

TRANSMISSION FAILURE, AGM STYLE

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Abstract This article provides a discussion of the principle of transmission of evidential support across entailment from the perspective of belief revision theory in the AGM tradition. After outlining and briefly defending a small number of basic principles of belief change, which include a number of belief contraction analogues of the Darwiche-Pearl postulates for iterated revision, a proposal is then made concerning the connection between evidential beliefs and belief change policies in rational agents. This proposal is found to be sufficient to establish the truth of a much-discussed intuition regarding transmission failure.

Keywords AGM - belief revision - iterated change - evidence - non-transitivity - transmission failure - Weak Ramsey Test

1 Introduction

The past few years have witnessed a significant amount of discussion surrounding the issue of whether or not evidential support ‘transmits’ over entailment and if not, in what precise circumstances transmission failure occurs. Initiated in so-called ‘mainstream’ epistemology¹, the debate was later picked up by formal epistemologists. Okasha (2004) attempted to translate the debate into probabilistic terms. This proposal was subsequently criticised by Chandler (2010a), who also offered an alternative analysis within the same framework. Admittedly, however, Chandler’s translation was itself less than satisfactory, assuming, for instance, the so-called ‘Lockean thesis’ (LT), which implies that the acceptability of a proposition supervenes on its probability of being true. But there is arguably good reason to be

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¹See for instance Wright (1985; 2000; 2002; 2003).

sceptical of this supervenience thesis and hence of LT itself (Chandler 2010b).²

In the present paper, we leave aside the debate over the probabilistic handling of transmission failure and consider the issue using an altogether different set of modeling tools: belief revision theory in the AGM tradition. This framework aims to model the statics and dynamics of the set of beliefs of a rational agent. Changes in view are effected by means of two operations. *Revision* consists in the adjustment of a corpus of beliefs to accommodate the acquisition of a new doxastic commitment, which might possibly conflict with the agent's previously held worldview. *Contraction*, on the other hand, involves the adjustment of the corpus to accommodate the retraction of a previously held commitment. These operations are subject to a number of well-known constraints, which aim to enforce an intuitive parsimony requirement on belief dynamics.³

With this shift in formal machinery comes a shift in focus. We move our attention away from the *semantics* of statements of evidential support—statements expressing the obtaining of relations of evidential support between two states of affairs—to the issue of their *rational believability*. As we shall see, however, there is an intimate relationship between these two issues.

It turns out that a very straightforward proposal concerning the relation between belief dynamics and rational acceptability of evidential sentences vindicates some important intuitions about transmission of support and its failure. This will hopefully showcase some of the important potential contributions that the still comparatively neglected field of belief revision theory can make to contemporary epistemology.

The structure of the paper is as follows. In Section 2, we introduce the basic modeling framework that we shall be using, spelling out and briefly discussing the assumptions used in the subsequent proofs. Drawing on a somewhat neglected suggestion from the AGM literature on the Ramsey Test for conditionals, Section 3 then outlines a proposal regarding the normative connection between beliefs about evidence and commitments to policies of belief change. This account is argued to have a number of plausible properties and to be left unscathed by a potential worry raised by a variant of Gärdenfors' well-known impossibility result for Ramsey Test conditionals. Section 4

²For further criticisms of Chandler (2010a), see Moretti (forthcoming).

³These include the so-called 'AGM' and 'Darwiche-Pearl' postulates; see Gärdenfors (1984) and Darwiche & Pearl (1997).

puts the suggestion to work on the problem at hand. The final section briefly concludes with a note on the scope of the achievements of the paper.

2 Revision, contraction and some constraints thereon

In what follows, \mathcal{L} will denote an arbitrary propositional language, constructed by means of the standard Boolean connectives $\{\vee, \wedge, \neg, \rightarrow\}$. We shall call the sentences in \mathcal{L} ‘factual’. Furthermore, we introduce the binary connective \triangleright and denote by \mathcal{L}_E the smallest extension of \mathcal{L} that includes all sentences of the form $\varphi \triangleright \psi$ (where $\varphi, \psi \in \mathcal{L}$).⁴ ‘ $A \triangleright B$ ’ is to be interpreted as meaning ‘ A is evidence for B ’. This language can be viewed as the agent’s ‘language of thought’: the set of sentences with the respect to which the agent can form judgments.

We shall use \mathbb{K} to denote the set of all rationally permissible belief sets K drawn from \mathcal{L}_E . We shall make the plausible and standard assumption that members of \mathbb{K} are (i) logically consistent and (ii) closed under logical consequence. In other words, we shall take it to be the case that

(CON) For all $K \in \mathbb{K}$, $\neg \wedge K \notin \text{Cn}(\emptyset)$

(CL) For all $K \in \mathbb{K}$, $\text{Cn}(K) \subseteq K$

where Cn is a consequence operator, mapping subsets of \mathcal{L}_E onto other such subsets. We shall not be assuming anything about the properties of this function aside from the extremely modest requirements of supraclassicality and satisfaction of the Deduction Theorem:

(SUP) For all $\Gamma \subseteq \mathcal{L}_E$, $\text{Cn}_0(\Gamma_0) \subseteq \text{Cn}(\Gamma)$, where Γ_0 denotes the factual subset of Γ and Cn_0 returns the set of its classical consequences

(DT) For all $A, B \in \mathcal{L}$ and $\Gamma \subseteq \mathcal{L}_E$, if $B \in \text{Cn}(\Gamma \cup \{A\})$, then $A \rightarrow B \in \text{Cn}(\Gamma)$

By customary abuse of notation, we write $\text{Cn}(A)$ for $\text{Cn}(\{A\})$.

