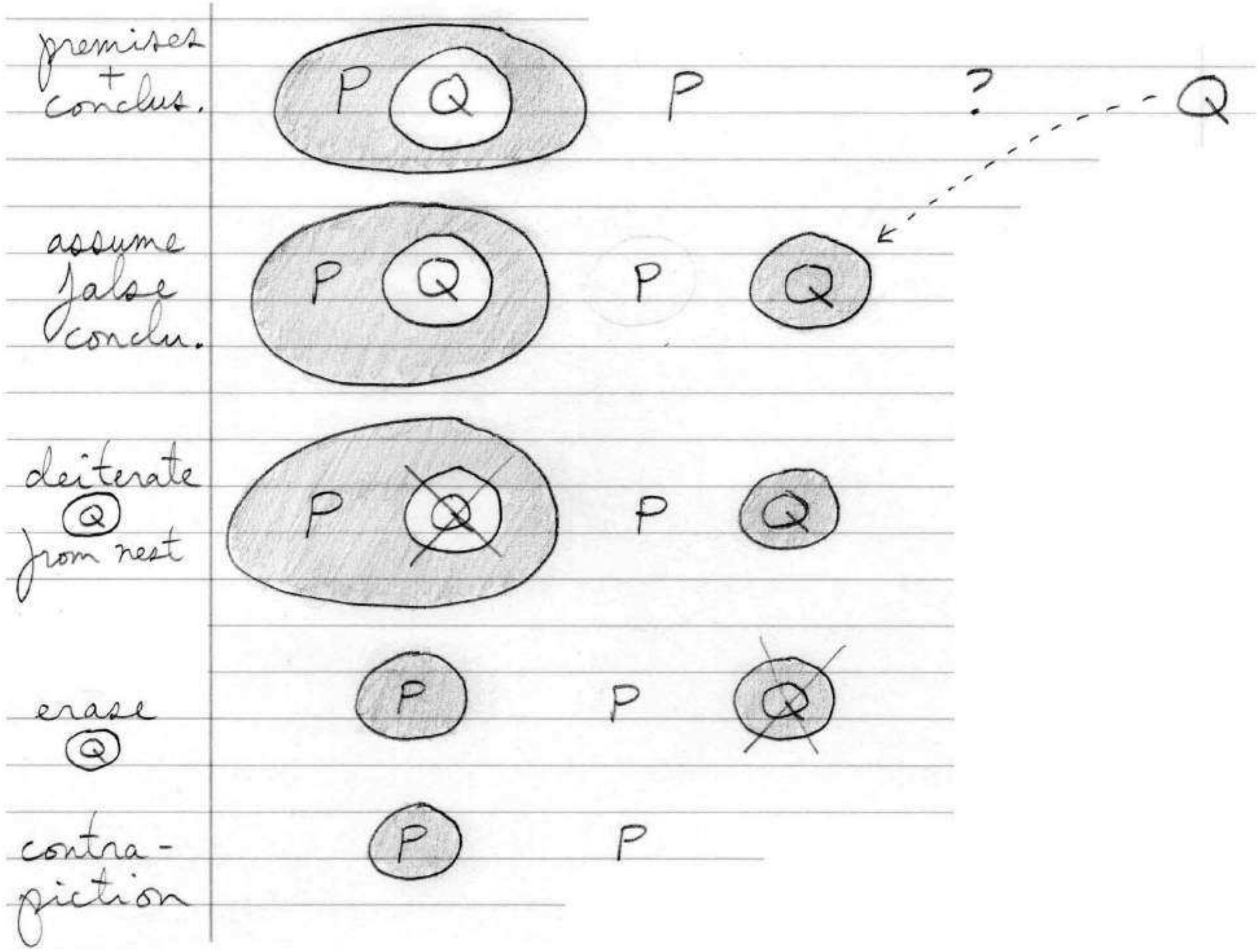


Consider once again this argument:

"If I go to the restaurant (P), then I will miss the game (Q). Now, I am going to the restaurant (P). Therefore, I will miss the game (Q)."

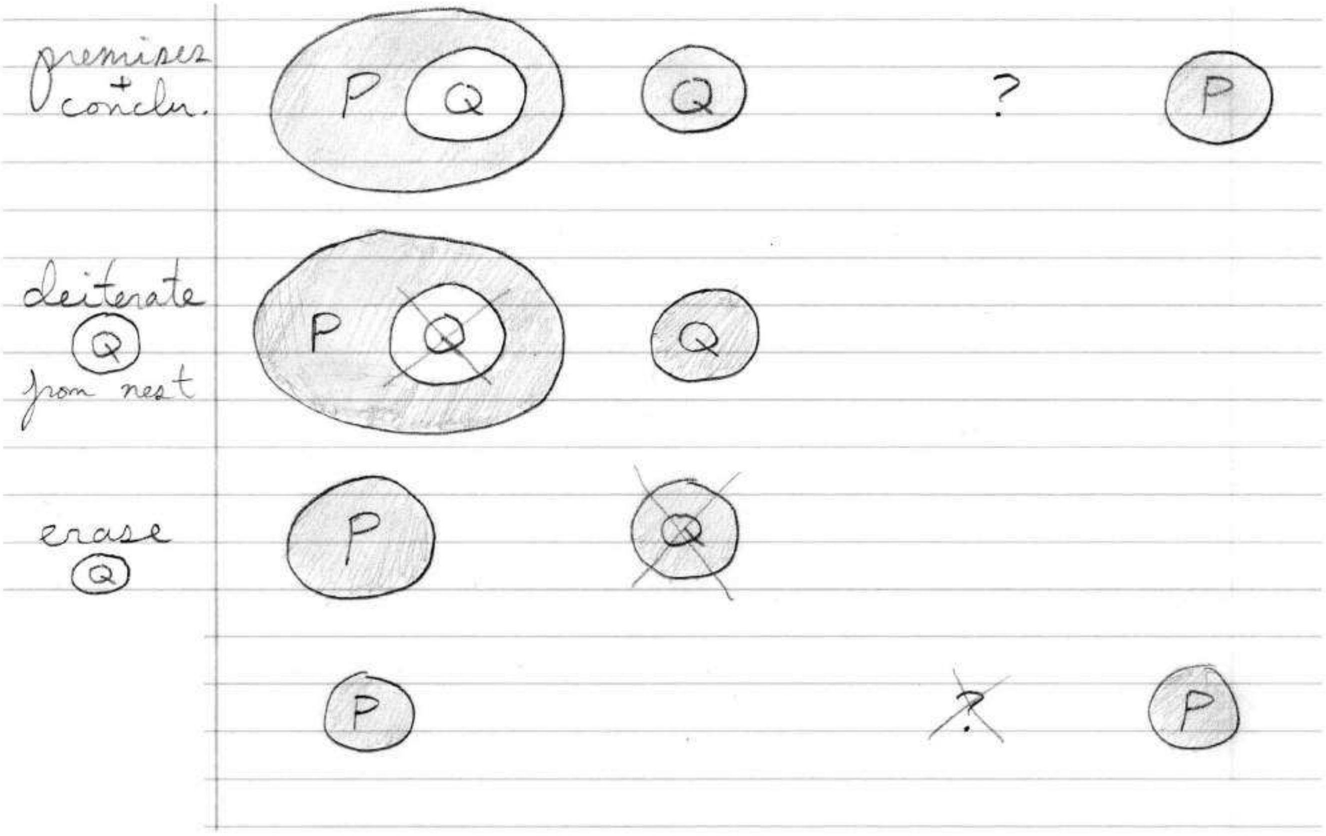
If we want to know whether the conclusion (after the "therefore") really follows, another way to find out is to assume that the conclusion does not follow, and then try to diagrammatically derive a contrapiction from this.

Indirect derivation of Modus Ponens



Derivation of Modus Tollens

"If she goes to the movies (P), then she wil miss her driving test (Q). Now, I happen to know that she did not miss her driving test (Q). So, I can infer that she did not go to the movies (P)."



