

Fuzzy Cookie-Cutter Classes

"Nothing is more important for teaching us to understand the concepts we have than constructing fictitious ones."

[Wittgenstein, *Last Writings*, Vol. I, par. 19.]

According to what might be called the 'empiricist' theory of logical, mathematical, and metaphysical necessity, necessity is produced by definitions, conventions, and practices. "Convention is the mother of necessity."

Correspondingly, one might say, the 'empiricist' theory of fuzziness is that it is the result of a gap, or incompleteness, in the rules, the 'grammar,' governing our language. That is to say, it is due to something we haven't done - an inability or abstinence on our part.

Are the empiricists right about these things? And, in particular, are they right about fuzziness?

In an effort to understand fuzziness better, let's think about certain 'fuzzy cookie-cutter' classes. A cookie-cutter class is a non-natural class. Its extension is 'cut out' more or less arbitrarily (in some sense) from the 'stuff' of the natural world. A 'fuzzy' cookie-cutter yields a fuzzy, non-natural, extension.

The G-Rock Creatures

They live on planet X, and have lived there for several thousand years. They are artifacts that, for most of the time since their origin, have survived by building rough replicas of themselves when replacements are needed. According to the standard version of the story, they are not very bright. Their principal occupation, and source of satisfaction, is identifying, gathering, and storing-up, 'G-rocks.'

A G-rock is a rock (presumably formed by natural geological processes) that goes through the G-creatures 'Big Screen' but fails to make it through the 'Small Screen.' That's more or less all there is by way of a definition of the sort of rocks they go for.

The rule, and the practice, that determines whether or not a given rock is a G-rock is a bit vague. Is a rock a G-rock if it manages to get through at least one creature's Big Screen at least once on a more or less standard roughly twenty second shake, and fails to get through at least one creature's Small Screen at least once under similar more or less standard conditions? [No answer.] Is it a non-G-rock if it fails to get through some one's Big Screen at least once under standard conditions? [No answer.] It is, I think, this vagueness that

generates the fuzzy fringe of the extension (and intention) of the term 'G-rock.'

Pretend some of the gods get interested in G-rocks. Zeus and Hera are considering the extension of the class. Where does the fuzziness start, and where does it fade out? How is this to be determined?

[Note that the class might drift as time passes. Perhaps, after twenty thousand years, none of the original three hundred G-rocks would pass the test. For instance, they might all be too small. The weave of both the Big Screens and the Small Screens have gradually gotten that much larger.

The gods are not interested in incoherent 'classes.' To avoid this problem, they study the extension of the class fixed (or determined) by the G-rock practice as it was at a particular time among certain creatures (e.g. such and such a five creature crew as it was last Tuesday.)]

The fuzziness (or, at least, the basic, first-order, fuzziness) around the class of G-rocks is generated by 'conflicting' test results. For instance, a given rock gets through the Big screen about fifty percent of the time, and never gets through the Small screen. Is it a G-rock? [No answer] The creatures don't know what to do with such rocks. They only have three classes to work with, and this rock doesn't quite go anywhere. Perhaps they hide it, and never mention it again. I give the claim that this is a G-rock a fuzziness score of .5. Obviously the claim that it isn't a G-rock gets the same score.

Supplementary Practices - Optional Attachments

It is permissible to believe that over the centuries the G-rock practice has acquired certain additions and refinements that make it 'work better.' The practice, has, in effect, been worn smooth - like a pebble in a river-bed.

Thus, for instance, it is an annoying fact that any alleged G-rock could, in principle, by sheer chance, fail to get through the Big-screen. Perhaps, during some standard twenty second shake, the rock just kept bouncing off the intersections in the screen's weave and thus missed the holes. Does this annoying fact show that **all** alleged G-rocks are 'really', sort of, **indeterminate cases** of G-rocks? The thought would distress the creatures (if they were capable of understanding it).

I prefer to think that the creatures have, somehow, developed a way around this problem. If a given rock almost always passes the G-rock test, they take it to be a perfectly good specimen of a G-rock. One failure in, say, a thousand trials, or one in one hundred, doesn't count against its claim to be a G-rock - the 'failure' is just a fluke.