Finally, we introduce belief revision models, which represent the set of rules governing the evolution of the beliefs of a particular rational agent through various episodes of contraction and revision. Formally, a belief revision model M is a pair of functions $\langle *, \dot{-} \rangle$, such

⁴We do not allow for iteration or embedding of evidential sentences in this model.

that $*$ (the revision function) and $\dot{-}$ (the contraction function) map elements of $\mathbb{K} \times \mathcal{L}$ onto \mathbb{K} .⁵ We denote by \mathbb{M} the set of all such models. For the purpose of establishing the results of the present paper, these functions will be assumed to satisfy only a fairly minimal set of constraints, holding, as for all subsequent principles to be mentioned in this paper, unless otherwise stated, for all $K \in \mathbb{K}$, $A, B, C \in \mathcal{L}$ and $*$, $\dot{-}$ that are members of some $M \in \mathbb{M}$.⁶ Regarding revision, we have:

(AGM*2) If $\neg A \notin \text{Cn}(\emptyset)$, then $A \in K * A$

(AGM*V) If $\neg A \in \text{Cn}(\emptyset)$ then $K * A = K$.

(AGM*3) If $B \in K * A$, then $B \in \text{Cn}(K \cup \{A\})$

(AGM*4) If $\neg A \notin K$ and $B \in \text{Cn}(K \cup \{A\})$, then $B \in K * A$

(CM) If $B \in K * A$ and $C \in K * A$, then $C \in K * A \wedge B$

(I*1) If $C \in \text{Cn}(A)$, then $B \in K * A$ iff $B \in (K * C) * A$

It is worth offering a brief informal gloss on these conditions and their motivations. The first condition simply states that revision by a factual non-contradiction yields a belief set that includes that belief, the qualification being of course included so as to ensure compatibility with (CON). The second condition deals with the case of revision by factual contradictions, telling us that such operations effect no change on the belief set: the input is ‘rejected’, so to speak. The third condition states that revision by a factual sentence does not lead to a gain in factual opinionation that goes beyond what the new belief would have entailed in conjunction with the old corpus. It is a principle of doxastic conservatism. The fourth and final condition essentially tells us that revision by a factual sentence that does not contradict the original corpus yields a new corpus that includes at least the joint factual consequences of the original corpus and the input to the revision operation. The guiding motivation here is a desire to avoid an unnecessary culling of beliefs in the course of a non belief-contravening revision: beliefs ought to be dropped only if they clash with the input to the revision process. In conjunction with (AGM*3), (AGM*4) states that the factual subset of the output of a non belief-contravening revision by a factual sentence is simply obtained by adding to the original corpus K the input to the revision

⁵Revision or contraction by evidential beliefs is not modeled here.

⁶It is important to note here the restriction to sentences in \mathcal{L} . Waiving this restriction may yield principles of dubious plausibility. We return to this point below.

process alongside any logical consequences that the latter may have in conjunction with K .

Since some of our results will depend on (AGM*4), it should be mentioned that there has been a some of controversy surrounding an immediate consequence of this condition, known as ‘Preservation’. Preservation states that states that, where A and B are factual sentences, if one believes that A and does not believe that $\neg B$, then the belief that A is preserved upon revising one’s corpus of beliefs by B . The intuitive force of the principle is perhaps best felt when considering its contrapositive: if one believes that A but would no longer believe A upon revision by B , one must be committed to $\neg B$. Another way of putting this is that one ought to believe the negation of potential Factual defeaters for one’s factual beliefs, a principle that would appear to be rather intuitive. The controversy over Preservation has its roots in a famous impossibility result concerning the Ramsey Test for conditionals (Gärdenfors 1986). The result demonstrates the joint incompatibility of Preservation, the fairly intuitive Ramsey Test for conditionals and a number of further principles governing belief dynamics. We shall return to this issue later on in the paper, since a strengthened impossibility result threatens to force us to chose between Preservation and a proposal that we shall offer concerning the relation between beliefs about evidence and commitments to policies of belief change.

Our fifth condition, (CM) or again ‘Cautious Monotony’, is a weakening of one of the so-called AGM ‘supplementary postulates’. It is closely related to a fairly uncontroversial homonymous condition that one finds in the literature on nonmonotonic logics. It states that, where A , B and C are factual sentences, if one were to come to believe both B and C upon revision by A , one would come to believe C upon revision by the conjunction of A and B . Whilst this condition is strongly intuitive, it is worth noting however that Rott (2006) offers what he claims to be an intuitive counterexample. Stalnaker (2009) provides a convincing reply.

The final condition, (I*1), is due to Darwiche and Pearl (1997), going by the name of ‘CR1’ in their nomenclature. It is uncontroversial in the belief revision literature. Again, it is a principle of conservatism, this time constraining gains and losses, under revision by factual sentences, of what are sometimes called ‘conditional beliefs’: beliefs that one would acquire upon revising one’s beliefs in certain ways. In informal terms, it states that revision by a given sentence C ought not make a difference to the beliefs that one would

have upon revision by any sentence C that A entails. It is motivated by considerations such as the following:

I have a circuit containing an adder and a multiplier. I believe both the adder and multiplier are working, hence the circuit as a whole is working. If someone were to tell me that the circuit failed, I would blame the multiplier, not the adder. . . . However, if someone tells me that the adder is bad, I would believe that the multiplier is fine. . . . Now, they tell me the circuit is faulty, and immediately after, that the adder is bad. . . . Plausible reasoning (and Postulate C1) . . . claim that I should change my mind because the only reason I blamed the multiplier was to explain the failing circuit. Otherwise, by my own admission, I would presume the multiplier is fine. Moreover, I also admitted that the two components do not affect each other. Hence, learning that the adder is bad perfectly explains away whatever reasons I had in blaming the multiplier; I should revert to my initial belief that the multiplier is fine. (*ibid*)