Indirect derivation of Modus Tollens

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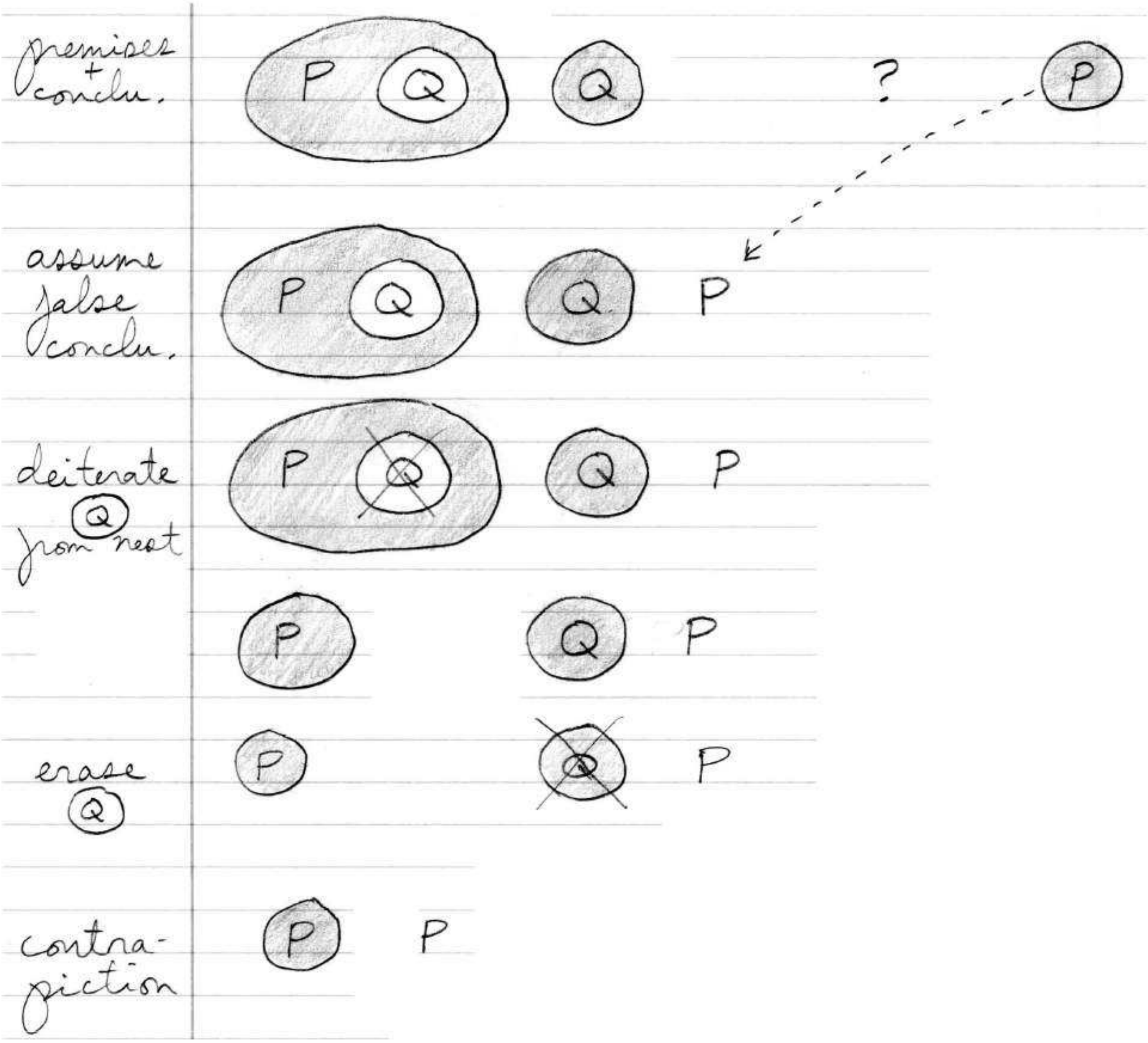
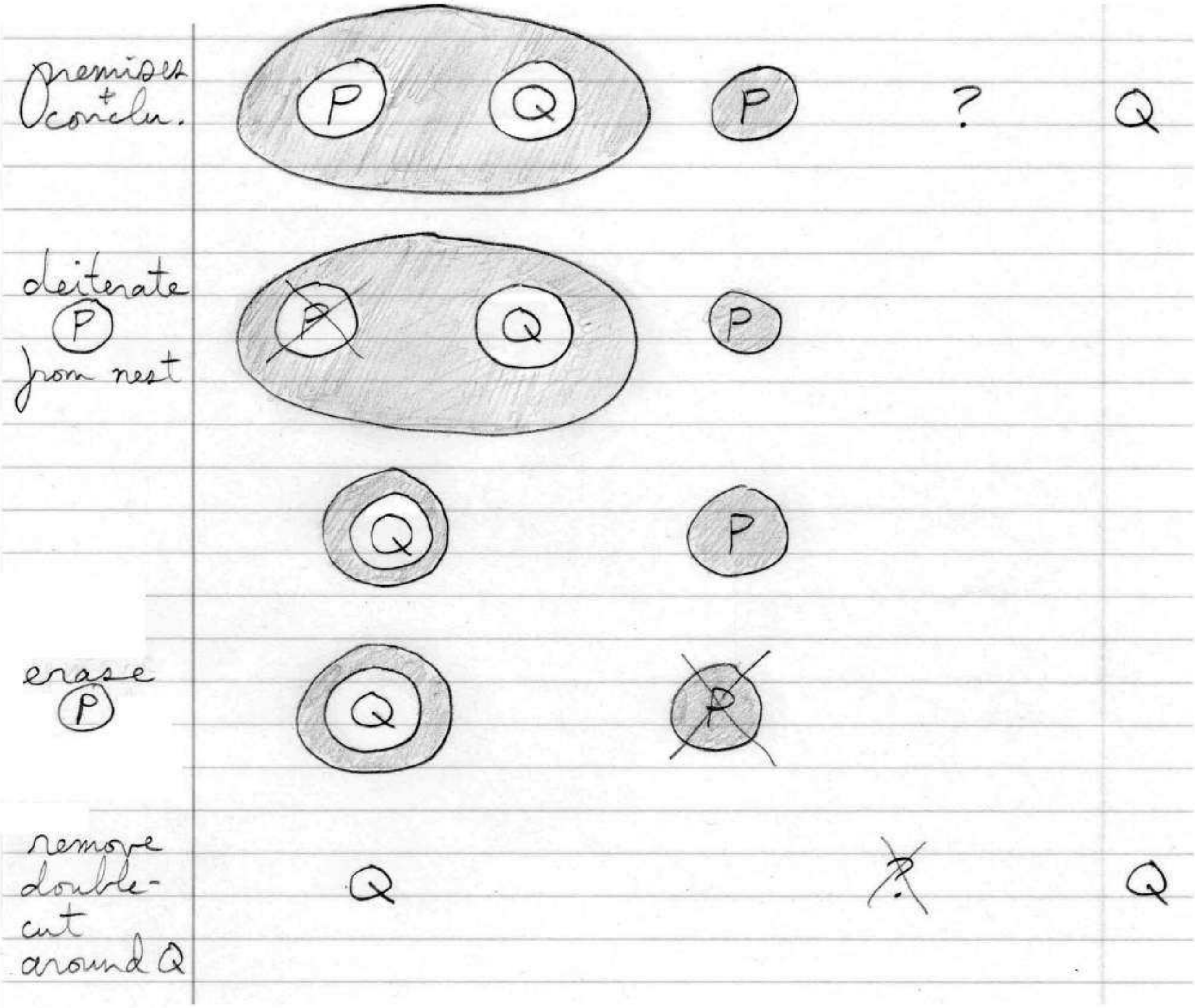


Derivation of disjunctive syllogism

"We are going to eat rice (P) or bread (Q). We are not going to eat rice (P). Therefore, we are going to eat bread (Q)."



Indirect derivation of disjunctive syllogism

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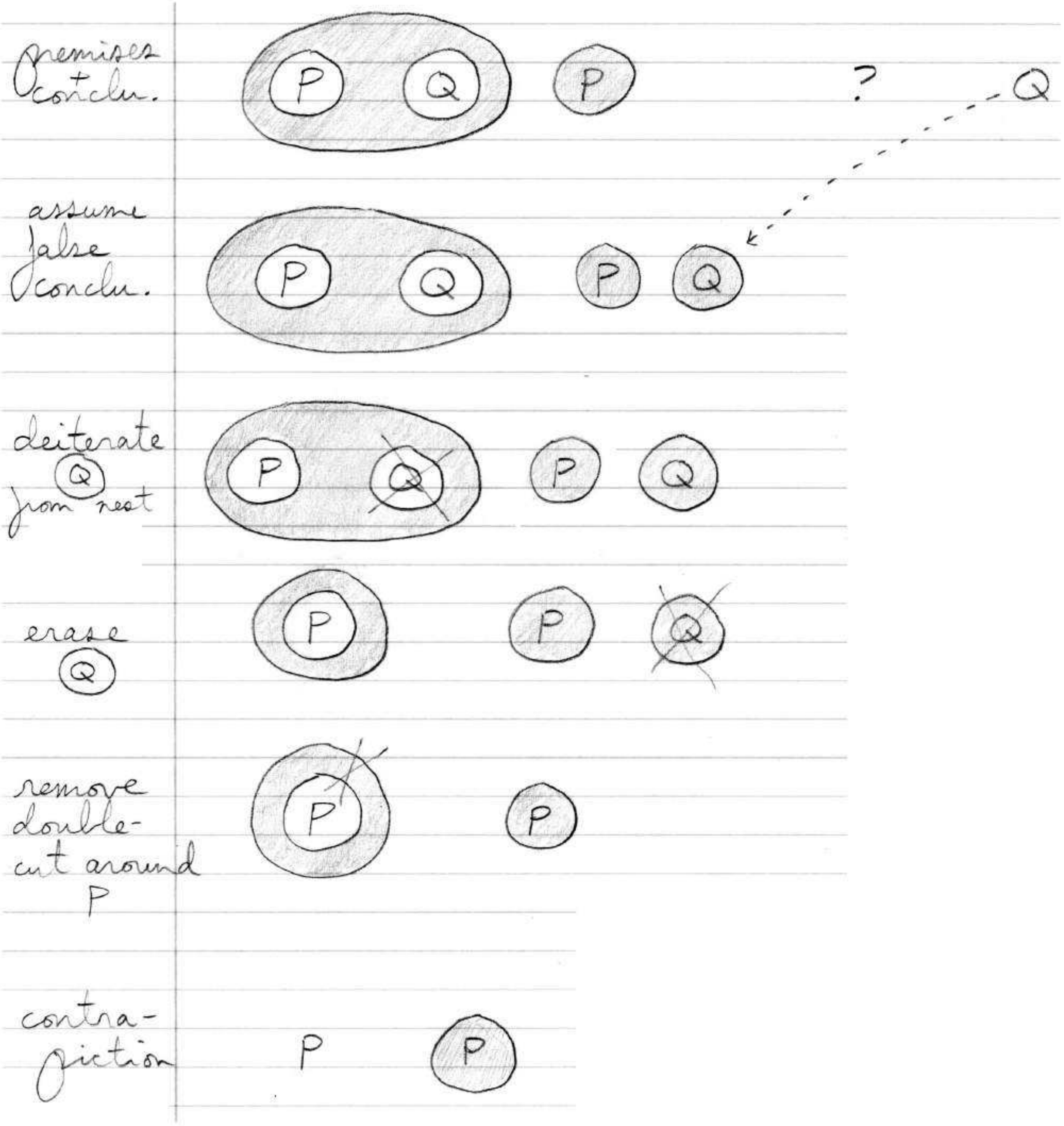
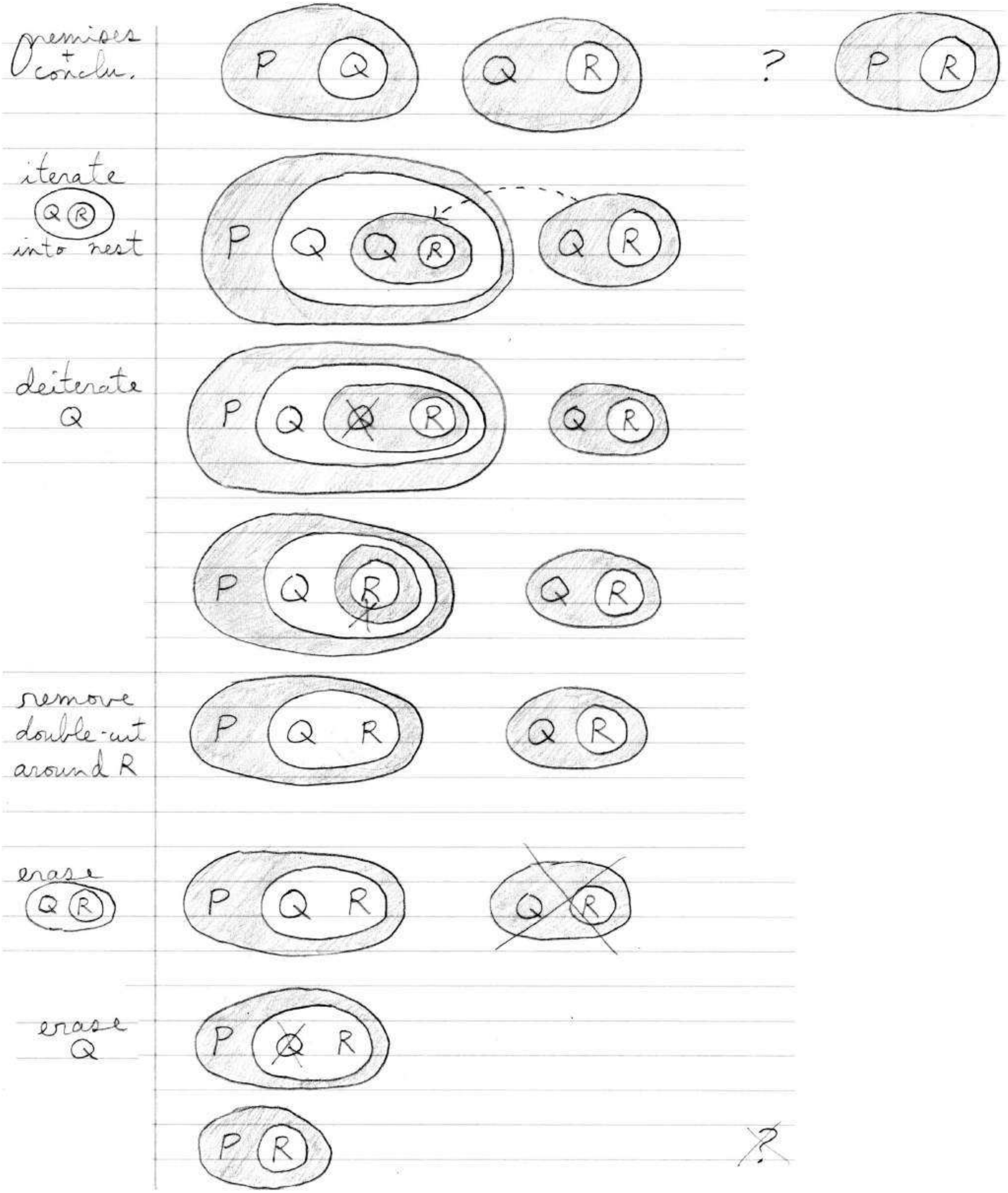