Occasionally the screens of one or more of the creatures become substantially divergent from the regular, run-of-the-mill screens (at a given time). The creature begins to claim that most of the presently recognized G-rocks are, in fact, too small. Or, perhaps the creature claims that many rocks rejected by the majority as 'too Big' are splendid examples of G-rocks. Sometimes, in these circumstances, the creatures as a group come to hold that the deviant creature (or, at least the creature's screens) is (are) defective. ("Go to the repair

shop and get yourself a new pair of screens!!") This modification of the basic practice introduces a sort of 'reflective equilibrium' into the system. The testing procedure determines what is and what is not a G-rock. But rocks that have passed the test themselves provide a check on the proper functioning of the procedure.

As we have already seen, the practice generates first order indeterminacy. Each of the three classes of rocks recognized by the creatures is fuzzy.

Let's concentrate on the fuzzy boundary between the F-rocks (too big/fat) and the G-rocks. Consider the assertions to the effect that a given rock is an G-rock. The truth values of these assertions as applied possible rocks range from 0 to 1. As I see it, some of the G-rocks should be given a score of 1. These are the 'paradigmatic' G-rocks. If a rock is thus paradigmatic, then the claim that it's 'Too Fat' is just plain false - score 0. The fact that the paradigmatic rock fails, or would fail, to get through the F-screen say once in a thousand trials is irrelevant. We (including the gods) fudge the numbers so as to reflect the relevant sociology. It's the practice that drives the extension and the structure of the fuzziness. (I'm assuming the practice includes the 'fluke' attachment - and 'reflective equilibrium'.)

Higher Order Borderline Cases

At each level in the ascent we confront two questions. The sequence begins like this:

(1a) Are there clear possible examples of first order borderline cases of G-rocks?

(1b) Could the practice be such as to yield no possible examples of first order borderline cases of G-rocks?

(2a) Are there clear possible examples of second order borderline cases of G-rocks?

(2b) Could the practice be such as to yield first order borderline cases of G-rocks, but no second order borderline cases?

(3a) Could the practice be such as to yield clear possible examples of third order borderline cases of G-rocks?

(3b) Could the practice be such as to yield first order, and second order, borderline cases of G-rocks, but no third order borderline cases?

And so on, without end.

The answer to question (1a) is clearly "Yes!". Question (1b) is a bit more problematic. Up until quite recently, I thought the answer was probably "No." After all, how could the relevant fuzziness be eliminated from the G-creature's practice? And, if this **were** possible, wouldn't the result be a radically different practice? So I thought. But a friend of mine has persuaded me that there is a fairly easy route to the elimination of borderline cases. All we have to do is make the G-rock practice highly 'subjective' or 'protagorian.' Suppose the practice goes like this: x is a G-rock relative to creature C1, at time t1, iff x passes (or would pass?) a G-rock test administered by C1 at t1. Isn't it likely that this protagorian variant of the practice **cannot** produce borderline cases?

At (2a) things begin to get more tricky. As I see it, clear cases of second order borderline cases of G-rocks are generated by fuzziness in the supplementary practices

adopted by the G-rock creatures employing the standard G-rock practice. Suppose, for example, that there is a considerable degree of vagueness in regard to how many 'fluke' failures to get through the Big screen are allowed. When do the 'flukes' begin to suggest that the rock in question is, sort-of, a borderline case of a G-rock? If one failure out of a thousand trials is plainly OK, how about ten failures, or twenty? Suppose that sometimes the creatures are a bit disturbed, and confused, by a rock that fails three times in a hundred trials, and sometimes not. The supplementary practice goes fuzzy here. This, I think, is a case of second order indeterminacy. What is indeterminate is whether or not the rock is an **indeterminate** case of a G-rock.

Are there **third** order borderline cases at the fringes of the second order borderline cases? What in the practice of the creatures would show, or suggest, such cases? It seems to me that **any** wavering, or indecision, in regard to whether or not a given proportion of 'failures' is small enough to be a mere 'fluke' is evidence of second order indeterminacy, not third. I don't see the need for, or even room for, third order indeterminacy here.

Let me hasten to add that I am not ruling out the possibility of third order, or even fourth order, indeterminacy in regard to G-rocks. Perhaps if we thought harder, and more clearly, about this matter we would see how such cases could occur.

Some ways of defining higher order borderline cases have the consequence that normal first order indeterminacy entails the presence of second order, third order, etc. indeterminacy - the whole infinite hierarchy. I take this to suggest that something has gone wrong, or, at least, that some other people and I are thinking of higher order indeterminacy in somewhat different ways.