Moving on to contraction, we assume the following modest principles:

(AGM*2) If $B \in K \dot{-} A$, then $B \in K$

(AGM*3) If $A \notin K$ or $A \in \text{Cn}(\emptyset)$, then $K \dot{-} A = K$

(AGM*4) If $A \notin \text{Cn}(\emptyset)$, then $A \notin K \dot{-} A$

(AGM*6) If $B \in K$, then $B \in \text{Cn}((K \dot{-} A) \cup \{A\})$

(I*2) If $C \in \text{Cn}(A)$, then $B \in K * A$ iff $B \in (K \dot{-} C) * A$

(I*4) If $B \notin K * A$, then $B \notin (K \dot{-} B) * A$

The first condition states that contraction by a factual sentence leads to no gain in opinionation. This requirement is in the same spirit as (AGM*3). The second condition states two degenerate cases in which a factual contraction operation leaves simply returns the original belief set: the first case is the one in which the factual sentence to be contracted by is already absent from the belief set, the second case involves contraction by a factual tautology, which cannot be removed on pains of violating (CL). The third condition mirrors (AGM*2): it

states that contraction by a factual non-tautology is successful, in the sense that the input to the operation is removed from the belief set.

The fourth condition is more noteworthy: it states, in conjunction with (AGM*3), (AGM*4) and (AGM⁻4), that removing a factual sentence A from a corpus of beliefs and then subsequently reinstating it leaves us with at least the same factual beliefs as the ones that we started off with—i.e. that we ‘recover’ the original factual beliefs. In conjunction with (AGM⁻2), it tells us that we wind up with exactly the original factual beliefs. Since the principle plays a role in the proofs that follow, it should be noted that, whilst Recovery is still considered to be part of belief revision orthodoxy, it has been vigorously opposed by some researchers in the field. The most vocal critic here has been Hansson, who has offered a series of putative counterexamples to the principle (see for instance Hansson 1992). These counterexamples are controversial, however. To illustrate, consider one of his best known cases:

I previously entertained the two beliefs ‘George is a criminal’ (A) and ‘George is a mass murderer’ (B). When I received information that induced me to give up the first of these beliefs (A), the second (B) had to go as well (since A would otherwise follow from B)... I then received new information that made me accept the belief "George is a shoplifter" (C). (Hansson 1999)

As Hansson points out, in case the final revision by C is non belief contravening, in conjunction with (AGM*3) and (AGM*4), Recovery entails that not only do I then come to believe C , but I also come to believe B . But this, he thinks, is counterintuitive: so much the worse for Recovery.

But in reply to this, one might well want to argue the following. Sure, *if* the retraction of A that he has in mind is to be modeled as a contraction by A , Recovery then entails that I will retain $A \rightarrow B$, i.e. the belief that either George is a mass murderer or he is no criminal at all, and hence that I will recover B upon revising by C . But since Hansson finds the upshot of the subsequent revision to be counterintuitive in the situation that he has in mind, this simply goes to show that the retraction that Hansson is considering is *not* properly modeled by contraction by A alone, but by a more radical excision that involves giving up not only A , but $A \rightarrow B$ as well.⁷ In other

⁷Perhaps a contraction by $\{A, A \rightarrow B\}$; see Spohn (2010) for a recent discussion of so-called ‘multiple contraction’, i.e. contraction by *sets* of sentences rather than single

words: the blame is not to be pinned on Recovery, but rather on Hansson's particular choice of contraction operation in the modelling of the example.⁸

Finally, the fifth and sixth conditions, (I⁻2) and (I⁻4) are the straightforward counterparts for contraction of two well-known constraints on revision, due to Darwiche and Pearl (1997). (I⁻2) has been implicitly endorsed by Nayak and his colleagues (Nayak et al 2006). Somewhat surprisingly, (I⁻4) does not appear to have yet been discussed in the literature. Both, however, are satisfied by the most popular constructive proposals for so-called 'iterated contraction'.^{9,10}

(I⁻2) tells us that one's dispositions, or lack of dispositions, to acquire beliefs upon revision by a factual sentence *A* ought not to be perturbed by contraction by any sentence *C* that it entails. To illustrate: The insurgents believe that the food supplies in the city that they are besieging are running short. In spite of this, they expect to have to maintain pressure for at least a couple more weeks. Of course, were they to receive information *both* confirming the scarcity of food supplies *and* substantiating current rumours of a spread of typhus within the city walls, they would conclude that the authorities will surrender by the end of the week. Upon further consideration of their evidence, however, they retract their judgment about the nutritional situation. Subsequently, they receive reliable reports of both severe rationing and an increasing number of cases of fever and delirium. According to (I⁻2) they ought to expect an imminent capitulation, as it indeed seems they should.

(I⁻4) states that a disposition to come to hold the factual belief *B* upon revision by a factual sentence *A* ought not be acquired as a result

sentences

⁸It has been suggested to me that one could perhaps interpret Glaister (2000) as offering a response along these lines.

⁹For instance: natural, aka 'conservative', contraction, lexicographic contraction and moderate, aka 'priority', contraction. See Ramachandran et al (forth.) for a recent exposition and comparison of these three approaches.