Derivation of hypothetical syllogism

"If you eat food (P), then you go to the bathroom (Q). If you go to the bathroom (Q), then you leave the kitchen at some point (R). We can conclude from this that, if you eat food (P), then you leave the kitchen at some point (R)."



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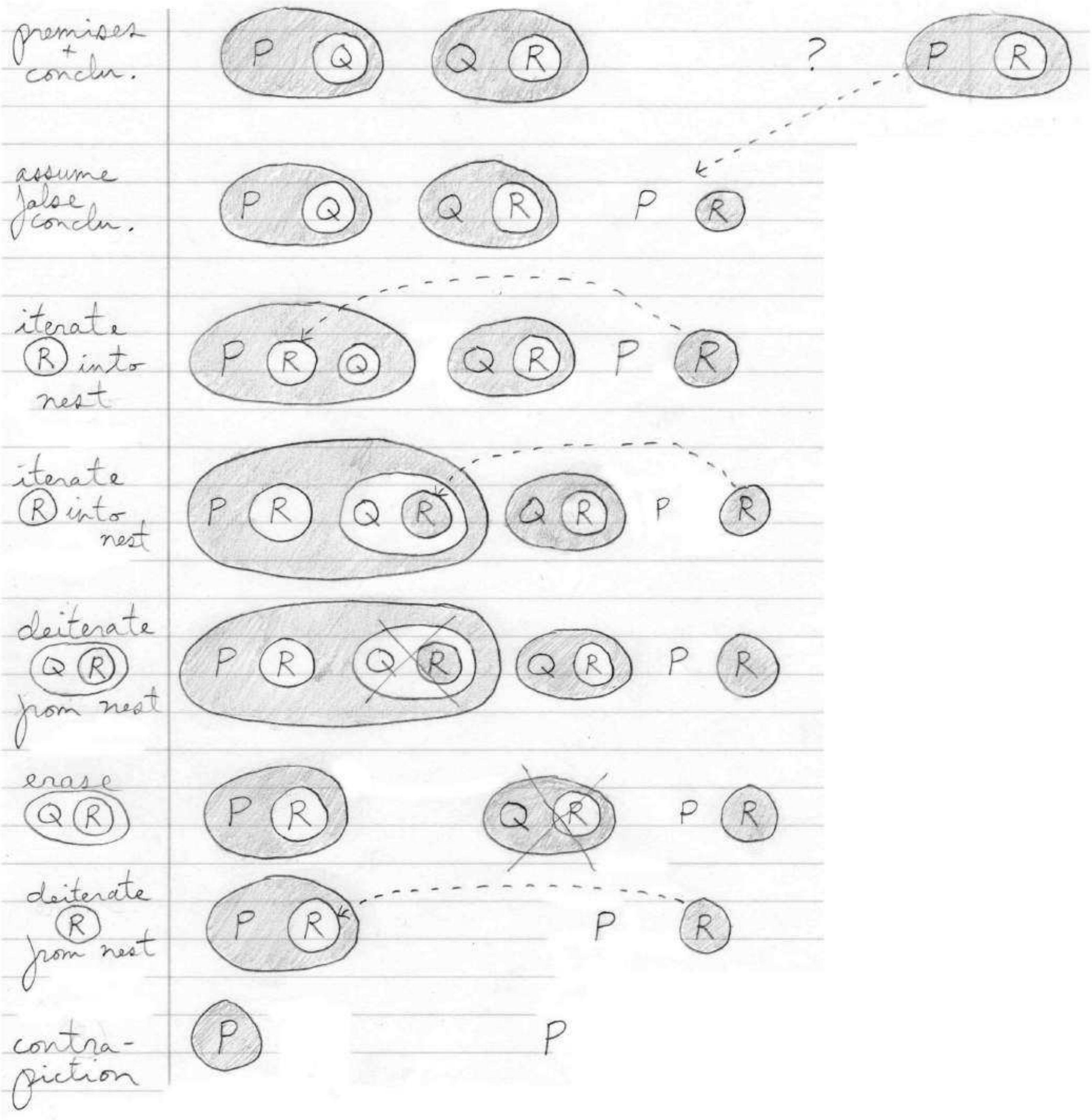
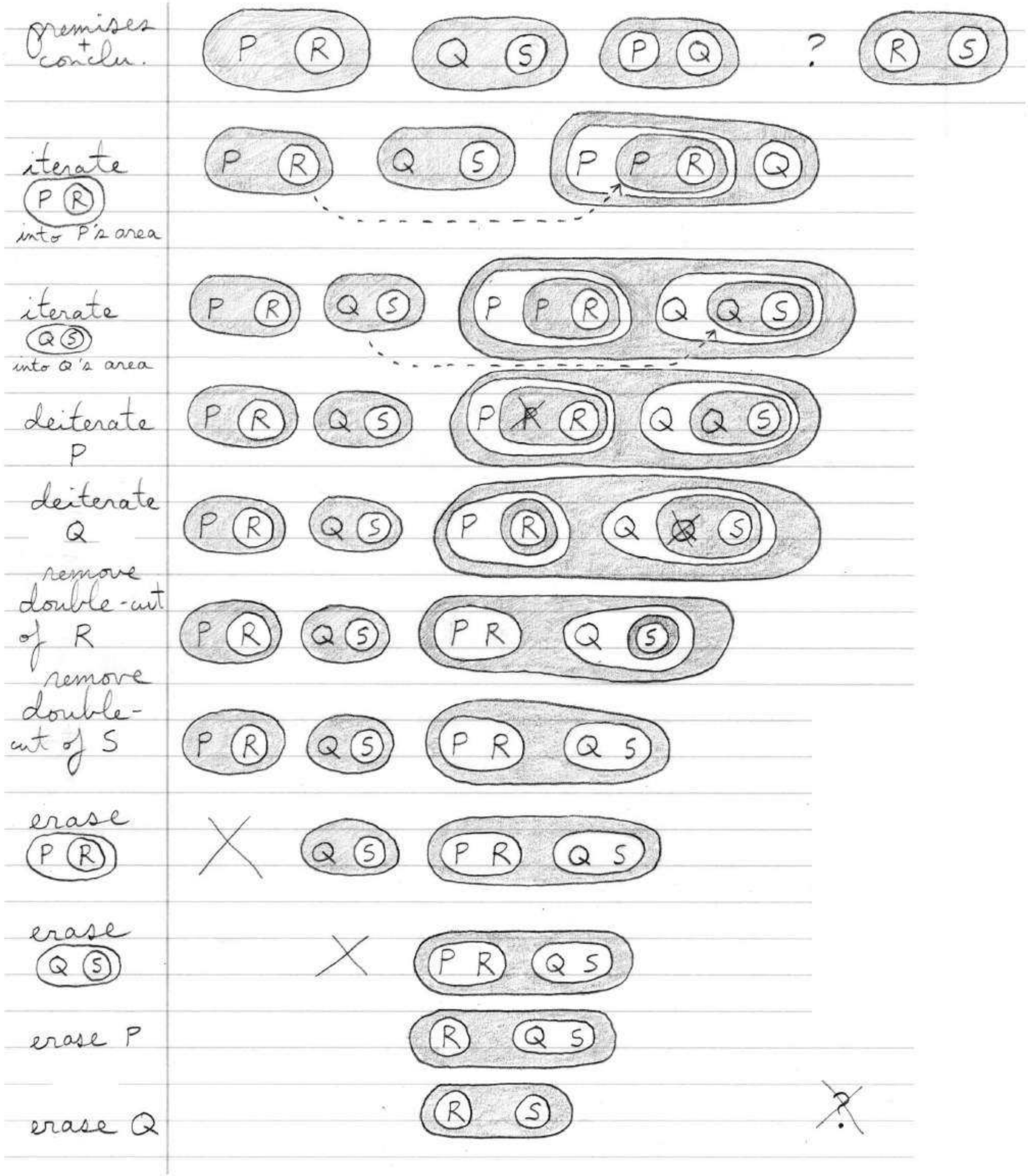


