

MODAL LOGIC

An Introduction

Zia Movahed



HERMES PUBLISHERS

PREFACE

This is an introduction to first-order modal logic. It is intended for readers with no knowledge of the subject, but acquainted with non-modal formal logic at elementary level.

My overall approach to the subject is three-pronged: proof-theoretically Fitch's style of natural deduction rules is employed; semantically what is now called Kripke semantics is used; and philosophically the most pertinent and motivating philosophical problems leading to different formal modal systems are discussed.

To make the book self-contained, it includes two chapters (2 and 10) on propositional and quantified logic with identity in which all material needed for understanding the rest of the book are covered.

Apart from chapter 1 which is a brief account of historical development of fundamental modal concepts and systems, the book consists of two parts: modal propositional logic and modal predicate logic with identity. Chapter 3 is devoted to an extensive discussion of the syntax of the basic system K. In this chapter, by extending Fitch's style of natural deduction rules to K, many sequents and theorems are proved. In chapter 4 the same is done for normal systems D, T, K4, B, S4, S5, Trive and Ver. The question of distinct modalities is raised in chapter 5 and for some systems answered. In chapter 6, to bring to light the distinguishing power of Kripke semantics. Leibnizian semantics is introduced and applied to the systems in chapters 3 and 4. Chapter 7 deals in detail with Kripke semantics or, to do justice to the other pioneer logicians, Kanger-Hintikka-Kripke semantics. Chapter 8 is wholly devoted to Kripke semantics for the modal systems in chapters 3 and 4. In chapter 9, to show some applications of modal logic, two philosophical problems, i.e. future contingency and the realist thesis that there are truths unknowable to us, are modally analyzed.

Part II is patterned like part I. Chapter 10, as I mentioned above is on nonmodal quantified logic with identity. In chapter 11 the formal language of modal quantified logic with identity ($Q\Box=$) is carefully examined, extensional and intensional languages are explained and general semantical features of $Q\Box=$ are discussed.

Chapter 12 introduces a set of systems in some of which all controversial sentences including the Barcan and the converse Barcan formulas as well as the Buridan formulas are derivable. In chapter 13 semantical and philosophical consequences of theorems derived in systems of chapter 12 are critically examined. Finally, Chapter 14 deals with semantics and syntax of the systems respecting Kripke's considerations in his seminal paper of 1963, i.e. systems devoid of all controversial theorems characteristic of systems of chapter 12. In fact the latter systems also came into literature in Kripke's paper of 1959. So these two papers proposed two extreme systems of quantified modal logic. Many other systems proposed since 1963 lie somewhere between these two. There are enough hints and references for the attentive reader of the book to construct a logic congenial to her/his taste and interest.

The book has two appendices. Appendix I contains solution to all exercises given in each chapter. This makes the book even more accessible to self-teaching readers. Appendix II is a brief introduction to Lewis's systems frequently referred to throughout the book.

Some other features of the book are:

1. Unlike some authors who put more emphasis on either syntax or semantics (mostly on the latter), I have treated both aspects equally.

2. The book includes many lexical and historical notes. In one of them (note 3, chapter 12) it is shown that famous formulas attributed to Ruth Barcan and Jean Buridan (C. 1300-after, 1358), French philosopher, had been discovered by the Persian philosopher Ibn-Sina (Avicenna, 980-1037), and discussed extensively by his successors before the European medieval logicians. Accordingly, it is suggested, by giving reliable references, that the formulas be renamed as Ibn-Sina-Barcan and Ibn-Sina-Buridan formulas. In fact as history has it, Buridan had access to the Moslems' writings through their Latin translations.

3. For readers willing to increase their knowledge of the subject a descriptive selected bibliography is added at the end of the book.

To the best of my knowledge, and notwithstanding the Moslems' rich tradition of modal logic, this is the first book on modern modal logic in Persian. Thus I have tried to provide the readers with fundamentals of the subject, and to avoid including second-hand materials and too much reformulation of the same points which

could have made the book voluminous without adding to the readers' understanding of the subject.

I would like to thank those who directly influenced this book in various ways. Firstly and mostly my thanks go to professor W. D. Hart who, through his stimulating lectures, introduced me to the subject. I have also benefited from his invaluable advice while I was writing this book. Secondly, I wish to express my gratitude to Professor Dagfinn Føllesdal for all I have learned from conversation with him and for his generosity in providing me with references otherwise inaccessible to me. Thirdly, my special thanks go to Professor Timothy Williamson who inspired me to see modal logic from a different angle.

Finally, I wish to thank Dr. Hossein Masoumi Hamedani and Dr. Mohammad Ardeshir who have read the final draft of the book and suggested many improvements. They, of course, are not responsible for my errors.

The job of putting the manuscript into computer was not easy. I am really indebted to Miss. Fatemeh Jafari Nazari for carrying the job patiently and skillfully. And last but not least my thanks are due to Mr. Lotfollah Saghravani, head of The Hermes Publishers without whose help and support the task would not be achieved.

Zia Movahed

July 10, 2002