¹⁰For the record, here are the analogues for the remaining two Darwiche and Pearl postulates, namely CR1 and CR3, respectively. The first of these is implicitly endorsed by Nayak et al (2006). The second, however, appears to have been overlooked in the literature. Again, these are both satisfied by the main constructive proposals regarding iterated contraction.

$$(I^{-}1) \text{ If } \neg C \in \text{Cn}(A), \text{ then } B \in K * A \text{ iff } B \in (K \dot{-} C) * A$$

$$(I^{-}3) \text{ If } \neg B \in K * A, \text{ then } \neg B \in (K \dot{-} B) * A$$

of a mere contraction by B . It is supported by the following kind of consideration: Bobby thinks that Rob stole the money and that he was strapped for cash. Bobby would however retract his accusation if he were to find out that Rob wasn't strapped for cash after all. Upon reconsideration of his evidence, he subsequently retracts his belief that Rob stole the money, but still holds that Rob was strapped for cash. Bobby then finds out that Rob wasn't strapped for cash after all. He now infers that Rob is guilty. Bobby's inference is clearly irrational, in line with the verdict delivered by (I 4).

3 Beliefs about evidence

It seems clear that there is a relation between one's beliefs in the truth of evidential statements and facts about belief dynamics. For instance, if one is not committed to B and holds that A is evidence for B , it would seem that one is thereby committed to come to believe that B upon finding out that A and nothing logically stronger. in other words:¹¹

(EV1) If $B \notin K$, then $A \triangleright B \in K$ iff $B \in K * A$

Could one perhaps say something stronger? One suggestion would be to simply turn the above claim into an unconditional one:

(EV2) $A \triangleright B \in K$ iff $B \in K * A$ ¹²

In other words: rational agents believe that A is evidence for B iff they would come to believe that B upon revising by A . This straightforward suggestion does admittedly have a few attractive properties. For instance, it immediately follows from (CL) and (AGM*2) that it entails that, for non-contradictory A , if $B \in \text{Cn}(A)$, then $A \triangleright B \in K$.¹³ But (EV2) is clearly inadequate. Why so? The following well-known fact should make this clear:

¹¹It should be noted that there is a somewhat looser sense of 'evidence' that can be found in the philosophical literature, most notably in the Bayesian confirmation-theoretic literature, that does not fit this claim. Here we mean by 'evidence' what could perhaps be termed 'evidence sufficient for rational belief given one's background assumptions'. And this, in any case, is the sense in which the term is used in the literature on transmission.

¹²This of course simply mirrors the standard 'Ramsey Test' proposal for conditionals.

¹³Although it then also follows, given (AGM*V), that arbitrary contradictions are taken to be evidence for whatever one happens to believe. This could be seen as an awkward result.

Observation 1: Given the right-to-left half of (EV2), (CL), (SUP), (CON) and (AGM*4), it follows that if $A, B \in K$, then $A \triangleright B \in K$.¹⁴

So whilst the proposal may indeed yield the intuitively correct result for agents who do not already believe that B , for those who do, the membership conditions on offer are patently too weak. Thankfully, it is not too difficult to find a more promising alternative suggestion:

(EV3) $A \triangleright B \in K$ iff $B \in (K \dot{-} B) * A$ ¹⁵

In other words: rational agents believe that A is evidence for B iff they would come to believe that B upon first contracting by B and then subsequently revising by A . This now takes care of the previously problematic case of agents who are already committed to B : it is easily verified that that it no longer follows that $A \triangleright B \in K$ if $A, B \in K$, even given the strongest constraints on contraction and revision that can be found in the literature. The proposal also shares with (EV2) the advantage of entailing (EV1), given a weak constraint on contraction:

Observation 2: Given (AGM $\dot{-}$ 3), (EV3) entails (EV1).

And as was the case with (EV2), it also entails, in conjunction with (CL) and (AGM*2), that, for non-contradictory A , if $B \in \text{Cn}(A)$, then $A \triangleright B \in K$.¹⁶

Before moving on, it is worth addressing a potential source of concern, alluded to in Section 2. In Gärdenfors (1987), we find the proof of a result stating that, given a number of principles of belief dynamics that Gärdenfors takes to be plausible, (EV4) entails:

(TRIV) There exists no $K \in \mathbb{K}$ such that, for pairwise inconsistent $A, B, C \in \mathcal{L}$, $\neg A, \neg B, \neg C \notin K$

Commitment to (TRIV) would certainly be bad news. The principle clearly offers an undesirable constraint on belief sets: surely it is permissible to have a belief set consistent with three pairwise incompatible sentences. Does this spell trouble for the current proposal?

¹⁴See for instance Rott (1986), who makes the point in relation to conditionals and the Ramsey Test. I believe that the point was first noted by Gärdenfors (1979).

¹⁵This is analogous to the variant of the Ramsey Test labeled '(R3)' by Gärdenfors (1987, p. 324) and first proposed proposed by Rott (1986).

Note that, by the contrapositive of (I $\dot{-}$ 2), if $A \triangleright B \in K$ according to (EV4), then $A \triangleright B \in K$ according to (EV2).

¹⁶The somewhat counterintuitive result noted in footnote 10 above is also avoided.