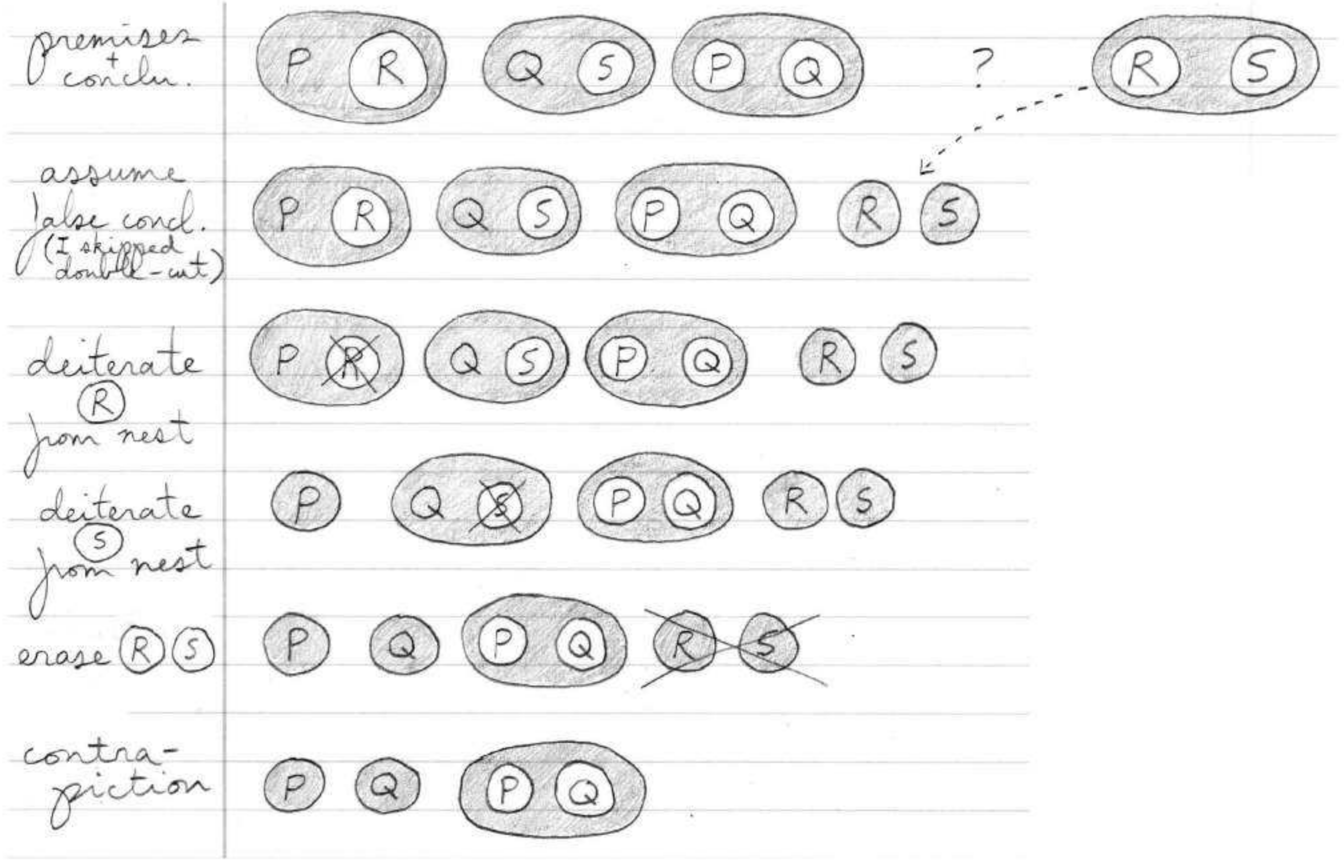
Derivation of constructive dilemma

"If he buys a television (P), then he will use up his savings (R). If he buys a car (Q), then he will max out his credit card (S). Now, we know that he will either buy a television (P) or buy a car (Q). Hence, we can be certain that he will either use up his savings (R) or max out his credit card (S)."



Indirect derivation of constructive dilemma

"If he buys a television (P), then he will use up his savings (R). If he buys a car (Q), then he will max out his credit card (S). Now, we know that he will either buy a television (P) or buy a car (Q). Hence, we can be certain that he will either use up his savings (R) or max out his credit card (S)."



The full range of Existential Graphs

- Four systems:

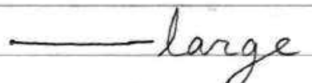
alpha ← looked at today	complete
beta	
gamma	
delta	unfinished

Glance the Beta Graphs

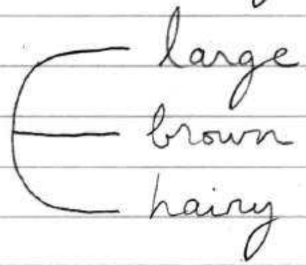
Beta graphs: First-order predicate
logic / quantifier logic

dot • means "something exists"

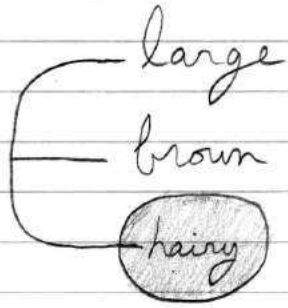
drag that dot into a "line of identity" (LI)
and connect it to a predicate



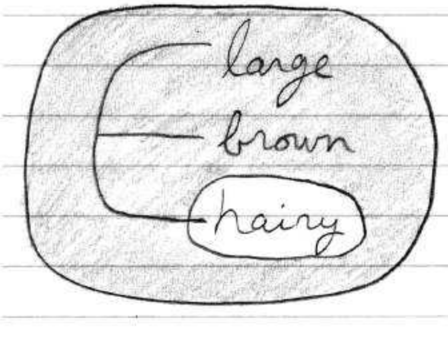
"something is large"



"There is something
that is large, brown,
and hairy."



"There is something
that is large,
brown, but not hairy."
(existential scope ↳ ∃)



"There is nothing that is
large, brown, but
not hairy."
(universal scope → ∀)

An important insight for semiotics...

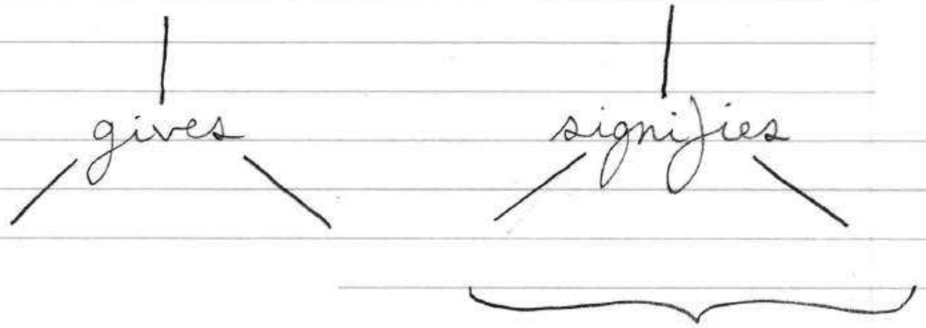
Some predicates have 1 "hook", e.g.:

— red

Some predicates have 2 hooks:

— father of —

Some predicates have 3 hooks:



this is one of Peirce's most lasting and important contributions to philosophy of signs (semiotics)

↓

signs are triadic relations

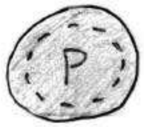
A glance at the Gamma and Delta Graphs

Gamma Graphs

"adds "broken" cut" →

possibly not-P

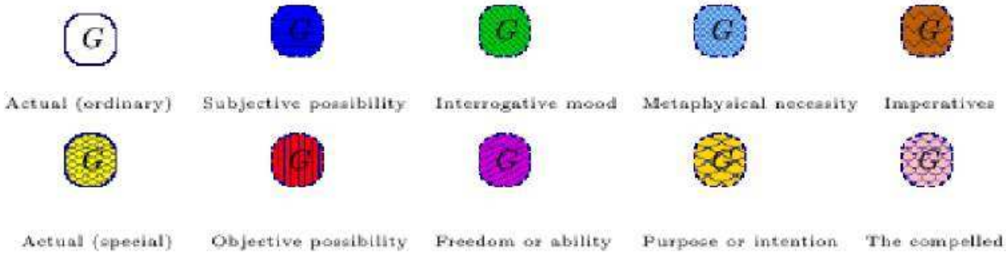
possibly not not-P
(possible that P)



not the case that possibly not-P
(necessary that P)

Delta Graphs

Colour-coded (let's not even go there...)



3.2.4.3. Tinctures with Identity Lines

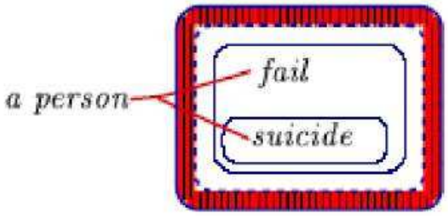
Combining LIs with modalities gives rise to a diagrammatic counterpart to a modal predicate logic. Now the *de dicto*/*de re* distinctions can be diagrammatised. How the 'cross-world' identity functions in such diagrams when quantification and modality are interspersed has not been discussed in the literature so far.