Well, whether or not it does hinges crucially on whether or not the ‘plausible’ principles of belief change involved in the derivation are indeed plausible. It is worth taking a closer look at what *exactly* it is that Gärdenfors proves. He shows that:

Observation 3: Given (CL), (CON), (SUP), (DT), (MON), (AGM $\dot{-}$ 3), (AGM*2), (AGM*3+), (AGM*4+), (EV3) entails (TRIV).¹⁷

where

(AGM*3+) For all $A \in \mathcal{L}$ and $B \in \mathcal{L}_E$, if $B \in K * A$, then $B \in \text{Cn}(K \cup \{A\})$

(AGM*4+) For all $A \in \mathcal{L}$ and $B \in \mathcal{L}_E$, if $\neg A \notin K$ and $B \in \text{Cn}(K \cup \{A\})$, then $B \in K * A$

(MON) For all $A, B \in \mathcal{L}$ and $\Gamma \subseteq \mathcal{L}_E$, if $A \in \text{Cn}(B)$, then $\text{Cn}(\Gamma \cup \{A\}) \subseteq \text{Cn}(\Gamma \cup \{B\})$

So Gärdenfors needs both a strengthening of *two* of our principles of revision, waiving our restriction that B be an element of \mathcal{L} , as well as an additional assumption regarding Cn . This is a significant further commitment. And indeed, to the extent that (EV4) has a certain degree of *prima facie* plausibility, it seems plausible to view Gärdenfors’s result as a reductio of such a strengthening of our assumptions. Furthermore, as Gärdenfors notes in his proof, the first two principles jointly entail that for all $K \in \mathbb{K}$ and $A, B \in \mathcal{L}$, $K * A \vee B \subseteq K * A$. But this is clearly too strong. For instance, to co-opt an example due to Lindström and Rabinowicz (1995, p. 153), let:

B : There is a blizzard in Bayreuth.

T : There is a tornado in Toulouse.

Assuming that K represent a state of complete agnosticism with respect to the weather, a typical belief set K might be such that $(\neg B \vee \neg T) \triangleright \neg T \in K * B$, but $(\neg B \vee \neg T) \triangleright \neg T \notin K * B \wedge T$.¹⁸

¹⁷Adapted from Gärdenfors (1987). A more explicit version of his original proof, which clearly states the various assumptions involved, is provided in the appendix.

¹⁸Note that all that we need here is the weak claim that such a K be rationally *permissible*. Rational obligation, though perhaps intuitive in the present example, is more than is required.

Note that Hansson (1992) voices similar concerns regarding (AGM*3+) and (AGM*4+), in relation to a conditional language.

4 Transmission and its failure

Informally put, the principle of transmission of warrant across entailment states that if A is evidence for B and B entails C , then A is thereby evidence for C . What *precisely* is meant by this is not entirely straightforward to clarify, and we shall briefly return to this issue in due course. But it is accepted that, *at the very least*, the principle states the following:

(TR1) If $C \in \text{Cn}(B)$, then $A \triangleright C \in \text{Cn}(A \triangleright B)$.

As mentioned in the introduction to this paper, there is a clear connection between the project of providing truth conditions for evidential statements and the project of outlining constraints on beliefs sets that contain the corresponding kinds of beliefs. One such connection is provided by (CL). It entails that if (TR1) is true, so too is the following:

(TR2) If $C \in \text{Cn}(B)$ and $A \triangleright B \in K$, then $A \triangleright C \in K$.

One might also be interested in the claim that the converse holds, i.e. that if (TR2) holds, then so to does (TR1). This of course does not follow from (CL), but it would follow from its converse, namely:

(CCL): For all $A \in \mathcal{L}$ and $\Gamma \subseteq \mathcal{L}_E$, if there is no $K \in \mathbb{K}$ such that $\Gamma \subseteq K$ and $A \notin K$, then $A \in \text{Cn}(\Gamma)$.

Arguably, (CCL) is no less plausible than (CL). Indeed, why require, for instance, that all agents who believe A also believe B when it is possible for A to be true whilst B is false?

Now both (TR1) and (TR2) receive some *prima facie* plausibility from consideration of cases such as the following (Wright 2002, p. 332):

Death Cap: A_D : 3 hours ago, Jones consumed a large risotto of death caps. B_D : Jones has absorbed a large quantity of amatoxins. C_D : Jones has absorbed a lethal quantity of toxins.

In this case, a reasonable belief set K_D might be such that $A_D \triangleright B_D \in K_D$ and $A_D \triangleright C_D \in K_D$.

But Wright and others have convincingly argued that (TR1) is false. One well-known intuitive counterexample is the following:

Zebra: A_Z : The animal in the enclosure is a black & white-striped four-legged equine creature. B_Z : The animal in the enclosure is a zebra. C_Z : The animal in the enclosure is not a cleverly painted mule.

Clearly, it is permissible to have a belief set K_Z , such that $A_Z \triangleright B_Z \in K_Z$ but $A_Z \triangleright C_Z \notin K_Z$. And to the extent that this is the case, (TR2), and hence, by (CL), (TR1) must be rejected.¹⁹ Another case in point, in a similar vein, is the following:

Moore: A_M : It looks like George has a hand. B_M : George has a hand. C_M : There is an external world.

Again, it is patently permissible to have a belief set K_M , such that $A_M \triangleright B_M \in K_M$ but $A_M \triangleright C_M \notin K_M$.

Let us say then, that K is a case of *transmission failure* iff for some $A, B, C \in \mathcal{L}$ such that $C \in \text{Cn}(B)$, we have $A \triangleright B \in K$, but $A \triangleright C \notin K$.²⁰ The first item of good news here is that (EV3) in conjunction with the strongest constraints on $*$ and \vdash available in the literature does not rule out cases of transmission failure.^{21, 22} And it is worth noting, furthermore, that the same *cannot* be said of our first proposal, (EV2). Indeed:

¹⁹Note that, given the principle according to which, for non-contradictory A , if $B \in \text{Cn}(A)$, then $A \triangleright B \in K$ (which, as we have seen, our proposal entails) and (CL), these cases are also counterexamples to *transitivity* of evidential support, i.e. the principle according to which $A \triangleright C \in \text{Cn}(\{A \triangleright B, B \triangleright C\})$.