3.2.4.4. Example ("Peirce's Puzzle")

Consider the sentence

There is a person who will commit suicide if she fails.

The diagram for this is



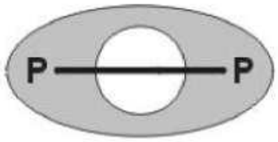
The sentence above is clearly different in meaning from

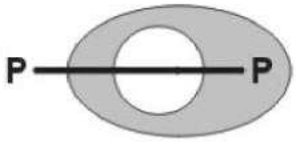
There is a person who will commit suicide if everyone fails.

Table 4. *EGIF and formulas for the above EGs*

Graph	EGIF	Formula
Left graph	[*x] [*y] ~[[?x ?y]]	$\exists x \exists y \sim(x=y)$
Middle graph	[*x] [*y] ~[(is ?x ?y)]	$\exists x \exists y \sim \mathrm{is}(x,y)$
Right graph	[*x] [*y] (P ?x) (P ?y) ~[is(?x ?y)]	$\exists x \exists y(\mathrm{P}(x) \wedge \mathrm{P}(y) \wedge \sim \mathrm{is}(x,y))$

As these examples show, an oval with a line through it can be read as negated equality. It is equivalent to the symbol ≠ in the algebraic notation, but it is so readable that there is no need for a special symbol. With a nest of two ovals, the graph on the left below denies that there are two Ps. The graph on the right asserts that there is exactly one P.





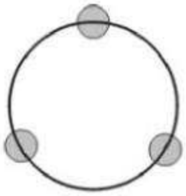
For the graph on the right, the outer part says that there is a P, and the shaded part denies that there is another P different from the first. Therefore, there must be exactly one P. Following is the EGIF:

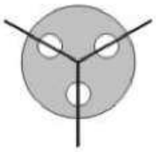
[*x] (P ?x) ~[[*y] (P ?y) ~[[?x ?y]]]

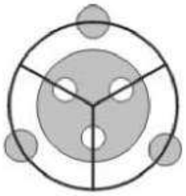
The direct translation of the EGIF to an English sentence or an algebraic formula would use two negations. But the double negation could also be read as an implication. Following are both translations:

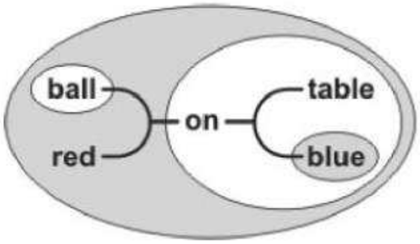
"There is a P, and there is no other P" — $\exists x(\mathrm{P}(x) \wedge \sim\exists y(\mathrm{P}(y) \wedge x \neq y))$.
"There's a P, and if there's any P, it's the same as the first" —
$\exists x(\mathrm{P}(x) \wedge \forall y(\mathrm{P}(y) \supset x=y))$.

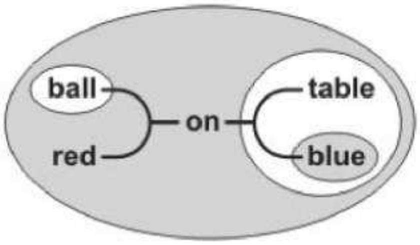
These examples can be extended with multiple lines of identity and negated equalities. The graph on the left below says there exist at least three things. The graph in the middle says there exist at most three things. The graph on the right, which combines the previous two, says there exist exactly three things:


at least 3


at most 3


exactly 3





Since each of these graphs has four ovals, there are several different ways of reading them. They make good classroom exercises for soliciting different readings from the students. The graph on the left can be read as a simple *if-then* sentence, as an *if* sentence with a disjunctive conclusion, or as a universally quantified sentence with a disjunctive body:

- "If there is a red thing that is not a ball, then it's on a table that is not blue."
- "If there is something red, then either it's a ball or it's on a table that is not blue."
- "Every red thing is a ball or is on a table that is not blue."

Since the graph on the right has two lines of identity (quantifiers) inside the shaded oval, the English pronouns must be supplemented with other words to distinguish them:

- "If a red thing that is not a ball is on something, then the latter is a table that is not blue."
- "If a red thing is on something, then either the former is a ball or the latter is a table that is not blue."
- "If a red thing *x* is on something *y*, then either *x* is a ball or *y* is a table that is not blue."
- "For every red thing *x* on something *y*, either *x* is a ball or *y* is a table that is not blue."

Letters or variables are used in the algebraic notation for logic or to supplement the pronouns of a natural language. Such supplements are necessary for a linear notation, but they are not needed in a graph notation that shows identity by direct connections.

These diagrams illustrate some fundamental principles of logic, which are true of any notation, but which are especially clear when expressed in EGs:

- Without any negations, the operators of conjunction and existence are sufficient to describe anything that exists. That includes all the experimental data of any branch of science, since negations can only be inferred, never observed directly.

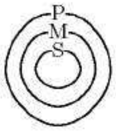


Fig. 6

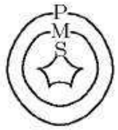


Fig. 7

Fig. 8

Fig. 9

The particular syllogisms are shown in Figs. 10–17. M is the middle term in all cases.

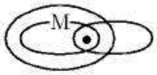


Fig. 10

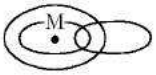


Fig. 11

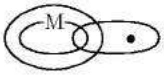


Fig. 12

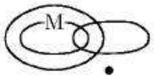


Fig. 13

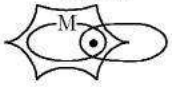


Fig. 14

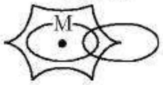


Fig. 15

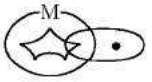


Fig. 16

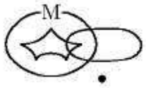


Fig. 17

In order to represent spurious syllogisms, we may make the dot upon the line of an oval to represent uncertainty whether the existing individual spoken of lies within or without the class that oval represents. Figs. 18–21 represent the spurious syllogisms. These figures seem to me useful in teaching. They show that in all cases one extreme (or the part of it spoken of) is outside, the other inside the middle term; and therein lies the force of the reasoning. The objection to Euler's diagrams that they are almost impracticable in complicated problem seems to me trifling; first, because their purpose is to show the nature of the syllogism, not to solve problems; and secondly, because any complicated problem is very readily broken up into a succession of problems with four terms each, or fewer.

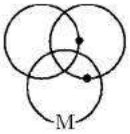


Fig. 18

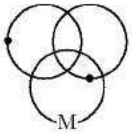


Fig. 19

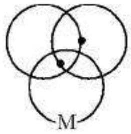


Fig. 20

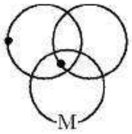


Fig. 21

The difficulty of representing the logic of relatives graphically, lies entirely in the circumstance that it is necessary to distinguish between

Some woman is adored by every catholic. and Every catholic adores some woman.

1. Let us take a sheet of paper, or blackboard, and say that anything we write upon it, unless we cut it off from the rest of the sheet by drawing an oval lightly around it, shall be considered to be affirmed by us, and therefore to be true.

2. If two propositions are written unenclosed on the sheet, both are affirmed.

3. We connect by a heavy line individuals whose identity we assert. Thus, Fig. 22, says "John adores Susan":

Fig. 22 John—adores—Susan

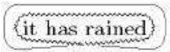
but Fig. 23 only says "John exists, and somebody adores Susan", for it differs from Fig. 22 only in not asserting the identity of John and the adorer of Susan.

Fig. 23 John— —adores—Susan

Hence, Fig. 24, asserts "Somebody adores somebody",

Fig. 24 —adores—

Fig. 30



Thus the less there is *odd*-ly *enclosed*, that is within an odd number of enclosing ovals, the more the proposition asserts. That is as it should be, since the less is excluded from the spread of depth, the greater the depth.

Now, the promised analysis of the proposition still hanging in the air, let us consider relative propositions. Fig. 31 asserts that there is something that is a catholic, and there is something that is a woman, and the former adores the latter. Fig. 32 asserts that if anything is a catholic there is something that adores a woman. Fig. 33 asserts that if anything is a catholic, that same individual adores a woman; that is, that every catholic adores a woman. On the same principle, Fig. 34 asserts that if anything is, it is a catholic, or everything is a catholic. Fig. 35 asserts that there is a certain woman and if anything is a catholic it adores *her*, or some woman is such that every catholic adores her.

Fig. 31

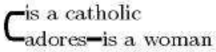


Fig. 32

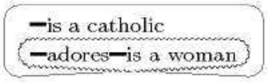


Fig. 33

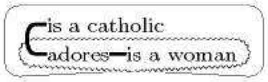


Fig. 34

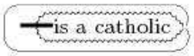
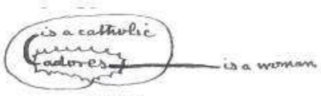


Fig. 35



[P.H.]

We see then that, interpreting these propositions in the exemplar "trope" (to revive an old Greek term of logic), that is, understanding that each speaks only of as many individuals as it contains of disconnected heavy lines, we are to determine these individuals, by beginning at the outside and going inward, and in that progress every heavy line we meet with evenly enclosed, refers to a *suitably chosen* individual, while every line which in its outermost part is oddly enclosed, refers to *any individual taken at pleasure.*

It is obvious that anything unenclosed, or evenly enclosed can be erased. Thus from Fig. 36, some catholic is obedient, we can infer Fig. 37 'something is a catholic and something is obedient' and thence again Fig. 38, 'something is a catholic'.

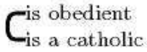


Fig. 36

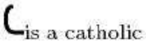


Fig. 37

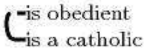


Fig. 38

In like matter, anything oddly enclosed can be inserted. Thus from Fig. 35 we can infer Fig. 39, some woman is adored by all obedient catholics.