²⁰This is a slight departure here from standard usage of the term. Cases of transmission failure are typically taken to be triples $\langle A, B, C \rangle$ of sentences, such that $C \in \text{Cn}(B)$, $A \triangleright B$ is true in some world, but $A \triangleright C$ is false in that world. Of course, given (CL), if K is a case of transmission failure according to the definition offered here, $\langle A, B, C \rangle$ is a case of transmission failure according to the standard definition.

²¹Proof omitted, but straightforward.

²²In fact, as Peter Fritz has perspicuously pointed out to me, given (CL), (AGM \vdash -3), (AMG*2), (AMG*4), and (EV3), (TR1) cannot hold on pains of ‘triviality’, i.e. of placing unwarranted constraints on \mathbf{K} . More precisely, given these assumptions, (TR1) entails the following:

(TRIV2) There is no $K \in \mathbf{K}$, and $A, B, C \in \mathcal{L}$ such that $A \in K$, $B, \neg B \notin K$ but $B \triangleright A \notin K$.

For the proof, we show that, in the presence of the relevant background conditions, if (TRIV2) is false, then so too is (TR1). Indeed, assume that $B \notin K$. It then follows by (CL) that $A \wedge B \notin K$. By (AGM \vdash -3) it then follows that $K \vdash A \wedge B = K$, and hence that $(K \vdash A \wedge B) * B = K * B$. Assume that $A \in K$ but that $\neg B \notin K$. It follows by (AGM*4) that $A \in K * B$ and hence that $A \in (K \vdash A \wedge B) * B$. Since, by assumption, $\neg B \notin K$, it follows by (CL) that $\neg B \notin \text{Cn}(\emptyset)$. So by (AGM*2), we also have $B \in (K \vdash A \wedge B) * B$. It then follows by (CL) that $A \wedge B \in (K \vdash A \wedge B) * B$. By the right-to-left direction of (EV3), $B \triangleright (A \wedge B) \in K$. Assume furthermore that $B \triangleright A \notin K$. Since $A \in \text{Cn}(A \wedge B)$, (TR1) must be false. \square

And of course, (TRIV2) is a clearly undesirable constraint to impose. Let A stand for the proposition that it is sunny today in Leuven (which I believe) and B stand for the proposition that a parcel has arrived for me this morning (which I suspend judgment on). It seems permissible for me to not hold that B would be evidence for A (indeed: I do not).

Observation 4: (EV2) and (CL) jointly entail (TR2).

So cases of transmission failure are *countenanced* by our assumptions. But can we say anymore in terms of their characterisation? Well it turns out that Wright has offered a much-discussed observation regarding Zebra/Moore -style cases of transmission failure (Wright 1985), namely that the relevant subjects would not believe that $A \triangleright B$ if they were not *already* committed to C ; offering them A as a supporting consideration for C would amount to begging the question.²³ This feature of the relevant beliefs sets has an extremely natural translation into our framework, namely: $A \triangleright B \notin K \dashv C$.

Intuitively, then, it would seem that $A \triangleright B \notin K \dashv C$ is a necessary and sufficient condition for K to be a case of transmission failure, given $C \in \text{Cn}(B)$ and $A \triangleright B \in K$. We can now finally present the principal result of this paper:

Observation 5: Given (AGM * 2), (AGM * 3), (AGM * V), (CM), (I*1), (AGM \dashv 2), (AGM \dashv 3), (AGM \dashv 4), (AGM \dashv 6), (I \dashv 2), (I \dashv 4), (EV4), (CL), (DT) and (SUP), it follows that: if $C \in \text{Cn}(B)$ and $A \triangleright B \in K$, then (1) $A \triangleright C \notin K$ iff (2) $A \triangleright B \notin K \dashv C$.

This observation both provides a formal vindication of our intuitions and an additional consideration in favour of (EV4).

5 Concluding remarks

Now interestingly, there may be reasons to think that (TR1) does not quite capture the principle of transmission, as discussed in the literature. It turns out that the consequent of (TR1), and hence the principle itself, would appear to be too weak. The following case, taken from Wright (2002), allegedly establishes the point:

Election: A_E : Jones has just placed an ‘X’ on a ballot paper.
 B_E : Jones has just voted. C_E : An election is taking place.

Here, according to Wright, we have a case of transmission failure because, whilst A_E does indeed provide evidential support for both B_E and C_E , it does not provide support for the latter *via* its providing support for the former, rather, in his terminology, it does so merely

²³This is not *quite* how Wright phrases it. His precise claim appears to be that, where $C \in \text{Cn}(B)$, $A \triangleright C \notin \text{Cn}(A \triangleright B)$ when one would be *entitled* to infer B from A only given prior *entitlement* to believe C (Wright 1985, p. 433).

‘directly’. This supposedly contrasts with Death Cap, in which A_D ’s support for C_D is accounted for by its support for B_D . But, to the extent that it is possible to make sense of Wright’s conceptual distinction, it is far from clear, however, how to formalise the required strengthening of the consequent of (TR1) so as to classify Election as an instance of transmission failure.

Of course, this does nothing to undermine the importance of the results established here, since we have proven the sufficiency of a given condition for the falsity of a consequence of the principle of transmission, and hence for the falsity of the principle itself. It remains the case, however, that it would be desirable to provide a more comprehensive treatment of the topic.