Fig. 39

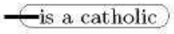


Fig. 40

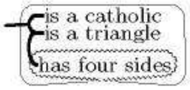


Fig. 41

This shows us what Fig. 40 must mean. For Fig. 41 follows from it. Namely, 'There is something which if it is a catholic and is a triangle has four sides'.

meaning. Thus, Figs. 46 and 47 have precisely the same meaning, namely,

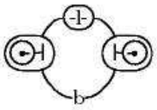


Fig. 46

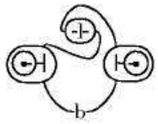


Fig. 47

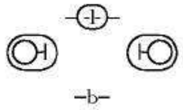


Fig. 48

"Either nobody loves anybody or everybody benefits everybody"; and therefore Fig. 48 ought to replace them both. Again, Figs. 33 and 49 have the same

Fig. 49

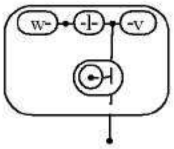
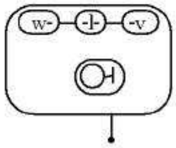


Fig. 50



meaning. For to say that some wise man loves some virtuous man is the same as to say that some virtuous man is loved by some wise man. Hence, both should be replaced by Fig. 50, where the attachment to the outer circle must be due to one of the loose ends of the dyads of nullity. So, Figs. 51, 52, and 53 have the same meaning, and should be replaced by Fig. 54.

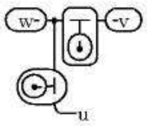


Fig. 51

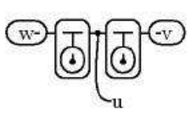


Fig. 52

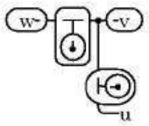


Fig. 53

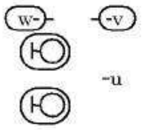


Fig. 54

In the third place, when, in consequence of omissions, a graph appears within two circles, the one immediately enclosing the other and nothing else, both ought to be dropped, as annulling each other. For in such a case, there are always two graphs either of which may be so doubly enclosed, without any difference in the meaning. For instance, Figs. 55 and 56 have the same meaning, that "All but the virtuous are loved by all the wise". Hence, they

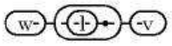


Fig. 55

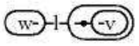


Fig. 56

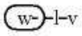


Fig. 57

Fig. 58

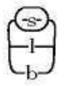


Fig. 59

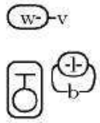


Fig. 60

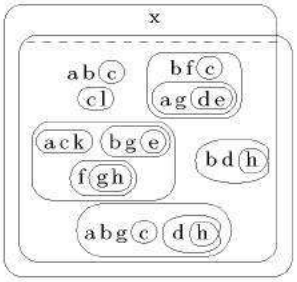
should be replaced by Fig. 57, which does not make meaningless distinctions, and thus avoids complications, and shows the real similarity in the modes of connection of l and v. On the same principle, Fig. 34 is to be replaced by Fig. 58; Fig. 36 by Fig. 59; and Fig. 38 by Fig. 60.

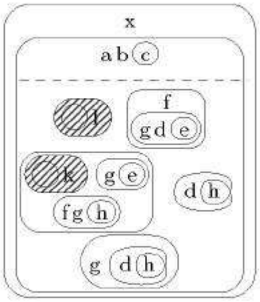
These simplifications consist in omitting features which diversify graphs without any corresponding diversification of meaning. The more diagrammatically perfect a system of representation, the less room it affords for different ways of expressing the same fact. Still further to carry out this idea, we ought, in the fourth place, when two triads of diversity are directly connected to shorten the bond of connection to nothing. Thus, Figs. 61 and 62,

x is scribed). In the upper region of each of the original replicas of the iterated enclosures scribe the graph selected, cancelling that graph throughout the lower region, as in Operation 3. In the upper region of each of the new replicas of the iterated enclosures scribe an enclosure containing only the graph selected, and throughout the lower region substitute a vacant enclosure for that graph, as in Operation 4.

When all these operations practicable have been performed, no alpha-contingent graph will remain in the lower regions. The graph remaining is to be understood as referring to the universe of alpha possibility, and to be accepted as true.

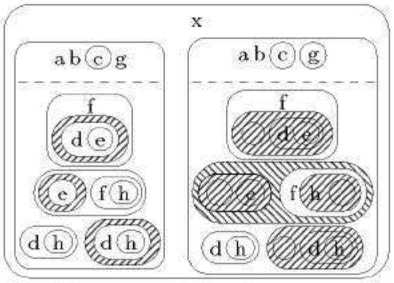
Example.



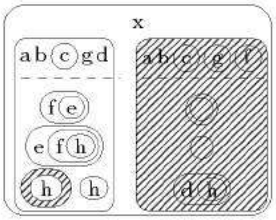


I first deal with abc together.

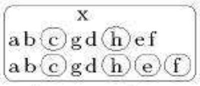
Practically I should next deal with (d(h)) although this would violate the rule. But when one has a mastery of the subject, one can see whether or not it would make any difference. But I will pursue the rule and will next deal with g.



On the g side, we can now deal with d by Operation 3. On the right we can deal with f by Operation 4. We thus get



The vacant enclosure on the right destroys the entire enclosure in which it is and it is easy to see that the h will destroy the other showing that the graph is impossible. But if the h had not occurred at the bottom of the original graph, we should have had the graph



showing that if the original graph were true abgd would be true c and h false and e and f both true or both false. [end S-32]

Existential Graphs: The Initial Conventions

MSS S-29, S-33.

1. It is agreed that whatever is written on a certain sheet shall represent objects reacting in pairs in a certain universe. Thus, Hamlet may be written if Hamlet reacts with the objects of that universe; otherwise Hamlet must not be written. "Man" can be written if there is a man in the universe. A

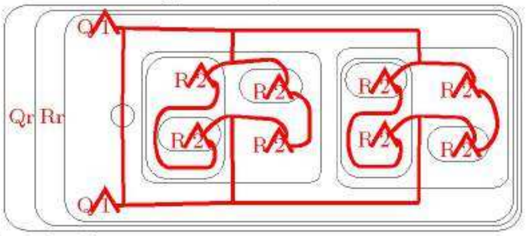
In order to ~~represent~~ make such forms of statement in graphs, I introduce ~~the following~~ certain spots which I term Potentials. They are shown on this diagram:

THE POTENTIALS.

A—p	means	A is a primary individual
A—q	means	A is a monadic character, or "quality"
A—r	means	A is a dyadic relation
A—s	means	A is a legisign
	means	A is a ~~monad~~ graph
	means	B possesses the quality A
	means	B is in the relation A to C
	means	B is in the triadic relation A to C for D.

It is obvious that the lines of identity on the left-hand side of the potentials are quite peculiar,[6] since the characters they denote are not, properly speaking, individuals. For that reason and others, to the left of the potentials I use selectives not ligatures.

As an example of the use of the potentials, we may take this graph, which expresses a theorem of great importance:[7]



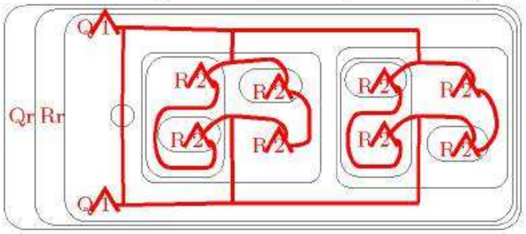
The proposition is that

> for every quality, Q, whatsoever, there is a dyadic relation, R, such that, taking any two different individuals both possessing this quality, Q, either the first stands in the relation R to some thing to which the second does not stand in that relation while there is nothing to which the second stands in that relation without the first standing in the same relation to it, or else it is just the other way, namely that the second stands in the relation, R, to which the first does not stand in that relation while there is nothing to

---

[6] **[Alt. S-31:]** ...since the characters which they denote are not properly speaking individuals. For that reason and others, I use selectives instead of lines of identity on the left of the potentials.

As an example of the use of the potentials, we may take this graph.

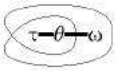


This states a theorem of immense importance which I will prove to you in another lecture, if there is time. It states that [end]

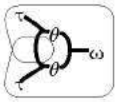
[7] [Marginal note, crossed out:] This diagram would be less confusing if the angles weren't so sharp and the lines so straight and parallel and right-angled.

more direct Secondness, and if there is a dyadic relation r such that every point of the line is in this relation, r, to one or other of the individuals A and B, then there is *ipso facto* a point which is in that relation r, to both A and B.

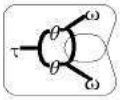
Beta Part.



There is a point in a dot.

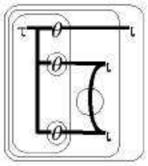


If in any dot there is a point A or a point B, these are identical.

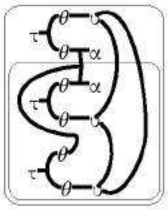


No point is in two dots.

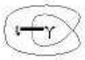
Taking any two individuals A and B and any relation R either there is a point of the line that is not R to A and is not R to B or there is a point of the line that is R to A and R to B.



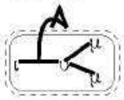
If there is a line of identity there are two individual lines of identity such that every point of the line is either in the one or in the other.



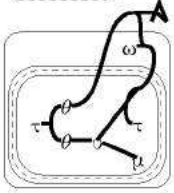
The question is whether this is not deducible from the last.



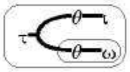
Every line of identity is a graph.



It is always permitted to scribe a line of identity on the sheet of assertion with its extremities attached to blanks.



If a dot is on the sheet of assertion it is permissible to attach to it a line of identity with the other extremity at a blank.

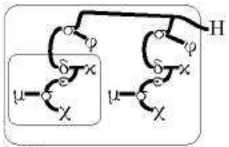


At whatever point on a line of identity there is a dot.

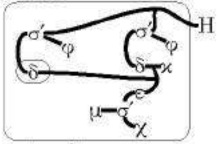
If a line of identity is scribable on the sheet of assertion, it is permitted to insert a point of teridentity upon any point of it.

Diagrams for Lecture 5.

2. The sheet of assertion if, by virtue of any fact, it carries an entire graph containing an enclosure whose area carries an entire graph which does not contain anything but blanks, then whatever permission permits whatever graph to be carried on the sheet contains that enclosure.



better



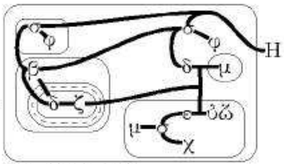
not quite sure

Gamma Graphs 3.

On the sheet of assertion if the entire graph is permitted to contain any graph not a blank nor a cut whose area carries as its entire graph

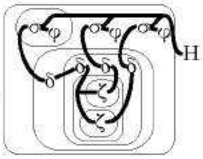
Use “sheet” for sheet of assertion
Use “carry” for “carry as the entire graph, a replica
Use “contain” for “contains as a part of the area”

3. The sheet if is it permitted to carry a graph$_1$ containing a graph$_2$ not a blank and not a cut whose area carries a blank is permitted to carry any graph replica not differing from that graph-replica$_1$ in any other respect than that it does not carry that graph-replica$_2$.



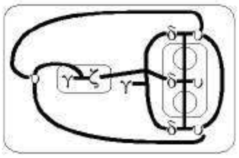
not quite sure

4. If the sheet of assertion is permitted to carry a graph-replica and is permitted to carry a second graph-replica and there is a third graph-replica that it is not permitted to carry than the third contains some part such that if anything is contained in it it is not a coreplica of any thing contained on the second and is not a coreplica of anything contained in the third.

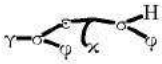


Gamma Graphs 4.

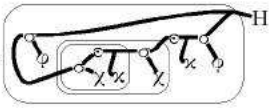
5. Any line of identity joining any two points A and B, C being any third point, is a coreplica of any graph-replica containing a line joining A and C and containing a line joining B and C and not containing anything else but a blank.



6. It is always permissible to draw a cut on the sheet of assertion with *some* graph in it.



7. If it is permissible that the sheet of assertion should carry a cut whose area carries nothing but a cut, then it is permissible that the sheet of assertion should carry the graph carried on the area of the last cut.



Gamma Graphs 5.

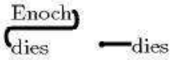
For example, from Fig. 148 we can infer Fig. 155, meaning "Enoch is a man and is either not a man or else dies".

From Fig. 76 can be inferred Fig. 98.

This may be called the rule of simple omission.

Third Rule. Within an even number of ovals anything may be illatively inserted which is already in the graph not within any other ovals.

Fig. 156



For example, from Fig. 149 can be inferred Fig. 156, or "Enoch dies and somebody dies".

Corollary 9. Within an odd number of enclosures anything can be illatively omitted which is elsewhere in the graph not within any other ovals.

Fig. 157

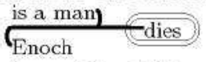


Fig. 158

For example from Fig. 155 can be inferred Fig. 157. And thence by the rule of simple omission can be inferred Fig. 158.

Corollary 10. Within an even number of enclosures, anything may be illatively carried into two ovals one within the other.

Fig. 159

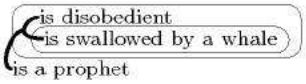


Fig. 160

For example, from Fig. 159 "there is a prophet who if he is disobedient is swallowed by a whale", we can infer Fig. 160 "somebody if he is disobedient is a prophet swallowed by a whale". This is a bad example. Take the following.

Fig. 161

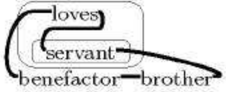


Fig. 162

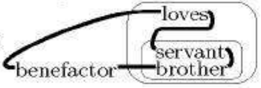


Fig. 161 means "There is a person who benefits a brother of a person, yet who loves only servants of that person (if he loves anybody)". Hence, we can infer Fig. 162, "There is a benefactor of a man who loves (if at all) only servants of a person of whom that man is a brother".

Corollary 11. If an oval contains nothing but ovals and each of these contain the same certain verb, then that verb can be taken out of all those ovals.

Fig. 163

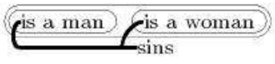


Fig. 164

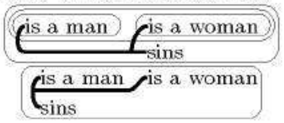


Fig. 165

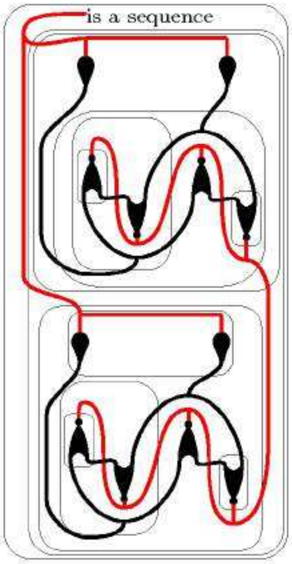
For example, given Fig. 163, meaning "Either some man sins or some woman sins", we can infer Fig. 164, meaning "Something which sins is either a man or a woman".

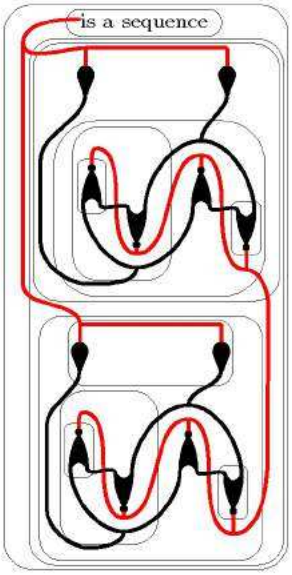
This is proved by contraposition. For the denial of Fig. 164 is Fig. 165.

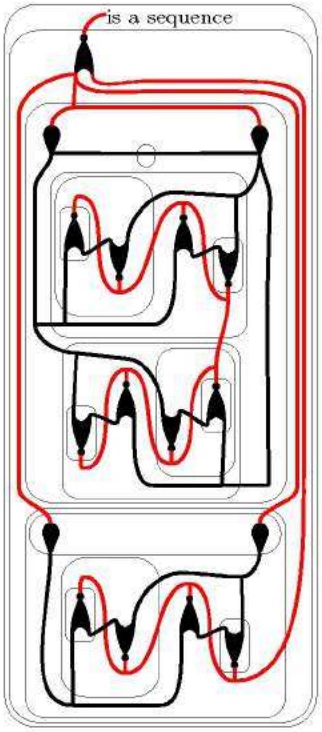
Rule of Existential Graphs. Any marked point on a cut is to be regarded as being in the place of the cut.

The doctrine of forks belongs to Rule III.

I. *Rule of Erasure and Insertion.*
  i. In even enclosures,
    1. Any line of identity may be broken at any point. Any part of a line from a loose end may be erased.
    2. Any detached partial graph may be erased.

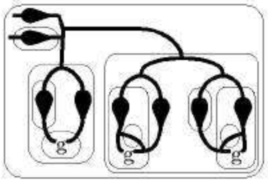




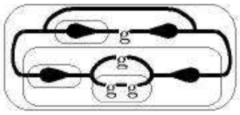


But the above leaves out an alternative, namely that it may be the *second* that is in the relation to the *first*.[138r]

1898 Aug 6 [139r]. The whole numbers form a sequence which has this property, where —g— means —exceeds—. The universe is that of the numbers.



Hence follows the important Fermatian theorem:



Addition of whole numbers defined by

$$0 + 0 = 0$$

Let Ex be no. next greater than x

$$x + Ex = E(x + y)$$
$$Ex + y = E(x + y)$$

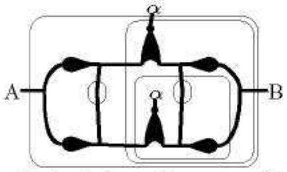
Multiplication is defined by

$$0 \times 0 = 0$$
$$x \times Ey = (x \times y) + x$$
$$Ex \times y = y + (x \times y)$$

Involution

$$x^1 = x$$
$$X^{Ex} = x \cdot x^y$$

1898 Aug 8 [140r].



As are at least as small in multitude than the *B*s.

1903 June 11 [236r]. Principle of contradiction (not) Nothing is other than itself

Fig. 13 asserts that some cat is A, which is to be understood as the proper name of this individual exclusively, and that cannot be precious.

Fig. 14 conveys the same meaning,—that a certain cat cannot be precious, without mention of the proper name.

Fig. 12

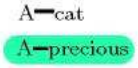


Fig. 13

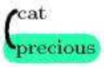


Fig. 14

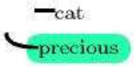


Fig. 15

Fig. 15 asserts that there is a cat and there is something that cannot be precious.

Fig. 16 asserts that there is a millionnaire and that there is a certain person to whom it is impossible that that millionnaire should give any cat unless that cat be precious.

Fig. 16

Fig. 17

Fig. 18

Fig. 17 asserts that there is a certain millionnaire and a certain cat and if that millionnaire gives that cat to anybody, it must be either that the cat is precious *or* the person to whom it is given must be a zoölogist. Fig. 18 differs from Fig. 17 in asserting that if the millionnaire gives the cat to anybody, the cat must be precious *and* the person to whom it is given must be a zoölogist.

Fig. 19 differs from Fig. 18 in asserting the consequence not if the millionnaire gives the cat to *anybody*, but only if he gives the cat to a certain existing person. If "cat" were on the *area* of the outer cut instead of on its *place*, it would mean that if the millionaire gives *any* cat, etc. I beg the reader will practice diagrammatizing a few statement by himself; for that is the only way to understand such a system. Having done so, he will be able to answer this question: Precisely what is the difference, if any, between the interpretations of Fig. 20 and Fig. 21.