Appendix

Proof of Observation 1: Assume, for conditional proof, that $A \in K$. By (CL), (SUP) and (CON) it follows that $\neg A \notin K$. Now assume, again for conditional proof, that $B \in K$ and hence that $B \in \text{Cn}(K \cup \{A\})$. It follows by (AGM*4) that $B \in K * A$. By the right-to-left half of (EV2), it then follows that $A \triangleright B \in K$. \square

Proof of Observation 2: Assume, for conditional proof, that $B \notin K$. By (AGM $\dot{-}$ 3), it follows that $K \dot{-} B = K$. So $B \in K * A$ iff $B \in (K \dot{-} B) * A$, and by (EV3), iff $A \triangleright B \in K$. \square

Proof of Observation 3:²⁴ Given (AGM $\dot{-}$ 3), (EV3) entails (EV1) (see Observation 2). Furthermore, (EV1) entails:

(WM) If $K \subseteq K'$, $A \vee C \notin K'$ and $C \in K * A$, then $C \in K' * A$.

Indeed, assume that $K \subseteq K'$ and $A \vee C \notin K'$. It follows that $A \vee C \notin K$, and hence, by (CL), that $A, C \notin K$. Assume furthermore that $C \in K * A$. It follows by (EV1), and the fact that $K \subseteq K'$, that $A \triangleright C \in K$, that $A \triangleright C \in K'$, and therefore, by (EV1), that $C \in K' * A$.

Now assume for reductio that $K \in \mathbb{K}$ is such that, for pairwise inconsistent $A, B, C \in \mathcal{L}$, $\neg A, \neg B, \neg C \notin K$.

The general proof strategy is as follows. We first derive (i) $B \in (K * A) * B \vee C$ and (ii) $C \in (K * A) * B \vee C$. By (CL), this then entails that $B \wedge C \in (K * A) * B \vee C$, which violates (CON). Here, we simply prove (i). The proof for (ii) is analogous.

²⁴Adapted from Gärdenfors (1987).

From (AGM*3+), (AGM*4+), (SUP) and (MON), it follows that $K * A \vee B \subseteq K * A$. Indeed consider an arbitrary $C \in K * A \vee B$. By (AGM*3+), it follows that $C \in \text{Cn}(K \cup \{A \vee B\})$. By (MON), we have $C \in \text{Cn}(K \cup \{A \vee B, A\})$. By (SUP), the latter is equal to $\text{Cn}(K \cup \{A\})$. Now, by hypothesis, $\neg A \notin K$, so by (AGM*4+), it follows that $C \in K * A$.

We now show that $B \vee C \notin K * A$, and hence, by (SUP) and (CL), that $(B \vee C) \vee B \notin K * A$. Assume for reductio that $B \vee C \in K * A$. Then by (AGM*3), $B \vee C \in \text{Cn}(K \cup \{A\})$. By (DT), $A \rightarrow (B \vee C) \in K$. Since A is inconsistent both with B and with C , it follows by (CL) that $\neg A \in K$, contrary to our assumptions.

Next, we prove that $B \in (K * A \vee B) * B \vee C$. We first show that $\neg(B \vee C) \notin K * A \vee B$. Assume for reductio that $\neg(B \vee C) \in K * A \vee B$. Then by (AGM*3), it follows that $\neg(B \vee C) \in \text{Cn}(K \cup \{A \vee B\})$. By (DT), we have $(A \vee B) \rightarrow \neg(B \vee C) \in K$. By (SUP), we have $\neg B \in \text{Cn}((A \vee B) \rightarrow \neg(B \vee C))$, so by (CL), it follows that $\neg B \in K$, contrary to our assumptions.

Now, since $\neg A, \neg B \notin K$, by (CL), $\neg A, \neg B \notin \text{Cn}(\emptyset)$, and hence, by (SUP), it follows that $\neg(A \vee B) \notin \text{Cn}(\emptyset)$. By (AGM*2), therefore, $A \vee B \in K * A \vee B$. From this, it follows by (SUP) that $B \in \text{Cn}(K * A \vee B \cup \{B \vee C\})$. By (AGM*4), and the fact that $\neg(B \vee C) \notin K * A \vee B$, it then follows that $B \in (K * A \vee B) * B \vee C$.

From the fact that $K * A \vee B \subseteq K * A$, $(B \vee C) \vee B \notin K * A$, and $B \in (K * A \vee B) * B \vee C$, which have all now been derived, it then follows by (WM) that (i) $B \in (K * A) * B \vee C$. \square

Proof of Observation 4: Assume, for conditional proof, that (i) $C \in \text{Cn}(B)$ and (ii) $A \triangleright B \in K$. By the left-to-right half of (EV2) and (ii), it follows that $B \in K * A$. From this, by (i) and (CL), we have $C \in K * A$. By the right-to-left half of (EV2), it then follows that $A \triangleright C \in K$. \square

Proof of Observation 5: If $C \in \text{Cn}(\emptyset)$, this is trivial. Indeed, by (AGM $\dot{-}$ 3), $K \dot{-} C = K$. So assuming $A \triangleright B \in K$, we just need to establish that $A \triangleright C \in K$. This is easy. Assume $C \in \text{Cn}(\emptyset)$. By (CL), $C \in (K \dot{-} C) * A$ and therefore, by the right-to-left direction of (EV4), $A \triangleright C \in K$.