Fig. 19

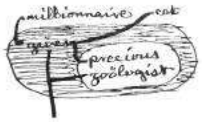
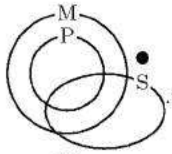


Fig. 20

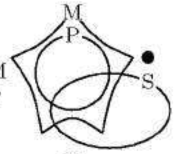
Fig. 21

Fig. 20 asserts that there is a certain millionaire and there is a certain lady to whom if that millionaire gives any cat he must be a zoölogist. Fig. 21 asserts that there is a millionaire and there is a lady, and there is also a lady who *may* be a different person or may be the same (it is not stated which). Call the ladies for convenience Lady A and Lady B. Now if the millionaire in question gives Lady A a cat, then Lady B must be a zoölogist. Are these two assertions the same in substance or not? One will naturally say that they are not the same in substance, because Fig. 21 might be true if the millionaire was entirely unacquainted with any lady zoölogist, but would only be induced to give a certain lady a cat because he knew that she had a sister who was a zoölogist, in which case Fig. 20 would not be true. But those pragmatists who maintain that a possibility has no reality if it never be realized, will be forced to say that the meanings of Fig. 20 and of Fig. 21 are precisely the same. For the precise denials of them have the same meaning. These denials are scribed in Figs. 22 and 23 respectively.



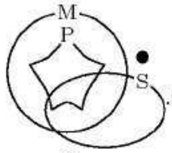
Any P is M
Something is neither S nor M
∴ Something is neither S nor P

Fig. 12



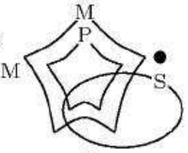
Any M is P
Some M is not S
∴ Some P is not S

Fig. 13



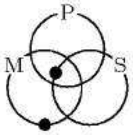
Everything is either M or P
Something is neither S nor M
∴ Some P is not S

Fig. 14



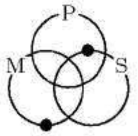
Any M is P
Some S is not M
∴ Some P is not S

Fig. 15



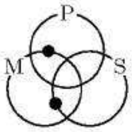
Something is either M or P
Something is neither S nor M
∴ Some S is not some not-P

Fig. 16



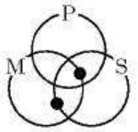
Something is neither M nor P
Some M is not S
∴ Some P is not S

Fig. 17



Some M is not P
Some S is not M
∴ Some S is not some P

Fig. 18



Some M is P
Some S is not M
∴ Some S is not some P

Fig. 19

A highly practical improvement on this method has been invented by Venn (*Symbolic Logic*, at *xi*). If four classes are involved in the premisses, Fig.20,  is first drawn, representing the sixteen classes into which the four classes by affirmation and negation divide the universe. Then those that the premisses render non-existent are blackened. C.S.P.

. . . are blackened, while those which the premisses positively declare existent can receive dots. If the premises involve six classes, the sheet of paper can be divided into four compartments and Fig.20 can be drawn in each of them. Various algebraic methods are easier in very complicated cases; but there is no danger of forgetting how to apply Venn's graphical method. These extreme rarity of problems which at all try its powers would probably cause it to be preferred in practice. For the logic of relatives, two systems of graphs have been proposed, the *entitative* and the *existential*, which have considerable theoretical interest. Existential graphs depend upon the following conventions:

A sheet of paper is taken to represent the universe of truth, and any proposition written on it, unenclosed, is represented as true. The enclosure of a proposition in a lightly drawn oval is understood as cutting it off from the universe of truth, so that it is represented as false. A heavily drawn line signifies that all the points on it are identical; and any point on it denotes an existing individual, not designate, where some branch of the line abuts upon its proper name. A heavy line is enclosed by an oval only if every point of it is entirely within the oval.

For example, the following, Fig.21,

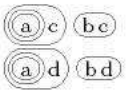
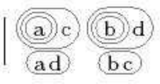
Fig. 21 What the Bible says

Supposed to be written on the field of truth means that all the Bible says is true.

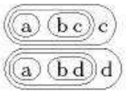
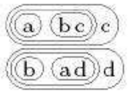
Fig. 22 Enoch's fate was singular.

Fig 22 means that that proposition is true.

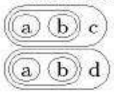
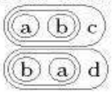
Fig. 23 (What the Bible says) Enoch's fate was singular.

Then iterate the other enclosure so that a letter that comes to be thrice enclosed shall also be once enclosed; thus:

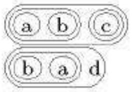



Then deiterate thus:

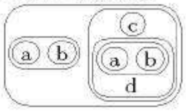



These are now just alike and I no longer need the double columns.

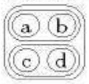
Put double cut round one of the singly enclosed letters



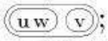
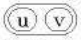
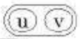
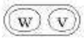
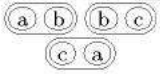
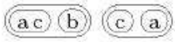
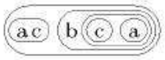
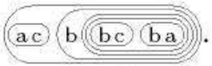
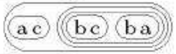
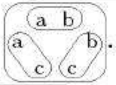
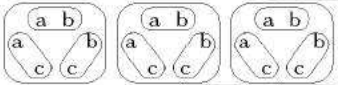
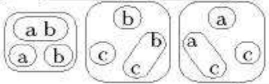
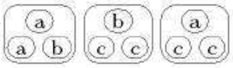
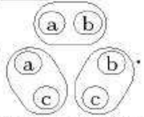
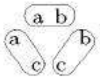
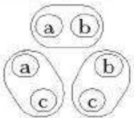
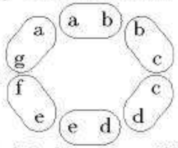
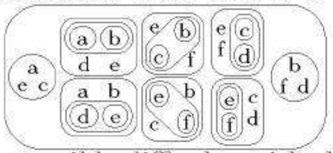
Now iterate



Now deiterate



which is $\overline{A}$.

Example 5. You saw, in the solution of the last example, that, whatever graphs u, v and w may be, from  follows ; so that the former is what is called the "condition" or "sufficient condition" of the latter. That it is also the *requisite* condition or "*condicio sine qua non*" is shown by simply iterating  and then from one instance deleting the evenly enclosed u, and from the other deleting the w, giving   so that   and  are logically equivalent. Therefore,  is logically equivalent to . But from this follows . For, first, iterating we get  and iterating twice again . Second, deleting the evenly enclosed b that stands alone, we get  and third, removing the double cut, we get  or . On the other hand twice iterating this, thus  and then simply deleting evenly enclosed letters we get  and by similar erasure we get  whence by deiteration . We thus find  and  to be logically equivalent. There are some numbers of letters exceeding 3 for which closely similar and symmetrical equivalences hold. But it is not always so. Thus  is equivalent to 

I think you will have no further possible difficulty with these examples; but [here] I will give the answers to the questions.

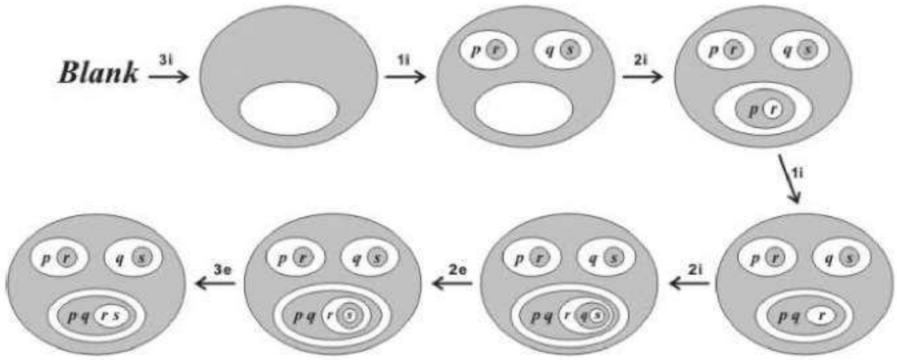
The proof in EGIF is more complex because of the need to relabel identifiers:

0. Starting graphs: [*y] (t ?y) ~[[*x] ~[(P ?x)]]
1. By 2i, iterate [*y] and change the defining label *y to a bound label ?y in the copy:
   [*y] (t ?y) ~[[?y] [*x] ~[(P ?x)]]
2. By 1i, insert the coreference [?x ?y]:
   [*y] (t ?y) ~[[?y] [*x] [?x ?y] ~[(P ?x)]]
3. Relabel *x and ?x to ?y, simplify [?y ?y] to [?y], and deiterate copies of [?y]:
   [*y] (t ?y) ~[~[(P ?y)]]
4. By 3e, erase the double negation: [*y] (t ?y) (P ?y)

In the *Principia Mathematica*, Whitehead and Russell proved the following theorem, which Leibniz called the *Praeclarum Theorema* (Splendid Theorem). It is one of the last and most complex theorems in propositional logic in the *Principia*, and the proof required a total of forty-three steps:

$((p\supset r) \wedge (q\supset s)) \supset ((p\wedge q) \supset (r\wedge s))$

With Peirce's rules, this theorem can be proved in just seven steps starting with a blank sheet of paper. Each step inserts or erases one graph, and the final graph is the statement of the theorem.



After only four steps, the graph looks almost like the desired conclusion, except for a missing copy of *s* in the innermost area. Since that area is positive, the only way to get *s* in there is by iterating some graph that contains *s* and erasing the parts that are not needed. Following is the EGIF version of the proof:

1. By 3i, draw a double negation around the blank: ~[~[ ]]
2. By 1i, insert the hypothesis in the negative area:
   ~[~[(p) ~[(r)]] ~[(q) ~[(s)]] ~[ ]]

3 take-away insights

Contrary to mainstream ...

① Logic doesn't have to be <u>all</u> symbolic

mix

conventional symbols for relata	non-conventional similarity-based icons for relations
(P, Q, etc.)	("and" as juxtaposition instead of ".")

② Graphs are transformed, not re-written. So, Diagrams can give us "a moving picture of the action of the mind in thought" (MS 298.1, 1905)

"[...] all deductive reasoning [...] involves an element of observation; namely, deduction consists in constructing an icon or diagram the relation of whose parts shall present a complete analogy with those of the parts of the objects of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts." ("On the Algebra of Logic," 1885 CP 3.363)

③ This view of deduction as diagramm. manip./observ. solves puzzle of how necess. reasoning ...

... contains all its results as soon as it starting assumptions + rules are made

yet

... "presents as rich and apparently unending a series of surprising discov. as any observ. science" (CP 3.363) →

Existential Graphs cookies, on the baking sheet of assertion

(Prepared by my partner, photographed by Cathy Bruce)