Also trivial is the case in which $\neg A \in \text{Cn}(\emptyset)$. Indeed, in this case, if $B \notin \text{Cn}(\emptyset)$, then $A \triangleright B \notin K$: by (AGM $\dot{-}$ V), we have $(K \dot{-} B) * A = K \dot{-} B$, and by (AGM $\dot{-}$ 4) we have $B \notin (K \dot{-} B)$, so $B \notin (K \dot{-} B) * A$ and hence, by the left-to-right direction of (EV4), $A \triangleright B \notin K$. If $B \in \text{Cn}(\emptyset)$, then, by (CL), $B \in ((K \dot{-} C) \dot{-} B) * A$,

and hence, by the right-to-left direction of (EV4), $A \triangleright B \in K \dot{-} C$. Furthermore, if $C \in \text{Cn}(B)$, then, by (CL), $C \in (K \dot{-} C) * A$ and therefore, by the right-to-left direction of (EV4), $A \triangleright C \in K$.

So we henceforth assume that C is contingent and that A is consistent: $C \notin \text{Cn}(\emptyset)$ and $\neg A \notin \text{Cn}(\emptyset)$.

The left-to-right direction of the biconditional, from (1) to (2), is trivial to establish. Indeed, assume $A \triangleright C \notin K$. By the right-to-left direction of (EV4), it follows that $C \notin (K \dot{-} C) * A$. Now assume $C \in \text{Cn}(B)$. It then follows by (CL) that $B \notin (K \dot{-} C) * A$. But, given (AGM $\dot{-}$ 4), (AGM $\dot{-}$ 3) and (CL), this is equivalent to $B \notin ((K \dot{-} C) \dot{-} B) * A$. Indeed, since $C \notin \text{Cn}(\emptyset)$, by (AGM $\dot{-}$ 4), $C \notin K \dot{-} C$. Since $C \in \text{Cn}(B)$, by (CL), $B \notin K \dot{-} C$. Therefore, by (AGM $\dot{-}$ 3), $(K \dot{-} C) \dot{-} B = (K \dot{-} C)$. Given this equivalence, by the left-to-right direction of (EV4), it follows that $A \triangleright B \notin K \dot{-} C$.

Regarding the other direction, from (2) to (1), we prove the result in two small steps. We first prove:

Lemma 1: If $A \triangleright B \in K$, then $B \in (K \dot{-} C) * A \wedge C$.

We then show that:

Lemma 2: If $B \in (K \dot{-} C) * A \wedge C$ and $A \triangleright B \notin K \dot{-} C$, then $A \triangleright C \notin K$.

Regarding Lemma 1, assume for conditional proof that $A \triangleright B \in K$. It follows, by the left-to-right direction of (EV4), that $B \in (K \dot{-} B) * A$. From the fact that $C \in \text{Cn}(B)$, it follows by (CL) that $C \in (K \dot{-} B) * A$. It then follows by (CM) that $B \in (K \dot{-} B) * A \wedge C$. Now assume for reductio that $B \notin (K \dot{-} C) * A \wedge C$. Since $C \in \text{Cn}(A \wedge C)$, it follows by (I $\dot{-}$ 2) that $B \notin K * A \wedge C$. By (I $\dot{-}$ 4) it then follows that $B \notin (K \dot{-} B) * A \wedge C$: contradiction.

Regarding Lemma 2, assume $A \triangleright B \notin K \dot{-} C$ for conditional proof. By the right-to-left direction of (EV4), it follows that $B \notin ((K \dot{-} C) \dot{-} B) * A$. As noted above, given (AGM $\dot{-}$ 4), (AGM $\dot{-}$ 3) and (CL), this is equivalent to $B \notin (K \dot{-} C) * A$. Given that $\neg A \notin \text{Cn}(\emptyset)$, by (AGM * 2), $A \in (K \dot{-} C) * A$. But by (SUP), $A \rightarrow B \in \text{Cn}(\{(A \wedge C) \rightarrow B, A \rightarrow C\})$. So it follows by (CL) that either:

- (i) $(A \wedge C) \rightarrow B \notin (K \dot{-} C) * A$, or
- (ii) $A \rightarrow C \notin (K \dot{-} C) * A$

Assume (i) for reductio. It follows, by (AGM*3), (CL) and (DT) that $B \notin ((K \dot{-} C) * A) * A \wedge C$. This is easy to show, by proving the contrapositive. Indeed, assume $B \in ((K \dot{-} C) * A) * A \wedge C$. By (AGM*3), we have $B \in \text{Cn}((K \dot{-} C) * A \cup \{A \wedge C\})$. By (DT), it follows that $(A \wedge C) \rightarrow B \in \text{Cn}((K \dot{-} C) * A)$. By (CL), $(A \wedge C) \rightarrow B \in (K \dot{-} C) * A$.

Now, by (I*1), it follows that $B \notin (K \dot{-} C) * A \wedge C$. Assume $B \in (K \dot{-} C) * A \wedge C$ for conditional proof: contradiction.

It therefore follows that (ii) $A \rightarrow C \notin (K \dot{-} C) * A$. But since, by (SUP), $A \rightarrow C \in \text{Cn}(C)$, it follows by (CL) that $C \notin (K \dot{-} C) * A$. Finally, by the left-to-right direction of (EV4), it follows that $A \triangleright C \notin K$. \square

Acknowledgments

The research for this article was funded by a Research Foundation – Flanders (FWO) postdoctoral research grant. Thanks to Peter Broessel, Franz Huber and James Joyce for helpful discussions. I am especially grateful to Peter Fritz for both providing me with the result cited in footnote 22 and for spotting an important mistake in an earlier version of this paper.

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